Light-front tomography of light and heavy mesons

Theory Center Seminar
Jefferson Lab, February 26, 2024

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1933

"WE COLLABORATE. I'M AN EXPERT, BUT NOT AN AUTHORITY, AND DR. GELPIS IS AN AUTHORITY, BUT NOT AN EXPERT."

Collaborators:

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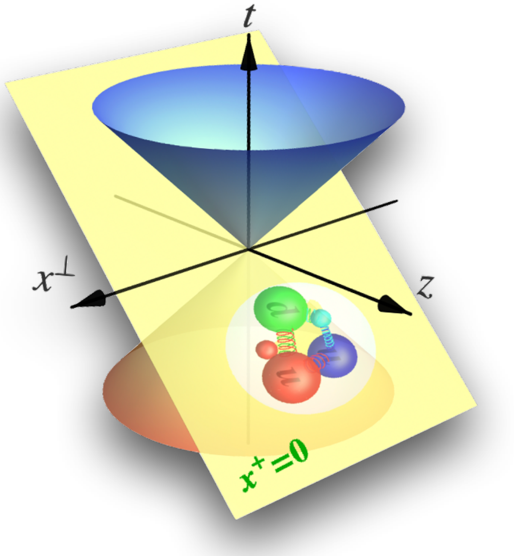
Before coming to the meat of the talk, namely Light Front Wave Functions, let's motivate them with LCDAs …



Light-Cone Distribution Amplitudes

- Hadron light-cone distribution amplitudes (LCDAs) have been introduced four decades ago in the context of the QCD description of hard exclusive reactions.

- The LCDAs are scale-dependent nonperturbative functions that can be interpreted as quantum-mechanical amplitudes.

- $\phi_M(x,\mu)$ is a probability amplitude that describes the momentum distribution of a quark and anti-quark in the bound-state's valence Fock state.
- x is the light-front momentum fraction: $\frac{k^+}{P^+}$ and μ is the renormalization scale.

Light-Front Distribution Amplitudes

QCD factorization in B decays involves matrix elements which are convolution integrals:

$$\langle \pi^+\pi^- | (\bar{u}b)_{\mathrm{V-A}} (\bar{d}u)_{\mathrm{V-A}} | \bar{B}_d \rangle \to \int_0^1 d\xi du dv \, \Phi_B(\xi)\, \Phi_\pi(u)\, \Phi_\pi(v)\, T(\xi, u, v; m_b)$$

The integrals are over a (hard) scattering kernel $\boldsymbol{T}$ (ξ,u,v,m) and light-cone distribution amplitudes (LCDA) expanded in Gegenbauer polynomials:

$$\varphi_\pi(x;\tau) = \varphi_\pi^{\mathrm{asy}}(x) \left[1 + \sum_{j=2,4,\ldots}^{\infty} a_j^{3/2}(\tau)\, C_j^{(3/2)}(2x-1) \right]$$

$$\varphi_\pi^{\mathrm{asy}}(x) = 6x(1-x)$$

- LCDA were poorly known for light mesons, in recent years improved determinations of the first two Gegenbauer moments of the pion and kaon, RQCD Collaboration, Bali et al. (2019).
- Next to nothing was known about heavy-light mesons, mostly models and asymptotic LCDA used.

Light-Front Distribution Amplitudes

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$$\boxed{\phi_M(x) \neq \phi^{\mathrm{asy}}(x) = 6x(1-x)}$$

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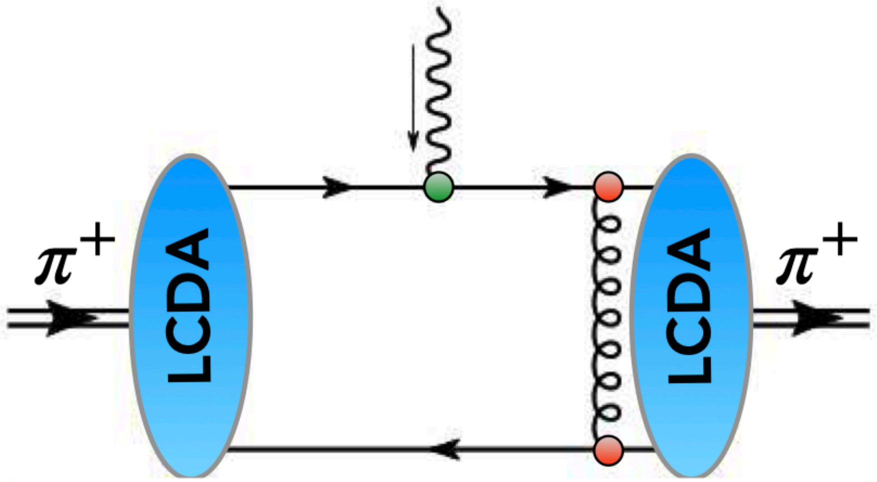
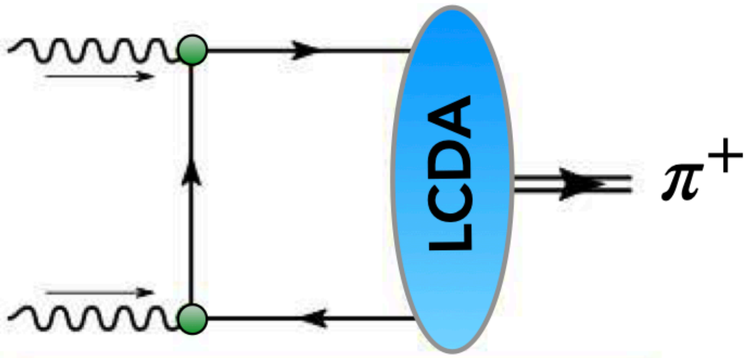
- Next to nothing was known about heavy-light mesons, mostly models and asymptotic LCDA used.

