Entanglement and Symmetries in Nuclear EFTs

- Ian Low
- Argonne/Northwestern
- Theory Center Seminar
- Feb 13th, 2023
Acknowledgement 1:

Acknowledgement 2:
Work collaborated with Qiaofeng Liu and Tom Mehen.
Entanglement is quantum world’s most prominent feature:

- It refers to the situation where a measurement on a subsystem will improve our knowledge on the rest of the system.

- A quantum state of a system is entangled if it cannot be written as a tensor-product state of its subsystems.

- Consider a bipartite system $\mathcal{H}_{12} = \mathcal{H}_1 \otimes \mathcal{H}_2$, a state vector $|\psi\rangle \in \mathcal{H}_{12}$ is entangled if there is no $|\psi_1\rangle \in \mathcal{H}_1$ and $|\psi_2\rangle \in \mathcal{H}_2$ such that

  $$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$$
Einstein famously attacked “entanglement” as spooky action at a distance:
The Nobel Prize in Physics 2022 was awarded jointly to Alain Aspect, John F. Clauser and Anton Zeilinger "for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science"
On the other hand, symmetry is among the most fundamental principles in physics:

- Powerful characterization of nature based on invariance under a specified group of transformations.
- Symmetries give rise to conserved quantities: energy, momentum, angular momentum, etc.
- Combining with quantum mechanics, there is a subtle realization of symmetry – spontaneous symmetry breaking.
- All known fundamental interactions are based on symmetry principles.
In modern perspectives,

• Gauge symmetry is not a real symmetry – it’s a reflection of the redundancy in our description of nature.

A massless spin-1 particle has only two degrees of freedom, + and - Helicities. However a Lorentz invariant description requires adding two fictitious degrees of freedom by hand.

• Global symmetries are believed to be either completely broken or only approximate.

Discrete symmetries like parity, time-reversal invariance are broken in nature, while approximate symmetries are ubiquitous, for example the isospin invariance.
But what is the origin of symmetry?
There are two historical perspectives:

**Beauty In, Garbage Out**
As we explore higher and higher energy regimes, we discover more and more symmetries. The symmetry is usually hidden or broken in low energies.

**Garbage In, Beauty Out**
At high energy level there is no symmetry. Rather symmetry emerges only at large distances, in the infrared. These are accidental symmetries.
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But neither explain whether symmetry can be the natural outgrowth of more fundamental principles.
In 2018 a paper from Seattle made a fascinating observation regarding emergent symmetries and entanglement suppression in low-energy QCD:

PHYSICAL REVIEW LETTERS 122, 102001 (2019)

**Entanglement Suppression and Emergent Symmetries of Strong Interactions**

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(Received 20 December 2018; published 14 March 2019)

Entanglement suppression in the strong-interaction \( S \) matrix is shown to be correlated with approximate spin-flavor symmetries that are observed in low-energy baryon interactions, the Wigner \( SU(4) \) symmetry for two flavors and an \( SU(16) \) symmetry for three flavors. We conjecture that

This raises the intriguing possibility of understanding symmetry from quantum entanglement!
John Wheeler famously coined the phrase:

*It from bit*: “All things physical are information-theoretic in origin”
Indeed, remarkable connections between fundamental physics and information science in the past decade. It is natural to ask:

Can symmetry come from qubit?
In this talk, we will use low-energy QCD as a playground to study:

• Unexpected, emerging (approximate) global symmetries in low-energy hadronic physics.

• Correlation between symmetry and entanglement suppression in non-relativistic 2-to-2 scattering.

• Elucidate the connection from an information-theoretic viewpoint.
Emergent symmetries in low-energy QCD:

• Schrodinger symmetry (non-relativistic conformal invariance)
  
  The largest symmetry group preserved by the Schrodinger equation, which includes Galilean boosts, scale and special conformal transformations.

