S-wave scattering of triply bottom baryons from lattice QCD

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Baryon-baryon interactions

- One of the central topics in hadron and nuclear physics. 

- Significant experimental effort in collecting NN scattering data. 
  YN and YY interactions are poorly understood. 
  Substantial theoretical efforts based on CQM and OBE models. 
  Fujiwara&Suzuki 2008, Rijken&Yamamoto 2010

- Hyperon formation $\Leftrightarrow$ Large nuclear densities in astrophysical objects 
  Neutron stars, supernovae, white dwarfs, etc. 
  Implications on the EOS and on the properties of these astrophysical objects. 
  Tolos&Fabbietti 2020

- A handful of experimental efforts using large nuclei reactions. 
  Inputs on LECs to EFTs $\Rightarrow$ nuclear many body calculations. 
  Epelbaum 2005, INT-NFPNP 2022, $0\nu\beta\beta$ PSWR 2022

- Despite several experimental efforts only one dibaryon bound state remains established: Deuteron. 
  $d^* (2380)$, the only other dibaryon feature confirmed $\rightarrow$ isoscalar spin 3 resonance. 
A handful of lattice QCD efforts on baryon-baryon scattering typically at $m_\pi > m^{\text{phys}}_\pi$.

see works by NPLQCD, HALQCD*, Mainz, CalLat, and others in the past decade.

Main focus on light and strange six quark systems: Deuteron, H-dibaryon, ...

Most recent calculations suggest these systems may have a near-threshold feature. Discretization effects could be crucial.  

Two different procedures in extracting the scattering amplitudes.

(a) Solving QM potentials derived from NBS wave functions

(b) Lüscher’s formalism involving finite volume spectrum extraction

In this work, we use procedure (b).

Scattering of decuplet baryons: towards an understanding of $d^*(2380)$ feature.

Predictions for 6 nonstrange dibaryon states based on flavor symmetry

⇒ Expects an energy degenerate spin 0 resonance.

Heavy cousins from the symmetric flavor multiplet: no near three or four particle thresholds.

Simple model studies ($\Omega\Omega$ scattering): widely different inferences.
Scattering of Ω-like baryons using lattice QCD

- No explicit light quark mass \((m_l)\) dependence and no low lying three or four particle thresholds. Leading \(m_l\) dependence could arise from pair produced \(2\pi\) vertices.
  Calculations at unphysically large \(m_l\): Relatively cheap calculations with clean signals.

- Early calculations of \(S\)-wave \(\Omega\Omega\) scattering using Lüscher’s formalism.
  Weakly repulsive interaction observed in the total spin 0.  
  
  Buchoff&Luu&Wasem 2012

- Investigations by HALQCD Collaboration
  - Moderate attraction in \(S\)-wave \(\Omega\Omega\) scattering  
  - Similar conclusions from near \(m_\pi^{\text{phys}}\) large volume study  
  - Similar observations for \(S\)-wave \(\Omega_{ccc}\Omega_{ccc}\) scattering  
    Study performed on ensemble used in 2018 study.  
  
  Yamada \textit{et al} 2015
  Gongyo \textit{et al} 2018
  Lyu \textit{et al} 2021

- Our plan is to investigate the \(m_q\) dependence in \(S\)-wave \(\Omega_{qqq}\Omega_{qqq}\) scattering
  - This talk covers the results and our inferences in the bottom sector.
  - Data production and early spectrum analysis going on for quark mass range \((m_s \lesssim m_q < m_b)\).

  Mathur&PM&Chakraborty, 2022
Lattice QCD: Basic idea

LQCD: A non-perturbative, gauge invariant regulator for the QCD path integrals.

- Quark fields $\psi_\alpha(x)$ on lattice sites
- Gauge fields as parallel transporters $U_\mu$
  Lives in the links. $U_\mu(x) = e^{igaA_\mu(x)}$
- $\bar{\psi}_\alpha^i(x)[U_\mu(x)]_{ij}\psi_\alpha^j(x + a\hat{\mu})$ is gauge invariant.
- Lattice spacing: UV cut off
- Lattice size: IR cut off

Employ Monte Carlo importance sampling methods on Euclidean metric for numerical studies.
QCD spectrum from Lattice QCD

• Aim: to extract the physical states of QCD.

