Multichannel GPD and CFF extraction from hard exclusive processes

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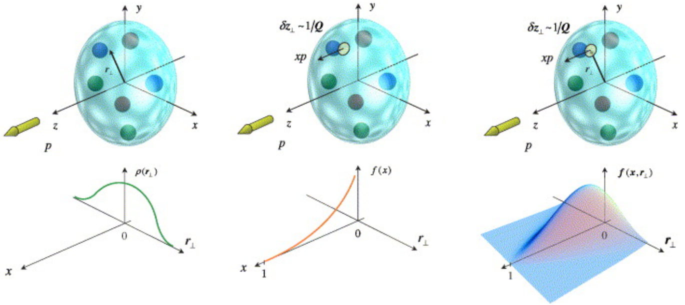
JLab cake seminar

Outline

Hadron femtography

- we want to uncover the inner structure of nucleons
  - 3D position of constituents
  - spin and angular momentum distribution
  - pressure...

Belitsky, Radyushkin

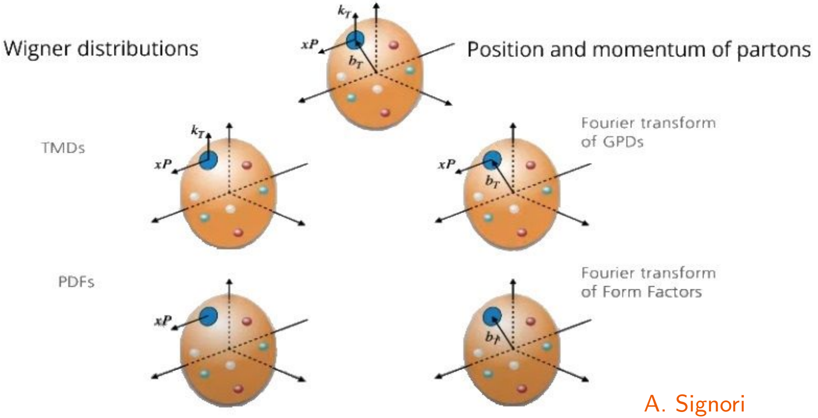


Form factors, transverse position of partons

PDFs, longitudinal momentum of partons

GPDs, transverse position and longitudinal momentum of partons

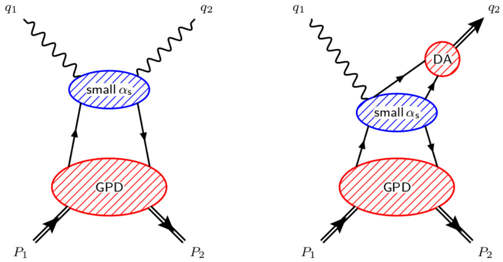
Hierarchy of distributions



A. Signori

Accessing GPDs

- exclusive processes such as DVCS and DVMP

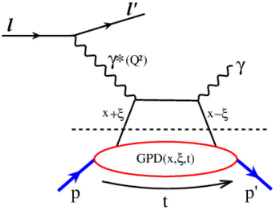


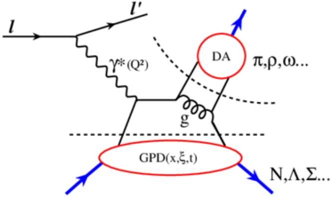
(a) DVCS (b) DVMP

- at leading twist four complex Compton form factors $\mathcal{H}\left(\xi,t,Q^2\right)$, $\mathcal{E}\left(\xi,t,Q^2\right)$, $\widetilde{\mathcal{H}}\left(\xi,t,Q^2\right)$, $\widetilde{\mathcal{E}}\left(\xi,t,Q^2\right)$
- also double DVCS, timelike Compton scattering, etc.

Factorization and GPDs

- factorization theorem [Collins et al. '97, '98]
- DVCS: transversal photon, DVMP: longitudinal photon and meson at twist-2





H. Moutarde et al.

- GPDs enter a convolution as the soft unperturbative part

DVCS

$$\ell(k_1) + N(P_1) \to \ell(k_2) + N(P_2) + \gamma(q_2)$$

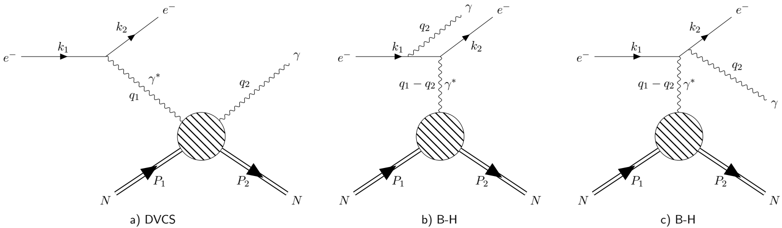


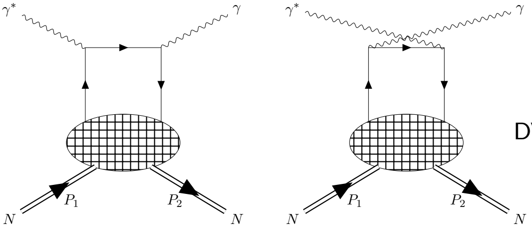
Figure: Deeply virtual Compton scattering and Bethe-Heitler process.

$$\frac{d^5\sigma}{dx_{\mathrm{B}}dyd\left|\Delta^2\right|d\phi d\varphi} = \frac{\alpha^3 x_{\mathrm{B}} y}{16\pi^2 Q^2 e^3 \sqrt{1+\epsilon^2}}\left(\left|\mathcal{T}_{DVCS}\right|^2 + \left|\mathcal{T}_{BH}\right|^2 + \mathcal{I}\right)$$

Dictionary

- virtual photon momentum $Q^2 = -q_1^2$
- momentum transfer: $t = \Delta^2 = (P_2 - P_1)^2$
- $q = \frac{1}{2}(q_1 + q_2), \quad \mathcal{Q}^2 = -q^2, \quad P = P_1 + P_2$
- Bjorken x: $x = \frac{Q^2}{2P_1 \cdot q_1}$
- generalized Bjorken variable: $\xi = \frac{\mathcal{Q}^2}{P \cdot q}$
- skewness $\eta = -\frac{\Delta \cdot q}{P \cdot q}$
- Bjorken limit
  $s = (P_1 + q_1)^2 \sim q_1^2 \to \infty, \quad -\Delta^2 \ll s, \quad \xi = \text{fixed}$

Compton form factors



a) s-channel b) u-channel

DVCS at LO.

