# Loop-String-Hadron on a Maximal Tree

## A tale of gauge invariance and flowers

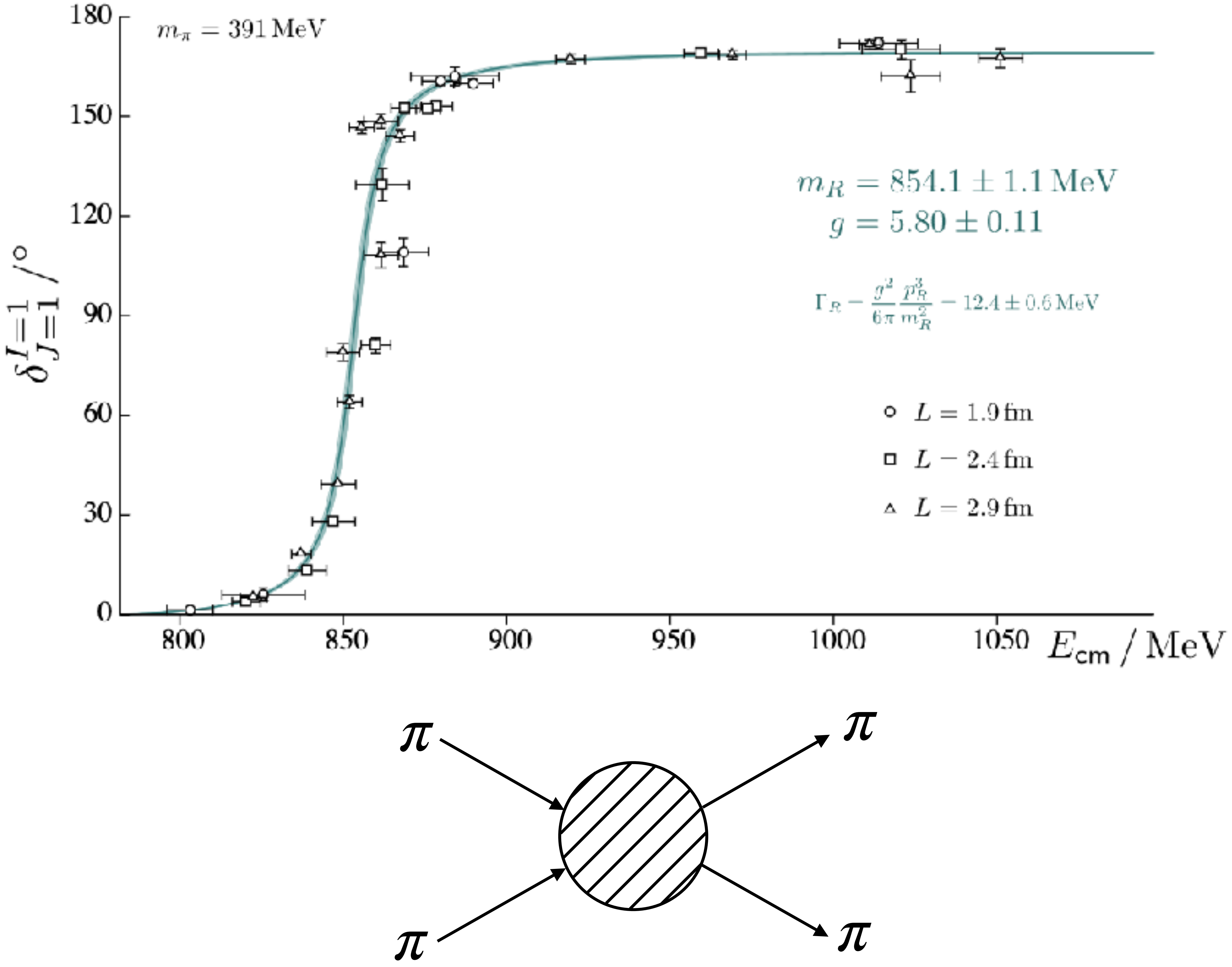
Based on work with Christian Bauer

Iván Mauricio Burbano Aldana
Advised by Christian Bauer and Raúl Briceño

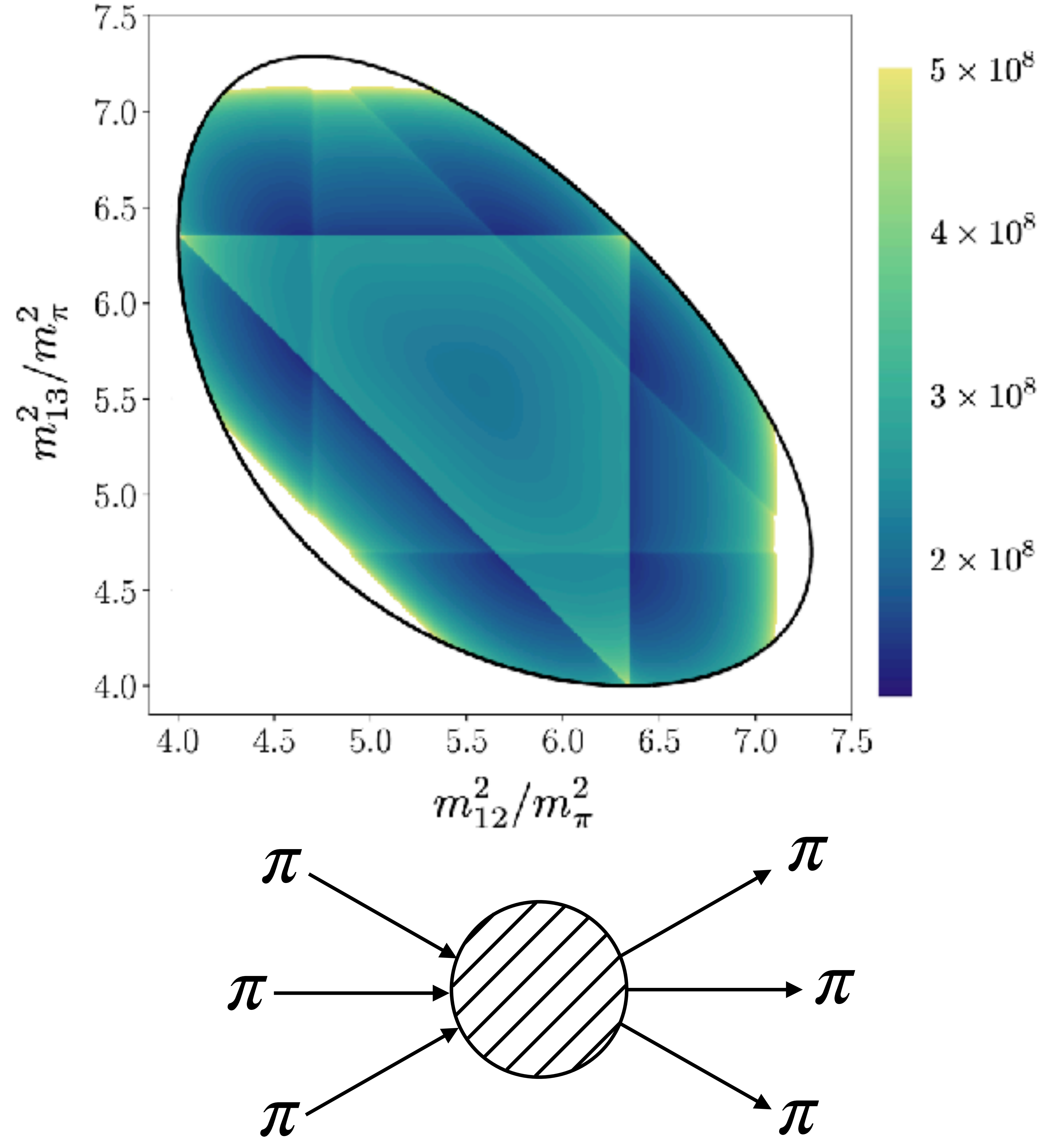
# Introduction

# Why am I here?
## To learn about “classical” lattice QCD!

Jozef J. Dudek, Robert G. Edwards, and Christopher E. Thomas [1212.0830](https://arxiv.org/abs/1212.0830).

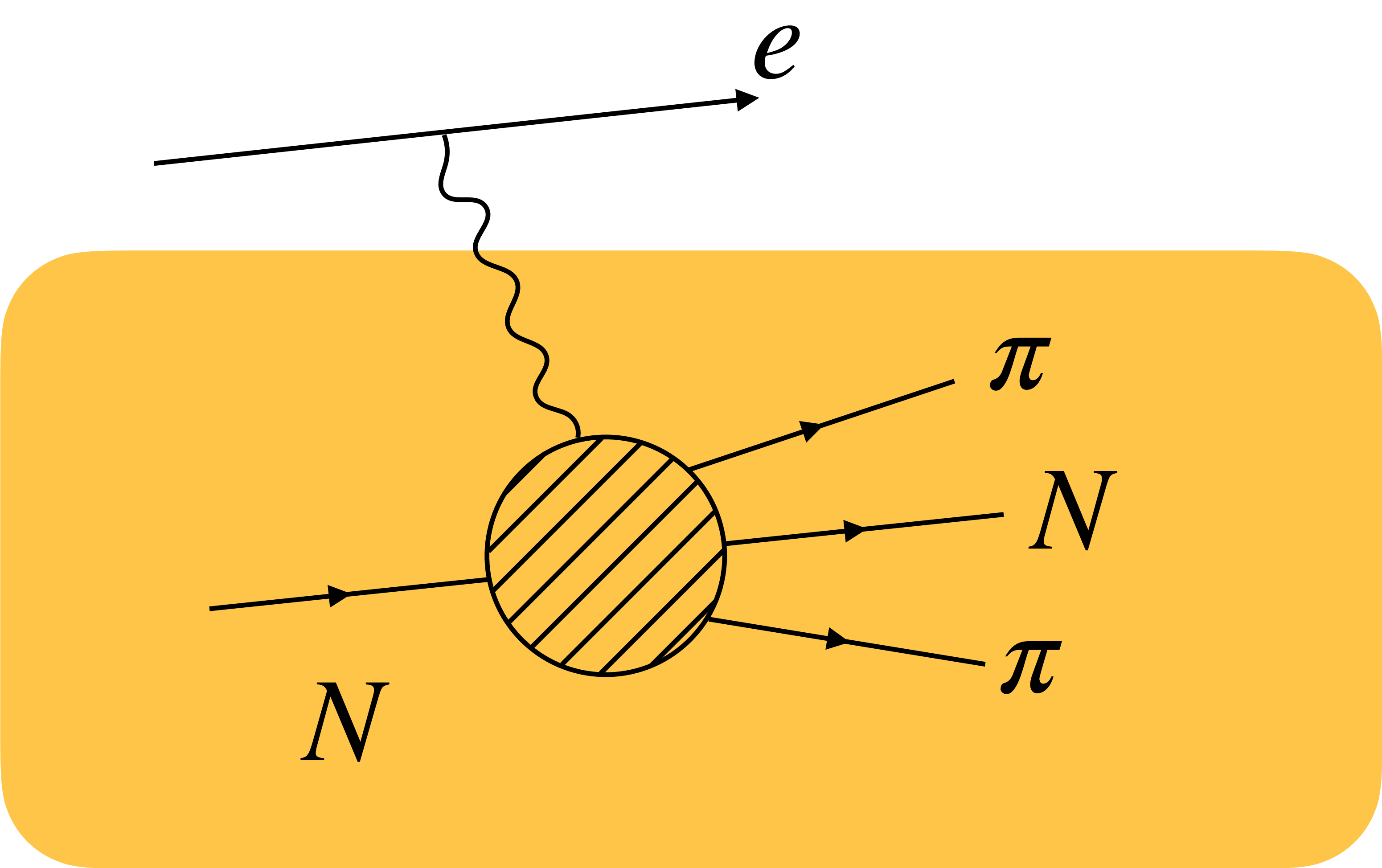



Maxwell T. Hansen, Raul A. Briceño, Robert G. Edwards, Christopher E. Thomas, and David J. Wilson. [2009.04931](https://arxiv.org/abs/2009.04931).

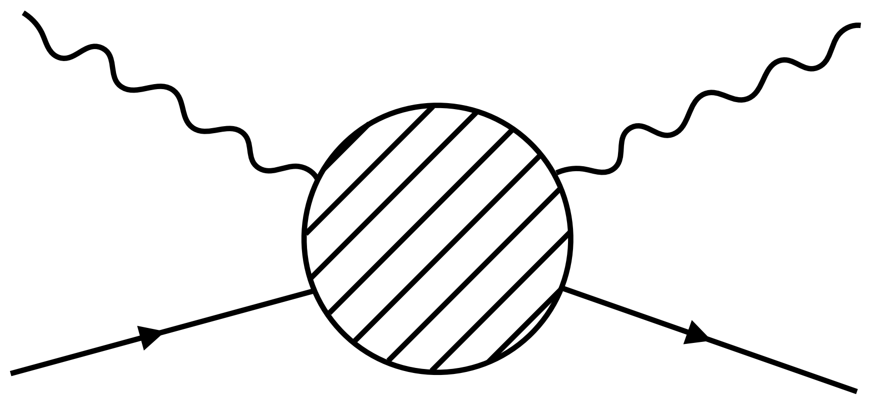


# Why Quantum Computers?

## Because doing scattering on Euclidean time is getting hard




Raul A. Briceño, Marco A. Carrillo, Juan V. Guerrero, Maxwell T. Hansen, and Alexandru M. Sturzu, 2112.01968.



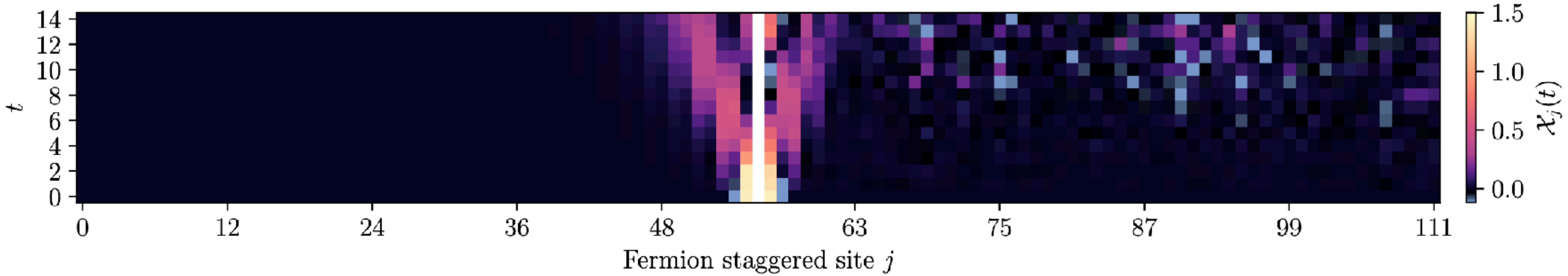
We could do this at arbitrarily high energies if we could compute:

$$\langle \vec{p}_f | j_M(t) j(0) | \vec{p}_i \rangle$$

In Minkowski space!

# Or we can be more literal…

Roland C. Farrell, Marc Illa, Anthony N. Ciavarella, and Martin J. Savage 2401.08044.



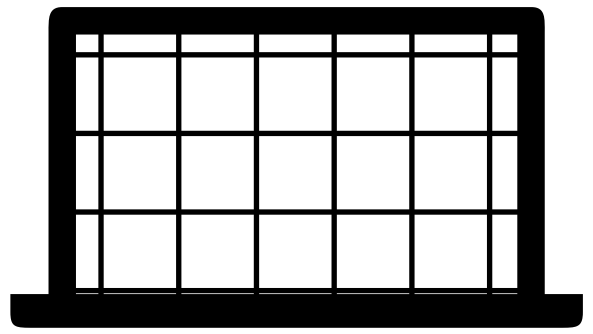

Classical Tensor Networks

Quantum computer (112 Qubits)

