The Nucleon Parton Distribution Functions from Lattice QCD

An exploratory study of distillation in the pseudo-PDF framework

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Outline

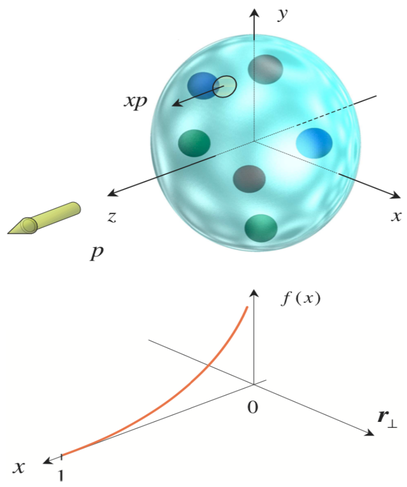
- intro & motivation
- theoretical framework
- lattice evaluation
- PDF results
- summary & outlook

Introduction

Motivation - why the PDFs?

Parton Distribution Functions (PDFs):

- part of ongoing efforts for deep understanding of internal quark and gluon structure of hadrons
- number density of "partons" with a longitudinal momentum fraction x of the nucleon's momentum
  → 1D object, necessary in 3D imaging of hadrons
- accessible in deep inelastic and semi-inclusive deep inelastic scattering



Generalized Parton Distributions (GPDs)
Transverse-momentum dependent (TMD) PDFs } complement 3D picture of hadrons

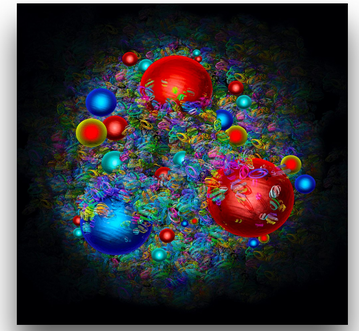
Important LHC applications Gao et.al., arXiv: 1709.04922

- predictions of various cross sections
- W-boson mass
- Strong coupling constant
- Higgs boson couplings and cross-sections (probe BSM physics)

Introduction

Motivation - why on the lattice?

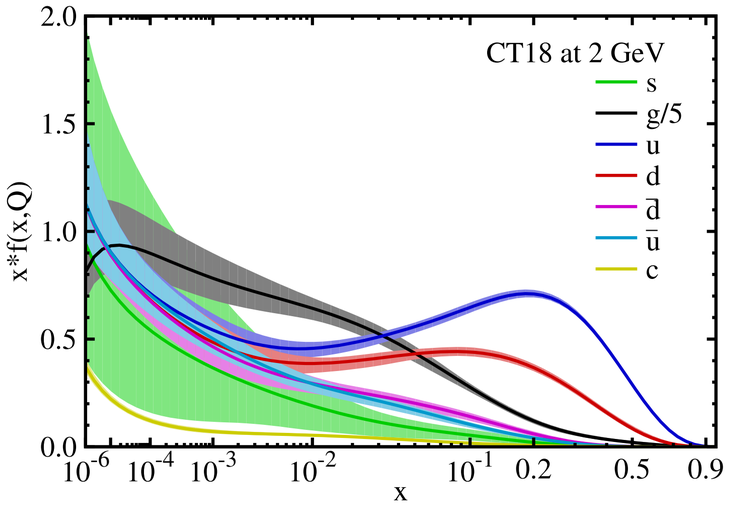
Hadrons have an immensely rich composition: Experimental extraction of information becomes more complex the deeper we look



Kovarik et.al., arXiv: 1905.06957

Accessing PDFs from experiments is an intricate procedure

Still, relatively good precision exists for unpolarized PDFs



Hou et.al. arXiv: 1912.10053

Lattice Evaluation

- PDFs: "hot" topic, deserve theoretical attention
- unpolarized PDFs → benchmark quantity
  - transversity PDFs not well constrained
  - GPDs and TMDs even more difficult to access
- first principles calculation

PDFs on the lattice

pseudo-PDF framework

Unpolarized PDF: non-local matrix element on the light-cone in Minkowski space

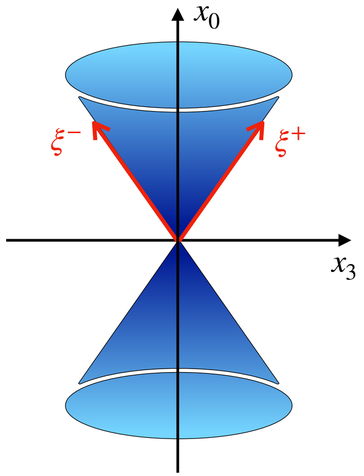
$$q(x) = \int_{-\infty}^{\infty} \frac{d\xi^-}{4\pi} e^{-ixP^+\xi^-} \langle N(P)|\bar{\psi}(\xi^-)\gamma^+ W(\xi^-,0)\psi(0)|N(P)\rangle$$

$$v^\pm = \frac{v^0 \pm v^3}{\sqrt{2}}$$

ξ^+, ξ^-: light-cone coordinates

Inaccessible on the lattice!

light-cone distances shrink to the origin in Euclidean space-time



Review: K. Cichy, M. Constantinou arXiv: 1811.07248

Methods for accessing x-dependence on the lattice:

- hadronic tensor K.F. Liu et.al. arXiv: hep-ph/9306299
- auxiliary quarks U. Aglietti et.al. arXiv: hep-ph/9806277, W. Detmold et al. arXiv: hep-lat/0507007, V. Braun et. al. arXiv: 0709.1348
- "good lattice cross sections" Y-Q. Ma and J. Qiu arXiv: 1404.6860, 1412.2688, 1709.03018
- "OPE without OPE" A.J. Chambers et.al. arXiv: 1703.01153
- quasi-distributions X. Ji arXiv: 1305.1539
- **pseudo-distributions** A. Radyushkin arXiv: 1705.01488, 1612.05170, 1710.08813, 1711.06031, 1801.02427, 1912.04244

PDFs on the lattice

pseudo-PDF framework

Matrix element in Euclidean space:

Wilson line

boosted nucleon states

$$M^\alpha(P,z) \equiv \langle N(P)|\bar{\psi}(z)\gamma^\alpha W(z,0)\psi(0)|N(P)\rangle$$

vector current: unpolarized PDFs

Lorentz decomposition

$$M^\alpha(P,z) = 2P^\alpha \mathcal{M}(\nu,z^2) + 2z^\alpha \mathcal{N}(\nu,z^2)$$

leading twist + $\mathcal{O}(z^2\Lambda^2_{\rm QCD})$

higher twist

kinematics

$P = (E,0,0,P_z)$

$z = (0,0,0,z_3)$

$\alpha = 0$

Ioffe-time distributions

$$\mathcal{M}(\nu,z_3^2) \sim \langle N(P_z)|\bar{\psi}(z_3)\gamma^0 W(z_3,0)\psi(0)|N(P_z)\rangle$$

$\nu \equiv P_z \cdot z_3$: "Ioffe time"

B. Ioffe, Phys.Lett. 30B, 123 (1969)

Lorentz invariant!