• Euclidean two point current-current correlation functions

\[ C_{ji}(t_f - t_i) = \langle 0 | O_j(t_f) \bar{O}_i(t_i) | 0 \rangle = \sum_n \frac{Z^n_i Z^n_j}{2m_n} e^{-m_n(t_f - t_i)} \]

where \( O_j(t_f) \) and \( \bar{O}_i(t_i) \) are the desired interpolating operators and \( Z^n_j = \langle 0 | O_j | n \rangle \).

• Effective mass defined as \( \log\left[ \frac{C(t)}{C(t+1)} \right] \)

• The ground state: from the exponential fall off at large times.

Non-linear fitting techniques.

Ω_{bbb}Ω_{bbb} scattering from lattice QCD
Interpolating operators

For $J^P = \frac{3}{2}^+$ $\Omega_{bbb}$, we utilize a nonrelativistic operator

$$O_{\Omega_{bbb}} |s_{1}, s_{2}, s_{3}\rangle \langle J, J_z |$$

$s_i$: quark spin

| $O_{\Omega_{bbb}}$ | $|s_{1}, s_{2}, s_{3}\rangle$ | $|J, J_z\rangle$ |
|--------------------|-------------------------------|------------------|
| $\chi_1$          | $|\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\rangle$ | $|\frac{3}{2}, \frac{3}{2}\rangle$ |
| $\chi_2$          | $|\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\rangle_S$ | $|\frac{3}{2}, -\frac{1}{2}\rangle$ |
| $\chi_3$          | $|\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\rangle_S$ | $|\frac{3}{2}, \frac{1}{2}\rangle$ |
| $\chi_4$          | $|\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\rangle$ | $|\frac{3}{2}, -\frac{3}{2}\rangle$ |

subscript $S$ implies symmetric structure

$J$ and $J_z$: total spin and its azimuthal component

The $\Omega_{bbb}\Omega_{bbb}$ operator is then constructed as

$$O_{D_{6b}} (x, t) = \chi_m (x, t) [CG]^{mn} \chi_n (x, t)$$

which for the scalar quantum numbers looks like

$$O_{D_{6b} J=0} = \frac{1}{2} \left[ \chi_1 \chi_4 + \chi_3 \chi_2 - \chi_2 \chi_3 - \chi_4 \chi_1 \right]$$
Compute matrices of two point correlation functions

\[ C_{ji}(t) = \langle 0 | \bar{O}_j(t) O_i(0) | 0 \rangle = \sum_n \frac{Z_i^n Z_j^{n*}}{2E_n} e^{-E_n(t)} \]

Fully bottom six quark systems near \( \Omega_{bbb} \Omega_{bbb} \) threshold, we use ‘only’ \( O \) of \( BB \) type, where each \( B \) (baryon) is color singlet projected.

Wick contractions to compute:
With 6 identical quarks there are \( 6! = 720 \) quark line contractions possible.
The trivial symmetry in the flavor sector \( \Rightarrow \) Only two contractions are unique.
Heavy quarks on the lattice

- Fermion action: \( \bar{\psi}(\gamma.D + m)\psi \)

- Only dimensionless quantities defined on the lattice

  Lattice fermion action: \( \bar{\psi}(x)(\gamma.D_L + am)\psi(x) \)

  Discretization (relativistic) \( \rightarrow D_L \); clover, staggered, overlap, domain-wall, etc.

  Discretization errors on observables \( O(am) \).

- Charm quarks: \( m_c \sim 1.275 \text{ GeV} \)
  \[ am_c = 0.5 \quad \Rightarrow \quad a \sim 0.075 \text{ fm} \quad am_c = 0.3 \quad \Rightarrow \quad a \sim 0.046 \text{ fm} \]

- Bottom quarks: \( m_b \sim 4.66 \text{ GeV} \)
  \[ am_b = 0.5 \quad \Rightarrow \quad a \sim 0.021 \text{ fm} \quad am_b = 0.3 \quad \Rightarrow \quad a \sim 0.013 \text{ fm} \]
Bottom quark action

✿ Bottom quarks realized using NRQCD formulation. Hamiltonian through $\mathcal{O}(\alpha \nu_b^4)$. Perturbatively tuned NRQCD co-efficients.


