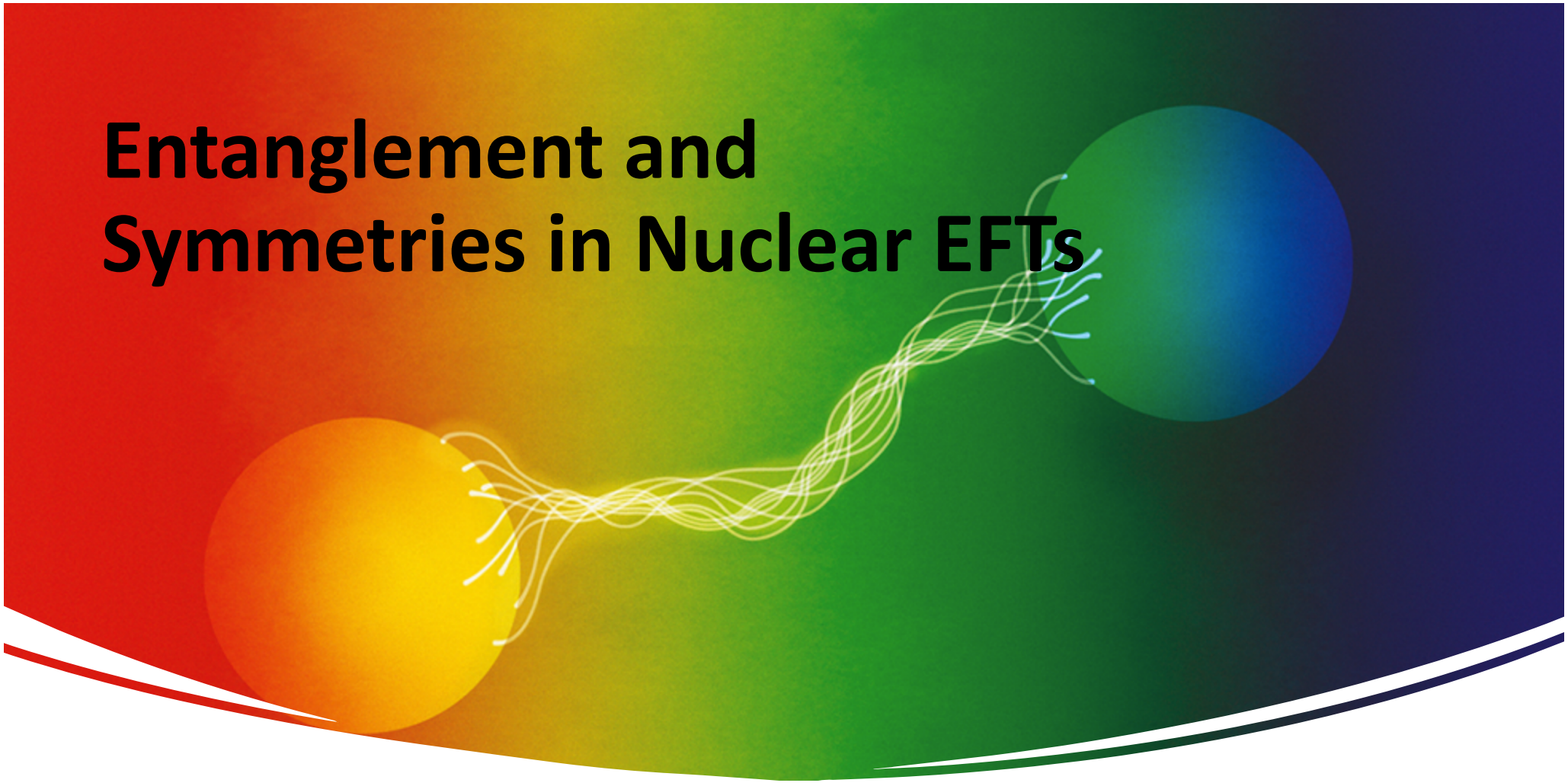
Entanglement and Symmetries in Nuclear EFTs



Jefferson Lab

- Ian Low
- Argonne/Northwestern

- Theory Center Seminar
- Feb 13th, 2023

Acknowledgement 1:

U.S. DEPARTMENT OF ENERGY | Office of Science

Office of Nuclear Physics

Acknowledgement 2:

Work collaborated with Qiaofeng Liu and Tom Mehen.

Entanglement is quantum world's most prominent feature:

- It refers to the situation where a measurement on a subsystem will improve our knowledge on the rest of the system.

- A quantum state of a system is entangled if it cannot be written as a tensor-product state of its subsystems.

- Consider a bipartite system $\mathcal{H}_{12} = \mathcal{H}_1 \otimes \mathcal{H}_2$, a state vector $|\psi\rangle \in \mathcal{H}_{12}$ is *entangled* if there is no $|\psi_1\rangle \in \mathcal{H}_1$ and $|\psi_2\rangle \in \mathcal{H}_2$ such that

$$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$$

Einstein famously attacked “entanglement” as spooky action at a distance:

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

A. Einstein B. Podolsky N. Rosen

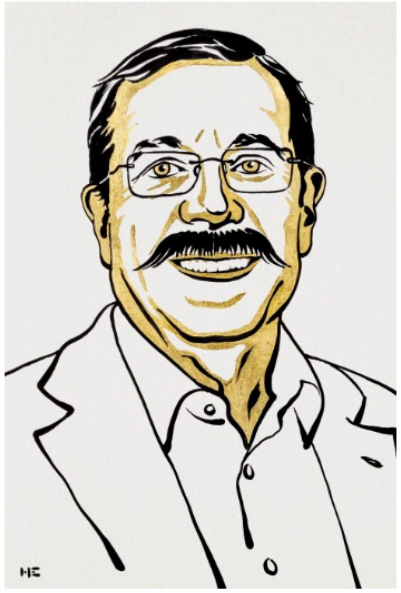
EINSTEIN ATTACKS QUANTUM THEORY

Scientist and Two Colleagues Find It Is Not ‘Complete’ Even Though ‘Correct.’

SEE FULLER ONE POSSIBLE

Believe a Whole Description of ‘the Physical Reality’ Can Be Provided Eventually.

The Nobel Prize in Physics 2022



Ill. Niklas Elmehed © Nobel Prize Outreach

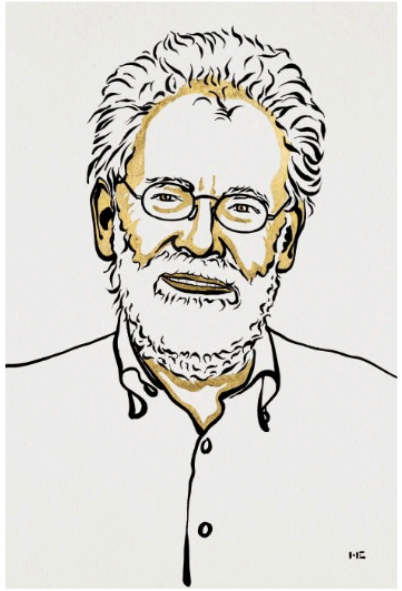
Alain Aspect

Prize share: 1/3

Ill. Niklas Elmehed © Nobel Prize Outreach

John F. Clauser

Prize share: 1/3



Ill. Niklas Elmehed © Nobel Prize Outreach

Anton Zeilinger

Prize share: 1/3

The Nobel Prize in Physics 2022 was awarded jointly to Alain Aspect, John F. Clauser and Anton Zeilinger "for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science"

On the other hand, symmetry is among the most fundamental principles in physics:

Powerful characterization of nature based on invariance under a specified group of transformations.

Symmetries give rise to conserved quantities: energy, momentum, angular momentum, etc.

Combining with quantum mechanics, there is a subtle realization of symmetry – spontaneous symmetry breaking.

All known fundamental interactions are based on symmetry principles.

In modern perspectives,

- Gauge symmetry is not a real symmetry – it's a reflection of the redundancy in our description of nature.

A massless spin-1 particle has only two degrees of freedom, + and - Helicities. However a Lorentz invariant description requires adding two fictitious degrees of freedom by hand.

- Global symmetries are believed to be either completely broken or only approximate.

Discrete symmetries like parity, time-reversal invariance are broken in nature, while approximate symmetries are ubiquitous, for example the isospin invariance.

But what is the origin of symmetry?

There are two historical perspectives:

Beauty In, Garbage Out –
As we explore higher and higher energy regimes, we discover more and more symmetries. The symmetry is usually hidden or broken in low energies.

Garbage In, Beauty Out –
At high energy level there is no symmetry. Rather symmetry emerges only at large distances, in the infrared. These are accidental symmetries.

But what is the origin of symmetry?

There are two historical perspectives:

Beauty In, Garbage Out –
As we explore higher and higher energy regimes, we discover more and more symmetries. The symmetry is usually hidden or broken in low energies.

Garbage In, Beauty Out –
At high energy level there is no symmetry. Rather symmetry emerges only at large distances, in the infrared. These are accidental symmetries.

But neither explain whether symmetry can be the natural outgrowth of more fundamental principles.

