S-wave scattering of triply bottom baryons from lattice QCD

M. Padmanath

IMSc Chennai

@JLab [online]
23^{rd} January 2023

with N. Mathur and D. Chakraborty

Based on article arXiv:2205.02862[hep-lat]

Baryon-baryon interactions

- One of the central topics in hadron and nuclear physics.

  c.f. Hashimoto&Tamura 2006, Clement 2016.

- Significant experimental effort in collecting NN scattering data.
  YN and YY interactions are poorly understood.
  Substantial theoretical efforts based on CQM and OBE models.

  Fujiwara&Suzuki 2008, Rijken&Yamamoto 2010

- Hyperon formation $\Leftarrow$ Large nuclear densities in astrophysical objects
  Neutron stars, supernovae, white dwarfs, etc.
  Implications on the EOS and on the properties of these astrophysical objects. Tolos&Fabbietti 2020

- A handful of experimental efforts using large nuclei reactions.
  Inputs on LECs to EFTs $\Rightarrow$ nuclear many body calculations.

  Epelbaum 2005, INT-NFPNP 2022, $0\nu\beta\beta$ PSWR 2022

- Despite several experimental efforts only one dibaryon bound state remains established: Deuteron.
  $d^*(2380)$, the only other dibaryon feature confirmed $\rightarrow$ isoscalar spin 3 resonance.

  WASA-at-COSY 2011, Clement 2016.

Baryon-baryon interactions from lattice QCD

- A handful of lattice QCD efforts on baryon-baryon scattering typically at $m_\pi > m_\pi^{phys}$.

  see works by NPLQCD, HALQCD*, Mainz, CalLat, and others in the past decade.

- Main focus on light and strange six quark systems: Deuteron, H-dibaryon, ...
  Most recent calculations suggest these systems may have a near-threshold feature.
  Discretization effects could be crucial. HALQCD '12, NPLQCD '17, Hörz *et al* '21, Green *et al* '21

- Two different procedures in extracting the scattering amplitudes.
  (a) *Solving QM potentials derived from NBS wave functions HALQCD
  (b) Lüscher's formalism involving finite volume spectrum extraction Briceño 2014

  In this work, we use procedure (b).

- Scattering of decuplet baryons: towards an understanding of $d^*(2380)$ feature.
  Predictions for 6 nonstrange dibaryon states based on flavor symmetry Dyson&Xuong 1964
  $\Rightarrow$ Expects an energy degenerate spin 0 resonance.

- Heavy cousins from the symmetric flavor multiplet: no near three or four particle thresholds.
  Simple model studies ($\Omega\Omega$ scattering): widely different inferences.

  Richard *et al* '20, Liu *et al* '21, Huang *et al* '20

Scattering of Ω-like baryons using lattice QCD

- No explicit light quark mass (m_l) dependence and no low lying three or four particle thresholds.
  Leading m_l dependence could arise from pair produced 2π vertices.
  Calculations at unphysically large m_l: Relatively cheap calculations with clean signals.

- Early calculations of S-wave $\Omega\Omega$ scattering using Lüscher's formalism.
  Weakly repulsive interaction observed in the total spin 0. Buchoff&Luu&Wasem 2012

- Investigations by HALQCD Collaboration
  - Moderate attraction in S-wave $\Omega\Omega$ scattering — Yamada *et al* 2015
  - Similar conclusions from near m_π^{phys} large volume study — Gongyo *et al* 2018
  - Similar observations for S-wave $\Omega_{ccc}\Omega_{ccc}$ scattering
    Study performed on ensemble used in 2018 study. — Lyu *et al* 2021

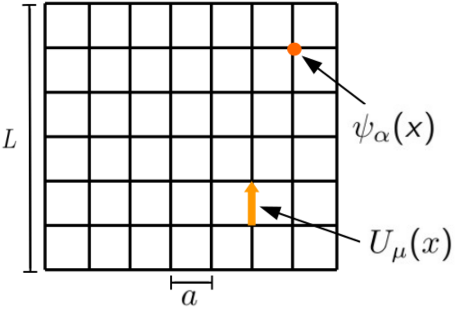
- Our plan is to investigate the m_q dependence in S-wave $\Omega_{qqq}\Omega_{qqq}$ scattering
  - This talk covers the results and our inferences in the bottom sector.
  - Data production and early spectrum analysis going on for quark mass range ($m_s \lesssim m_q < m_b$).

