Manifestation of anomalies in Deep Virtual Compton Scattering

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Based on:
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Outline

• Chiral & trace anomalies in QCD
• Anomaly in polarized DIS & proton’s spin puzzle: History
• QCD Compton Scattering: Calculation of box diagrams
  ▪ Polarized case
  ▪ Unpolarized case
Recap on chiral anomaly in QCD:

- Lagrangian invariant under global chiral rotation: \( \psi \rightarrow e^{i\alpha \gamma_5} \psi \)

- Axial-vector current: \( J_5^\mu = \sum_f \bar{\psi}_f \gamma^\mu \gamma_5 \psi_f \)
Recap on chiral anomaly in QCD:

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- Axial-vector current: $J_5^\mu = \sum_f \bar{\psi}_f \gamma^\mu \gamma_5 \psi_f$

- But measure of the path integral is not invariant, which breaks the conservation of the axial current

K. Fujikawa, PRL 1979
Anomaly equation:

\[
\partial_\mu j_5^\mu = -\frac{n_f \alpha_s}{4\pi} F^{\mu\nu} \tilde{F}_{\mu\nu} \quad \tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}
\]

A fundamental property of axial-vector current is the anomaly equation.
Chiral anomaly

Anomaly equation:

\[ \partial_\mu J_5^\mu = -\frac{n_f \alpha_s}{4\pi} F^{\mu\nu} \tilde{F}_{\mu\nu} \]
\[ \tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} \]

A fundamental property of axial-vector current is the anomaly equation.

Adler – Bell - Jackiw chiral anomaly

Famous example: ABJ anomaly contribution to \( \pi^0 \rightarrow 2\gamma \)

In the chiral limit, without the anomaly, \( \pi^0 \) does not decay!

\[ \tau = (0.84 \pm 0.04) \times 10^{-16} \text{ s} \]
Chiral anomaly

Anomaly equation:

$$\partial_\mu J_5^\mu = -\frac{n_f \alpha_s}{4\pi} F_{\mu\nu} \tilde{F}_{\mu\nu}$$

$$\tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$$

A fundamental property of axial-vector current is the anomaly equation

A perturbative solution to anomaly equation:

Calculation in off-forward kinematics ($l = p_2 - p_1$):

$$\langle p_2 | J_5^\mu | p_1 \rangle = \frac{n_f \alpha_s}{4\pi} \left( \frac{i l^\mu}{l^2} \right) p_2 | F_\alpha^\alpha \tilde{F}_\alpha^\alpha | p_1 \rangle$$

Triangle diagram is dominated by infra-red pole
Chiral anomaly

**Axial Form Factors:**

\[
\langle p_2 | J_5^\mu | p_1 \rangle = \bar{u}(p_2) \left[ \gamma^\mu \gamma_5 g_A(t^2) + \frac{v^\mu \gamma_5}{2M} g_P(t^2) \right] u(p_1)
\]

**Calculation in off-forward kinematics** \((l = p_2 - p_1):\)

\[
\langle p_2 | J_5^\mu | p_1 \rangle = \frac{n_f g_A}{4\pi} \frac{i\not{l}^\mu}{l^2} p_2 | F_\alpha^\gamma F_\alpha^\beta | p_1 \rangle
\]

Triangle diagram is dominated by infra-red pole

**A fundamental property of axial vector current is the anomaly equation**

**Massless pole in pseudo scalar Form Factor?**

\[ g_P(t^2) \sim \frac{1}{t^2} \]

**A perturbative solution**

\[ g_A(0) = \Delta \Sigma \] : Fraction of proton spin carried by quarks
Chiral anomaly

Axial Form Factors:

\[ \langle P_2 | J_5^\mu | P_1 \rangle = \bar{u}(P_2) \left[ \gamma_\mu \gamma_5 g_A(l^2) + \frac{l_\mu \gamma_5}{2M} g_P(l^2) \right] u(P_1) \]

A fundamental property of axial-vector current is the anomaly equation:

A perturbative solution:

\[ g_A(0) = \Delta \Sigma \] : Fraction of proton spin carried by quarks

Calculation in off-forward kinematics \( (l = p_2 - p_1) \):

\[ \langle p_2 | J_5^\mu | p_1 \rangle = \frac{n_f \alpha_s}{4\pi} \left( \frac{i l_\mu}{l^2} \right) p_2 | F_\alpha^{\alpha\beta} \tilde{F}_\alpha^{\alpha\beta} | p_1 \rangle \]

Triangle diagram is dominated by infra-red pole

In QCD, we expect:

\[ g_P(l^2) \sim \frac{1}{l^2 - m^2_{\eta'}} \]

Massless pole in pseudo scalar Form Factor? \( g_P(l^2) \sim \frac{1}{l^2} \)
Chiral anomaly

Taking divergence of axial-vector matrix element:

\[ 2M g_A (l^2) + \frac{l^2}{2M} g_P (l^2) = \frac{i \langle P_2 | \frac{n f_\alpha \sigma}{4\pi} \tilde{F} (l^2) | P_1 \rangle}{\bar{u}(P_2) \gamma_5 u(P_1)} \]

Pole cancellation at Form Factor level:

\[ \frac{g_P (l^2)}{2M} = \frac{-2Mg_A (l^2)}{l^2} + \frac{i \langle P_2 | \frac{n f_\alpha \sigma}{4\pi} \tilde{F} (l^2) | P_1 \rangle}{l^2 \bar{u}(P_2) \gamma_5 u(P_1)} \]

In QCD, we expect: \[ g_P (l^2) \sim \frac{1}{l^2 - m_\eta^2} \]

Same pole as what one naively gets from perturbation theory
Recap on trace anomaly in QCD:

- Lagrangian invariant under scale transformation: \( x^\mu \rightarrow e^\sigma x^\mu, \quad \phi \rightarrow e^{-D\sigma} \phi \)

- Dilatation current: \( D^\mu = \Theta^{\mu\nu} x_\nu \)

Energy Momentum Tensor (EMT)

\[
\Theta^{\mu\nu} = -F^{\mu\lambda} F_{\lambda}^{\nu} + \frac{\eta^{\mu\nu}}{4} F^2 + i \bar{\psi} \gamma^{(\mu} \overleftrightarrow{D}^{\nu)} \psi
\]
Recap on trace anomaly in QCD:

- Lagrangian invariant under scale transformation: $x^\mu \to e^\sigma x^\mu$, $\phi \to e^{-D\sigma} \phi$

- Dilatation current: $D^\mu = \Theta^{\mu\nu} x_\nu$ \hspace{1cm} $\Theta^{\mu\nu}$: Energy Momentum Tensor (EMT)

- Conformal symmetry explicitly broken by quantum effects

$$\partial_\mu D^\mu = \Theta_\mu^\mu \neq 0$$
Recap on trace anomaly in QCD:

- A quantum anomaly in the trace of its energy momentum tensor (conformal anomaly) breaks conformal invariance

Trace anomaly:

\[ \Theta_\mu = \frac{\beta(g)}{2g} F^\mu_\nu F_\mu^\nu \]

\[ \Theta^{\mu \nu} : \text{Energy Momentum Tensor (EMT)} \]
Recap on trace anomaly in QCD:

Major scientific goal of EIC: \[ \Theta^\mu_\mu = \frac{\beta(g)}{2g} F^{\mu\nu} F_{\mu\nu} \]

How does the mass of the nucleon arise?

