Meson-meson scattering at large N_c

Jorge Baeza-Ballesteros

In collaboration with P. Hernández and F. Romero-López
Based on arXiv/2202.02291 and ongoing work

IFIC, University of Valencia-CSIC

Jefferson Lab - 25th September 2023

Overview

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1. The large N_c limit of QCD
2. Chiral Perturbation Theory
3. Scattering in the lattice
4. $\pi\pi$ scattering at threshold
5. Meson-meson scattering at large N_c
6. Summary and outlook

The large N_c limit of QCD

QCD simplifies at **large N_c**

$N_c \to \infty \quad N_f = \text{const}$

→ Keeps **non-perturbative features**

→ Has **predictive power** in the non-perturbative regime

→ Lattice allows to study **subleading N_c effects**

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Long-term goal: Understand subleading N_c effects in the lattice:

- $K \to (\pi\pi)_{I=0,2}$ [Donini et al. 2020]
- Resonances ⟶ Stable ($\Gamma \sim 1/N_c$)
- Exotic states (tetraquarks...) [Coleman 1985, Weinberg 2013]

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Possible **exotic resonances**

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Nature of tetraquarks at large N_c

[Coleman 1985, Weinberg 2013]

Meson-meson scattering at large N_c

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↘ N_c scaling of **scattering amplitudes**
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Constrain LECs from ChPT

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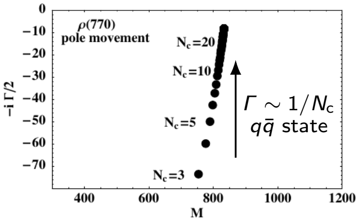
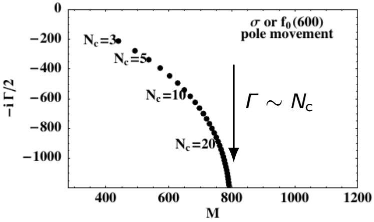
Constrain LECs from ChPT

Large N_c + **Unitarized ChPT** ⟶ N_c scaling of resonances [Peláez, 2004]





Meson-meson scattering at large N_c

$N_f = 4$ ($m_u = m_d = m_s = m_c$)

Used to study $K \to \pi\pi$

[Donini et al. 2020]

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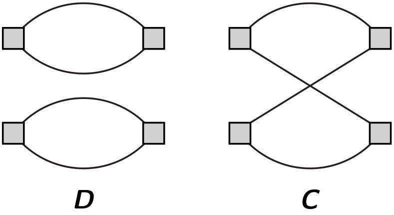
→ **7 scattering channels**

$$15 \otimes 15 = \mathbf{84}\,(\boldsymbol{SS}) \oplus 45 \oplus 45 \oplus \mathbf{20}\,(\boldsymbol{AA}) \oplus 15 \oplus 15 \oplus 1$$

$\pi^+\pi^+$ (84, SS)

$D_s^+\pi^+ - D^+K^+$ (20, AA)

$$C_{SS} = D - C + (p_1 \leftrightarrow p_2)$$
$$C_{AA} = D + C + (p_1 \leftrightarrow p_2)$$



$\pi\pi$ scattering at large N_c

We can characterize the N_c and N_f scaling of diagrams at large N_c

Large N_c **counting rules**:

- Color loops $\sim N_c$
- Vertex $\sim g \sim N_c^{-1/2}$
- Internal quark loops $\sim N_f$

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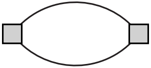
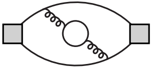
$C_\pi =$  $+$  $+ \quad \ldots$

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$= \quad aN_c$

$a, b \sim \mathcal{O}(1)$ constants

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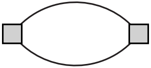
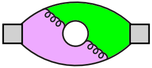
$C_\pi =$  $+$  $+ \quad \ldots$

$= \quad aN_c \quad + \quad bg^4 N_c^2 N_f$

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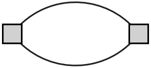
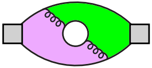
$C_\pi =$  $+$  $+ \quad \ldots$

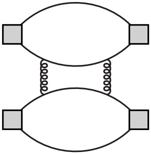
$= \quad aN_c \quad + \quad bN_f \quad + \quad \mathcal{O}(N_c^{-1})$

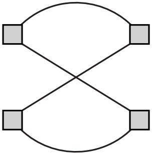
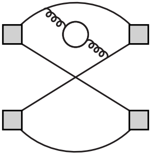
$a, b \sim \mathcal{O}(1)$ constants

$\pi\pi$ scattering at large N_c

$c, d, e, f \sim \mathcal{O}(1)$ constants

Two pions:

$D = C_\pi^2 +$  $+ \ \dots \ = C_\pi^2 + d + \mathcal{O}(N_c^{-1})$

$C =$  $+$  $+ \ \dots \ = e N_c + f N_f + \mathcal{O}(N_c^{-1})$

$\pi\pi$ scattering at large N_c

$c, d, e, f \sim \mathcal{O}(1)$ constants

$\tilde{a}, \tilde{b}, \tilde{c} \sim \mathcal{O}(1)$ constants

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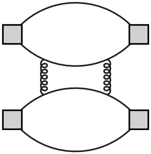

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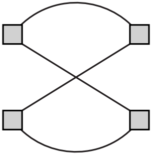

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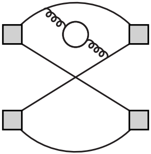

$+ \quad \ldots \quad = eN_c + fN_f + \mathcal{O}(N_c^{-1})$

Scattering length in the *SS* and *AA* channels:

$$a_0^{SS,AA} = \mp \frac{1}{N_c}\left(\tilde{a} + \tilde{b}\frac{N_f}{N_c} \pm \tilde{c}\frac{1}{N_c}\right) + \mathcal{O}(N_c^{-3})$$