In 2018 a paper from Seattle made a fascinating observation regarding emergent symmetries and entanglement suppression in low-energy QCD:

PHYSICAL REVIEW LETTERS **122,** 102001 (2019)

Entanglement Suppression and Emergent Symmetries of Strong Interactions

Silas R. Beane,[1] David B. Kaplan,[2] Natalie Klco,[1,2] and Martin J. Savage[2]
[1]*Department of Physics, University of Washington, Seattle, Washington 98195-1560, USA*
[2]*Institute for Nuclear Theory, University of Washington, Seattle, Washington 98195-1550, USA*

(Received 20 December 2018; published 14 March 2019)

Entanglement suppression in the strong-interaction S matrix is shown to be correlated with approximate spin-flavor symmetries that are observed in low-energy baryon interactions, the Wigner $SU(4)$ symmetry for two flavors and an $SU(16)$ symmetry for three flavors. We conjecture that

This raises the intriguing possibility of understanding symmetry from quantum entanglement!

John Wheeler famously coined the phrase:

It from bit : "All things physical are information-theoretic in origin"

INFORMATION, PHYSICS, QUANTUM: THE SEARCH FOR LINKS

John Archibald Wheeler * †

Abstract

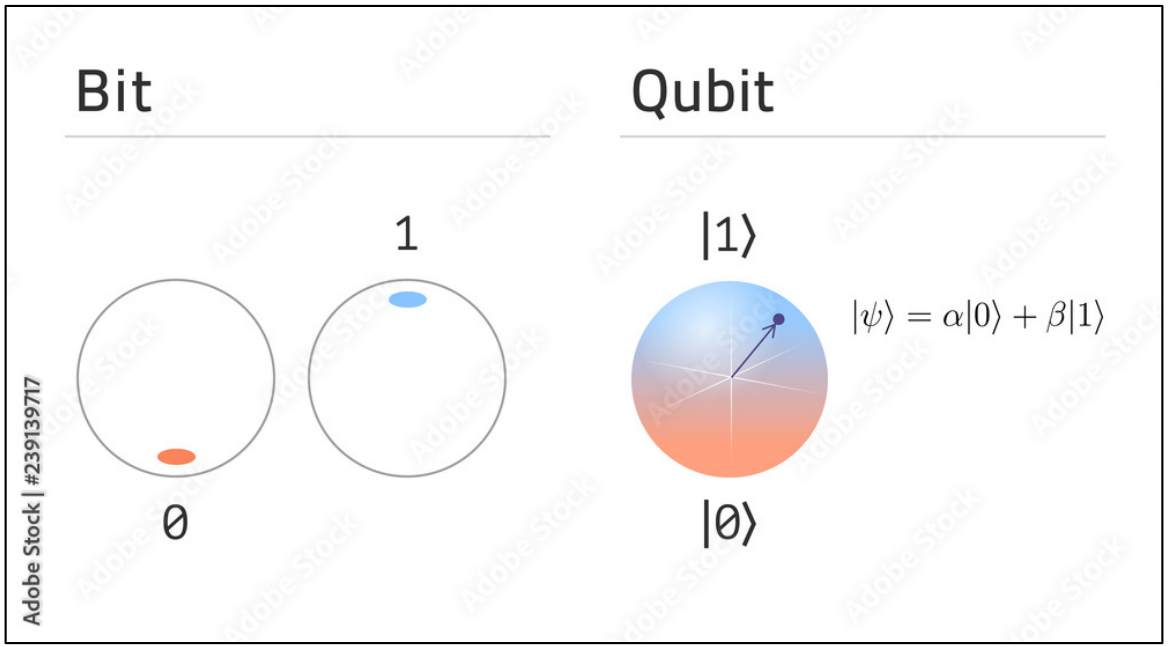
This report reviews what quantum physics and information theory have to tell us about the age-old question, How come existence? No escape is evident from four

winnowing: **It from bit**. Otherwise put, every **it** — every particle, every field of force, even the spacetime continuum itself — derives its function, its meaning, its very existence entirely — even if in some contexts indirectly — from the apparatus-elicited answers to yes or no questions, binary choices [52], **bits**.

Indeed, remarkable connections between fundamental physics and information science in the past decade.

It is natural to ask:

Can symmetry come from qubit?



In this talk, we will use low-energy QCD as a playground to study:

- Unexpected, emerging (approximate) global symmetries in low-energy hadronic physics.

- Correlation between symmetry and entanglement suppression in non-relativistic 2-to-2 scattering.

- Elucidate the connection from an information-theoretic viewpoint.

Emergent symmetries in low-energy QCD:

- Schrodinger symmetry (non-relativistic conformal invariance)

The largest symmetry group preserved by the Schrodinger equation, which includes Galilean boosts, scale and special conformal transformations.

- Spin-flavor symmetries

Possible only in non-relativistic theories due to Coleman-Mandula theorem.
Examples include $SU(2N_f)$ quark spin-flavor symmetries.
There's also Wigner's "supermultiplet" SU(4) spin-flavor symmetry among neutrons and protons.

SU($2N_f$) spin-flavor symmetries are symmetries of non-relativistic quark model and date back to 1960s:

SU(4): $$\mathbf{4} = \begin{pmatrix} u\uparrow \\ u\downarrow \\ d\uparrow \\ d\downarrow \end{pmatrix}$$

SU(6): $$\mathbf{6} = \begin{pmatrix} u\uparrow \\ u\downarrow \\ d\uparrow \\ d\downarrow \\ s\uparrow \\ s\downarrow \end{pmatrix}$$

Ample evidence for SU(4) and SU(6) in nature:

- Masses
- Magnetic moments and transitions
- Semilepton currents
- Meson-baryon and baryon-baryon couplings

SU($2N_f$) spin-flavor symmetries are symmetries of non-relativistic quark model and date back to 1960s:

SU(4): $$\mathbf{4} = \begin{pmatrix} u\uparrow \\ u\downarrow \\ d\uparrow \\ d\downarrow \end{pmatrix}$$

SU(6): $$\mathbf{6} = \begin{pmatrix} u\uparrow \\ u\downarrow \\ d\uparrow \\ d\downarrow \\ s\uparrow \\ s\downarrow \end{pmatrix}$$

Ample evidence for SU(4) and SU(6) in nature:

- Masses
- Magnetic moments and transitions
- Semilepton currents
- Meson-baryon and baryon-baryon couplings

They can be derived from QCD in the large N_c limit!

Dashen, Monohar (1993); Dashen, Jenkins, Manohar (1994); Kaplan, Savage (1995)

In low-energy nuclear physics, there is a different SU(4) symmetry first observed by Wigner:

JANUARY 15, 1937 PHYSICAL REVIEW VOLUME 51

On the Consequences of the Symmetry of the Nuclear Hamiltonian on the Spectroscopy of Nuclei

E. WIGNER*
Princeton University, Princeton, New Jersey

(Received October 23, 1936)

The structure of the multiplets of nuclear terms is investigated, using as first approximation a Hamiltonian which does not involve the ordinary spin and corresponds to equal forces between all nuclear constituents, protons and neutrons. The multiplets turn out to have a

In this case the neutron and proton fill out a "supermultiplet":

$$N = \begin{pmatrix} p\uparrow \\ p\downarrow \\ n\uparrow \\ n\downarrow \end{pmatrix}$$

In low-energy nuclear physics, there is a different SU(4) symmetry first observed by Wigner:

JANUARY 15, 1937 PHYSICAL REVIEW VOLUME 51

On the Consequences of the Symmetry of the Nuclear Hamiltonian on the Spectroscopy of Nuclei

E. WIGNER*
Princeton University, Princeton, New Jersey

(Received October 23, 1936)

The structure of the multiplets of nuclear terms is investigated, using as first approximation a Hamiltonian which does not involve the ordinary spin and corresponds to equal forces between all nuclear constituents, protons and neutrons. The multiplets turn out to have a

In this case the neutron and proton fill out a "supermultiplet":

$$N = \begin{pmatrix} p\uparrow \\ p\downarrow \\ n\uparrow \\ n\downarrow \end{pmatrix}$$

In the EFT language, Wigner's SU(4) is accidental in that, after imposing the SU(4) quark spin-flavor symmetry, the only remaining operator has this symmetry.

So far these spin-flavor symmetries can be argued from the large N_c limit.

There are emergent symmetries beyond the large N_c :

- Unnaturally large scattering lengths in low-energy NN scattering in the s-wave, which include $^1S_0$ and $^3S_1$ channels.

  The S-matrix for a single channel is

$$S = e^{2i\delta(p)} = 1 + i\frac{Mp}{2\pi}\mathcal{A}\ , \qquad \mathcal{A} = \frac{4\pi}{M}\frac{1}{p\cot\delta - ip}$$

  It is long known that it's $p \cot \delta$ which admits an expansion in 1/p, the Effective Range Expansion (ERE):

$$p\cot\delta = -\frac{1}{a} + \frac{1}{2}r_0\,p^2 + \cdots = -\frac{1}{a} + \frac{1}{2}\Lambda^2\sum_{n=0}^{\infty} r_n\left(\frac{p^2}{\Lambda^2}\right)^n$$

The scattering length

So far these spin-flavor symmetries can be argued from the large N_c limit.