Efforts detailed in a decade worth of reports:

Fundamentally important in QCD: Trace anomaly is the origin of hadron masses

\[ \langle P | \Theta^\mu_\mu | P \rangle = 2M^2 \]
Recap on trace anomaly in QCD:

- A quantum anomaly in the trace of its energy momentum tensor (conformal anomaly) breaks conformal invariance

**Trace anomaly:**

\[
\Theta_\mu = \frac{\beta(g)}{2g} F^\mu\nu F_{\mu\nu}
\]

\(\Theta^{\mu\nu}\) : Energy Momentum Tensor (EMT)

A perturbative solution to anomaly equation:

**Calculation in off-forward kinematics \((l = p_2 - p_1)\):**

\[
\langle p_2 | \Theta_{\text{QCD}}^{\mu\nu} | p_1 \rangle = -\frac{e^2}{24\pi^2 l^2} \left(p^\mu p^\nu + \frac{l^\mu l^\nu}{4} - \frac{l^2 g^{\mu\nu}}{4} \right) \langle p_2 | F^{a\beta} F_{a\beta} | p_1 \rangle
\]

Triangle diagram is dominated by infra-red pole
Recap on trace anomaly in QCD:

**Trace anomaly**

A quantum anomaly in the trace of its energy momentum tensor (conformal anomaly) breaks conformal invariance.

\[ \langle P_2 | \Theta_{\mu \nu} | P_1 \rangle = \frac{1}{M} \bar{u}(P_2) \left[ p_{\mu} p_{\nu} A_f + (A_f + B_f) \frac{P^{(\mu} i\sigma^{\nu)} l_p}{2} + \frac{D_f}{4} \left( l^{\mu} l^{\nu} - g^{\mu \nu} l^2 \right) + M^2 \bar{C} f g^{\mu \nu} \right] u(P_1) \]

**Massless poles in Gravitational Form Factors?**

\[ A_f(l^2), B_f(l^2), D_f(l^2) \sim \frac{1}{l^2} \]

**A perturbative solution to anomaly equation:**

Giannotti, Mottola (2009)

**Calculation in off-forward kinematics** \((l = p_2 - p_1)\):

\[ \langle p_2 | \Theta_{QED}^{\mu \nu} | p_1 \rangle = -\frac{e^2}{24\pi^2 l^2} \left( p_{\mu} p_{\nu} + \frac{l_{\mu} l_{\nu} - l^2 g_{\mu \nu}}{4} \right) \langle p_2 | F^{\alpha \beta} F_{\alpha \beta} | p_1 \rangle \]

Triangle diagram is dominated by infra-red pole.
Recap on trace anomaly in QCD:

- Trace anomaly: A quantum anomaly in the trace of its energy momentum tensor (conformal anomaly) breaks conformal invariance.

A perturbative solution to anomaly equation:

In QCD, we expect:

\[
\frac{1}{l^2} \rightarrow \frac{1}{l^2 - m_G^2}
\]

Massless poles in Gravitational Form Factors?

\[
A_f(l^2), B_f(l^2), D_f(l^2) \sim \frac{1}{l^2}
\]

Triangle diagram is dominated by infra-red pole

Glueball mass generations

Calculation in off-forward kinematics:

\[
\langle p_2 | \Theta_{QED}^{\mu \nu} | p_1 \rangle = -\frac{e^2}{24\pi^2l^2} \left( p^\mu p^\nu + \frac{l^\mu l^\nu - l^2 g^\mu \nu}{4} \right) \langle p_2 | F^{\alpha \beta} F_{\alpha \beta} | p_1 \rangle
\]
The role of chiral anomaly in polarized DIS is a well-known old story
Anomaly in polarized DIS: History

Polarized DIS & proton's spin puzzle:

$g_1$ can be extracted from longitudinal double spin asymmetry:

$$A_{LL} = \frac{\mu^\uparrow p^\uparrow - \mu^\uparrow p^\downarrow}{\mu^\uparrow p^\uparrow + \mu^\uparrow p^\downarrow}$$

$$\sim \left(1 + \frac{\sigma_L}{\sigma_T}\right) \frac{2xg_1}{F_2}$$

First moment of $g_1$:

$$\int_0^1 dx g_1(x) = \frac{1}{9} (\Delta u + \Delta d + \Delta s)$$

$$+ \frac{1}{12} (\Delta u - \Delta d) + \frac{1}{36} (\Delta u + \Delta d - 2\Delta s)$$

$$+ \mathcal{O}(\alpha_s)$$

$g_A(0) = \Delta \Sigma$ : Fraction of proton spin carried by quarks

Deep inelastic scattering (DIS) experiments showed that quarks carry only about 30% of the proton’s spin:

$\Delta \Sigma \approx 0.32$, which is much smaller than predicted by the quark model $\Delta \Sigma \sim 1$ - spin puzzle
Anomaly in polarized DIS: History

Polarized DIS & proton’s spin puzzle:

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A_{LL} = \frac{\mu^\uparrow p^\uparrow - \mu^\uparrow p^\downarrow}{\mu^\uparrow p^\uparrow + \mu^\uparrow p^\downarrow} \approx \left(1 + \frac{\sigma_L}{\sigma_T}\right) \frac{2xg_1}{F_2}
\]

First moment of \( g_1 \):

\[
\int_0^1 dx g_1(x) = \frac{1}{9}(\Delta u + \Delta d + \Delta s) + \frac{1}{12}(\Delta u + \Delta d + \Delta s) + \mathcal{O}(\alpha_s)
\]

Calculate “box diagram”, which is a controversial diagram.
Anomaly in polarized DIS: History

Box diagram

One-loop correction to $g_1$ (gluon channel):

$$g_1(x) \sim \frac{\alpha_s}{2\pi} \left( \ln \frac{Q^2}{m_q^2} \Delta P_{qg}(x) + \delta C_{qq}(x) \right) \otimes \Delta G(x)$$

Polarized DGLAP splitting function: $\Delta P_{qg}(x) = 2x - 1$

Hard coefficient function (mass regularization):

$$\delta C_{qq}(x) = (2x - 1) \left( \ln \frac{1-x}{x} - 1 \right) + 2(1-x)$$

A lot of controversy over this term in the past

(same result in DR)
Anomaly in polarized DIS: History

A solution of spin problem?

The “1 − x” comes from the infrared region of the box diagram:

\[ (1 - x) \int_0^{Q^2} dk_1^2 \frac{m_q^2}{(k_1^2 + m_q^2)^2} = \text{finite!} \]

But the coefficient function is supposed to be dominated by UV physics ...

Polarized DGLAP splitting function: \( \Delta P_{qq}(x) = 2x - 1 \)

Hard coefficient function (mass regularization):

\[ \delta C_{qq}(x) = (2x - 1) \left( \ln \frac{1 - x}{x} - 1 \right) + 2(1 - x) \]

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Anomaly in polarized DIS: History

A solution of spin problem?

The “1 − α” comes from the infrared region of the box diagram. One-loop correction to $g^2 (gluon channel):$

Polarized DGLAP splitting function:

Hard coefficient function (mass regularization):

A lot of controversy over this term in the past

Consider this “anomalous” contribution as a part of “intrinsic spin”:

$$\Delta \tilde{\Sigma} = \Delta \Sigma + \frac{n_f \alpha_s}{2\pi} \Delta G$$

Expect $\Delta \tilde{\Sigma} \sim 1$

If $\Delta G$ is large & positive, this can explain the smallness of $\Delta \Sigma$!
Anomaly in polarized DIS: History

Critique 1

Gluonic contribution to $g_1$ and its relationship to the spin-dependent parton distributions

Geoffrey T. Bodwin and Jianwei Qiu*

usual forms of the quark sum rules. We conclude that the size of the gluonic contribution to the first moment of $g_1$ is entirely a matter of the convention used in defining the quark distributions.

Polarized DGLAP splitting function: $\Delta P_{qq}(x) = 2x - 1$

Hard coefficient function (mass regularization):

$$\delta C_{qq}(x) = (2x - 1) \left( \ln \frac{1 - x}{x} - 1 \right) + 2(1 - x)$$

A lot of controversy over this term in the past
Calculation in forward kinematics: No infrared pole!

