Haag’s theorem and the status of particles in QFTs with interactions

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Philosophy of Physics: Some of my interests

▶ epistemology: theories about knowledge
How to formulate QFT mathematically is a central foundational question. What role does mathematics play in physical theories? What is the instrumental value of mathematical rigor?

What methods have been used to develop physical theories? (e.g., analogical reasoning) Virtues and pitfalls?
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  Assuming that our physical theories aim to represent the world, what sort of world do they depict?
  e.g., Does ‘particle physics’ describe particles?
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Informed by history of physics: What can we learn from past successes and failures?
Overview

1. Haag’s theorem

2. The status of particles in QFTs with interactions
Haag’s theorem (1955)

Haag-Hall-Wightman theorem (1957)

Haag (1955):
plausibility argument that a representation of the CCRs for a free field cannot be unitarily related at any time to a representation of the CCRs for an interacting field

Hall and Wightman (1957):
inspired by Haag, present a proof that if a representation of the CCRs for one field is unitarily related at any time to a representation of the CCRs for another field, then the first four VEVs (four-point Wightman functions) are equal* (i.e., presumably the two field theories are identical)

* for a free and an interacting field, the Jost-Schroer theorem establishes that all VEVs (n-point Wightman functions) are equal (or Greenberg (1959) offers a proof specific to Hall-Wightman theorem)
Haag-Hall-Wightman theorem (1957)

PART I

(Pl.1) **Irreducible reps of ETCCRs:** Each pair of fields \((\phi_j, \pi_j)\) gives an irreducible representation of the ETCCRs

(Pl.2) **Euclidean covariance of fields:** Fields transform appropriately under Euclidean transformations induced by unitary transformations \(U_j(a, R)\)

(Pl.3) **Existence of Euclidean invariant states:** There exist unique normalizable states (vacua) \(|0_j\rangle\) invariant under \(U_j(a, R)\)

(Pl.4) **Fields unitarily related by \(V\) at some \(t\):**
\[
\phi_2(x, t) = V\phi_1(x, t)V^{-1}; \quad \pi_2(x, t) = V\pi_1(x, t)V^{-1}
\]

Then

(Cl.1) \(U_2(a, R) = VU_1(a, R)V^{-1}\)

(Cl.2) \(c|0_2\rangle = V|0_1\rangle\) where \(|c| = 1\)
PART II

(PII.1) Assumptions of PART I.

(PII.2) **Poincaré covariance of fields**: Fields transform appropriately under Poincaré transformations induced by unitary transformations $T_j(a, \Lambda)$

(PII.3) States $|0_j\rangle$ are Poincaré invariant

(PII.4) No states of negative energy exist

Then

(CII) The first four VEVs of the two fields are equal (i.e., four-point Wightman functions):

$$\langle 0_2 | \phi_2(x_1), \ldots \phi_2(x_n) | 0_2 \rangle = \langle 0_1 | \phi_1(x_1), \ldots \phi_1(x_n) | 0_1 \rangle$$

where $n = 1, 2, 3$ or $4$
How to respond to the Hall-Wightman-Haag theorem?

- not possible to accept the conclusion
How to respond to the Hall-Wightman-Haag theorem?

- not possible to accept the conclusion
- renormalize
  i.e., stick to the physical principles set out in the assumptions of the theorem, but find a method of making the infinite predictions entailed by these assumptions finite
- axiomatize
  i.e., revise the physical principles set out in the assumptions of the theorem

Reject the assumption that

(v) The fields are related at some time $t$ by a unitary transformation $V$:

$$\phi_2(x, t) = V\phi_1(x, t) V^{-1}, \quad \pi_2(x, t) = V\pi_1(x, t) V^{-1}$$
True, but not very illuminating:

- “the interaction picture only exists if there are no interactions”
- “unitarily inequivalent representations are physically relevant for QFT”

Why? what is the diagnosis of the problem raised by Haag’s theorem specifically? (there are lots of obstacles to representing interactions in QFT!)
Sloganeering

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Better slogans:

- “vacuum polarization necessarily occurs in relativistic QFTs”
  \[(H_\text{I} | 0_F \rangle \neq 0)\]
- “the representation determines the dynamics”
What is the diagnosis of the problem raised by Haag’s theorem?

- relativistic assumptions required
- infinite number of degrees of freedom required
- pertains to infrared divergences (i.e., long distances)
The underlying factor responsible for Haag’s theorem in Poincaré covariant field theories

**Difference in structure of Lie algebras**

**Galilean:**
When $H_I$ commutes with the other generators, taking $H_F \Rightarrow H_F + H_I$ does not affect structure of Lie algebra
e.g.,

\[
[K, H_F] = [K, H_F + H_I] = iP
\]
and $H_F |0\rangle_F = (H_F + H_I) |0\rangle_F = 0$

**Poincaré:**
Even when $H_I$ commutes with the other generators, taking $H_F \Rightarrow H_F + H_I$ affects structure of the Lie algebra
e.g.,

\[
[P, K] = -iH_F = -iH_F - iH_I
\]
The Fock representation for a free system (of mass $m$) in Minkowski space supports a *quanta* interpretation

- Eigenvectors of total number op $N$ have appropriate relativistic energies for $n$ particle states
- $|0\rangle$ invariant under unitary representation of Poincaré group ("‘looks the same to all observers’’ [in inertial motion]")
Further questions (to be bracketed):

- To what extent do quanta possess particulate properties? (e.g., localizable, bear labels)
- What happens in other circumstances? (e.g., accelerating observers, non-stationary spacetimes)

Review article: DF, “Particles in QFT”
https://philsci-archive.pitt.edu/20083/
A quanta interpretation is not available for interacting systems

The Fock representation for a free system cannot be used to represent an interacting system. (Haag’s theorem)

Mathematical representations for interacting systems cannot be interpreted as directly describing quanta.

**Conclusion:** QFTs with interactions do not represent quanta (i.e., particles are not fundamental entities according to the theory)

Conclusions

1. Haag’s theorem: “vacuum polarization necessarily occurs in relativistic QFTs” \( (H_I |0_F \rangle \neq 0) \)
   ▶ relativistic assumptions required: in particular, different roles for \( H \) in the Galilean group vs. Poincaré group
   ▶ infinite number of degrees of freedom required
   ▶ pertains to infrared divergences (i.e., long distances)

2. An implication of Haag’s theorem is that QFTs with interactions do not represent quanta (i.e., particles are not fundamental entities according to the theory)