Same scaling for other scattering observables

Overview

Chiral Perturbation Theory (ChPT)

ChPT describes QCD in terms of pseudo-Goldstone bosons (**pions**)

$$\phi = \begin{pmatrix} \pi^0 + \frac{\eta_0}{\sqrt{3}} + \frac{\eta_c}{\sqrt{6}} & \sqrt{2}\pi^+ & \sqrt{2}K^+ & \sqrt{2}D^0 \\ \sqrt{2}\pi^- & -\pi^0 + \frac{\eta_0}{\sqrt{3}} + \frac{\eta_c}{\sqrt{6}} & \sqrt{2}K^0 & \sqrt{2}D^+ \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2\eta_0}{\sqrt{3}} + \frac{\eta_c}{\sqrt{6}} & \sqrt{2}D_s^+ \\ \sqrt{2}\bar{D}^0 & \sqrt{2}D^- & \sqrt{2}D_s^- & -\frac{3\eta_c}{\sqrt{6}} \end{pmatrix} \qquad (N_\mathrm{f} = 4)$$

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Most general lagrangian with QCD symmetries

$$\mathcal{L}_2 = \frac{F^2}{4}\mathrm{Tr}[\partial_\mu U \partial^\mu U^\dagger] + \frac{F^2 B_0}{2}\mathrm{Tr}[\chi U^\dagger + \chi^\dagger U] \qquad \text{(2 LECs)}$$

$F^2 \sim \mathcal{O}(N_c)$

$B_0, M_\pi \sim \mathcal{O}(1)$

$$\mathcal{L}_4 = \sum_{i=0}^{12} L_i O_i \qquad L_i \sim \mathcal{O}(N_c) \text{ or } \mathcal{O}(1) \qquad \text{(13 LECs)}$$

ChPT at large N_c

The η' needs to be included

$$M^2_{\eta'} = M^2_\pi + \frac{2N_f\chi_{top}}{F^2_\pi} \xrightarrow[\text{large } N_c]{F^2_\pi\sim\mathcal{O}(N_c)} M^2_\pi + \ldots$$

[Witten-Veneciano]

Large N_c or U(N_f) ChPT [Kaiser, Leutwyler 2000]:

- Include η' in pion matrix

$$\phi|_{U(N_f)} = \phi|_{U(N_f)} + \eta'\mathbb{1}$$

- Leutwyler counting scheme

$$\mathcal{O}(m_q) \sim \mathcal{O}(M_\pi^2) \sim \mathcal{O}(k^2) \sim \mathcal{O}(N_c^{-1})$$

$\pi\pi$ scattering in ChPT

$\pi\pi$ **scattering at LO in ChPT** [Weinberg 1979]

$$k \cot \delta_0 = \frac{1}{a_0} + \dots$$

$$M_\pi a_0^{SS} = -\frac{M_\pi^2}{16\pi F_\pi^2} \propto -\frac{1}{N_c}$$

$$M_\pi a_0^{AA} = +\frac{M_\pi^2}{16\pi F_\pi^2} \propto +\frac{1}{N_c}$$

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$\pi\pi$ scattering at NLO in large N_c ChPT [JBB at al. 2022]

$$M_\pi a_0^{SS,AA} = \mp\frac{M_\pi^2}{16\pi F_\pi^2} + f_{SS,AA}(M_\pi, F_\pi, L_{SS,AA}, K_{SS,AA})$$

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Large N_c $\longrightarrow$

$$L_{SS} = N_c L^{(0)} + N_f L_c^{(1)} - L_a^{(1)} + \mathcal{O}(N_c^{-1})$$
$$L_{AA} = \underbrace{N_c L^{(0)} + N_f L_c^{(1)}}_{\text{Same sign}} \underbrace{+L_a^{(1)}}_{\text{Opposite sign}} + \mathcal{O}(N_c^{-1})$$

Overview

QCD in the lattice

Lattice QCD is a first-principles approach to the strong interaction

$$\langle O[\phi]\rangle = \frac{1}{Z}\int \mathcal{D}\phi\, \mathrm{e}^{-S[\phi]}\, O[\phi]$$

$\phi \equiv$ quark, gluons

$S[\phi] \equiv$ QCD action

$O[\phi] \equiv$ observable

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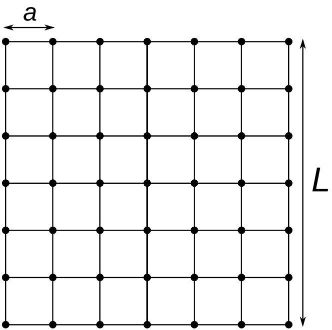
Finite, discretized spacetime

+

Montecarlo methods



Meson-meson scattering in the lattice

Finite volume
- No asymptotic states
- Stationary multiparticle states

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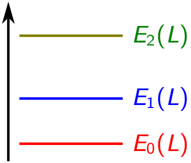
Finite-volume spectrum



Quantization condition (QC) ⟷

Infinite-volume scattering amplitudes

Two-particle energy spectrum

Use a **basis of operators**, $O_i(t)$, with the correct quantum numbers

$$C_{ij}(t) = \langle O_i(t) O_j(0)^\dagger \rangle \qquad O_i \sim \pi(\boldsymbol{k}_1)\pi(\boldsymbol{k}_2)$$