Calculation in off-forward kinematics \((l = p_2 - p_1)\):

\[
(p_2|J_5^\mu|p_1) = \frac{n_f \alpha_s}{4\pi} \frac{il^\mu}{l^2} p_2| F_\alpha^\alpha \tilde{F}_\alpha^\beta |p_1)
\]

Triangle diagram is dominated by infra-red pole

\[
\frac{1}{2p^+} \langle p, \pm | j^+_5 | p, \pm \rangle = \mp \frac{\alpha_s N_f}{2\pi} (Carlitz, Collins, Mueller)
\]

The \(g_1\) problem: Deep inelastic electron scattering and the spin of the proton*

R.L. Jaffe and Aneesh Manohar**

not local in the sense required. It is therefore not necessarily protected from infrared effects, in particular from the non-perturbative effects which give the \(\eta'\) a mass*.

regularization-independent. In QCD, the pole at \(l^2 = 0\) is unphysical and is cancelled by non-triangle contributions to the matrix element of \(A^\mu_{\alpha\beta}\). With the aid
Anomaly in polarized DIS: History

Critique 2

Calculation in forward kinematics: $l = p_2 - p_1$

Box diagram can be viewed as a non-local generalization of triangle diagram

If triangle is dominated by anomaly pole, trace of that should be visible in box diagram

Calculation in off-forward kinematics $(l = p_2 - p_1)$:

$$\langle p_2 | J_5^\mu | p_1 \rangle = \frac{n_f \alpha_s}{4\pi} \frac{i\lambda^\mu}{l^2} \langle p_2 | F_{\alpha\beta}^a \tilde{F}_{\alpha\beta}^a | p_1 \rangle$$

Triangle diagram is dominated by infra-red pole

Polarized DGLAP splitting function:

Hard coefficient function (mass regularization):

A lot of controversy over this term in the past (Carlitz, Collins, Mueller)

But the coefficient function is supposed to be

R.L. JAFFE and Aneesh MANOHAR**

not local in the sense required. It is therefore not necessarily protected from infrared effects, in particular from the non-perturbative effects which give the $\gamma'$ a mass*.

regularization-independent. In QCD, the pole at $l^2 = 0$ is unphysical and is cancelled by non-triangle contributions to the matrix element of $A_{\mu}^0$. With the aid
Imprint of Anomalies in DIS

First calculation of box diagram with $l^2 \neq 0$:

Anomaly equation:

The role of the chiral anomaly in polarized deeply inelastic scattering I: Finding the triangle graph inside the box diagram in Bjorken and Regge asymptotics

Andrey Tarasov$^{1,2}$ and Raju Venugopalan$^3$

The role of the chiral anomaly in polarized deeply inelastic scattering II: Topological screening and transitions from emergent axion-like dynamics

Andrey Tarasov$^{1,2}$ and Raju Venugopalan$^3$

Andrey & Raju demonstrated within world-line formalism that to capture the physics of anomaly we need to calculate everything in off-forward kinematics for polarized DIS

Calculation in off-forward kinematics ($l = p_2 - p_1$):

$$\langle p_2 | J_5^\mu | p_1 \rangle = \frac{n_f \alpha_s}{4\pi} \frac{\mathcal{F}_{\alpha\beta}}{l^2} \langle p_2 | F_{\alpha\beta} | p_1 \rangle$$

Triangle diagram is dominated by infra-red pole
Imprint of Anomalies in QCD Compton scattering

Andrey & Raju demonstrated within world-line formalism that to capture the physics of anomaly we need to calculate everything in off-forward kinematics for polarized DIS.

arXiv: 2210.13419 (2022)

Chiral and trace anomalies in Deeply Virtual Compton Scattering

Shohini Bhattacharya,1,* Yoshitaka Hatta,1,2,† and Werner Vogelsang3,‡

We explored the physics of anomaly in DVCS using Feynman-diagram approach.
Imprint of Anomalies in QCD Compton scattering

Kinematics:

\[ q_1 = q + \frac{l}{2} \]
\[ q_2 = q - \frac{l}{2} \]
\[ p_1 = p - \frac{l}{2} \]
\[ p_2 = p + \frac{l}{2} \]
\[ t = l^2 \neq 0 \]

Calculation of imaginary part of anti-symmetric/symmetric \((\mu, \nu)\) of Compton amplitude with non-zero \(t\)
Imprint of Anomalies in QCD Compton scattering

Kinematics:

\[ q_1 = q + \frac{l}{2} \quad q_2 = q - \frac{l}{2} \]

- Usual rationale is that keeping \( t = l^2 \neq 0 \) produces higher twist corrections \( \sim \frac{t}{Q^2} \)

But there are surprises ...

\[ p_1 = p - \frac{l}{2} \quad p_2 = p + \frac{l}{2} \]

Calculation of imaginary part of anti-symmetric/symmetric \((\mu, \nu)\) of Compton amplitude with non-zero \( t \)
Imprint of Anomalies in QCD Compton scattering

Antisymmetric part of Compton amplitude

\[-\epsilon^{\alpha \beta \mu \nu} P_\beta \text{Im} T_{\mu \nu}^{\text{asy}}\]
Imprint of Anomalies in QCD Compton scattering

Antisymmetric part of Compton amplitude

\[-e^{\alpha\beta\mu\nu} P_{\beta} \text{Im} T_{\mu\nu}^{\text{asym}} \approx \frac{1}{2\pi} \frac{\alpha_s}{2} \left( \sum e_j^2 \right) \bar{u}(P_2) \left[ \Delta P_{qg} \ln \frac{Q^2}{l^2} + \delta C_g^{\text{off}} \right] \otimes \Delta G(x_B) \gamma^\alpha \gamma_5 + \frac{l^2}{l^2} \delta C_g^{\text{anom}} \otimes \tilde{F}(x_B) \gamma_5 \right] u(P_1)\]

Expected terms:

Splitting function

\[\Delta P_{qg}(\hat{x}) = 2T_R(2\hat{x} - 1)\]

Coefficient function

\[\delta C_g^{\text{off}}(\hat{x}) = 2T_R(2\hat{x} - 1) \left( \ln \frac{1}{\hat{x}(1 - \hat{x})} - 1 \right)\]

Recall: In DR, one obtains

\[\Delta P_{qg} - \frac{1}{\epsilon} + \delta C_g^{\text{MS}}\]

\[\delta C_g^{\text{MS}}(\hat{x}) = 2T_R(2\hat{x} - 1) \left( \ln \frac{1 - \hat{x}}{\hat{x}} - 1 \right) + 4T_R(1 - \hat{x})\]
Imprint of Anomalies in QCD Compton scattering

Antisymmetric part of Compton amplitude

\[-e^{\alpha\beta\mu\nu} P_\beta \text{Im} T_{\mu\nu}^{\text{asym}} \approx \frac{1}{2\pi} \frac{\alpha_s}{l^2} \sum_i e_i^2 \bar{u}(P_2) \left[ \left( \Delta P_{qg} \ln \frac{Q^2}{l^2} + \delta C_g^{\text{off}} \right) \otimes \Delta G(x_B) \gamma^\alpha \gamma_5 + \frac{l^2}{l^2} \delta C_g^{\text{anom}} \otimes \tilde{F}(x_B) \gamma_5 \right] u(P_1)\]

Pole term
In agreement with Tarasov, Venugopalan

Coefficient function \[\delta C_g^{\text{anom}}(\hat{x}) = 4T_R(1 - \hat{x})\]

