Manifestation of anomalies in Deep Virtual Compton Scattering

Shohini Bhattacharya

RIKEN BNL/BNL

10 April 2023

In Collaboration with:

Yoshitaka Hatta (BNL)
Werner Vogelsang (Tubingen U.)

Based on:

arXiv:2210.13419 , In preparation

Jefferson Lab

Theory Seminars

Outline

- Chiral & trace anomalies in QCD
- Anomaly in polarized DIS & proton's spin puzzle : History
- QCD Compton Scattering: Calculation of box diagrams
    - Polarized case
    - Unpolarized case

Chiral anomaly

Recap on chiral anomaly in QCD:

- **Lagrangian invariant under global chiral rotation** $\psi \to e^{i\alpha\gamma_5}\psi$
- **Axial-vector current:** $J_5^\mu = \sum_f \bar{\psi}_f \gamma^\mu \gamma_5 \psi_f$

Chiral anomaly

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- **Axial-vector current:** $J_5^\mu = \sum_f \bar{\psi}_f \gamma^\mu \gamma_5 \psi_f$
- **But measure of the path integral is not invariant, which breaks the conservation of the axial current**

K. Fujikawa, PRL 1979

Chiral anomaly

Anomaly equation:

$$\partial_\mu J_5^\mu = -\frac{n_f \alpha_s}{4\pi} F^{\mu\nu} \tilde{F}_{\mu\nu} \qquad \tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$$

A fundamental property of axial-vector current is the anomaly equation

Chiral anomaly

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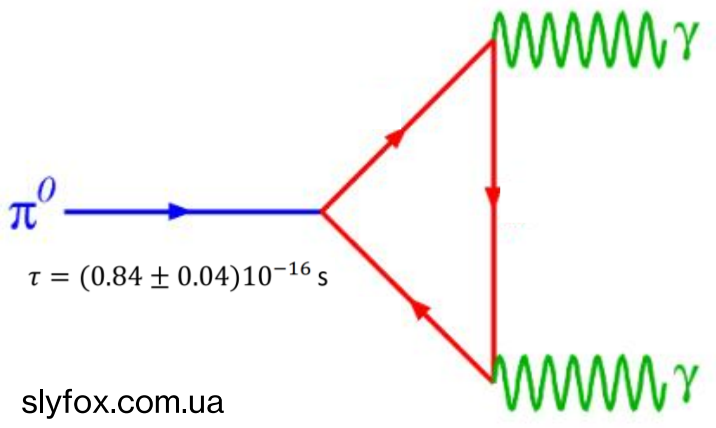
Adler – Bell - Jackiw chiral anomaly

Famous example: ABJ anomaly contribution to $\pi^0 \to 2\gamma$



In the chiral limit, without the anomaly,

π^0 **does not decay!**

Chiral anomaly

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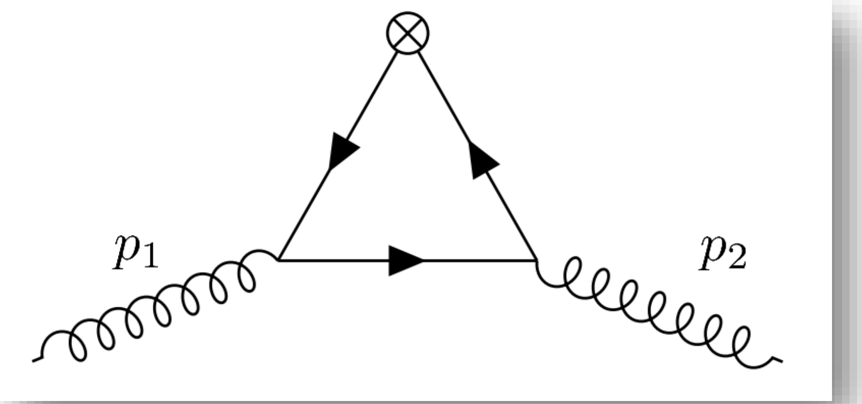
A perturbative solution to anomaly equation:



Calculation in off-forward kinematics $(l = p_2 - p_1)$**:**

$$\langle p_2 | J_5^\mu | p_1 \rangle = \frac{n_f \alpha_s}{4\pi} \frac{i l^\mu}{l^2} \langle p_2 | F_a^{\alpha\beta} \tilde{F}^a_{\alpha\beta} | p_1 \rangle$$

Triangle diagram is dominated by infra-red pole

Chiral anomaly

Axial Form Factors:

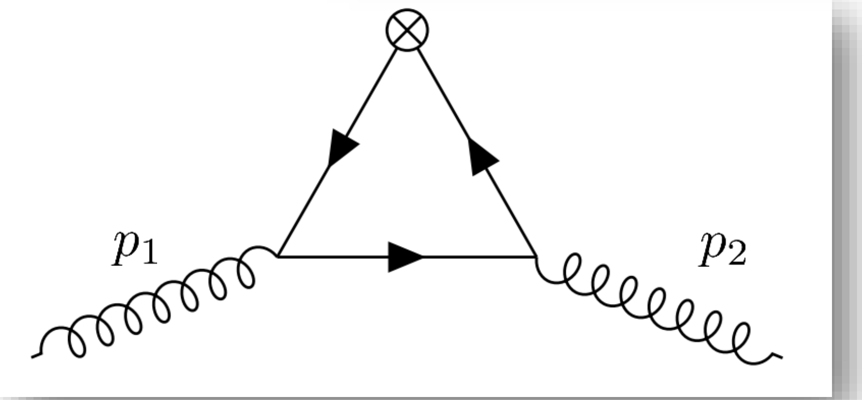
$$\langle P_2|J_5^\mu|P_1\rangle = \bar{u}(P_2)\left[\gamma^\mu\gamma_5 g_A(l^2) + \frac{l^\mu\gamma_5}{2M} g_P(l^2)\right]u(P_1)$$

A fundamental property of axial-vector current is the anomaly equation

Massless pole in pseudo scalar Form Factor? $g_P(l^2) \sim \frac{1}{l^2}$

A perturbative solutio

$g_A(0) = \Delta\Sigma$ **: Fraction of proton spin carried by quarks**



Calculation in off-forward kinematics $(l = p_2 - p_1)$**:**

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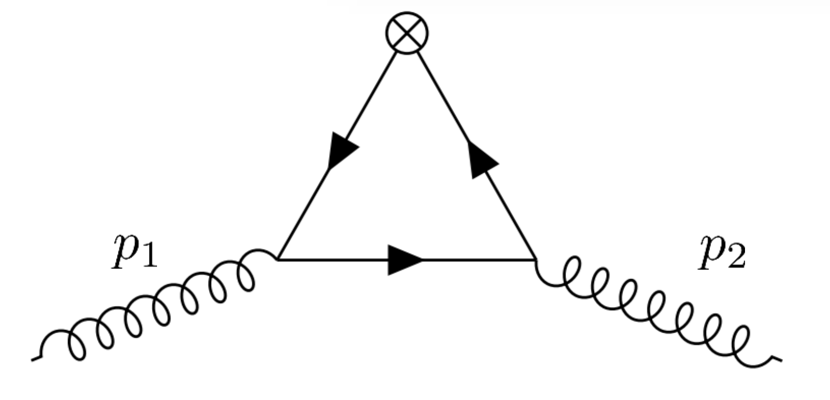
In QCD, we expect: $g_P(l^2) \sim \frac{1}{l^2 - m_{\eta'}^2}$

eta meson mass generation

A perturbative solutio

$g_A(0) = \Delta\Sigma$: Fraction of proton spin carried by quarks



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Chiral anomaly

Taking divergence of axial-vector matrix element:

$$2Mg_A(l^2) + \frac{l^2}{2M} g_P(l^2) = \frac{i\langle P_2|\frac{n_f\alpha_s}{4\pi}F\tilde{F}(l^2)|P_1\rangle}{\bar{u}(P_2)\gamma_5 u(P_1)}$$

Massless pole in pseudo scalar Form Factor? $g_P(l^2) \sim \frac{1}{l^2}$

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Pole cancellation at Form Factor level:

...ction of proton spin carried by quarks

$$\frac{g_P(l^2)}{2M} = -\frac{2Mg_A(l^2)}{l^2} + \frac{i\langle P_2|\frac{n_f\alpha_s}{4\pi}F\tilde{F}(l^2)|P_1\rangle}{l^2\bar{u}(P_2)\gamma_5 u(P_1)}$$

$$\approx -\frac{i}{l^2}\left(\frac{\langle P_2|\frac{n_f\alpha_s}{4\pi}F\tilde{F}|P_1\rangle}{\bar{u}(P_2)\gamma_5 u(P_1)}\Bigg|_{l^2=0} - \frac{\langle P_2|\frac{n_f\alpha_s}{4\pi}F\tilde{F}|P_1\rangle}{\bar{u}(P_2)\gamma_5 u(P_1)}\right)$$

...ion in off...rward kinematics $(l = p_2 - p_1)$:

$$\ldots 1\rangle = \frac{n_f\alpha_s}{4\pi}\frac{il^\mu}{l^2}\langle p_2|F_a^{\alpha\beta}\tilde{F}^a_{\alpha\beta}|p_1\rangle$$

Same pole as what one naively gets from perturbation theory

Trace anomaly

Recap on trace anomaly in QCD:

- **Lagrangian invariant under scale transformation** $x^\mu \to e^\sigma x^\mu \quad \phi \to e^{-D\sigma}\phi$

- **Dilatation current:** $D^\mu = \Theta^{\mu\nu} x_\nu$

Energy Momentum Tensor (EMT)

$$\Theta^{\mu\nu} = -F^{\mu\lambda}F^{\nu}{}_{\lambda} + \frac{\eta^{\mu\nu}}{4}F^2 + i\bar{\psi}\gamma^{(\mu}\overleftrightarrow{D}^{\nu)}\psi$$

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- **Dilatation current:** $D^\mu = \Theta^{\mu\nu} x_\nu$ $\Theta^{\mu\nu}$ **: Energy Momentum Tensor (EMT)**

- **Conformal symmetry explicitly broken by quantum effects**

$$\partial_\mu D^\mu = \Theta^\mu_\mu \neq 0$$

Trace anomaly

Recap on trace anomaly in QCD:

- A quantum anomaly in the trace of its energy momentum tensor (conformal anomaly) breaks conformal invariance

Trace anomaly:

$$\Theta^{\mu}_{\mu} = \frac{\beta(g)}{2g} F^{\mu\nu} F_{\mu\nu}$$

$\Theta^{\mu\nu}$: Energy Momentum Tensor (EMT)

Trace anomaly

Recap on trace anomaly in QCD:

Major scientific goal of EIC: in the trace of its ener... ...aly)

...variance

How does the mass of the nucleon arise?

$$\Theta^{\mu}_{\mu} = \frac{\beta(g)}{2g} F^{\mu\nu} F_{\mu\nu}$$

$\Theta^{\mu\nu}$: Energy Momentu...

Efforts detailed in a decade worth of reports:

Fundamentally important in QCD: Trace anomaly is the origin of hadron masses

$$\langle P|\Theta^{\mu}_{\mu}|P\rangle = 2M^2$$

Trace anomaly

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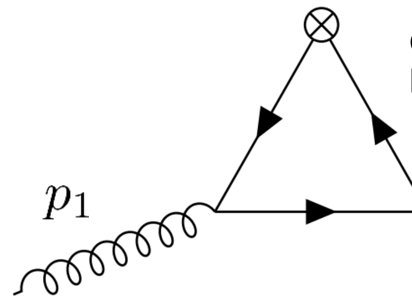
A perturbative solution to anomaly equation:



Giannotti, Mottola (2009)

Calculation in off-forward kinematics $(l = p_2 - p_1)$**:**

$$\langle p_2 | \Theta^{\mu\nu}_{\rm QED} | p_1 \rangle = -\frac{e^2}{24\pi^2 l^2} \left(p^\mu p^\nu + \frac{l^\mu l^\nu - l^2 g^{\mu\nu}}{4} \right) \langle p_2 | F^{\alpha\beta} F_{\alpha\beta} | p_1 \rangle$$

Triangle diagram is dominated by infra-red pole

Trace anomaly

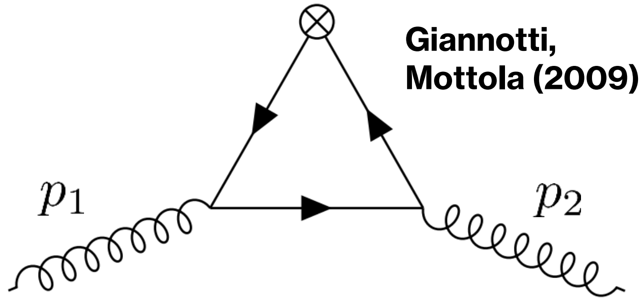
Gravitational Form Factors:

$$\langle P_2|\Theta_f^{\mu\nu}|P_1\rangle = \frac{1}{M}\bar{u}(P_2)\left[P^\mu P^\nu A_f + (A_f+B_f)\frac{P^{(\mu}i\sigma^{\nu)\rho}l_\rho}{2} + \frac{D_f}{4}(l^\mu l^\nu - g^{\mu\nu}l^2) + M^2\bar{C}_f g^{\mu\nu}\right]u(P_1)$$