  Mathur&PM&Chakraborty, 2022

Lattice QCD: Basic idea

LQCD : A non-perturbative, gauge invariant regulator for the **QCD** path integrals.

- Quark fields $\psi_\alpha(x)$ on lattice sites
- Gauge fields as parallel transporters U_μ
  Lives in the links. $U_\mu(x) = e^{igaA_\mu(x)}$
- $\bar{\psi}^i_\alpha(x)[U_\mu(x)]_{ij}\psi^j_\alpha(x+a\hat{\mu})$ is gauge invariant.
- Lattice spacing : UV cut off
- Lattice size : IR cut off



Employ Monte Carlo importance sampling methods on Euclidean metric for numerical studies.

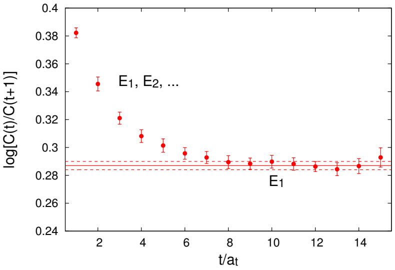
QCD spectrum from Lattice QCD

- Aim : to extract the physical states of QCD.

- Euclidean two point current-current correlation functions

$$C_{ji}(t_f - t_i) = \langle 0|\mathcal{O}_j(t_f)\bar{\mathcal{O}}_i(t_i)|0\rangle = \sum_n \frac{Z_i^{n*} Z_j^n}{2m_n} e^{-m_n(t_f - t_i)}$$

where $\mathcal{O}_j(t_f)$ and $\bar{\mathcal{O}}_i(t_i)$ are the desired interpolating operators and $Z_j^n = \langle 0|\mathcal{O}_j|n\rangle$.



- Effective mass defined as $\log[\frac{C(t)}{C(t+1)}]$

- The ground state : from the exponential fall off at large times.
  Non-linear fitting techniques.

Interpolating operators

- For $J^P = \frac{3}{2}^+$ Ω_{bbb}, we utilize a nonrelativistic operator

Basak *et al* 2005

$\mathcal{O}_{\Omega_{bbb}}$	$\lvert s_1, s_2, s_3 \rangle$	$\lvert J, J_z \rangle$
χ_1	$\lvert +\frac{1}{2}, +\frac{1}{2}, +\frac{1}{2} \rangle$	$\lvert 3/2, +3/2 \rangle$
χ_2	$\lvert +\frac{1}{2}, +\frac{1}{2}, -\frac{1}{2} \rangle_S$	$\lvert 3/2, +1/2 \rangle$
χ_3	$\lvert +\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \rangle_S$	$\lvert 3/2, -1/2 \rangle$
χ_4	$\lvert -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \rangle$	$\lvert 3/2, -3/2 \rangle$

s_i: quark spin

subscript S implies symmetric structure

J and J_z: total spin and its azimuthal component

- The $\Omega_{bbb}\Omega_{bbb}$ operator is then constructed as

$$\mathcal{O}_{\mathcal{D}_{6b}}(x,t) = \chi_m(x,t)\ [CG]^{mn}\ \chi_n(x,t),$$

which for the scalar quantum numbers looks like

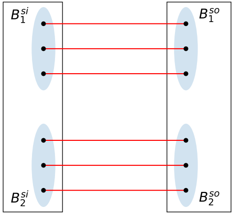
$$\mathcal{O}_{\mathcal{D}_{6b}^{J=0}} = \frac{1}{2}\left[\chi_1\chi_4 + \chi_3\chi_2 - \chi_2\chi_3 - \chi_4\chi_1\right]$$