There are emergent symmetries beyond the large N_c :

- Unnaturally large scattering lengths in low-energy NN scattering in the s-wave, which include $^1S_0$ and $^3S_1$ channels.

  The S-matrix for a single channel is

$$S = e^{2i\delta(p)} = 1 + i\frac{Mp}{2\pi}\mathcal{A}\,, \qquad \mathcal{A} = \frac{4\pi}{M}\frac{1}{p\cot\delta - ip}$$

  It is long known that it's $p \cot \delta$ which admits an expansion in 1/p, the Effective Range Expansion (ERE):

$$p\cot\delta = -\frac{1}{a} + \frac{1}{2}r_0\,p^2 + \cdots = -\frac{1}{a} + \frac{1}{2}\Lambda^2\sum_{n=0}^{\infty} r_n\left(\frac{p^2}{\Lambda^2}\right)^n$$

The scattering length

$^1S_0$: a_0 = -23.7 fm
$^3S_1$: a_1 = 5.4 fm → Deuteron!
$1/m_\pi$ = 1.4 fm

In the limit the scattering length *a* diverges, the system has no scale and there's the non-relativistic conformal invariance. Mehen, Stewart, Wise (1999)

At the infinitesimal level,

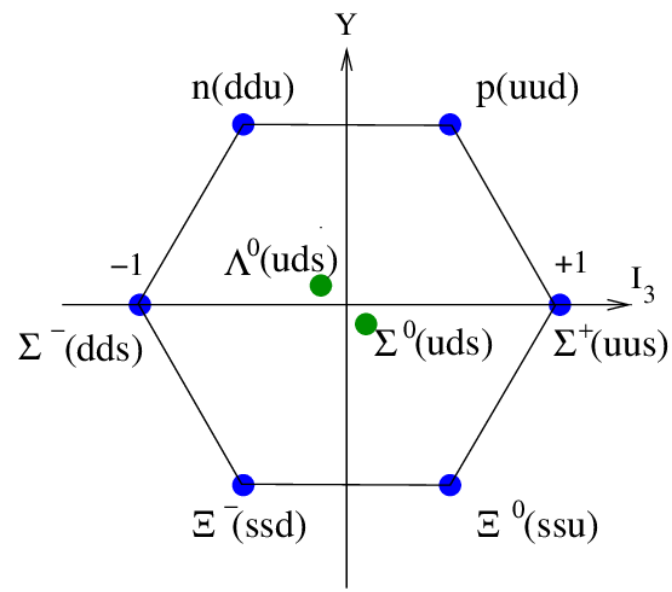
$$\begin{aligned}
&\text{boosts:} && \vec{x}\,' = \vec{x} + \vec{v}\,t \ , \quad t' = t \ , \\
&\text{scale:} && \vec{x}\,' = \vec{x} + s\vec{x} \ , \quad t' = t + 2st \ , \\
&\text{conformal:} && \vec{x}\,' = \vec{x} - ct\vec{x} \ , \quad t' = t - ct^2 \ ,
\end{aligned}$$

So NN scattering has approximate Schrodinger symmetry.

WHO ORDERED THAT??!

- Nucleons are part of spin-1/2 octet baryons:

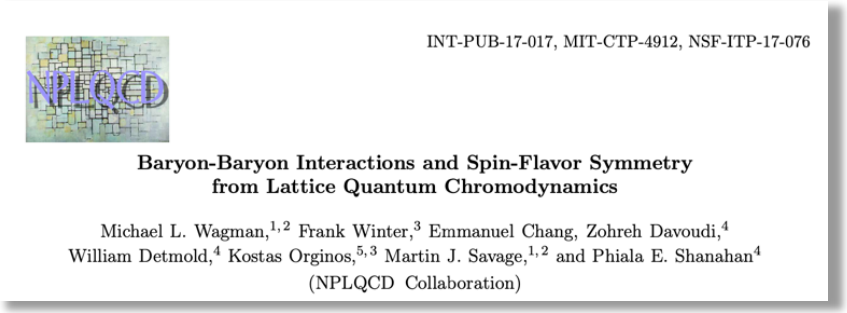
$$B = \begin{pmatrix} \Sigma^0/\sqrt{2} + \Lambda/\sqrt{6} & \Sigma^+ & p \\ \Sigma^- & -\Sigma^0/\sqrt{2} + \Lambda/\sqrt{6} & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda \end{pmatrix}$$



$$\mathcal{L}_{\rm LO}^{n_f=3} = -\frac{c_1}{f^2}\langle B_i^\dagger B_i B_j^\dagger B_j\rangle - \frac{c_2}{f^2}\langle B_i^\dagger B_j B_j^\dagger B_i\rangle - \frac{c_3}{f^2}\langle B_i^\dagger B_j^\dagger B_i B_j\rangle - \frac{c_4}{f^2}\langle B_i^\dagger B_j^\dagger B_j B_i\rangle$$
$$- \frac{c_5}{f^2}\langle B_i^\dagger B_i\rangle\langle B_j^\dagger B_j\rangle - \frac{c_6}{f^2}\langle B_i^\dagger B_j\rangle\langle B_j^\dagger B_i\rangle, \qquad i,j = \uparrow,\downarrow$$

Savage, Wise (1995)

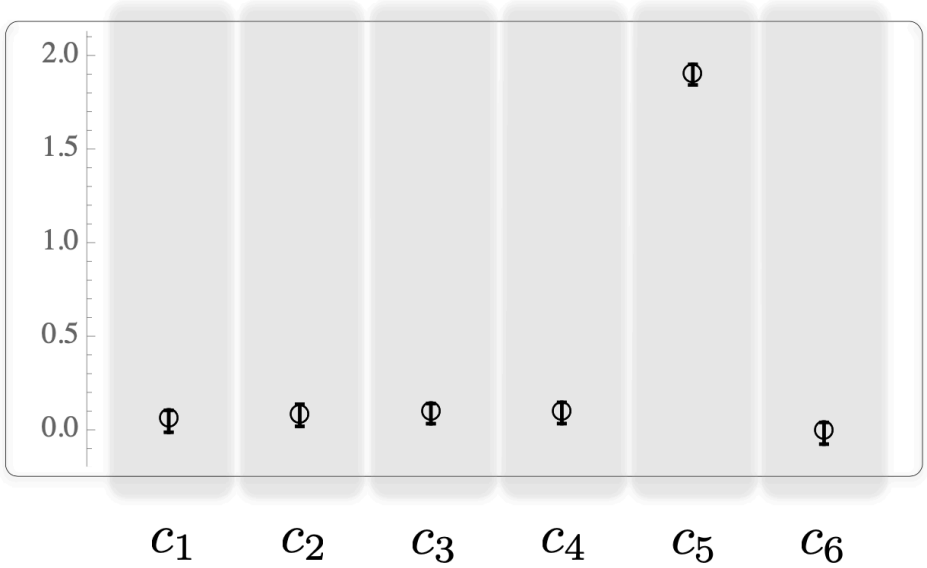
Lattice QCD could compute the six Wilson coefficients under some special circumstances:



INT-PUB-17-017, MIT-CTP-4912, NSF-ITP-17-076

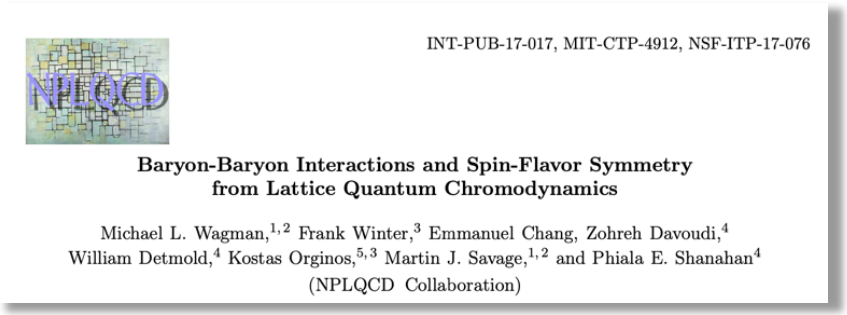
Baryon-Baryon Interactions and Spin-Flavor Symmetry from Lattice Quantum Chromodynamics

Michael L. Wagman,[1,2] Frank Winter,[3] Emmanuel Chang, Zohreh Davoudi,[4] William Detmold,[4] Kostas Orginos,[5,3] Martin J. Savage,[1,2] and Phiala E. Shanahan[4]
(NPLQCD Collaboration)



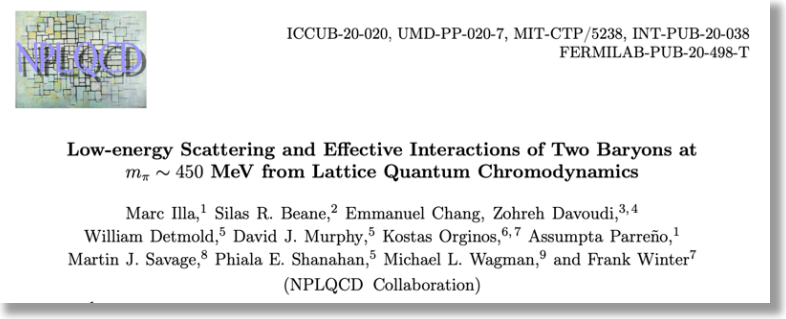
m_π = 804 MeV

Lattice QCD could compute the six Wilson coefficients under some special circumstances:


INT-PUB-17-017, MIT-CTP-4912, NSF-ITP-17-076

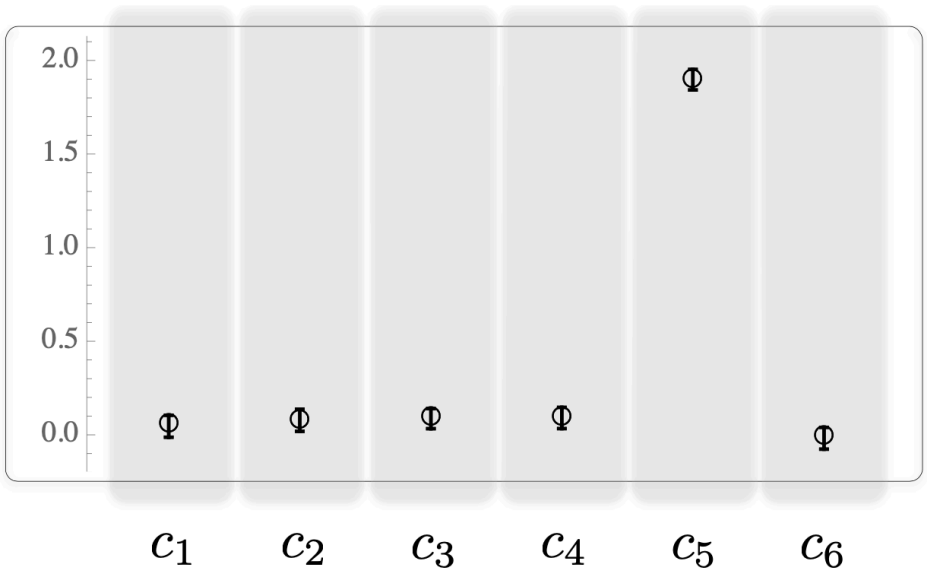
Baryon-Baryon Interactions and Spin-Flavor Symmetry from Lattice Quantum Chromodynamics

Michael L. Wagman,[1,2] Frank Winter,[3] Emmanuel Chang, Zohreh Davoudi,[4] William Detmold,[4] Kostas Orginos,[5,3] Martin J. Savage,[1,2] and Phiala E. Shanahan[4]
(NPLQCD Collaboration)

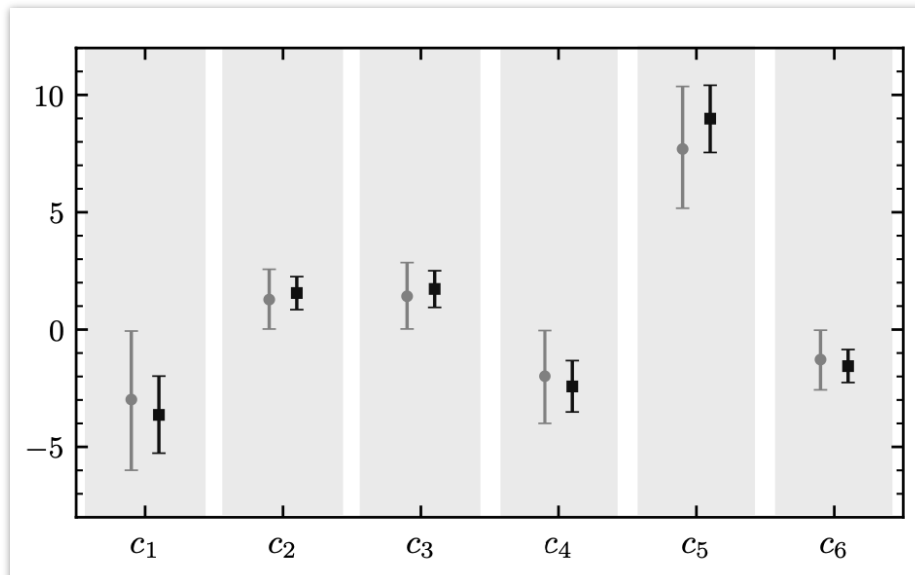

ICCUB-20-020, UMD-PP-020-7, MIT-CTP/5238, INT-PUB-20-038
FERMILAB-PUB-20-498-T

Low-energy Scattering and Effective Interactions of Two Baryons at $m_\pi \sim 450$ MeV from Lattice Quantum Chromodynamics

Marc Illa,[1] Silas R. Beane,[2] Emmanuel Chang, Zohreh Davoudi,[3,4] William Detmold,[5] David J. Murphy,[5] Kostas Orginos,[6,7] Assumpta Parreño,[1] Martin J. Savage,[8] Phiala E. Shanahan,[5] Michael L. Wagman,[9] and Frank Winter[7]
(NPLQCD Collaboration)



m_π = 804 MeV



m_π = 450 MeV

In the limit where all coefficients but c_5 are vanishing:

$$\mathcal{L}_{\rm LO}^{n_f=3} = -\frac{c_1}{f^2}\langle B_i^\dagger B_i B_j^\dagger B_j\rangle - \frac{c_2}{f^2}\langle B_i^\dagger B_j B_j^\dagger B_i\rangle - \frac{c_3}{f^2}\langle B_i^\dagger B_j^\dagger B_i B_j\rangle - \frac{c_4}{f^2}\langle B_i^\dagger B_j^\dagger B_j B_i\rangle$$
$$-\frac{c_5}{f^2}\langle B_i^\dagger B_i\rangle\langle B_j^\dagger B_j\rangle - \frac{c_6}{f^2}\langle B_i^\dagger B_j\rangle\langle B_j^\dagger B_i\rangle, \qquad i,j=\uparrow,\downarrow$$

The remaining operator can be re-written,

$$\mathcal{B} = (n_\uparrow, n_\downarrow, p_\uparrow, p_\downarrow, \dots)\ , \qquad \mathcal{L} = -c_5\left(\mathcal{B}^\dagger\mathcal{B}\right)^2$$

which is invariant under an SU(16) spin-flavor symmetry

$$\mathcal{B} \to U\mathcal{B}\ , \qquad U^\dagger U = 1$$

There is no large N_c explanation!

To discuss entanglement suppression, we need to quantify the amount of entanglement → Entanglement Measure!

Many possibilities for Entanglement Measure. For bipartite systems:

von Neumann entropy: $$E(\rho) = -\mathrm{Tr}(\rho_1 \ln \rho_1) = -\mathrm{Tr}(\rho_2 \ln \rho_2)$$

Linear entropy: $$E(\rho) = -\mathrm{Tr}(\rho_1(\rho_1 - 1)) = 1 - \mathrm{Tr}\rho_1^2$$

$$\rho = |\psi\rangle\langle\psi| \qquad \rho_{1/2} = \mathrm{Tr}_{2/1}(\rho)$$

The common property is that the entanglement measure vanishes for a product state $|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$, but attains the maximum for maximally entangled states.

For a system with two spin-1/2 particles, let's define the "computational basis:"

$$\{|\uparrow\uparrow\rangle\,,|\uparrow\downarrow\rangle\,,|\downarrow\uparrow\rangle\,,|\downarrow\downarrow\rangle\}$$

Then for a general normalized state,

$$|\psi\rangle = \alpha\,|\uparrow\uparrow\rangle + \beta\,|\uparrow\downarrow\rangle + \gamma\,|\downarrow\uparrow\rangle + \delta\,|\downarrow\downarrow\rangle\,, \qquad |\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1\ ,$$

The reduced density matrix and linear entropy are

$$\rho_1 = \mathrm{Tr}_2\,|\psi\rangle\,\langle\psi| = \begin{pmatrix} |\alpha|^2 + |\beta|^2 & \alpha\gamma^* + \beta\delta^* \\ \alpha^*\gamma + \beta^*\delta & |\gamma|^2 + |\delta|^2 \end{pmatrix}\,,$$

Easy to check that $E(|\psi\rangle) = 1 - \mathrm{Tr}_1\rho_1^2 = 2|\alpha\delta - \beta\gamma|^2.$

1. It vanishes for a product state.
2. Maximal entanglement is 1/2, which is the case for the Bell states:

$$(|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle)/\sqrt{2} \qquad (|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle)/\sqrt{2}$$

Entanglement is a property of the quantum state.

But we are more interested in the ability of a *quantum-mechanical operator* (i.e. the S-matrix) to entangle → the Entanglement Power

The entanglement power measures the ability of an operator *U* to generate entanglement by averaging over all product states:

$$E(U) = \overline{E(U\,|\psi_1\rangle \otimes |\psi_2\rangle)},$$

Entanglement is a property of the quantum state.