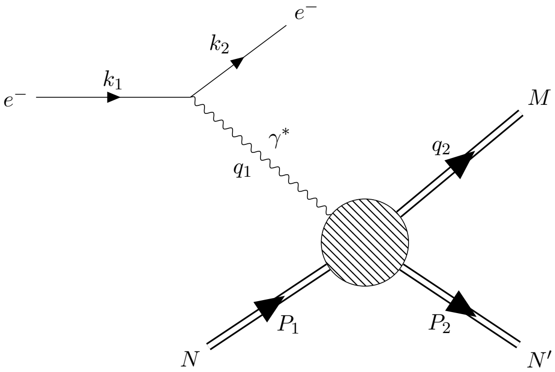
$$\mathrm{d}\sigma \propto |\mathcal{T}_{\mathrm{DVCS}}|^2 = \frac{2\left(2-2y+y^2\right)}{y^2Q^2\left(2-x_{\mathrm{B}}\right)^2}\left[4\left(1-x_{\mathrm{B}}\right)\left(|\mathcal{H}|^2+|\widetilde{\mathcal{H}}|^2\right)-x_{\mathrm{B}}^2\left(\mathcal{H}\mathcal{E}^*+\mathcal{E}\mathcal{H}^*+\widetilde{\mathcal{H}}\widetilde{\mathcal{E}}^*+\widetilde{\mathcal{E}}\widetilde{\mathcal{H}}^*\right)\right.$$
$$\left.-\left(x_{\mathrm{B}}^2+\left(2-x_{\mathrm{B}}\right)^2\frac{\Delta^2}{4M^2}\right)|\mathcal{E}|^2-x_{\mathrm{B}}^2\frac{\Delta^2}{4M^2}|\widetilde{\mathcal{E}}|^2\right], \quad y=\frac{Q^2}{xs}$$

CFFs:

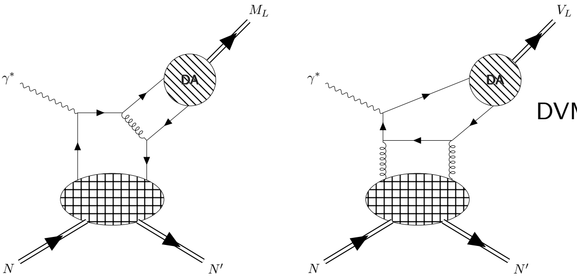
$$\mathcal{F}^A\left(\xi,\Delta^2,Q^2\right)=\int_{-1}^{1}\frac{dx}{2\xi}\underbrace{{}^AT\left(x,\xi\middle|\alpha_{\mathrm{s}}\left(\mu_{\mathrm{R}}\right),\frac{Q^2}{\mu_{\mathrm{F}}^2}\right)}_{\text{hard scale}}\underbrace{F^A\left(x,\eta,\Delta^2,\mu_{\mathrm{F}}^2\right)}_{\text{soft scale}},\ A\in\{q,g\}$$

DVMP

$$\ell(k_1) + N(P_1) \to \ell(k_2) + N'(P_2) + M(q_2)$$



Transition form factors



a) quark subprocess   b) gluon subprocess

DVMP at LO.

$$\mathrm{d}\sigma \propto \frac{16(1-y)}{y^2\,(2-x_{\mathrm{B}})^2}\left[4\,(1-x_{\mathrm{B}})\,|\mathcal{H}|^2 - x_{\mathrm{B}}^2\,(\mathcal{H}\mathcal{E}^* + \mathcal{E}\mathcal{H}^*) - \left(x_{\mathrm{B}}^2 + (2-x_{\mathrm{B}})^2\,\frac{\Delta^2}{4M^2}\right)|\mathcal{E}|^2\right]$$

$$\mathcal{F}^A\left(\xi,\Delta^2,Q^2\right) = \frac{fC_{\mathrm{F}}}{QN_{\mathrm{c}}}\int_{-1}^{1}\frac{\mathrm{d}x}{2\xi}\int_0^1 \mathrm{d}v\;\underbrace{\varphi(v)}_{\text{soft scale}}\;\underbrace{{}^A T\left(x,v,\xi\middle|\alpha_{\mathrm{s}}\left(\mu_{\mathrm{R}}\right),\frac{Q^2}{\mu_{\mathrm{F}}^2},\frac{Q^2}{\mu_{\varphi}^2},\frac{Q^2}{\mu_{\mathrm{R}}^2}\right)}_{\text{hard scale}}\;\underbrace{F^A\left(x,\eta,\Delta^2\right)}_{\text{soft scale}}$$

Twist-2 generalized parton distributions

- Quark sector:

$$\langle P_2 | \overline{\psi}(z_1^-)\gamma^+\psi(z_2^-) | P_1 \rangle = \int_{-1}^{1} dx e^{-ixP\cdot z}\left[h^+ H^q(x,\eta,\Delta^2) + e^+ E^q(x,\eta,\Delta^2)\right]$$

$$\langle P_2 | \overline{\psi}(z_1^-)\gamma^+\gamma^5\psi(z_2^-) | P_1 \rangle = \int_{-1}^{1} dx e^{-ixP^+z^-}\left[\widetilde{h}^+ \widetilde{H}^q(x,\eta,\Delta^2) + \widetilde{e}^+ \widetilde{E}^q(x,\eta,\Delta^2)\right]$$

- Gluon sector:

$$\langle P_2 | F_a^{+\mu}(z_1^-) g_{\mu\nu} F_b^{\nu+}(z_2^-) | P_1 \rangle = \int_{-1}^{1} dx e^{-ixP^+z^-}\left[h^+ H^g(x,\eta,\Delta^2) + e^+ E^g(x,\eta,\Delta^2)\right]$$

$$\left\langle P_2 \left| F_a^{+\mu}(z_1^-) i\epsilon_{\mu\nu}^{\perp} F_a^{\nu+}(z_2^-) \right| P_1 \right\rangle = \int_{-1}^{1} dx e^{-ixP^+z^-}\left[\widetilde{h}^+ \widetilde{H}^g(x,\eta,\Delta^2) + \widetilde{e}^+ \widetilde{E}^g(x,\eta,\Delta^2)\right]$$