Spreading of a wavepacket in the Schwinger model

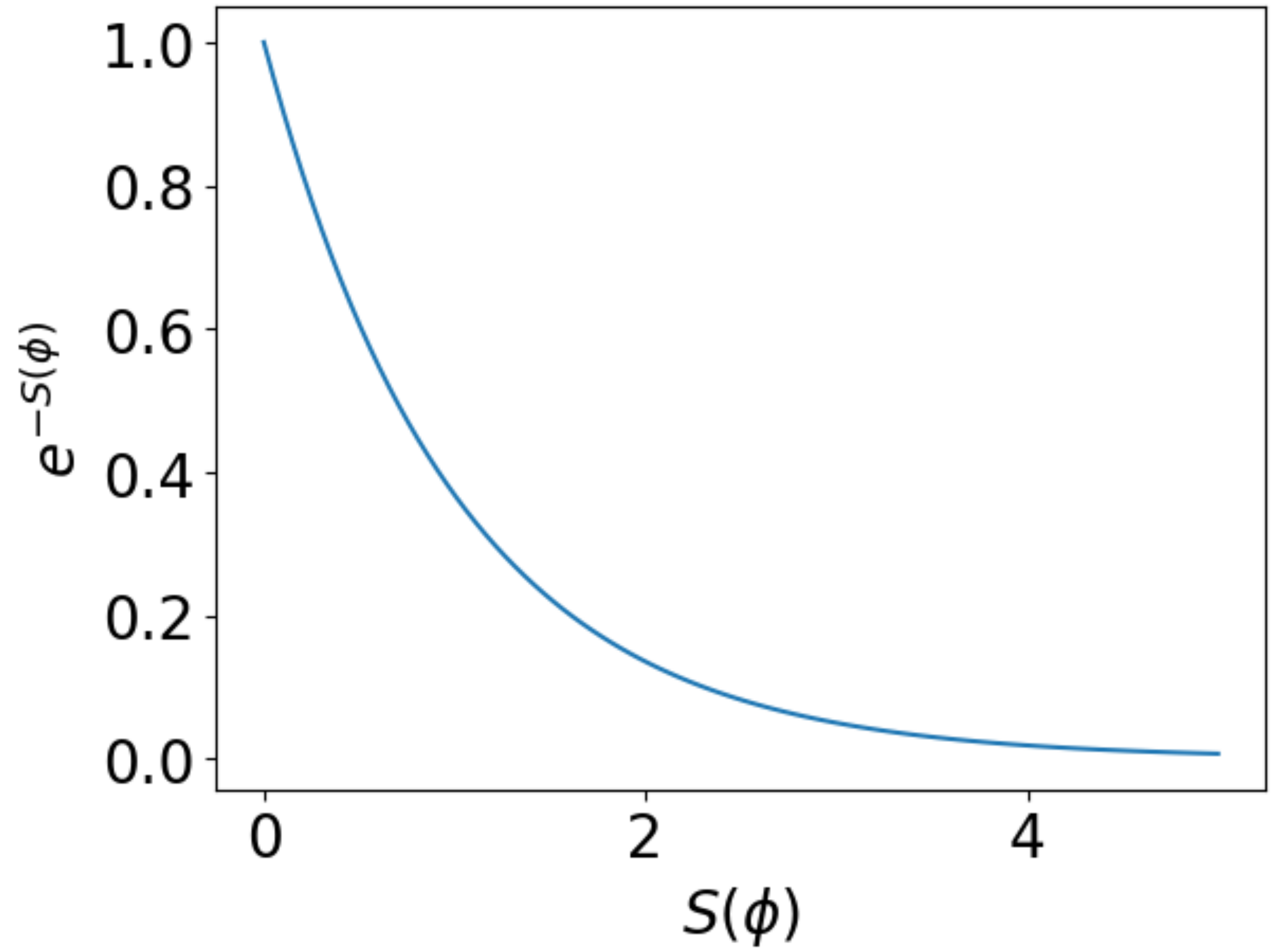
# Lattice QCD on Quantum Computers

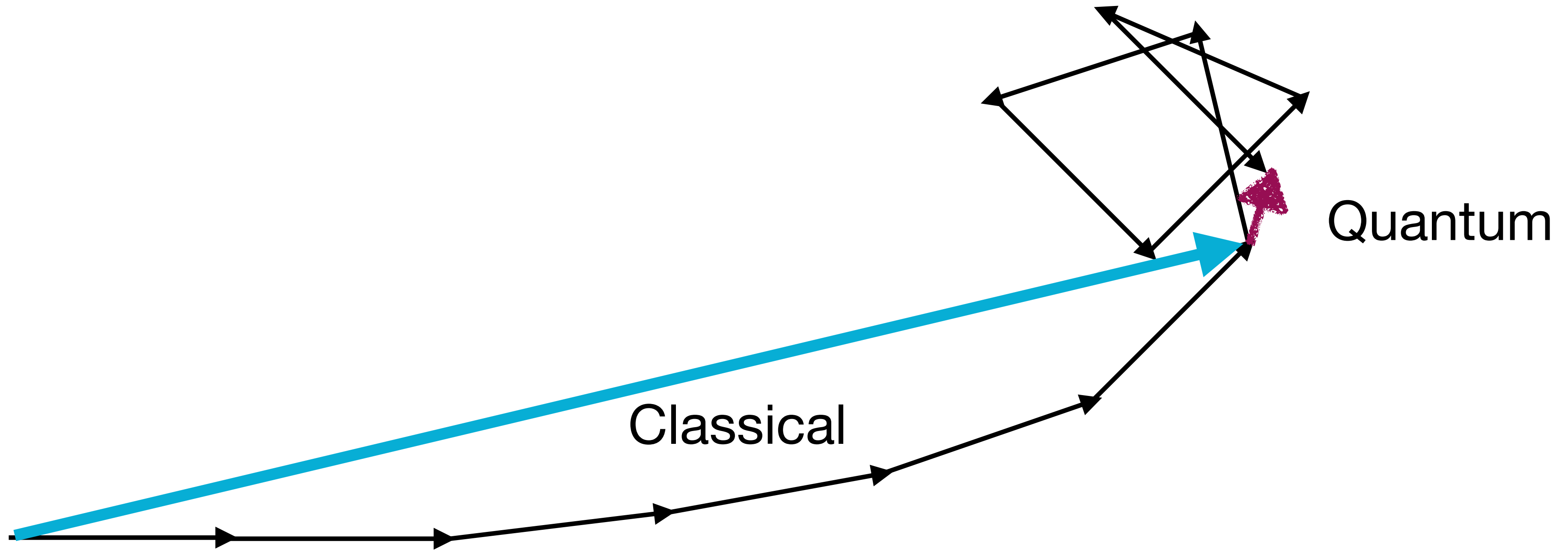
## Real Time Evolution



$$\int \mathscr{D}\phi e^{-S_E(\phi)} = \langle \varphi_f | e^{-\beta H} | \varphi_i \rangle$$

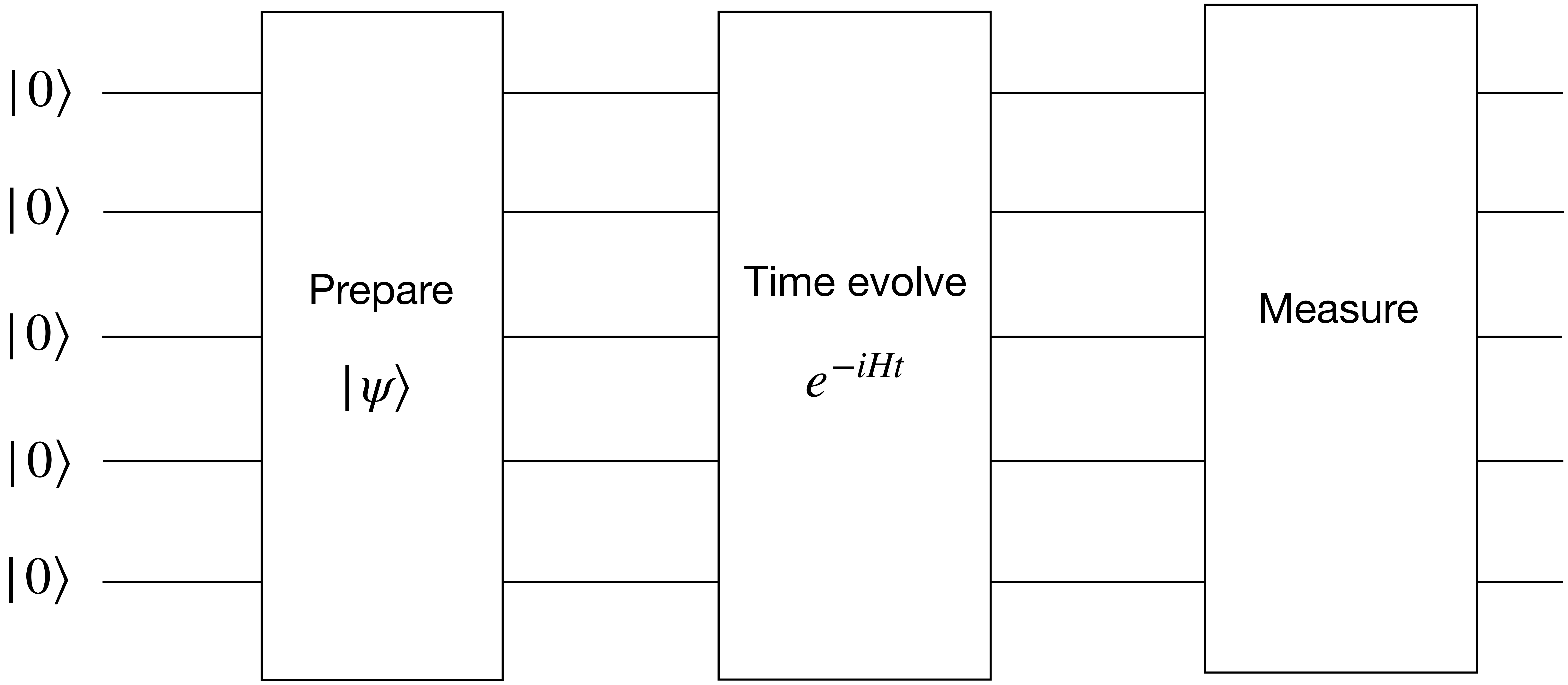
$$\int \mathscr{D}\phi e^{iS(\phi)} = \langle \varphi_f | e^{-iHt} | \varphi_i \rangle$$



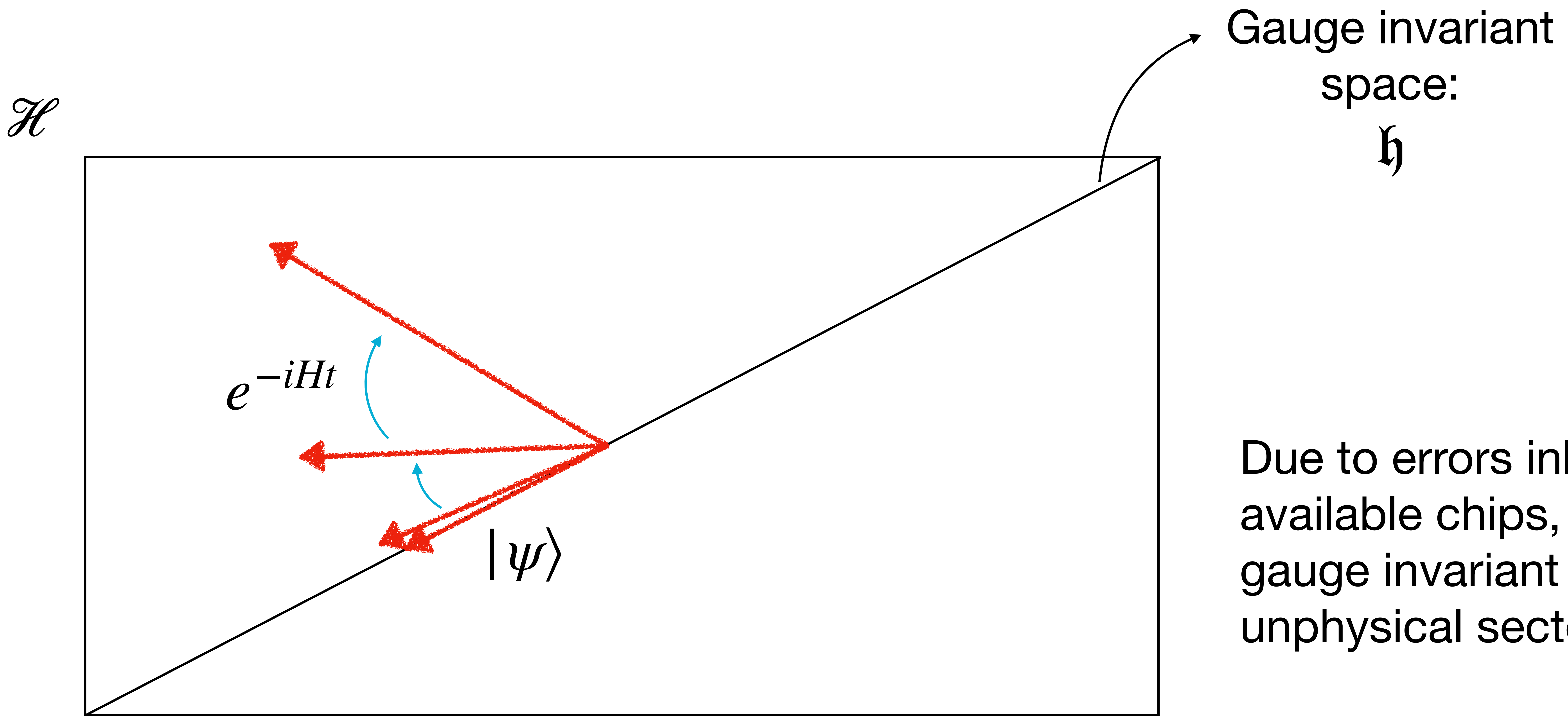



# Primer on Quantum Computation

## Shut up and evolve!



# Difficulty: Gauge Symmetry




Due to errors inherent in the current available chips, even starting with a gauge invariant state we can drift into the unphysical sector of the Hilbert space.

# Comparison Between Formulations

## Everything here comes with caveats!

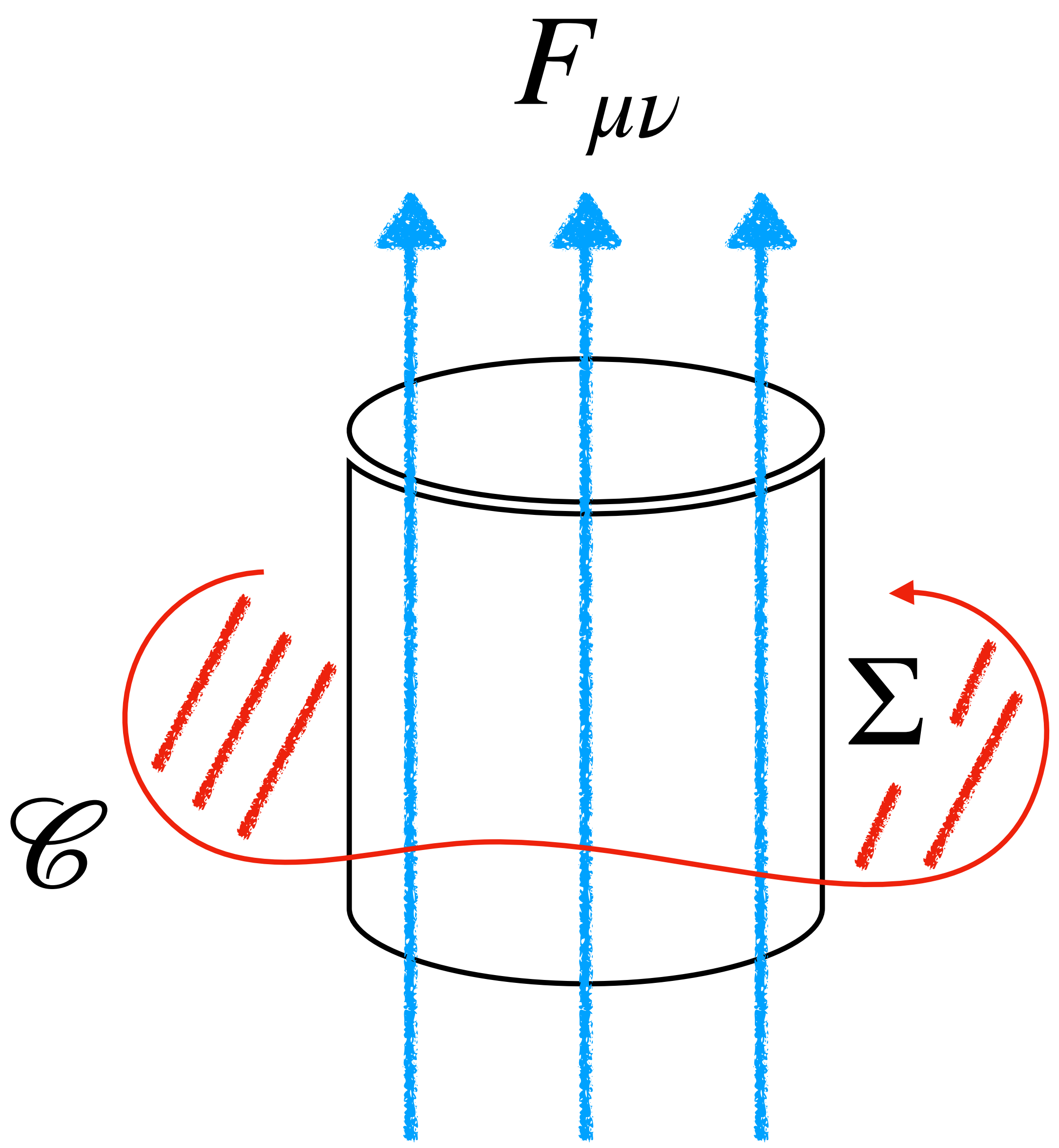
| | electric Kogut-Susskind | prepotentials | Loop-String-Hadron | magnetic maximal-tree | Loop-String-Hadron on flower |
|---|---|---|---|---|---|
| **gauge Invariant** | Non-Abelian per vertex | Abelian per link | Abelian per link | Non-Abelian on a single vertex | ✔ |
| **low coupling** | ✘ | ✘ | ✘ | ✔ | (✔) |
| **discrete quantum numbers** | ✔ | ✔ | ✔ | ✘ | ✔ |
| **local dynamics** | ✔ | ✔ | ✔ | ✘ | ✘ |

# Why do we need gauge “symmetry”?

## Proposal: because of locality

Let’s give it up!