Twist-4 GPD:

\[\tilde{F}(x, l^2) = \frac{iP^+}{\bar{u}(P_2)\gamma_5 u(P_1)} \int \frac{d^4z}{2\pi} e^{ixP\cdot z^-} \langle P_2 | F_a^{\mu\nu}(-z^-/2) \tilde{F}_a^{\mu\nu}(z^-/2) | P_1 \rangle\]

(Non-local) chiral anomaly manifests itself in high energy scattering amplitude

Same controversial “1-x” term as in previous calculations! (see earlier slide)
Imprint of Anomalies in QCD Compton scattering

Antisymmetric part of Compton amplitude

\[-e^{\alpha_\beta\mu\nu} \, P_{\beta} \, \text{Im} T^{\text{asym}}_{\mu\nu} \approx \frac{1}{2} \frac{\alpha_s}{2\pi} \left( \sum_f \epsilon_f^2 \right) \bar{u}(P_2) \left( \Delta P_{\gamma g} \ln \frac{Q^2}{l^2} + \delta C^\text{off}_g \right) \otimes \Delta G(x_B) \gamma^\alpha \gamma_5 + \frac{l^\alpha}{l^2} \delta C^\text{anom}_{g} \otimes \tilde{F}(x_B) \gamma_5 \right) u(P_1)\]

**But no suppression in** \(1/Q^2\)!  
**Twist-4 GPD**

The QCD factorization theorem: Collins, Freund; Ji, Osborne (1998)

\[-e^{\alpha_\beta\mu\nu} \, P_{\beta} \, \text{Im} T^{\text{asym}}_{\mu\nu} = \frac{1}{2} \sum_f \epsilon_f^2 \bar{u}(P_2) \left[ \gamma^\alpha \gamma_5 (\tilde{H}_f(x_B, \xi, l^2) + \tilde{H}_f(-x_B, \xi, l^2)) + \frac{l^\alpha \gamma_5}{2M} (\tilde{E}_f^{\text{bare}}(x_B, \xi, l^2) + \tilde{E}_f^{\text{bare}}(-x_B, \xi, l^2)) \right] u(P_1)\]

**Twist-2 GPDs** to all orders

(Non-local) chiral anomaly manifests itself in high energy scattering amplitude & apparently breaks QCD factorization
Imprint of Anomalies in QCD Compton scattering

Antisymmetric part of Compton amplitude

\[ -\epsilon^{\alpha\beta\mu\nu} P_\beta \text{Im} T^{\text{asym}}_{\mu\nu} \approx \frac{\alpha_s}{2\pi} \left( \sum_f e_f^2 \right) \bar{u}(P_2) \left[ \left( \Delta P_{qg} \ln \frac{Q^2}{l^2} + \delta C_g^{\text{off}} \right) \otimes \Delta G(x_B) \gamma^\alpha \gamma_5 + \frac{l^\alpha}{l^2} C_g^{\text{anom}} \otimes \tilde{F}(x_B) \gamma_5 \right] u(P_1) \]

Anomalous contribution to GPD $\tilde{E}$ at one loop

The QCD factorization theorem: Collins, Freund; Ji, Osborne (1998)

\[ -\epsilon^{\alpha\beta\mu\nu} P_\beta \text{Im} T^{\text{asym}}_{\mu\nu} = \frac{1}{2} \sum_f e_f^2 \bar{u}(P_2) \left[ \gamma^\alpha \gamma_5 (\tilde{H}_f(x_B, \xi, l^2) + \tilde{H}_f(-x_B, \xi, l^2)) + \frac{l^\alpha \gamma_5}{2M} \tilde{E}_f^{\text{bare}}(x_B, \xi, l^2) + \tilde{E}_f^{\text{bare}}(-x_B, \xi, l^2) \right] u(P_1) + O(\alpha_s) + O(1/Q^2), \]

Twist-2 GPDs to all orders

(Non-local) chiral anomaly manifests itself in high energy scattering amplitude & apparently breaks QCD factorization
Imprint of Anomalies in QCD Compton scattering

Antisymmetric part of Compton amplitude

Imprint of Anomalies in QCD Compton scattering

\[
\text{Imprint of Anomalies in QCD Compton scattering}
\]

ONE-LOOP QCD CORRECTIONS TO DEEPLY-VIRTUAL COMPTON SCATTERING: THE PARTON HELICITY-INDEPENDENT CASE

Xiangdong Ji and Jonathan Osborne

Predictions from conformal algebra for the deeply virtual Compton scattering.

A.V. Belitsky D. Müller

NLO Corrections to Deeply-Virtual Compton Scattering

L. Mankiewicz*, G. Piller*, E. Stein*, M. Vänttinen* and T. Weigl*

NLO corrections to timelike, spacelike and double deeply virtual Compton scattering.

B. Pire¹ and L. Szymanski² and J. Wagner²
Elusive pole

Pole was unnoticed in the GPD literature because one typically assumes

\[ l^\mu = -2 \xi p^\mu \rightarrow t = l^2 = 0 \]

before loop integration

Usual rationale: Corrections supposedly higher twist

\[ \frac{t}{Q^2} \]

Non-trivial chiral anomaly manifests itself in high energy scattering amplitudes and

NLO corrections to timelike, spacelike and double deeply virtual Compton scattering.

B. Pire, L. Szymanowski, and J. Wagner
Imprint of Anomalies in QCD Compton scattering

Antisymmetric part of Compton amplitude

Elusive pole

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Usual rationale: Corrections supposedly higher twist

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However, box diagram is power-divergent in the IR!

Still, pole was never seen before because:

\[ \langle p_2 | F^{\mu\nu} \tilde{F}_{\mu\nu} | p_1 \rangle \propto \epsilon^{\mu\nu\alpha\beta} l_\mu p_\nu \epsilon_{1\alpha} \epsilon_{2\beta} \]

\[ \rightarrow 0 \quad \text{when} \quad l^\mu \propto p^\mu \]
Perturbative calculations suggest that massless poles are induced in GPD $\tilde{E}$.

However, we know there are no massless poles in axial form factor (moment of GPD $\tilde{E}$).

$$g_P(l^2) = \int dx \tilde{E}(x) \sim \frac{1}{l^2}$$

The QCD factorization theorem: Collins, Freund; Ji, Osborne (1998)

Deeply tied to the UA(1) problem: Why is the $\eta'$ so massive (957 MeV)?

Twist-2 GPDs to all orders

(Non-local) chiral anomaly manifests itself in high energy scattering amplitude & apparently breaks QCD factorization
Redefine

\[
\tilde{E}_f(x_B, l^2) + \tilde{E}_f(-x_B, l^2) = \tilde{E}_f^{\text{bare}}(x_B, l^2) + \tilde{E}_f^{\text{bare}}(-x_B, l^2) + \frac{\alpha_s}{2\pi} \frac{2M}{l^2} \delta C_g^{\text{anom}} \otimes \tilde{F}(x_B, l^2)
\]

“Bare GPD” (tree level)

Perturbative pole (one loop)

Twist-2 GPDs to all orders

(Non-local) chiral anomaly manifests itself in high energy scattering amplitude & apparently breaks QCD factorization

Imprint of Anomalies in QCD Compton scattering

The QCD factorization theorem: Collins, Freund, Ji, Osborne (1998)

\[
-e^{\beta \nu} P_\beta \text{Im} T^{\text{asym}} = \frac{1}{2} \sum_f \epsilon_f^2 \bar{u}(P_2) \left[ \gamma^\alpha \gamma_5 (\tilde{H}_f(x_B, \xi, l^2) + \tilde{H}_f(-x_B, \xi, l^2)) + \frac{\alpha_s}{2M} \delta C_g^{\text{anom}} \otimes \tilde{F}(x_B, l^2) \right] u(P_1) + \mathcal{O}(\alpha_s) + \mathcal{O}(1/Q^2),
\]
Imprint of Anomalies in QCD Compton scattering

The QCD factorization theorem:

\[ p_1 = p - \frac{1}{2} \]
\[ p_2 = p + \frac{1}{2} \]

Antisymmetric part of Compton amplitude

Redefine

Perturbative pole in GPD

\[ \int \frac{dz^-}{4\pi} e^{i\hat{p} \cdot z^-} (p_2 \bar{\psi}(-z^-/2)\gamma^+\gamma_5\psi(z^-/2)|p_1) \bigg|_{\text{pole}} \sim \frac{\alpha_s}{2\pi} \frac{2i\ell^+}{l^2} (1 - \hat{x}) \otimes \delta(1 - \hat{x}) \epsilon_{1\ell} \epsilon_{2\ell}^* \]

Same pole in one-loop calculation!