- H and E parity even, $\widetilde{H}$ and $\widetilde{E}$ parity odd, all chiral even
- additional transversity twist-2 GPDs, chiral odd

GPD properties

- forward limit $\Delta = 0$

$$H^q\left(x, \eta = 0, \Delta^2 = 0\right) = f^q(x) = q(x)\theta(x) - \overline{q}(-x)\theta(-x)$$

$$\widetilde{H}^q\left(x, \eta = 0, \Delta^2 = 0\right) = \Delta f^q(x) = \Delta q(x)\theta(x) + \Delta\overline{q}(-x)\theta(-x)$$

- first moments

$$\int_{-1}^{1} \mathrm{d}x H^q(x, \eta, t) = F_1^q(t), \quad \int_{-1}^{1} \mathrm{d}x E^q(x, \eta, t) = F_2^q(t)$$

- impact parameter distribution

$$q\left(x, \mathbf{b}_\perp\right) = \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} e^{-i\mathbf{b}_\perp \cdot \Delta_\perp} H^q\left(x, 0, t = -\Delta_\perp^2\right)$$

- Ji's sum rule

$$\left\langle J_q^3 \right\rangle = \frac{1}{2} \int_{-1}^{1} dx x \left[H_q(x, \xi, t = 0) + E_q(x, \xi, t = 0)\right]$$

Modelling GPDs

GPD evolution

- evolution in x space complicated, mixing of components

$$\frac{1}{x^{p_a}}\frac{\partial H^a\left(x,\eta,t,\mu^2\right)}{\partial\log\left(\mu^2\right)} = \alpha_s\left(\mu^2\right)\sum_{b\in\{q,g\}}\int_x^1\frac{\mathrm{d}z}{\eta}K^{ab,(0)}\left(\frac{z}{\eta},\frac{\eta}{x}\right)\frac{H^b\left(z,\eta,t,\mu^2\right)}{z^{p_b}}$$

- we use conformal moments

$$F_n(\eta,t) = \int_{-1}^{1} dx c_n(x,\eta)F(x,\eta,t)$$

$$c_n(x,\eta) = \eta^n\frac{\Gamma\left(\frac{3}{2}\right)\Gamma(1+n)}{2^n\Gamma\left(\frac{3}{2}+n\right)}C_n^{\frac{3}{2}}\left(\frac{x}{\eta}\right)$$

- $C_n^{3/2}$ Gegenbauer polynomials for quarks (5/2 for gluons)
- analytic continuation $n\to j\in\mathbb{C}$
- evolution diagonal in j space at LO

$$\mu\frac{d}{d\mu}F_j^q\left(\eta,t,\mu^2\right) = -\frac{\alpha_s(\mu)}{2\pi}\gamma_j^{(0)}F_j^q\left(\eta,t^2,\mu^2\right)$$

Valence quark GPDs

- valence quarks modeled in x space ($q = u, d$) at crossover line $x = \eta$ (no Q^2 evolution)

$$\mathfrak{Im}\mathcal{H}(\xi, t) \stackrel{LO}{=} \pi \left[\frac{4}{9} H^{u_{\rm val}}(\xi, \xi, t) + \frac{1}{9} H^{d_{\rm val}}(\xi, \xi, t) + \frac{2}{9} H^{\textsf{sea}} (\xi, \xi, t) \right]$$

$$H(x, x, t) = nr2^{\alpha} \left(\frac{2x}{1+x} \right)^{-\alpha(t)} \left(\frac{1-x}{1+x} \right)^{b} \frac{1}{\left(1 - \frac{1-x}{1+x} \frac{t}{M^2} \right)^{p}} .$$

$$\alpha_v(t) = 0.43 + 0.85t/\mathrm{GeV}^2$$

- fixed parameters: n from PDFs, $\alpha(t)$ Regge trajectory, p counting rules

Sea quark and gluon GPDs

- sea quarks modelled in j space
- $SO(3)$ partial waves expansion

$$F_j(\eta, t) = \sum_{\substack{J=J_{\min} \\ \text{even}}}^{j+1} F_j^J(t)\eta^{j+1-J}\hat{d}_{\alpha,\beta}^J(\eta), \quad J = j{+}1, j{-}1, j{-}3, \ldots$$

- leading contribution

$$H_j^a(\eta = 0, t) = N^a \frac{\mathrm{B}\left(1 - \alpha^a + j, \beta^a + 1\right)}{\mathrm{B}\left(2 - \alpha^a, \beta^a + 1\right)} \frac{\beta(t)}{1 - \frac{t}{\left(m_j^a\right)^2}},$$

$$\left(m_j^a\right)^2 = \frac{1 + j - \alpha^a}{\alpha'^a}, \quad \beta(t) = \left(1 - \frac{t}{M^2}\right)^{-p}, \quad a = \{s, g\}$$

- full NLO QCD Q^2 evolution

- partial wave expansion implemented simply in Mellin-Barnes integral

$$\boldsymbol{\mathcal{H}} = \frac{1}{2i}\int_{c-i\infty}^{c+i\infty} dj \xi^{-j-1}\left[i+\tan\left(\frac{\pi j}{2}\right)\right]\times$$
$$\times\left[[\mathbb{C}\otimes\mathbb{E}]_j + [\mathbb{C}\otimes\mathbb{E}]_{j+2}\boldsymbol{S} + [\mathbb{C}\otimes\mathbb{E}]_{j+4}\boldsymbol{T}\right]\boldsymbol{H}_j^{(l)}$$

- 10-15 parameters

Dispersion relations

- CFFs constrained by dispersion relations

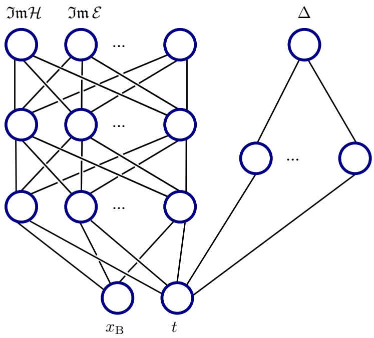
$$\mathfrak{Re}\,\mathcal{H}(\xi,t) \stackrel{LO}{=} \Delta(t)+\frac{1}{\pi}\mathrm{P.V.}\int_0^1 \mathrm{d}x \left(\frac{1}{\xi-x}-\frac{1}{\xi+x}\right)\mathfrak{Im}\,\mathcal{H}(x,t)$$