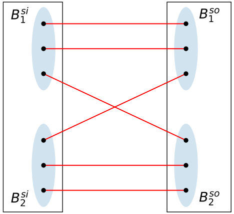
Correlation functions and Wick contractions

- Compute matrices of two point correlation functions

$$C_{ji}(t) = \langle 0|\bar{\mathcal{O}}_j(t)\mathcal{O}_i(0)|0\rangle = \sum_n \frac{Z_i^n Z_j^{n*}}{2E_n} e^{-E_n(t)}$$

- Fully bottom six quark systems near $\Omega_{bbb}\Omega_{bbb}$ threshold, we use 'only' $\mathcal{O}$ of BB type, where each B (baryon) is color singlet projected.
- Wick contractions to compute: With 6 identical quarks there are $6! = 720$ quark line contractions possible. The trivial symmetry in the flavor sector $\Rightarrow$ Only two contractions are unique.





Heavy quarks on the lattice

- Fermion action: $\bar{\psi}(\gamma.\mathbf{D} + \mathbf{m})\psi$

- Only dimensionless quantities defined on the lattice

  Lattice fermion action: $\bar{\psi}(\mathbf{x})(\gamma.\mathbf{D_L} + \mathbf{am})\psi(\mathbf{x})$

  Discretization (relativistic) $\rightarrow \mathbf{D_L}$; clover, staggered, overlap, domain-wall, etc.

  Discretization errors on observables $\mathcal{O}(\mathbf{am})$.

- Charm quarks: $m_c \sim 1.275$ GeV

  $am_c = 0.5 \quad \Rightarrow \quad a \sim 0.075$ fm $\qquad am_c = 0.3 \quad \Rightarrow \quad a \sim 0.046$ fm

- Bottom quarks: $m_b \sim 4.66$ GeV

  $am_b = 0.5 \quad \Rightarrow \quad a \sim 0.021$ fm $\qquad am_b = 0.3 \quad \Rightarrow \quad a \sim 0.013$ fm

Bottom quark action

- Bottom quarks realized using NRQCD formulation. Hamiltonian through $\mathcal{O}(\alpha\nu_b^4)$. Perturbatively tuned NRQCD co-efficients.

  HPQCD Collaboration, arXiv:1110.6887, Hammant *et al*, arXiv:1303.3234.

- Tuned using the spin averaged mass of 1S bottomonium

  $\Delta E_{hfs}^{1S,\bar{b}b} = 63(3)(3)$ MeV [Lat.cont.] $\Delta E_{hfs}^{1S,\bar{b}b} = 62(1)$ MeV [Expt.]

  El-Khadra *et al*, arXiv:9604004[hep-lat]

- We extract the energy splittings

$$\Delta E_{\Omega_{bbb}} = M^L_{\Omega_{bbb}} - 1.5 M_{\overline{1S}} \qquad \& \qquad \Delta E_{\mathcal{D}_{6b}} = M^L_{\mathcal{D}_{6b}} - 2M_{\Omega_{bbb}}$$

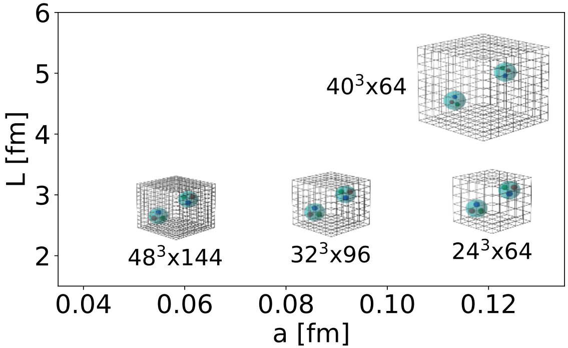
- Following a continuum extrapolation performed for $\Delta E_{\Omega_{bbb}}$

  $M^{phys}_{\Omega_{bbb}} = 14366(7)(9)$ MeV determined from $\Delta E^{cont}_{\Omega_{bbb}} + 1.5 M^{phys}_{\overline{1S}}$

Lattice QCD ensembles

- Ensembles : N_f =2+1+1 HISQ (MILC): $24^3.64$, $32^3.96$, $48^3.144$, & $40^3.64$
  a =0.1207(11), 0.0888(8), 0.0582(5), and 0.1189(7) fm respectively using r_1 parameter.