Redundancy

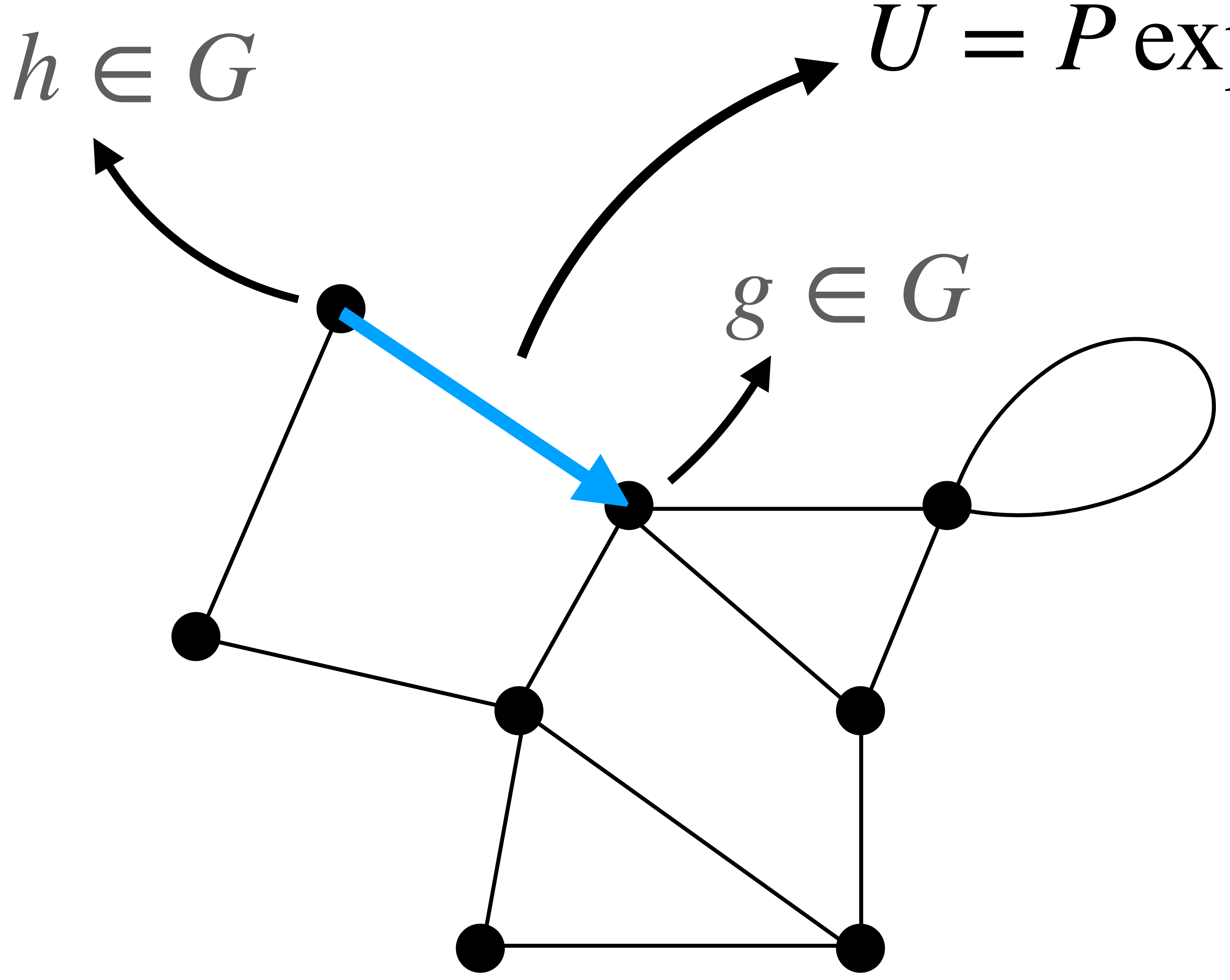



$$\int_{\mathscr{C}} A = \int_{\Sigma} F$$

# Yang-Mills Background

# Hamiltonian Lattice Yang-Mills

## Kogut-Susskind: Classical kinematics




Space

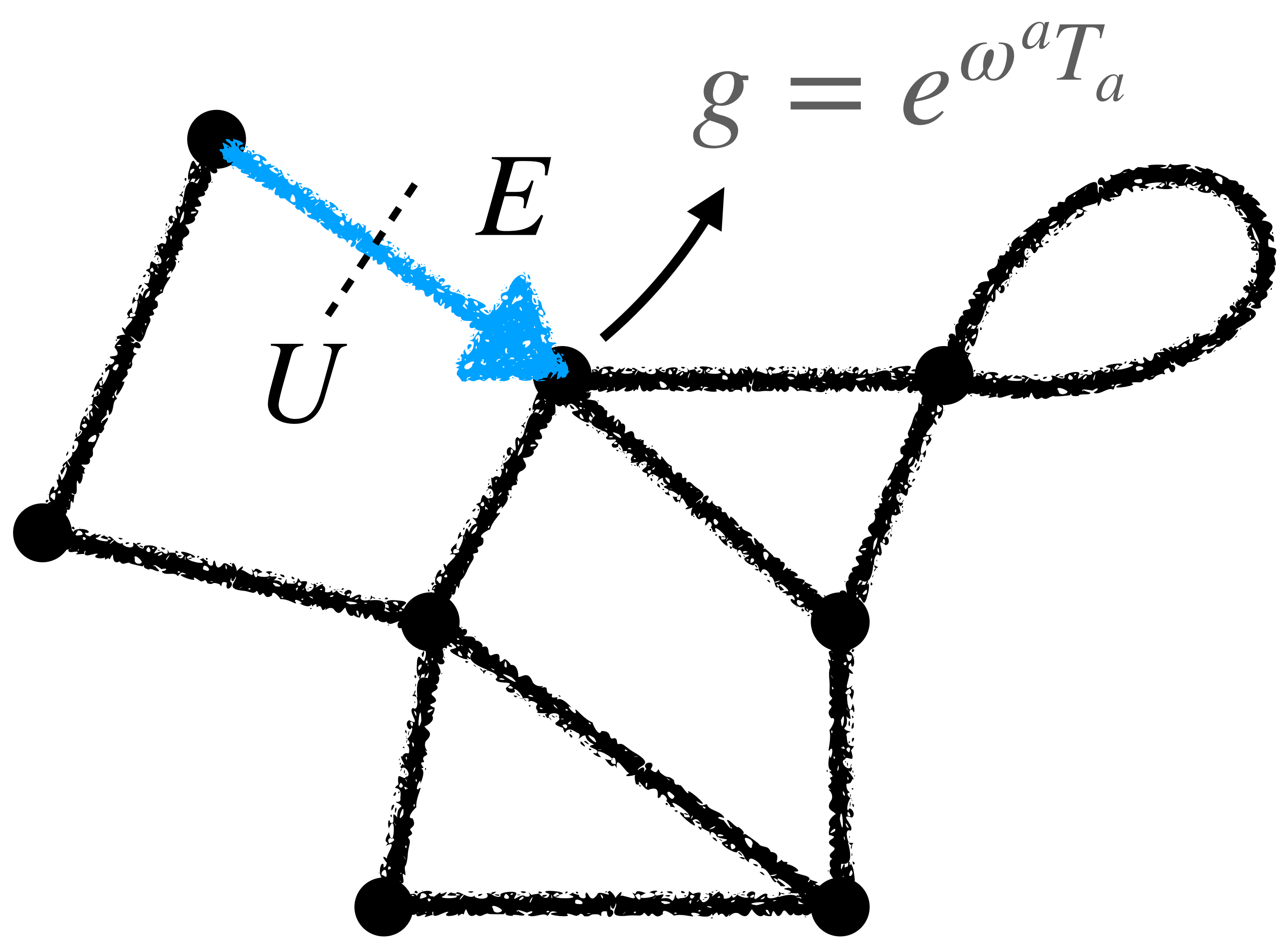
$$U = P\exp\int_e A \in G = U(1), SU(2), SU(3), SU(N), \ldots$$

Gauge Transformation: $U \longmapsto gUh^{-1}$

For me link = edge

# Hamiltonian Lattice Yang-Mills

## Kogut-Susskind: Quantum kinematics

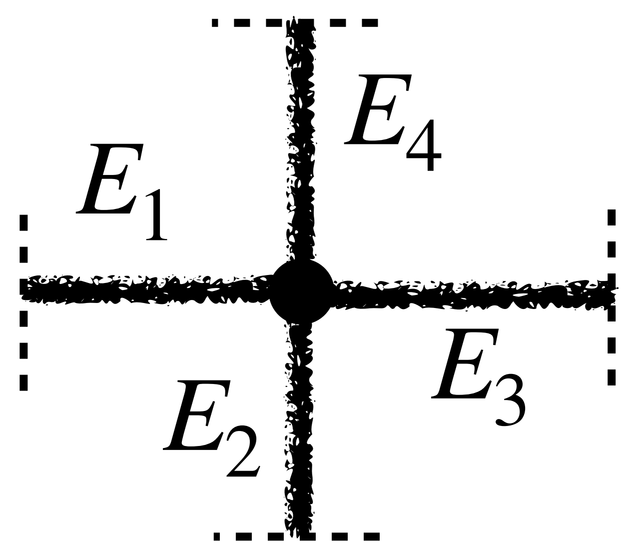



Magnetic basis: (analogous to traditional lattice QCD)

States $|\psi\rangle$ are wavefunctions that assign to each configuration of Wilson lines $U, \ldots$ a probability amplitude density $\langle U, \ldots | \psi \rangle$

Electric Operators: $E = E_a T^a$

$$\langle U, \ldots | e^{-i\omega^a E_a} | \psi \rangle = \langle g^{-1} U, \ldots | \psi \rangle$$




Gauss' Law: $0 = E_1 + E_2 + E_3 + E_4 \longleftrightarrow \int_{S^2} \mathrm{d}\vec{S} \cdot \vec{E}$

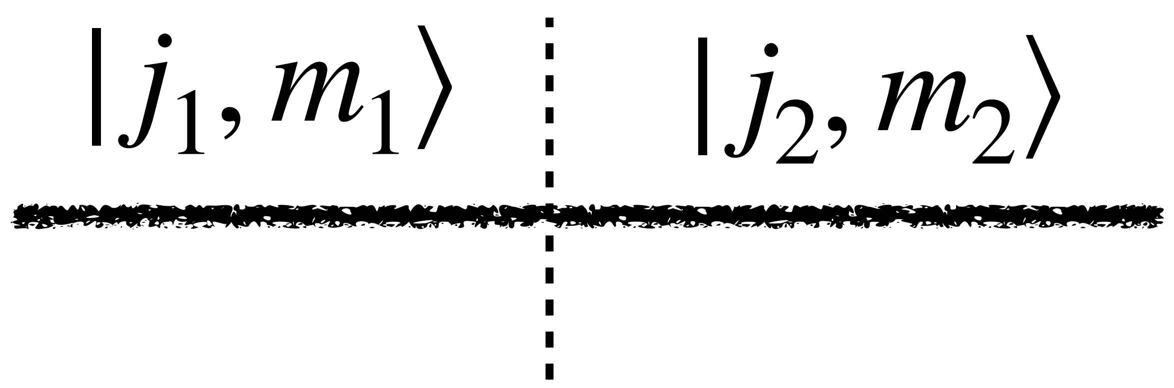
# Hamiltonian $SU(2)$ Yang-Mills

## Electric Basis

**Peter-Weyl Theorem:** The space of wavefunctions is spanned by the matrix elements of every representation.

$$\langle U | \psi \rangle := (-1)^{j-m_1} \sqrt{2j+1} \langle j, -m_1 | R^{(j)}(U) | j, m_2 \rangle$$

New way of thinking of states:




$+$

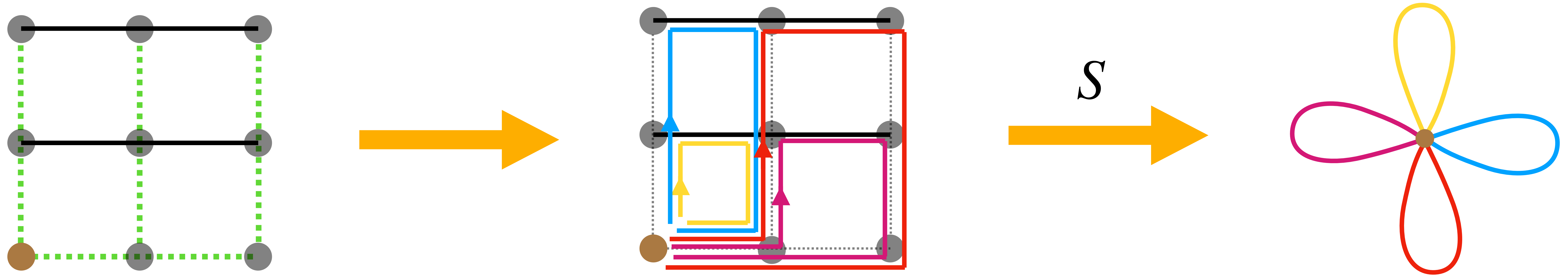
Abelian Gauss' Law: $j_1 = j_2$

# Conclusions

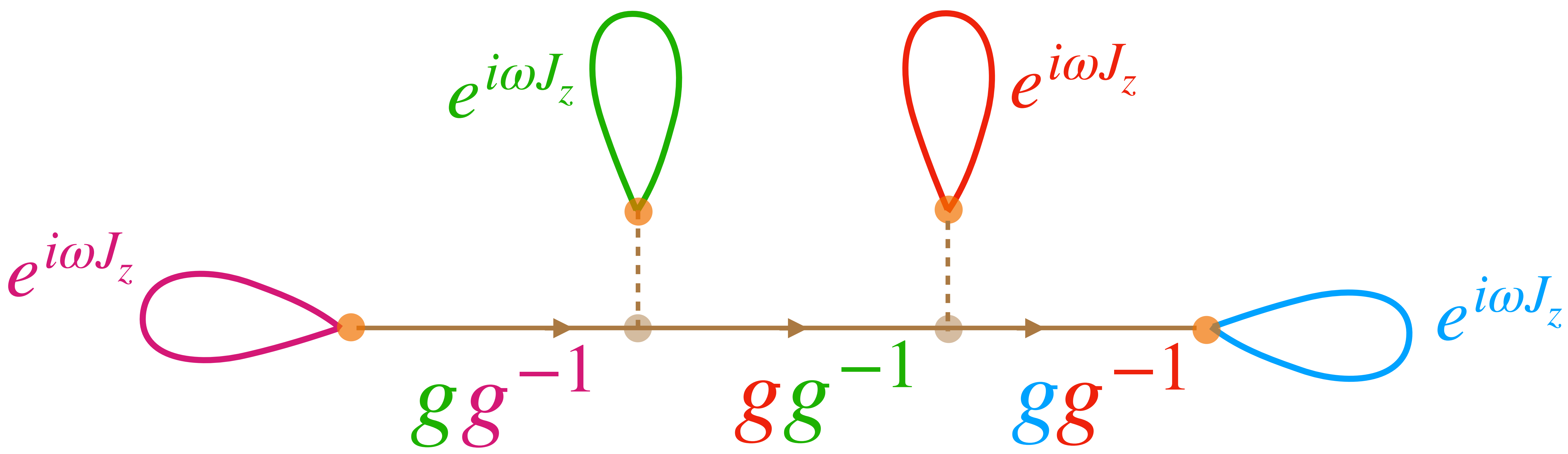
## Spoiler alert!

Christian W. Bauer, Irian D'Andrea, Marat Freytsis, and Dorota M. Grabowska 2307.11829.

1. We can (almost) fully gauge fix using a maximal tree into a flower

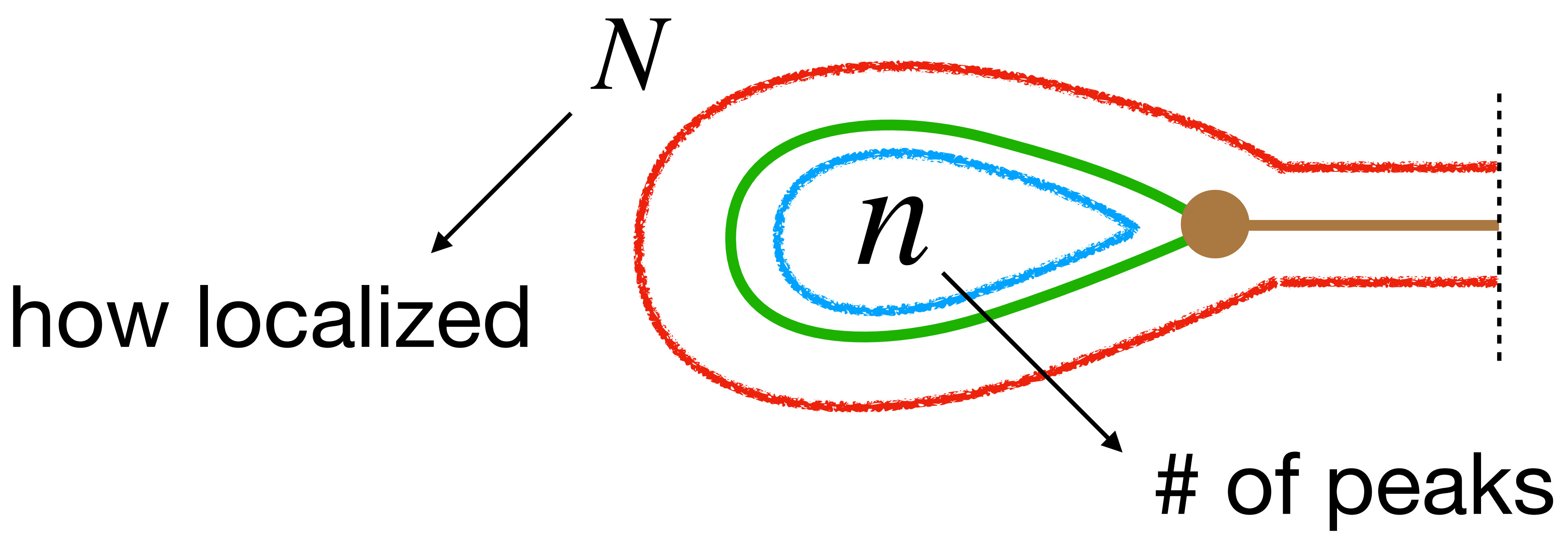


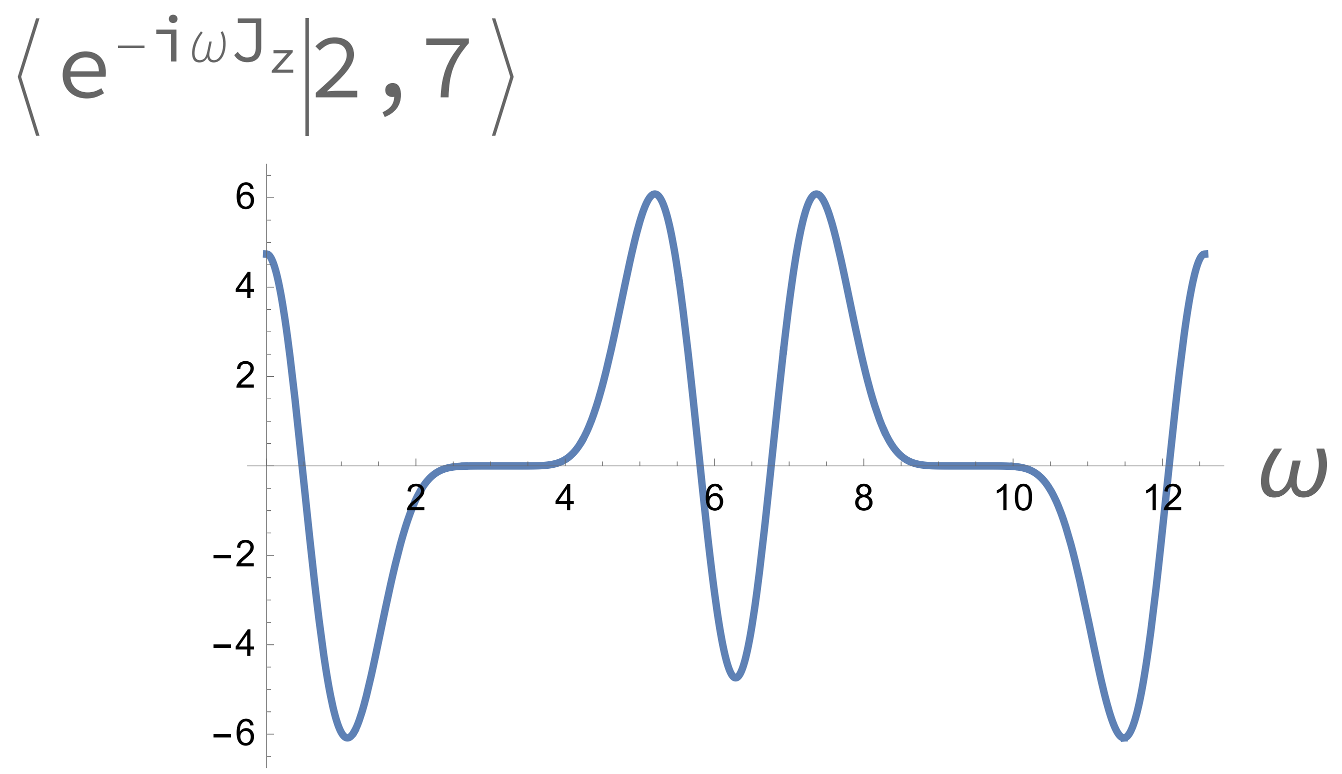

2. We can split the flower into a branch to avoid Mandelstam constraints



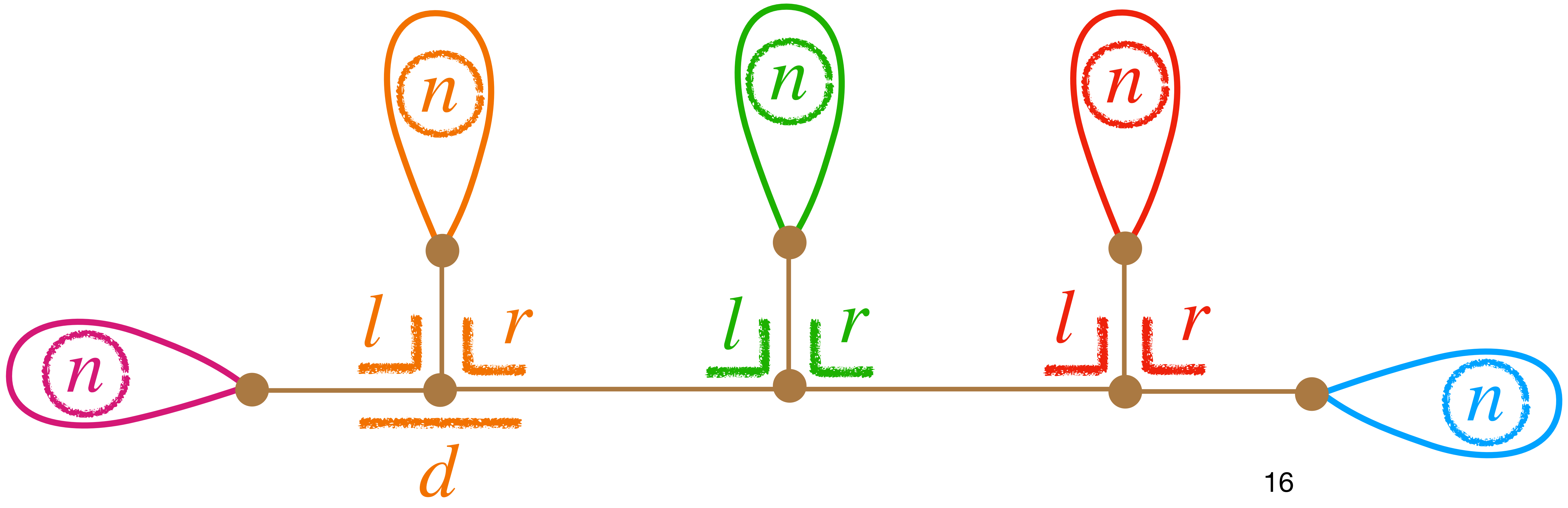
# Conclusions

3. Leafs induce special functions that translate between electric and magnetic bases




4. The system can be described through $3((\text{\# of plaquettes}) - 1)$ quantum numbers




A single constraint:

$$d > \max\{r - l, r - l + l - r\}$$

# Hamiltonian $SU(2)$ Yang-Mills

## Prepotential Formulation

Let us think of the spin 1/2 states as different bosonic species

$\uparrow$ $\qquad$ $\downarrow$

$a_\uparrow^\dagger |0\rangle \qquad a_\downarrow^\dagger |0\rangle$

Can recover all other states in any representation

$$|j, m\rangle \propto \left(a_\uparrow^\dagger\right)^{j+m} \left(a_\downarrow^\dagger\right)^{j-m} |0\rangle$$