- subtraction constant model

$$\Delta(t) = \frac{C}{\left(1-\frac{t}{M_C^2}\right)^2}$$

- $\Delta_{\mathcal{H}}(t) = -\Delta_{\mathcal{E}}(t)$, $\Delta_{\tilde{\mathcal{H}}}(t) = \Delta_{\tilde{\mathcal{E}}}(t) = 0$
- only imaginary part of CFFs and one subtraction constant $\Delta(t)$ are modelled

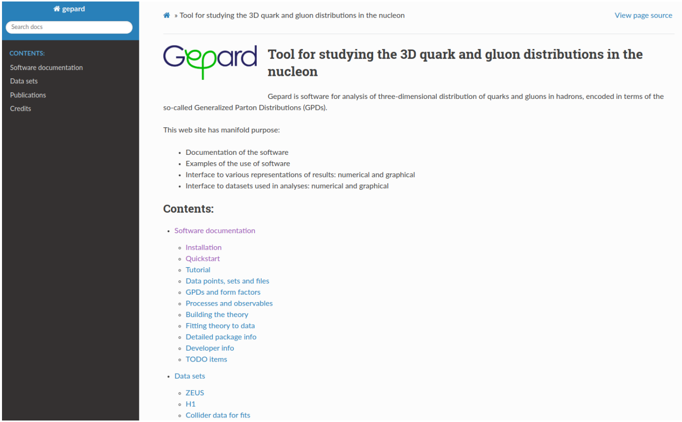
Neural networks constrained by dispersion relations



- Only imaginary part of CFFs and one subtraction constant $\Delta(t)$ are parametrized by neural nets

Gepard

https://gepard.phy.hr



gepard

Search docs

CONTENTS:

- Software documentation
- Data sets
- Publications
- Credits

» Tool for studying the 3D quark and gluon distributions in the nucleon — View page source

Tool for studying the 3D quark and gluon distributions in the nucleon

Gepard is software for analysis of three-dimensional distribution of quarks and gluons in hadrons, encoded in terms of the so-called Generalized Parton Distributions (GPDs).

This web site has manifold purpose:

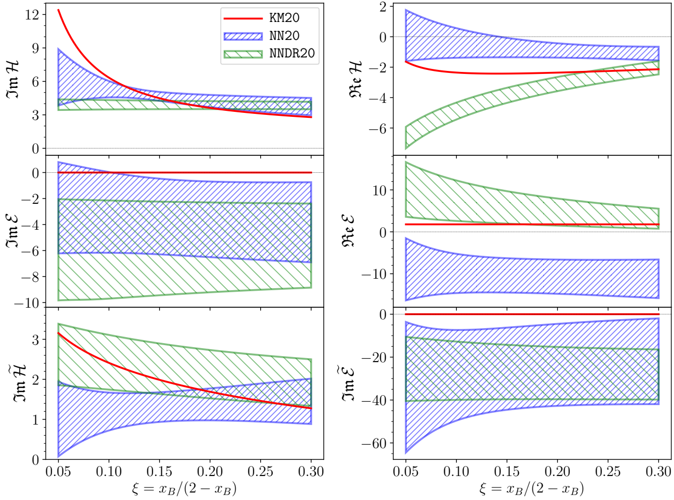
- Documentation of the software
- Examples of the use of software
- Interface to various representations of results: numerical and graphical
- Interface to datasets used in analyses: numerical and graphical

Contents:

Results

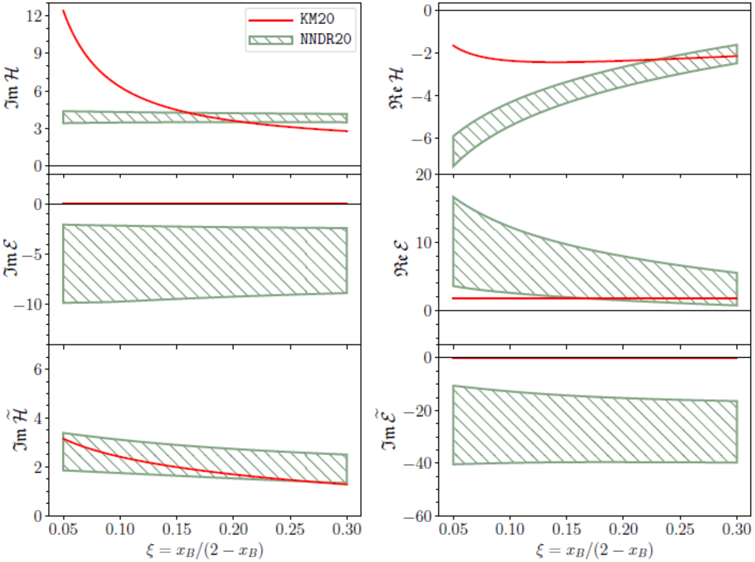
Extraction of 6 CFFs

[M. Č., K. Kumerički, A. Schäfer, '20], from JLab Hall A data



$\mathcal{Q}^2 = 4$ GeV^2
$t = -0.2$ GeV^2

Proton DVCS

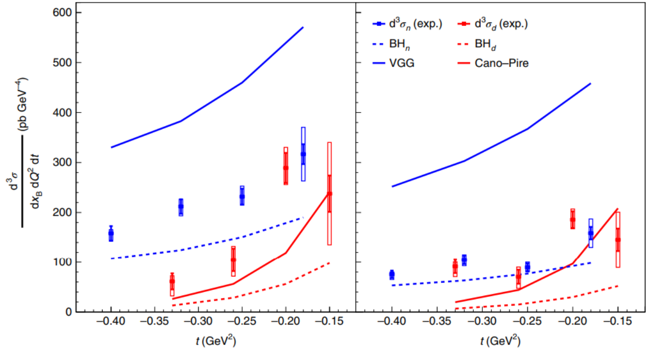


$Q^2 = 4$ GeV^2

$t = -0.2$ GeV^2

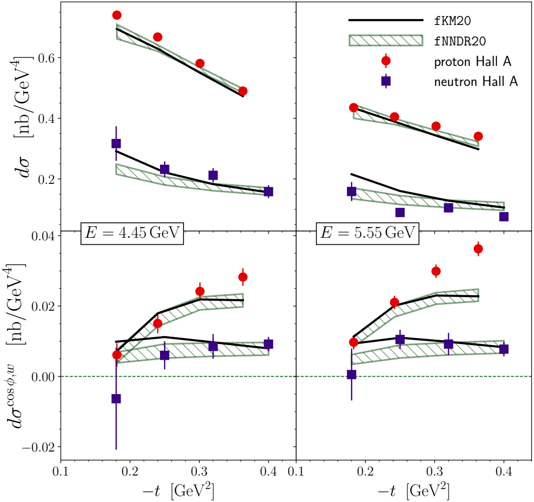
Neutron DVCS

[Benali et al. '20], DVCS off a deuterium target



Using isospin symmetry (e.g. $H^{\text{val}}_{u,\text{proton}} = H^{\text{val}}_{d,\text{neutron}}$) we combine proton and neutron DVCS data to separate up and down quark contributions to CFFs.

- separate model for each flavor CFF: $\mathcal{H}_u$, $\mathcal{H}_d$
- fKM20 "physical" flavored model, fNNDR neural nets and dispersion relations

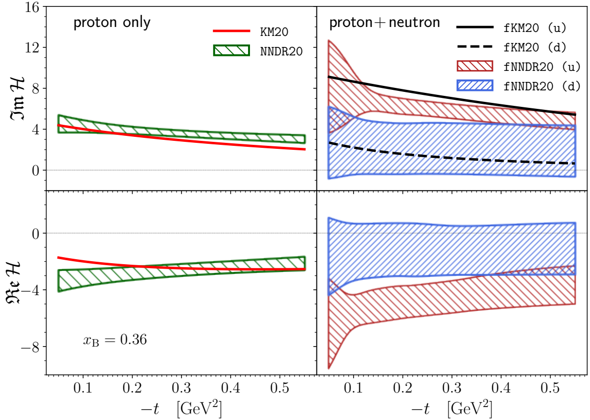


$x_B = 0.36$
$\mathcal{Q}^2 = 1.75\ \mathrm{GeV}^2$

Flavor CFFs

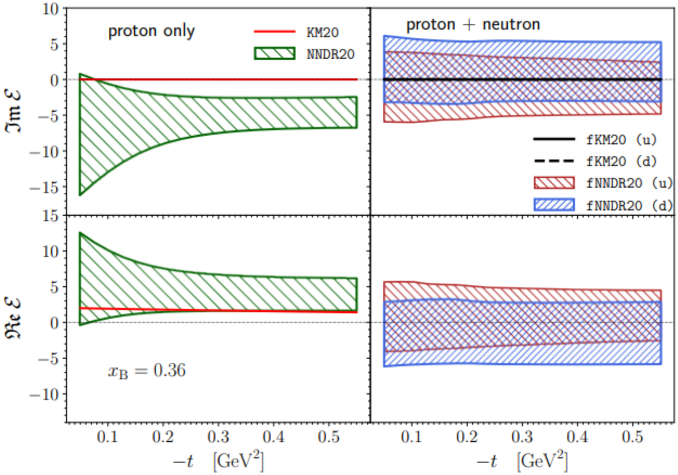
- up and down contributions to CFF $\mathcal{H}$ cleanly separated

$$\mathcal{H} = \frac{4}{9}\mathcal{H}_u + \frac{1}{9}\mathcal{H}_d$$



$x_B = 0.36$
$Q^2 = 4$ GeV^2

- $\mathcal{E}$ cannot be separated



CLAS 12 GeV predictions

- proton and neutron beam spin asymmetry

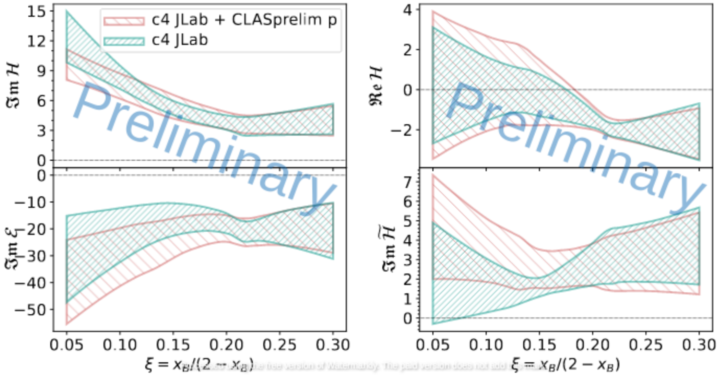
$$A_{LU} = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} \propto \mathfrak{Im} \left\{ F_1 \mathcal{H} + \xi \left(F_1 + F_2 \right) \widetilde{\mathcal{H}} - \frac{\Delta^2}{4M^2} F_2 \mathcal{E} \right\} \sin(\phi)$$

- we analyse harmonics with beam energy 10.4 GeV
- physical model only assumes isospin rotation

$$\mathcal{H}_n^{\text{val}} = \frac{2e_d^2 + e_u^2}{2e_u^2 + e_d^2} \mathcal{H}^{\text{val}} = \frac{2}{3} \mathcal{H}^{\text{val}} \, , \quad \mathcal{H}_n^{\text{sea}} = \mathcal{H}^{\text{sea}}$$

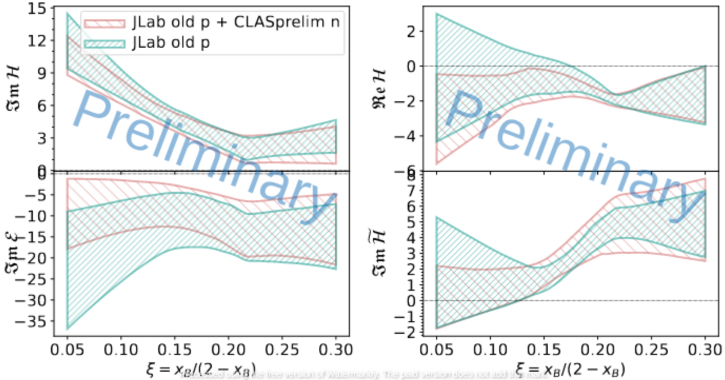
New proton data

We model $\mathfrak{Im}\mathcal{H}$, $\mathfrak{Re}\mathcal{H}$, $\mathfrak{Im}\mathcal{E}$ and $\mathfrak{Im}\widetilde{\mathcal{H}}$, and use old JLab data and new proton CLAS12 data