ν, z_3 : Lorentz scalars

FT: Along constant z_3 line

$$\mathcal{M}(\nu,z_3^2) = \int_{-1}^{1} dx e^{i\nu x}\mathcal{P}(x,z_3^2) \Rightarrow \mathcal{P}(x,z_3^2) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu e^{-i\nu x}\mathcal{M}(\nu,z_3^2)$$

pseudo-PDF

$$q(x) = \lim_{z_3\to 0}\mathcal{P}(x,z_3^2)$$ (ignoring divergences)

PDF

$\mathcal{M}(\nu,z_3^2)$ features:

- standard log. divergence
- power divergence (Wilson line)
- mult. renormizable to all orders in PT

T. Ishikawa et.al. arXiv: 1609.02018

J. W. Chen et. al. arXiv: 1707.03107

PDFs on the lattice

pseudo-PDF framework

Steps towards x-dependence

i) cancel divergences - renormalization

K. Orginos et.al. arXiv: 1706.05373
J. Karpie et. al. arXiv: 1710.08288
J. Karpie et. al. arXiv: 1807.10933

$$\mathfrak{M}(\nu, z_3^2) = \frac{\mathcal{M}(P_z z_3, z_3^2)}{\mathcal{M}(P_z z_3, z_3^2)|_{P_z=0}}$$

$$\left(Z_V(P_z) = \frac{1}{\mathcal{M}(P_z z_3, z_3^2)|_{z_3=0}} \right)$$

V^0 renormalization

$$\mathfrak{M}(\nu, z_3^2) = \left(\frac{\mathcal{M}(P_z z_3, z_3^2)}{\mathcal{M}(P_z z_3, z_3^2)|_{z_3=0}} \right) \times \left(\frac{\mathcal{M}(P_z z_3, z_3^2)|_{P_z=0, z_3=0}}{\mathcal{M}(P_z z_3, z_3^2)|_{P_z=0}} \right)$$

reduced-ITD

ii) matching to light-cone ITDs

1-loop matching equation:

$$\mathfrak{M}(\nu, z^2) = \mathcal{Q}(\nu, \mu^2) + \frac{\alpha_s C_F}{2\pi} \int_0^1 du\, \mathcal{Q}(u\nu, \mu^2) \left[\ln\left(z^2 \mu^2 \frac{e^{2\gamma_E+1}}{4} \right) \mathbf{B}[u] + \mathbf{L}[u] \right]$$

light-cone ITD

evolution to common scale $1/z_0$

conversion to $\overline{\mathrm{MS}}$ scheme

A. Radyushkin arXiv: 1801.02427
J.-H.Zhang et. al. arXiv: 1801.03023
T. Izubuchi et. al. arXiv: 1801.3917

iii) reconstruction of x-dependence

F. T. in ν-space: $\mathcal{Q}(\nu, \mu^2) = \int_{-1}^{1} dx e^{i\nu x} q(x, \mu^2)$

valence quark

$$\mathfrak{Re}\, \mathcal{Q}(\nu, \mu^2) = \int_0^1 dx \cos(\nu x) q_v(x, \mu^2)$$

$q + \bar{q}$

$$\mathfrak{Im}\, \mathcal{Q}(\nu, \mu^2) = \int_0^1 dx \sin(\nu x) q_{v2s}(x, \mu^2)$$

Inverse problem…!

A. Radyushkin arXiv: 1705.01488

Lattice evaluation

Correlation functions: "Distillation" as a smearing method

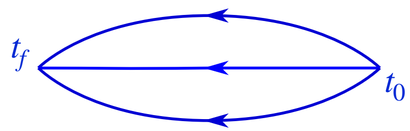
Smearing: Increase overlap of hadron interpolating fields with ground state

$$\mathcal{X}_B(\vec{x},t) = \epsilon^{abc}\mathcal{S}_{\sigma_1\sigma_2\sigma_3}(\mathcal{D}_1 q_1)^a_{\sigma_1}(\vec{x},t)(\mathcal{D}_2 q_2)^b_{\sigma_2}(\vec{x},t)(\mathcal{D}_1 q_3)^c_{\sigma_3}(\vec{x},t) \qquad q_{\rm sm}(\vec{x},t) = \mathcal{F}[U(\vec{x},t);\vec{y}]q(\vec{y},t)$$

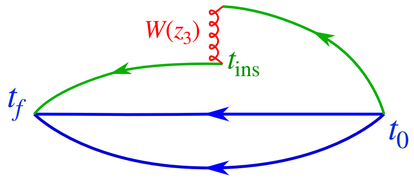
"Standard" techniques: Gauge-covariant, based on 3D lattice Laplacian

$$-\nabla^2_{\vec{x},\vec{y}}(t) = 6\delta_{\vec{x}\vec{y}} - \sum_{j=1}^{3}\left[U_j(\vec{x},t)\delta_{\vec{x}+\hat{j},\vec{y}} + U_j^\dagger(\vec{x}-\hat{j},t)\delta_{\vec{x}-\hat{j},\vec{y}}\right] \qquad \mathcal{F}_{\alpha_s,N_s} = \left(1+\frac{\alpha_s\nabla^2}{N_s}\right)^{N_s}$$

$$C_{\rm 2pt}(t_f,t_0) = \sum_{\vec{x},\vec{y}}\langle\mathcal{X}(\vec{x},t_f)\bar{\mathcal{X}}(\vec{y},t_0)\rangle$$



$$C_{\rm 3pt}(t_f,t_{\rm ins},t_0;\Gamma) = \sum_{\vec{x},\vec{z},\vec{y}}\langle\mathcal{X}(\vec{x},t_f)\,[\bar{q}\gamma^0 W q](\vec{z},t)\,\bar{\mathcal{X}}(\vec{y},t_0)\rangle$$



Distillation: Smearing operator → low-rank eigenvector representation

M. Peardon et. al. arXiv: 0905.2160

$$\Box_{\vec{x},\vec{y}}(t) \equiv \sum_{k=1}^{N} v_k(\vec{x},t)v_k^\dagger(\vec{y},t) = V(t)V^\dagger(t)\ ,\ -\nabla^2 v_k = \lambda_k v_k \Longrightarrow q_{\rm sm}(\vec{x},t) = \Box_{\vec{x},\vec{y}}(t)q(\vec{y},t)$$

perambulators: $\tau(t_f,t_0) = V^\dagger(t_f)M^{-1}(t_f,t_0)V(t_0)$

elementals: $\Phi^{i,j,k}_{\sigma_1\sigma_2\sigma_3}(t) = \epsilon^{abc}\mathcal{S}_{\sigma_1\sigma_2\sigma_3}(\mathcal{D}_1 v_i(t))^a(\mathcal{D}_2 v_j(t))^b(\mathcal{D}_1 v_k(t))^c$

gen. perambulators: $\tilde{\tau}(t_f,t_{\rm ins},t_0) = V^\dagger(t_f)M^{-1}(t_f,t_{\rm ins})\Gamma(t_{\rm ins})M^{-1}(t_{\rm ins},t_0)V(t_0)$

inversions: $S^{(i)}_\alpha(y;t_0) = M^{-1}(y;\vec{z},t)_{\alpha\beta}v_i(\vec{z})\delta_{\beta\beta_0}\delta_{tt_0} = M^{-1}(y;\vec{z},t_0)_{\alpha\beta_0}v_i(\vec{z})$

Pros:

- factorization of correlation functions
- various spin structures *without* additional inversions
- mom. projection at the sink *and* the source
- reusability of building blocks

Cons:

- extra step: compute eigenvectors of ∇^2
- big files/storage

Lattice evaluation

Simulation details

Lattice action / configurations: W&M / JLab

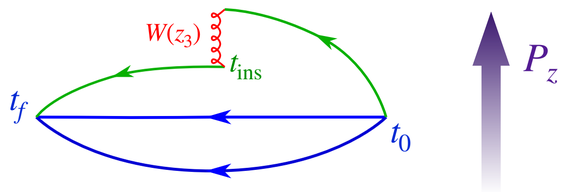
- Wilson-Clover fermion action, $N_f = 2 + 1$
- lattice volume: $32^3 \times 64$
- $\beta = 6.3$, $a = 0.091\,\text{fm}\,(a^{-1} = 2.16\,\text{GeV})$
- $m_\pi = 0.350\,\text{GeV}$, $m_N = 1.160\,\text{GeV}$
- $L = 2.9\,\text{fm}$, $m_\pi L = 5.15$