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Solve **generalized eigenvalue problem**

$$C^{-1/2}(t_0)C(t)C^{-1/2}(t_0)v_n = \lambda_n(t)v_n \longrightarrow \lambda_n(t) \xrightarrow{T \gg t \gg t_0} A_n e^{-E_n t}$$

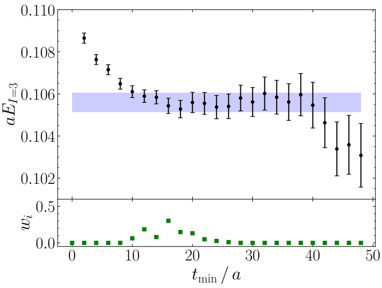
Fit for **different fit ranges** and extract the energies from **plateaux**

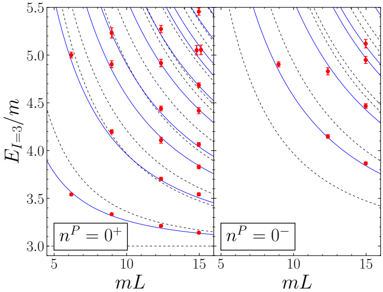
Two-particle energy spectrum

Average plateaux using **Akaike Information Criterion** [Jay, Neil, 2020]

$$w_i \propto \exp\left[-\frac{1}{2}\left(\chi^2 - 2N + 2N_{\text{par}}\right)\right]$$

- Reduces human bias
- Allows to automatically find plateaux for accurate data



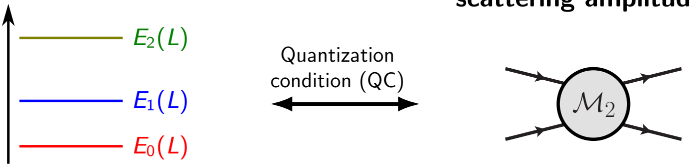


Two- and three-particle scattering in the O(3) model, $\gtrsim$ 1500 energy levels [JBB, Hansen (in preparation)]

Two-particle quantization condition

Finite-volume spectrum

Infinite-volume scattering amplitudes



Two-particle QC [Lüscher 1986, Rummukainen and Gotlieb 1995, He et al. 2005]:

Single-channel, *s*-wave ⟶

$$k \cot \delta_0 = \frac{2}{\gamma L \pi^{1/2}} \mathcal{Z}_{00}^{\boldsymbol{P}}\left(\frac{kL}{2\pi}\right)$$

Overview

Our lattice ensembles

Goal: N_c scaling of $\pi\pi$ scattering and match to ChPT

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Ensembles with $N_f = 4$ dynamical quarks for $N_c = 3-6$ generated using **HiRep** [Del Debbio et al., 2010]

Summary of ensembles [Hernández et al., 2019]

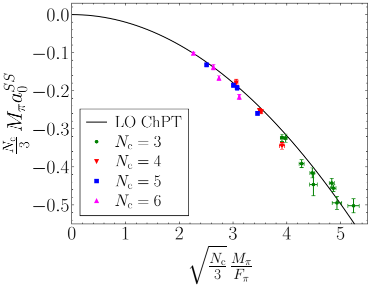
$a = 0.075$ fm $\rightarrow [N_c = 3-6] \times$ [4 or 5 values of M_π] = 17 ensembles
$a = 0.065$ fm $\rightarrow [N_c = 3] \times$ [2 values of M_π] = 2 ensembles
$a = 0.059$ fm $\rightarrow [N_c = 3] \times$ [2 values of M_π] = 2 ensembles

$$M_\pi = 350\text{--}590\,\text{MeV}$$

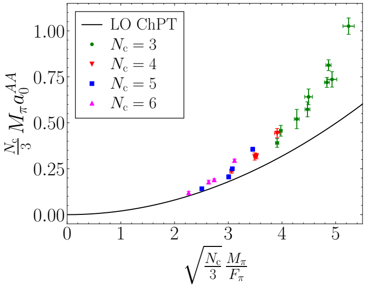
$\pi\pi$ scattering lengths

Goal: N_c scaling of $\pi\pi$ scattering and match to ChPT

SS channel (repulsive)

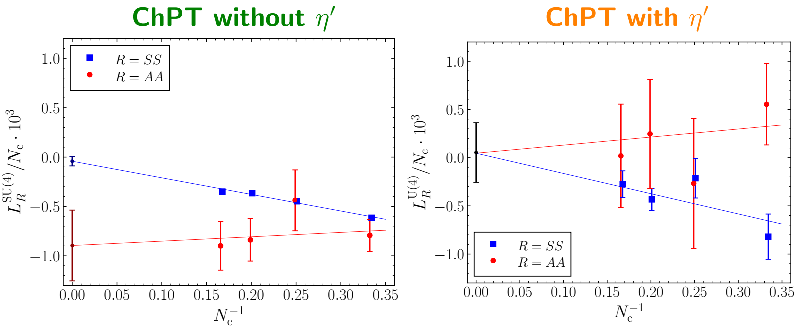


AA channel (attractive)