Perhaps not an ad hoc argument!

Chiral anomaly manifests itself in high energy scattering amplitude & apparently breaks QCD factorization

The pole “belongs” to GPD
Imprint of Anomalies in QCD Compton scattering

The QCD factorization theorem: Collins, Freund, Ji, Osborne (1998)

The QCD factorization theorem: Collins, Freund, Ji, Osborne (1998)

Redefine

\[ \hat{E}_f(x_B, t^2) + \hat{E}_f(-x_B, t^2) = \hat{E}^{\text{bare}}_f(x_B, t^2) + \hat{E}^{\text{bare}}_f(-x_B, t^2) + \frac{\alpha_s}{2\pi} \frac{2M}{l^2} \delta \mathcal{C}_g^{\text{anom}} \otimes \tilde{F}(x_B, t^2) \]

“Bare GPD” (tree level) Perturbative pole (one loop)

Postulate that the perturbative pole cancels the pre-existing pole in “bare” GPD:

\[ \hat{E}^{\text{bare}}_f(x_B, t^2) + \hat{E}^{\text{bare}}_f(-x_B, t^2) \approx -\frac{\alpha_s}{2\pi} \frac{2M}{l^2} \delta \mathcal{C}_g^{\text{anom}} \otimes \tilde{F}(x_B, t^2 = 0) \]

Postulate that the “renormalized” GPD integrates to \( g_P(l^2) \):

\[ g_P(l^2) = \sum_f \int_{-1}^{1} dx \hat{E}_f(x, l^2) = \sum_f \int_{0}^{1} dx (\hat{E}_f(x, l^2) + \hat{E}_f(-x, l^2)) \]
Imprint of Anomalies in QCD Compton scattering

The QCD factorization theorem:

Collins, Freund; Ji, Osborne (1998)

Twist-

2 GPDs to all orders

Redefine

\[ \tilde{E}_f(x_B, l^2) + \tilde{E}_f(-x_B, l^2) = \tilde{E}_f^{\text{bare}}(x_B, l^2) + \tilde{E}_f^{\text{bare}}(-x_B, l^2) + \frac{\alpha_s}{2\pi} \frac{2M}{l^2} \delta C_g^{\text{anom}} \otimes \tilde{F}(x_B, l^2) \]

"Bare GPD" (tree level)  Perturbative pole (one loop)

Postulate that the "renormalized" GPD integrates to \( g_P(l^2) \):

\[ g_P(l^2) = \sum_f \int_{-1}^1 dx \tilde{E}_f(x, \xi, l^2) = \sum_f \int_{-1}^1 dx (\tilde{E}_f(x, \xi, l^2) + \tilde{E}_f(-\xi, l^2)) \]

Pole cancellation at \( \int dx \):

\[ g_P(l^2) \]

We find:

\[ \frac{g_P(l^2)}{2M} = -\frac{i}{l^2} \left( \frac{\langle P_2 \frac{\rho_{QG}}{4\pi} F \tilde{F} | P_1 \rangle}{\bar{u}(P_2) \gamma_5 u(P_1)} \right|_{l^2=0} - \frac{\langle P_2 \frac{\rho_{QG}}{4\pi} F \tilde{F} | P_1 \rangle}{\bar{u}(P_2) \gamma_5 u(P_1)} \]
Imprint of Anomalies in QCD Compton scattering

The QCD factorization theorem: Collins, Freund, Ji, Osborne (1998)

Twist -2 GPDs to all orders

"We demonstrate that the dynamical interplay between the physics of the anomaly, and that of the isosinglet pseudoscalar $U_A(1)$ sector of QCD resolves both problems simultaneously: the lifting of the $\bar{\eta}$ pole by topological mass generation of the $\eta'$ and the cancellation of the anomaly pole"

- Tarasov, Venugopalan

See also Jaffe Manohar, 1990
Imprint of Anomalies in QCD Compton scattering

Symmetric part of Compton amplitude \((\xi \neq 0)\)
Imprint of Anomalies in QCD Compton scattering

Symmetric part of Compton amplitude \((\xi \neq 0)\)

Pole! (New result)

\[
(H_f(x_B, \xi, l^2) - H_f(-x_B, \xi, l^2)) = (H_f^\text{bare}(x_B, \xi, l^2) - H_f^\text{bare}(-x_B, \xi, l^2)) + \frac{\alpha_s}{2\pi} \frac{1}{l^2} C_{\text{anom}} \otimes \mathcal{F}(x_B, \xi, l^2)
\]

\[
(E_f(x_B, \xi, l^2) - E_f(-x_B, \xi, l^2)) = (E_f^\text{bare}(x_B, \xi, l^2) - E_f^\text{bare}(-x_B, \xi, l^2)) - \frac{\alpha_s}{2\pi} \frac{1}{l^2} C_{\text{anom}} \otimes \mathcal{F}(x_B, \xi, l^2)
\]

“Bare GPD” (tree level)

Perturbative pole (one loop)
Imprint of Anomalies in QCD Compton scattering

**Symmetric part of Compton amplitude** \((\xi \neq 0)\)

**Pole! (New result)**

\[
(H_f(x_B, \xi, l^2) - H_f(-x_B, \xi, l^2)) = (H_f^{\text{bare}}(x_B, \xi, l^2) - H_f^{\text{bare}}(-x_B, \xi, l^2)) + \frac{\alpha_s}{2\pi} \frac{1}{l^2} C^{\text{anom}} \otimes' \mathcal{F}(x_B, \xi, l^2)
\]

\[
(E_f(x_B, \xi, l^2) - E_f(-x_B, \xi, l^2)) = (E_f^{\text{bare}}(x_B, \xi, l^2) - E_f^{\text{bare}}(-x_B, \xi, l^2)) - \frac{\alpha_s}{2\pi} \frac{1}{l^2} C^{\text{anom}} \otimes' \mathcal{F}(x_B, \xi, l^2)
\]

**Hatta, Zhao (2020); Radyushkin, Zhao (2021)**

**Twist-4 GPD:**

\[
\mathcal{F}(x, \xi, l^2) = -4xP^+ M \int \frac{dz^-}{2\pi} e^{ixP^+ z} \frac{\langle P_2 | F^{\mu\nu}(-z^-/2) F_{\mu\nu}(z^-/2) | P_1 \rangle}{\bar{u}(P_2) u(P_1)}
\]

“Bare GPD” (tree level)

Perturbative pole (one loop)

(Non-local) trace anomaly manifests itself in high energy scattering amplitude & apparently breaks QCD factorization
Imprint of Anomalies in QCD Compton scattering

Symmetric part of Compton amplitude \((\xi \neq 0)\)

Pole! (New result)

\[
(H_f(x_B, \xi, l^2) - H_f(-x_B, \xi, l^2)) = (H_f^{\text{bare}}(x_B, \xi, l^2) - H_f^{\text{bare}}(-x_B, \xi, l^2)) + \frac{\alpha_s}{2\pi} \frac{1}{l^2} C_{\text{anom}} \otimes' \mathcal{F}(x_B, \xi, l^2)
\]