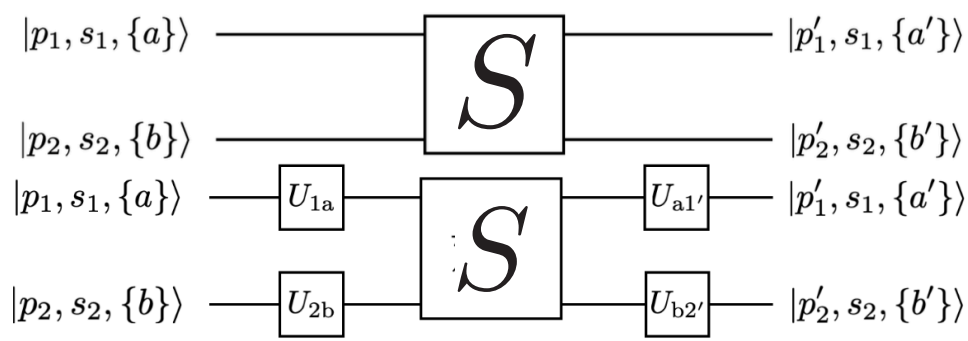
But we are more interested in the ability of a *quantum-mechanical operator* (i.e. the S-matrix) to entangle → the Entanglement Power

The entanglement power measures the ability of an operator *U* to generate entanglement by averaging over all product states:

$$E(U) = \overline{E(U\,|\psi_1\rangle \otimes |\psi_2\rangle)},$$

There is, however, a notion of equivalent class:

$$U \sim U' \qquad \text{if} \qquad U = (U_1 \otimes U_2)U'(V_1 \otimes V_2)$$



Modulo the equivalent class, there are two and only two minimally entangling operators, which in the computational basis,

$$\mathbf{1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad \mathrm{SWAP} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} .$$

Identity gate: do nothing.
SWAP gate: interchange the qubits.

In terms of Pauli matrices,

$$\mathrm{SWAP} = (1 + \boldsymbol{\sigma} \cdot \boldsymbol{\sigma})/2 \ , \qquad \boldsymbol{\sigma} \cdot \boldsymbol{\sigma} \equiv \sum_a \boldsymbol{\sigma}^a \otimes \boldsymbol{\sigma}^a \ .$$

Low, Mehen: 2104.10835

In this notation, the S-matrix for low-energy neutron-proton scattering in the s-wave can be written as

$$S = \frac{1}{2}\left(e^{2i\delta_1} + e^{2i\delta_0}\right)\mathbf{1} + \frac{1}{2}\left(e^{2i\delta_1} - e^{2i\delta_0}\right)\ \mathrm{SWAP},$$

δ_0 and δ_1 are the scattering phases in the $^1S_0$ and $^3S_1$ channels.

Conditions for the S-matrix to minimize entanglement:

1. $S = 1$ if $\delta_0 = \delta_1$ ➡ SU(4) or SU(16) spin-flavor sym.
2. $S = \mathrm{SWAP}$ if $|\delta_0 - \delta_1| = \pi/2$ ➡ Schrodinger sym.

Low, Mehen: 2104.10835

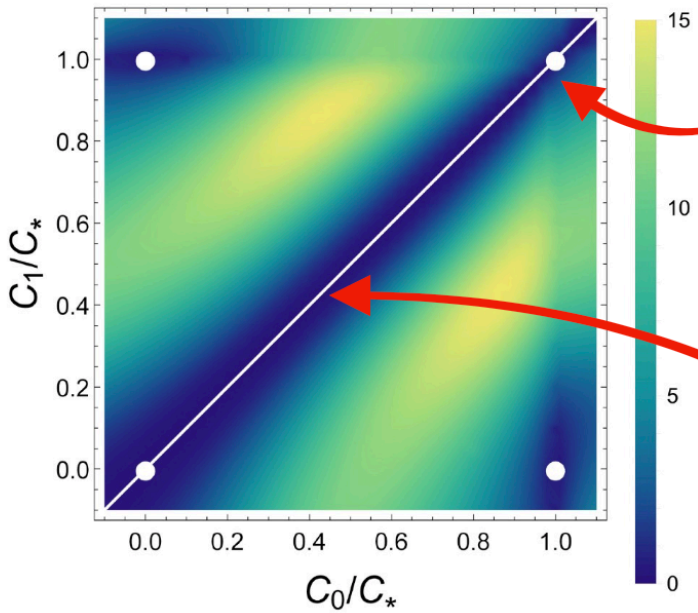
This is precisely the observation of the Seattle group:

$$\mathcal{L}_6 = -\tfrac{1}{2}C_S(N^\dagger N)^2 - \tfrac{1}{2}C_T(N^\dagger \vec{\sigma} N)^2$$

$$^1S_0: \qquad \bar{C}_0 = (C_S - 3C_T)$$

$$^3S_1: \qquad \bar{C}_1 = (C_S + C_T)$$

$$\mathcal{E}(\hat{\mathbf{S}}) = \frac{1}{6}\,\sin^2\left(2(\delta_1 - \delta_0)\right)$$



Conformal fixed points (zero or infinite scattering lengths, $\delta_i = 0, \pi/2$)

$SU(4)_{Wigner}$ symmetry line ($\delta_0 = \delta_1$)

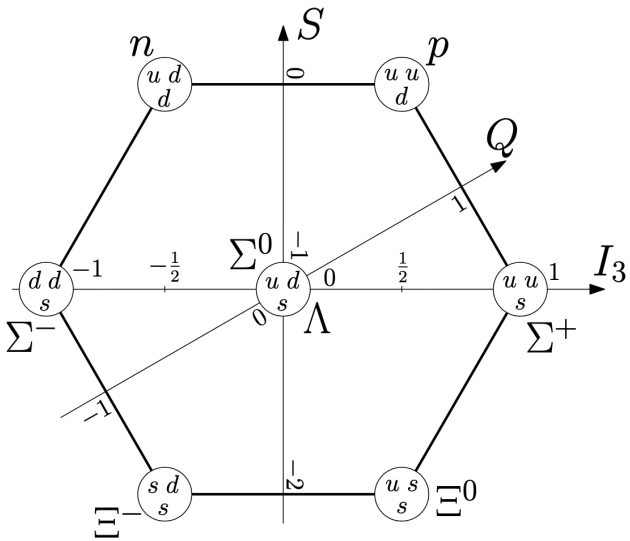
Slide by D.B. Kaplan

Let's extend the analysis to other spin-1/2 baryons, which have a rich theoretical structure and phenomenology:

-- Scattering of two baryons in general will change flavors, unlike in the nucleon scattering.

-- In the limit of exact $SU(N_f)$ flavor symmetry, Pauli exclusion principle forces the two-baryon wave function to be totally anti-symmetric → an interesting interplay between flavor and spin.

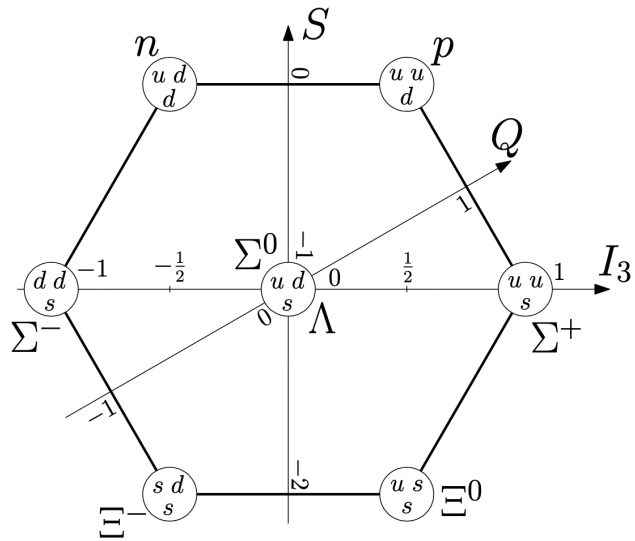
We will focus on the spin-1/2 octet baryons:



2-to-2 scattering contains 64 channels, but group theory says:

$$\mathbf{8} \otimes \mathbf{8} = \underbrace{\mathbf{27} \oplus \mathbf{8}_S \oplus \mathbf{1}}_{\text{Symmetric in flavors}} \oplus \underbrace{\mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{8}_A}_{\text{Anti-Symmetric in flavors}}$$

We will focus on the spin-1/2 octet baryons:



2-to-2 scattering contains 64 channels, but group theory says:

$$\mathbf{8} \otimes \mathbf{8} = \underbrace{\mathbf{27} \oplus \mathbf{8}_S \oplus \mathbf{1}}_{\text{Symmetric in flavors}} \oplus \underbrace{\mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{8}_A}_{\text{Anti-Symmetric in flavors}}$$

Pauli Exclusion Principle!

Symmetric in flavors → Anti-Symmetric in spins ($^1S_0$)!

Anti-Symmetric in flavors → Symmetric in spins ($^3S_1$)!

Recall strong interaction preserves charge (Q) and strangeness (S)

→ Classify the scattering channel into sectors with definitive (Q, S).