Fits at fixed N_c

We fit to ChPT at fixed N_c for each channel separately,

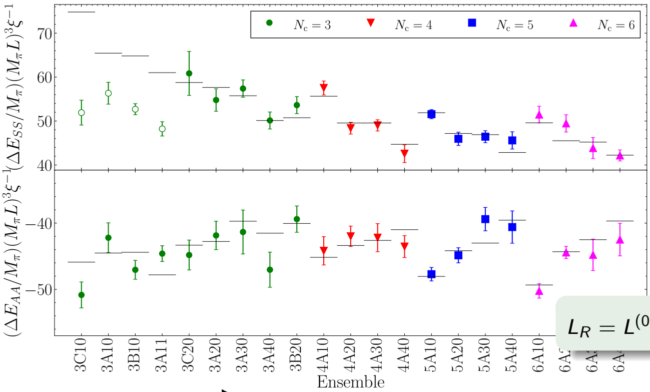


⋆ $L_{SS,AA}$ are well described by leading and subleading N_c terms

⋆ Only U(4) ChPT is consistent with the common large N_c limit

Simultaneous chiral and N_c fit

Simultaneous chiral and N_c fit of both channels to U(4) ChPT,



***SS* channel**

***AA* channel**

$$L_R = L^{(0)} N_c + L_R^{(1)} + \dots$$

Decreasing $\xi = M_\pi^2/(4\pi F_\pi)^2$

$$\frac{L_{SS}}{N_c} \times 10^3 = -0.02(8) - \frac{1.79(19)}{N_c} \qquad \frac{L_{AA}}{N_c} \times 10^3 = -0.02(8) + \frac{1.7(4)}{N_c}$$

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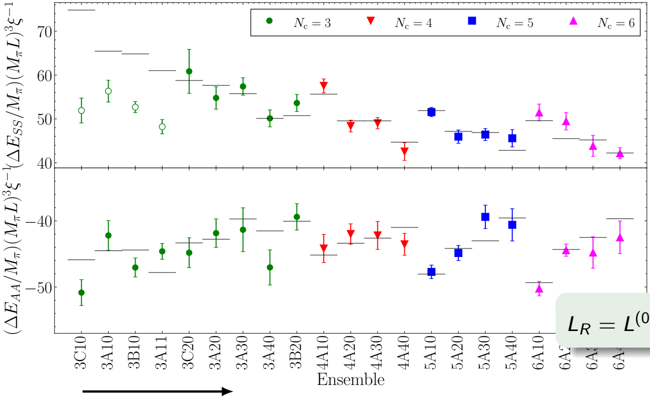

***SS* channel**

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$$L_R = L^{(0)} N_c + L_R^{(1)} + \dots$$

Decreasing $\xi = M_\pi^2/(4\pi F_\pi)^2$

$$\text{Large } N_c \longrightarrow \frac{L_{SS,AA}}{N_c} \times 10^3 = -0.02(8) - 0.01(5)\frac{N_f}{N_c} \mp 1.76(20)\frac{1}{N_c}$$

Comparison to lattice and phenomenology

Compare to lattice and experimental results for the ***SS* channel**,

1. Change N_f: $\frac{L_{SS}}{N_c} \times 10^3 = -0.02(8) - 0.01(5)\frac{N_f}{N_c} - 1.76(20)\frac{1}{N_c}$
2. Translate to SU(N_f)
3. Change renormalization scale, $\mu = 1.40(12)\,\text{GeV} \to 0.77\,\text{GeV}$

Comparison to lattice and phenomenology

Compare to lattice and experimental results for the ***SS* channel**,

1. Change N_f: $\frac{L_{SS}}{N_c} \times 10^3 = -0.02(8) - 0.01(5)\frac{N_f}{N_c} - 1.76(20)\frac{1}{N_c}$
2. Translate to SU(N_f)
3. Change renormalization scale, $\mu = 1.40(12)\,\text{GeV} \to 0.77\,\text{GeV}$

$N_f = 3$

$$L_{SS}^{N_c=3,N_f=3} \times 10^3 = -1.14(22)_{\text{stat}}(11)_\mu$$ (this work)

$$L_{SS}^{N_c=3,N_f=3} \times 10^3 = -0.9(1.5)$$ [Bijnens, Ecker 2014]

$N_f = 2$

$$\ell_{I=2} = 4.3(1.2)_{\text{stat}}(0.5)_\mu$$ (this work)

$$\ell_{I=2} = 3.79(0.61)_{\text{stat}} \left({}^{+1.34}_{-0.11}\right)_{\text{sys}}$$ [Feng, et al. 2010]

$$\ell_{I=2} = 4.65(0.85)_{\text{stat}}(1.07)_{\text{sys}}$$ [Helmes, et al. 2015]

$$\ell_{I=2} = 512\pi^2 L_{SS}^{N_c=3,N_f=2}$$

Overview

$\pi\pi$ scattering at large N_c

AA channel is **attractive** $\longrightarrow$ **Possible tetraquark**

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Recently found **exotic states at LHCb** [LHCb 2020, 2022]:

$$J = 0: \begin{array}{l} T^{0}_{cs0}(2900) \text{ in } D^{+}K^{-} \\ T^{++}_{c\bar{s}0}(2900) \text{ and } T^{0}_{c\bar{s}0}(2900) \text{ in } D_s^{\pm}\pi^{+} \end{array} \longrightarrow \textbf{\textit{AA} channel}$$

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$J = 1$: $T^0_{cs1}(2900)$ in D^+K^- $\longrightarrow$ $84 \oplus \mathbf{45} \oplus \mathbf{45} \oplus 20 \oplus 15 \oplus 15 \oplus 1$

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Below $D_s^*\rho$ threshold $\longrightarrow$ Described as **meson-meson bound states**

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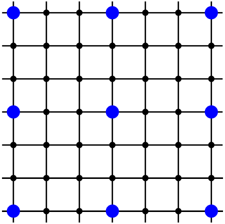
Goal: N_c scaling of meson-meson scattering + tetraquark

Lattice computations

$N_c = 3, 4, 5, 6$ ensembles with $a \sim 0.075$ fm and $M_\pi \sim 590$ MeV

Operator basis: $\pi\pi$ + $\rho\rho$ + local tetraquark

- Local tetraquark operators ⟶ **Point sources in a regular subgrid $\tilde{\Lambda}$**



$$T(\boldsymbol{P}) \propto \sum_{\boldsymbol{x}\in\tilde{\Lambda}} e^{-i\boldsymbol{P}\boldsymbol{x}}\, T(\boldsymbol{x})$$