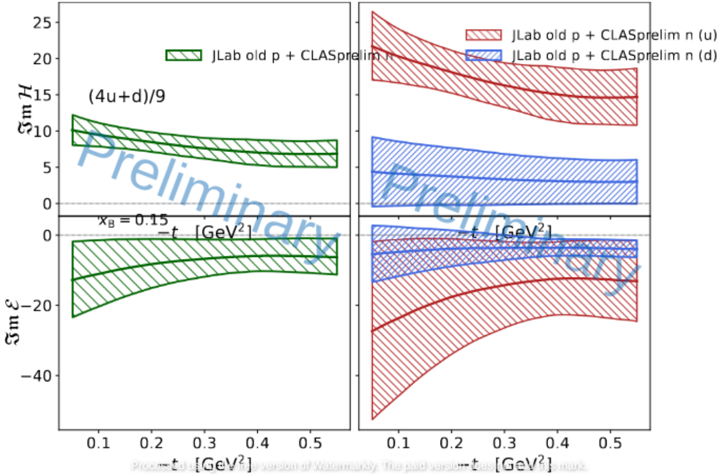


New neutron data

We model $\mathfrak{Im}\mathcal{H}$ and $\mathfrak{Im}\mathcal{E}$ as flavour dependent, $\mathfrak{Re}\mathcal{H}$ and $\mathfrak{Im}\widetilde{\mathcal{H}}$ as flavour agnostic, and use old JLab data and new neutron CLAS12 data



New flavour separation



NLO DIS+DVCS+DVMP small-x global fit

- First global fits to DIS+DVCS+DVMP HERA collider data [Lautenschlager, Müller, Schäfer, '13, unpublished!]
- hard scattering amplitude corrected in the meantime [Duplančić, Müller, Passek-Kumerički '17]
- [M. Č. et al., '23] preliminary results for NLO DIS+DVCS+DVMP small-x global fit
- we also studied LO fits, fits to DIS+DVCS and fits to DIS+DVMP
- what are the effects of NLO corrections?
- can we get universal GPDs regardless of DVCS and DVMP data?

Cross sections

- DVCS

$$\frac{d\sigma^{\gamma^* N \to \gamma N}}{d\Delta^2} \approx \frac{\pi \alpha_{em}^2}{(W^2+Q^2)^2} \left[|\mathcal{H}|^2 + |\widetilde{\mathcal{H}}|^2 - \frac{\Delta^2}{4M^2} |\mathcal{E}|^2 \right]$$

- DVMP

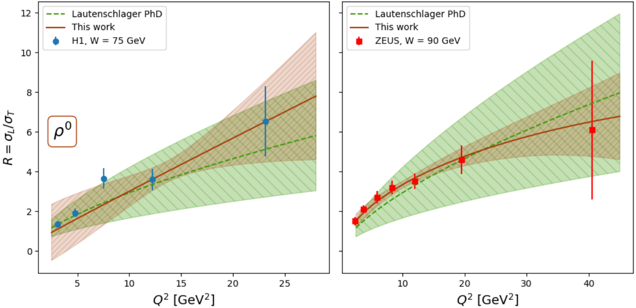
$$\frac{d\sigma^{\gamma^* N \to V N}}{d\Delta^2} \approx \frac{4\pi^2 \alpha_{em} x_B^2}{Q^4} \left[|\mathcal{H}|^2 - \frac{\Delta^2}{4M^2} |\mathcal{E}|^2 \right]$$

- for $|\Delta^2| < 1$ GeV^2 CFF $\mathcal{E}$ suppressed by $-\frac{\langle \Delta^2 \rangle}{4M^2} \approx 5 \times 10^{-2}$
- for $\widetilde{\mathcal{H}}$ Regge intercept $\alpha(0) \approx 1/2$, for $\mathcal{H}$ $\alpha(0) \approx 1$, $\widetilde{\mathcal{H}}$ also suppressed
- we ignore valence contributions, only singlet $\mathcal{H}$
- asymptotic distribution amplitude, dominant term in conformal space $\varphi_0 \approx 1$

Changes to original analysis

- for DVMP cut-off at $Q^2 \geq 10$ GeV^2 instead of 4
- no ϕ production
- different parameter constraints
- Q^2 and W dependence in $R = \sigma_L/\sigma_T$

$$R\left(Q^2\right) = \frac{Q^2}{m_V^2}\left(1 + a\frac{Q^2}{m_V^2}\right)^{-p} \to R(W,Q^2) = \frac{Q^2}{m_\rho^2}\left(1 + a\frac{Q^2}{m_\rho^2}\right)^{-p}\left(1 - b\frac{Q^2}{W}\right)$$



Data & χ^2/N_{pts}

- DIS data: H1 F_2
- DVCS: H1 and ZEUS data, $Q^2 \geq 5.0$ GeV^2
- DVMP: H1 nd ZEUS ρ^0 production, $Q^2 \geq 10.0$ GeV^2
- no t dependence

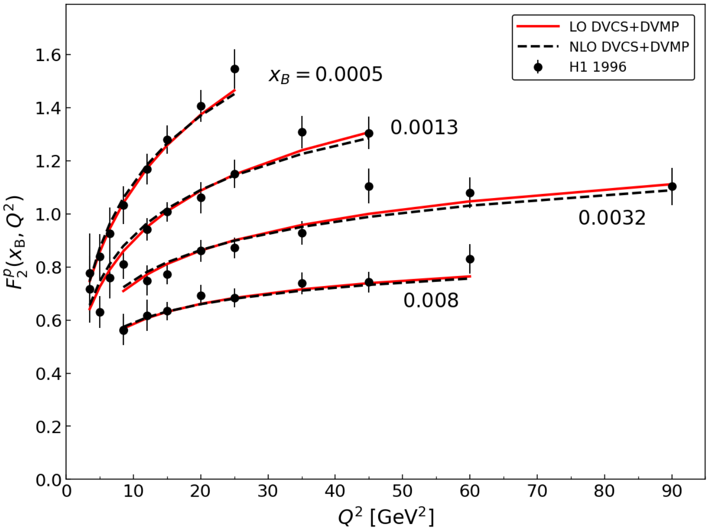
Dataset	N_{pts}	LO DVCS	LO DVMP	LO ALL	NLO DVCS	NLO DVMP	NLO ALL
DIS	85	0.6	0.6	0.6	0.8	0.8	0.8
DVCS	27	0.4	12252.2	0.6	0.6	27269.6	0.8
DVMP	45	49.7	3.1	3.3	11.7	1.5	1.7
Total	157	14.6	2108.3	1.4	3.9	4690.6	1.1