• Spin-flavor symmetries
  
  Possible only in non-relativistic theories due to Coleman-Mandula theorem. Examples include SU(2N_f) quark spin-flavor symmetries. There’s also Wigner’s “supermultiplet” SU(4) spin-flavor symmetry among neutrons and protons.
SU(2N_f) spin-flavor symmetries are symmetries of non-relativistic quark model and date back to 1960s:

\[
\begin{align*}
\text{SU(4):} & \quad 4 = \begin{pmatrix} u \uparrow \\ u \downarrow \\ d \uparrow \\ d \downarrow \end{pmatrix} \\
\text{SU(6):} & \quad 6 = \begin{pmatrix} u \uparrow \\ u \downarrow \\ d \uparrow \\ d \downarrow \\ s \uparrow \\ s \downarrow \end{pmatrix}
\end{align*}
\]

Ample evidence for SU(4) and SU(6) in nature:
- Masses
- Magnetic moments and transitions
- Semilepton currents
- Meson-baryon and baryon-baryon couplings
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- Masses
- Magnetic moments and transitions
- Semilepton currents
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They can be derived from QCD in the large \( N_c \) limit!

Dashen, Monohar (1993); Dashen, Jenkins, Manohar (1994); Kaplan, Savage (1995)
In low-energy nuclear physics, there is a different SU(4) symmetry first observed by Wigner:

In this case the neutron and proton fill out a "supermultiplet":

\[ N = \begin{pmatrix} p \uparrow \\ p \downarrow \\ n \uparrow \\ n \downarrow \end{pmatrix} \]
In low-energy nuclear physics, there is a different SU(4) symmetry first observed by Wigner:

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In the EFT language, Wigner’s SU(4) is accidental in that, after imposing the SU(4) quark spin-flavor symmetry, the only remaining operator has this symmetry.
So far these spin-flavor symmetries can be argued from the large $N_c$ limit. There are emergent symmetries beyond the large $N_c$:

- Unnaturally large scattering lengths in low-energy NN scattering in the s-wave, which include $^1S_0$ and $^3S_1$ channels.

The S-matrix for a single channel is

$$S = e^{2i\delta(p)} = 1 + i \frac{Mp}{2\pi} A,$$

$$A = \frac{4\pi}{M} \frac{1}{p \cot \delta - ip}$$

It is long known that it’s $p \cot \delta$ which admits an expansion in $1/p$, the Effective Range Expansion (ERE):

$$p \cot \delta = -\frac{1}{a} + \frac{1}{2} r_0 p^2 + \cdots = -\frac{1}{a} + \frac{1}{2} \Lambda^2 \sum_{n=0}^{\infty} r_n \left( \frac{p^2}{\Lambda^2} \right)^n$$

The scattering length
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---

The scattering length

$^1S_0 : a_0 = -23.7 \text{ fm}$

$^3S_1 : a_1 = 5.4 \text{ fm} \rightarrow \text{Deuteron!}$

$1/m_\pi = 1.4 \text{ fm}$
In the limit the scattering length $a$ diverges, the system has no scale and there’s the non-relativistic conformal invariance. Mehen, Stewart, Wise (1999)

At the infinitesimal level,

- boosts: $\vec{x}' = \vec{x} + \vec{v}t$, $t' = t$,
- scale: $\vec{x}' = \vec{x} + s\vec{x}$, $t' = t + 2st$,
- conformal: $\vec{x}' = \vec{x} - ct\vec{x}$, $t' = t - ct^2$,

So NN scattering has approximate Schrödinger symmetry.

WHO ORDERED THAT??!
Nucleons are part of spin-1/2 octet baryons:

\[ B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}} \Lambda \end{pmatrix} \]

\[ \mathcal{L}^{n_f=3}_{\text{LO}} = -\frac{c_1}{f^2} \langle B_i^\dagger B_i B_i^\dagger B_j \rangle - \frac{c_2}{f^2} \langle B_i^\dagger B_j B_j^\dagger B_i \rangle - \frac{c_3}{f^2} \langle B_i^\dagger B_i^\dagger B_i B_i \rangle - \frac{c_4}{f^2} \langle B_i^\dagger B_j^\dagger B_j B_i \rangle - \frac{c_5}{f^2} \langle B_i^\dagger B_i \rangle \langle B_j^\dagger B_j \rangle - \frac{c_6}{f^2} \langle B_i^\dagger B_j \rangle \langle B_j^\dagger B_i \rangle, \quad i, j = \uparrow, \downarrow \]