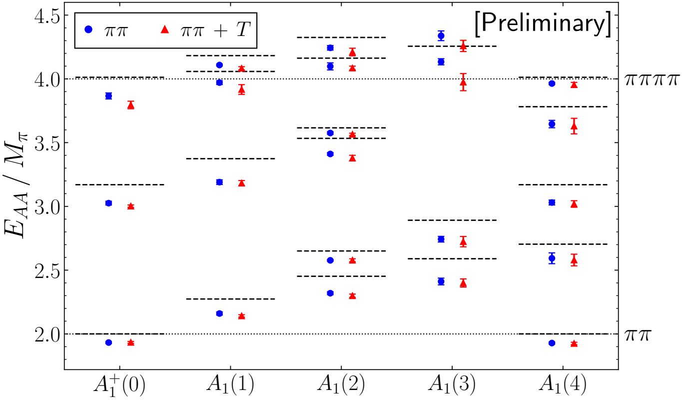
$$T(x) \sim \bar{d}\Gamma_1 u\, \bar{s}\Gamma_2 c - \bar{s}\Gamma_1 u\, \bar{d}\Gamma_2 c$$

Quantum numbers of AA channel

Finite-volume energies: AA channel

We study the effect of different operators for $N_c = 3$:

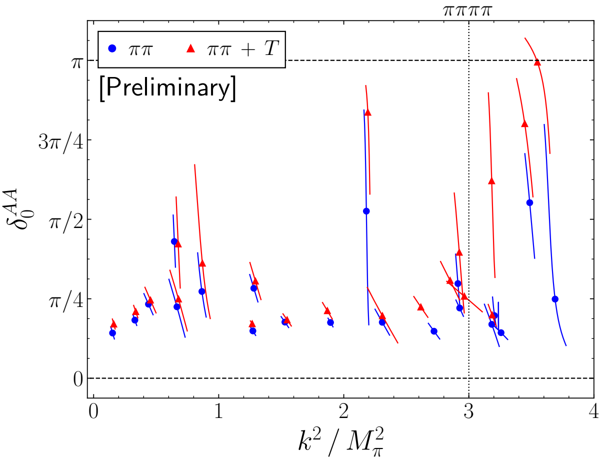
$\pi\pi$ vs $\pi\pi$ + Local tetraquarks



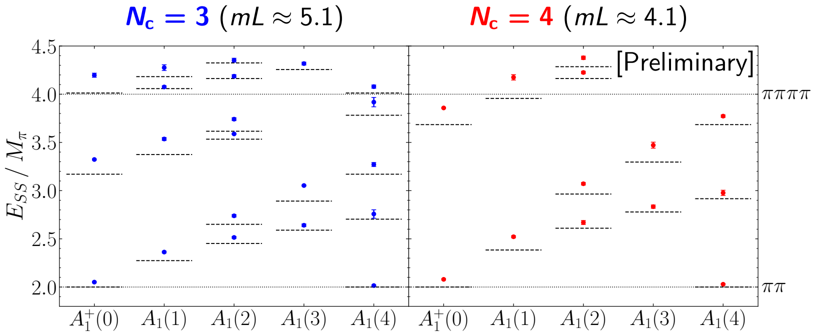
↳ Cubic-group irreps

Scattering phase shift: *AA* channel

Compare amplitude for $\pi\pi$ vs $\pi\pi$ + local tetraquarks

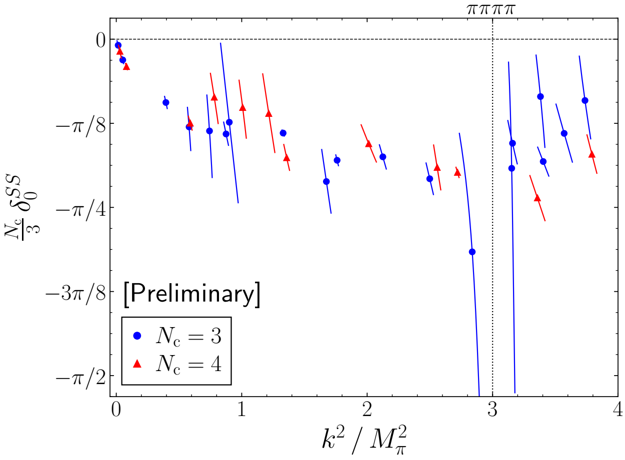


Finite-volume energies: SS channel ($\pi\pi$ only)



Scattering phase shift: SS channel ($\pi\pi$ only)

Expected large N_c scaling: $\boldsymbol{\mathcal{M} \sim N_c^{-1}}$



Overview

Summary and outlook

Goal: N_c scaling of meson-meson scattering

- The large N_c limit allows to study the non-pertubartive regime of QCD, but subleading effects may be large
- We have studied $\pi\pi$ scattering near threshold and matched it to U(4) ChPT
- We are working on an energy-dependent study of mesons-meson scattering, searching for possible tetraquark resonance

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Thank you for your attention!