The pole "belongs" to GPD

Perturbative pole in GPD

\[
\int \frac{dk^- d^2k_\perp}{(2\pi)^{3-2\epsilon}} \text{Tr}[\gamma_\alpha(k^+ + \frac{l}{2})\gamma^\beta(k^+ - \frac{l}{2})\gamma^0(k_\perp - \frac{l}{2})] \frac{\epsilon_\alpha(p + \frac{l}{2}) \epsilon_\beta(p - \frac{l}{2})}{(p - k)^2(k - l/2)^2(k + l/2)^2}
\]

Same pole in one-loop calculation!
Imprint of Anomalies in QCD Compton scattering

**Polynomiality:**
Symmetric part of Compton amplitude \( (\xi \neq 0) \)

**Structure of convolution:**

\[
C_{\text{anom}} \otimes' F(x_B, \xi, l^2) \equiv \int_{x_B}^1 \frac{dx}{x} K(\hat{\mu}, \hat{\nu})F(x, \xi, l^2) - \frac{\theta(\xi - x_B)}{2} \int_{-1}^1 \frac{dx}{x} L(\hat{\mu}, \hat{\nu})F(x, \xi, l^2)
\]

\[
K(\hat{\mu}, \hat{\nu}) = 2T_R \frac{-1 + \hat{\mu}}{1 - \hat{\nu}^2}
\]

\[
L(\hat{\mu}, \hat{\nu}) = 2T_R \frac{-1 + \hat{\nu}}{1 - \hat{\mu}^2}
\]

**Twist-4 GPD:**
\[
F(x, \xi, l^2) = -4xP^+ M \int \frac{dz^-}{2\pi} \frac{e^{i x P^+ z}}{u(z^-/2) u(z^-/2)} P_2 \bar{u}(z^-/2) u(P_1)
\]

Hatta, Zhao (2020); Radyushkin, Zhao (2021)

**Non-zero for non-zero skewness**

**“Bare GPD” (tree level)**

**Perturbative pole (one loop)**

(Non-local) trace anomaly manifests itself in high energy scattering amplitude & apparently breaks QCD factorization
Imprint of Anomalies in QCD Compton scattering

**Polynomiality:**
Symmetric part of Compton amplitude \( (\xi \neq 0) \)

**Structure of convolution:**
\[
C^\text{anom} \otimes' F(x_B, \xi, l^2) \equiv \int_{x_B}^{1} \frac{dx}{x} K(\hat{x}, \hat{\xi}) F(x, \xi, l^2) - \frac{\theta(x - x_B)}{2} \int_{-1}^{1} \frac{dx}{x} L(\hat{x}, \hat{\xi}) F(x, \xi, l^2)
\]

**Example**
\[
\sum_f e_f^2 x_B (H_f^\text{bare}(x_B, \xi, l^2) - H_f^\text{bare}(-x_B, \xi, l^2)) \approx \frac{\alpha_s}{2\pi} \left( \sum_f e_f^2 \right) \frac{x_B}{l^2} \left[ C^\text{anom} \otimes' F(x_B, \xi, l^2 = 0) \right]
\]

**Twist-4 GPD:**
\[
\sum_f e_f^2 (A_f^\text{bare}(l^2) + \xi^2 D_f^\text{bare}(l^2)) \approx \frac{T_R\alpha_s}{12\pi l^2} \left( \sum_f e_f^2 \right) \left( \frac{\langle P|F_{\alpha\beta}(i\not{D} + \not{P}) F_{\alpha\beta}|P\rangle}{(P^+)^2} + \xi^2 \langle P|F^2|P\rangle \right)
\]
Imprint of Anomalies in QCD Compton scattering

Symmetric part of Compton amplitude \((\xi \neq 0)\)

**Pole! (New result)**

\[
(H_f(x_B, \xi, l^2) - H_f(-x_B, \xi, l^2)) = (H_f^{\text{bare}}(x_B, \xi, l^2) - H_f^{\text{bare}}(-x_B, \xi, l^2)) + \frac{\alpha_s}{2\pi} \frac{1}{l^2} C^{\text{anom}} \otimes' \mathcal{F}(x_B, \xi, l^2)
\]

\[
(E_f(x_B, \xi, l^2) - E_f(-x_B, \xi, l^2)) = (E_f^{\text{bare}}(x_B, \xi, l^2) - E_f^{\text{bare}}(-x_B, \xi, l^2)) - \frac{\alpha_s}{2\pi} \frac{1}{l^2} C^{\text{anom}} \otimes' \mathcal{F}(x_B, \xi, l^2)
\]

“Bare GPD” (tree level)

Perturbative pole (one loop)
Imprint of Anomalies in QCD Compton scattering

Symmetric part of Compton amplitude \((\xi \neq 0)\)

We proposed a possible scenario of pole cancellation in order to justify factorization

**glueball mass generations**

D term

\[
D_f(l^2) \approx M \frac{T_R n_f \alpha_s}{6\pi l^2} \left( \frac{\langle P_2 | F^2 | P_1 \rangle}{\bar{u}(P_2) u(P_1)} \right) \bigg|_{l^2=0} - \frac{\langle P_2 | F^2 | P_1 \rangle}{\bar{u}(P_2) u(P_1)}
\]

\[A(l^2), B(l^2), D(l^2) \sim \frac{1}{l^2 - m_G^2}\]

(Non-local) trace anomaly manifests itself in high energy scattering amplitude & apparently breaks QCD factorization

Hatta, Zhao (2020); Radyushkin, Zhao (2020)
Imprint of Anomalies in QCD Compton scattering

Real part of Compton amplitude: (SB, Hatta, Vogelsang, In Preparation)
Imprint of Anomalies in QCD Compton scattering

Real part of Compton amplitude: (SB, Hatta, Vogelsang, In Preparation)

Example: Antisymmetric case

\[ \text{Pole in real part!} \]

\[ \sim \frac{16\langle F \tilde{F} \rangle}{l^2(-1 + \xi^2)} \left( -(-1 + x) \ln \frac{x - 1}{x} + (x - \xi) \ln \frac{x - \xi}{x} \right) - (x \to -x) \]

\[ + \tilde{g}_{qq}(\hat{x}, \hat{\xi}) \ln \frac{Q^2}{\Lambda^2} \]

\[ + \tilde{C}_1^g(\hat{x}, \hat{\xi}) \]

\[ 2T_R \int_0^1 \frac{dx}{x} \left(1 - \hat{x}\right) \ln \frac{\hat{x} - 1}{x} + (\hat{x} - \hat{\xi}) \ln \frac{\hat{x} - \hat{\xi}}{\hat{x}} - (\hat{x} \to -\hat{x}) \tilde{F}(x, \xi, t) \]

\[ = \int_0^1 dy \tilde{C}_0(y, x_B) \left[ \int_0^1 \frac{dx}{x} \tilde{K} \left( y, x, \xi \right) \tilde{F}(x, \xi, t) - \theta(\xi - y) \int_0^1 \frac{dx}{x} \tilde{L} \left( y, x, \xi \right) \tilde{F}(x, \xi, t) \right] \]

Same structure for convolution!
Even in real part, same mechanism should cancel pole as in imaginary part
Imprint of Anomalies in QCD Compton scattering

Real part of Compton amplitude:  (SB, Hatta, Vogelsang, In Preparation)