Runs performed at JLab computer clusters and at Frontera (TACC) using the Chroma software suite and in-house analysis packages

Parameters and statistics:

- 350 gauge configurations
- distillation vectors: $N_{ev} = 64$
- 6 $t_{sep} \equiv (t_f - t_0) = [4a, 6a, \ldots, 14a] \rightarrow 0.4\ldots1.3\,\text{fm}$
- 4 time sources $t_0/a = [t_0^i, +T/4, +T/2, +3T/4]$
- boost momentum $P_z = 2\pi/L \times [0, \pm 1, \ldots, \pm 6] \rightarrow 0\ldots2.55\,\text{GeV}$
- displacements $z_3 = 0, \pm 1a, \ldots, \pm 8a \rightarrow 0\ldots0.73\,\text{fm}$
- **5600 statistics**

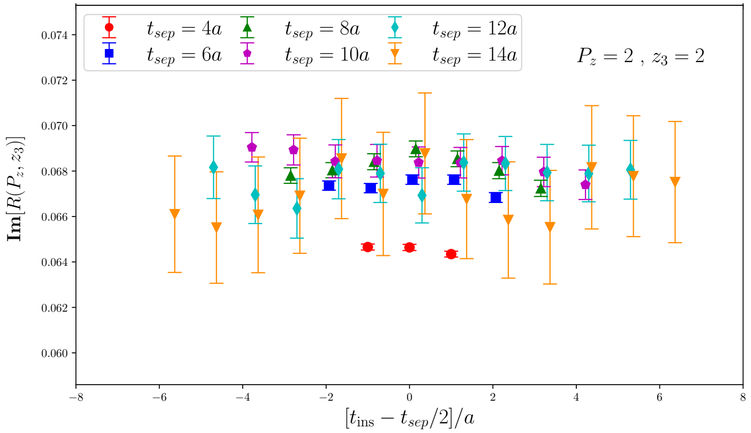
$$C_{3\text{pt}}(t_f, t_{\text{ins}}, t_0; \Gamma) = \sum_{\vec{x},\vec{z},\vec{y}} \langle \mathcal{X}(\vec{x}, t_f)\ [\bar{q}\gamma^0 W q](\vec{z}, t)\ \bar{\mathcal{X}}(\vec{y}, t_0)\rangle$$



Lattice evaluation

Matrix elements: ground state dominance

Ratio of 3pt/2pt functions: $R(P_z, z_3; t_{sep}, t_{ins}) = \frac{C_{3\text{pt}}(P_z, z_3; t_{sep}, t_{ins})}{C_{2\text{pt}}(P_z, t_{sep})} \xrightarrow[t_{ins}-t_0 \gg a]{t_{sep}-t_{ins} \gg a} \mathcal{M}(\nu, z_3^2)$

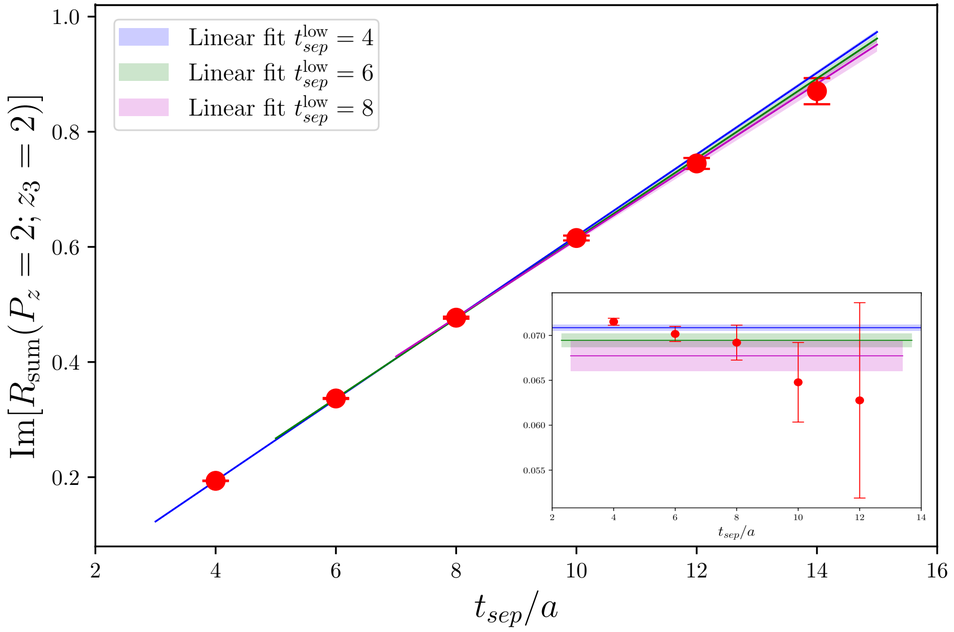


L. Maiani et. al. Nucl.Phys. B293 (1987)

i) Summation method

$$R_{\text{sum}}(P_z, z_3; t_{sep}) = \sum_{t_{ins}=a}^{t_{sep}-a} R(P_z, z_3; t_{sep}, t_{ins})$$
$$= c_1 + \mathcal{M}(P_z, z_3) \times t_{sep} + \mathcal{O}\left(e^{-\Delta E t_{sep}}\right)$$

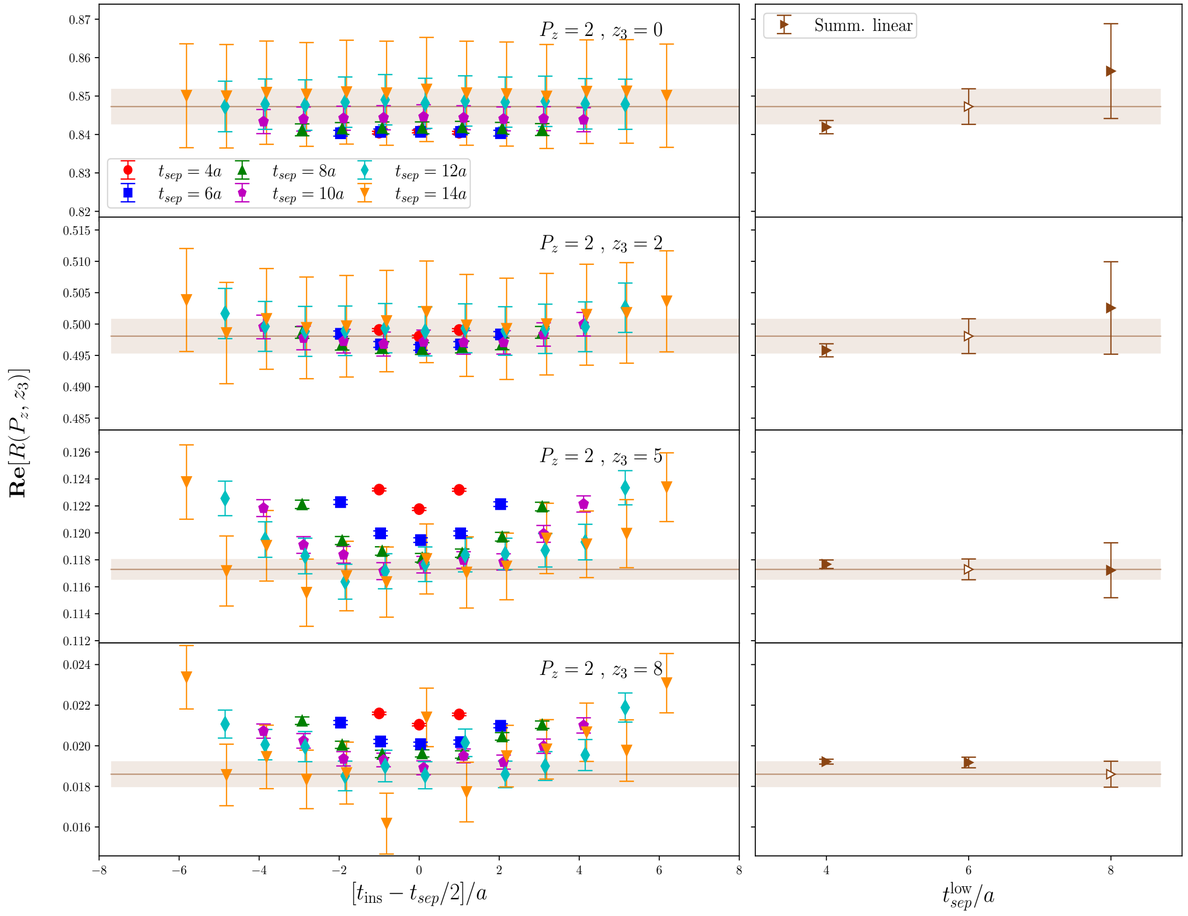
Perform linear fits, increase lower value of t_{sep} until convergence