✿ Tuned using the spin averaged mass of 1S bottomonium

$$\Delta E_{1S, bb}^{hfs} = 63(3)(3) \text{ MeV} \quad [\text{Lat.cont.}] \quad \Delta E_{1S, \bar{b}b}^{1S} = 62(1) \text{ MeV} \quad [\text{Expt.}]$$

El-Khadra et al, arXiv:9604004[hep-lat]

✿ We extract the energy splittings

$$\Delta E_{\Omega_{bbb}} = M^L_{\Omega_{bbb}} - 1.5M_{1S} \quad \& \quad \Delta E_{D6b} = M^L_{D6b} - 2M_{\Omega_{bbb}}$$

✿ Following a continuum extrapolation performed for $\Delta E_{\Omega_{bbb}}$

$$M^{phys}_{\Omega_{bbb}} = 14366(7)(9) \text{ MeV determined from } \Delta E_{\Omega_{bbb}}^{cont} + 1.5M^{phys}_{1S}$$

$\Omega_{bbb} \Omega_{bbb}$ scattering from lattice QCD

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Lattice QCD ensembles

- Ensembles: $N_f = 2+1+1$ HISQ (MILC): $24^3.64$, $32^3.96$, $48^3.144$, & $40^3.64$
  - $a = 0.1207(11)$, $0.0888(8)$, $0.0582(5)$, and $0.1189(7) \text{ fm}$ respectively using $r_1$ parameter.

Three ‘different’ lattice spacing and two ‘different’ volumes ($\sim 2.85 \text{ fm}$ and $4.76 \text{ fm}$).
Possible continuum extrapolation and finite-volume scattering analysis.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{lattice_diagram.png}
\end{figure}
Signal quality and quark smearing

- Our prime focus on the finite volume ground state energy
- Limit ourselves to two point correlation functions
  \[ C_\mathcal{O}(t_f - t_i) = \sum_{\vec{x}_f} e^{-i\vec{p} \cdot \vec{x}_f} \langle 0 | \mathcal{O} (\vec{x}_f, t_f) \bar{\mathcal{O}}(t_i) | 0 \rangle \]
- Quark smearing: ‘wall-smeared’ quark source (in \( \bar{\mathcal{O}}_i \)) and point quark sink (in \( \mathcal{O}_j \)).
  Asymmetric correlation functions: Effective energies rising from below.
  Excited state contributions with different sign that for the ground state.
Wall smearing and other smearing procedures

- Widely used in several calculations in determining ground state energies

  Mathur, MP, et al 2012-22, HALQCD

- Other symmetric and asymmetric smearing procedures to affirm our findings.

  wall, Gaussian, box [Hudspith 2020] sink smearing.

- Checking the $t_{\text{min}}$ dependence of the fits
**Multiple fitting procedures**

- Standard single exponential fitting against Matrix Prony algorithm
  - Kunis&Peter&Römer 2015, NPLQCD 2012, Buchoff&Luu&Wase 2012
- Comparison with the results from fits to the ratio of correlators

\[ \Delta E = M(2\Omega_{bbb}) - M(D_{6b}) \]

\( \Omega_{bbb} \) scattering from lattice QCD

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Lattice results for energy splitting

Table presents the energy splitting $\Delta E_{D_{6\overline{6}}} = M_{D_{6\overline{6}}}^{L} - 2M_{\Omega_{bb\overline{b}}}$

- Clear energy gap between the noninteracting and interacting cases.
- Similar energy splittings across all four ensembles.
- Negative energy splittings indicating attractive interaction.
Finite volume spectrum and infinite volume physics

* On a finite volume Euclidean lattice: Discrete energy spectrum
  Cannot constrain infinite volume scattering amplitude away from threshold. Maiani-Testa 1990

* Non-interacting two-hadron levels are given by
  \[ E(L) = \sqrt{m_1^2 + \vec{k}_1^2} + \sqrt{m_2^2 + \vec{k}_2^2} \]
  where \( \vec{k}_{1,2} = \frac{2\pi}{L}(n_x, n_y, n_z) \).

* Switching on the interaction: \( \vec{k}_{1,2} \neq \frac{2\pi}{L}(n_x, n_y, n_z) \). e.g. in 1D \( \vec{k}_{1,2} = \frac{2\pi}{L}n + \frac{2}{L}\delta(k) \).