Massless poles in Gravitational Form Factors?

$$A_f(l^2), B_f(l^2), D_f(l^2) \sim \frac{1}{l^2}$$

A perturbative solution to anomaly equation:



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Massless poles in Gravitational Form Factors?

$$A_f(l^2), B_f(l^2), D_f(l^2) \sim \frac{1}{l^2}$$

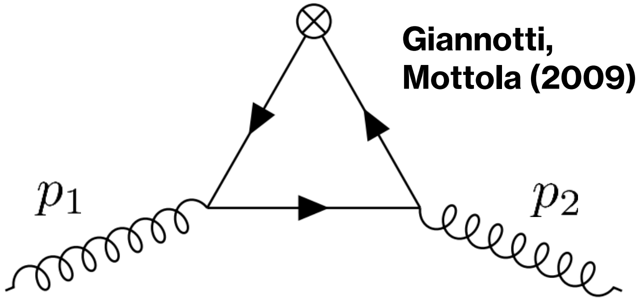
In QCD, we expect:

$$\frac{1}{l^2} \to \frac{1}{l^2 - m_G^2}$$

EMT)

glueball mass generations

A perturbative solution to anomaly equation:



Giannotti, Mottola (2009)

Calculation in off ... $p_1)$**:**

Fujita, Hatta, Sugimoto, Ueda (2022)
Mamo, Zahed (2019)

$$\langle p_2|\Theta^{\mu\nu}_{\rm QED}|p_1\rangle = -\frac{e^2}{24\pi^2 l^2}\left(p^\mu p^\nu + \frac{l^\mu l^\nu - l^2 g^{\mu\nu}}{4}\right)\langle p_2|F^{\alpha\beta}F_{\alpha\beta}|p_1\rangle$$

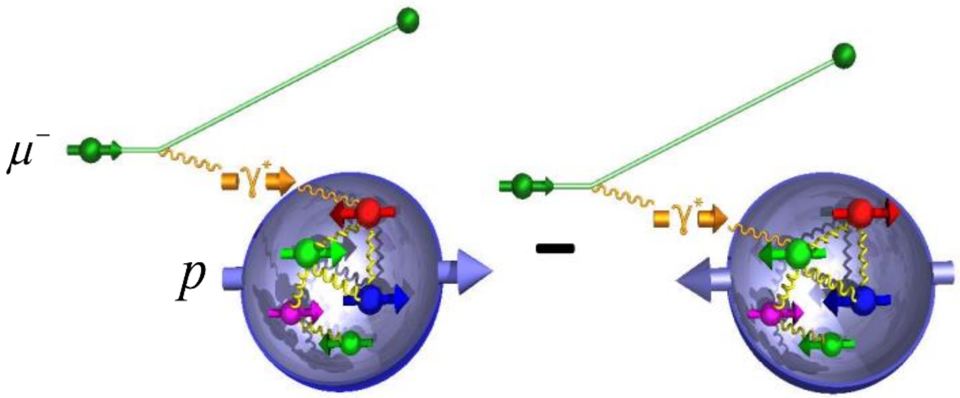
Triangle diagram is dominated by infra-red pole

Anomaly in polarized DIS: History

The role of chiral anomaly in polarized DIS is a well-known old story

Anomaly in polarized DIS: History

Polarized DIS & proton's spin puzzle:



g_1 **can be extracted from longitudinal double spin asymmetry:**

$$A_{LL} = \frac{\mu^\uparrow p^\downarrow - \mu^\uparrow p^\uparrow}{\mu^\uparrow p^\uparrow + \mu^\uparrow p^\downarrow} \sim \left(1 + \frac{\sigma_L}{\sigma_T}\right)\frac{2x g_1}{F_2}$$

First moment of g_1**:**

$$\Delta\Sigma$$

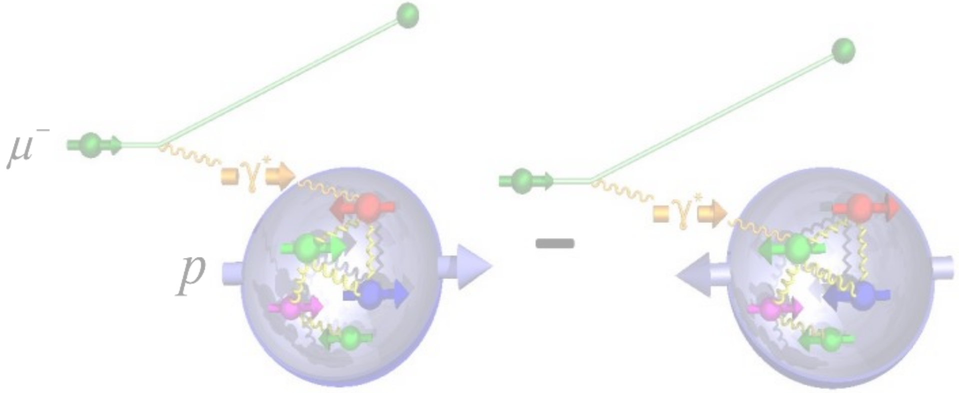
$g_A(0) = \Delta\Sigma$ **: Fraction of proton spin carried by quarks**

$$\int_0^1 dx g_1(x) = \frac{1}{9}(\Delta u + \Delta d + \Delta s) + \frac{1}{12}(\Delta u - \Delta d) + \frac{1}{36}(\Delta u + \Delta d - 2\Delta s) + \mathcal{O}(\alpha_s)$$

Deep inelastic scattering (DIS) experiments showed that quarks carry only about 30% of the proton's spin: $\Delta\Sigma \approx 0.32$, which is much smaller than predicted by the quark model $\Delta\Sigma \sim 1$ - spin puzzle

Anomaly in polarized DIS: History

Polarized DIS & proton's spin puzzle:



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$$+ \frac{1}{12}(\Delta u \quad 1$$

$$+ \mathcal{O}(\alpha_s)$$

Calculate "box diagram", which is a controversial diagram

Deep inelastic scattering (DIS) expe that
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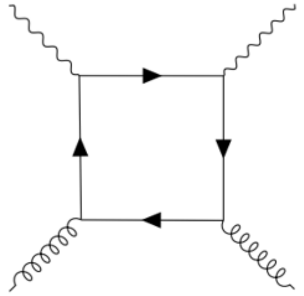
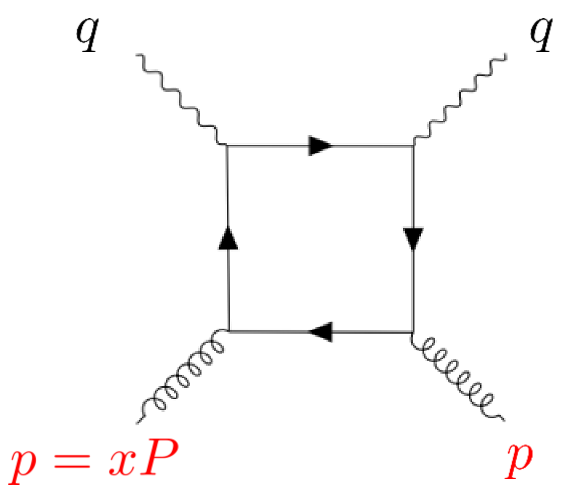


Anomaly in polarized DIS: History



Box diagram

One-loop correction to g_1 (gluon channel):

$$g_1(x) \sim \frac{\alpha_s}{2\pi}\left(\ln\frac{Q^2}{m_q^2}\Delta P_{qg}(x) + \delta C_{qg}(x)\right) \otimes \Delta G(x)$$

Polarized DGLAP splitting function: $\Delta P_{qg}(x) = 2x - 1$

Hard coefficient function (mass regularization):

A lot of controversy over this term in the past

$$\delta C_{qg}(x) = (2x-1)\left(\ln\frac{1-x}{x} - 1\right) + 2(1-x)$$

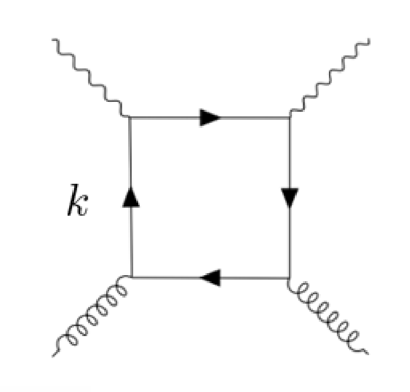
(same result in DR)

Anomaly in polarized DIS: History

A solution of spin problem?

The "$1-x$" comes from the infrared region of the box diagram:

$$(1-x)\int_0^{Q^2} dk_\perp^2 \frac{m_q^2}{(k_\perp^2+m_q^2)^2} = \text{finite!}$$



$p = xP$ $\quad p$

But the coefficient function is supposed to be dominated by UV physics ...

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Anomaly in polarized DIS: History

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The "$1-x$" comes from the infrared region of the box dia

THE ANOMALOUS GLUON CONTRIBUTION TO POLARIZED LEPTOPRODUCTION

G. ALTARELLI and G.G. ROSS [1]
CERN, CH-1211 Geneva 23, Switzerland

Received 29 June 1988

$p = xP$ $\quad p$

But the coefficient function is supposed to be dominated by

THE ROLE OF THE AXIAL ANOMALY IN MEASURING SPIN-DEPENDENT PARTON DISTRIBUTIONS

R.D. CARLITZ
Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, PA 15260, USA

J.C. COLLINS
Department of Physics, Illinois Institute of Technology, Chicago, IL 60616, USA

and

A.H. MUELLER
Department of Physics, Columbia University, New York, NY 10027, USA

Received 22 August 1988

Consider this "anomalous" contribution as a part of "intrinsic spin":

$$\Delta\tilde{\Sigma} = \Delta\Sigma + \frac{n_f \alpha_s}{2\pi}\Delta G$$

lot of controversy over this term
the past

$$-1\Big) + 2(1-x)$$

Expect $\Delta\tilde{\Sigma} \sim 1$

If ΔG is large & positive, this can explain the smallness of $\Delta\Sigma$!

Anomaly in polarized DIS: History

Critique 1

The "$1-x$" comes from the infrared region of the box diagram:

Gluonic contribution to g_1 and its relationship to the spin-dependent parton distributions

Geoffrey T. Bodwin and Jianwei Qiu*

usual forms of the quark sum rules. We conclude that the size of the gluonic contribution to the first moment of g_1 is entirely a matter of the convention used in defining the quark distributions.

Bu

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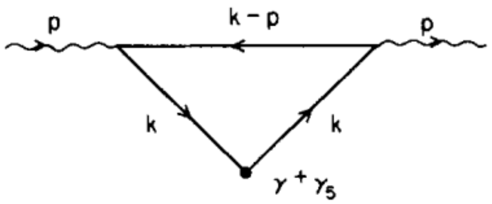
Anomaly in polarized DIS: History

Critique 2

Calculation in forward kinematics:

No infrared pole!



$$\frac{1}{2p^+}\langle p, \pm | j_5^+ | p, \pm \rangle = \mp \frac{\alpha_s N_f}{2\pi}$$

(Carlitz, Collins, Mueller)

$(1-x)\int_0^{Q^2} d\ldots \frac{\ldots}{(k_\perp^2+m_q^2)^2}$

But the coefficient function is supposed to be

Polarized DGLAP splitting function:

THE g_1 PROBLEM: DEEP INELASTIC ELECTRON SCATTERING AND THE SPIN OF THE PROTON*

R.L. JAFFE and Aneesh MANOHAR**

not local in the sense required. It is therefore not necessarily protected from infrared effects, in particular from the non-perturbative effects which give the η' a mass★.

regularization-independent. In QCD, the pole at $l^2 = 0$ is unphysical and is cancelled by non-triangle contributions to the matrix element of A_μ^0. With the aid

Jaffe, Manohar (1990)

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Triangle diagram is dominated by infra-red pole

Anomaly in polarized DIS: History

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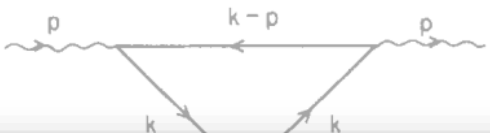

$$\frac{1}{2p^+}\langle p,\pm|j_5^+|p,\pm\rangle = \mp\frac{\alpha_s N_f}{2\pi}$$

Box diagram can be viewed as a non-local generalization of triangle diagram

If triangle is dominated by anomaly pole, trace of that should be visible in box diagram

...TTERING
AND THE SPIN OF THE PROTON*

R.L. JAFFE and Aneesh MANOHAR**

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Jaffe, Manohar (1990)

Imprint of Anomalies in DIS

First calculation of box diagram with $l^2 \neq 0$:

Anomaly equation:

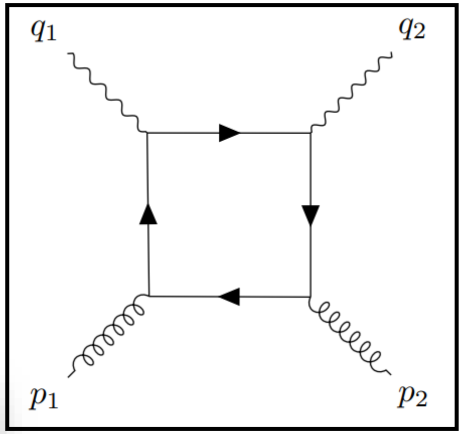
The role of the chiral anomaly in polarized deeply inelastic scattering I: Finding the triangle graph inside the box diagram in Bjorken and Regge asymptotics

Andrey Tarasov[1,2] and Raju Venugopalan[3]

A fundamental prop

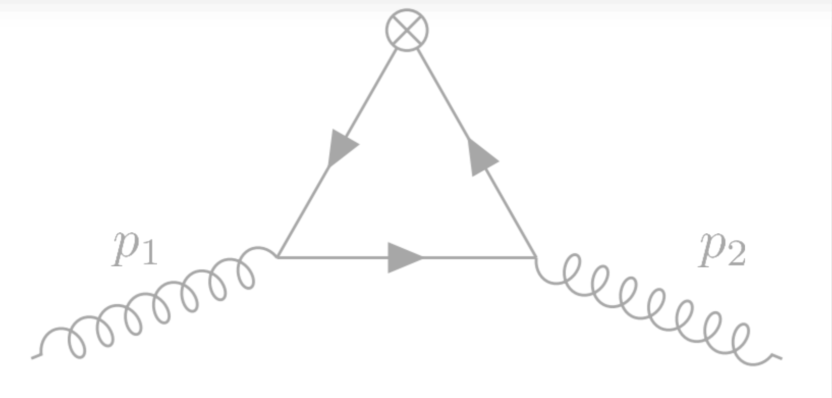
The role of the chiral anomaly in polarized deeply inelastic scattering II: Topological screening and transitions from emergent axion-like dynamics

Andrey Tarasov[1,2] and Raju Venugopalan[3]



Box diagram

Andrey & Raju demonstrated within world-line formalism that to capture the physics of anomaly we need to calculate everything in off-forward kinematics for polarized DIS



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Triangle diagram is dominated by infra-red pole

Imprint of Anomalies in QCD Compton scattering

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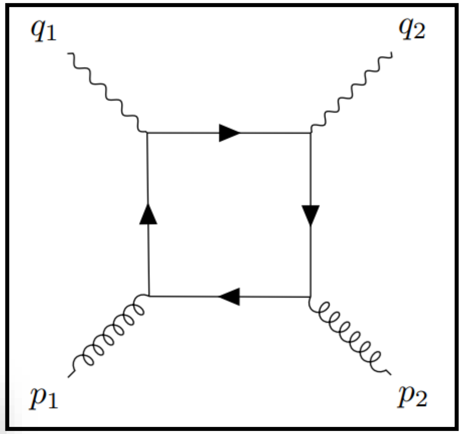

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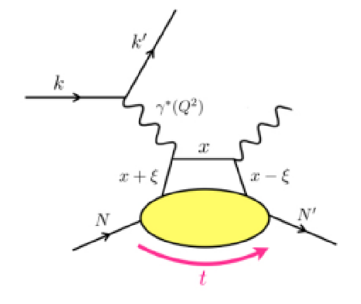


arXiv: 2210.13419 (2022)

Chiral and trace anomalies in Deeply Virtual Compton Scattering

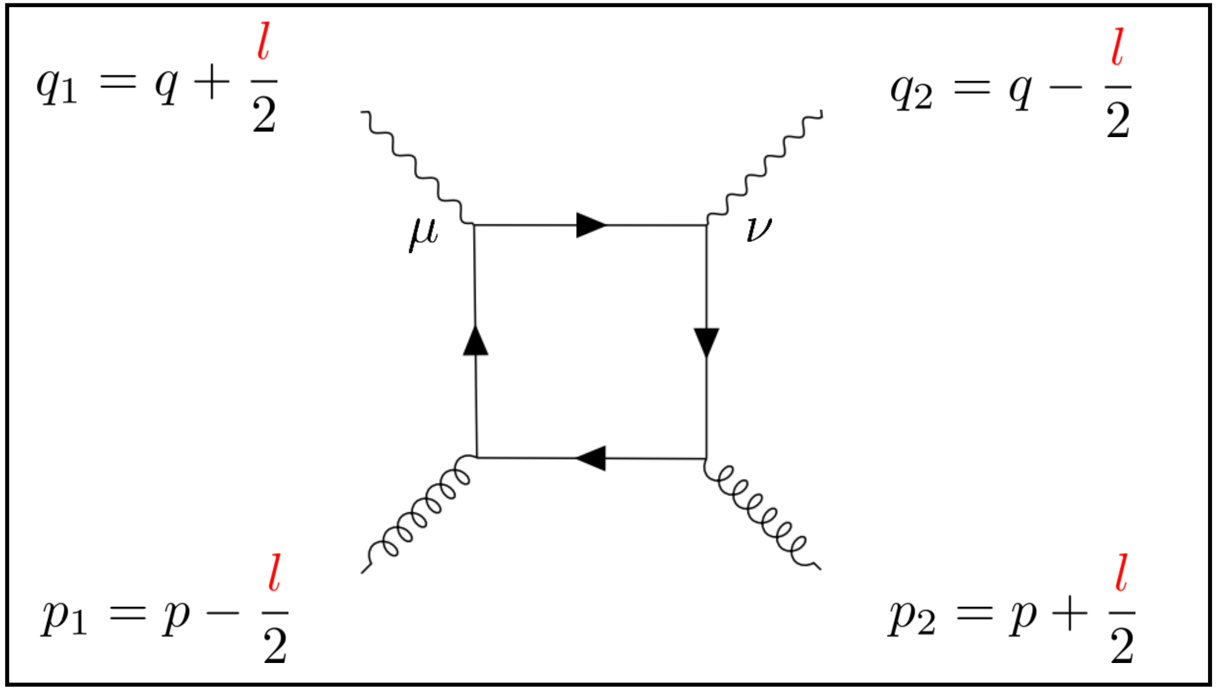
Shohini Bhattacharya,[1,*] Yoshitaka Hatta,[1,2,†] and Werner Vogelsang[3,‡]

$(l = p_2 - p_1)$:

We explored the physics of anomaly in DVCS using Feynman-diagram approach

Imprint of Anomalies in QCD Compton scattering

Kinematics:



$$t = l^2 \neq 0$$

Calculation of imaginary part of anti-symmetric/symmetric (μ, ν) of Compton amplitude with non-zero t

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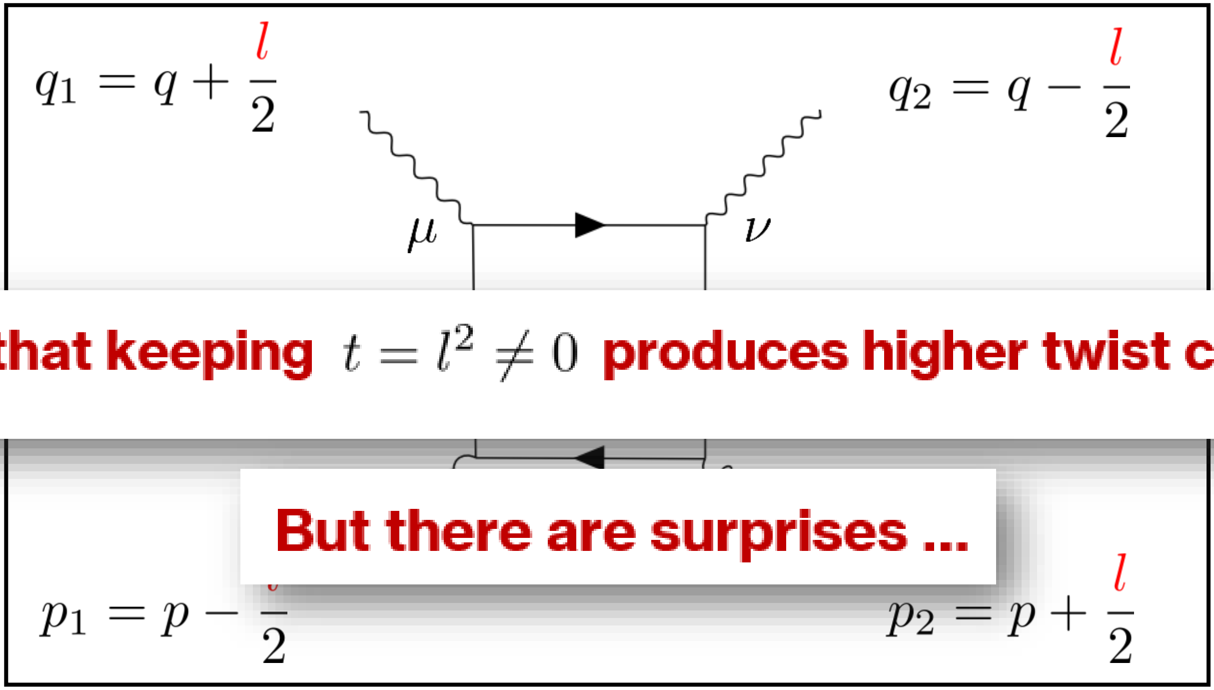

Usual rationale is that keeping $t = l^2 \neq 0$ produces higher twist corrections $\sim \frac{t}{Q^2}$

But there are surprises ...

Calculation of imaginary part of anti-symmetric/symmetric (μ, ν) of Compton amplitude with non-zero t

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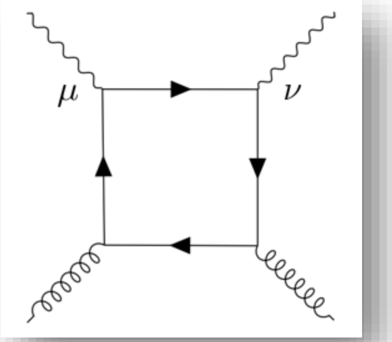

Antisymmetric part of Compton amplitude

$$-\epsilon^{\alpha\beta\mu\nu}P_\beta \mathrm{Im}T^{\mathrm{asym}}_{\mu\nu}$$

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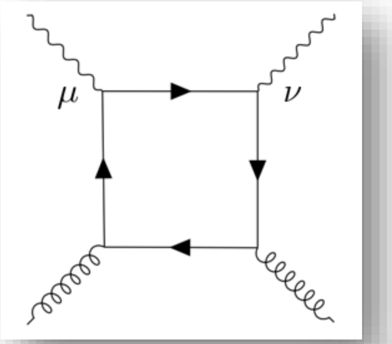


Antisymmetric part of Compton amplitude

$$-\epsilon^{\alpha\beta\mu\nu} P_\beta \mathrm{Im} T^{\mathrm{asym}}_{\mu\nu} \approx \frac{1}{2}\frac{\alpha_s}{2\pi}\left(\sum_f e_f^2\right)\bar{u}(P_2)\left[\left(\Delta P_{qg}\ln\frac{Q^2}{-l^2}+\delta C_g^{\mathrm{off}}\right)\otimes \Delta G(x_B)\gamma^\alpha\gamma_5 + \frac{l^\alpha}{l^2}\delta C_g^{\mathrm{anom}}\otimes \tilde{\mathcal{F}}(x_B)\gamma_5\right] u(P_1)$$

Polarized gluon distribution

Expected terms:

Splitting function $\Delta P_{qg}(\hat{x}) = 2T_R(2\hat{x}-1)$

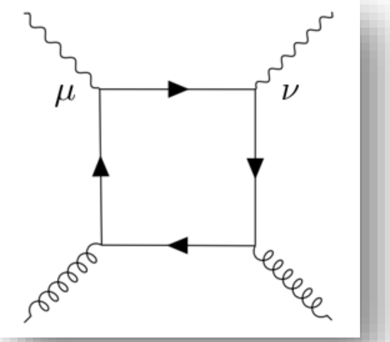
Coefficient function $\delta C_g^{\mathrm{off}}(\hat{x}) = 2T_R(2\hat{x}-1)\left(\ln\frac{1}{\hat{x}(1-\hat{x})}-1\right)$

Recall: In DR, one obtains

$$\Delta P_{qg}\frac{-1}{\epsilon} + \delta C_g^{\overline{\mathrm{MS}}}$$

$$\delta C_g^{\overline{\mathrm{MS}}}(\hat{x}) = 2T_R(2\hat{x}-1)\left(\ln\frac{1-\hat{x}}{\hat{x}}-1\right) + 4T_R(1-\hat{x})$$

Imprint of Anomalies in QCD Compton scattering



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Pole term
In agreement with Tarasov, Venugopalan

Coefficient function $\delta C_g^{\mathrm{anom}}(\hat{x}) = 4T_R(1-\hat{x})$

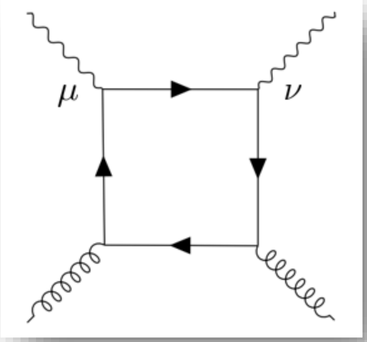
Same controversial "1-x" term as in previous calculations! (see earlier slide)

Twist-4 GPD:

$$\tilde{\mathcal{F}}(x,l^2) = \frac{iP^+}{\bar{u}(P_2)\gamma_5 u(P_1)}\int\frac{dz^-}{2\pi}e^{ixP^+z^-}\langle P_2|F_a^{\mu\nu}(-z^-/2)\tilde{F}^a_{\mu\nu}(z^-/2)|P_1\rangle$$

(Non-local) chiral anomaly manifests itself in high energy scattering amplitude

Imprint of Anomalies in QCD Compton scattering



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Twist-4 GPD

But no suppression in $1/Q^2$!