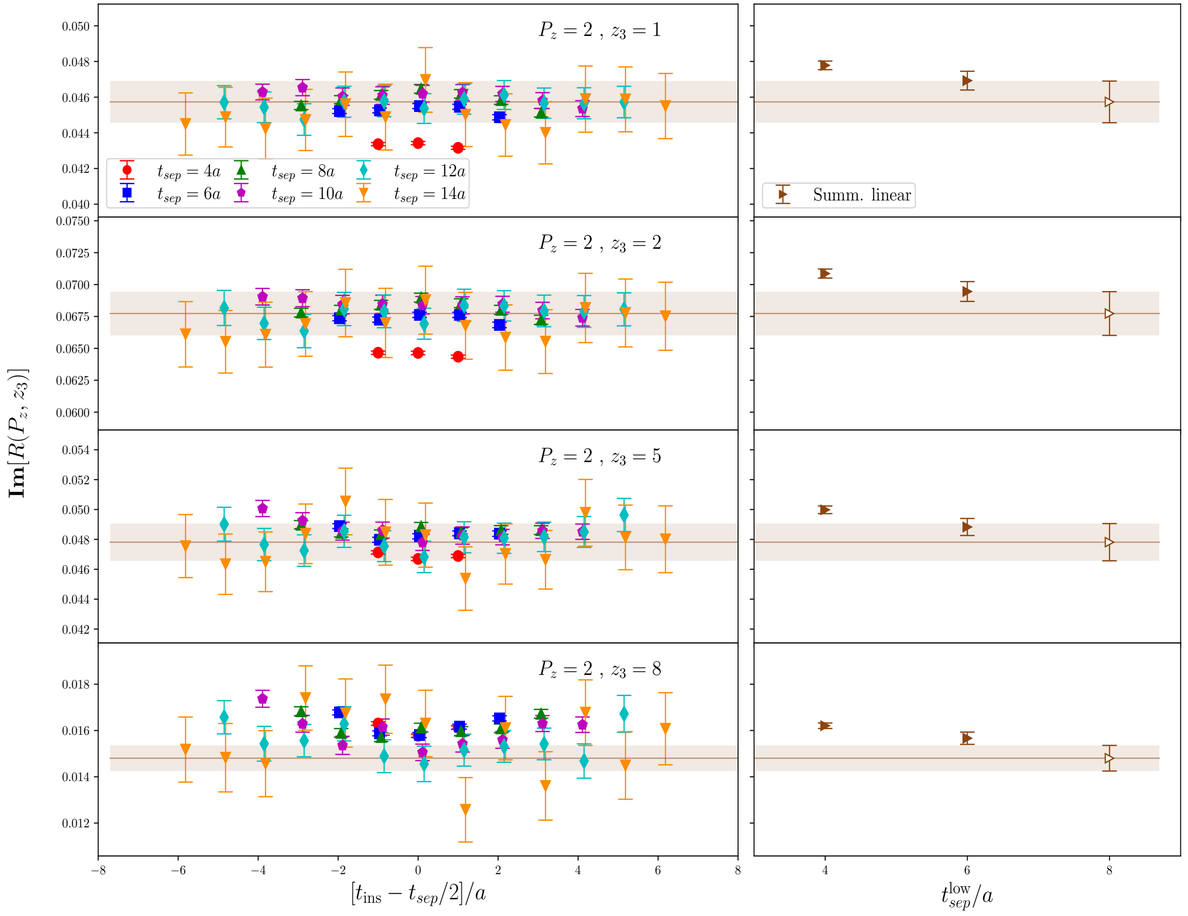


ii) Plateau method (ask for more!)