	Q	S
nn	0	0
np	1	0
pp	2	0
$n\Sigma^-$	-1	-1
$n\Lambda$ $n\Sigma^0$ $p\Sigma^-$	0	-1
$p\Lambda$ $p\Sigma^0$ $n\Sigma^+$	1	-1
$p\Sigma^+$	2	-1

	Q	S
$\Sigma^-\Sigma^-$	-2	-2
$\Sigma^-\Lambda$ $\Sigma^-\Sigma^0$ $n\Xi^-$	-1	-2
$\Sigma^+\Sigma^-$ $\Sigma^0\Sigma^0$ $\Lambda\Sigma^0$ $\Lambda\Lambda$ $n\Xi^0$ $p\Xi^-$	0	-2
$\Sigma^+\Lambda$ $\Sigma^+\Sigma^0$ $p\Xi^0$	1	-2
$\Sigma^+\Sigma^+$	2	-2

	Q	S
$\Sigma^-\Xi^-$	-2	-3
$\Sigma^-\Xi^0$ $\Xi^-\Sigma^0$ $\Xi^-\Lambda$	-1	-3
$\Xi^-\Sigma^+$ $\Xi^0\Lambda$ $\Xi^0\Sigma^0$	0	-3
$\Xi^0\Sigma^+$	1	-3
$\Xi^-\Xi^-$	-2	-4
$\Xi^-\Xi^0$	-1	-4
$\Xi^0\Xi^0$	0	-4

The S-matrix is block-diagonal among different (Q,S) sectors.

Liu, Low, Mehen: 2210.12085

To warm up, let's consider consider the 1-dimensional sectors with non-identical baryons:

Baryon pairs	symmetric flavor irrep	anti-symmetric flavor irrep
np, $\Sigma^-\Xi^-$, $\Sigma^+\Xi^0$	$\mathbf{27}$	$\overline{\mathbf{10}}$
$n\Sigma^-$, $p\Sigma^+$, $\Xi^-\Xi^0$	$\mathbf{27}$	$\mathbf{10}$

Using the n-p scattering to set the stage:

$$S = \underbrace{\frac{1-\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}}{4}}_{\text{Spin projector into } {}^1S_0 \text{ channel}} \otimes \mathrm{P}_{\mathbf{27}}\, e^{2i\delta_{27}} + \underbrace{\frac{3+\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}}{4}}_{\text{Spin projector into } {}^3S_1 \text{ channel}} \otimes \mathrm{P}_{\overline{\mathbf{10}}}\, e^{2i\delta_{\overline{10}}}$$

P_R: the projector into the SU(3) R-irrep.

In the flavor eigenbasis $\{|pn\rangle, |np\rangle\}$

$$\mathrm{P}_{\mathbf{27}} = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix} \qquad \mathrm{P}_{\overline{\mathbf{10}}} = \begin{pmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{pmatrix}$$

Recall $\mathrm{SWAP} = (1 + \boldsymbol{\sigma} \cdot \boldsymbol{\sigma})/2$ and

$$\frac{1 - \boldsymbol{\sigma} \cdot \boldsymbol{\sigma}}{4} = \frac{1}{2}(1 - \mathrm{SWAP}) \ , \qquad \frac{3 + \boldsymbol{\sigma} \cdot \boldsymbol{\sigma}}{4} = \frac{1}{2}(1 + \mathrm{SWAP})$$

Moreover,

$$\mathrm{P}_{\mathbf{27}} = \frac{1}{2}(1 + \overline{\mathrm{SWAP}}) \ , \qquad \mathrm{P}_{\overline{\mathbf{10}}} = \frac{1}{2}(1 - \overline{\mathrm{SWAP}})$$

where $\overline{\mathrm{SWAP}} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ interchanges the two incoming flavor states.

In the end,

$$S = \frac{1 - \mathrm{SWAP}}{2} \otimes \frac{1 + \overline{\mathrm{SWAP}}}{2} e^{2i\delta_{27}} + \frac{1 + \mathrm{SWAP}}{2} \otimes \frac{1 - \overline{\mathrm{SWAP}}}{2} e^{2i\delta_{\overline{10}}}$$

$\underline{\mathrm{SWAP}}$ = SWAP operator in the spin space
$\overline{\mathrm{SWAP}}$ = SWAP operator in the flavor space

$$\delta_{27} = \delta_{\overline{10}} = \delta \ : \qquad S = \frac{e^{2i\delta}}{2}\left(1 \otimes 1 - \mathrm{SWAP} \otimes \overline{\mathrm{SWAP}}\right)$$

$$\delta_{27} = \delta_{\overline{10}} \pm \frac{\pi}{2} = \delta \ : \ S = \frac{e^{2i\delta}}{2}\left(\mathrm{SWAP} \otimes 1 - 1 \otimes \overline{\mathrm{SWAP}}\right)$$

At first sight these two S-matrices don't appear to be minimally entangling….

$$\delta_{27} = \delta_{\overline{10}} = \delta \ : \qquad S = \frac{e^{2i\delta}}{2}\left(1 \otimes 1 - \mathrm{SWAP} \otimes \overline{\mathrm{SWAP}}\right)$$

$$\delta_{27} = \delta_{\overline{10}} \pm \frac{\pi}{2} = \delta \ : \ S = \frac{e^{2i\delta}}{2}\left(\mathrm{SWAP} \otimes 1 - 1 \otimes \overline{\mathrm{SWAP}}\right)$$

At first sight these two S-matrices don't appear to be minimally entangling….

But there is an interesting interplay due to Fermi-Dirac statistics:

$$\mathrm{SWAP} \otimes \overline{\mathrm{SWAP}}\ |N_1, s_1; N_2, s_2\rangle = |N_2, s_2; N_1, s_1\rangle = -|N_1, s_1; N_2, s_2\rangle$$

$$\Rightarrow \qquad \mathrm{SWAP} \otimes \overline{\mathrm{SWAP}} = -1 \otimes 1$$

$$1 \otimes \overline{\mathrm{SWAP}}\ |N_1, s_1; N_2, s_2\rangle = -\mathrm{SWAP} \otimes 1\ |N_1, s_1; N_2, s_2\rangle$$

$$\delta_{27} = \delta_{\overline{10}} = \delta \ : \qquad S = \frac{e^{2i\delta}}{2}\left(1 \otimes 1 - \mathrm{SWAP} \otimes \overline{\mathrm{SWAP}}\right) \boxed{= e^{2i\delta}\ 1 \otimes 1}$$

$$\delta_{27} = \delta_{\overline{10}} \pm \frac{\pi}{2} = \delta \ : \ S = \frac{e^{2i\delta}}{2}\left(\mathrm{SWAP} \otimes 1 - 1 \otimes \overline{\mathrm{SWAP}}\right) \boxed{= e^{2i\delta}\ \mathrm{SWAP} \otimes 1}$$

At first sight these two S-matrices don't appear to be minimally entangling….

But there is an interesting interplay due to Fermi-Dirac statistics:

$$\mathrm{SWAP} \otimes \overline{\mathrm{SWAP}}\ |N_1, s_1; N_2, s_2\rangle = |N_2, s_2; N_1, s_1\rangle = -|N_1, s_1; N_2, s_2\rangle$$

$$\Rightarrow \qquad \mathrm{SWAP} \otimes \overline{\mathrm{SWAP}} = -1 \otimes 1$$

$$1 \otimes \overline{\mathrm{SWAP}}\ |N_1, s_1; N_2, s_2\rangle = -\mathrm{SWAP} \otimes 1\ |N_1, s_1; N_2, s_2\rangle$$

In the end we do get minimally entangling S-matrices, as expected!

The analysis extends to other more complicated (Q,S) sectors and we are able to obtain conditions on the scattering phases under which the 2-to-2 scattering in each (Q, S) sector is minimally entangled:

(Q,S) sectors	Minimal Entanglement Conditions
np $\Sigma^-\Xi^-$ $\Sigma^+\Xi^0$	$\delta_{27} = \delta_{\overline{10}}$ or $\delta_{27} = \delta_{\overline{10}} \pm \frac{\pi}{2}$
$n\Sigma^-$ $p\Sigma^+$ $\Xi^-\Xi^0$	$\delta_{27} = \delta_{10}$ or $\delta_{27} = \delta_{10} \pm \frac{\pi}{2}$
$(p\Lambda,\ p\Sigma^0,\ n\Sigma^+)$ $(n\Lambda,\ n\Sigma^0,\ p\Sigma^-)$ $(\Sigma^-\Lambda,\ \Sigma^-\Sigma^0,\ n\,\Xi^-)$ $(\Sigma^+\Lambda,\ \Sigma^+\Sigma^0,\ p\,\Xi^0)$ $(\Sigma^-\Xi^0, \Xi^-\Sigma^0, \Xi^-\Sigma^0)$ $(\Xi^-\Sigma^+, \Xi^0\Lambda, \Xi^0\Sigma^0)$	$\delta_{27} = \delta_{8_S} = \delta_{10} \pm \frac{\pi}{2} = \delta_{\overline{10}} \pm \frac{\pi}{2} = \delta_{8_A} \pm \frac{\pi}{2}$ or $\delta_{27} = \delta_{8_S} = \delta_{10} = \delta_{\overline{10}} = \delta_{8_A}$
$(\Sigma^+\Sigma^-,\ \Sigma^0\Sigma^0,\ \Lambda\Sigma^0,\ \Xi^-p,\ \Xi^0n,\ \Lambda\Lambda)$	$\delta_{27} = \delta_{8_S} = \delta_1 = \delta_{10} = \delta_{\overline{10}} = \delta_{8_A}$ or $\delta_{27} = \delta_{8_S} = \delta_1 = \delta_{10} \pm \frac{\pi}{2} = \delta_{\overline{10}} \pm \frac{\pi}{2} = \delta_{8_A} \pm \frac{\pi}{2}$

TABLE III. Conditions in each flavor sector for the S-matrix to be minimally entangling. An Identity gate is achieved when all the phases are equal, while a SWAP gate is when the phases differ by $\pi/2$.