MILC Collaboration, arXiv:1212.4768



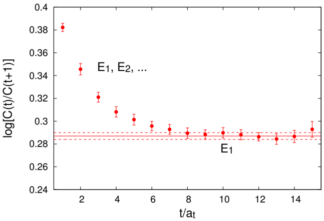
- Three 'different' lattice spacing and two 'different' volumes ($\sim$ 2.85fm and 4.76 fm). Possible continuum extrapolation and finite-volume scattering analysis.

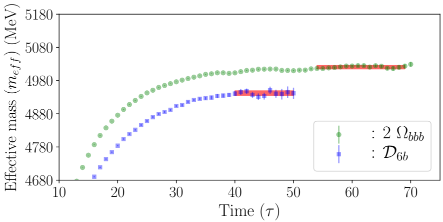
Signal quality and quark smearing

- Our prime focus on the finite volume ground state energy
- Limit ourselves to two point correlation functions

$$C_{\mathcal{O}}(t_f - t_i) = \sum_{\vec{x}_f} e^{-i\vec{p}.\vec{x}_f} \langle 0|\mathcal{O}(\vec{x}_f, t_f)\bar{\mathcal{O}}(t_i)|0\rangle$$

- Quark smearing: 'wall-smeared' quark source (in $\bar{\mathcal{O}}_i$) and point quark sink (in $\mathcal{O}_j$).
  Asymmetric correlation functions: Effective energies rising from below.
  Excited state contributions with different sign that for the ground state.





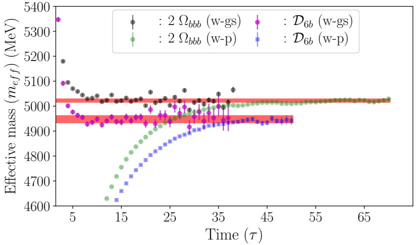
Wall smearing and other smearing procedures

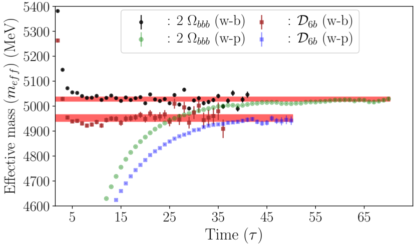
- Widely used in several calculations in determing ground state energies

  Mathur, MP, *et al* 2012-22, HALQCD

- Other symmetric and asymmetric smearing procedures to affirm our findings.

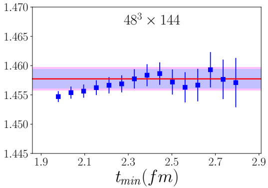
  wall, Gaussian, box [Hudspith 2020] sink smearing.

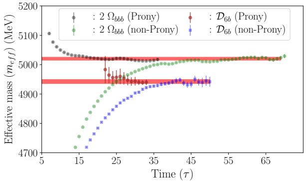




- Checking the t_{min} dependence of the fits

Multiple fitting procedures

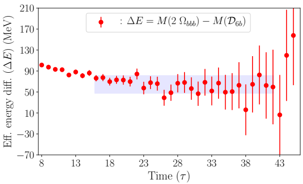


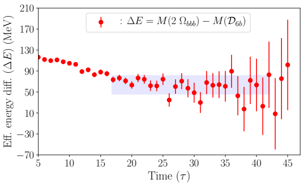


- Standard single exponential fitting against Matrix Prony algorithm

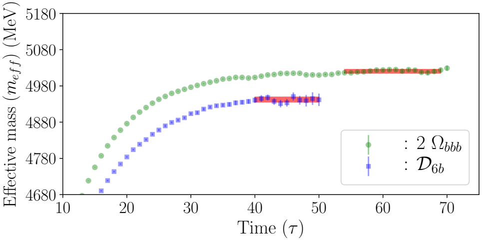
Kunis&Peter&Römer 2015, NPLQCD 2012, Buchoff&Luu&Wasem 2012