* Lüscher’s formula relates finite volume level shifts \( \leftrightarrow \) infinite volume phase shifts. Lüscher 1991

* Generalizations of Lüscher’s formalism: c.f. Briceño 2014
Scattering amplitude parametrization

- Scattering amplitude: 
  \[ S = 1 + i \frac{4k}{E_{cm} t} \]

- For the \( \Omega_{bbb} \Omega_{bbb} \) system [total spin equals 0], and assuming only \( S \)-wave,
  \[ t^{-1} = 2 \tilde{K}^{-1} - i \frac{2k}{E_{cm}}, \quad \text{with} \quad \tilde{K}^{-1} = k \cdot \cot \delta(k) \]

  Bound state constraint: \( k \cdot \cot \delta(k) = -\sqrt{-k^2} \)

- Lüscher’s prescription: \( k \cdot \cot \delta(k) = F(k) \), where \( F(k^2) \) is a known mathematical function.
  \( k^2 \) is determined from each extracted energy splitting as 
  \[ k^2 = \frac{\Delta E_{D6}^L}{4} (\Delta E_{D6}^L + 4M_{\Omega_{bbb}}^{phys}) \]

- We parametrize \( k \cdot \cot \delta(k) \) with a constant (inverse scattering length “\(-1/a_0\)”).
  The remnant lattice spacing “\(a\)” dependence: “\(-1/a_0[0] - a/a_0[1]\)”.
  Fits performed with and without “\(a\)” dependence.
Amplitude analysis and binding energy estimate

- Fits with \(-1/a_0^{[0]} - a/a_1^{[1]}\) is found to be the best with \(\chi^2/d.o.f. = 0.7/2\)
  \[a_0^{[0]} = 0.18^{+0.02}_{-0.02}\) fm, \(a_1^{[1]} = -0.18^{+0.18}_{-0.11}\) fm

- Constraint \(k \cdot \cot \delta(k) = -\sqrt{-k^2}\) gives us a bound state pole with
  \[\Delta E_{D_6b}^{cont} = -81^{(+14)}_{(-16)}(14)\) MeV.

Using \(M_{\Omega_{bbb}}^{phys} = 14366(7)(9)\) MeV, we compute the mass of this bound state as
  \[M_{D_6b}^{phys} = 2M_{\Omega_{bbb}}^{phys} + \Delta E_{D_6b}^{cont} = 28651^{(+16)}_{(-17)}(15)\) MeV.
A heavy object such as $D_{6b}^{--}$ could be compact. Coulombic repulsion could be significant.

$V_s$: Using a multi-Gaussian attractive potential such that $\Delta E_{D_{6b}}^{cont} = -81(^{+14}_{-16})(14)$ MeV.

Assuming an electric charge distribution as determined in Can et al 2015, we determine $V_{em}$ for $D_{6b}^{--}$.

Compare the radial probability of the ground state wavefunction for $V_s$ and $V_s + V_{em}$.

Coulombic potential hardly influences where the probabilities peak. Maximum associated change in $\Delta E_{D_{6b}}^{cont}$ found to be 10 MeV.
Other systematic uncertainties

- Lattice QCD configurations: 2+1+1 HISQ, improved to $\mathcal{O}(\alpha_s a^2)$
  - $b$ quarks: NRQCD Hamiltonian with pert. imp. coefficients up to $\mathcal{O}(\alpha_s v^4)$
  - 1S bottomonium hyperfine splitting with an uncertainty < 6 MeV.

- Energy splittings are used as inputs to FV analysis.
  - Significantly reduced correlated uncertainties.

- Multiple fitting procedures to identify the correct plateau.
  - Statistical and fit-window uncertainties added in quadrature.

- Convolved through Lüscher’s analysis + continuum extrapolation: $(+14\,-16)$ MeV.

- Possible excited state effects using different smearing programs: < 8 MeV.

- Continuum extrapolation fit forms, scale setting, quark mass tuning and EM corrections: < 12 MeV.

\[ \Delta E^{cont}_{D_{6b}} = -81(+14\,-16)(14) \text{ MeV} \]
Summary

- $S$-wave $\Omega_{bbb}\Omega_{bbb}$ scattering: Strong evidence for a deeply bound state.

\[ \Delta E_{D_{\bar{6}b}}^{cont} = -81^{+14}_{-16}(14) \text{ MeV} \]

- A rigorous systematic investigation involving faithful identification of real plateau, finite-volume scattering analysis and a continuum extrapolation.

- Calculations on similar systems by Buchoff et al, & HALQCD collaboration in the light strange and charm sector: indicate only weak interactions in those.

- Our correlator data in the charm and strange sector is being produced. We plan to make a study of mass dependence from our results. Strange and charm involve relativistic quarks: More expensive calculations.

- A study involving symmetric correlation functions would still be beneficial to reaffirm our findings.
Thank you
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