Lattice evaluation

Matrix elements: select data



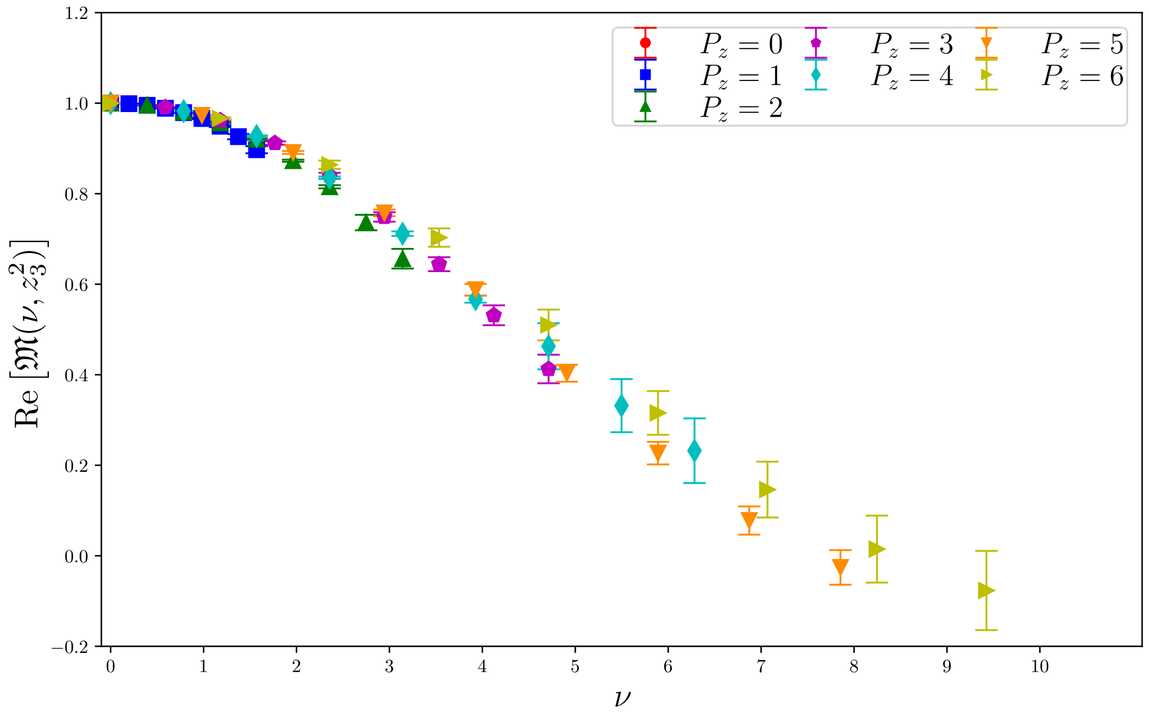


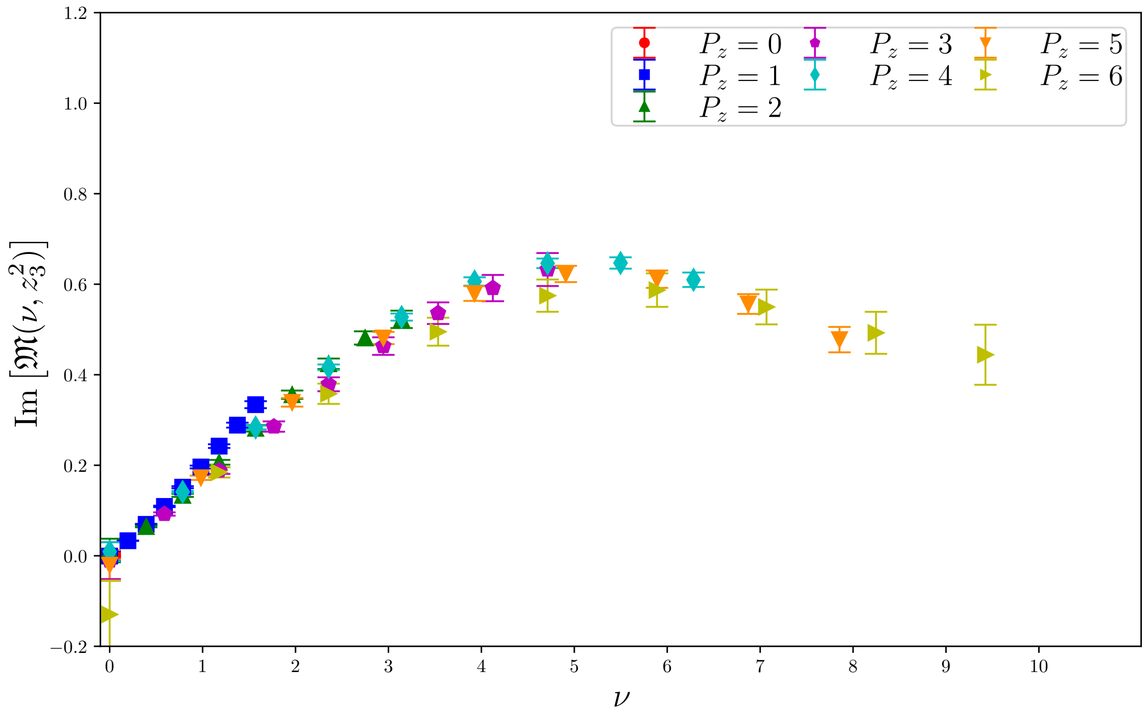
PDF results

Pseudo-ITDs

$$\mathfrak{M}(\nu, z_3^2) = \left(\frac{\mathcal{M}(P_z z_3, z_3^2)}{\mathcal{M}(P_z z_3, z_3^2)|_{z_3=0}}\right) \times \left(\frac{\mathcal{M}(P_z z_3, z_3^2)|_{P_z=0, z_3=0}}{\mathcal{M}(P_z z_3, z_3^2)|_{P_z=0}}\right)$$

- get rid of logarithmic divergences
- renormalization of vector current
- higher-twist contaminations $\mathcal{O}(z_3^2 \Lambda_{\mathrm{QCD}}^2)$ partially suppressed





PDF results

Evolution and matching to $\overline{\mathrm{MS}}$: Light-cone ITDs

$$\mathcal{Q}(\nu,\mu^2) = \mathfrak{M}(\nu,z^2) - \frac{\alpha_s C_F}{2\pi}\int_0^1 du\,\mathfrak{M}(u\nu,z^2)\left[\ln\left(z^2\mu^2\frac{e^{2\gamma_E+1}}{4}\right)\mathbf{B}[u] + \mathbf{L}[u]\right]$$

light-cone ITD

evolution to common scale $1/z_0$

conversion to $\overline{\mathrm{MS}}$ scheme

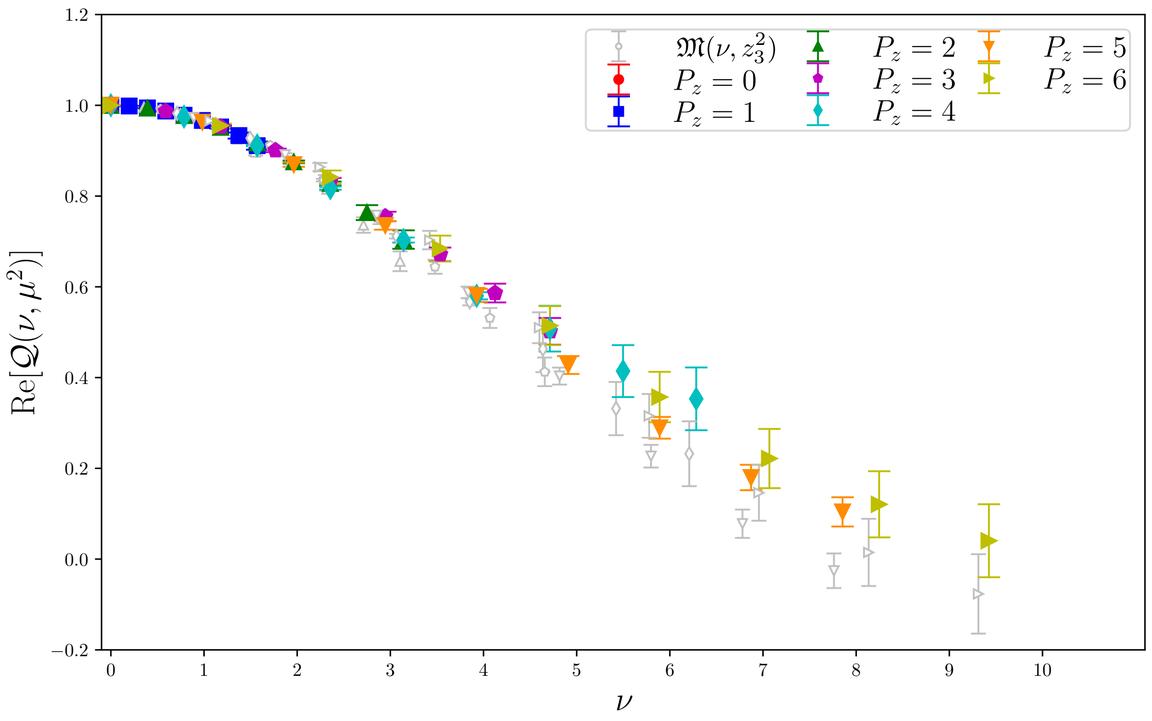
$$B(u) = \left[\frac{1+u^2}{1-u}\right]_+$$

$$L(u) = \left[4\frac{\ln(1-u)}{1-u} - 2(1-u)\right]_+$$

$$\int_0^1 \mathfrak{M}(u\nu)[f(u)]_+ = \int_0^1 (\mathfrak{M}(u\nu) - \mathfrak{M}(\nu))\,f(u)$$

Common scale: $z_0(\mu = 2\mathrm{GeV}) = \dfrac{2e^{-\gamma_E-1/2}}{\mu} = 0.067\,\mathrm{fm}\,,\ \dfrac{1}{z_0} = 2.94\mathrm{GeV}$