- Comparison with the results from fits to the ratio of correlators





Lattice results for energy splitting



Ensemble	$\Delta E_{\mathcal{D}_{6b}}$ [MeV]
$24^3 \times 64$	$-61(11)$
$32^3 \times 96$	$-68(9)$
$40^3 \times 64$	$-62(7)$
$48^3 \times 144$	$-71(7)$

- Table presents the energy splitting $\Delta E_{\mathcal{D}_{6b}} = M^L_{\mathcal{D}_{6b}} - 2M_{\Omega_{bbb}}$
- Clear energy gap between the noninteracting and interacting cases.
- Similar energy splittings across all four ensembles.
- Negative energy splittings indicating attractive interaction.

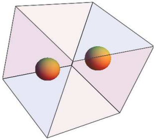
Finite volume spectrum and infinite volume physics

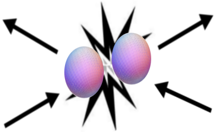
- On a finite volume Euclidean lattice : Discrete energy spectrum
  Cannot constrain infinite volume scattering amplitude away from threshold. Maiani-Testa 1990

- Non-interacting two-hadron levels are given by
  $$E(L) = \sqrt{m_1^2 + \vec{k}_1^2} + \sqrt{m_2^2 + \vec{k}_2^2} \text{ where } \vec{k}_{1,2} = \tfrac{2\pi}{L}(n_x, n_y, n_z).$$

- Switching on the interaction: $\vec{k}_{1,2} \neq \frac{2\pi}{L}(n_x, n_y, n_z)$. *e.g.* in 1D $\vec{k}_{1,2} = \frac{2\pi}{L}n + \frac{2}{L}\delta(k)$.

- Lüscher's formula relates finite volume level shifts $\Leftrightarrow$ infinite volume phase shifts. Lüscher 1991





- Generalizations of Lüscher's formalism: *c.f.* Briceño 2014

Scattering amplitude parametrization

- Scattering amplitude: $S = 1 + i\frac{4k}{E_{cm}t}$

- For the $\Omega_{bbb}\Omega_{bbb}$ system [total spin equals 0], and assuming only S-wave,

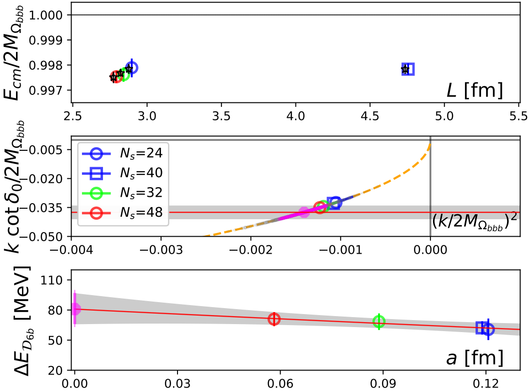
$$t^{-1} = \frac{2\tilde{K}^{-1}}{E_{cm}} - i\frac{2k}{E_{cm}}, \quad \text{with} \quad \tilde{K}^{-1} = k.cot\delta(k)$$

Bound state constraint: $k.cot\delta(k) = -\sqrt{-k^2}$

- Lüscher's prescription: $k.cot\delta(k) = \mathcal{F}(k)$, where $\mathcal{F}(k^2)$ is a known mathematical function.
  k^2 is determined from each extracted energy splitting as $k^2 = \frac{\Delta E^L_{D_{6b}}}{4}(\Delta E^L_{D_{6b}} + 4M^{phys}_{\Omega_{bbb}})$

- We parametrize $k.cot\delta(k)$ with a constant (inverse scattering length "$-1/a_0$").
  The remnant lattice spacing "a" dependence: "$-1/a_0^{[0]} - a/a_0^{[1]}$".
  Fits performed with and without "a" dependence.