Example: Antisymmetric case

\[
\sum_{l} \left( \frac{16 \langle F F \rangle}{l^2 (1 + \xi^2)} \left( (-1 + x) \ln \frac{x - 1}{x} + (x - \xi) \ln \frac{x - \xi}{x} \right) - (x \to -x) \right)
\]

\[+ \tilde{\kappa}_{qg}(\hat{x}, \hat{\xi}) \ln \frac{Q^2}{l^2}\]

\[+ \tilde{G}_{1}^{g}(\hat{x}, \hat{\xi})\]

Reproduced the known logarithms from literature

\[
\tilde{\kappa}_{qg}(\hat{x}, \hat{\xi}) = \frac{2\hat{x} - 1 - \hat{\xi}^2}{2(1 - \hat{\xi}^2)^2} \ln \frac{\hat{x} - 1}{\hat{x}} - \frac{\hat{x} - \hat{\xi}}{(1 - \hat{\xi}^2)^2} \ln \frac{\hat{x} - \hat{\xi}}{\hat{x}} - (\hat{x} \to -\hat{x})
\]

Ji, Osborne; Belitsky, Mueller
Imprint of Anomalies in QCD Compton scattering

Real part of Compton amplitude: (SB, Hatta, Vogelsang, In Preparation)

Example: Antisymmetric case

\[
\sim \frac{16 \langle F \tilde F \rangle}{l^2 (-1 + \xi^2)} \left( -(-1 + x) \ln \frac{x - 1}{x} + (x - \xi) \ln \frac{x - \xi}{x} \right) - (x \to -x)
\]

\[+ \tilde k_{qg}(\hat x, \hat \xi) \ln \frac{Q^2}{l^2}\]

\[+ \tilde C^g_1(\hat x, \hat \xi) \quad \text{Coefficient function}\]
Imprint of Anomalies in QCD Compton scattering

Quark-channel diagrams in DVCS:  (SB, Hatta, Vogelsang, In Preparation)
Imprint of Anomalies in QCD Compton scattering

Quark-channel diagrams in DVCS:  (SB, Hatta, Vogelsang, In Preparation)

Example: Antisymmetric case

\[ \sim \frac{1}{\hat{l}^2} + \tilde{\kappa}_{qq}(\hat{x}, \hat{\xi}) \ln \frac{Q^2}{\hat{l}^2} + \delta\tilde{C}^q_1(\hat{x}, \hat{\xi}) - (\hat{x} \rightarrow -\hat{x}) \]

No pole!
Imprint of Anomalies in QCD Compton scattering

Quark-channel diagrams in DVCS:  (SB, Hatta, Vogelsang, In Preparation)

Example: Antisymmetric case

\[
\tilde{\kappa}_{qq}(\hat{x}, \hat{\xi}) \sim \frac{1}{l^2} \left[ \tilde{\kappa}_{qq}(\hat{x}, \hat{\xi}) \ln \frac{Q^2}{l^2} \right] + \delta \tilde{C}_1^q(\hat{x}, \hat{\xi}) - (\hat{x} \rightarrow -\hat{x})
\]

Reproduced the known logarithms from literature

\[
\tilde{\kappa}_{qq}(\hat{x}, \hat{\xi}) = \frac{3}{2(1 - \hat{x})} + \frac{\hat{x}^2 + 1 - 2\hat{\xi}^2}{(1 - \hat{\xi}^2)(1 - \hat{x})} \ln \frac{\hat{x} - 1}{\hat{x}} - \frac{(\hat{x} - \hat{\xi})(1 + \hat{x}^2 + 2\hat{x}\hat{\xi})}{(1 - \hat{x}^2)(1 - \hat{\xi}^2)} \ln \frac{\hat{x} - \hat{\xi}}{\hat{x}} - (\hat{x} \rightarrow -\hat{x})
\]

Ji, Osborne; Belitsky, Mueller
Imprint of Anomalies in QCD Compton scattering

Quark-channel diagrams in DVCS:  (SB, Hatta, Vogelsang, In Preparation)

Example: Antisymmetric case

\[ \sim \frac{1}{\ell^2} + \kappa_{qq}(\hat{x}, \hat{\xi}) \ln \frac{Q^2}{-\ell^2} + \delta \tilde{C}_1^q(\hat{x}, \hat{\xi}) \quad (\hat{x} \rightarrow -\hat{x}) \]

Coefficient function

\[ \delta \tilde{C}_1^q(\hat{x}, \hat{\xi}) = - \frac{\left(\frac{Q^2}{-\ell^2}\right)^\epsilon}{\epsilon^2(1 - \hat{x})} + \frac{3}{2\epsilon(1 - \hat{x})} + \cdots - (\hat{x} \rightarrow -\hat{x}) \]

Unexpected double IR pole
Imprint of Anomalies in QCD Compton scattering

Quark-channel diagrams in DVCS: (SB, Hatta, Vogelsang, In Preparation)

Example: Antisymmetric case

Usually, double poles are cancelled by real gluon emissions

Here, we don’t have such a contribution

Coefficient function

$$\delta \tilde{C}_1^q(\hat{x}, \hat{\xi}) = -\frac{(Q^2)}{-l^2} \epsilon \frac{3 (Q^2)}{-l^2} \epsilon \frac{(1 - \hat{x})}{2\epsilon(1 - \hat{x})} + \cdots - (\hat{x} \rightarrow -\hat{x})$$

Unexpected double IR pole
Imprint of Anomalies in QCD Compton scattering

Quark-channel diagrams in DVCS: (SB, Hatta, Vogelsang, In Preparation)

Example: Antisymmetric case

\[
\sim \frac{1}{l^2} + \bar{\kappa}_{qq} (\hat{x}, \hat{\xi}) \ln \frac{Q^2}{l^2} + \delta \tilde{C}_1^q (\hat{x}, \hat{\xi}) - (\hat{x} \to -\hat{x})
\]

Coefficient function

\[
\delta \tilde{C}_1^q (\hat{x}, \hat{\xi}) = -\frac{\left( \frac{Q^2}{-l^2} \right)^\epsilon}{\epsilon^2 (1 - \hat{x})} - \frac{3 \left( \frac{Q^2}{-l^2} \right)^\epsilon}{2\epsilon (1 - \hat{x})} + \cdots - (\hat{x} \to -\hat{x})
\]

Unexpected single IR pole
Imprint of Anomalies in QCD Compton scattering

Quark-channel diagrams in DVCS: (SB, Hatta, Vogelsang, In Preparation)

It looks like factorization is broken due to the unexpected double, single IR poles

But, when you compute GPD itself, you find the same double, single IR poles! These poles can be systematically absorbed into GPD

\[ \delta \tilde{C}_1^q(\hat{x}, \hat{\xi}) = -\frac{\left( \frac{Q^2}{-l^2} \right)^\epsilon}{\epsilon^2(1 - \hat{x})} - \frac{3}{2\epsilon(1 - \hat{x})} \left( \frac{Q^2}{-l^2} \right)^\epsilon + \cdots - (\hat{x} \to -\hat{x}) \]
Imprint of Anomalies in QCD Compton scattering

Quark-channel diagrams in DVCS: (SB, Hatta, Vogelsang, In Preparation)

Remarks:

• Off-forwardness is an alternative factorization scheme that clarifies the physics of anomaly

But, when you compute GPD itself, you find the same double, single IR poles! These poles can be systematically absorbed into GPD

• Novel connection between twist 2 & twist 4 sectors at the density level due to anomaly

\[
\delta \hat{C}_1^q(x, \hat{\xi}) = -\frac{(Q^2)\epsilon}{\epsilon^2(1-x)} - \frac{3}{2\epsilon(1-x)} + \cdots (\hat{x} \to -\hat{x})
\]
Summary

• Revisited QCD factorization for Compton scattering: Crucial topic for ongoing & future experiments including at EIC