Parameters

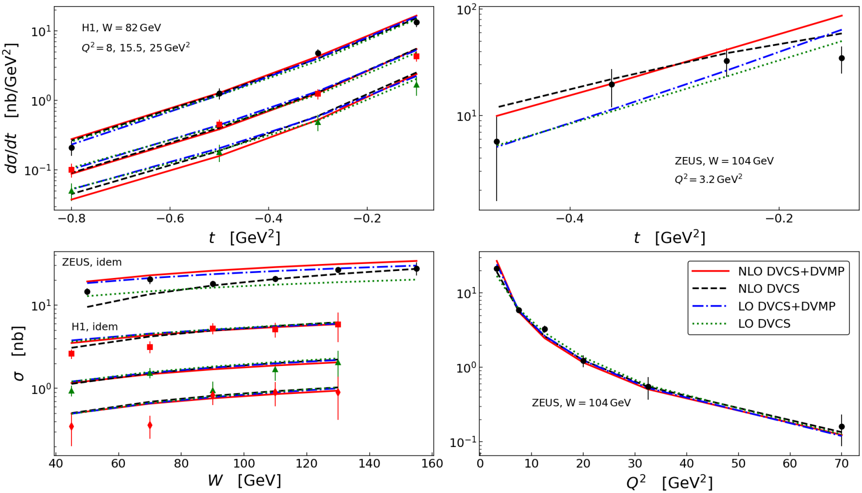
- pure DVMP fits prefer large quark skewness, rely on cancelation from gluons
- subleading waves were larger than leading ones, unstable
- DVMP more sensitive to gluons, we leave more freedom to gluon subleading PWs
- we want a decrease in contribution for PW expansion

	n^{sea}	α_0^{sea}	α'_{sea}	m^2_{sea}	s_2^{sea}	s_4^{sea}	α_0^G	α'_G	m^2_G	s_2^G	s_4^G
unit		1	GeV^{-2}	GeV^2	1	1	1	GeV^{-2}	GeV^2	1	1
initial	0.15	1.00	0.15	0.70	-0.20	0.00	1.00	0.15	0.70	0.00	0.00
limits			(0.0,1.0)	(0,3)	(-0.3,0.3)	(-0.1,0.1)		(0.0,1.0)	(0,3)	(-3.0,3.0)	(-1.0,1.0)
final	0.168	1.128	0.125	0.412	0.280	-0.044	1.099	0.000	0.145	2.958	-0.951
uncert.	0.002	0.011	0.040	0.050	0.032	0.010	0.011	0.010	0.008	0.039	0.025