Amplitude analysis and binding energy estimate



- Fits with "$-1/a_0^{[0]} - a/a_0^{[1]}$" is found to be the best with $\chi^2/d.o.f. = 0.7/2$

$$a_0^{[0]} = 0.18(^{+0.02}_{-0.02})\ \text{fm}, \quad a_0^{[1]} = -0.18(^{+0.18}_{-0.11})\ \text{fm}^2$$

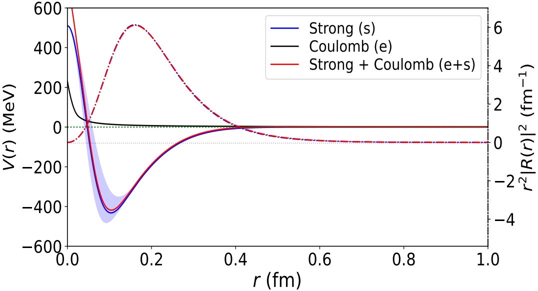
- Constraint $k.cot\delta(k) = -\sqrt{-k^2}$ gives us a bound state pole with

$$\Delta E^{cont}_{\mathcal{D}_{6b}} = -81(^{+14}_{-16})(14)\ \text{MeV}.$$

Using $M^{phys}_{\Omega_{bbb}} = 14366(7)(9)$ MeV, we compute the mass of this bound state as

$$M^{phys}_{\mathcal{D}_{6b}} = 2M^{phys}_{\Omega_{bbb}} + \Delta E^{cont}_{\mathcal{D}_{6b}} = 28651(^{+16}_{-17})(15)\ \text{MeV}$$

Possible Coulombic repulsion



- A heavy object such as $\mathcal{D}_{6b}^{--}$ could be compact. Coulombic repulsion could be significant.
- V_s: Using a multi-Gaussian attractive potential such that $\Delta E^{cont}_{\mathcal{D}_{6b}} = -81(^{+14}_{-16})(14)$ MeV.
  HALQCD 2021
- Assuming an electric charge distribution as determined in Can *et al* 2015, we determine V_{em} for $\mathcal{D}_{6b}^{--}$.
- Compare the radial probability of the ground state wavefunction for V_s and $V_s + V_{em}$.
- Coulombic potential hardly influences where the probabilities peak. Maximum associated change in $\Delta E^{cont}_{\mathcal{D}_{6b}}$ found to be 10 MeV.

Other systematic uncertainties

- Lattice QCD configurations: 2+1+1 HISQ, improved to $\mathcal{O}(\alpha_s a^2)$
  b quarks: NRQCD Hamiltonian with pert. imp. coefficients up to $\mathcal{O}(\alpha_s v^4)$
  1S bottomonium hyperfine splitting with an uncertainty < 6 MeV.
- Energy splittings are used as inputs to FV analysis.
  Significantly reduced correlated uncertainties.
- Multiple fitting procedures to identify the correct plateau.
  Statistical and fit-window uncertainties added in quadrature.
- Convolved through Lüscher's analysis + continuum extrapolation: $\left(^{+14}_{-16}\right)$ MeV.
- Possible excited state effects using different smearing programs: < 8 MeV.
- Continuum extrapolation fit forms, scale setting, quark mass tuning and
  EM corrections: < 12 MeV.

$$\Delta E^{cont}_{\mathcal{D}_{6b}} = -81\left(^{+14}_{-16}\right)(14) \text{ MeV}$$

Summary

- S-wave $\Omega_{bbb}\Omega_{bbb}$ scattering: Strong evidence for a deeply bound state.

$$\Delta E^{cont}_{\mathcal{D}_{6b}} = -81(^{+14}_{-16})(14)\text{ MeV}$$

- A rigorous systematic investigation involving faithful identification of real plateau, finite-volume scattering analysis and a continuum extrapolation.

- Calculations on similar systems by *Buchoff et al*, & *HALQCD* collaboration in the light strange and charm sector: indicate only weak interactions in those.

- Our correlator data in the charm and strange sector is being produced. We plan to make a study of mass dependence from our results. Strange and charm involve relativistic quarks: More expensive calculations.

- A study involving symmetric correlation functions would still be beneficial to reaffirm our findings.

Thank you

$\overline{1S}$ $\bar{b}b$ hyperfine splitting