Liu, Low, Mehen: 2210.12085

To investigate what emerging symmetries appear, it's most convenient to use the EFT Lagrangian, where the symmetry is manifest:

$$\mathcal{L}_{\rm LO}^{n_f=3} = -\frac{c_1}{f^2}\langle B_i^\dagger B_i B_j^\dagger B_j\rangle - \frac{c_2}{f^2}\langle B_i^\dagger B_j B_j^\dagger B_i\rangle - \frac{c_3}{f^2}\langle B_i^\dagger B_j^\dagger B_i B_j\rangle - \frac{c_4}{f^2}\langle B_i^\dagger B_j^\dagger B_j B_i\rangle$$
$$- \frac{c_5}{f^2}\langle B_i^\dagger B_i\rangle\langle B_j^\dagger B_j\rangle - \frac{c_6}{f^2}\langle B_i^\dagger B_j\rangle\langle B_j^\dagger B_i\rangle,$$

These Wilson coefficients can be projected into SU(3)-symmetric Wilson coefficients:

Relation between scattering phase and Wilson coefficient:

$$p\cot\delta_i = -\left(\mu + \frac{4\pi}{MC_i}\right)$$

For natural scattering length, set μ = 0.

$$C_{27} = c_1 - c_2 + c_5 - c_6 \ ,$$
$$C_{8_S} = -\frac{2}{3}c_1 + \frac{2}{3}c_2 - \frac{5}{6}c_3 + \frac{5}{6}c_4 + c_5 - c_6 \ ,$$
$$C_1 = -\frac{1}{3}c_1 + \frac{1}{3}c_2 - \frac{8}{3}c_3 + \frac{8}{3}c_4 + c_5 - c_6 \ ,$$
$$C_{\overline{10}} = c_1 + c_2 + c_5 + c_6 \ ,$$
$$C_{10} = -c_1 - c_2 + c_5 + c_6 \ ,$$
$$C_{8_A} = \frac{3}{2}c_3 + \frac{3}{2}c_4 + c_5 + c_6 \ .$$

Wagman et. al.: 1706.06550

From these relations it is straightforward to obtain conditions for entanglement suppressions on the Wilson coefficients:

Flavor subspaces	Minimal Entanglement Conditions
np $\Sigma^-\Xi^-$ $\Sigma^+\Xi^0$	$c_2 = -c_6 \quad \text{or} \quad c_1 + c_5 = -\frac{2\pi}{M\mu},\ c_2 + c_6 = \pm\frac{2\pi}{M\mu}$
$n\Sigma^-$ $p\Sigma^+$ $\Xi^-\Xi^0$	$c_1 = c_6 \quad \text{or} \quad -c_2 + c_5 = -\frac{2\pi}{M\mu},\ c_1 - c_6 = \pm\frac{2\pi}{M\mu}$
$(p\Lambda,\ p\Sigma^0,\ n\Sigma^+)$ $(n\Lambda,\ n\Sigma^0,\ p\Sigma^-)$ $(\Sigma^-\Lambda,\ \Sigma^-\Sigma^0,\ n\Xi^-)$ $(\Sigma^+\Lambda,\ \Sigma^+\Sigma^0,\ p\Xi^0)$ $(\Sigma^-\Xi^0, \Xi^-\Sigma^0, \Xi^-\Sigma^0)$ $(\Xi^-\Sigma^+, \Xi^0\Lambda, \Xi^0\Sigma^0)$	$c_1 = -c_2 = -\frac{1}{2}c_3 = \frac{1}{2}c_4 = c_6$ or $c_1 = -c_2 = -\frac{1}{2}c_3 = \frac{1}{2}c_4 = -c_5 - \frac{2\pi}{M\mu} = c_6 \pm \frac{2\pi}{M\mu}$
$(\Sigma^+\Sigma^-,\ \Sigma^0\Sigma^0,\ \Lambda\Sigma^0,\ \Xi^-p,\ \Xi^0n,\ \Lambda\Lambda)$	$c_1 = c_2 = c_3 = c_4 = c_6 = 0 \quad \text{or}$ $c_1 = c_2 = c_3 = c_4 = 0, c_5 = -2\pi/M\mu, c_6 = \pm 2\pi/M\mu$

TABLE IV. Minimal entanglement conditions on Wilson coefficients in each flavor subspace.

Liu, Low, Mehen: 2210.12085

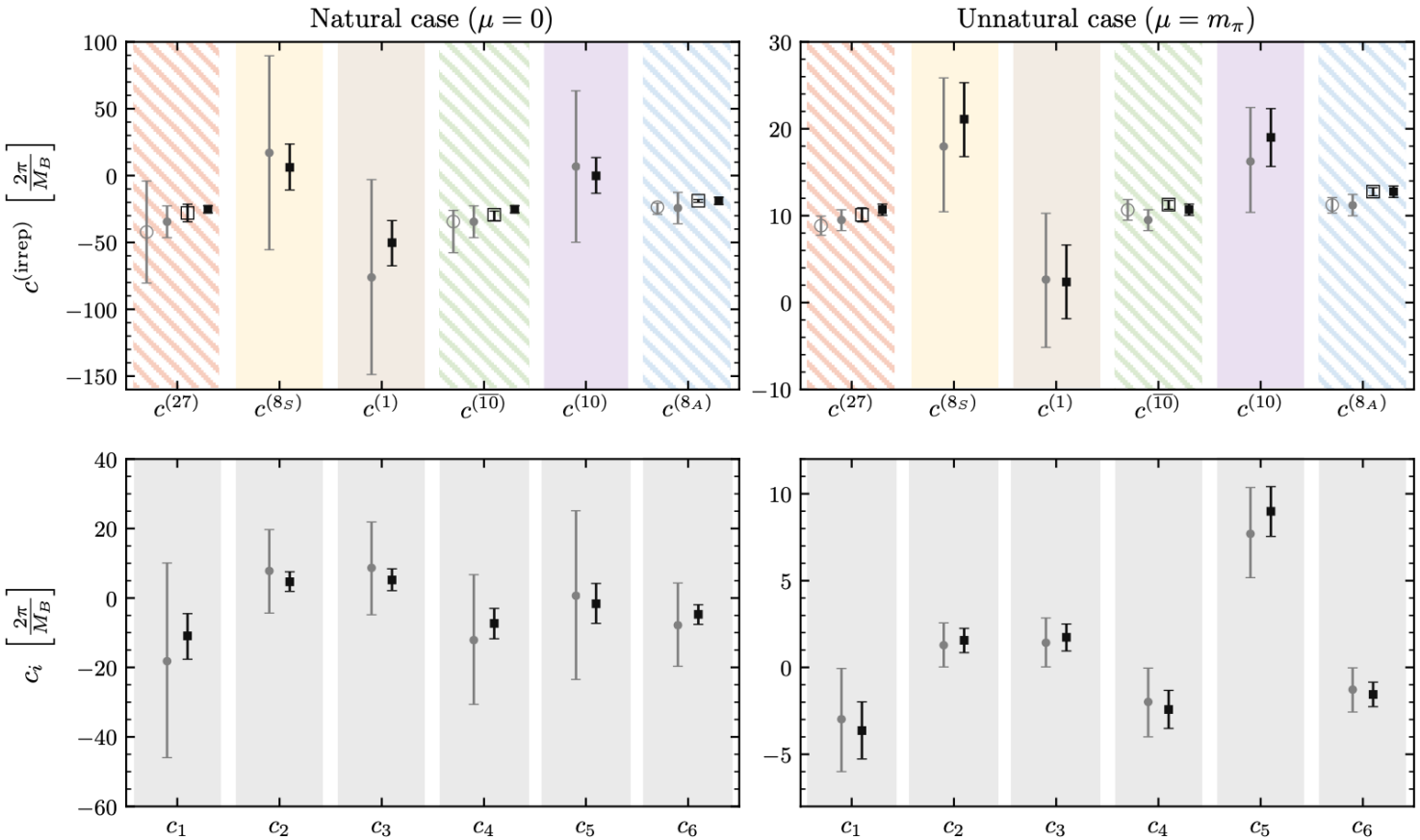
A summary table on the emerging symmetries:

Flavor Subspace	Symmetry of Lagrangian
np $\Sigma^-\Xi^-$ $\Sigma^+\Xi^0$	$SU(6)$ spin-flavor symmetry or conformal symmetry in $\mathbf{27}$ and $\overline{\mathbf{10}}$ irrep channels
$n\Sigma^-$ $p\,\Sigma^+$ $\Xi^-\Xi^0$	conjugate of $SU(6)$ spin-flavor symmetry or conformal symmetry in $\mathbf{27}$ and $\mathbf{10}$ irrep channels
$(p\Lambda,\ p\Sigma^0,\ n\Sigma^+)$ $(n\Lambda,\ n\Sigma^0,\ p\Sigma^-)$ $(\Sigma^-\Lambda,\ \Sigma^-\Sigma^0,\ n\,\Xi^-)$ $(\Sigma^+\Lambda,\ \Sigma^+\Sigma^0,\ p\,\Xi^0)$ $(\Sigma^-\Xi^0, \Xi^-\Sigma^0, \Xi^-\Sigma^0)$ $(\Xi^-\Sigma^+, \Xi^0\Lambda, \Xi^0\Sigma^0)$	$SO(8)$ flavor symmetry or conformal symmetry in $\mathbf{27}$, $\mathbf{8_S}$, $\mathbf{8_A}$, $\mathbf{10}$ and $\overline{\mathbf{10}}$ irrep channels
$(\Sigma^+\Sigma^-,\ \Sigma^0\Sigma^0,\ \Lambda\Sigma^0,\ \Xi^-p,\ \Xi^0n,\ \Lambda\Lambda)$	$SU(16)$ symmetry or $SU(8)$ and conformal symmetry

TABLE V. Symmetries predicted by entanglement minimization in each flavor sector.