- treat each z_3 independently
- need $\mathfrak{M}(u\nu,\mu^2) \to$ polynomial parametrization w.r.t. ν
- evolution/matching processes separately or combined can give rise to different systematics
- 1-loop matching: ~10% effect to pseudo-ITD data
- small higher-twist effects



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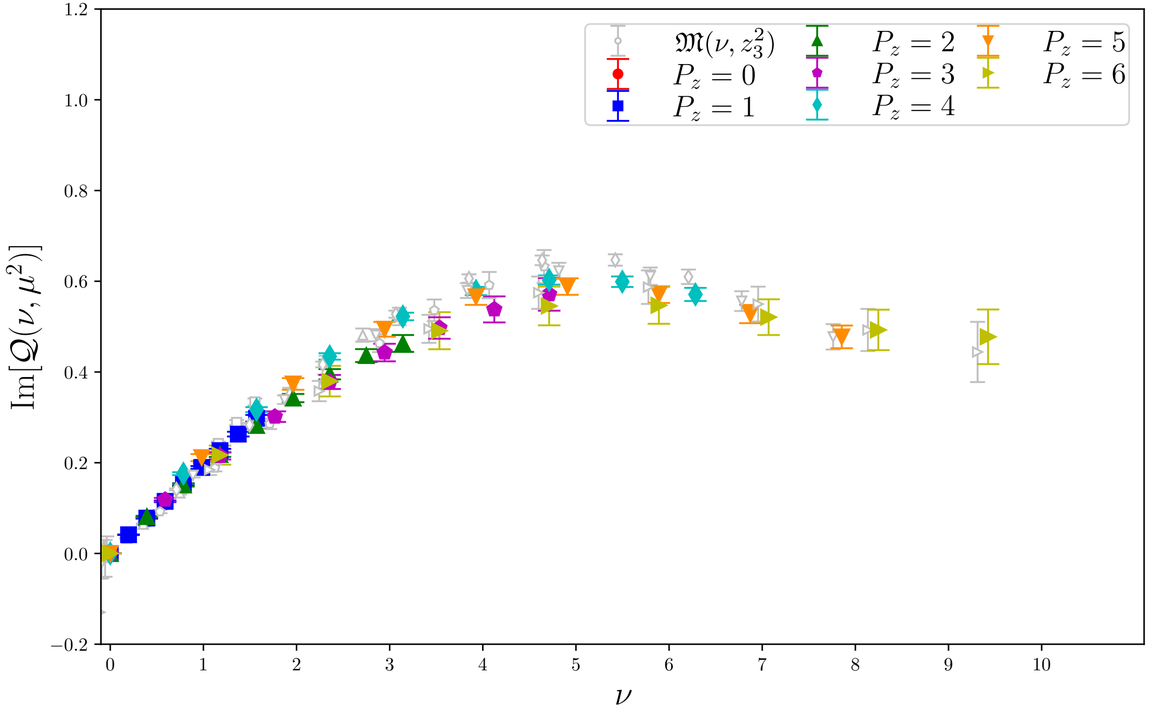


PDF results

Full picture: x-dependence

discrete data in ν-space

$$\mathcal{Q}(\nu,\mu^2) = \int_{-1}^{1} dx e^{i\nu x} q(x,\mu^2)$$

continuous function in x-space

Inverse problem! Way around:

- choose a physically meaningful functional form for $q(x,\mu^2)$
- introduce model bias in determination, but…
- general form:

$$q_v(x) = \frac{x^\alpha (1-x)^\beta (1+\gamma\sqrt{x}+\delta x)}{B(\alpha+1,\beta+1)+\gamma B(\alpha+3/2,\beta+1)+\delta B(\alpha+2,\beta+1)}$$

$B(i,j)$: beta-function

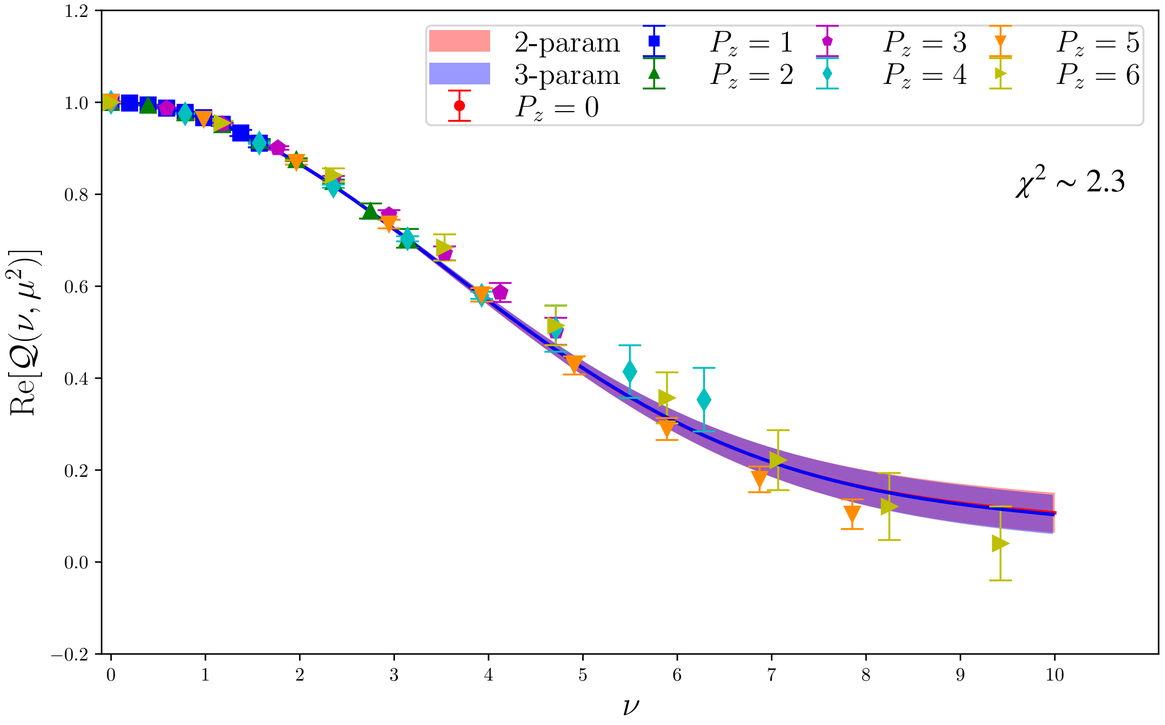
i) 2-parameter fit: $\alpha,\beta \neq 0, \gamma,\delta = 0$

ii) 3-parameter fit: $\alpha,\beta,\delta \neq 0, \gamma = 0$

$$\chi^2 = \sum_{\nu=0}^{\nu_{\max}} \frac{Q(\nu,\mu^2) - Q_{\rm FT}(\nu,\mu^2)}{\sigma_Q^2}, \quad Q_{\rm FT}(\nu,\mu^2) = \int_0^1 dx \cos(\nu x) q_v(x)$$

valence quark distribution:

$$\mathrm{Re}\,\mathcal{Q}(\nu,\mu^2) = \int_0^1 dx \cos(\nu x) q_v(x,\mu^2)$$



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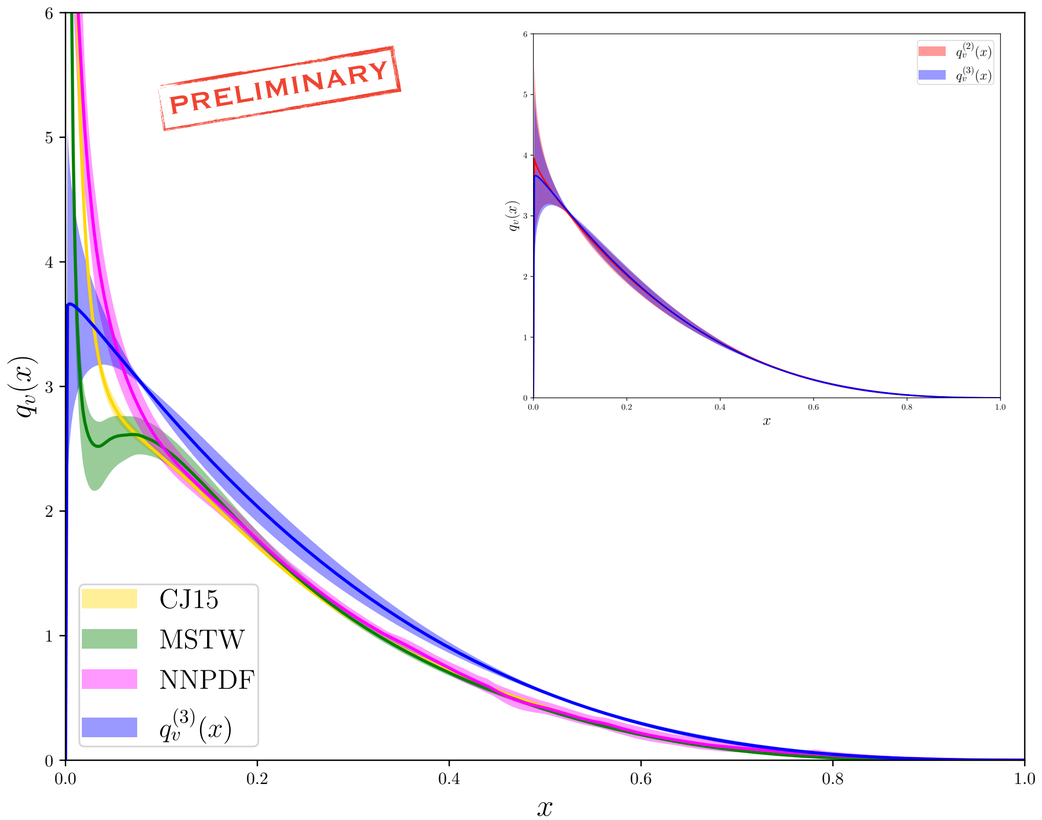


CJ15: A. Accardi et. al. arXiv: 1602.03154
MSTW: A.D.Martin etl al. arXiv: 0901.0002
NNPDF: R.D.Ball et. al. arXiv: 1706.00428

Conclusions

Summary & Outlook

- Lattice QCD via pseudo-PDF framework: in good position to give reliable insight on nucleon PDFs
  - Distillation: valuable alternative to standard smearing techniques
  - controllable statistical uncertainties and excited state effects
- Some stretch between lattice and phenomenological x-dependence exists:
  - lattice artifacts → finite volume and lattice spacing, non-physical simulations
  - parametrization of PDFs → be systematic: try different approaches, compare with other lattice evaluations

On the horizon:

- results from two other ensembles, one at almost physical pion mass → study volume effects, remove systematics
- plans for analyzing ensembles with smaller lattice spacing → continuum extrapolation
- helicity, transversity distributions & GPDs on the way!
- coming soon: disconnected diagrams

Conclusions

Summary & Outlook

- Lattice QCD via pseudo-PDF framework: in good position to give reliable insight on nucleon PDFs
  - Distillation: valuable alternative to standard smearing techniques
  - controllable statistical uncertainties and excited state effects
- Some stretch between lattice and phenomenological x-dependence exists:
  - lattice artifacts → finite volume and lattice spacing, non-physical simulations
  - parametrization of PDFs → be systematic: try different approaches, compare with other lattice evaluations

On the horizon:

- results from two other ensembles, one at almost physical pion mass → study volume effects, remove systematics
- plans for analyzing ensembles with smaller lattice spacing → continuum extrapolation
- helicity, transversity distributions & GPDs on the way!
- coming soon: disconnected diagrams

Thank you

Bonus!