# Hamiltonian $SU(2)$ Yang-Mills

## What have we gained?

Indices 😀

$$a_A^\dagger = \begin{pmatrix} a_\uparrow^\dagger \\ a_\downarrow^\dagger \end{pmatrix}$$

$$\epsilon^{AB}\psi_A\phi_B = \psi_A\phi^A = \psi\phi$$

Gauge invariance $\longleftrightarrow$ No free indices!

# Hamiltonian $SU(2)$ Yang-Mills

## Crazy Repackaging

Physical

$$A^A{}_B = \begin{cases} a^A & B = - \\ a^{\dagger A} & B = + \end{cases}$$

Unphysical

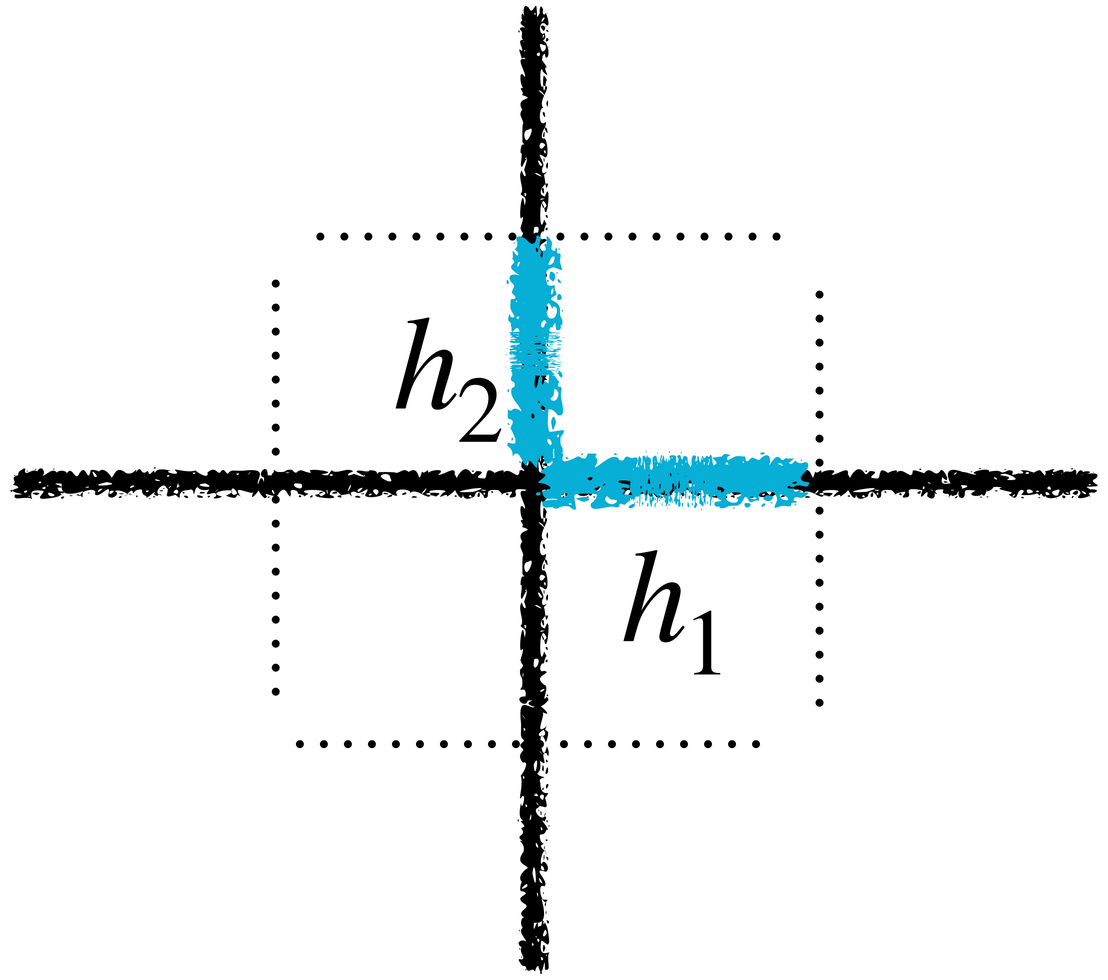
Properties:

$$[A^A{}_B, A^C{}_D] = \epsilon^{AC}\epsilon_{BD} \qquad (A^\dagger)^A{}_B = A_B{}^A \qquad AA^\dagger = N + 1$$

# Hamiltonian $SU(2)$ Yang-Mills
## Loop-String-Hadron (LSH) formulation

Loop operators:

$$\mathcal{L}_{AB}(h_1, h_2) = A_{CA}(h_2) A^C{}_B(h_1)$$




Creation: $A = B = +$

Number: $A = +, B = -, h_1 = h_2$

Indrakshi Raychowdhury, and Jesse R. Stryker 1912.06133.

# Dynamics

$$H = H_E + H_B$$

These are the plaquettes with a timelike direction

$$H_E = \frac{g^2}{4} \sum_{\text{half edges } h} E(h)^2$$

$$H_B = \frac{1}{2g^2} \sum_{\text{plaquettes } \gamma} \mathrm{tr}(U(\gamma))$$

strong coupling

weak coupling

# These can be written in LSH terms

$$E_a = a^\dagger \sigma_a a \longrightarrow E(h)^2 = \frac{\mathscr{L}_{+-}(h,h)}{2}\left(\frac{\mathscr{L}_{+-}(h,h)}{2} + 1\right)$$

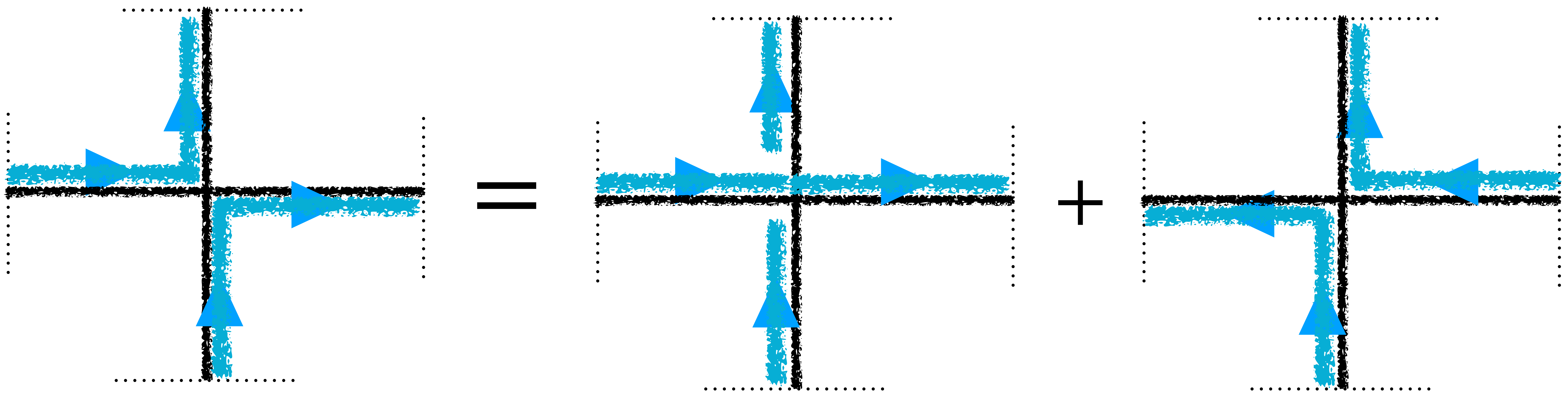
$$U = \frac{1}{\sqrt{N+1}}A \longrightarrow U^A{}_B(h_1,h_2) = U^A{}_C(h_2)(\sigma_x)^C{}_D U_B{}^D(h_1)$$

# Manipulating Graphs

# Virtual Point-Splitting

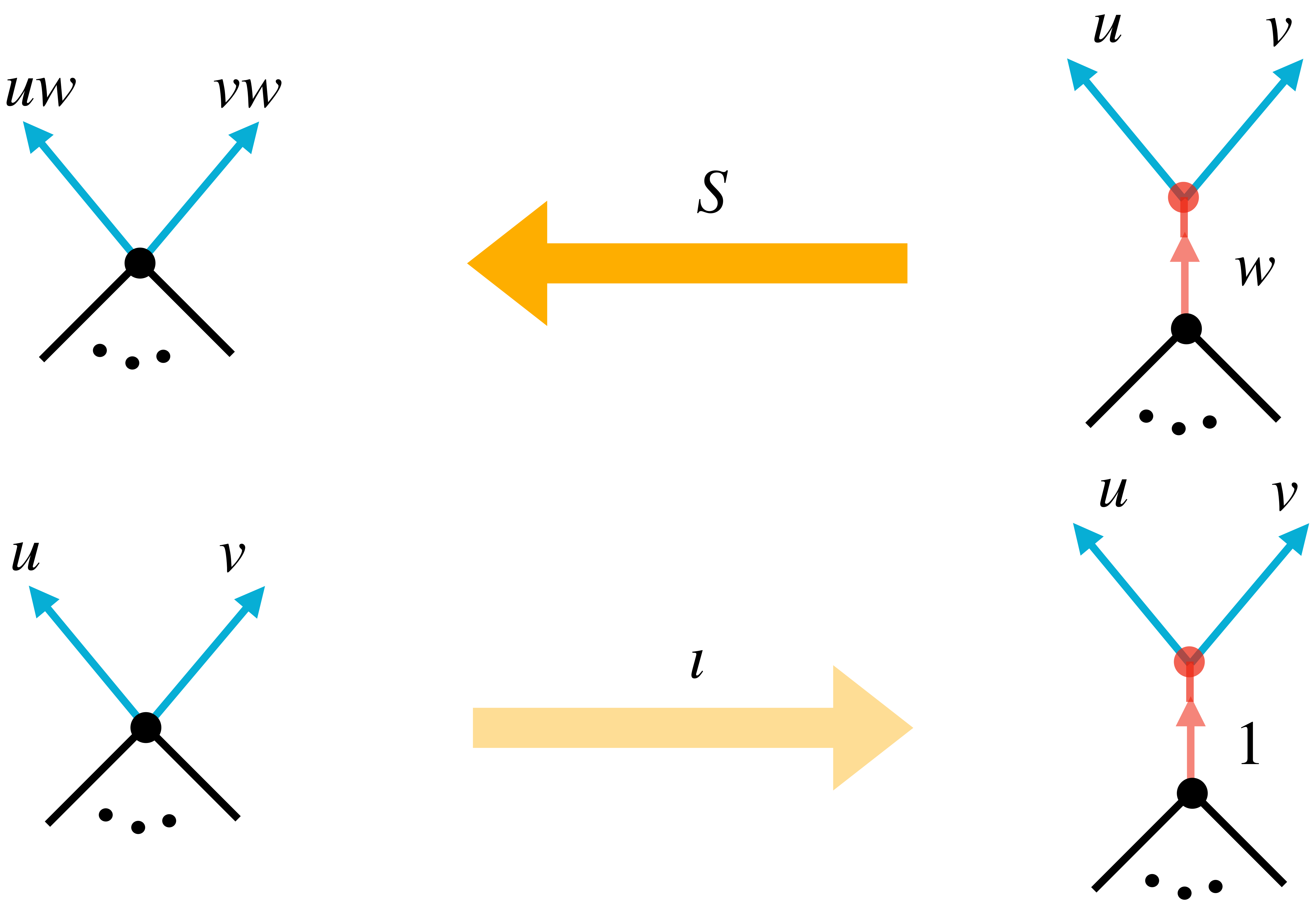
## Mandelstam Constraints

In the presence of vertices with valency higher than 4, the LSH creation operators don't create linearly independent states.



# Virtual Point-Splitting

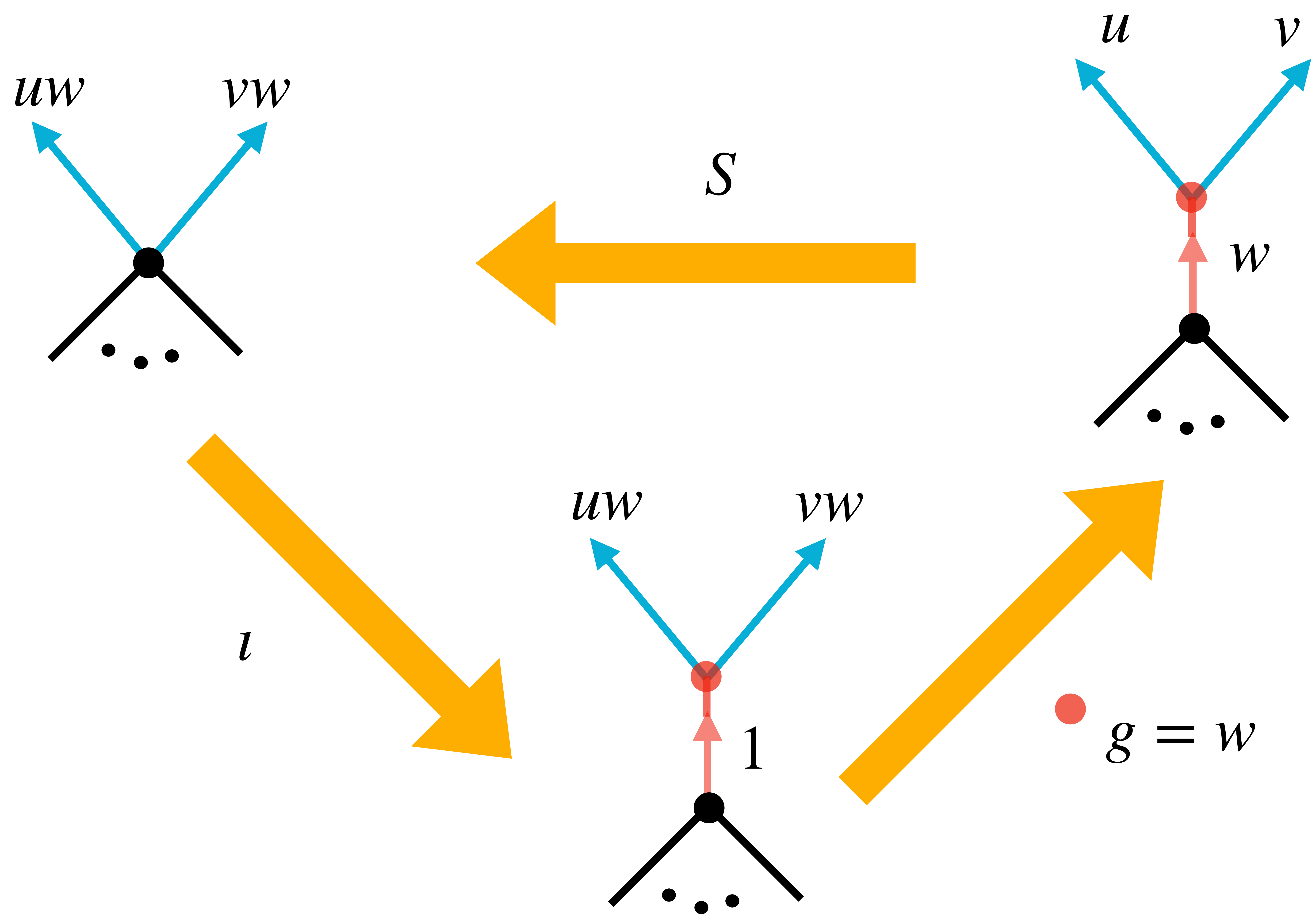
## Solution




This is not an equivalence at face value!