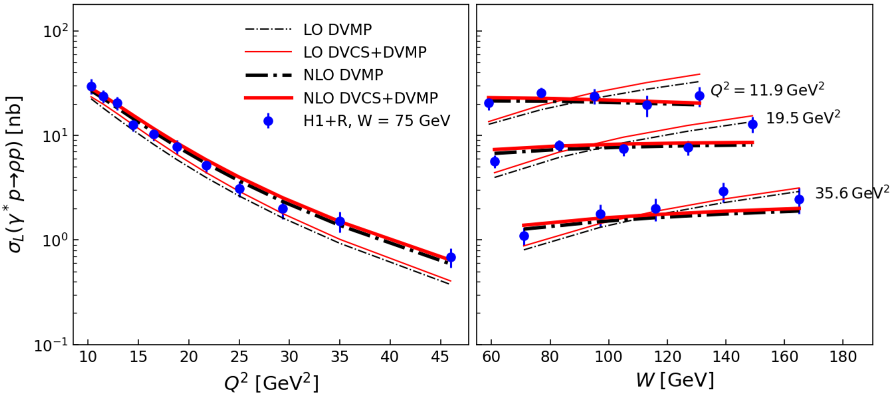
DIS F_2 data description

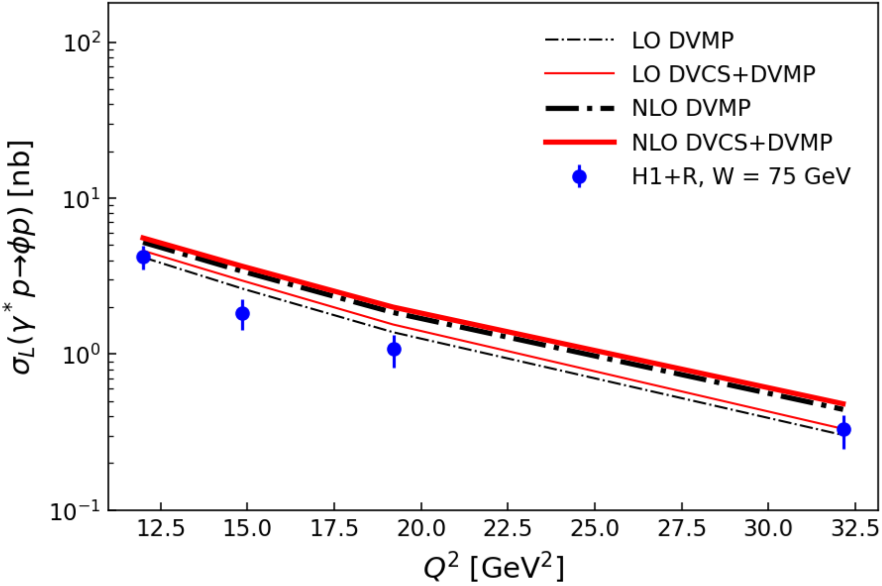


DVCS data description

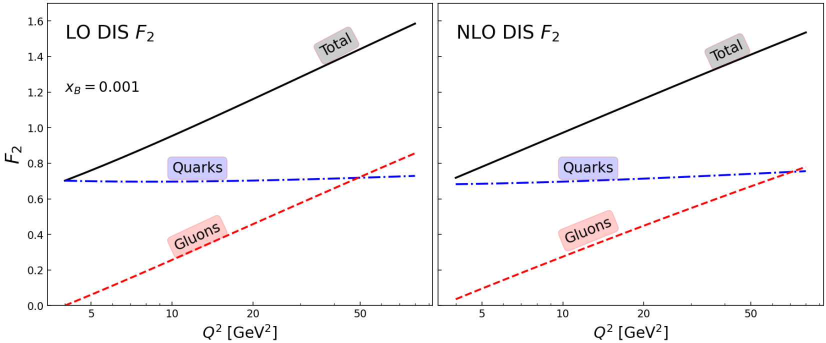


DVMP data description



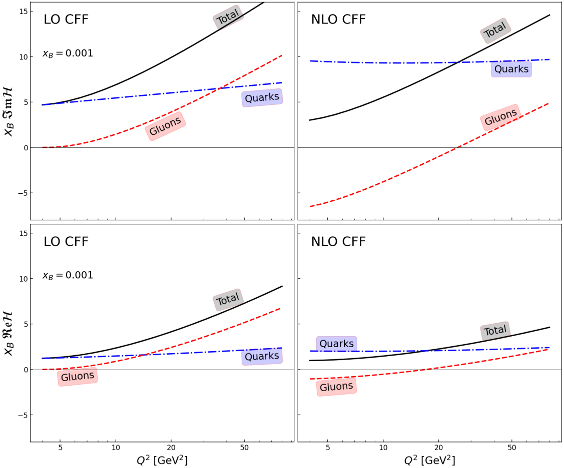

LO DVMP
LO DVCS+DVMP
NLO DVMP
NLO DVCS+DVMP
H1+R, W = 75 GeV
$\sigma_L(\gamma^* p \to \phi p)$ [nb]
10^2
10^1
10^0
10^{-1}
12.5
15.0
17.5
20.0
22.5
25.0
27.5
30.0
32.5
Q^2 [GeV^2]

Quark and gluon contributions: DIS



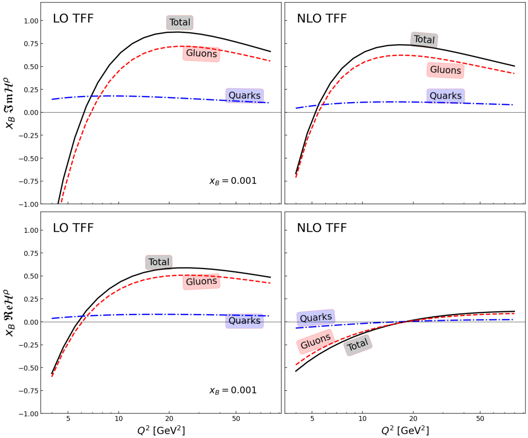
- at LO gluons do not contribute at low Q^2
- not much changes at NLO

Quark and gluon contributions: DVCS



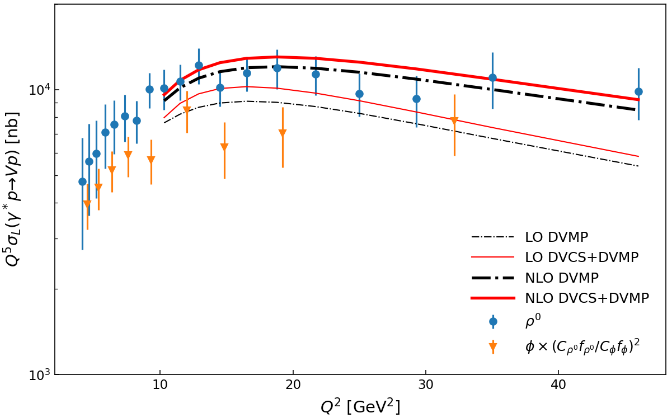
- at LO gluons do not contribute at low Q^2
- at NLO gluons negative at low Q^2

Quark and gluon contributions: DVMP



- at LO gluons dominate at low Q^2
- at NLO a much different story, gluons negative at low Q^2, dominate at large Q^2

DVMP Q^2 scalling

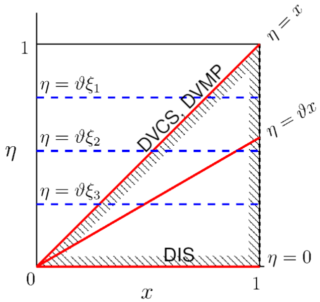


- theory scales roughly as Q^5 after $\sim$10 GeV^2

Skewness

- skewness: ratio of GPD to corresponding PDF

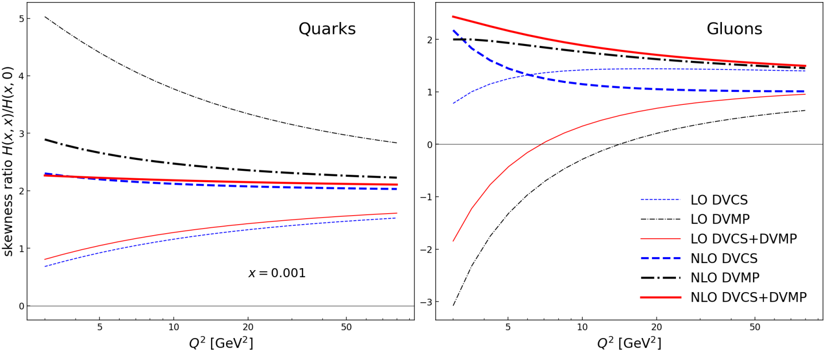
$$r = \frac{H(x, \eta = x)}{q(x)}$$



- conformal (Shuvaev) values, PDFs completely specify GPDs:

$$r^q \approx 1.65, \quad r^G \approx 1$$

Skewness at LO and NLO



Universal GPD structure emerges at NLO!

Conclusion

- extraction of 6 CFFs from 2020 Hall A data, flavour separation of CFF $\mathcal{H}$
- old models do not reproduce CLAS12 data
- some flavor separation of CFFs possible
- stable low-x fits for higher Q^2 and careful L-T separation
- universal GPD structure emerges at NLO