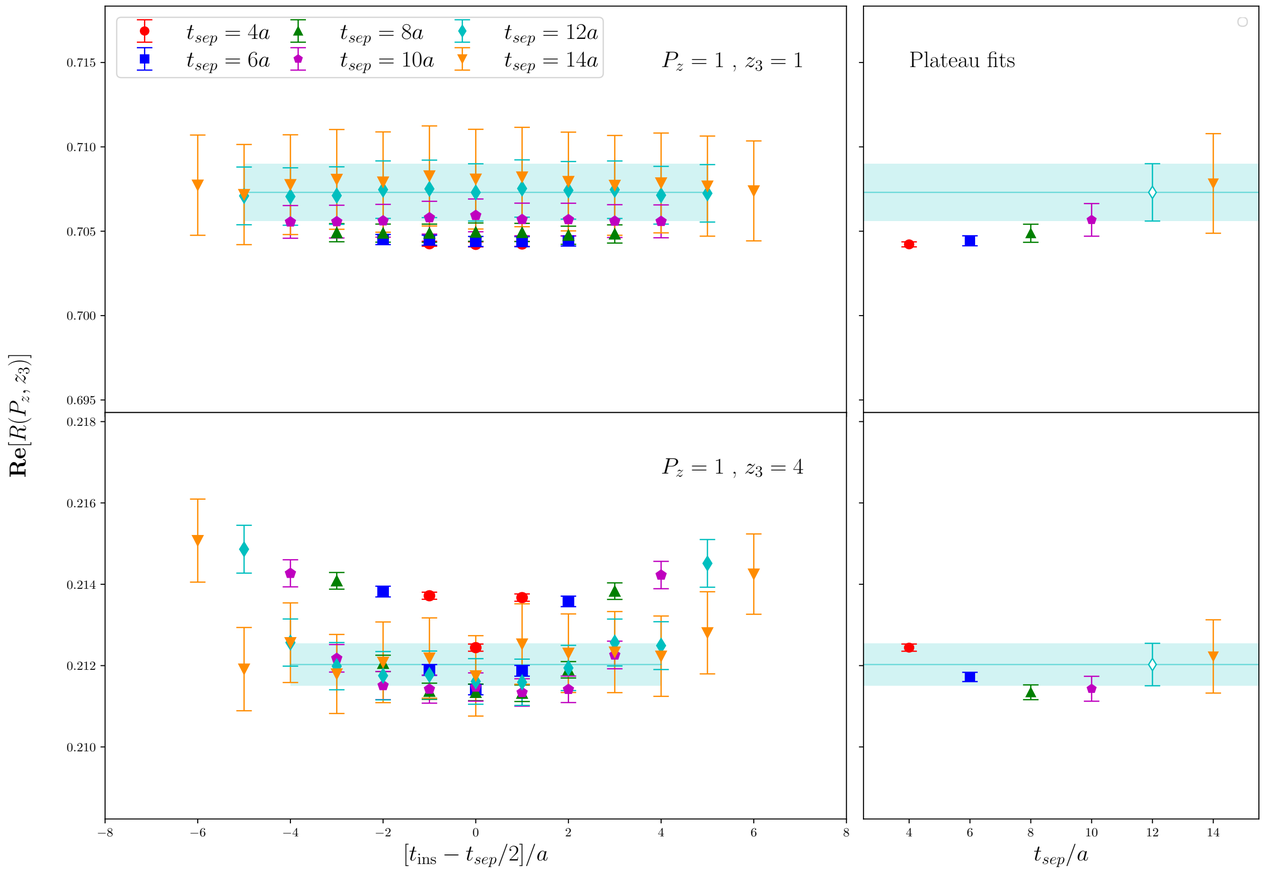
(Bonus!) Matrix elements

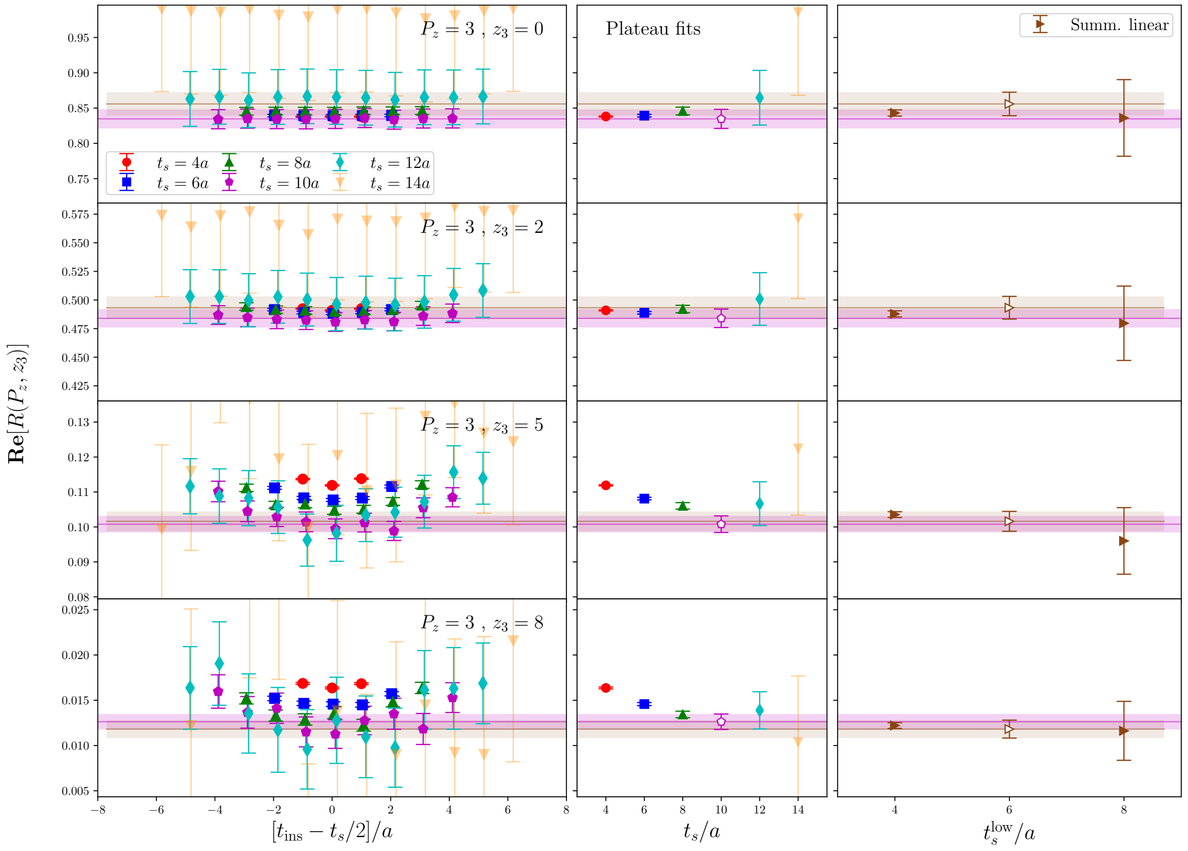
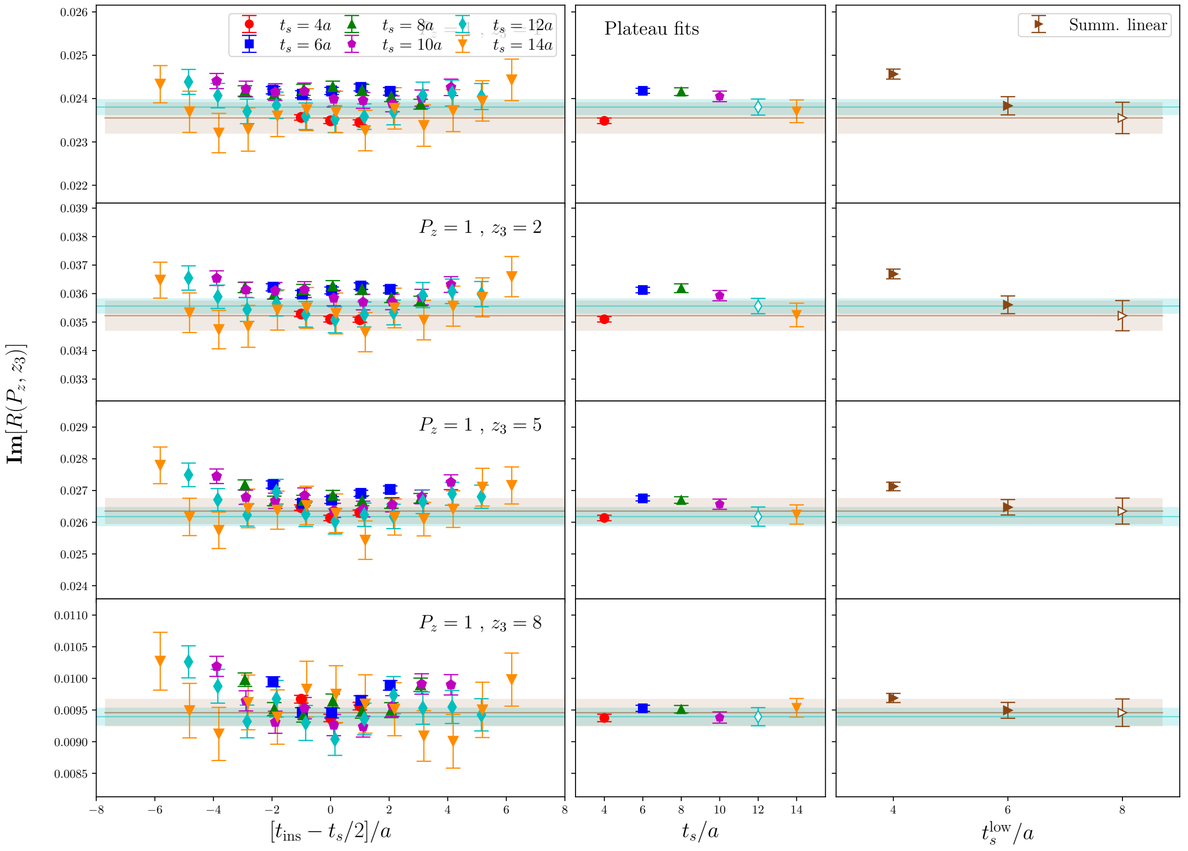
Ensuring ground state dominance

Ratio of 3pt/2pt functions: $R(P_z, z_3; t_{sep}, t_{ins}) = \dfrac{C_{3\mathrm{pt}}(P_z, z_3; t_{sep}, t_{ins})}{C_{2\mathrm{pt}}(P_z, t_{sep})}$

i) Plateau method

- for each t_{sep} , perform constant fits for multiple ranges
- decrease each fit range by one point on each side
- best fit: longest for which $\chi^2 \leq 0.9$



(Bonus!) Matrix elements

Ensuring ground state dominance