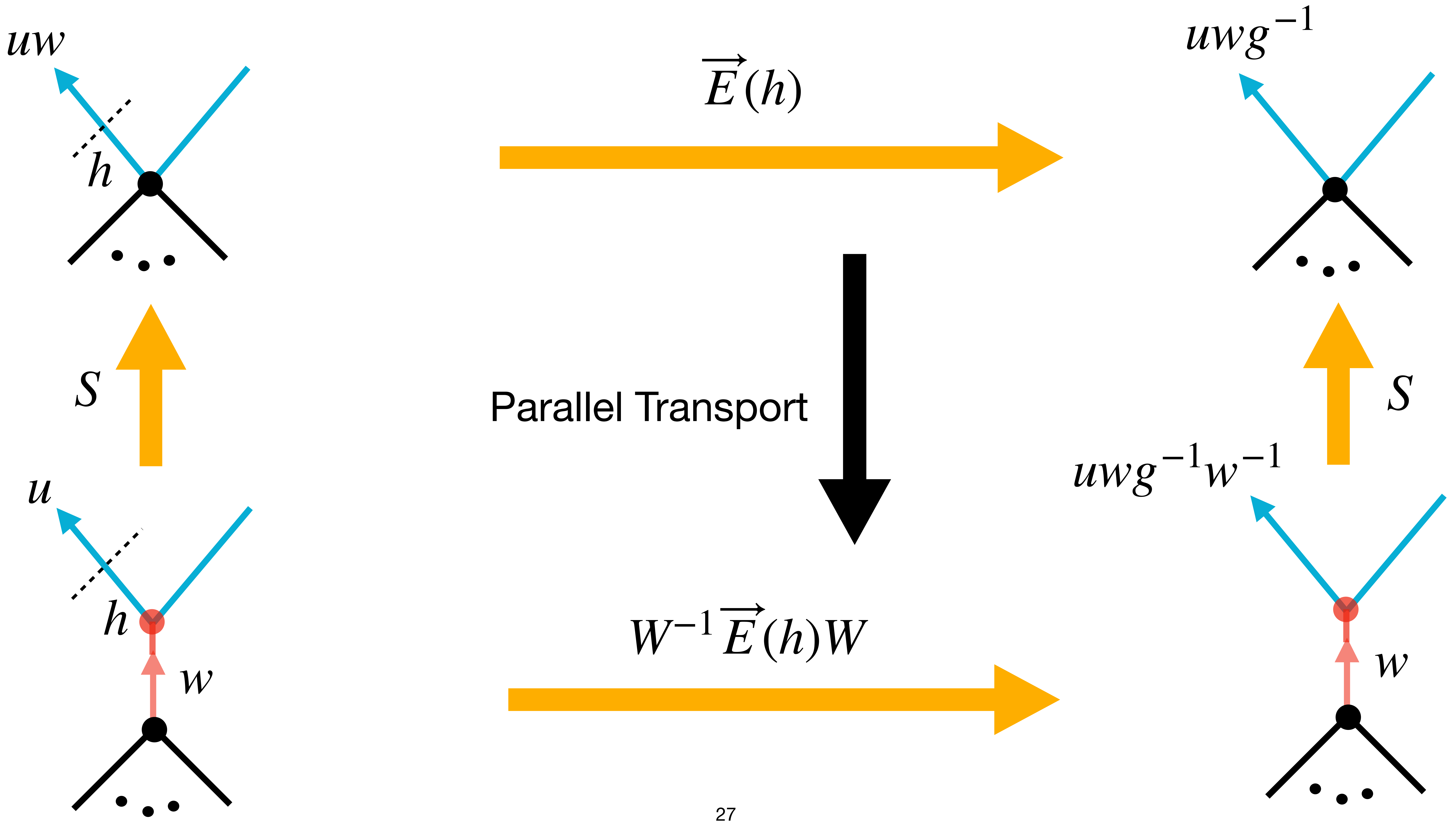
# Virtual Point-Splitting

## Equivalence up to gauge invariance



# Virtual Point-Splitting

## Electric behavior



# Coarsening of a graph

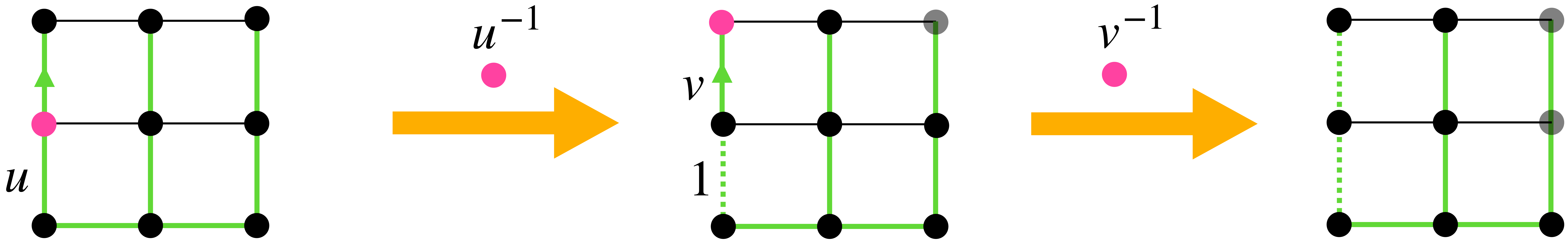

**Theorem:** The theories are equivalent (even at the quantum level) as long as

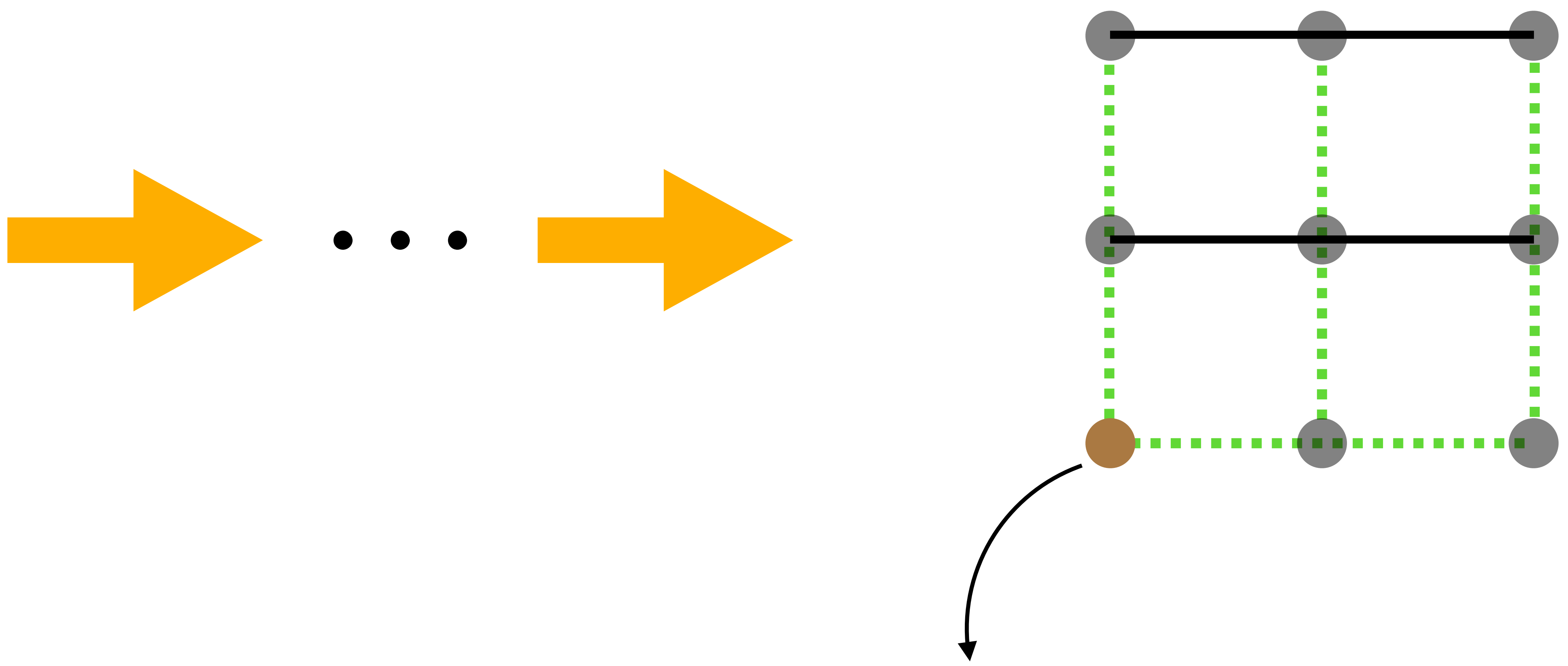
I) $S$ is surjective (which almost always happens) and

II) all configurations with the same coarsening are gauge equivalent to one another

The example above is **not** an equivalence (think about the orbit of any configuration with the identity at the loop).

# Maximal Tree Gauge-Fixing

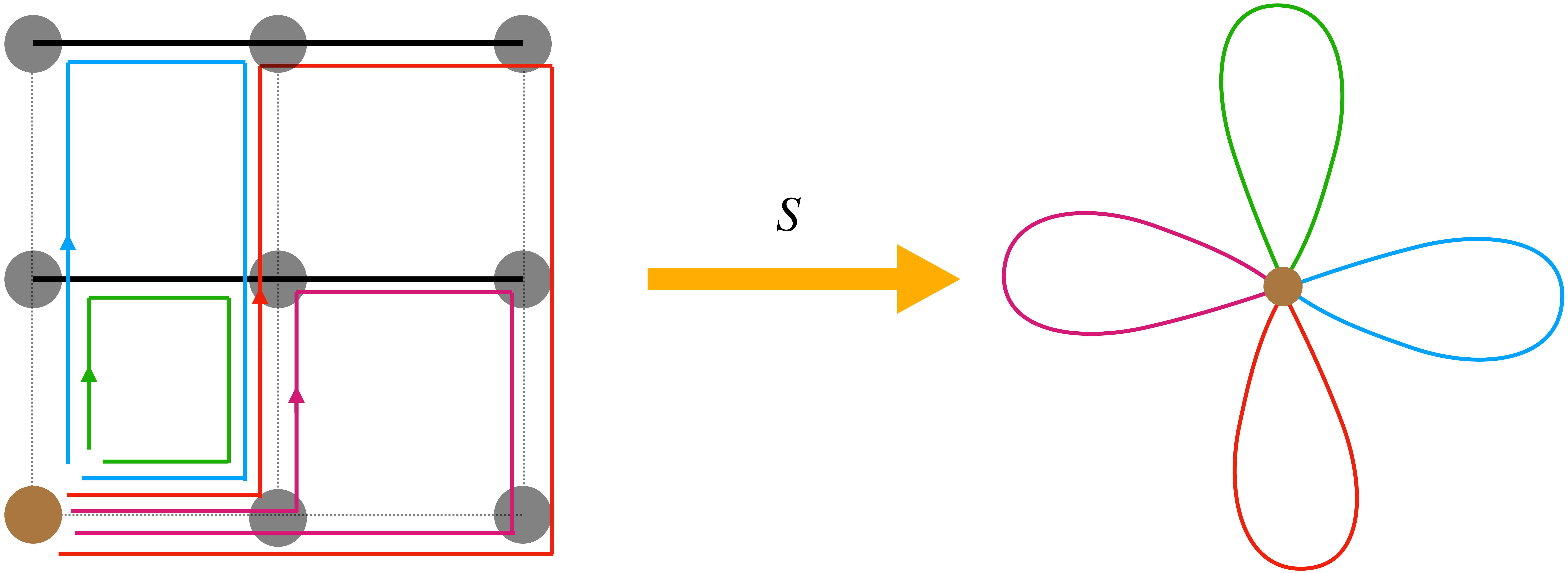



Only one single gauge degree of freedom remaining

# Enter Flowers

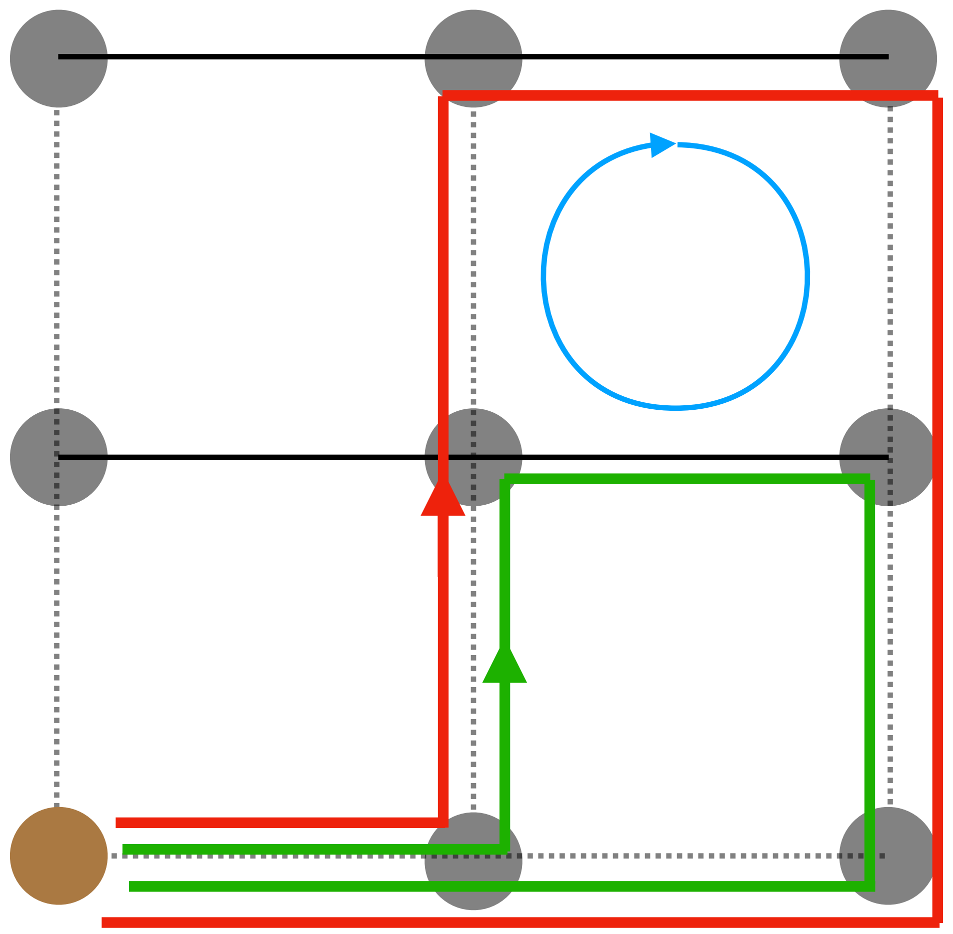
## Coarse graining through maximal trees



Christian W. Bauer, Irian D'Andrea, Marat Freytsis, and Dorota M. Grabowska 2307.11829.

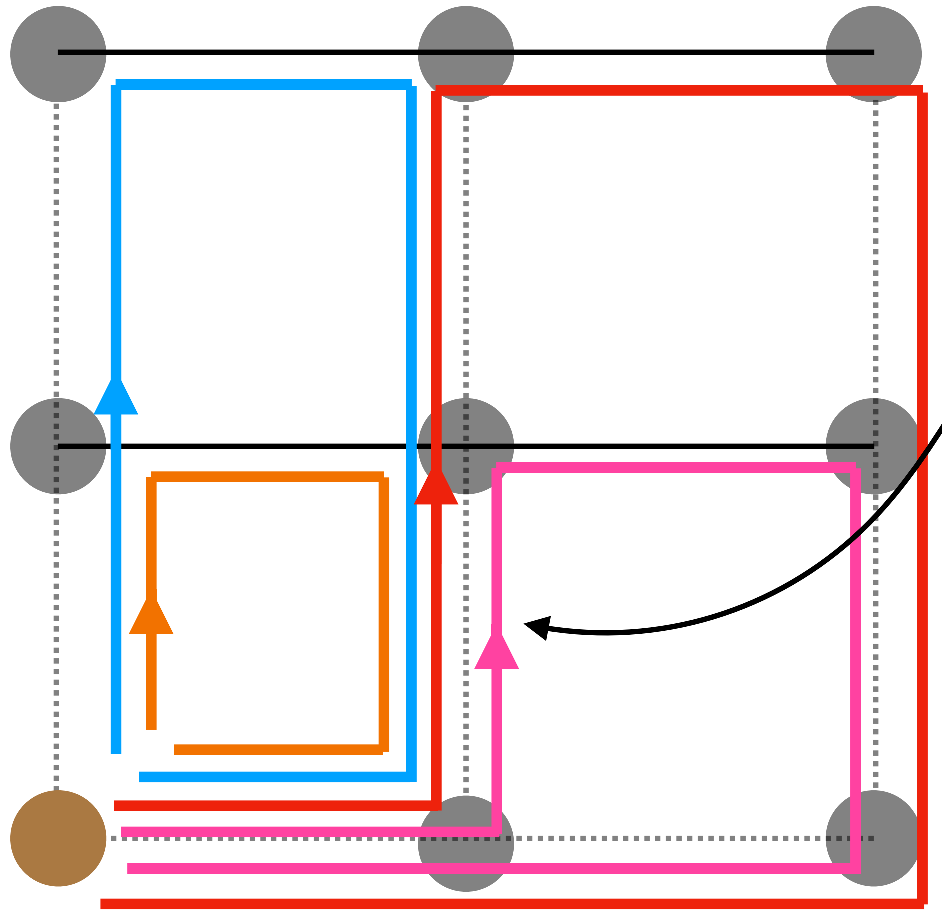
# Flower Hamiltonian

Magnetic Hamiltonian



$$U = U^{-1} U$$

Electric Hamiltonian

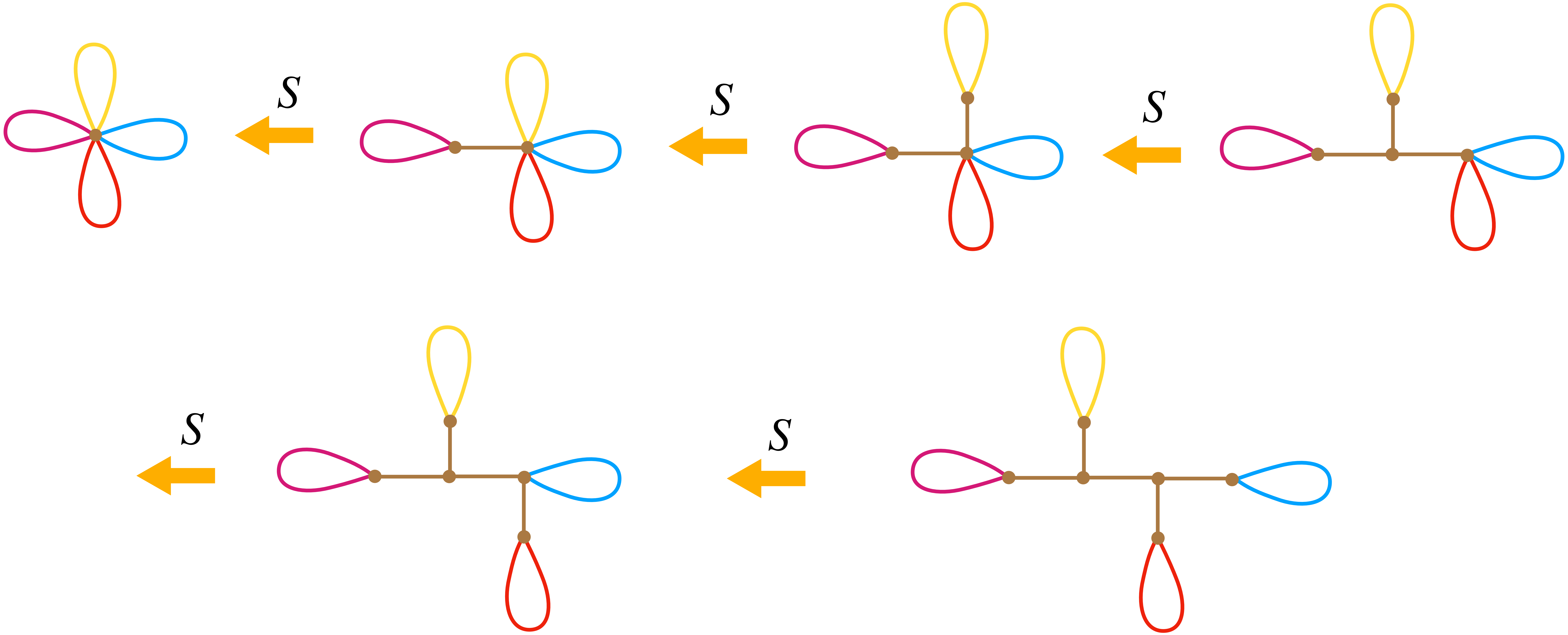


$$\vec{E}^2 \Longrightarrow \left(\vec{E}(f) + \vec{E}(f) - \vec{E}(i) - \vec{E}(i)\right)^2$$