• Importance to understand off-forward poles originating from chiral & trace anomalies

\[ T^{\mu\nu} \sim \frac{\langle F \tilde{F} \rangle}{l^2}, \quad \frac{\langle FF \rangle}{l^2} \quad \text{Unnoticed in literature} \]
Summary

Perturbative calculations suggest that massless poles are induced in GPDs $\tilde{E}, H, E$

- Revisited QCD factorization for Compton scattering: Crucial topic for ongoing & future experiments including at EIC

However, we know there are no massless poles in axial and gravitational form factors (moments of GPDs)

- Importance to understand off-forward poles originating from chiral & trace anomalies

$$T^{\mu\nu} \sim \frac{\langle F \tilde{F} \rangle}{l^2}, \quad \frac{\langle F F \rangle}{l^2}$$

Unnoticed in literature
Summary

Perturbative calculations suggest that massless poles are induced in GPDs $\tilde{E}, H, E$

- Revisited QCD factorization for Compton scattering: Crucial topic for ongoing & future experiments including at EIC

However, we know there are no massless poles in axial and gravitational form factors (moments of GPDs)

We proposed a possible scenario of pole cancellation

This has to do with eta-meson & glueball mass generations

Importance to understand off-forward poles originating from chiral & trace anomalies

cf, the $\eta'$ mass problem

Antisymmetric case:

Symmetric case:
Summary

• Revisited QCD factorization for Compton scattering: Crucial topic for ongoing & future experiments including at EIC

• Importance to understand off-forward poles originating from chiral & trace anomalies

\[ T^\mu_\nu \sim \frac{\langle F \bar{F} \rangle}{l^2}, \quad \frac{\langle FF \rangle}{l^2} \quad \text{Unnoticed in literature} \]

Off-forwardness is a physical factorization scheme that elucidates the physics of anomaly

Profound physical implication of poles; touches questions on mass generations, Chiral symmetry breaking, ...
Summary & outlook

- Revisited QCD factorization for Compton scattering: Crucial topic for ongoing & future experiments including at EIC
  - Unnoticed in literature

- Importance to understand off-forward poles originating from chiral & trace anomalies
  - Off-forwardness is a physical factorization scheme that elucidates the physics of anomaly
  - Profound physical implication of poles; touches questions on mass generations, Chiral symmetry breaking, ...

Novel connections between DVCS & chiral/trace anomalies:
This could be a new & potentially rich avenue for GPD research

Imprint of anomaly on other physical processes:
(Example: Deeply-virtual meson production)
Backup slides
Imprint of Anomalies in QCD Compton scattering

FIG. 1: Box diagrams for the Compton amplitude in off-forward kinematics.
Imprint of Anomalies in QCD Compton scattering

\[
\frac{g_P(l^2)}{2M} \approx -\frac{2M \Delta \Sigma}{l^2 - m_{\eta'}^2}, \quad i \frac{\langle P_2 \mid \frac{n_f {\alpha}_s}{4\pi} F \tilde{F} \mid P_1 \rangle}{\bar{u}(P_2) \gamma_5 u(P_1)} \approx -2M \Delta \Sigma \frac{m_{\eta'}^2}{l^2 - m_{\eta'}^2}.
\]
Example: Antisymmetric part of Compton amplitude

\[
F_1^{\text{eff}}(x_B, l) \approx \frac{1}{2} \frac{\alpha_s}{2\pi} \left( \sum_f e_f^2 \right) \left[ \left( P_{qg} \ln \frac{Q^2}{-l^2} + C_{1g}^{\text{eff}} \right) \otimes g(x_B) + \frac{1}{l^2} C_{\text{anom}} \otimes' \mathcal{F}(x_B, \xi, l^2) \frac{u(P_2) u(P_1)}{2M} \right],
\]

\[
F_2^{\text{eff}}(x_B, l) \approx x_B \frac{\alpha_s}{2\pi} \left( \sum_f e_f^2 \right) \left[ \left( P_{qg} \ln \frac{Q^2}{-l^2} + C_{2g}^{\text{eff}} \right) \otimes g(x_B) + \frac{1}{l^2} C_{\text{anom}} \otimes' \mathcal{F}(x_B, \xi, l^2) \frac{u(P_2) u(P_1)}{2M} \right].
\] (31)

We recognize the expected structure of the one-loop corrections associated with the unpolarized gluon PDF \( g(x) \), with the splitting function \( P_{qg}(\hat{x}) = 2T_R((1-\hat{x})^2 + \hat{x}^2) \). The coefficient functions are given by

\[
C_{1g}(\hat{x}) = 2T_R((1-\hat{x})^2 + \hat{x}^2) \left( \ln \frac{1}{\hat{x}(1-\hat{x})} - 1 \right),
\]

\[
C_{2g}(\hat{x}) = 2T_R((1-\hat{x})^2 + \hat{x}^2) \left( \ln \frac{1}{\hat{x}(1-\hat{x})} - 1 \right) + 8T_R \hat{x}(1-\hat{x}).
\] (32)

In addition, we find a pole \( 1/l^2 \) in both \( F_1^{\text{eff}} \) and \( F_2^{\text{eff}} \) (but not in the difference \( F_2^{\text{eff}} - 2x_B F_1^{\text{eff}} \) relevant to the longitudinal structure function), with the following convolution formula

\[
C_{\text{anom}} \otimes' \mathcal{F}(x_B, \xi, l^2) = \int_{x_B}^1 \frac{dx}{x} K(\hat{x}, \hat{\xi}) \mathcal{F}(x, \xi, l^2) - \frac{\theta(\xi - x_B)}{2} \int_{-1}^1 \frac{dx}{x} L(\hat{x}, \hat{\xi}) \mathcal{F}(x, \xi, l^2),
\] (33)

where

\[
K(\hat{x}, \hat{\xi}) = 2T_R \frac{\hat{x}(1-\hat{x})}{1-\hat{\xi}^2}, \quad L(\hat{x}, \hat{\xi}) = 2T_R \frac{\hat{\xi}(\hat{\xi} - \hat{x})}{1-\hat{\xi}^2}.
\] (34)
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Polynomiality:

\[
\int_0^1 dx_B x_B \left( H_f(x_B, \xi, l^2) - H_f(-x_B, \xi, l^2) \right) = \int_{-1}^1 dx_B x_B H_f(x_B, \xi, l^2) = A_f(l^2) + \xi^2 D_f(l^2),
\]

\[
\int_0^1 dx_B x_B \left( E_f(x_B, \xi, l^2) - E_f(-x_B, \xi, l^2) \right) = \int_{-1}^1 dx_B x_B E_f(x, \xi, l^2) = B_f(l^2) - \xi^2 D_f(l^2).
\]

\[
\sum_f e_f^2 \left( A_f^{\text{bare}}(l^2) + \xi^2 D_f^{\text{bare}}(l^2) \right) \approx \frac{T_R \alpha_s}{12\pi l^2} \left( \sum_f e_f^2 \right) \left( \frac{\langle P | F^{\alpha\beta}(i \vec{D}^+)^2 F_{\alpha\beta} | P \rangle}{(P^+)^2} + \xi^2 \langle P | F^2 | P \rangle \right),
\]

\[
\sum_f e_f^2 \left( B_f^{\text{bare}}(l^2) - \xi^2 D_f^{\text{bare}}(l^2) \right) \approx -\frac{T_R \alpha_s}{12\pi l^2} \left( \sum_f e_f^2 \right) \left( \frac{\langle P | F^{\alpha\beta}(i \vec{D}^+)^2 F_{\alpha\beta} | P \rangle}{(P^+)^2} + \xi^2 \langle P | F^2 | P \rangle \right).
\]

(Non-local) trace anomaly manifests itself in high energy scattering amplitude & possibly breaks QCD factorization