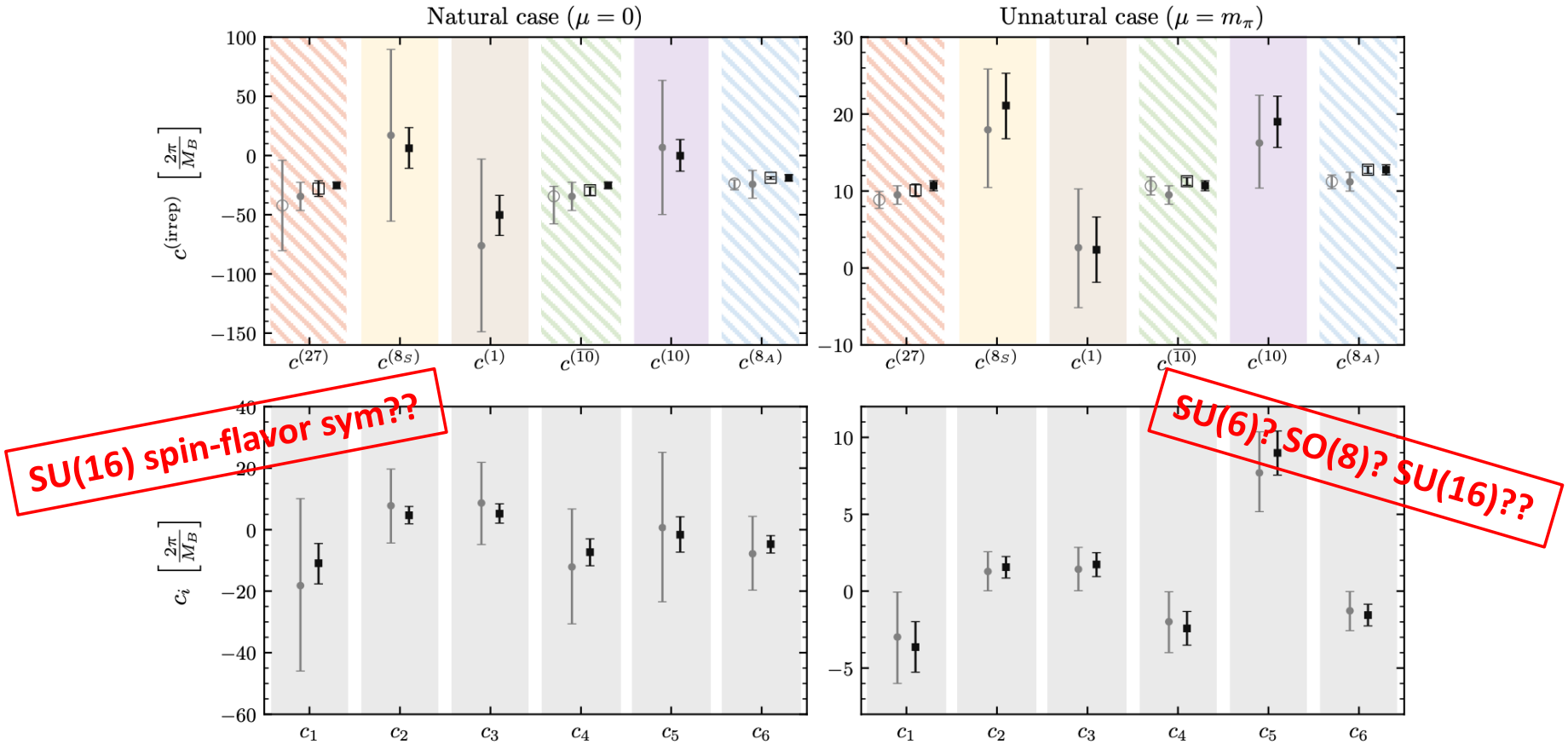
These Wilson coefficients can be simulated using lattice QCD:

NPLQCD: 2009.12357



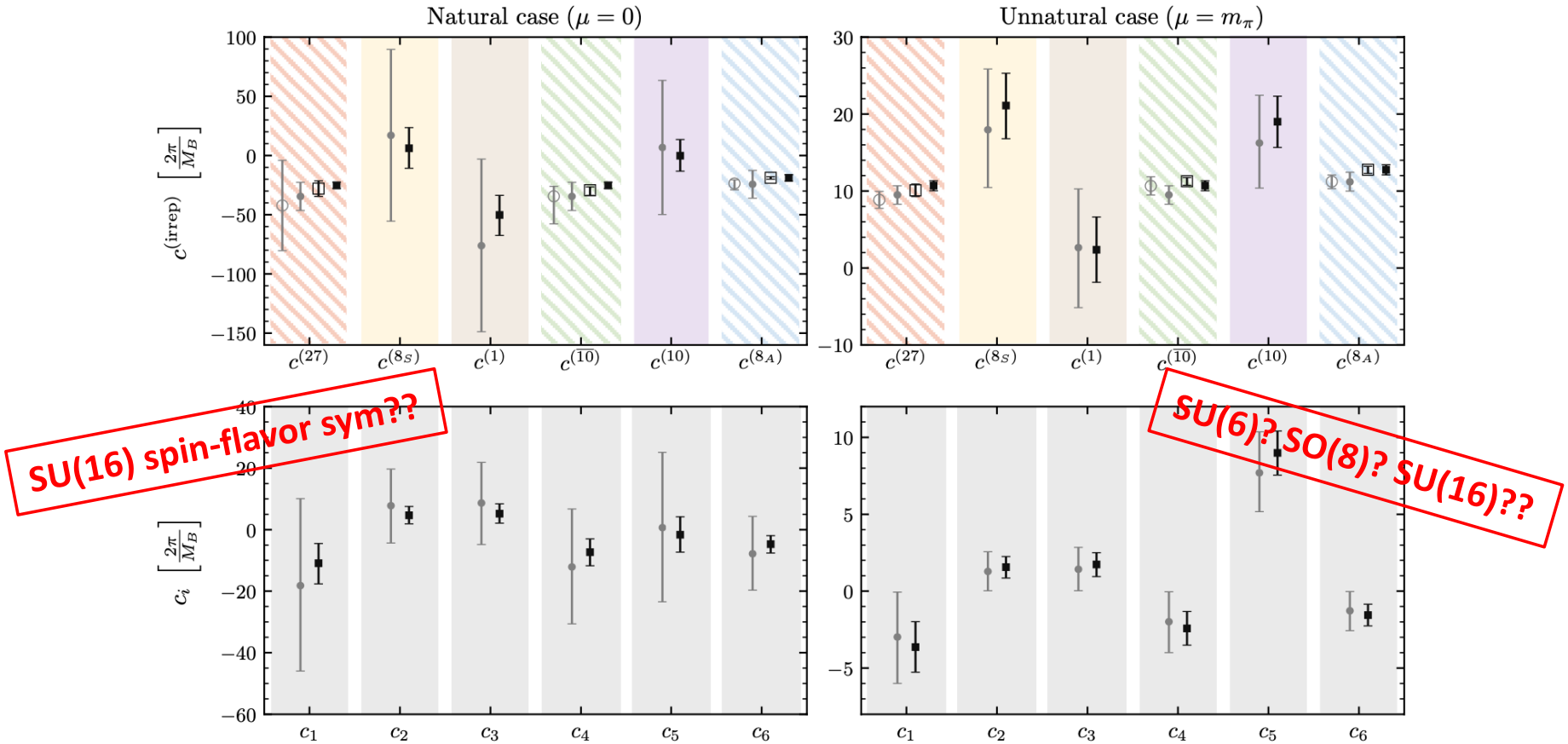
These Wilson coefficients can be simulated using lattice QCD:

NPLQCD: 2009.12357



These Wilson coefficients can be simulated using lattice QCD:

NPLQCD: 2009.12357



Alternatively, can we directly measure the scattering phases in hyperon-hyperon scattering?

Outlook

In pursuit of a new paradigm:

Can symmetry be the outgrowth of more fundamental principles?

The answer appears to be a tantalizing YES!

Less entanglement ⟺ More symmetry

- What about spontaneously broken symmetries? And gauge symmetry?
- We often invoke symmetry to explain naturalness. Can we understand (un)naturalness through quantum information?