# Haag's theorem and the status of particles in QFTs with interactions

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# Philosophy of Physics: Some of my interests

- **epistemology**: theories about knowledge
  How to formulate QFT mathematically is a central foundational question. What role does *mathematics* play in physical theories? What is the instrumental value of mathematical rigor?

  What *methods* have been used to develop physical theories? (e.g., analogical reasoning) Virtues and pitfalls?

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Informed by history of physics: What can we learn from past successes and failures?

# Overview

1. Haag's theorem
2. The status of particles in QFTs with interactions

# Haag's theorem (1955)

(1) *CCR's.*

$$[\phi_j(\mathbf{x},t),\pi_j(\mathbf{x}',t)] = i\delta(\mathbf{x}-\mathbf{x}') \qquad j=1,2$$
$$[\phi_j(\mathbf{x},t),\phi_j(\mathbf{x}',t)] = [\pi_j(\mathbf{x},t),\pi_j(\mathbf{x}',t)] = 0.$$

(2) *Euclidean transformations.* Euclidean transformations $(\mathbf{a},\mathbf{R})$ are induced by unitary transformations $U_j(\mathbf{a},\mathbf{R})$:

$$U_j(\mathbf{a},\mathbf{R})\phi_j(\mathbf{x},t)U_j^{-1}(\mathbf{a},\mathbf{R}) = \phi_j(\mathbf{R}\mathbf{x}+\mathbf{a},t)$$
$$U_j(\mathbf{a},\mathbf{R})\pi_j(\mathbf{x},t)U_j^{-1}(\mathbf{a},\mathbf{R}) = \pi_j(\mathbf{R}\mathbf{x}+\mathbf{a},t)$$

(3) *Vacua.* There exist unique normalizable states $|0_j\rangle$ invariant under Euclidean transformations

$$U_j(\mathbf{a},\mathbf{R})|0_j\rangle = |0_j\rangle$$

(4) *V(t).* At some time $t$ the fields are related by a unitary transformation $V(t)$:

$$\phi_2(\mathbf{x},t) = V(t)\phi_1(\mathbf{x},t)V^{-1}(t)$$
$$\pi_2(\mathbf{x},t) = V(t)\pi_1(\mathbf{x},t)V^{-1}(t)$$

(5) *Poincaré transformations.* Poincaré transformations $(a,\Lambda)$ of the fields are induced by unitary transformations $T_j(a,\Lambda)$:

$$T_j(a,\Lambda)\phi_j(x) = \phi_j(\Lambda x+a), \qquad j=1,2$$

(6) *Poincaré invariance of vacua.* The states $|0_j\rangle$ are Poincaré invariant

$$T_j(a,\Lambda)|0_j\rangle = |0_j\rangle$$

(7) *No states of negative energy exist.*

{T} = theoretical postulates of (unrenormalized) interaction picture representation

**Haag's Theorem:**

{T} ➡ No interaction

Earman and Fraser (2006), "Haag's Theorem and its Implications for the Foundations of Quantum Field Theory," *Erkenntnis* 64(3): 305-344.

# Haag-Hall-Wightman theorem (1957)

**Haag (1955)**:

plausibility argument that a representation of the CCRs for a free field cannot be unitarily related at any time to a representation of the CCRs for an interacting field

**Hall and Wightman (1957)**:

inspired by Haag, present a proof that if a representation of the CCRs for one field is unitarily related at any time to a representation of the CCRs for another field, then the first four VEVs (four-point Wightman functions) are equal* (i.e., presumably the two field theories are identical)

* for a free and an interacting field, the Jost-Schroer theorem establishes that all VEVs (n-point Wightman functions) are equal (or Greenberg (1959) offers a proof specific to Hall-Wightman theorem)

# Haag-Hall-Wightman theorem (1957)

PART I

(PI.1) **Irreducible reps of ETCCRs**: Each pair of fields $(\phi_j, \pi_j)$ gives an irreducible representation of the ETCCRs

(PI.2) **Euclidean covariance of fields**: Fields transform appropriately under Euclidean transformations induced by unitary transformations $U_j(\mathbf{a}, \mathbf{R})$

(PI.3) **Existence of Euclidean invariant states**: There exist unique normalizable states (vacua) $|0_j\rangle$ invariant under $U_j(\mathbf{a}, \mathbf{R})$

(PI.4) **Fields unitarily related by $V$ at some $t$**:
$\phi_2(\mathbf{x}, t) = V\phi_1(\mathbf{x}, t)V^{-1}$; $\pi_2(\mathbf{x}, t) = V\pi_1(\mathbf{x}, t)V^{-1}$

Then

(CI.1) $U_2(\mathbf{a}, \mathbf{R}) = VU_1(\mathbf{a}, \mathbf{R})V^{-1}$

(CI.2) $c|0_2\rangle = V|0_1\rangle$ where $|c| = 1$

# Haag-Hall-Wightman theorem (1957)

PART II

(PII.1) Assumptions of PART I.