The QCD factorization theorem: Collins, Freund; Ji, Osborne (1998)

$$-\epsilon^{\alpha\beta\mu\nu}P_\beta \mathrm{Im}T^{\mathrm{asym}}_{\mu\nu} = \frac{1}{2}\sum_f e_f^2\bar{u}(P_2)\left[\gamma^\alpha\gamma_5(\tilde{H}_f(x_B,\xi,l^2)+\tilde{H}_f(-x_B,\xi,l^2))+\frac{l^\alpha\gamma_5}{2M}(\tilde{E}_f^{\mathrm{bare}}(x_B,\xi,l^2)+\tilde{E}_f^{\mathrm{bare}}(-x_B,\xi,l^2))\right]u(P_1)$$
$$+\mathcal{O}(\alpha_s)+\mathcal{O}(1/Q^2)\,,$$

Twist-2 GPDs to all orders

(Non-local) chiral anomaly manifests itself in high energy scattering amplitude & apparently breaks QCD factorization

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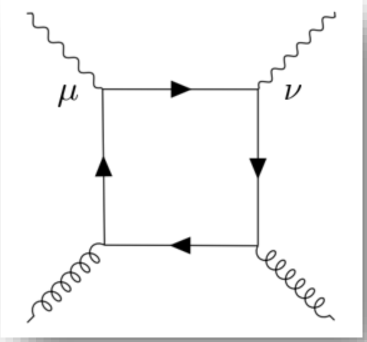

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Anomalous contribution to GPD $\tilde{E}$ at one loop

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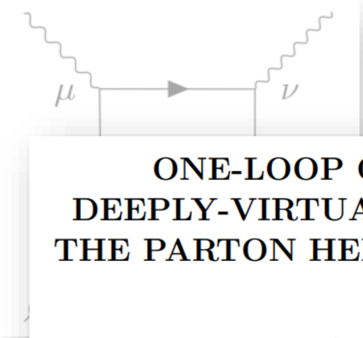

Antisymmetric part ...

Elusive pole

$$\sum_f e_f^2 \Big) \bar{u}(P_2)\left[\left(\Delta P_{qg}\ln\frac{Q^2}{-l^2}+\delta C_g^{\rm off}\right)\otimes\Delta G(x_B)\gamma^\alpha\gamma_5+\frac{l^\alpha}{l^2}\delta C_g^{\rm anom}\otimes\tilde{\mathcal{F}}(x_B)\gamma_5\right]u(P_1)$$

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ONE-LOOP QCD CORRECTIONS TO DEEPLY-VIRTUAL COMPTON SCATTERING: THE PARTON HELICITY-INDEPENDENT CASE

Xiangdong Ji and Jonathan Osborne

..., Osborne (1998)

Predictions from conformal algebra for the deeply virtual Compton scattering.

A.V. Belitsky D. Müller

$$\ldots \tilde{H}_f(x_B,\xi,l^2)+\tilde{H}_f(-x_B,\xi,l^2)\big)+\frac{l^\alpha\gamma_5}{2M}\big(\tilde{E}_f^{\rm bare}(x_B,\xi,l^2)+\tilde{E}_f^{\rm bare}(-x_B,\xi,l^2)\big)\Big]u(P_1)$$

NLO Corrections to Deeply-Virtual Compton Scattering *

L. Mankiewicz[†a], G. Piller[a], E. Stein[b], M. Vänttinen[a] and T. Weigl[a]

Twist-2 GPDs

... chiral anomaly manifests itself in high energy scattering amplitude &

... ation

NLO corrections to timelike, spacelike and double deeply virtual Compton scattering.

B. Pire[1] and L. Szymanowski[2] and J. Wagner[2]

Imprint of Anomalies in QCD Compton scattering

Elusive pole

Antisymmetric part

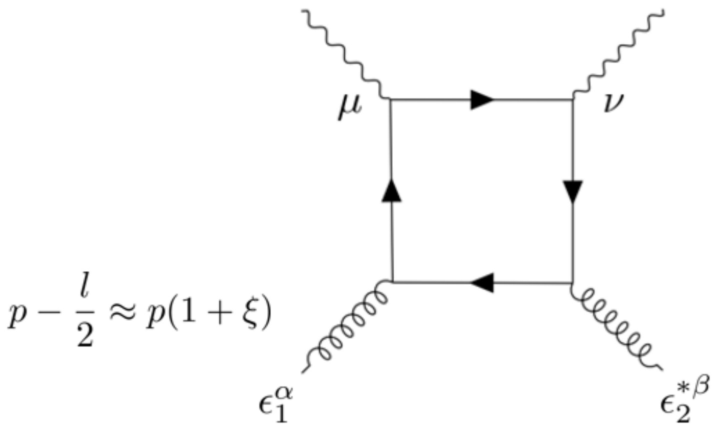
ONE-LOOP QCD CORRECTIONS TO

Pole was unnoticed in the GPD literature because one typically assumes

$$l^\mu = -2\xi p^\mu \;\to\; t = l^2 = 0$$

before loop integration

Usual rationale: Corrections supposedly higher twist $\dfrac{t}{Q^2}$



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Twist-2 GPDs

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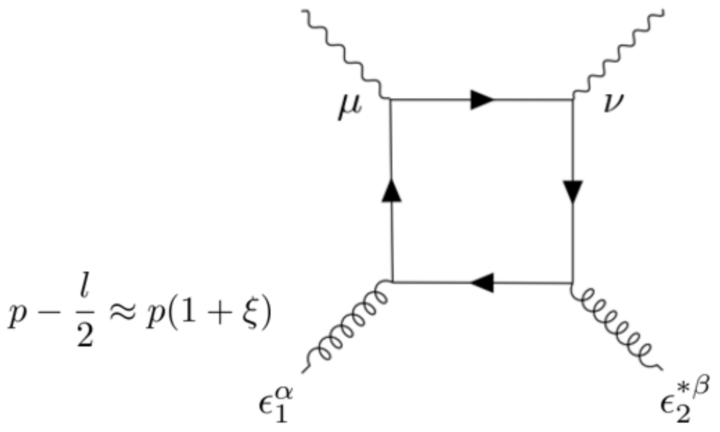


However, box diagram is power-divergent in the IR!

$$\frac{l^\alpha}{l^2}\delta C_g^{\rm anom} \otimes \tilde{\mathcal{F}}(x_B)\gamma_5$$

Still, pole was never seen before because:

$$\langle p_2|F^{\mu\nu}\tilde{F}_{\mu\nu}|p_1\rangle \propto \epsilon^{\mu\nu\alpha\beta} l_\mu p_\nu \epsilon_{1\alpha}\epsilon^*_{2\beta}$$

$\rightarrow 0$ **when** $l^\mu \propto p^\mu$

Imprint of Anomalies in QCD Compton scattering

Perturbative calculations suggest that massless poles are induced in GPD $\tilde{E}$

However, we know there are no massless poles in axial form factor (moment of GPD $\tilde{E}$)

$$g_P(l^2) = \int dx \tilde{E}(x) \sim \frac{1}{l^2}$$

...ous contribution to GPD $\tilde{E}$ at one loop

The QCD factorization theorem: Collins, Freund; Ji, Osborne (1998)

Deeply tied to the UA(1) problem: Why is the η' so massive (957 MeV!)?

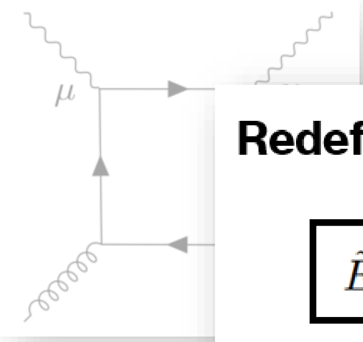
$+\mathcal{O}(\alpha_s) + \mathcal{O}(1/Q^2)$,

Twist-2 GPDs
to all orders

(Non-local) chiral anomaly manifests itself in high energy scattering amplitude &

apparently breaks QCD factorization

Imprint of Anomalies in QCD Compton scattering



Antisymmetric p

Justifying factorization

Redefine

$$\tilde{E}_f(x_B,l^2)+\tilde{E}_f(-x_B,l^2) = \tilde{E}_f^{\rm bare}(x_B,l^2)+\tilde{E}_f^{\rm bare}(-x_B,l^2) + \frac{\alpha_s}{2\pi}\frac{2M}{l^2}\delta C_g^{\rm anom}\otimes\tilde{\mathcal{F}}(x_B,l^2)$$

"Bare GPD" (tree level)

Perturbative pole (one loop)

$\otimes \tilde{\mathcal{F}}(x_B)\gamma_5 \Big] u(P_1)$

one loop

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$$-\epsilon^{\alpha\beta\mu\nu}P_\beta {\rm Im}T^{\rm asym}_{\mu\nu} = \frac{1}{2}\sum_f e_f^2\bar{u}(P_2)\Big[\gamma^\alpha\gamma_5(\tilde{H}_f(x_B,\xi,l^2)+\tilde{H}_f(-x_B,\xi,l^2)) + \frac{l^\alpha\gamma_5}{2M}(\tilde{E}_f^{\rm bare}(x_B,\xi,l^2)+\tilde{E}_f^{\rm bare}(-x_B,\xi,l^2))\Big]u(P_1)$$
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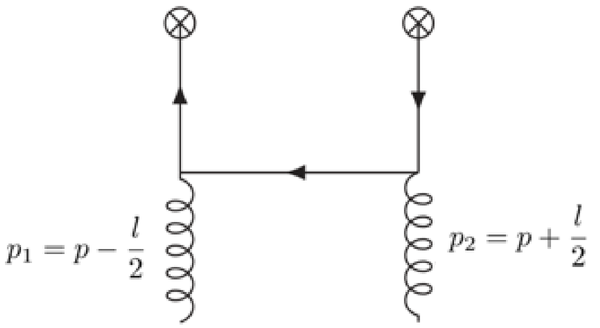


Perturbative pole in GPD

$$\int \frac{dz^-}{4\pi} e^{i\hat{x}p^+z^-} \langle p_2|\bar{\psi}(-z^-/2)\gamma^+\gamma_5\psi(z^-/2)|p_1\rangle\Big|_{\text{pole}} \sim \frac{\alpha_s}{2\pi} T_R \frac{2il^+}{l^2}(1-\hat{x}) \otimes \delta(1-\hat{x})\epsilon^{\epsilon_1\epsilon_2^* lp}$$

Same pole in one-loop calculation!

Perhaps not an ad hoc argument !