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\[ m_\pi = 804 \text{ MeV} \]
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\[ m_\pi = 450 \text{ MeV} \]
In the limit where all coefficients but $c_5$ are vanishing:

$$\mathcal{L}_{\text{LO}}^{n_f=3} = -\frac{c_1}{f^2} \langle B_i^\dagger B_i B_j^\dagger B_j \rangle - \frac{c_2}{f^2} \langle B_i^\dagger B_j^\dagger B_j B_i \rangle - \frac{c_3}{f^2} \langle B_i^\dagger B_j^\dagger B_i B_j \rangle - \frac{c_4}{f^2} \langle B_i^\dagger B_j^\dagger B_j B_i \rangle$$

- $\frac{c_5}{f^2} \langle B_i^\dagger B_i \rangle \langle B_j^\dagger B_j \rangle - \frac{c_6}{f^2} \langle B_i^\dagger B_j \rangle \langle B_j^\dagger B_i \rangle$, $i, j = \uparrow, \downarrow$

The remaining operator can be re-written,

$$\mathcal{B} = (n_\uparrow, n_\downarrow, p_\uparrow, p_\downarrow, \ldots) \quad , \quad \mathcal{L} = -c_5 (\mathcal{B}^\dagger \mathcal{B})^2$$

which is invariant under an SU(16) spin-flavor symmetry

$$\mathcal{B} \rightarrow U \mathcal{B} \quad , \quad U^\dagger U = 1$$

There is no large $N_c$ explanation!
To discuss entanglement suppression, we need to quantify the amount of entanglement \( \rightarrow \) Entanglement Measure!

Many possibilities for Entanglement Measure. For bipartite systems:

**von Neumann entropy:**

\[
E(\rho) = -\text{Tr}(\rho_1 \ln \rho_1) = -\text{Tr}(\rho_2 \ln \rho_2)
\]

**Linear entropy:**

\[
E(\rho) = -\text{Tr}(\rho_1 (\rho_1 - 1)) = 1 - \text{Tr} \rho_1^2
\]

\[
\rho = |\psi\rangle \langle \psi| \quad \quad \rho_{1/2} = \text{Tr}_{2/1}(\rho)
\]

The common property is that the entanglement measure vanishes for a product state \( |\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \), but attains the maximum for maximally entangled states.
For a system with two spin-1/2 particles, let’s define the “computational basis:”

\[ \{|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle\} \]

Then for a general normalized state,

\[ |\psi\rangle = \alpha |\uparrow\uparrow\rangle + \beta |\uparrow\downarrow\rangle + \gamma |\downarrow\uparrow\rangle + \delta |\downarrow\downarrow\rangle, \quad |\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1, \]

The reduced density matrix and linear entropy are

\[ \rho_1 = \text{Tr}_2 |\psi\rangle \langle \psi| = \begin{pmatrix} |\alpha|^2 + |\beta|^2 & \alpha\gamma^* + \beta\delta^* \\ \alpha^*\gamma + \beta^*\delta & |\gamma|^2 + |\delta|^2 \end{pmatrix}, \]

Easy to check that

\[ E(|\psi\rangle) = 1 - \text{Tr}_1 \rho_1^2 = 2|\alpha\delta - \beta\gamma|^2. \]

1. It vanishes for a product state.
2. Maximal entanglement is 1/2, which is the case for the Bell states:

\[ (|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle)/\sqrt{2} \quad (|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle)/\sqrt{2} \]
Entanglement is a property of the quantum state.