Two-particle $\pi\pi$ isospin states

$$\mathbf{84}\,(\boldsymbol{SS}) \oplus \mathbf{45}\,(\boldsymbol{SA}) \oplus \mathbf{45}\,(\boldsymbol{AS}) \oplus \mathbf{20}\,(\boldsymbol{AA}) \oplus 15 \oplus 15 \oplus 1$$

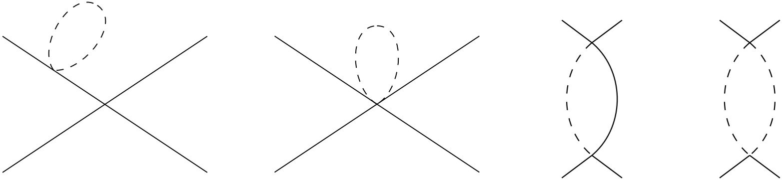
$$O_{SS}(p_1,p_2) = \frac{1}{2}\left[\pi^+(p_1)D_s^+(p_2) + \pi^+(p_2)D_s^+(p_1) + K^+(p_1)D^+(p_2) + K^+(p_2)D^+(p_1)\right]$$

$$O_{SA}(p_1,p_2) = \frac{1}{2}\left[\pi^+(p_1)D_s^+(p_2) - \pi^+(p_2)D_s^+(p_1) - K^+(p_1)D^+(p_2) + K^+(p_2)D^+(p_1)\right]$$

$$O_{AS}(p_1,p_2) = \frac{1}{2}\left[\pi^+(p_1)D_s^+(p_2) - \pi^+(p_2)D_s^+(p_1) + K^+(p_1)D^+(p_2) - K^+(p_2)D^+(p_1)\right]$$

$$O_{AA}(p_1,p_2) = \frac{1}{2}\left[\pi^+(p_1)D_s^+(p_2) + \pi^+(p_2)D_s^+(p_1) - K^+(p_1)D^+(p_2) - K^+(p_2)D^+(p_1)\right]$$

We must consider the following loop diagrams



$$\mathcal{T}_{AA}^{\mathrm{U}(N_f)} = \mathcal{T}_{AA}^{\mathrm{SU}(N_f)} + \frac{4M_\pi^4}{F_\pi^4 N_f}\left(1+\frac{2}{N_f}\right)F_{\eta'\pi}(k^2) - \frac{4M_\pi^4}{F_\pi^4 N_f^2}F_{\eta'\eta'}(k^2) + \frac{2M_\pi^4}{F_\pi^4}\left[K_{AA} + K'_{AA}\left(\frac{k^2}{M_\pi}\right)^2 + \dots\right]$$

$$\mathcal{T}_{SS}^{\mathrm{U}(N_f)} = \mathcal{T}_{SS}^{\mathrm{SU}(N_f)} - \frac{4M_\pi^4}{F_\pi^4 N_f}\left(1-\frac{2}{N_f}\right)F_{\eta'\pi}(k^2) - \frac{4M_\pi^4}{F_\pi^4 N_f^2}F_{\eta'\eta'}(k^2) + \frac{2M_\pi^4}{F_\pi^4}\left[K_{SS} + K'_{SS}\left(\frac{k^2}{M_\pi}\right)^2 + \dots\right]$$

Large N_c limit of U(N_f) ChPT

$M^2_{\eta'} \to M^2_\pi$ limit:

$$M_\pi a_0^{SS} = -\frac{M_\pi^2}{16\pi F_\pi^2}\left[1 - \frac{16 M_\pi^2}{F_\pi^2} L_{SS} + K_{SS}\left(\frac{M_\pi^2}{F_\pi^2}\right)^2 + \frac{M_\pi^2}{8F_\pi^2\pi^2}\log\frac{M_\pi^2}{\mu^2}\right]$$

$$M_\pi a_0^{AA} = \frac{M_\pi^2}{16\pi F_\pi^2}\left[1 - \frac{16 M_\pi^2}{F_\pi^2} L_{AA} + K_{AA}\left(\frac{M_\pi^2}{F_\pi^2}\right)^2 - \frac{M_\pi^2}{8F_\pi^2\pi^2}\log\frac{M_\pi^2}{\mu^2}\right]$$

$M^2_{\eta'} \gg M^2_\pi$ to match SU(N_f) and U(N_f) LECs:

$$\left[L_{SS}^{(1)}\right]_{SU(N_f)} = \left[L_{SS}^{(1)}\right]_{U(N_f)} - \frac{1}{8N_f^2(4\pi)^2}(1+N_f\lambda_0-\lambda_0)$$

$$\left[L_{AA}^{(1)}\right]_{SU(N_f)} = \left[L_{AA}^{(1)}\right]_{U(N_f)} + \frac{1}{8N_f^2(4\pi)^2}(1-N_f\lambda_0-\lambda_0)$$

$$\lambda_0 = \log\frac{M^2_{\eta'} - M^2_\pi}{\mu^2}$$

$\pi\pi$ scattering in ChPT

$\pi\pi$ scatering amplitudes for N_f flavours are known to NNLO [Weinberg 1979, Gasser, Leutwyler 1985, Bijnens, Lu 2011]

NLO

$$M_\pi a_0^{SS} = -\frac{M_\pi^2}{16\pi F_\pi^2}\left[1 - \frac{16M_\pi^2}{F_\pi^2}L_{SS} + \frac{M_\pi^2}{8F_\pi^2\pi^2N_f^2} - \frac{M_\pi^2}{8F_\pi^2\pi^2N_f} + \frac{M_\pi^2}{8F_\pi^2\pi^2N_f^2}\log\frac{M_\pi^2}{\mu^2} - \frac{M_\pi^2}{8F_\pi^2\pi^2N_f}\log\frac{M_\pi^2}{\mu^2} + \frac{M_\pi^2}{8F_\pi^2\pi^2}\log\frac{M_\pi^2}{\mu^2}\right]$$

$$L_R = L^{(0)}N_c + L_R^{(1)} + \dots$$

$$M_\pi a_0^{AA} = \frac{M_\pi^2}{16\pi F_\pi^2}\left[1 - \frac{16M_\pi^2}{F_\pi^2}L_{AA} - \frac{M_\pi^2}{8F_\pi^2\pi^2N_f^2} - \frac{M_\pi^2}{8F_\pi^2\pi^2N_f} - \frac{M_\pi^2}{8F_\pi^2\pi^2N_f^2}\log\frac{M_\pi^2}{\mu^2} - \frac{M_\pi^2}{8F_\pi^2\pi^2N_f}\log\frac{M_\pi^2}{\mu^2} - \frac{M_\pi^2}{8F_\pi^2\pi^2}\log\frac{M_\pi^2}{\mu^2}\right]$$