Parallel Transport to ●

# Point-Splitting Flowers

## Correcting for high-valency vertices



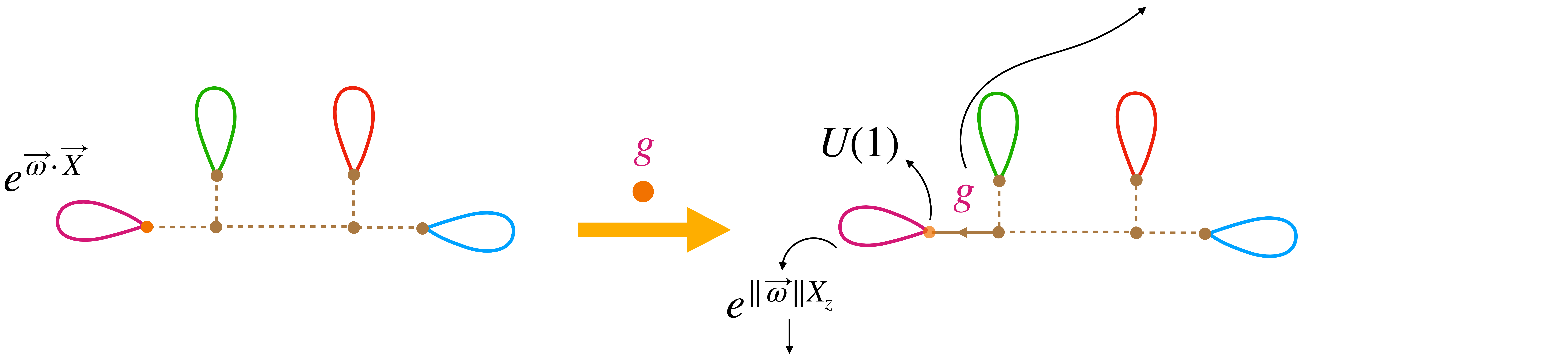
# Yang-Mills on the Flower

# Magnetic Degrees of Freedom

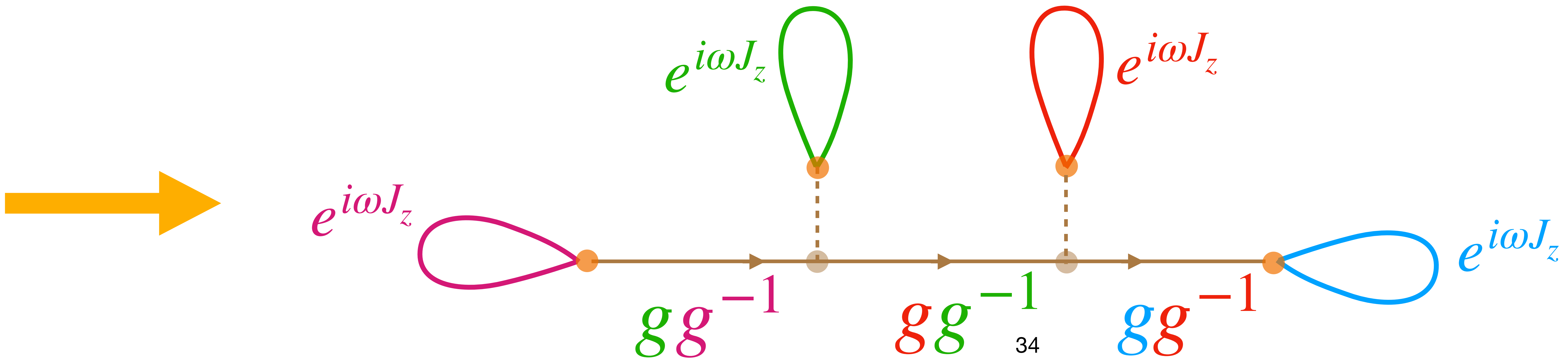
Orientation of $\overrightarrow{\omega}$

↓

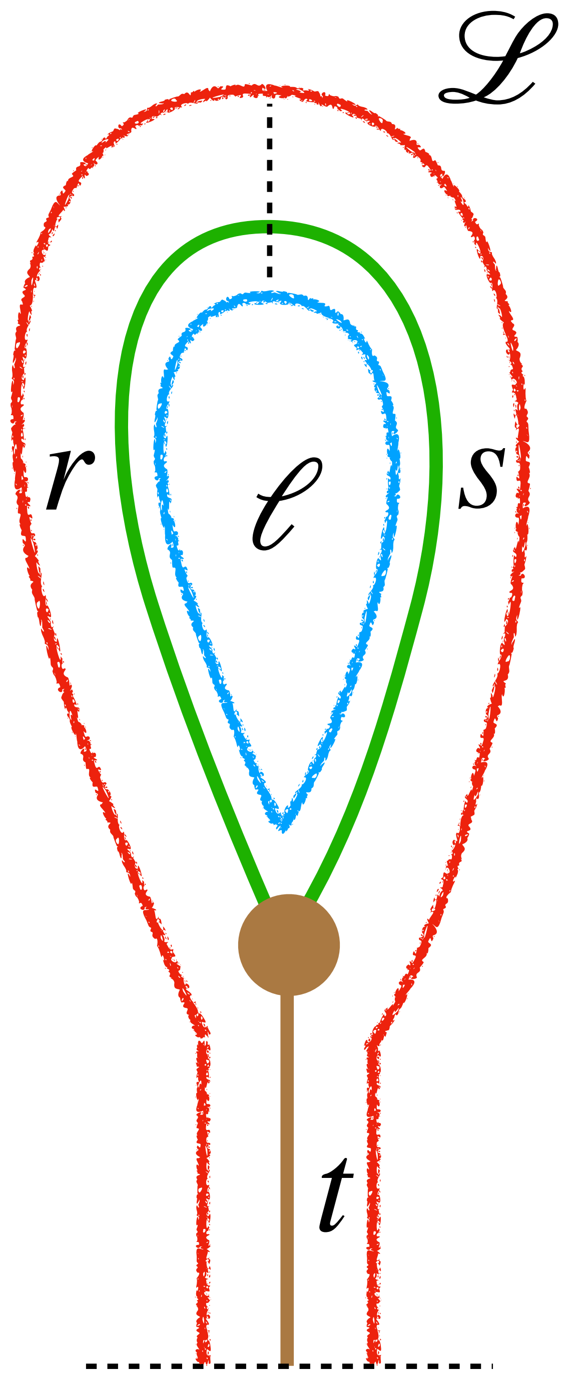
Delocalized at low coupling




Christian W. Bauer, Irian D'Andrea, Marat Freytsis, and Dorota M. Grabowska 2307.11829. → Localized at low coupling (i.e. near the continuum limit)



# LSH on a Leaf



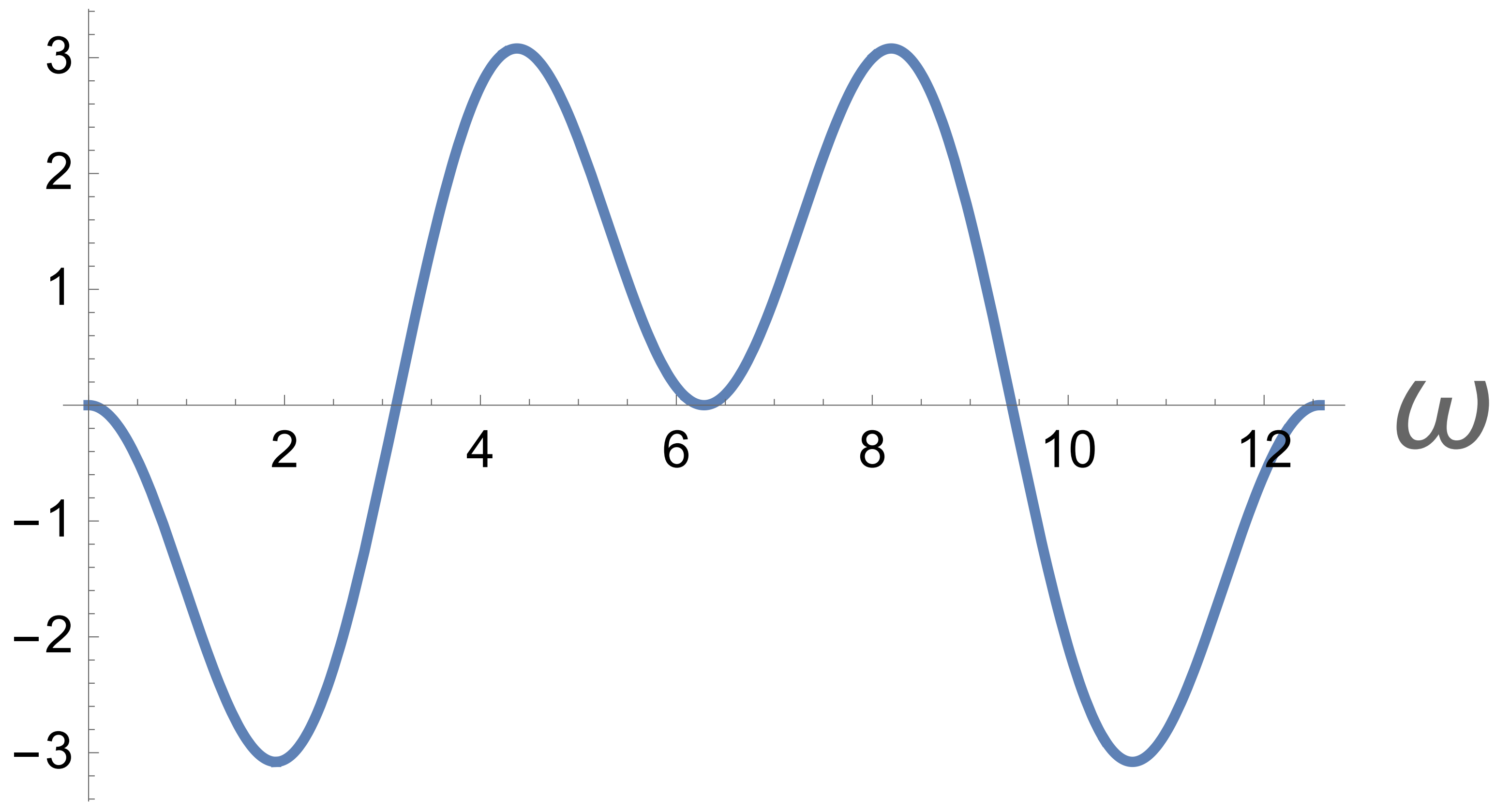

$$\ell = a^\dagger(r)a^\dagger(s) \qquad \mathscr{L} = (a^\dagger(s)a^\dagger(t))(a^\dagger(r)a^\dagger(t))$$

$$|n, N\rangle \propto \ell^n \mathscr{L}^N |0\rangle$$

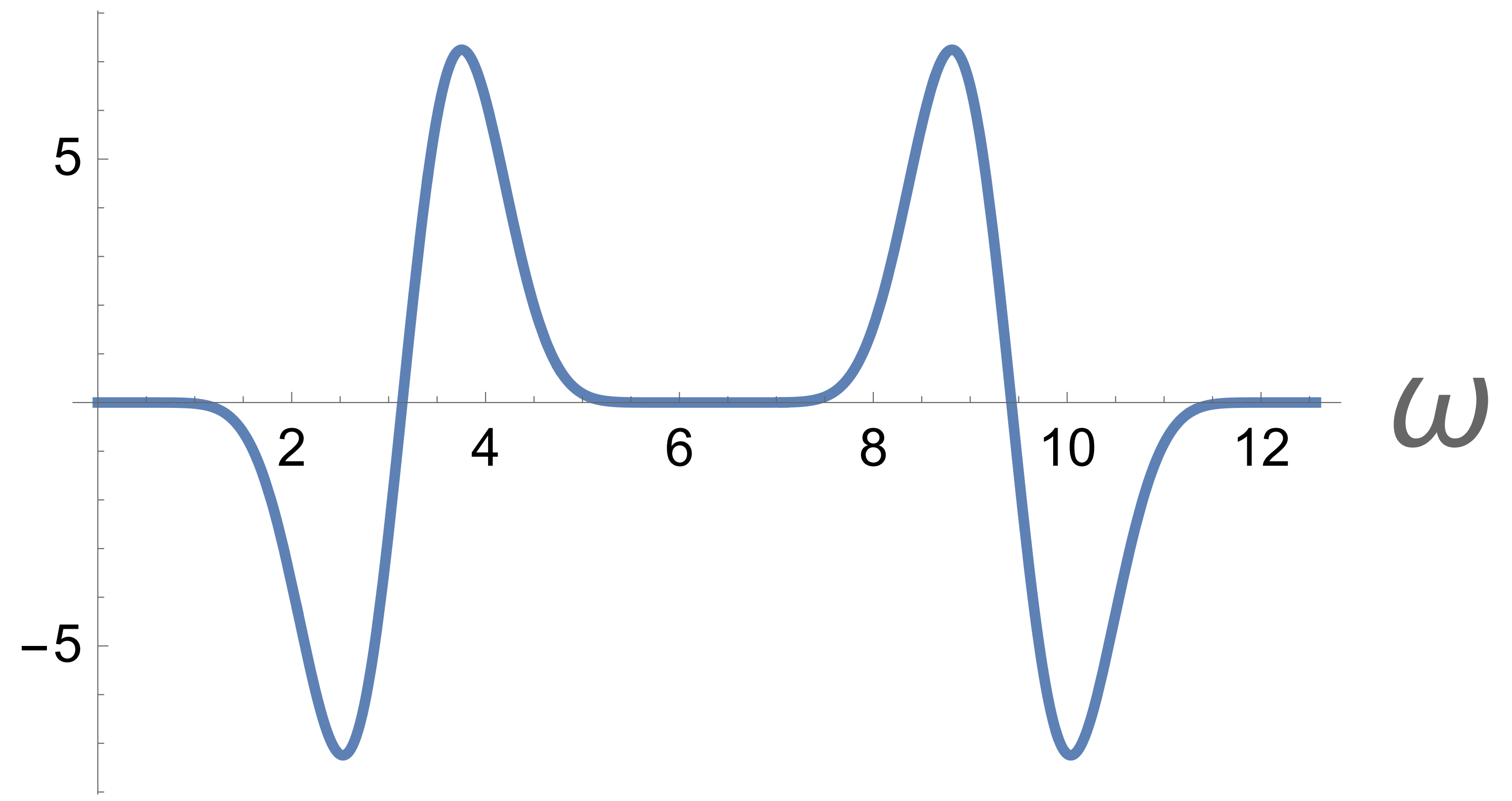
$$\langle e^{-i\omega J_z} | n, N\rangle = \sqrt{N+n+1} \sum_{m=-\frac{n+N}{2}}^{\frac{n+N}{2}} e^{-i\omega m} \langle N,0| \left( \left| \frac{n+N}{2}, -m \right\rangle \otimes \left| \frac{n+N}{2}, m \right\rangle \right)$$

# Special Function

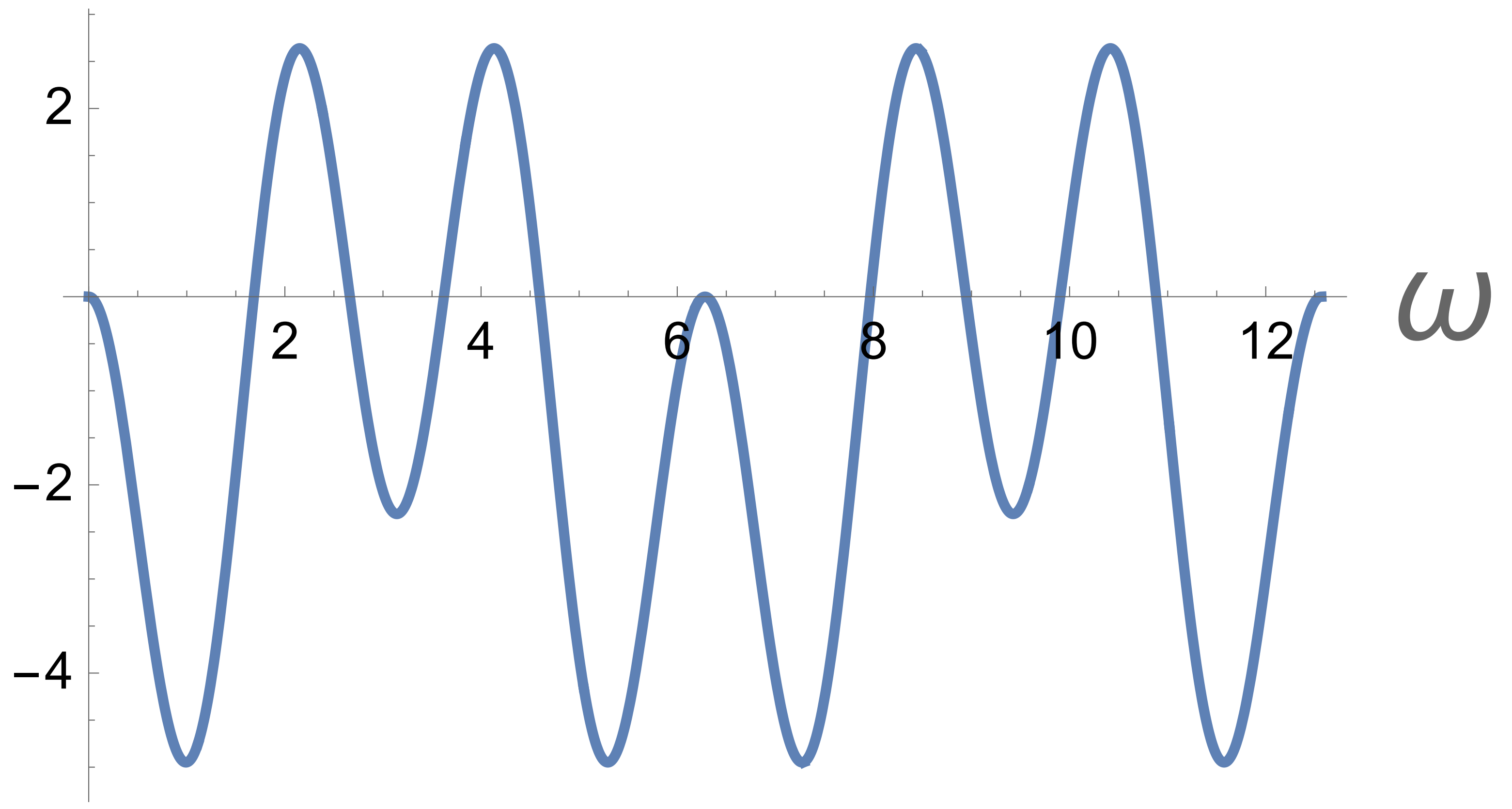
$\mathrm{Re}\langle e^{-i\omega J_z} | 1, 2 \rangle$