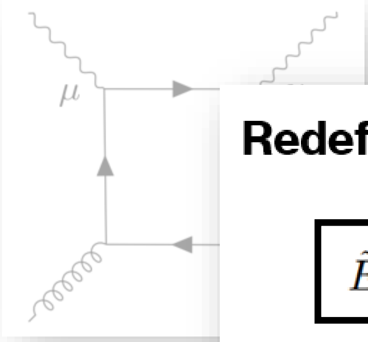
to all orders

chiral anomaly manifests itself in high energy scattering amplitude &

apparently breaks QCD factorization

The pole "belongs" to GPD

Imprint of Anomalies in QCD Compton scattering



Justifying factorization

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$$\tilde{E}_f(x_B,l^2)+\tilde{E}_f(-x_B,l^2)=\tilde{E}_f^{\text{bare}}(x_B,l^2)+\tilde{E}_f^{\text{bare}}(-x_B,l^2)+\frac{\alpha_s}{2\pi}\frac{2M}{l^2}\delta C_g^{\text{anom}}\otimes\tilde{\mathcal{F}}(x_B,l^2)$$

"Bare GPD" (tree level) — Perturbative pole (one loop)

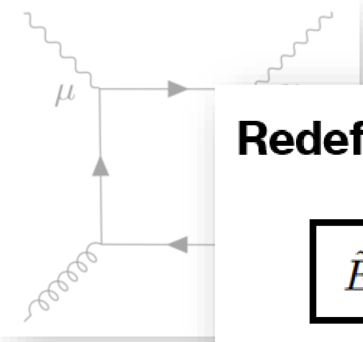
Postulate that the perturbative pole cancels the pre-existing pole in "bare" GPD:

$$\tilde{E}_f^{\text{bare}}(x_B,l^2)+\tilde{E}_f^{\text{bare}}(-x_B,l^2)\approx-\frac{\alpha_s}{2\pi}\frac{2M}{l^2}\delta C_g^{\text{anom}}\otimes\tilde{\mathcal{F}}(x_B,l^2=0)$$

Postulate that the "renormalized" GPD integrates to $g_P(l^2)$:

$$g_P(l^2)=\sum_f\int_{-1}^{1}dx\tilde{E}_f(x,\xi,l^2)=\sum_f\int_0^1 dx(\tilde{E}_f(x,\xi,l^2)+\tilde{E}_f(-x,\xi,l^2))$$

Imprint of Anomalies in QCD Compton scattering



Antisymmetric p

Justifying factorization

Redefine

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“Bare GPD” (tree level)

Perturbative pole (one loop)

$\otimes \tilde{\mathcal{F}}(x_B)\gamma_5 \big] u(P_1)$

one loop

The QCD factorization theorem: Collins, Freund; Ji, Osborne (1998)

Pole cancellation at $\int dx$

We find:
$$\frac{g_P(l^2)}{2M} = -\frac{i}{l^2}\left(\frac{\langle P_2|\frac{n_f\alpha_s}{4\pi}F\tilde{F}|P_1\rangle}{\bar{u}(P_2)\gamma_5 u(P_1)}\bigg|_{l^2=0} - \frac{\langle P_2|\frac{n_f\alpha_s}{4\pi}F\tilde{F}|P_1\rangle}{\bar{u}(P_2)\gamma_5 u(P_1)}\right)$$

See earlier slide

energy scattering amplitude &

orization

Postulate that the “renormalized” GPD integrates to $g_P(l^2)$:

$$g_P(l^2) = \sum_f \int_{-1}^{1} dx \tilde{E}_f(x,\xi,l^2) = \sum_f \int_0^1 dx(\tilde{E}_f(x,\xi,l^2)+\tilde{E}_f(-x,\xi,l^2))$$

Imprint of Anomalies in QCD Compton scattering

Connection between twist 2 & twist 4 GPDs due to anomaly

Justifying factorization

Redefine

$$\tilde{E}_f(x_B,l^2)+\tilde{E}_f(-x_B,l^2)=\tilde{E}_f^{\rm bare}(x_B,l^2)+\tilde{E}_f^{\rm bare}(-x_B,l^2)+\frac{\alpha_s}{2\pi}\frac{2M}{l^2}\delta C_g^{\rm anom}\otimes\tilde{\mathcal{F}}(x_B,l^2)$$

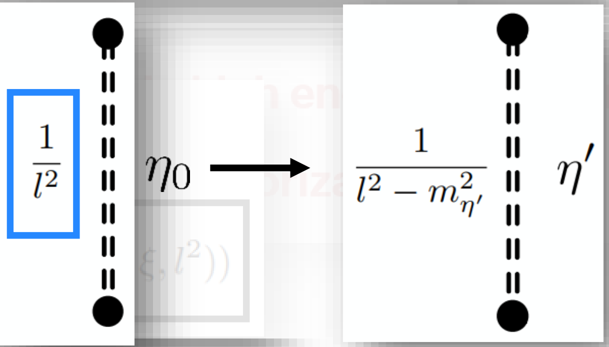
"Bare GPD" (tree level) — Perturbative pole (one loop)

Pole cancellation at $\int dx$

We find:

$$\frac{g_P(l^2)}{2M}=-\left[\frac{2M\Delta\Sigma}{l^2}-\frac{2M\Delta\Sigma}{l^2}+\frac{2M\Delta\Sigma}{l^2-m_{\eta'}^2}\right]\sim\frac{1}{l^2-m_{\eta'}^2}$$

See earlier slide

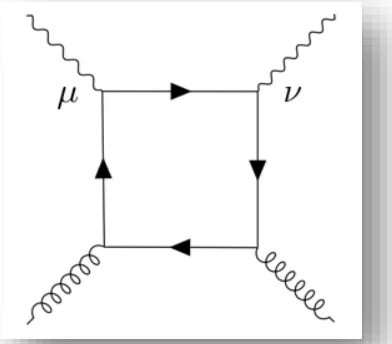


"We demonstrate that the dynamical interplay between the physics of the anomaly, and that of the isosinglet pseudoscalar $U_A(1)$ sector of QCD resolves both problems simultaneously: the lifting of the $\bar{\eta}$ pole by topological mass generation of the η' and the cancellation of the anomaly pole"

- Tarasov, Venugopalan

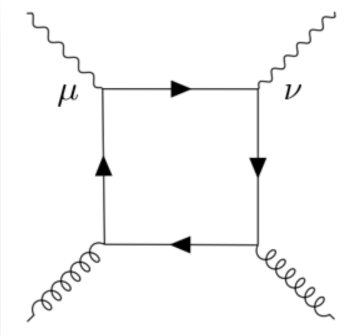
See also Jaffe Manohar, 1990

Imprint of Anomalies in QCD Compton scattering



Symmetric part of Compton amplitude $(\xi \neq 0)$

Imprint of Anomalies in QCD Compton scattering



Symmetric part of Compton amplitude $(\xi \neq 0)$

Pole! (New result)

$$\left(H_f(x_B,\xi,l^2) - H_f(-x_B,\xi,l^2)\right) = \left(H_f^{\rm bare}(x_B,\xi,l^2) - H_f^{\rm bare}(-x_B,\xi,l^2)\right) + \frac{\alpha_s}{2\pi}\frac{1}{l^2}C^{\rm anom} \otimes' \mathcal{F}(x_B,\xi,l^2)$$

$$\left(E_f(x_B,\xi,l^2) - E_f(-x_B,\xi,l^2)\right) = \left(E_f^{\rm bare}(x_B,\xi,l^2) - E_f^{\rm bare}(-x_B,\xi,l^2)\right) - \frac{\alpha_s}{2\pi}\frac{1}{l^2}C^{\rm anom} \otimes' \mathcal{F}(x_B,\xi,l^2)$$

“Bare GPD” (tree level)

Perturbative pole (one loop)

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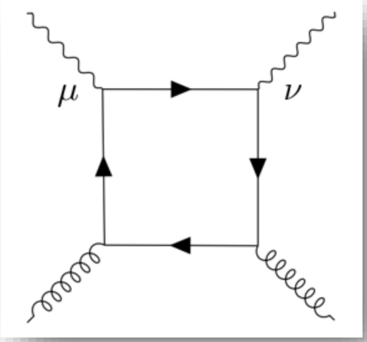

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Hatta, Zhao (2020);
Radyushkin, Zhao (2021)

Twist-4 GPD:

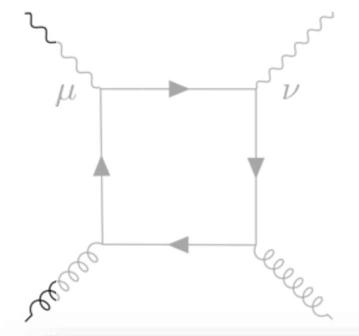
$$\mathcal{F}(x,\xi,l^2) = -4xP^+M\int\frac{dz^-}{2\pi}e^{ixP^+z^-}\frac{\langle P_2|F^{\mu\nu}(-z^-/2)F_{\mu\nu}(z^-/2)|P_1\rangle}{\bar{u}(P_2)u(P_1)}$$

"Bare GPD" (tree level)

Perturbative pole (one loop)

(Non-local) trace anomaly manifests itself in high energy scattering amplitude & apparently breaks QCD factorization

Imprint of Anomalies in QCD Compton scattering



Symmetric part of Compton amplitude $(\xi \neq 0)$

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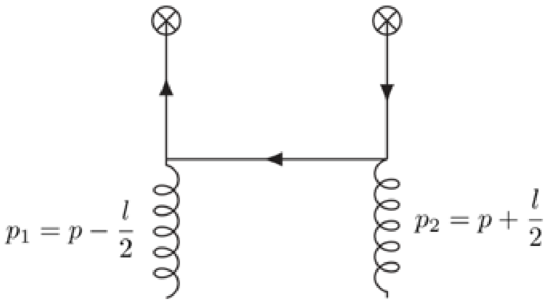
$$(H_f(x_B,\xi,l^2) - H_f(-x_B,\xi,l^2)) = (H_f^{\rm bare}(x_B,\xi,l^2) - H_f^{\rm bare}(-x_B,\xi,l^2)) + \frac{\alpha_s}{2\pi}\frac{1}{l^2} C^{\rm anom} \otimes' \mathcal{F}(x_B,\xi,l^2)$$

The pole "belongs" to GPD



Perturbative pole in GPD

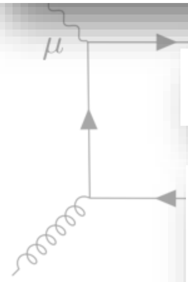
$$\int \frac{dk^- d^{2-2\epsilon}k_\perp}{(2\pi)^{3-2\epsilon}} \frac{\mathrm{Tr}[\gamma_\alpha(\not k + \not l/2)\gamma^+(\not k - \not l/2)\gamma^\beta(\not k - \not p)]}{(p-k)^2(k-l/2)^2(k+l/2)^2}\epsilon^*_\alpha(p+l/2)\epsilon_\beta(p-l/2)\Big|_{\rm pole} \sim \frac{1}{l^2}\frac{x(1-x)}{1-\xi^2}\langle FF\rangle$$

Same pole in one-loop calculation!

Imprint of Anomalies in QCD Compton scattering

Polynomiality:

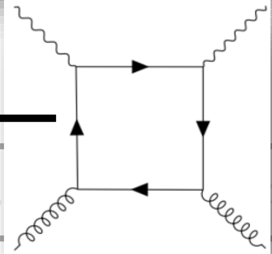
Symmetric part of Compton amplitude $(\xi \neq 0)$



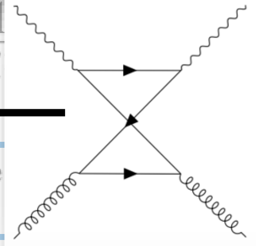
Structure of convolution:

$$C^{\text{anom}} \otimes' \mathcal{F}(x_B, \xi, l^2) \equiv \int_{x_B}^{1} \frac{dx}{x} K(\hat{x}, \hat{\xi}) \mathcal{F}(x, \xi, l^2) - \frac{\theta(\xi - x_B)}{2} \int_{-1}^{1} \frac{dx}{x} L(\hat{x}, \hat{\xi}) \mathcal{F}(x, \xi, l^2)$$

$$K(\hat{x}, \hat{\xi}) = 2T_R \frac{\hat{x}(1-\hat{x})}{1-\hat{\xi}^2}$$



$$L(\hat{x}, \hat{\xi}) = 2T_R \frac{\hat{x}(\hat{\xi}-\hat{x})}{1-\hat{\xi}^2}$$



Nonzero for nonzero skewness

“Bare GPD” (tree level)

Perturbative pole (one loop)

Hatta, Zhao (2020); Radyushkin, Zhao (2021)

Twist-4 GPD:

$$\mathcal{F}(x, \xi, l^2) = -4xP^+M \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \frac{\langle P_2 | F^{\mu\nu}(-z^-/2) F_{\mu\nu}(z^-/2) | P_1 \rangle}{\bar{u}(P_2) u(P_1)}$$

(Non-local) trace anomaly manifests itself in high energy scattering amplitude & apparently breaks QCD factorization

Imprint of Anomalies in QCD Compton scattering

Polynomiality:

Symmetric part of Compton amplitude $(\xi \neq 0)$

Structure of convolution:

$$C^{\text{anom}} \otimes' \mathcal{F}(x_B, \xi, l^2) \equiv \int_{x_B}^{1} \frac{dx}{x} K(\hat{x}, \hat{\xi}) \mathcal{F}(x, \xi, l^2) - \frac{\theta(\xi - x_B)}{2} \int_{-1}^{1} \frac{dx}{x} L(\hat{x}, \hat{\xi}) \mathcal{F}(x, \xi, l^2)$$

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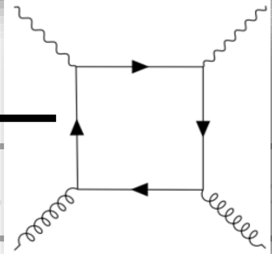
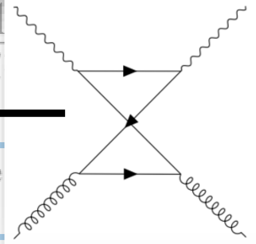


$$L(\hat{x}, \hat{\xi}) = 2T_R \frac{\hat{x}(\hat{\xi}-\hat{x})}{1-\hat{\xi}^2}$$



Nonzero for nonzero skewness

Hatta, Zhao (2020); Radyushkin, Zhao (2021)

Twist-4 GPD:

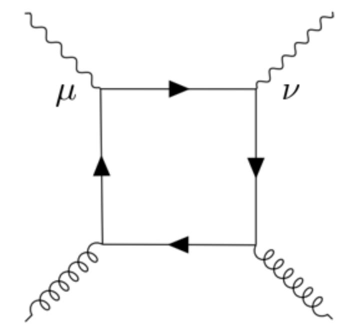
Example

$$\sum_f e_f^2 x_B (H_f^{\text{bare}}(x_B, \xi, l^2) - H_f^{\text{bare}}(-x_B, \xi, l^2)) \approx -\frac{\alpha_s}{2\pi} \left(\sum_f e_f^2 \right) \frac{x_B}{l^2} C^{\text{anom}} \otimes' \mathcal{F}(x_B, \xi, l^2 = 0)$$

Polynomiality

$$\sum_f e_f^2 (A_f^{\text{bare}}(l^2) + \xi^2 D_f^{\text{bare}}(l^2)) \approx \frac{T_R \alpha_s}{12\pi l^2} \left(\sum_f e_f^2 \right) \left(\frac{\langle P | F^{\alpha\beta} (i \overleftrightarrow{D}^+)^2 F_{\alpha\beta} | P \rangle}{(P^+)^2} + \xi^2 \langle P | F^2 | P \rangle \right)$$

Imprint of Anomalies in QCD Compton scattering



Symmetric part of Compton amplitude $(\xi \neq 0)$

Pole! (New result)

$$\left(H_f(x_B,\xi,l^2) - H_f(-x_B,\xi,l^2)\right) = \left(H_f^{\rm bare}(x_B,\xi,l^2) - H_f^{\rm bare}(-x_B,\xi,l^2)\right) + \frac{\alpha_s}{2\pi}\frac{1}{l^2}C^{\rm anom} \otimes' \mathcal{F}(x_B,\xi,l^2)$$

$$\left(E_f(x_B,\xi,l^2) - E_f(-x_B,\xi,l^2)\right) = \left(E_f^{\rm bare}(x_B,\xi,l^2) - E_f^{\rm bare}(-x_B,\xi,l^2)\right) - \frac{\alpha_s}{2\pi}\frac{1}{l^2}C^{\rm anom} \otimes' \mathcal{F}(x_B,\xi,l^2)$$

“Bare GPD” (tree level)

Perturbative pole (one loop)

Imprint of Anomalies in QCD Compton scattering

Symmetric part of Compton amplitude $(\xi \neq 0)$

We proposed a possible scenario of pole cancellation in order to justify factorization

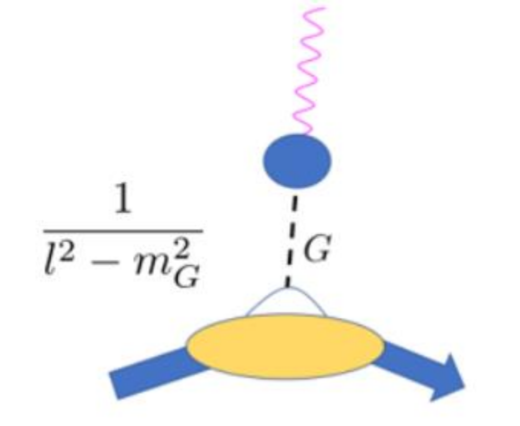
glueball mass generations

$(H_f(x_B,\xi,l^2) - H_f(-x_B,\xi,l^2)) = (H_f^{\text{bare}}(x_B,\xi,l^2) - H_f^{\text{bare}}(-x_B,\xi,l^2))$

$+ \frac{\alpha_s}{2\pi}\frac{1}{l^2} C^{\text{anom}} \otimes' \mathcal{F}(x_B,\xi,l^2)$

D term

$$\sum_f D_f(l^2) \approx M \frac{T_R n_f \alpha_s}{6\pi l^2}\left(\frac{\langle P_2|F^2|P_1\rangle}{\bar{u}(P_2)u(P_1)}\bigg|_{l^2=0} - \frac{\langle P_2|F^2|P_1\rangle}{\bar{u}(P_2)u(P_1)} \right)$$



Hatta, Zhao (2020);
Radyushkin, Zhao (2

$\mathcal{F}(x,\xi,l^2) = -$

$$A(l^2),\ B(l^2),\ D(l^2) \sim \frac{1}{l^2 - m_G^2}$$

level)

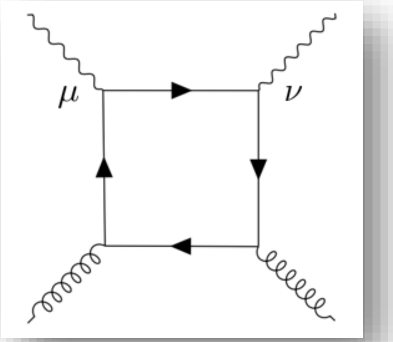
Perturbative pole (one loop)

(Non-local) trace anomaly manifests itself in high energy scattering amplitude & apparently breaks QCD factorization

Imprint of Anomalies in QCD Compton scattering

Real part of Compton amplitude: **(SB, Hatta, Vogelsang, In Preparation)**

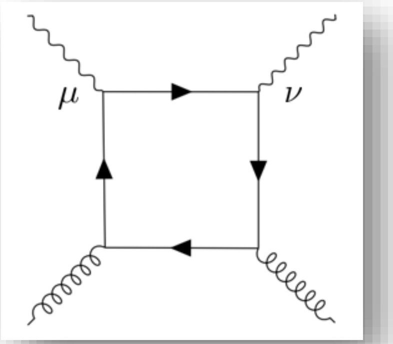
PRELIMINARY



Imprint of Anomalies in QCD Compton scattering

Real part of Compton amplitude: **(SB, Hatta, Vogelsang, In Preparation)**

PRELIMINARY



Example: Antisymmetric case

Pole in real part!

$$\sim \frac{16\langle F\tilde{F}\rangle}{l^2(-1+\xi^2)}\left(-(-1+x)\ln\frac{x-1}{x} + (x-\xi)\ln\frac{x-\xi}{x}\right) - (x \to -x)$$

$$+\tilde{\kappa}_{qg}(\hat{x},\hat{\xi})\ln\frac{Q^2}{-l^2}$$

$$+\tilde{C}_1^g(\hat{x},\hat{\xi})$$

$$2T_R\int_0^1 \frac{dx}{x}\frac{(1-\hat{x})\ln\frac{\hat{x}-1}{\hat{x}} + (\hat{x}-\hat{\xi})\ln\frac{\hat{x}-\hat{\xi}}{\hat{x}} - (\hat{x}\to -\hat{x})}{(1-\hat{\xi}^2)}\tilde{\mathcal{F}}(x,\xi,t)$$

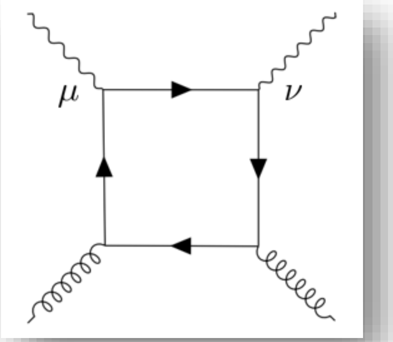
$$= \int_0^1 dy\tilde{C}_0(y,x_B)\left[\int_y^1 \frac{dx}{x}\tilde{K}\left(\frac{y}{x},\frac{\xi}{x}\right)\tilde{\mathcal{F}}(x,\xi,t) - \theta(\xi-y)\int_0^1 \frac{dx}{x}\tilde{L}\left(\frac{y}{x},\frac{\xi}{x}\right)\tilde{\mathcal{F}}(x,\xi,t)\right]$$

Same structure for convolution!
Even in real part, same mechanism should cancel pole as in imaginary part

Imprint of Anomalies in QCD Compton scattering

Real part of Compton amplitude: **(SB, Hatta, Vogelsang, In Preparation)**

PRELIMINARY



Example: Antisymmetric case

$$\sim \frac{16\langle F\tilde{F}\rangle}{l^2(-1+\xi^2)}\left(-(-1+x)\ln\frac{x-1}{x}+(x-\xi)\ln\frac{x-\xi}{x}\right)-(x\to -x)$$

$$+\tilde{\kappa}_{qg}(\hat{x},\hat{\xi})\ln\frac{Q^2}{-l^2}$$

$$+\tilde{C}_1^g(\hat{x},\hat{\xi})$$

Reproduced the known logarithms from literature

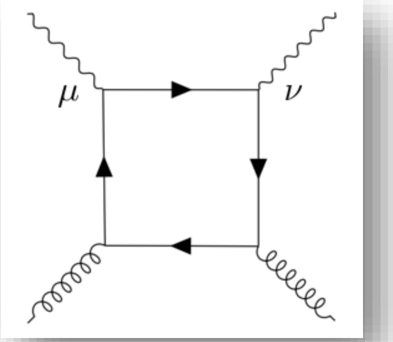
$$\tilde{\kappa}_{qg}(\hat{x},\hat{\xi}) = \frac{2\hat{x}-1-\hat{\xi}^2}{2(1-\hat{\xi}^2)^2}\ln\frac{\hat{x}-1}{\hat{x}}-\frac{\hat{x}-\hat{\xi}}{(1-\hat{\xi}^2)^2}\ln\frac{\hat{x}-\hat{\xi}}{\hat{x}}-(\hat{x}\to -\hat{x})$$

Ji, Osborne; Belitsky, Mueller

Imprint of Anomalies in QCD Compton scattering

Real part of Compton amplitude: **(SB, Hatta, Vogelsang, In Preparation)**

PRELIMINARY



Example: Antisymmetric case

$$\sim \frac{16\langle F\tilde{F}\rangle}{l^2(-1+\xi^2)}\left(-(-1+x)\ln\frac{x-1}{x}+(x-\xi)\ln\frac{x-\xi}{x}\right)-(x\to -x)$$

$$+\tilde{\kappa}_{qg}(\hat{x},\hat{\xi})\ln\frac{Q^2}{-l^2}$$

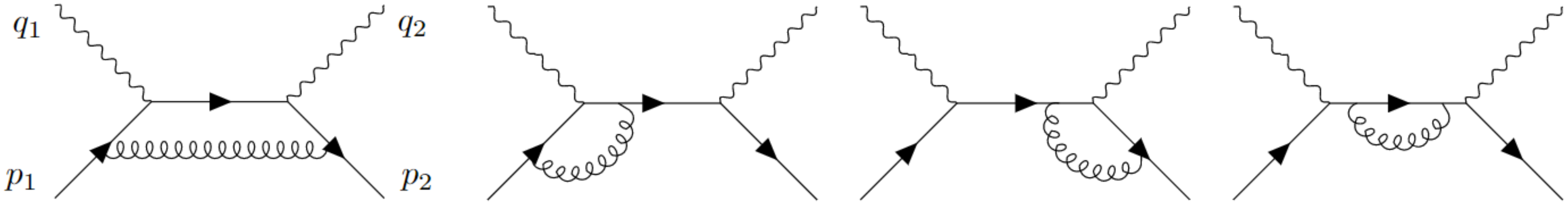
$$+\tilde{C}_1^g(\hat{x},\hat{\xi})$$

Coefficient function

Imprint of Anomalies in QCD Compton scattering

Quark-channel diagrams in DVCS: **(SB, Hatta, Vogelsang, In Preparation)**

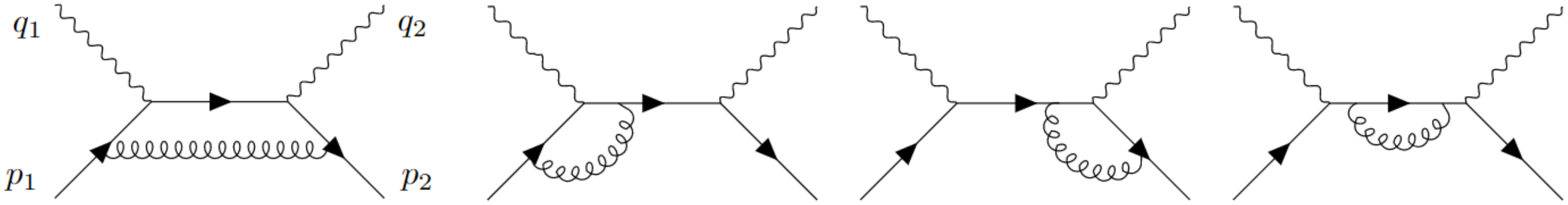
PRELIMINARY



Imprint of Anomalies in QCD Compton scattering

Quark-channel diagrams in DVCS: **(SB, Hatta, Vogelsang, In Preparation)**

PRELIMINARY



Example: Antisymmetric case

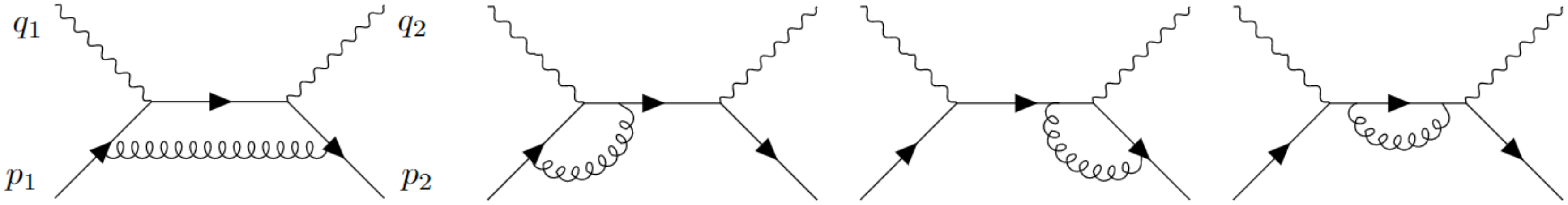
$$\sim \frac{1}{l^2} + \tilde{\kappa}_{qq}(\hat{x},\hat{\xi}) \ln\frac{Q^2}{-l^2} + \delta\tilde{C}_1^q(\hat{x},\hat{\xi}) - (\hat{x} \to -\hat{x})$$

No pole!