But we are more interested in the ability of a quantum-mechanical operator (i.e. the S-matrix) to entangle \( \rightarrow \) the Entanglement Power

The entanglement power measures the ability of an operator \( U \) to generate entanglement by averaging over all product states:

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E(U) = \overline{E(U |\psi_1\rangle \otimes |\psi_2\rangle)},
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\]

There is, however, a notion of equivalent class:

\[
U \sim U' \quad \text{if} \quad U = (U_1 \otimes U_2)U'(V_1 \otimes V_2)
\]

![Diagram of entanglement process]
Modulo the equivalent class, there are two and only two minimally entangling operators, which in the computational basis,

\[ 1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \text{SWAP} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \]

Identity gate: do nothing.
SWAP gate: interchange the qubits.

In terms of Pauli matrices,

\[ \text{SWAP} = (1 + \sigma \cdot \sigma) / 2 , \quad \sigma \cdot \sigma \equiv \sum_a \sigma^a \otimes \sigma^a . \]

Low, Mehen: 2104.10835
In this notation, the S-matrix for low-energy neutron-proton scattering in the s-wave can be written as

\[ S' = \frac{1}{2} (e^{2i\delta_1} + e^{2i\delta_0}) \mathbf{1} + \frac{1}{2} (e^{2i\delta_1} - e^{2i\delta_0}) \text{ SWAP}, \]

\( \delta_0 \) and \( \delta_1 \) are the scattering phases in the \( ^1S_0 \) and \( ^3S_1 \) channels.

**Conditions for the S-matrix to minimize entanglement:**

1. \( S = 1 \) if \( \delta_0 = \delta_1 \) \( \rightarrow \) SU(4) or SU(16) spin-flavor sym.
2. \( S = \text{SWAP} \) if \( |\delta_0 - \delta_1| = \pi/2 \) \( \rightarrow \) Schrödinger sym.

Low, Mehen: 2104.10835
This is precisely the observation of the Seattle group:

\[ \mathcal{L}_6 = -\frac{1}{2} C_S (N^\dagger N)^2 - \frac{1}{2} C_T (N^\dagger \bar{\sigma} N)^2 \]

\[ ^1S_0 : \quad \bar{C}_0 = (C_S - 3C_T) \]

\[ ^3S_1 : \quad \bar{C}_1 = (C_S + C_T) \]

\[ \mathcal{E}(\hat{S}) = \frac{1}{6} \sin^2 (2(\delta_1 - \delta_0)) \]

**Conformal fixed points**

(zero or infinite scattering lengths, \( \delta_i = 0, \pi/2 \))

**SU(4)_Wigner symmetry line**

(\( \delta_0 = \delta_1 \))
Let’s extend the analysis to other spin-1/2 baryons, which have a rich theoretical structure and phenomenology:

-- Scattering of two baryons in general will change flavors, unlike in the nucleon scattering.

-- In the limit of exact SU($N_f$) flavor symmetry, Pauli exclusion principle forces the two-baryon wave function to be totally anti-symmetric → an interesting interplay between flavor and spin.
We will focus on the spin-1/2 octet baryons:

2-to-2 scattering contains 64 channels, but group theory says:

\[ 8 \otimes 8 = 27 \oplus 8_S \oplus 1 \oplus 10 \oplus \overline{10} \oplus 8_A \]

*Symmetric in flavors*  
*Anti-Symmetric in flavors*
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2-to-2 scattering contains 64 channels, but group theory says:

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**Pauli Exclusion Principle!**

- Symmetric in flavors
- Anti-Symmetric in flavors
- Anti-Symmetric in spins ($^1S_0$)!
- Symmetric in spins ($^3S_1$)!
Recall strong interaction preserves charge (Q) and strangeness (S) → Classify the scattering channel into sectors with definitive (Q, S).

The S-matrix is block-diagonal among different (Q,S) sectors.