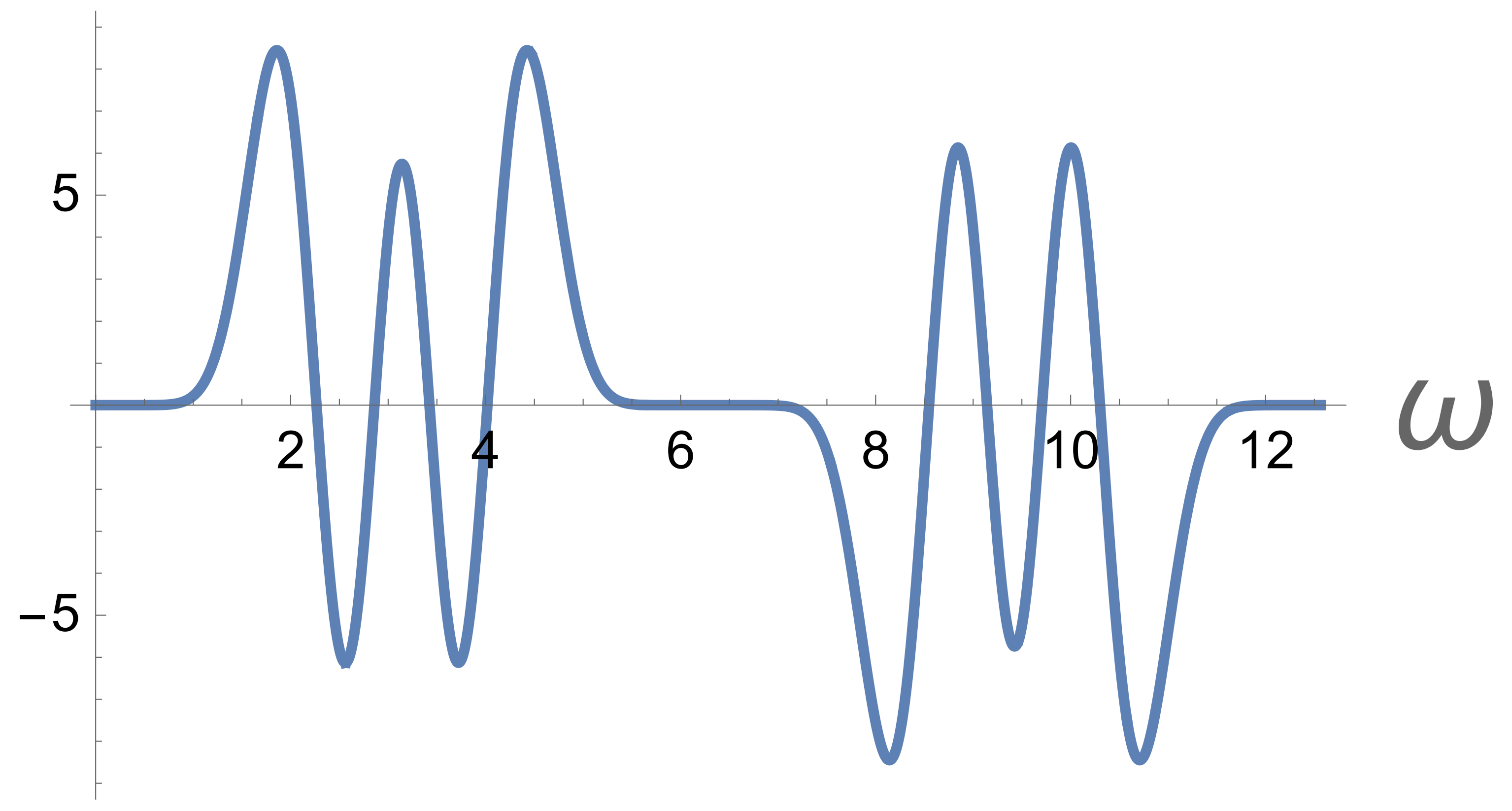

$\mathrm{Re}\langle e^{-i\omega J_z} | 1, 10 \rangle$



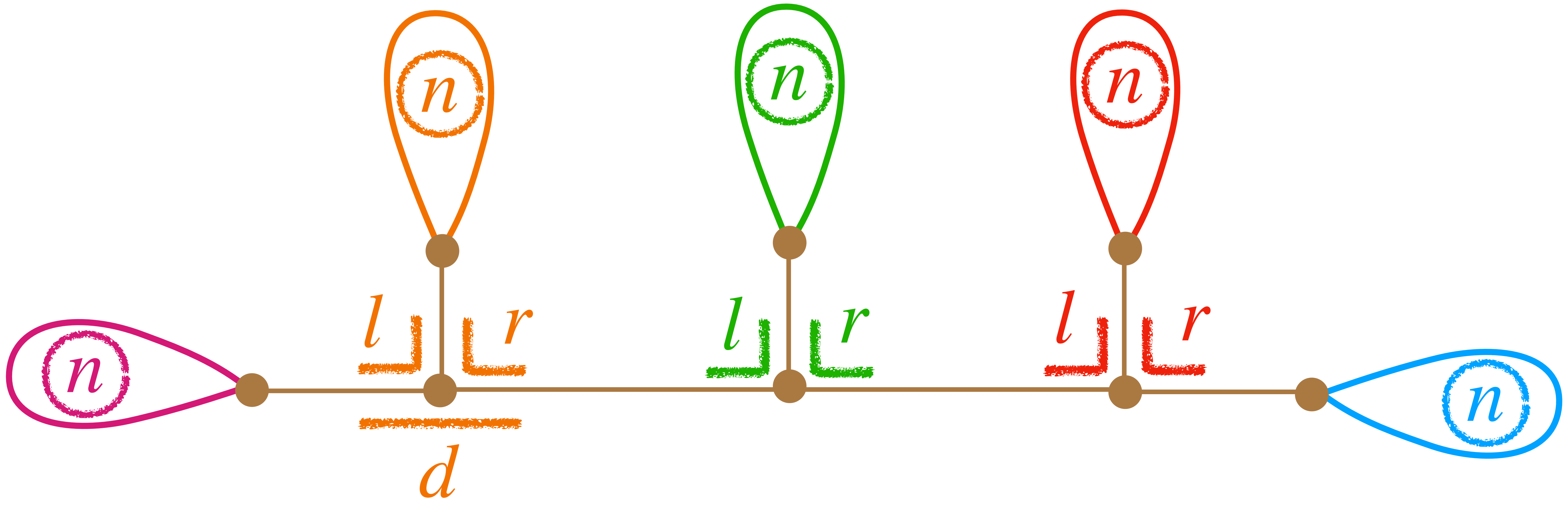

$\mathrm{Re}\langle e^{-i\omega J_z} | 4, 2 \rangle$




$\mathrm{Im}\langle e^{-i\omega J_z} | 4, 11 \rangle$



# LSH Degrees of Freedom




A single constraint: $d > \max\{r - l, r - l + l - r\}$

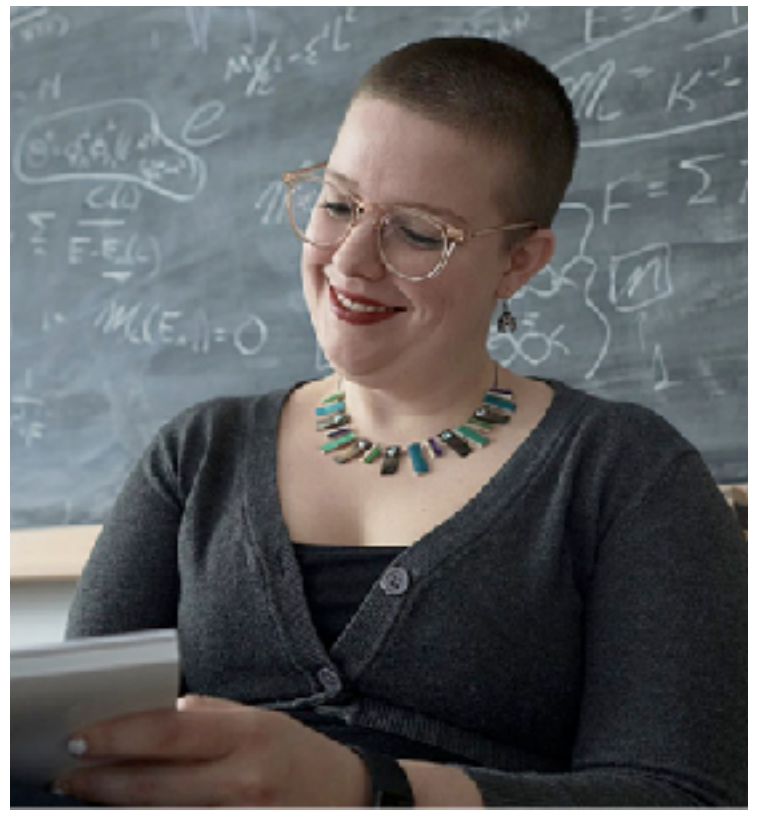
# Conclusions

| | magnetic Kogut-Susskind | electric Kogut-Susskind | prepotentials | Loop-String-Hadron | magnetic maximal-tree | Loop-String-Hadron on flower |
|---|---|---|---|---|---|---|
| **gauge Invariant** | Non-Abelian per vertex | Non-Abelian per vertex | Abelian per link | Abelian per link | Non-Abelian on a single vertex | ✓ |
| **low coupling** | ✓ | ✗ | ✗ | ✗ | ✓ | (✓) |
| **discrete quantum numbers** | ✗ | ✓ | ✓ | ✓ | ✗ | ✓ |
| **local dynamics** | ✓ | ✓ | ✓ | ✓ | ✗ | ✗ |

# Thanks!

Raúl Briceño

Christian Bauer



Dorota Grabowska

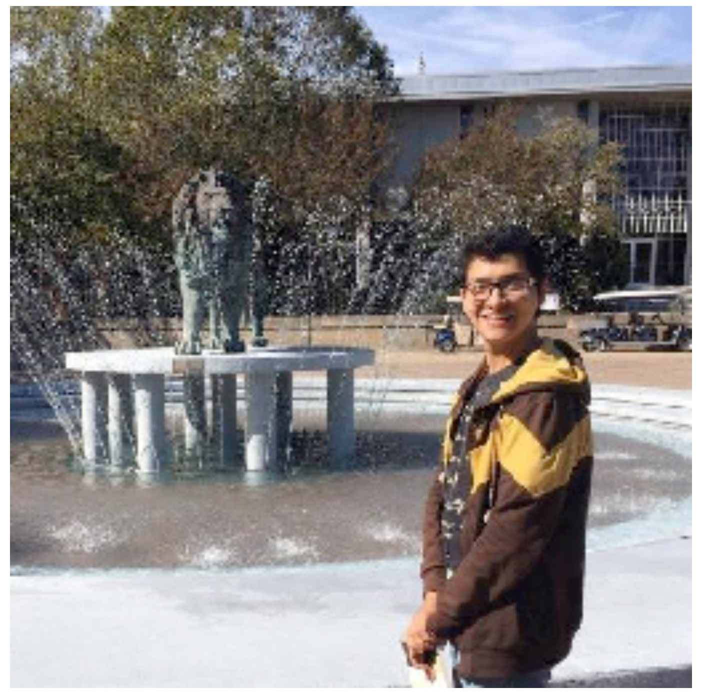
André Walker-Loud

Dimitra Pefkou

Jesse Stryker

Anthony Ciavarella

Irian D'Andrea

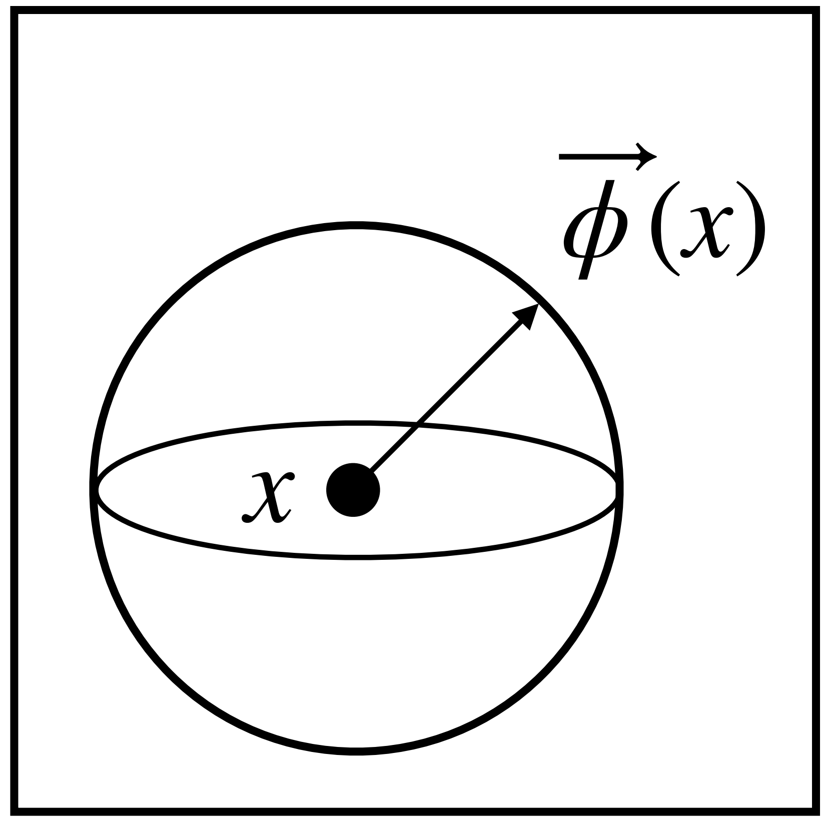


Marco Carrillo

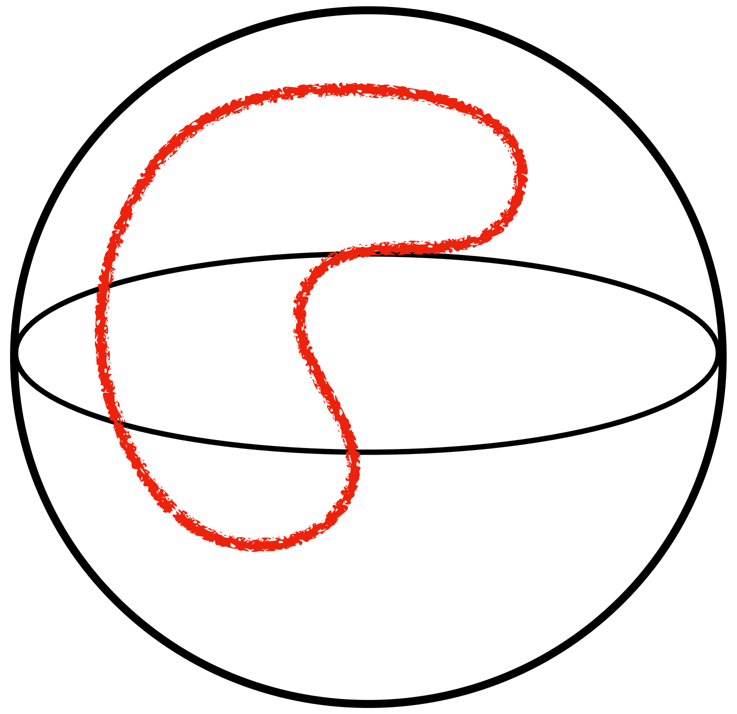
# My other work…
# Recovering Scattering Amplitudes from Monte Carlo Simulations

# O(3) Model Primer

Spacetime or some material




String Theory



$$S = \frac{1}{2g^2}\int d^2x\left(\partial\vec{\phi}\right)^2$$

Properties:

1. Asymptotic freedom: at $E=\infty$ the theory has two free particles.
2. Confinement: at any finite $E$ the two particles above are confined into an $O(3)$ vector multiplet of particles.
3. Dynamical mass generation: classical conformal invariance is broken at the quantum level.
4. Integrable