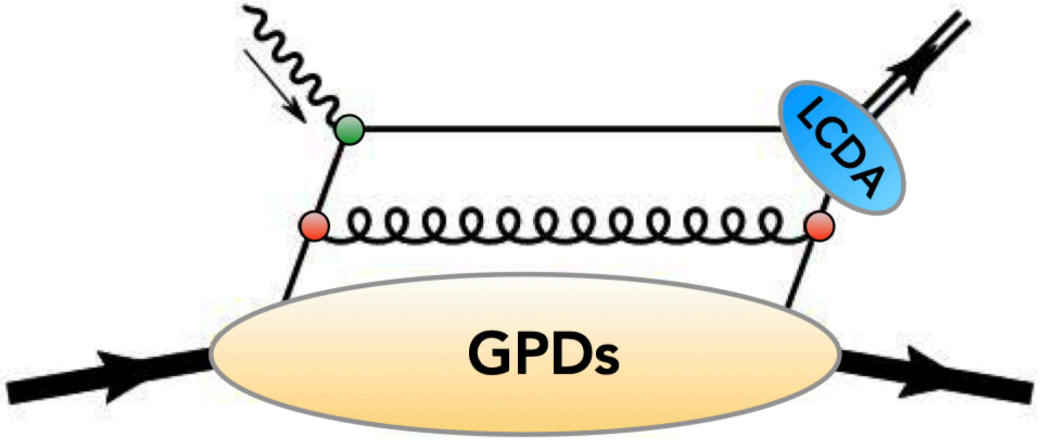
Hard exclusive scattering processes

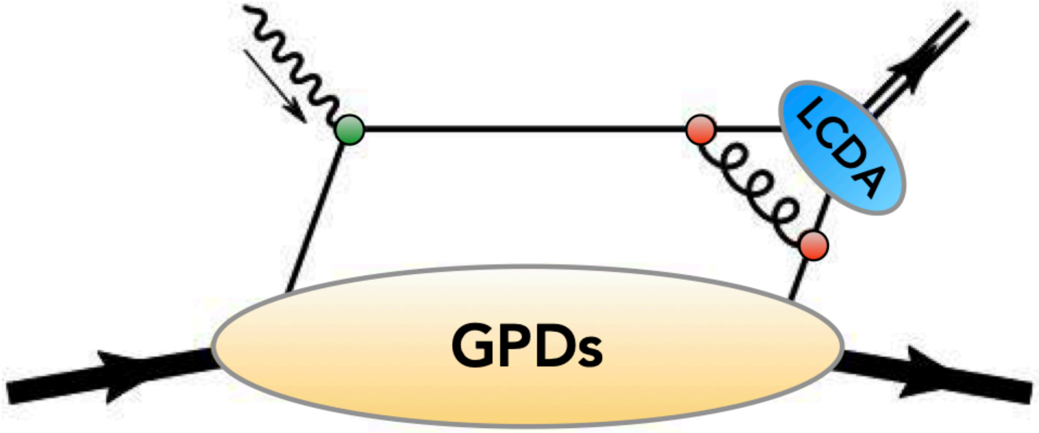


$$Q^2F_\pi(Q^2) \to 16\pi f_\pi^2\,\alpha_s(Q^2)$$



$$Q^2F_{\gamma^*\gamma\pi}(Q^2) \to 2f_\pi$$





Charged current production of charmed mesons

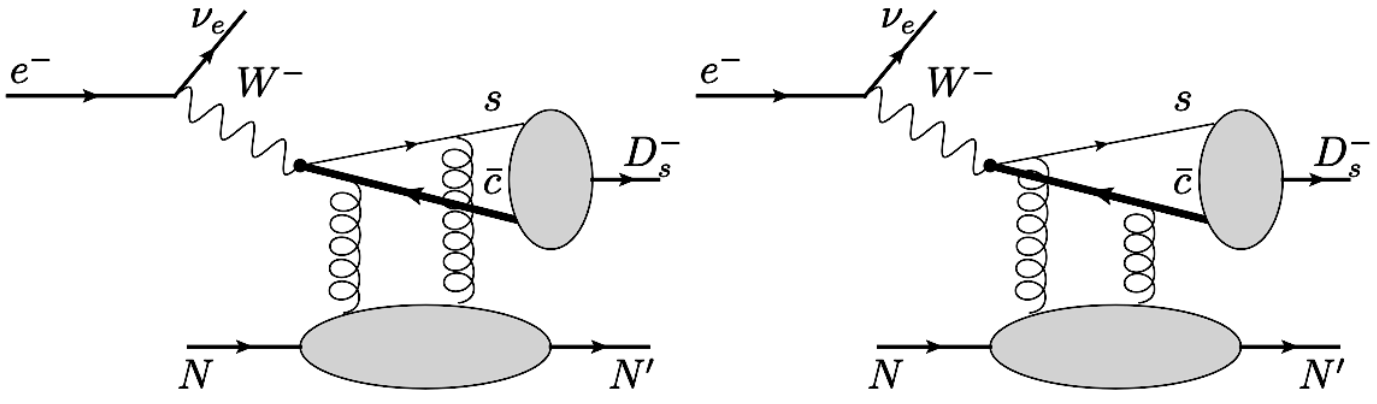


FIG. 1: Feynman diagrams for the factorized amplitude for the $e^- + N \to \nu_e + D_s^- + N'$ process involving the gluon GPDs; the thick line represents the heavy anti-quark $\bar{c}$.

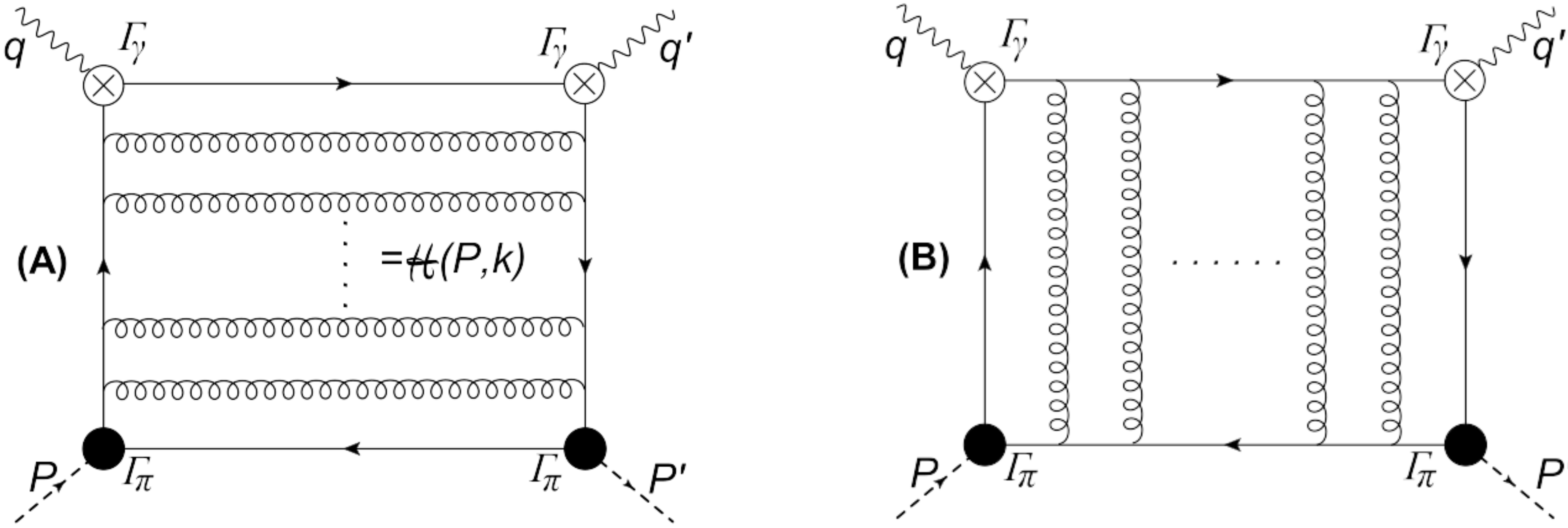
B. Pire, L. Szymanowski, and J. Wagner, Phys. Rev. D 104 (2021)

- Initial work by Siddikov and Schmidt for π and ρ production, Phys. Rev. D 99 (2019).
- Claim that their numerical estimates show that production could be studied electron-induced processes at EIC.
- Recently extended do D_s and D_s^* mesons in collinear QCD where generalized gluon distributions factorize from perturbatively calculable coefficient functions.
- Required input: pseudoscalar and vector meson *distribution amplitudes*.