Imprint of Anomalies in QCD Compton scattering

Quark-channel diagrams in DVCS: (SB, Hatta, Vogelsang, In Preparation)

PRELIMINARY



Example: Antisymmetric case

$$\sim \frac{1}{l^2} \quad + \tilde{\kappa}_{qq}(\hat{x},\hat{\xi}) \ln \frac{Q^2}{-l^2} + \delta \tilde{C}_1^q(\hat{x},\hat{\xi}) - (\hat{x} \to -\hat{x})$$

Reproduced the known logarithms from literature

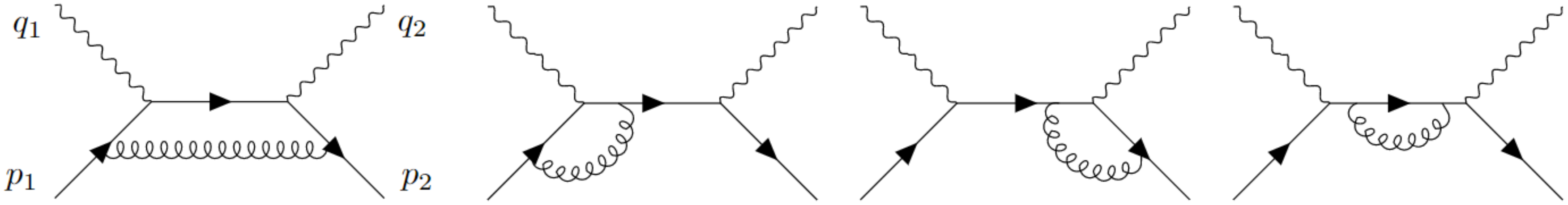
$$\tilde{\kappa}_{qq}(\hat{x},\hat{\xi}) = \frac{3}{2(1-\hat{x})} + \frac{\hat{x}^2+1-2\hat{\xi}^2}{(1-\hat{\xi}^2)(1-\hat{x})} \ln \frac{\hat{x}-1}{\hat{x}} - \frac{(\hat{x}-\hat{\xi})(1+\hat{x}^2+2\hat{x}\hat{\xi})}{(1-\hat{x}^2)(1-\hat{\xi}^2)} \ln \frac{\hat{x}-\hat{\xi}}{\hat{x}} - (\hat{x} \to -\hat{x})$$

Ji, Osborne; Belitsky, Mueller

Imprint of Anomalies in QCD Compton scattering

Quark-channel diagrams in DVCS: **(SB, Hatta, Vogelsang, In Preparation)**

PRELIMINARY



Example: Antisymmetric case

$$\sim \frac{1}{l^2} + \tilde{\kappa}_{qq}(\hat{x},\hat{\xi}) \ln \frac{Q^2}{-l^2} + \delta\tilde{C}_1^q(\hat{x},\hat{\xi}) - (\hat{x} \to -\hat{x})$$

Coefficient function

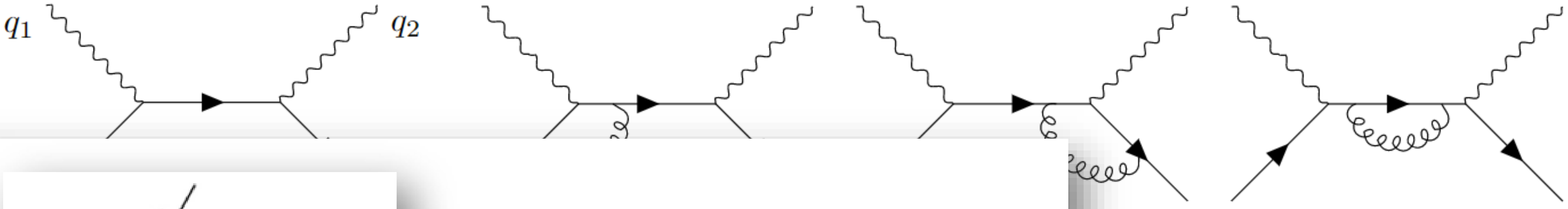
$$\delta\tilde{C}_1^q(\hat{x},\hat{\xi}) = -\frac{\left(\frac{Q^2}{-l^2}\right)^{\epsilon}}{\epsilon^2(1-\hat{x})} - \frac{3\left(\frac{Q^2}{-l^2}\right)^{\epsilon}}{2\epsilon(1-\hat{x})} + \cdots - (\hat{x} \to -\hat{x})$$

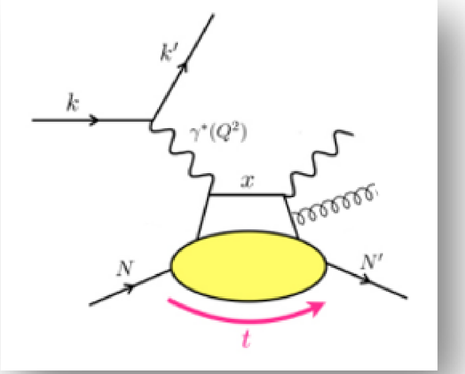
Unexpected double IR pole

Imprint of Anomalies in QCD Compton scattering

Quark-channel diagrams in DVCS: **(SB, Hatta, Vogelsang, In Preparation)**

PRELIMINARY





Usually, double poles are cancelled by real gluon emissions

Here, we don't have such a contribution

Examp

$(\hat{x} \to -\hat{x})$

Coefficient function

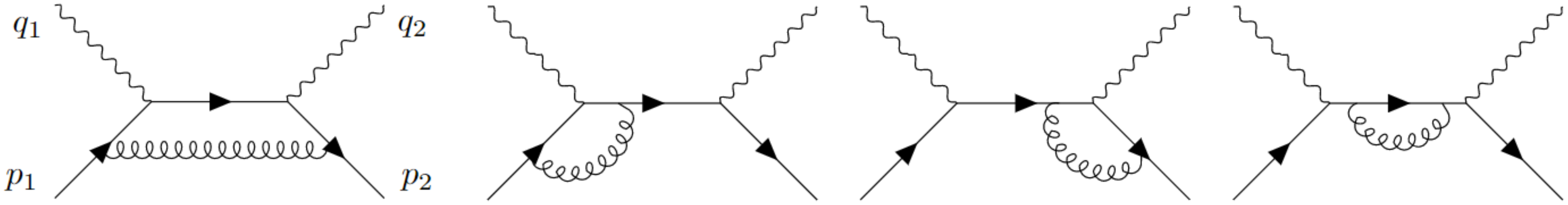
$$\delta\tilde{C}_1^q(\hat{x},\hat{\xi}) = -\frac{\left(\frac{Q^2}{-l^2}\right)^\epsilon}{\epsilon^2(1-\hat{x})} - \frac{3\left(\frac{Q^2}{-l^2}\right)^\epsilon}{2\epsilon(1-\hat{x})} + \cdots - (\hat{x} \to -\hat{x})$$

Unexpected double IR pole

Imprint of Anomalies in QCD Compton scattering

Quark-channel diagrams in DVCS: **(SB, Hatta, Vogelsang, In Preparation)**

PRELIMINARY



Example: Antisymmetric case

$$\sim \frac{1}{l^2} + \tilde{\kappa}_{qq}(\hat{x},\hat{\xi}) \ln \frac{Q^2}{-l^2} + \delta\tilde{C}_1^q(\hat{x},\hat{\xi}) - (\hat{x} \to -\hat{x})$$

Coefficient function

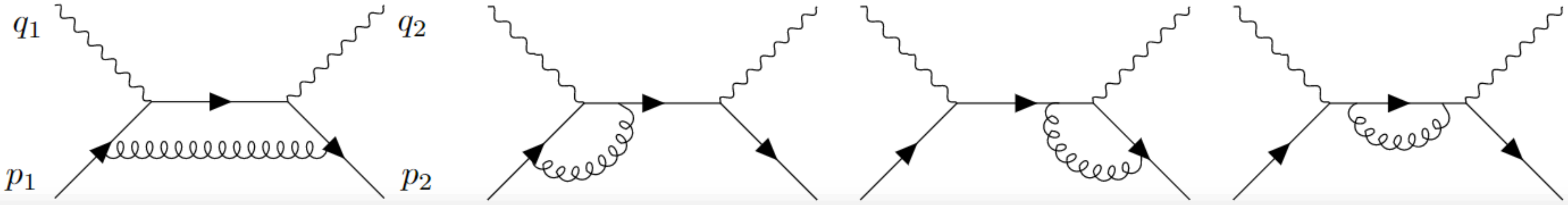
Unexpected single IR pole

$$\delta\tilde{C}_1^q(\hat{x},\hat{\xi}) = -\frac{\left(\frac{Q^2}{-l^2}\right)^\epsilon}{\epsilon^2(1-\hat{x})} - \frac{3\left(\frac{Q^2}{-l^2}\right)^\epsilon}{2\epsilon(1-\hat{x})} + \cdots - (\hat{x} \to -\hat{x})$$

Imprint of Anomalies in QCD Compton scattering

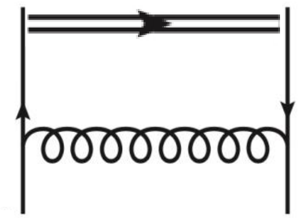
Quark-channel diagrams in DVCS: (SB, Hatta, Vogelsang, In Preparation)

PRELIMINARY



It looks like factorization is broken due to the unexpected double, single IR poles

But, when you compute GPD itself, you find the same double, single IR poles!
These poles can be systematically absorbed into GPD



Unexpected single IR pole

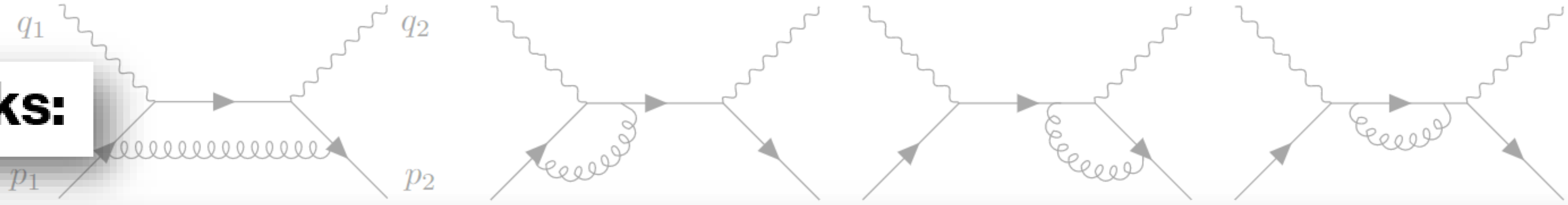
$$\delta\tilde{C}_1^q(\hat{x},\hat{\xi}) = -\frac{\left(\frac{Q^2}{-l^2}\right)^{\epsilon}}{\epsilon^2(1-\hat{x})} - \frac{3\left(\frac{Q^2}{-l^2}\right)^{\epsilon}}{2\epsilon(1-\hat{x})} + \cdots - (\hat{x} \to -\hat{x})$$

Unexpected double IR pole

Imprint of Anomalies in QCD Compton scattering

Quark-channel diagrams in DVCS: (SB, Hatta, Vogelsang, In Preparation)

PRELIMINARY



Remarks:

It loo

- **Off-forwardness is an alternative factorization scheme that clarifies the physics of anomaly**

But, when you compute GPD itself, you find the same double, single IR poles!