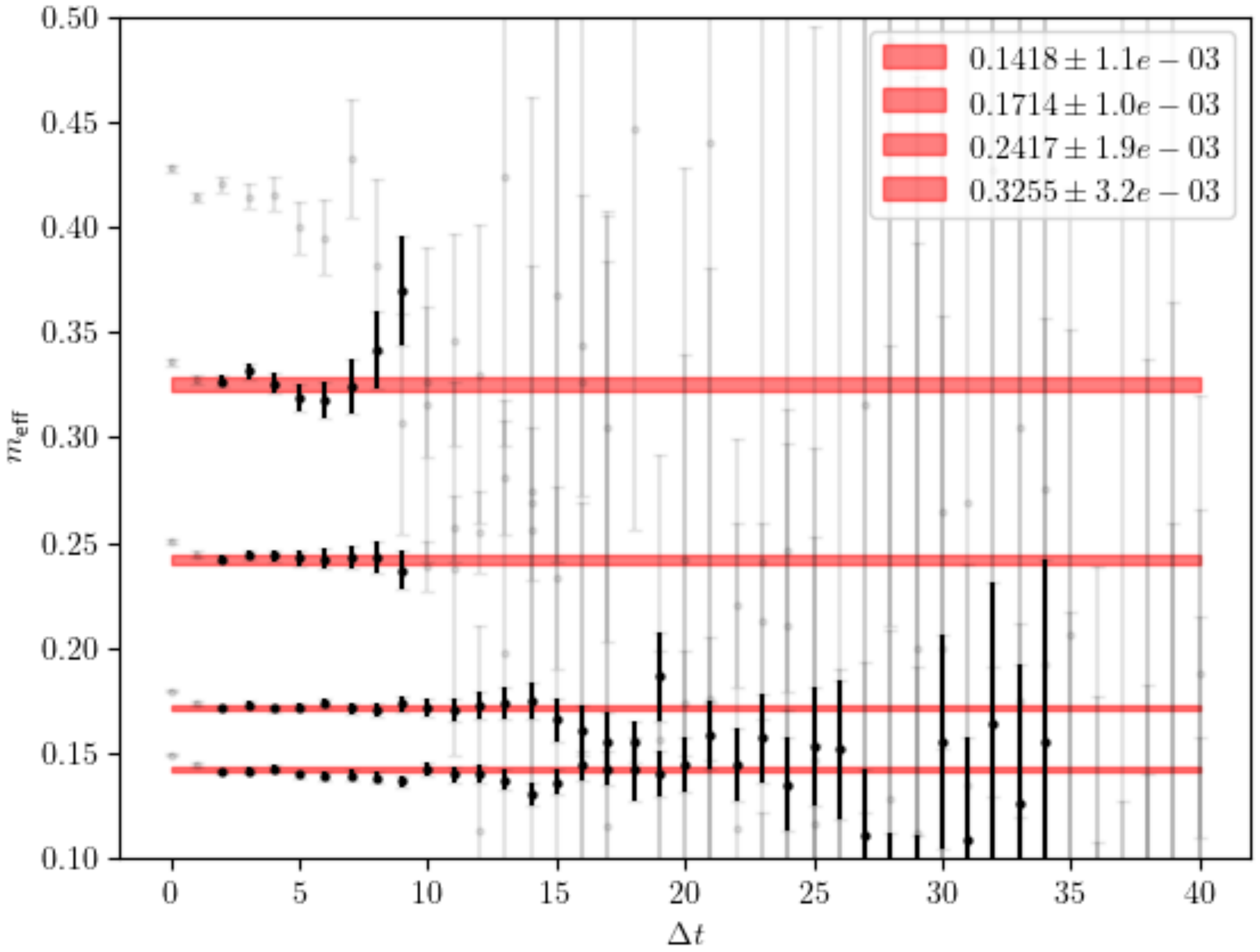
↓

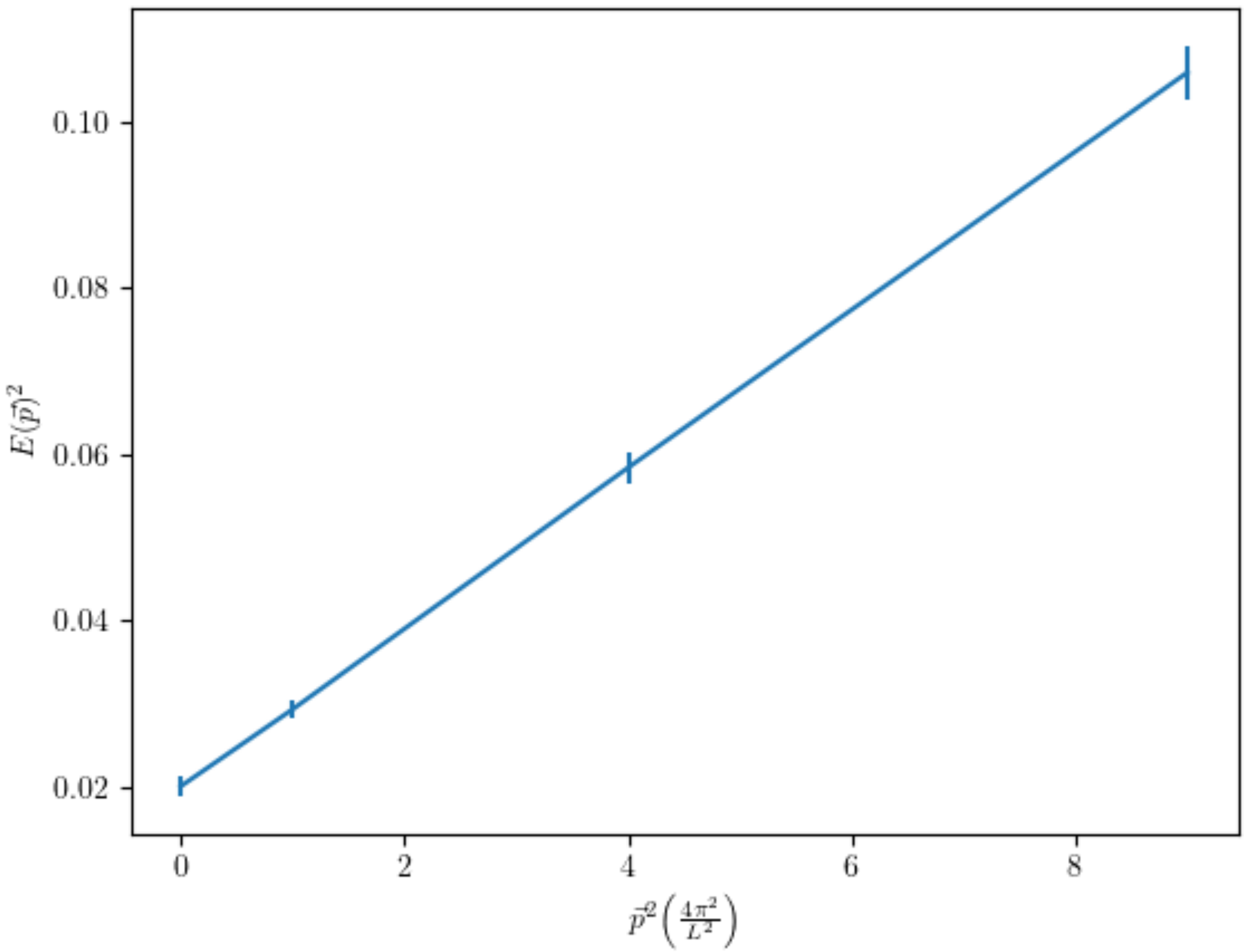
A very interesting model to study Yang-Mills and QCD!

# One Particle

We can recover the one-particle spectrum from the two-point function

$$\left\langle \vec{\phi}(0,-p)\cdot\vec{\phi}(t,p)\right\rangle = \langle 0|\,\vec{\phi}(t,-p)\cdot\vec{\phi}(0,p)\,|0\rangle = \sum_n e^{-E_n t}\,|\langle 0|\,\vec{\phi}(0,-p)\,|n\rangle|^2$$



# Generalized Eigenvalue Problem

**Problem:** Given a state $|n\rangle$ and a list of operators $\{O_a\}$, which linear combination $|\psi\rangle = v^a O_a|0\rangle$ best approximates $|n\rangle$?

**Idea:** Two point functions give us

$$\sum_{a,b} v^a v^{b*} \langle O_a(t) O_b^*(0)\rangle = \langle \psi | e^{-tH} | \psi \rangle = \sum_n e^{-tE_n} |\langle \psi | n \rangle|^2$$

↓ Minimize keeping $\langle \psi | e^{-t_0 H} | \psi \rangle$ constant

$$\langle O_a(t) O_b^*(0)\rangle v^b = \lambda \langle O_a(t_0) O_b^*(0)\rangle v^b$$

$\lambda \to e^{-tE_n}$

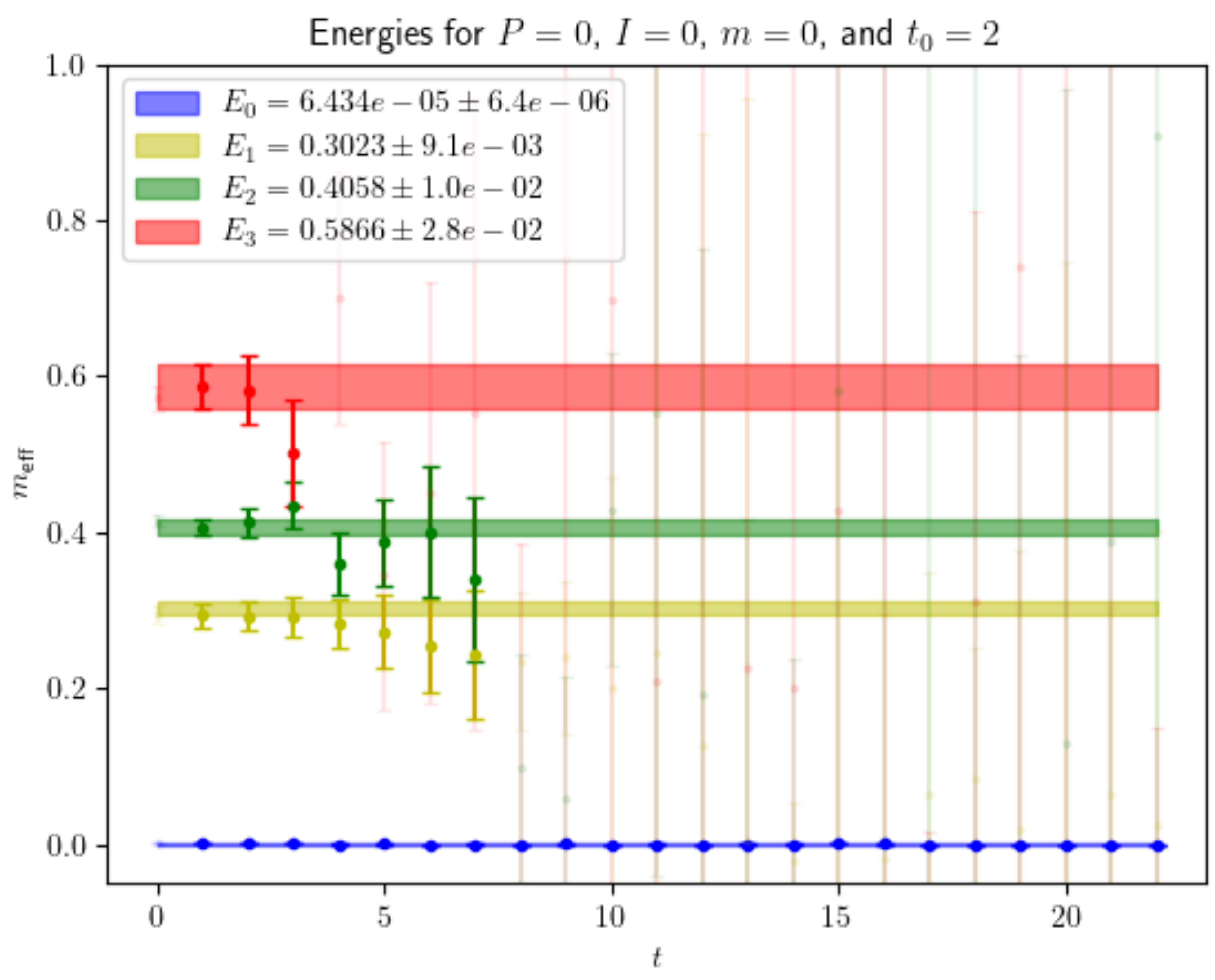
$v^b \to$ "Dual to $\langle 0 | O_a | n \rangle$"

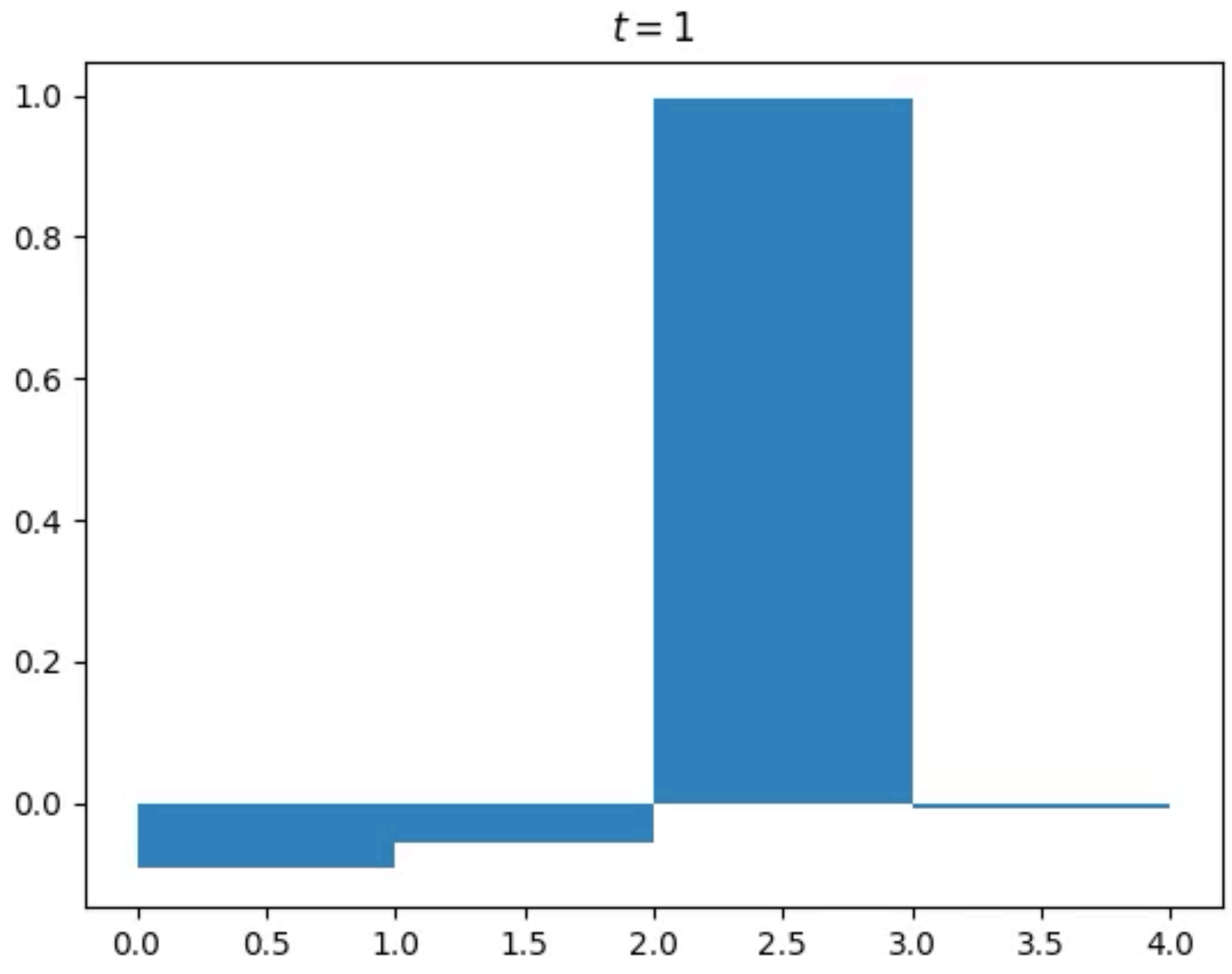
# Two Particle

Operator list:

Scalar, vector or symmetric traceless index

$$\{O^A(t,P,p) = P^A_{ab}\phi^a(t,P+p/2)\phi^b(t,P-p/2)\}_p$$



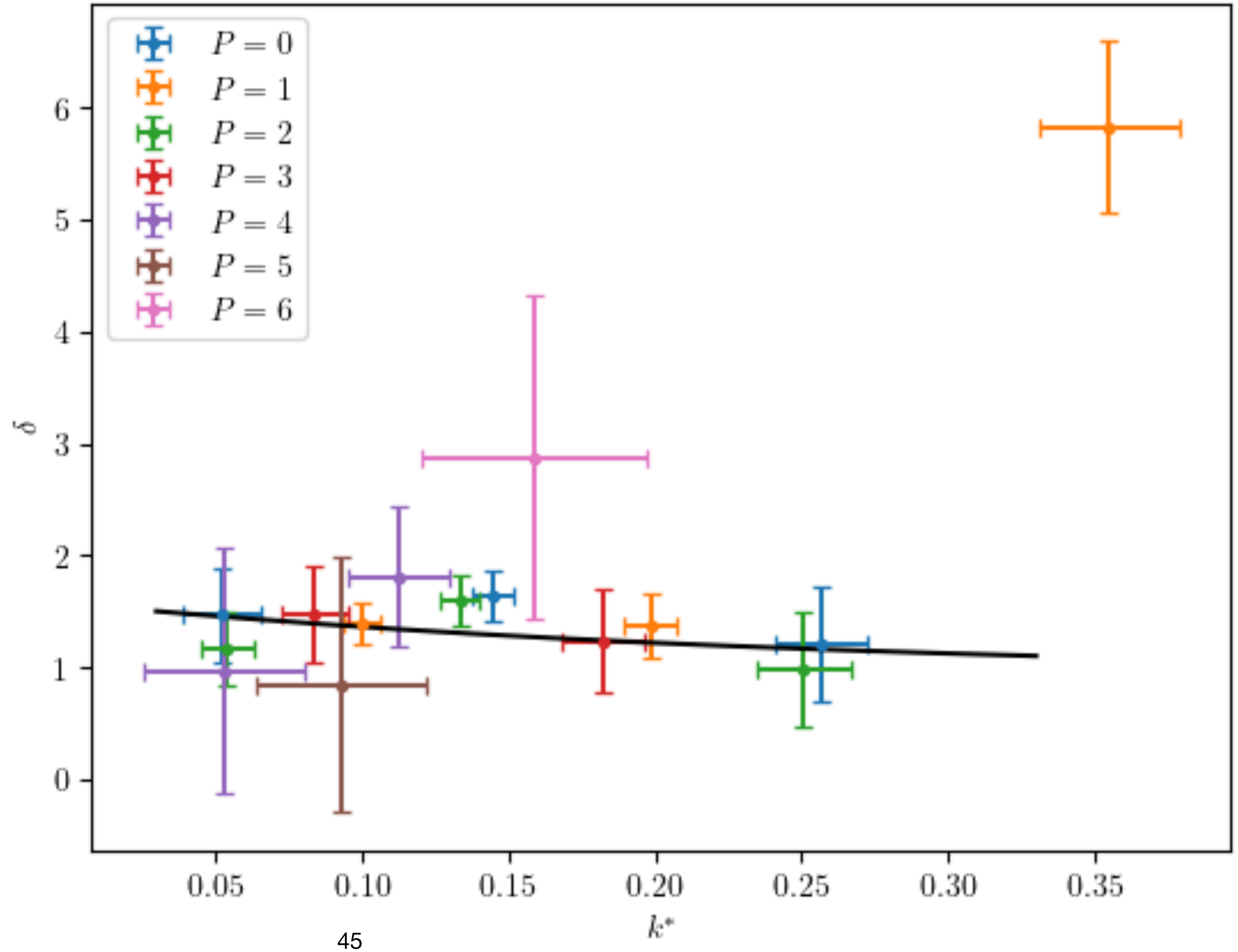
# Lüscher Quantization Condition

## Recovering scattering from finite-volume spectrum

$$\det(\mathscr{M} + F^{-1}) = 0$$

In 1+1 D

$$\delta + \gamma L k^* = (2n + \eta)\pi$$



# One Particle Form Factor

$$\langle p, a \,|\, J^b_\mu(0) \,|\, q, c\rangle = \epsilon^{abc}(p+q)_\mu F(Q^2)$$

LSZ-like arguments

$$|F(Q^2)|^2 = \frac{E_p E_q}{(p+k)^2_\mu} \frac{|\epsilon_{abc}\langle \phi^a(T,p) J^b_\mu(t,-p-q)\phi^b(0,q)\rangle|^2}{\langle \vec{\phi}(2T,p)\cdot\vec{\phi}(2t,-p)\rangle\langle\vec{\phi}(2t,q)\cdot\vec{\phi}(0,-q)\rangle}$$