Liu, Low, Mehen: 2210.12085
To warm up, let’s consider the 1-dimensional sectors with non-identical baryons:

<table>
<thead>
<tr>
<th>Baryon pairs</th>
<th>symmetric flavor irrep</th>
<th>anti-symmetric flavor irrep</th>
</tr>
</thead>
<tbody>
<tr>
<td>( np, \Sigma^-\Xi^-, \Sigma^+\Xi^0 )</td>
<td>27</td>
<td>10</td>
</tr>
<tr>
<td>( n\Sigma^-, p\Sigma^+, \Xi^-\Xi^0 )</td>
<td>27</td>
<td>10</td>
</tr>
</tbody>
</table>

Using the n-p scattering to set the stage:

\[
S = \frac{1 - \mathbf{\sigma} \cdot \mathbf{\sigma}}{4} \otimes P_{27} e^{2i\delta_{27}} + \frac{3 + \mathbf{\sigma} \cdot \mathbf{\sigma}}{4} \otimes P_{10} e^{2i\delta_{10}}
\]

Spin projector into \( ^1S_0 \) channel
Spin projector into \( ^3S_1 \) channel

\( P_R \): the projector into the SU(3) R-irrep.

In the flavor eigenbasis \( \{|pn\>, |np\>\} \)

\[
P_{27} = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix} \quad P_{10} = \begin{pmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{pmatrix}
\]
Recall $\text{SWAP} = (1 + \sigma \cdot \sigma)/2$ and

$$\frac{1 - \sigma \cdot \sigma}{4} = \frac{1}{2}(1 - \text{SWAP}) \quad \text{and} \quad \frac{3 + \sigma \cdot \sigma}{4} = \frac{1}{2}(1 + \text{SWAP})$$

Moreover,

$$P_{27} = \frac{1}{2}(1 + \overline{\text{SWAP}}), \quad P_{10} = \frac{1}{2}(1 - \overline{\text{SWAP}})$$

where $\overline{\text{SWAP}} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ interchanges the two incoming flavor states.

In the end,

$$S = \frac{1 - \text{SWAP}}{2} \otimes \frac{1 + \overline{\text{SWAP}}}{2} e^{2i\delta_{27}} + \frac{1 + \text{SWAP}}{2} \otimes \frac{1 - \overline{\text{SWAP}}}{2} e^{2i\delta_{10}}$$

**SWAP** = SWAP operator in the spin space

**SWAP** = SWAP operator in the flavor space
\[ \delta_{27} = \delta_{10} = \delta : \quad S = \frac{e^{2i\delta}}{2} \left( 1 \otimes 1 - \text{SWAP} \otimes \overline{\text{SWAP}} \right) \]

\[ \delta_{27} = \delta_{10} \pm \frac{\pi}{2} = \delta : \quad S = \frac{e^{2i\delta}}{2} \left( \text{SWAP} \otimes 1 - 1 \otimes \overline{\text{SWAP}} \right) \]

At first sight these two S-matrices don’t appear to be minimally entangling....
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But there is an interesting interplay due to Fermi-Dirac statistics:

\[
\text{SWAP} \otimes \overline{\text{SWAP}} \ |N_1, s_1; N_2, s_2\rangle = |N_2, s_2; N_1, s_1\rangle = -|N_1, s_1; N_2, s_2\rangle
\]

\[
\text{SWAP} \otimes \overline{\text{SWAP}} = -1 \otimes 1
\]

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\]

In the end we do get minimally entangling S-matrices, as expected!
The analysis extends to other more complicated \((Q,S)\) sectors and we are able to obtain conditions on the scattering phases under which the 2-to-2 scattering in each \((Q, S)\) sector is minimally entangled:

<table>
<thead>
<tr>
<th>((Q, S)) sectors</th>
<th>Minimal Entanglement Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>(np)</td>
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</tr>
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<td>(p\Sigma^+)</td>
<td></td>
</tr>
<tr>
<td>(\Xi^-\Xi^0)</td>
<td></td>
</tr>
<tr>
<td>((p\Lambda, p\Sigma^0, n\Sigma^+))</td>
<td></td>
</tr>
<tr>
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<td></td>
</tr>
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<td></td>
</tr>
<tr>
<td>((\Xi^-\Sigma^+, \Xi^0\Lambda, \Xi^0\Sigma^0))</td>
<td></td>
</tr>
<tr>
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TABLE III. Conditions in each flavor sector for the S-matrix to be minimally entangling. An Identity gate is achieved when all the phases are equal, while a SWAP gate is when the phases differ by \(\pi/2\).
To investigate what emerging symmetries appear, it's most convenient to use the EFT Lagrangian, where the symmetry is manifest:

\[ \mathcal{L}_{\text{LO}}^{n_f=3} = -\frac{c_1}{f^2} \langle B_i^\dagger B_i B_j^\dagger B_j \rangle - \frac{c_2}{f^2} \langle B_i^\dagger B_j B_j^\dagger B_i \rangle - \frac{c_3}{f^2} \langle B_i^\dagger B_j^\dagger B_i B_j \rangle - \frac{c_4}{f^2} \langle B_i^\dagger B_i B_j^\dagger B_j \rangle - \frac{c_5}{f^2} \langle B_i^\dagger B_i \rangle \langle B_j^\dagger B_j \rangle - \frac{c_6}{f^2} \langle B_i^\dagger B_j \rangle \langle B_j^\dagger B_i \rangle, \]

These Wilson coefficients can be projected into SU(3)-symmetric Wilson coefficients:

Relation between scattering phase and Wilson coefficient:

\[ p \cot \delta_i = - \left( \mu + \frac{4\pi}{MC_i} \right) \]

For natural scattering length, set \( \mu = 0. \)

\[ C_{27} = c_1 - c_2 + c_5 - c_6 , \]
\[ C_{8_S} = -\frac{2}{3} c_1 + \frac{2}{3} c_2 - \frac{5}{6} c_3 + \frac{5}{6} c_4 + c_5 - c_6 , \]
\[ C_1 = -\frac{1}{3} c_1 + \frac{1}{3} c_2 - \frac{5}{3} c_3 + \frac{8}{3} c_4 + c_5 - c_6 , \]
\[ C_{10} = c_1 + c_2 + c_5 + c_6 , \]
\[ C_{10} = -c_1 - c_2 + c_5 + c_6 , \]
\[ C_{8_A} = \frac{3}{2} c_3 + \frac{3}{2} c_4 + c_5 + c_6 . \]

Wagman et. al.: 1706.06550
From these relations it is straightforward to obtain conditions for entanglement suppressions on the Wilson coefficients:

<table>
<thead>
<tr>
<th>Flavor subspaces</th>
<th>Minimal Entanglement Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( np )</td>
<td>( c_2 = -c_6 ) or ( c_1 + c_5 = -\frac{2\pi}{M\mu}, c_2 + c_6 = \pm \frac{2\pi}{M\mu} )</td>
</tr>
<tr>
<td>( \Sigma^- \Xi^- )</td>
<td></td>
</tr>
<tr>
<td>( \Sigma^+ \Xi^0 )</td>
<td></td>
</tr>
<tr>
<td>( n\Sigma^- )</td>
<td>( c_1 = c_6 ) or ( -c_2 + c_5 = -\frac{2\pi}{M\mu}, c_1 - c_6 = \pm \frac{2\pi}{M\mu} )</td>
</tr>
<tr>
<td>( p\Sigma^+ )</td>
<td></td>
</tr>
<tr>
<td>( \Xi^- \Xi^0 )</td>
<td></td>
</tr>
<tr>
<td>( (p\Lambda, p\Sigma^0, n\Sigma^+) )</td>
<td>( c_1 = -c_2 = -\frac{1}{2}c_3 = \frac{1}{2}c_4 = c_6 ) or ( c_1 = -c_2 = -\frac{1}{2}c_3 = \frac{1}{2}c_4 = -c_5 - \frac{2\pi}{M\mu} = c_6 \pm \frac{2\pi}{M\mu} )</td>
</tr>
<tr>
<td>( (n\Lambda, n\Sigma^0, p\Sigma^-) )</td>
<td></td>
</tr>
<tr>
<td>( (\Sigma^-\Lambda, \Sigma^-\Sigma^0, n\Xi^-) )</td>
<td></td>
</tr>
<tr>
<td>( (\Sigma^+\Lambda, \Sigma^+\Sigma^0, p\Xi^0) )</td>
<td></td>
</tr>
<tr>
<td>( (\Xi^-\Xi^0, \Xi^-\Sigma^0, \Xi^-\Sigma^0) )</td>
<td></td>
</tr>
<tr>
<td>( (\Xi^-\Xi^+, \Xi^0\Lambda, \Xi^0\Sigma^0) )</td>
<td></td>
</tr>
<tr>
<td>( (\Sigma^+\Sigma^-, \Sigma^0\Sigma^0, \Lambda\Sigma^0, \Xi^-p, \Xi^0n, \Lambda\Lambda) )</td>
<td>( c_1 = c_2 = c_3 = c_4 = c_6 = 0 ) or ( c_1 = c_2 = c_3 = c_4 = 0, c_5 = -\frac{2\pi}{M\mu}, c_6 = \pm \frac{2\pi}{M\mu} )</td>
</tr>
</tbody>
</table>