Parton Distribution Functions

- Contrary to Light-Cone Distribution Functions, these are measurable objects.
- Model approaches to calculation commonly based on “handbag” diagrams for photon-pion forward Compton scattering in Bjorken limit.

$$q_f(x) = \frac{1}{4\pi}\int d\lambda\, e^{-ixP\cdot n\lambda}\langle \pi(P)|\bar{\psi}_f(\lambda n)\, \not{n}\, \psi_f(0)|\pi(P)\rangle$$

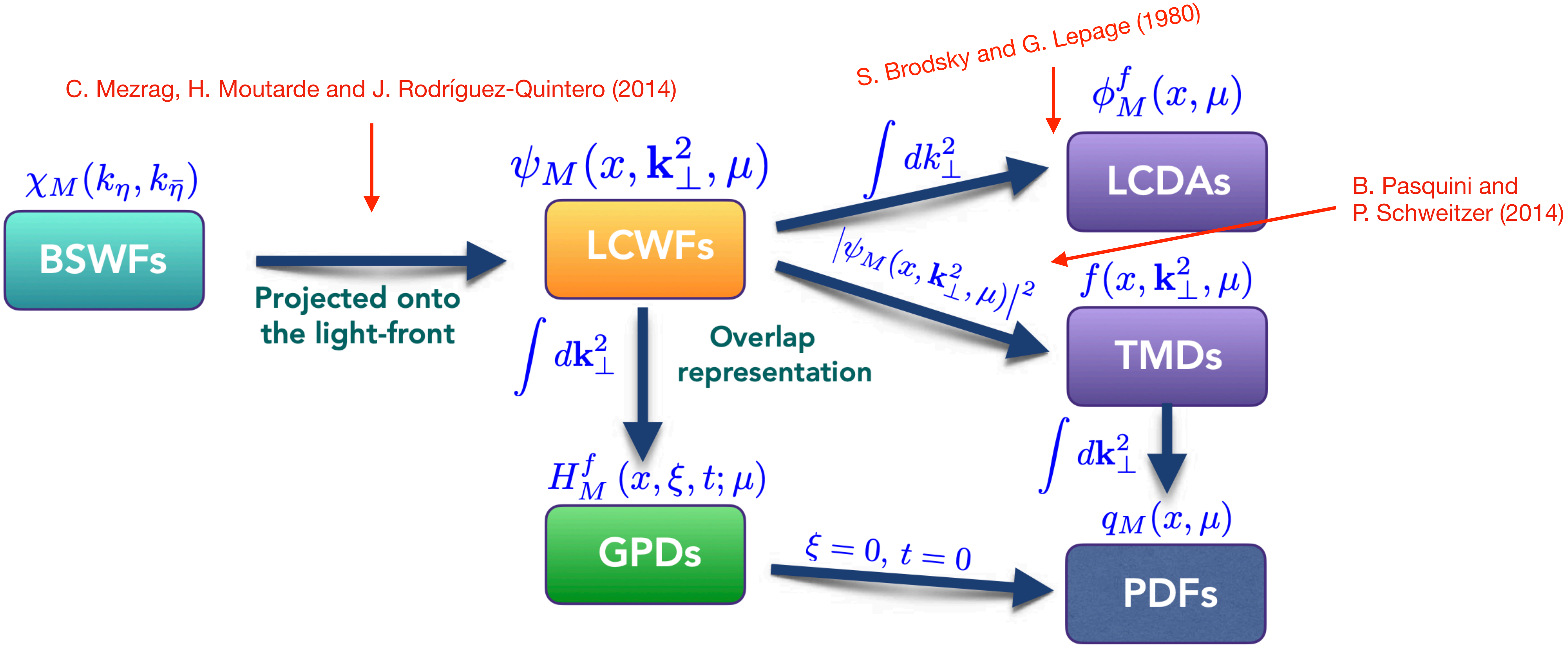


L. Chang, C. Mezrag, H. Moutarde, C. D. Roberts & J. Rodríguez-Quintero, PLB (2014)

What if we could obtain all kind of distribution functions from one unified calculation ?

Light-Front Wave Functions

With a particular projection of the Bethe-Salpeter wave functions we arrive the light-front wave functions, a more general object to describe probability amplitudes.



C. Mezrag, H. Moutarde and J. Rodríguez-Quintero (2014)

S. Brodsky and G. Lepage (1980)

B. Pasquini and
P. Schweitzer (2014)

$\chi_M(k_\eta, k_{\bar\eta})$

BSWFs

Projected onto the light-front

$\psi_M(x, \mathbf{k}_\perp^2, \mu)$

LCWFs

$\int dk_\perp^2$

$\phi_M^f(x, \mu)$

LCDAs

$|\psi_M(x, \mathbf{k}_\perp^2, \mu)|^2$

$f(x, \mathbf{k}_\perp^2, \mu)$

TMDs

$\int d\mathbf{k}_\perp^2$

Overlap representation

$H_M^f(x, \xi, t; \mu)$

GPDs

$\xi = 0,\ t = 0$

$\int d\mathbf{k}_\perp^2$

$q_M(x, \mu)$

PDFs

Light-Front Wave Functions

- M. Burkardt, X.-D. Ji, and F. Yuan (2002) showed that for pseudoscalar mesons there are two independent light front wave functions for the leading Fock state, with $l_z = 0$ and $l_z = 1$.
- Two-particle Fock-state configuration is given by: $|M\rangle = |M\rangle_{l_z=0} + |M\rangle_{|l_z|=1}$

$$|M\rangle_{l_z=0} = i \int \frac{d^2 \boldsymbol{k}_T}{2(2\pi)^3} \frac{dx}{\sqrt{x\bar{x}}} \psi_0\left(x, \boldsymbol{k}_T^2\right) \frac{\delta_{ij}}{\sqrt{3}} \frac{1}{\sqrt{2}}$$
$$\left[b^\dagger_{f\uparrow i}\left(x, \boldsymbol{k}_T\right) d^\dagger_{h\downarrow j}\left(\bar{x}, \bar{\boldsymbol{k}}_T\right) - b^\dagger_{f\downarrow i}\left(x, \boldsymbol{k}_T\right) d^\dagger_{h\uparrow j}\left(\bar{x}, \bar{\boldsymbol{k}}_T\right)\right] |0\rangle$$

$$|M\rangle_{|l_z|=1} = i \int \frac{d^2 \boldsymbol{k}_T}{2(2\pi)^3} \frac{dx}{\sqrt{x\bar{x}}} \psi_1\left(x, \boldsymbol{k}_T^2\right) \frac{\delta_{ij}}{\sqrt{3}} \frac{1}{\sqrt{2}}$$
$$\left[k_T^- b^\dagger_{f\uparrow i}\left(x, \boldsymbol{k}_T\right) d^\dagger_{h\uparrow j}\left(\bar{x}, \bar{\boldsymbol{k}}_T\right) + k_T^+ b^\dagger_{f\downarrow i}\left(x, \boldsymbol{k}_T\right) d^\dagger_{h\downarrow j}\left(\bar{x}, \bar{\boldsymbol{k}}_T\right)\right] |0\rangle$$

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The LFWFs are obtained from the Bethe-Salpeter wave function via the light front projections:

$$\psi_0\left(x, \mathbf{k}_\perp^2\right) = \sqrt{3}i \int \frac{dk^+ dk^-}{\pi} \delta\left(xP^+ - k^+\right) \mathrm{Tr}_D\left[\gamma^+ \gamma_5 \chi(k, P)\right]$$