Explicit N_f scaling is not the expected at large N_c

Large N_c: $a_0^R \propto \mp\frac{1}{N_c}\left(\tilde{a} + \tilde{b}\frac{N_f}{N_c} \mp \tilde{c}\frac{1}{N_c}\right) + \mathcal{O}(N_c^{-3})$

$\tilde{a}, \tilde{b}, \tilde{c} \sim \mathcal{O}(1)$ constants

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$\tilde{a}, \tilde{b}, \tilde{c} \sim \mathcal{O}(1)$ constants

$\pi\pi$ scattering in large N_c ChPT

We have computed $\mathcal{M}_{SS}$ and $\mathcal{M}_{AA}$ to NNLO in U(N_f) ChPT

$$M_\pi a_0^{SS} = -\frac{M_\pi^2}{16\pi F_\pi^2}\Bigg[1 - \frac{16M_\pi^2}{F_\pi^2}L_{SS} + K_{SS}\left(\frac{M_\pi^2}{F_\pi^2}\right)^2 + \frac{M_\pi^2}{4F_\pi^2\pi^2N_f^2} + \frac{M_\pi^2}{8F_\pi^2\pi^2}\log\frac{M_\pi^2}{\mu^2}$$
$$-\frac{M_\pi^2}{8F_\pi^2\pi^2N_f} - \frac{M_\pi^2+M_{\eta'}^2}{M_\pi^2-M_{\eta'}^2}\frac{M_\pi^2}{8F_\pi^2\pi^2N_f^2}\log\frac{M_\pi^2}{\mu^2} + \frac{M_{\eta'}^2}{M_\pi^2-M_{\eta'}^2}\frac{M_\pi^2}{8F_\pi^2\pi^2N_f}\log\frac{M_\pi^2}{\mu^2}$$
$$+\frac{M_\pi^2+M_{\eta'}^2}{M_\pi^2-M_{\eta'}^2}\frac{M_\pi^2}{8F_\pi^2\pi^2N_f^2}\log\frac{M_{\eta'}^2}{\mu^2} - \frac{M_{\eta'}^2}{M_\pi^2-M_{\eta'}^2}\frac{M_\pi^2}{8F_\pi^2\pi^2N_f}\log\frac{M_{\eta'}^2}{\mu^2}\Bigg]$$

$L_R|_{\mathrm{U}(N_f)} \neq L_R|_{\mathrm{SU}(N_f)}$

$$\frac{K_{SS}}{N_c^2} = \frac{K_{AA}}{N_c^2} + \mathcal{O}(N_c^{-1})$$

...and similarly for $M_\pi a_0^{AA}$

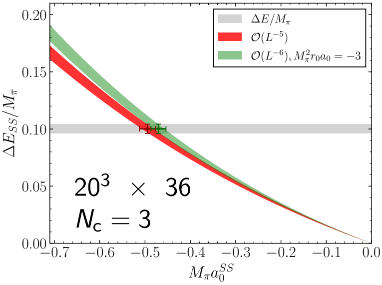
Expected N_f scaling at large N_c (as $M_{\eta'}^2 \to M_\pi^2$)

$$L_{SS} = N_c L^{(0)} + N_f L_c^{(1)} - L_a^{(1)} + \mathcal{O}(N_c^{-1})$$
$$L_{AA} = \underbrace{N_c L^{(0)} + N_f L_c^{(1)}}_{\text{Same sign}} \underbrace{+ L_a^{(1)}}_{\text{Opposite sign}} + \mathcal{O}(N_c^{-1})$$

Lüscher's formalism

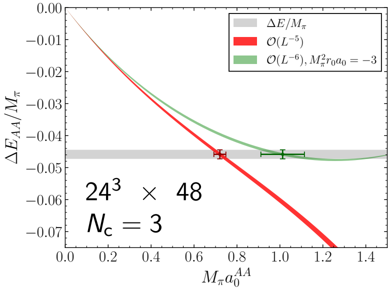
Check convergence of threshold expansion

SS channel



✓ Good convergence

AA channel



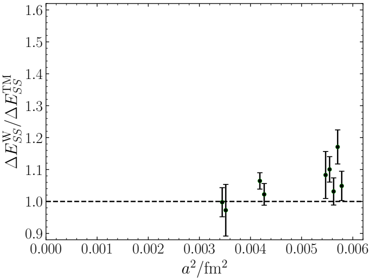
✗ Not converging at $\mathcal{O}(L^{-6})$ for large ξ/small volume.

$$\xi = \frac{M_\pi^2}{(4\pi F_\pi)^2}$$

Discretization effects for $N_c = 3$

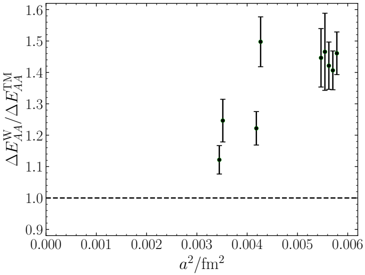
Compare both regularizations ⟶ Must be equal in the continuum