(PII.2) **Poincaré covariance of fields**: Fields transform appropriately under Poincaré transformations induced by unitary transformations $T_j(a, \Lambda)$

(PII.3) **States $|\mathbf{0_j}\rangle$ are Poincaré invariant**

(PII.4) **No states of negative energy exist**

Then

(CII) The first four VEVs of the two fields are equal (i.e., four-point Wightman functions):

$$\langle 0_2|\phi_2(x_1), \ldots \phi_2(x_n)|0_2\rangle = \langle 0_1|\phi_1(x_1), \ldots \phi_1(x_n)|0_1\rangle$$
$$\text{where } n = 1,\ 2,\ 3 \text{ or } 4$$

# How to respond to the Hall-Wightman-Haag theorem?

- not possible to accept the conclusion

# How to respond to the Hall-Wightman-Haag theorem?

- not possible to accept the conclusion
- renormalize
  i.e., stick to the physical principles set out in the assumptions of the theorem, but find a method of making the infinite predictions entailed by these assumptions finite
- axiomatize
  i.e., revise the physical principles set out in the assumptions of the theorem
  reject the assumption that
  (v) The fields are related at some time $t$ by a unitary transformation $V$:

$$\phi_2(\mathbf{x}, t) = V\phi_1(\mathbf{x}, t)V^{-1}, \qquad \pi_2(\mathbf{x}, t) = V\pi_1(\mathbf{x}, t)V^{-1}$$

# Sloganeering

True, but not very illuminating:

- "the interaction picture only exists if there are no interactions"
- "unitarily inequivalent representations are physically relevant for QFT"

Why? what is the diagnosis of the problem raised by Haag's theorem specifically? (there are lots of obstacles to representing interactions in QFT!)

# Sloganeering

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Better slogans:

- "vacuum polarization *necessarily* occurs in relativistic QFTs" ($H_I|0_F\rangle \neq 0$)
- "the representation determines the dynamics"

# What is the diagnosis of the problem raised by Haag's theorem?

- relativistic assumptions required
- infinite number of degrees of freedom required
- pertains to infrared divergences (i.e., long distances)

# The underlying factor responsible for Haag's theorem in Poincaré covariant field theories

## Difference in structure of Lie algebras

**Galilean**:
When $H_I$ commutes with the other generators, taking $H_F \Rightarrow H_F + H_I$ does not affect structure of Lie algebra
e.g.,

$$[K, H_F] = [K, H_F + H_I] = iP \tag{1}$$

and $H_F|0\rangle_F = (H_F + H_I)|0\rangle_F = 0$

**Poincaré**:
Even when $H_I$ commutes with the other generators, taking $H_F \Rightarrow H_F + H_I$ affects structure of the Lie algebra
e.g.,

$$[P, K] = -iH_F = -iH_F - iH_I \tag{2}$$

# Quanta interpretation of QFT

The Fock representation for a free system (of mass $m$) in Minkowski space supports a *quanta* interpretation

- Eigenvectors of total number op $N$ have appropriate relativistic energies for $n$ particle states
- $|0\rangle$ invariant under unitary representation of Poincaré group ("‘looks the same to all observers" [in inertial motion])

# Quanta interpretation of QFT

Further questions (to be bracketed):

- To what extent do quanta possess particulate properties? (e.g., localizable, bear labels)
- What happens in other circumstances? (e.g., accelerating observers, non-stationary spacetimes)

Review article: DF, “Particles in QFT”
https://philsci-archive.pitt.edu/20083/

# A quanta interpretation is not available for interacting systems

The Fock representation for a free system cannot be used to represent an interacting system. (Haag's theorem)

Mathematical representations for interacting systems cannot be interpreted as directly describing quanta.

**Conclusion:** QFTs with interactions do not represent quanta (i.e., particles are not fundamental entities according to the theory)

DF, "The fate of 'particles' in quantum field theories with interactions," *Studies in the History and Philosophy of Modern Physics* 39 (2008): 841-859.

# Conclusions

1. Haag's theorem: "vacuum polarization *necessarily* occurs in relativistic QFTs" ($H_I|0_F\rangle \neq 0$)
   - relativistic assumptions required: in particular, different roles for $H$ in the Galilean group vs. Poincaré group
   - infinite number of degrees of freedom required
   - pertains to infrared divergences (i.e., long distances)

2. An implication of Haag's theorem is that QFTs with interactions do not represent quanta (i.e., particles are not fundamental entities according to the theory)