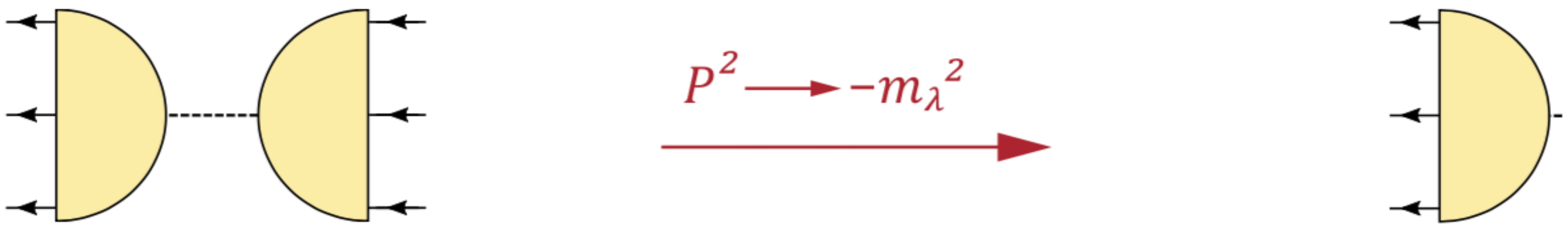
$$\psi_1\left(x, \mathbf{k}_\perp^2\right) = -\frac{\sqrt{3}i}{\mathbf{k}_\perp^2} \int \frac{dk^+ dk^-}{\pi} \delta\left(xP^+ - k^+\right) \mathrm{Tr}_D\left[i\sigma_{+i} k_T^i \gamma_5 \chi(k, P)\right]$$

C. Mezrag, H. Moutarde, and J. Rodriguez-Quintero, Few Body Syst. 57 (2016)
C. Shi and I. C. Cloët, Phys. Rev. Lett. 122, 082301 (2019)

$$\left(\Box_x + m^2\right) G(x,y) = -\delta(x-y)$$

Green functions in functional approaches to QCD

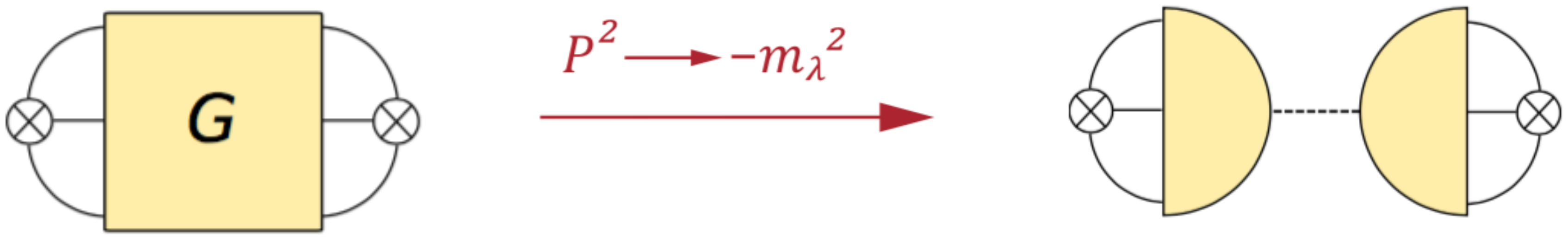
Everything we need to know is encoded in *n*-point Green functions



$$\mathbf{G}_{\alpha\beta\gamma;\alpha'\beta'\gamma'} \simeq \sum_{\lambda} \frac{\mathbf{\Psi}^{\lambda}_{\alpha\beta\gamma}\,\bar{\mathbf{\Psi}}^{\lambda}_{\alpha'\beta'\gamma'}}{P^2 + m_\lambda^2}$$

$$\mathbf{\Psi}^{\lambda}_{\alpha\beta\gamma} = \langle 0|\, T\psi_\alpha\psi_\beta\psi_\gamma \,|\lambda\rangle$$

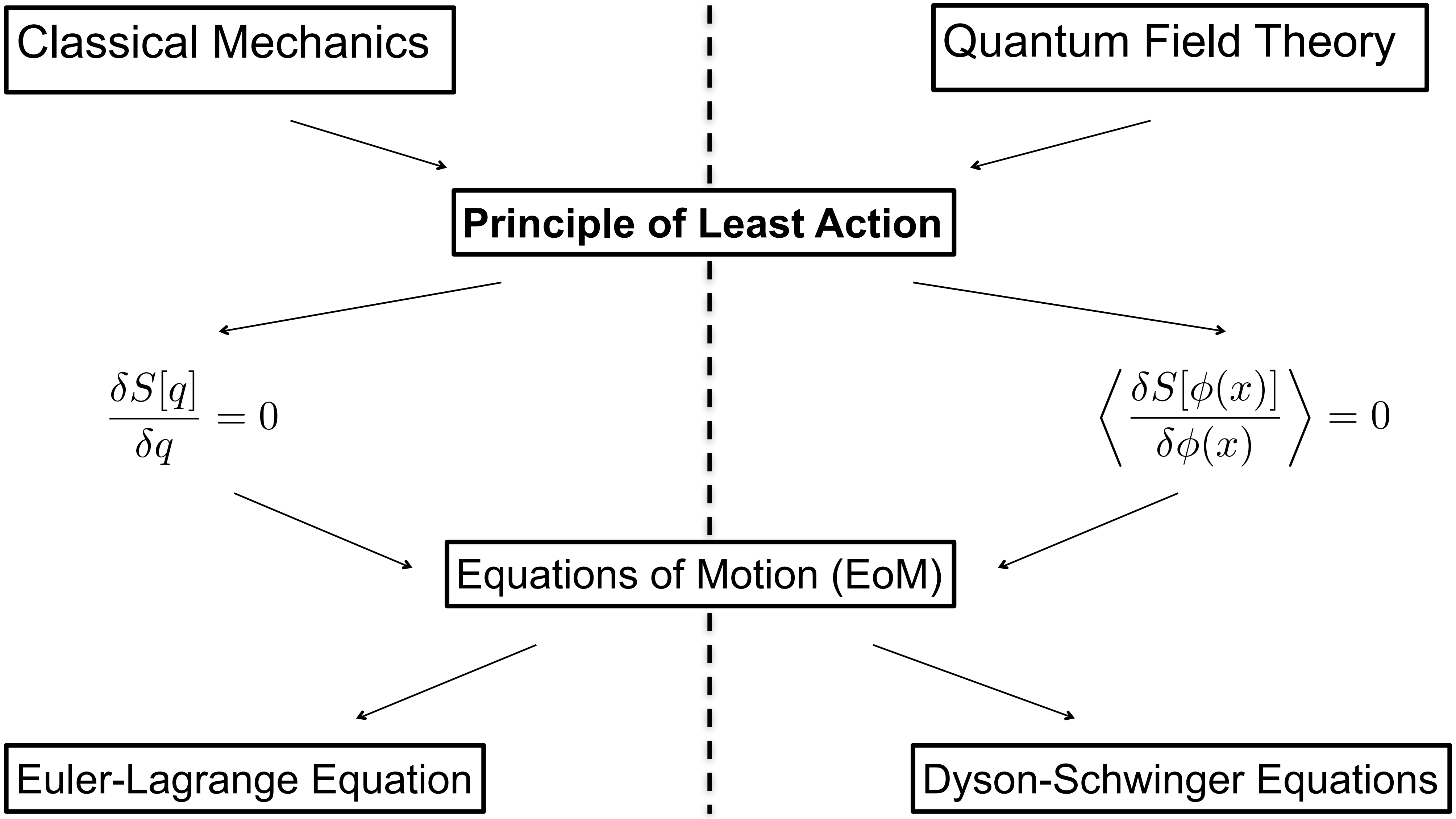
Spectral decomposition: extract baryon poles from quark 6-point functions ⟹ Residue at pole: Bethe-Salpeter wave function, contains all information about baryon