SS channel



Small $\mathcal{O}(a^2)$ effects

AA channel



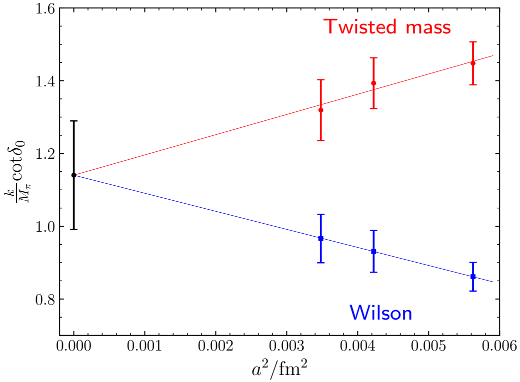
Large $\mathcal{O}(a^2)$ effects

AA-channel: Continuum extrapolation for $N_c = 3$

Continuum extrapolation of $k \cot \delta_0$ for $N_c = 3$ in 3 steps:

1. Extrapolation to $k/M_\pi = -0.08$ using Effective Range Expansion and $M_\pi^2 r_0 a_0 \in [-5, -1]$
2. Interpolation to $\xi = 0.14$
3. Constrained continuum extrapolation

LO ChPT: $M_\pi^2 r_0 a_0 = -3$



- ⋆ Large $\mathcal{O}(a^2)$ effects for both regularizations
- ⋆ Use TM fermions
- ⋆ Wilson-ChPT inspired parametrization

$$\Delta \mathcal{M}_{AA} = 32\pi^2 a^2 W \xi$$

$[W \sim \mathcal{O}(N_c^0)]$

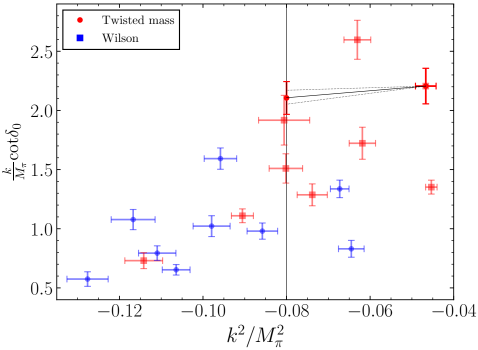
- ⋆ $W = 42(29)\ \mathrm{fm}^{-2}$

First step of continuum extrapolation

Extrapolation to $k/M_\pi^2 = -0.08$ using Effective Range Expansion with $M_\pi^2 r_0 a_0 \in [-5, -1]$

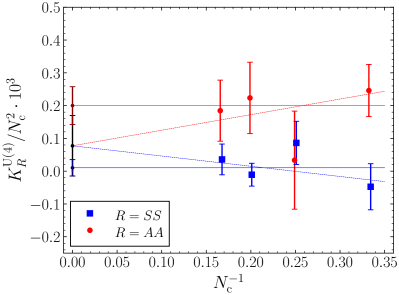
LO ChPT:
$M_\pi^2 r_0 a_0 = -3$

$$\text{ERE: } \frac{k}{M_\pi} \cot\delta_0 = \frac{1}{M_\pi a_0}\left[1 + \frac{1}{2}(M_\pi^2 r_0 a_0)\left(\frac{k^2}{M_\pi^2}\right)\right]$$



Fits at fixed N_c: Case of K

We fit to ChPT at fixed N_c for each channel separately,



Well described by:

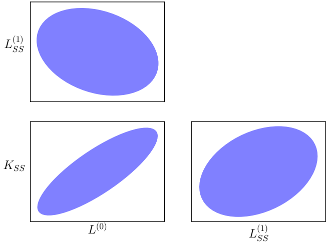
- ⋆ Different N_c^2 terms
- ⋆ Common N_c^2 term + subleading dependency

Results from global fits

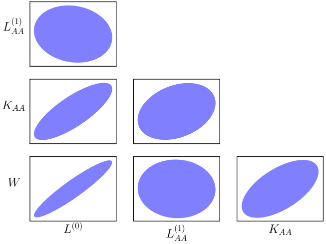
Single channel fits

Fit	$L^{(0)} \cdot 10^3$	$L^{(1)} \cdot 10^3$	$K/N_c^2 \cdot 10^5$	W	χ^2/dof
SS, SU(4)	-0.04(1.3)	-1.70(18)	-	-	12.8/15
AA, SU(4)	-1.22(19)	0.8(4)	-	-94(15)	38.5/19
SS, U(4)	-0.01(7)	-1.78(20)	1.2(2.5)	-	12.2/14
AA, U(4)	-0.1(4)	1.8(4)	21(5)	-32(23)	22.5/18

SS channel, U(4) ChPT



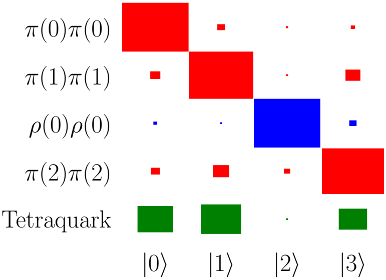
AA channel, U(4) ChPT



Effects of the operators in the basis

Eigenvectors of the GEVP provide intuition on the effect of each operator

AA channel, $N_c = 3$, $A_1^+(0)$, $M_\rho \sim 1.7 M_\pi$
Area $\propto$ Relative overlap



Negligible effect on $\pi\pi$

Significant overlap with $\pi\pi$

Finite-volume energies: AA channel

We study the effects of different operators for $N_c = 3$:

$\pi\pi$ + Local tetraquarks vs $\pi\pi$ + $\rho\rho$ + Local tetraquarks