**TABLE IV.** Minimal entanglement conditions on Wilson coefficients in each flavor subspace.
A summary table on the emerging symmetries:

<table>
<thead>
<tr>
<th>Flavor Subspace</th>
<th>Symmetry of Lagrangian</th>
</tr>
</thead>
<tbody>
<tr>
<td>$np$</td>
<td>$SU(6)$ spin-flavor symmetry or conformal symmetry in 27 and 10 irrep channels</td>
</tr>
<tr>
<td>$\Sigma^- \Xi^-$</td>
<td>conjugate of $SU(6)$ spin-flavor symmetry or conformal symmetry in 27 and 10 irrep channels</td>
</tr>
<tr>
<td>$\Sigma^+ \Xi^0$</td>
<td>$SO(8)$ flavor symmetry or conformal symmetry in 27, 8_S, 8_A, 10 and 10 irrep channels</td>
</tr>
<tr>
<td>$(p\Lambda, p\Sigma^0, n\Sigma^+)$</td>
<td>$SU(16)$ symmetry or $SU(8)$ and conformal symmetry</td>
</tr>
<tr>
<td>$(n\Lambda, n\Sigma^0, p\Sigma^-)$</td>
<td></td>
</tr>
<tr>
<td>$(\Sigma^- \Lambda, \Sigma^- \Sigma^0, n \Xi^-)$</td>
<td></td>
</tr>
<tr>
<td>$(\Sigma^+ \Lambda, \Sigma^+ \Sigma^0, p \Xi^0)$</td>
<td></td>
</tr>
<tr>
<td>$(\Xi^- \Xi^0, \Xi^- \Sigma^0, \Xi^- \Sigma^0)$</td>
<td></td>
</tr>
<tr>
<td>$(\Xi^- \Sigma^+, \Xi^0 \Lambda, \Xi^0 \Sigma^0)$</td>
<td></td>
</tr>
</tbody>
</table>

TABLE V. Symmetries predicted by entanglement minimization in each flavor sector.
These Wilson coefficients can be simulated using lattice QCD:

NPLQCD: 2009.12357
These Wilson coefficients can be simulated using lattice QCD:

\[ \text{SU(6)} \quad \text{SO(8)} \quad \text{SU(16)} \]

\[ \text{SU(16) spin-flavor sym?} \]

\[ \text{SU(6)? SO(8)? SU(16)?} \]

\[ \text{NPLQCD: 2009.12357} \]
These Wilson coefficients can be simulated using lattice QCD:

Alternatively, can we directly measure the scattering phases in hyperon-hyperon scattering?
Outlook

In pursuit of a new paradigm:

Can symmetry be the outgrowth of more fundamental principles?

The answer appears to be a tantalizing YES!

• What about spontaneously broken symmetries? And gauge symmetry?
• We often invoke symmetry to explain naturalness. Can we understand (un)naturalness through quantum information?