These poles can be systematically absorbed into GPD

ion

- **Novel connection between twist 2 & twist 4 sectors at the density level due to anomaly**

Unexpected single IR pole

$$\delta \tilde{C}_1^q(\hat{x},\hat{\xi}) = -\frac{\left(\frac{Q^2}{-l^2}\right)^{\epsilon}}{\epsilon^2(1-\hat{x})} - \frac{3\left(\frac{Q^2}{-l^2}\right)^{\epsilon}}{2\epsilon(1-\hat{x})} + \cdots - (\hat{x} \to -\hat{x})$$

Unexpected double IR pole

Summary

- **Revisited QCD factorization for Compton scattering: Crucial topic for ongoing & future experiments including at EIC**

- **Importance to understand off-forward poles originating from chiral & trace anomalies**

$$T^{\mu\nu} \sim \frac{\langle F\tilde{F}\rangle}{l^2}, \quad \frac{\langle FF\rangle}{l^2}$$

Unnoticed in literature

Summary

Perturbative calculations suggest that massless poles are induced in GPDs $\tilde{E},\ H,\ E$

- Revisited QCD factorization for Compton scattering: Crucial topic for ongoing &

However, we know there are no massless poles in axial and gravitational form factors (moments of GPDs)

- Importance to understand off-forward poles originating from chiral & trace anomalies

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Unnoticed in literature

Summary

Perturbative calculations suggest that massless poles are induced in GPDs $\tilde{E},\ H,\ E$

- Revisited QCD factorization for Compton scattering: Crucial topic for ongoing &

However, we know there are no massless poles in axial and gravitational form factors (moments of GPDs)

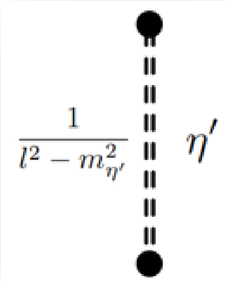
We proposed a possible scenario of pole cancellation

This has to do with eta-meson & glueball mass generations

- Importance to understand off-forward poles originating from chiral & trace anomalies

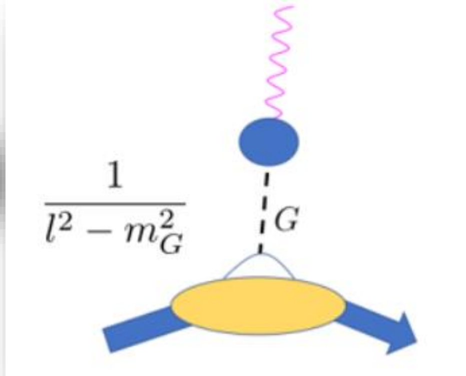
Antisymmetric case:

cf, the η' mass problem



$$g_P \sim \int dx \tilde{E}(x) \sim \frac{1}{l^2 - m_{\eta'}^2}$$

Symmetric case:



$$A(l^2),\ B(l^2),\ D(l^2) \sim \frac{1}{l^2 - m_G^2}$$

Summary

- Revisited QCD factorization for Compton scattering: Crucial topic for ongoing & future experiments including at EIC

- Importance to understand off-forward poles originating from chiral & trace anomalies

$$T^{\mu\nu} \sim \frac{\langle F\tilde{F}\rangle}{l^2}, \quad \frac{\langle FF\rangle}{l^2}$$ Unnoticed in literature

Off-forwardness is a physical factorization scheme that elucidates the physics of anomaly

Profound physical implication of poles; touches questions on mass generations, Chiral symmetry breaking, ...

Summary & outlook

• ... ng &

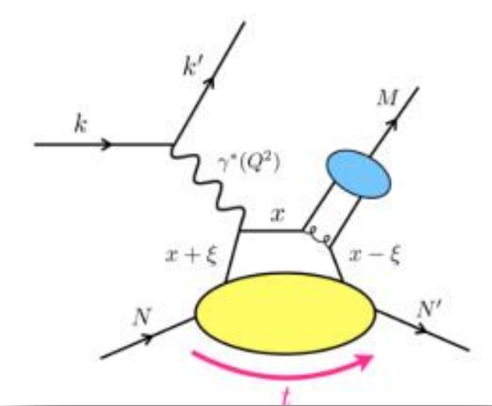
Novel connections between DVCS & chiral/trace anomalies:
This could be a new & potentially rich avenue for GPD research

• Importan... l & trace anomalies

Imprint of anomaly on other physical processes:

(Example: Deeply-virtual meson production)

Unnoticed in literature



Off-forwardness is a physical ... that elucidates the physics of anomaly

Profound physical implicatio... estions on mass generations,
Chiral symmetry breaking, ...

Backup slides

Imprint of Anomalies in QCD Compton scattering

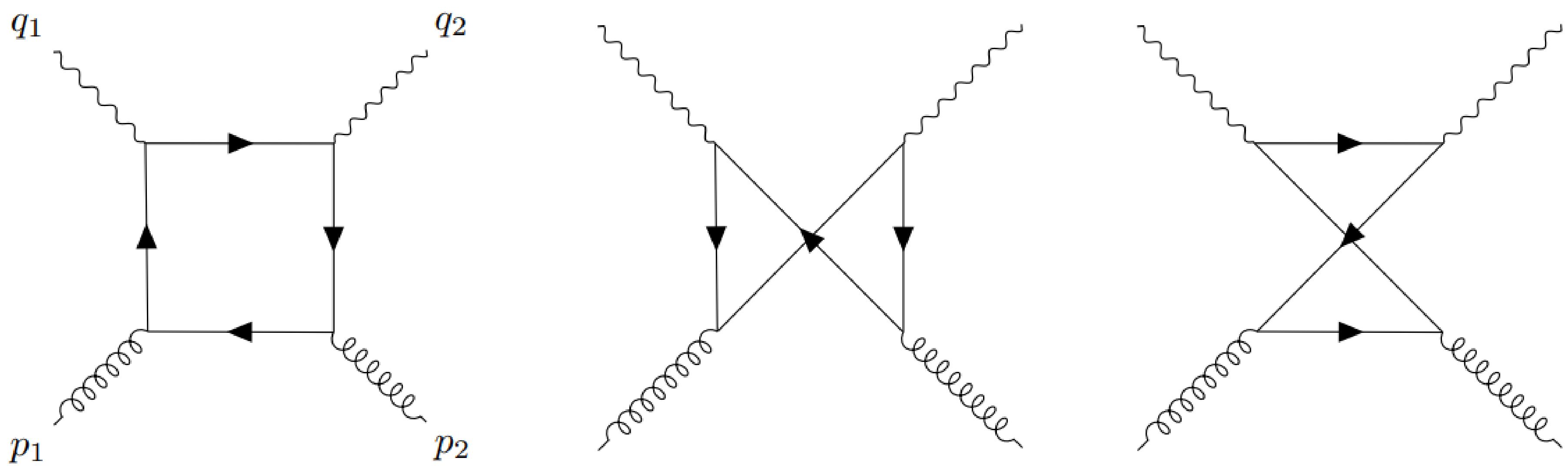
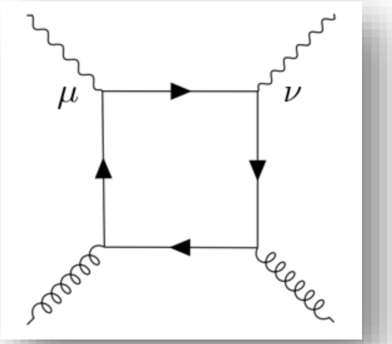


FIG. 1: Box diagrams for the Compton amplitude in off-forward kinematics.

Imprint of Anomalies in QCD Compton scattering

$$\frac{g_P(l^2)}{2M} \approx \frac{-2M\Delta\Sigma}{l^2 - m_{\eta'}^2}\,, \qquad i\frac{\langle P_2|\frac{n_f\alpha_s}{4\pi}F\tilde{F}|P_1\rangle}{\bar{u}(P_2)\gamma_5 u(P_1)} \approx -2M\Delta\Sigma\frac{m_{\eta'}^2}{l^2 - m_{\eta'}^2}\,.$$

Imprint of Anomalies in QCD Compton scattering



Symmetric case:

Example: Antisymmetric part of Compton amplitude

$-\epsilon^{\alpha\beta\mu\nu}P_\beta$

$$\bar{F}_1^{\rm off}(x_B,l)\approx\frac{1}{2}\frac{\alpha_s}{2\pi}\left(\sum_f e_f^2\right)\left[\left(P_{qg}\ln\frac{Q^2}{-l^2}+C_{1g}^{\rm off}\right)\otimes g(x_B)+\frac{1}{l^2}C^{\rm anom}\otimes'\mathcal{F}(x_B,\xi,l^2)\frac{\bar{u}(P_2)u(P_1)}{2M}\right],$$

$$\bar{F}_2^{\rm off}(x_B,l)\approx x_B\frac{\alpha_s}{2\pi}\left(\sum_f e_f^2\right)\left[\left(P_{qg}\ln\frac{Q^2}{-l^2}+C_{2g}^{\rm off}\right)\otimes g(x_B)+\frac{1}{l^2}C^{\rm anom}\otimes'\mathcal{F}(x_B,\xi,l^2)\frac{\bar{u}(P_2)u(P_1)}{2M}\right].\qquad(31)$$

We recognize the expected structure of the one-loop corrections associated with the unpolarized gluon PDF $g(x)$, with the splitting function $P_{qg}(\hat{x})=2T_R((1-\hat{x})^2+\hat{x}^2)$. The coefficient functions are given by

$$C_{1g}^{\rm off}(\hat{x})=2T_R((1-\hat{x})^2+\hat{x}^2)\left(\ln\frac{1}{\hat{x}(1-\hat{x})}-1\right),$$

$$C_{2g}^{\rm off}(\hat{x})=2T_R((1-\hat{x})^2+\hat{x}^2)\left(\ln\frac{1}{\hat{x}(1-\hat{x})}-1\right)+8T_R\hat{x}(1-\hat{x}).\qquad(32)$$

In addition, we find a pole $1/l^2$ in both $\bar{F}_1^{\rm off}$ and $\bar{F}_2^{\rm off}$ (but not in the difference $\bar{F}_2^{\rm off}-2x_B\bar{F}_1^{\rm off}$ relevant to the longitudinal structure function), with the following convolution formula

$$C^{\rm anom}\otimes'\mathcal{F}(x_B,\xi,l^2)\equiv\int_{x_B}^1\frac{dx}{x}K(\hat{x},\hat{\xi})\mathcal{F}(x,\xi,l^2)-\frac{\theta(\xi-x_B)}{2}\int_{-1}^1\frac{dx}{x}L(\hat{x},\hat{\xi})\mathcal{F}(x,\xi,l^2),\qquad(33)$$

where

$$K(\hat{x},\hat{\xi})=2T_R\frac{\hat{x}(1-\hat{x})}{1-\hat{\xi}^2},\qquad L(\hat{x},\hat{\xi})=2T_R\frac{\hat{x}(\hat{\xi}-\hat{x})}{1-\hat{\xi}^2}.\qquad(34)$$

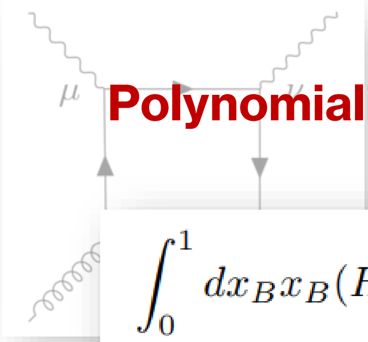
Antisymmetric case:

$$A\otimes B(x_B)\equiv\int_{x_B}^1\frac{dx}{x}A\left(\frac{x_B}{x}\right)B(x)$$

$-\epsilon^{\alpha\beta\mu\nu}P_\beta{\rm Im}T_{\mu\nu}^{\rm asym}\;=$

Twist-2 G

Imprint of Anomalies in QCD Compton scattering



Symmetric part of Compton amplitude $(\xi \neq 0)$

Polynomiality:

Pole! (New result)

$$\int_0^1 dx_B x_B (H_f(x_B,\xi,l^2) - H_f(-x_B,\xi,l^2)) = \int_{-1}^1 dx_B x_B H_f(x_B,\xi,l^2) = A_f(l^2) + \xi^2 D_f(l^2)\,,$$

$$\int_0^1 dx_B x_B (E_f(x_B,\xi,l^2) - E_f(-x_B,\xi,l^2)) = \int_{-1}^1 dx_B x_B E_f(x,\xi,l^2) = B_f(l^2) - \xi^2 D_f(l^2)\,,$$

$-x_B,\xi,l^2))$

$C^{\rm anom} \otimes' \mathcal{F}(x_B,\xi,l^2)$

$(E_f(x_B,\xi,l^2) - E_f(-x_B,\xi,l^2)) = (E_f^{\rm bare}(x_B,\xi,l^2) - E_f^{\rm bare}(-x_B,\xi,l^2))$

$$\sum_f e_f^2 (A_f^{\rm bare}(l^2) + \xi^2 D_f^{\rm bare}(l^2)) \approx \frac{T_R \alpha_s}{12\pi l^2}\left(\sum_f e_f^2\right)\left(\frac{\langle P|F^{\alpha\beta}(i\overleftrightarrow{D}^+)^2 F_{\alpha\beta}|P\rangle}{(P^+)^2} + \xi^2 \langle P|F^2|P\rangle\right),$$

$$\sum_f e_f^2 (B_f^{\rm bare}(l^2) - \xi^2 D_f^{\rm bare}(l^2)) \approx -\frac{T_R \alpha_s}{12\pi l^2}\left(\sum_f e_f^2\right)\left(\frac{\langle P|F^{\alpha\beta}(i\overleftrightarrow{D}^+)^2 F_{\alpha\beta}|P\rangle}{(P^+)^2} + \xi^2 \langle P|F^2|P\rangle\right).$$

$\xi, l^2)$

$\mathcal{F}(x,\xi,l^2) = -4x$

Perturbative pole (one loop)

(Non-local) trace anomaly manifests itself in high energy scattering amplitude &

possibly breaks QCD factorization