$\sim \exp(-m_N t)$

Lattice: extract baryon poles from correlators ⟹ exponential Euclidean time decay

NONPERTURBATIVE CONTINUUM TOOLS FOR QCD



Classical Mechanics

Quantum Field Theory

Principle of Least Action

$$\frac{\delta S[q]}{\delta q} = 0$$

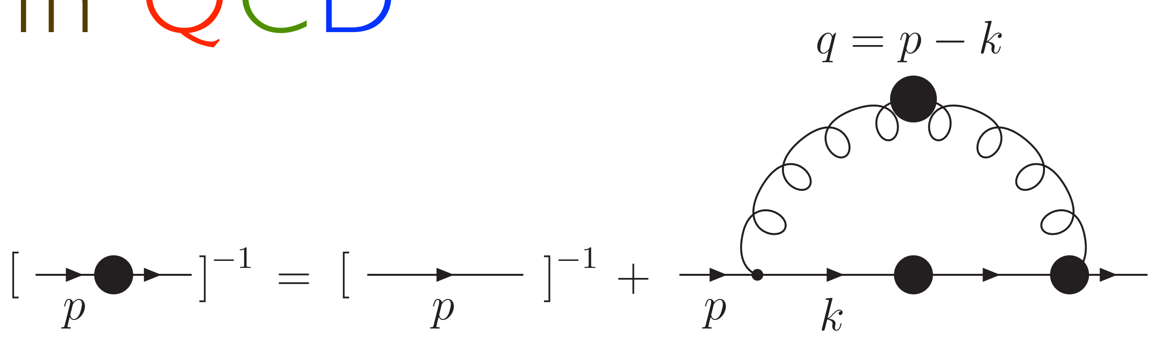
$$\left\langle \frac{\delta S[\phi(x)]}{\delta \phi(x)} \right\rangle = 0$$

Equations of Motion (EoM)

Euler-Lagrange Equation

Dyson-Schwinger Equations

Quark-Gap Equation in QCD



The propagator can be obtained from QCD's gap equation: the Dyson-Schwinger equation (DSE) for the dressed-fermion self-energy, which involves the set of infinitely many coupled equations.

$$
\begin{aligned}
S^{-1}(p) &= Z_2(i\gamma\cdot p + m^{\mathrm{bm}}) + \Sigma(p) \;:=\; i\gamma\cdot p\, A(p^2) + B(p^2) \\
\Sigma(p) &= Z_1 \int^{\Lambda} \frac{d^4q}{(2\pi)^4}\, g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S(q)\Gamma^a_\nu(q,p)
\end{aligned}
$$

with the *running* mass function $M(p^2) = B(p^2)/A(p^2)$.

$D_{\mu\nu}$: dressed-gluon propagator

$\Gamma^a_\nu(q,p)$: dressed quark-gluon vertex

Z_2 : quark wave function renormalization constant

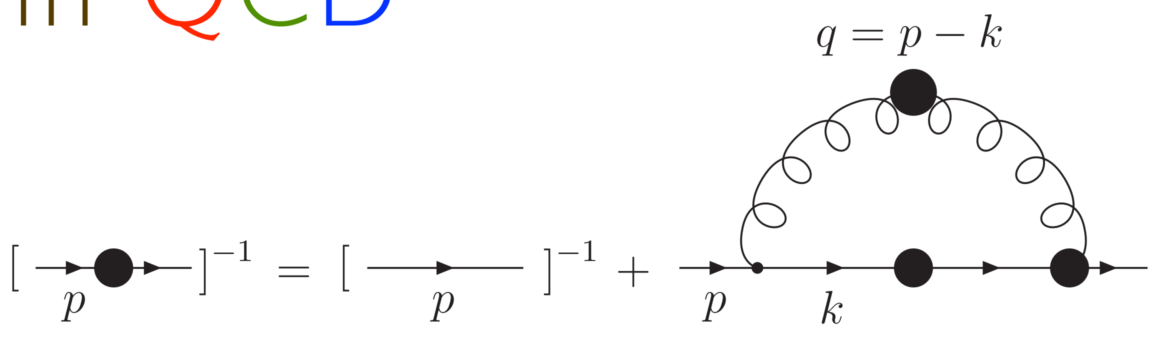
Z_1 : quark-gluon vertex renormalization constant

Each satisfies it's own DSE !

$$S^{-1}(p)|_{p^2=\zeta^2} = i\gamma\cdot p + m(\zeta)$$

where ζ is the renormalization point.

Quark-Gap Equation in QCD



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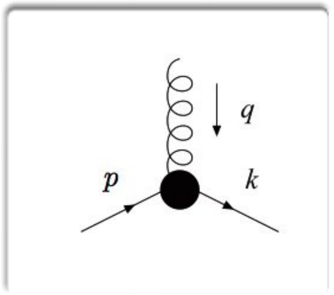

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$\Gamma^a_\nu(q,p)$: dressed quark-gluon vertex

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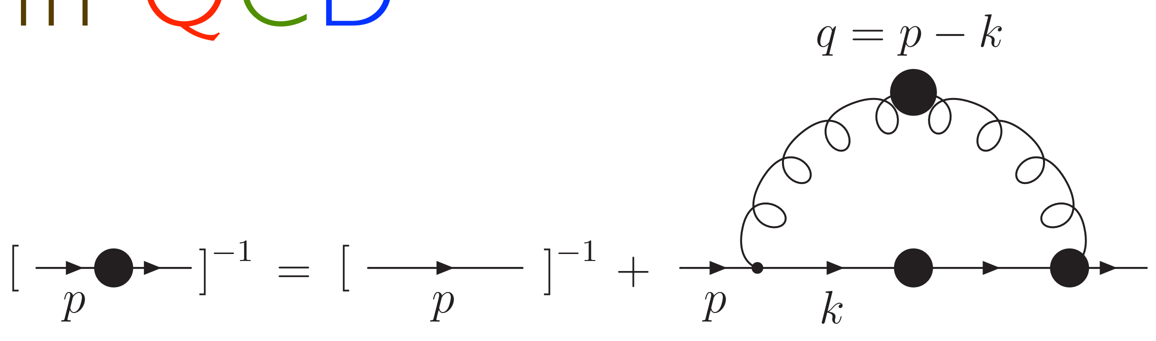
Quark-Gap Equation in QCD



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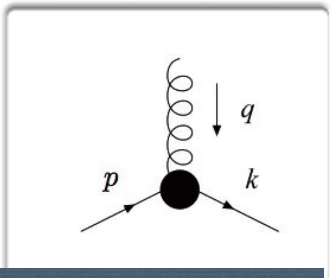

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Running Quark Mass

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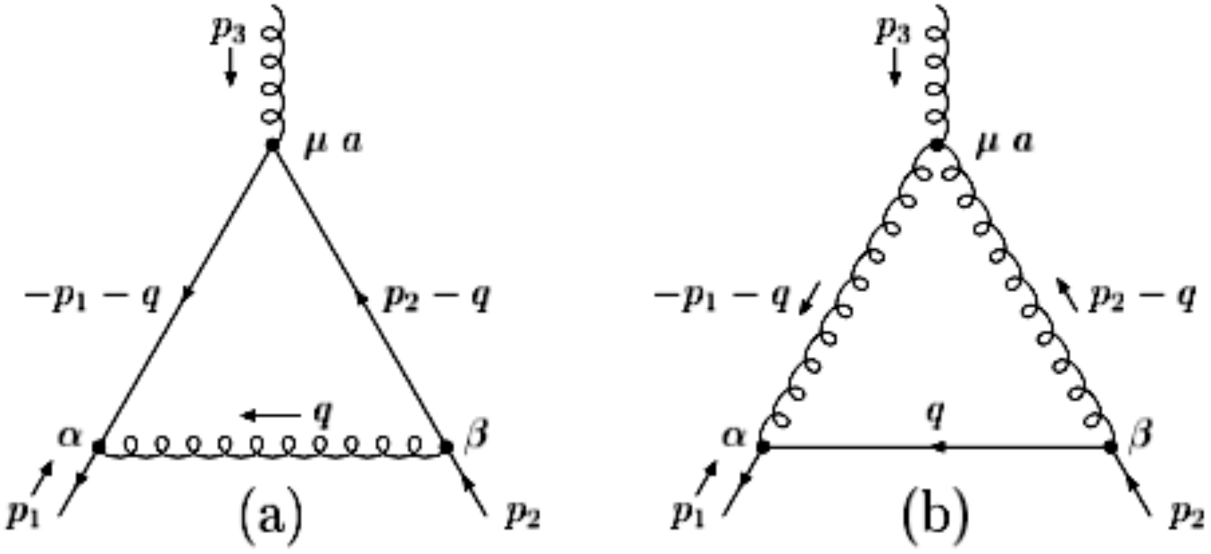
Nonperturbative quark-gluon vertex: tensor structure



a) *Abelian* correction at one loop
b) *Non-Abelian* correction at one loop

- The quark-gluon vertex in a tree-order is just $i\,\frac{\lambda_i}{2}\,\gamma_\mu$.
- However, already at one loop the Dirac-tensor structure is very complex.

Davydychev, *Osland and Saks* (2001)

Nonperturbative quark-gluon vertex: tensor structure

The fermion-gauge-boson vertex can be decomposed into longitudinal and transverse components: $\Gamma_\mu(k,p) = \Gamma^L_\mu(k,p) + \Gamma^T_\mu(k,p)$.

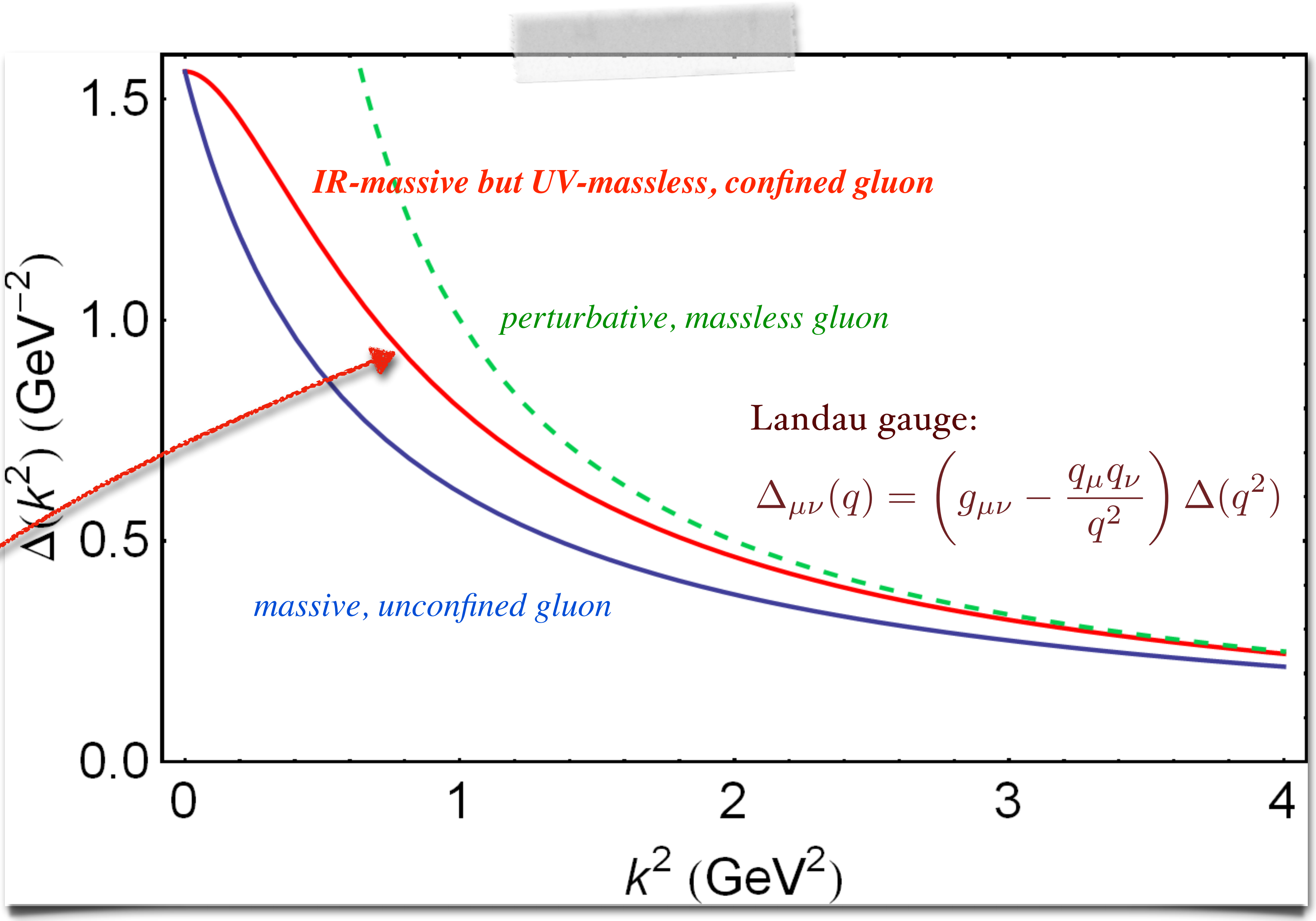
$$\Gamma^L_\mu(k,p) = \sum_{i=1}^{4} \lambda_i(k^2,p^2)\, L^i_\mu(k,p)$$

$$\Gamma^T_\mu(k,p) = \sum_{i=1}^{8} \tau_i(k^2,p^2)\, T^i_\mu(k,p)$$

$$\Gamma_\mu(k,p)\Big|_{k^2=p^2=q^2=\mu^2} = \gamma_\mu$$

$$q \cdot \Gamma^T_\mu(k,p) = 0$$

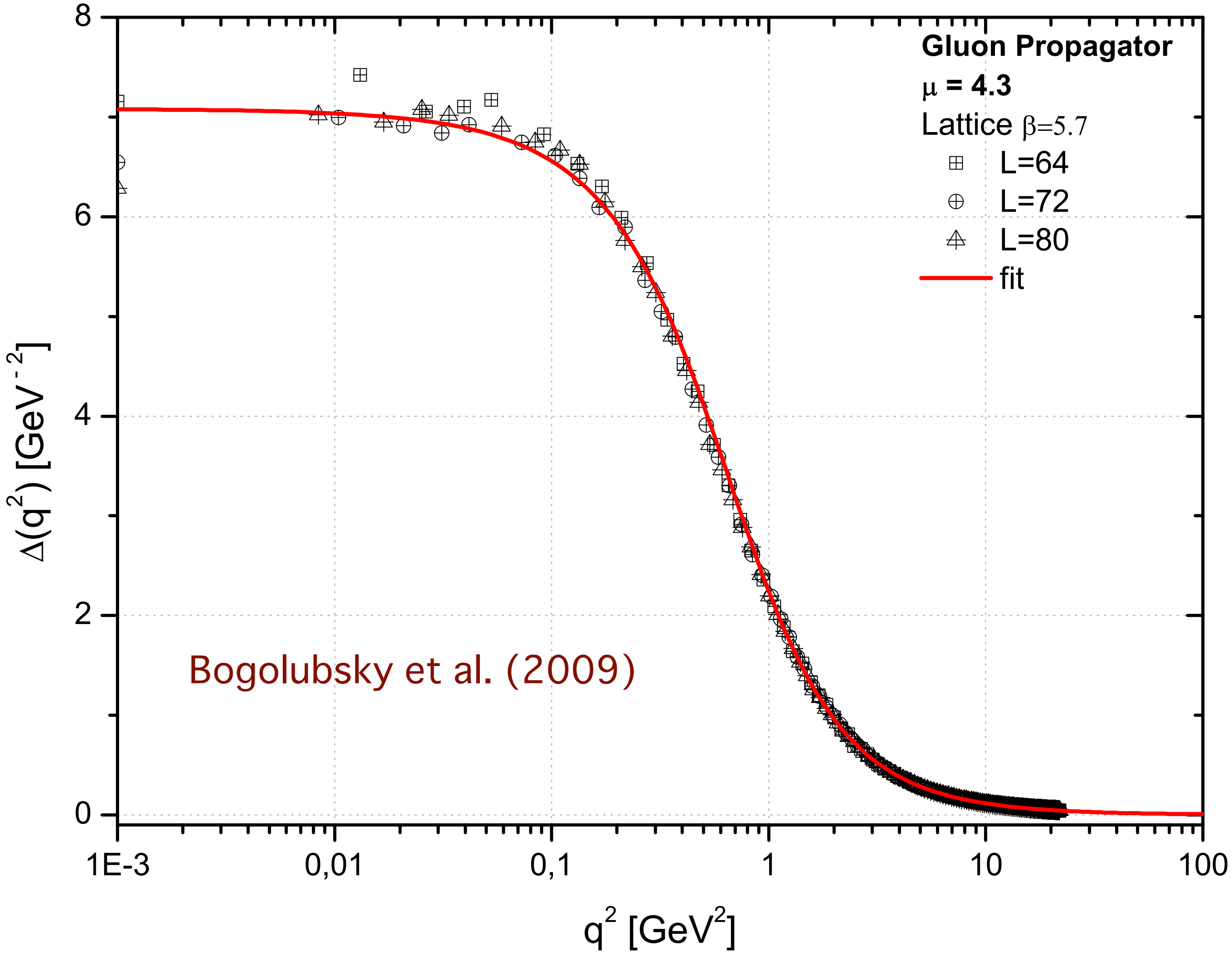
Gluon infrared behavior



Inflection point of mass function signals absence of Källen-Lehmann spectral representation ⇒ violation of reflection positivity ⇒ Confinement

A. Bashir, L. Chang, I. Cloët, B. E., Y. X. Liu, C.D. Roberts & P. Tandy (2012)

Lattice QCD gluon dressing function



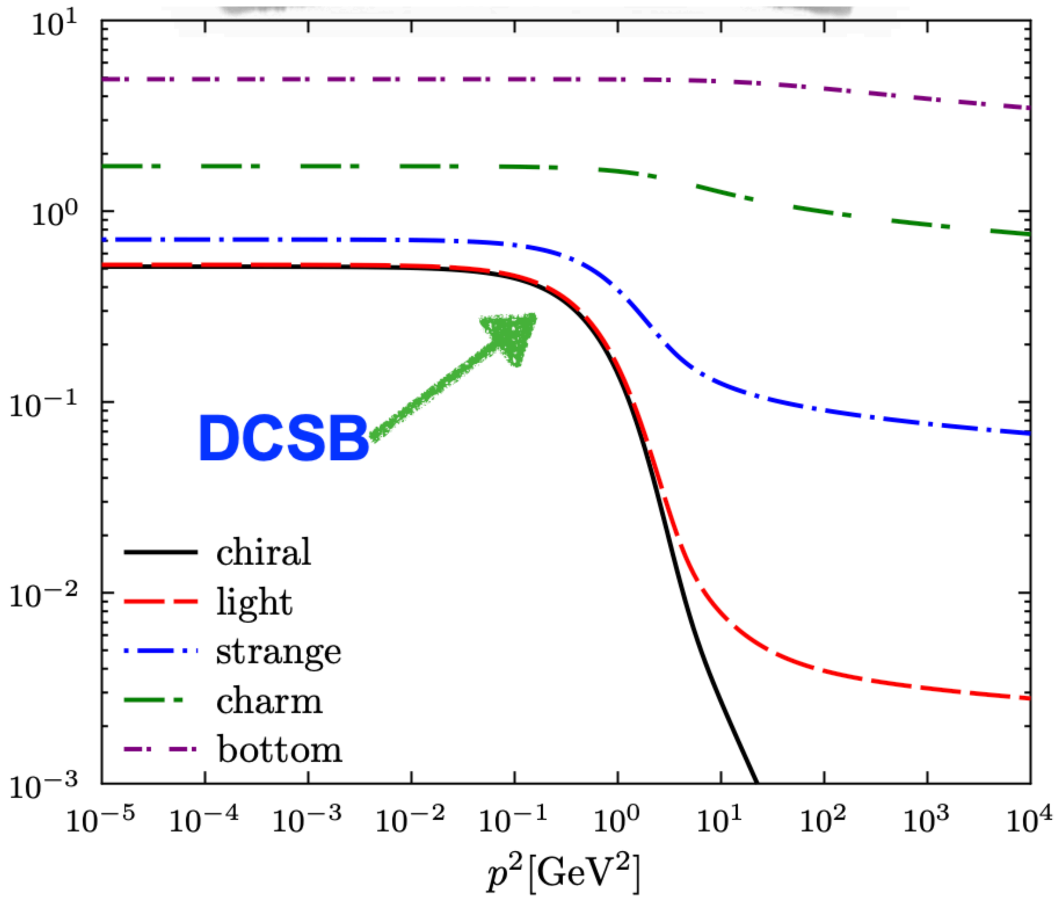
Landau gauge:

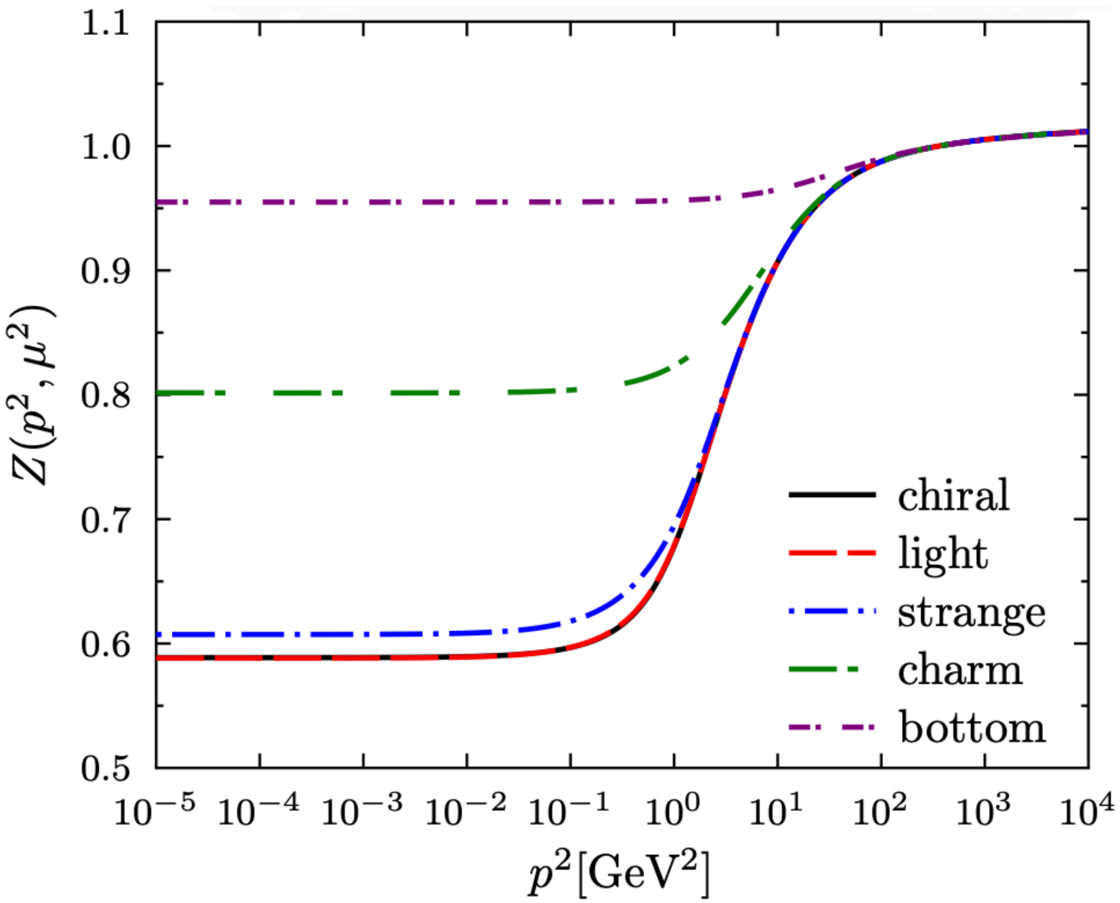
$$\Delta_{\mu\nu}(q) = \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\Delta(q^2)$$

Use effective interaction which reproduces Lattice QCD and DSE results for gluon-dressing function: *infrared massive fixed point; ultraviolet massless propagator.*

DSE solutions
Flavor dependence

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



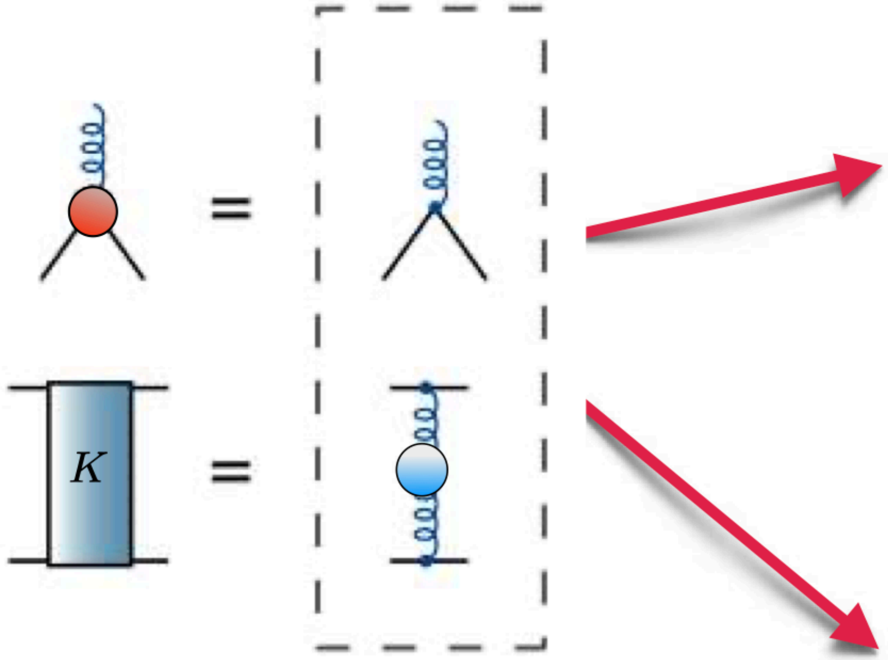


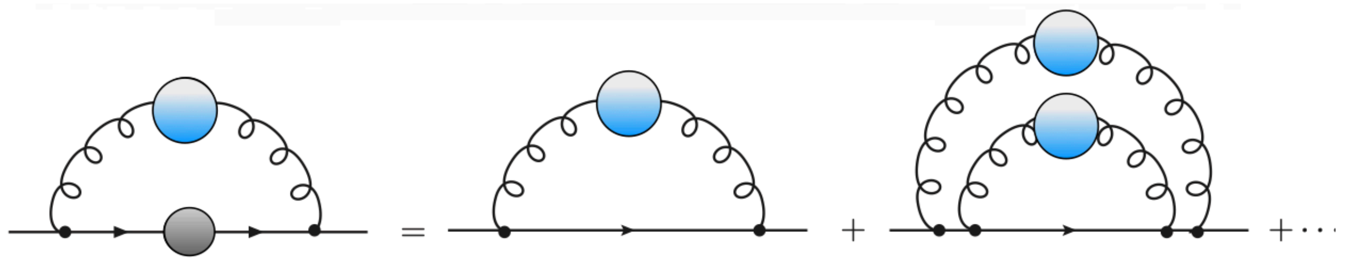
L. Albino, A. Bashir, B.E., L.X. Gutiérrez Guerrero, E. Rojas PRD 100 (2019)

L. Albino, A. Bashir, B.E., E. Rojas, F. E. Serna, R.C. Silveira, JHEP11 (2021)

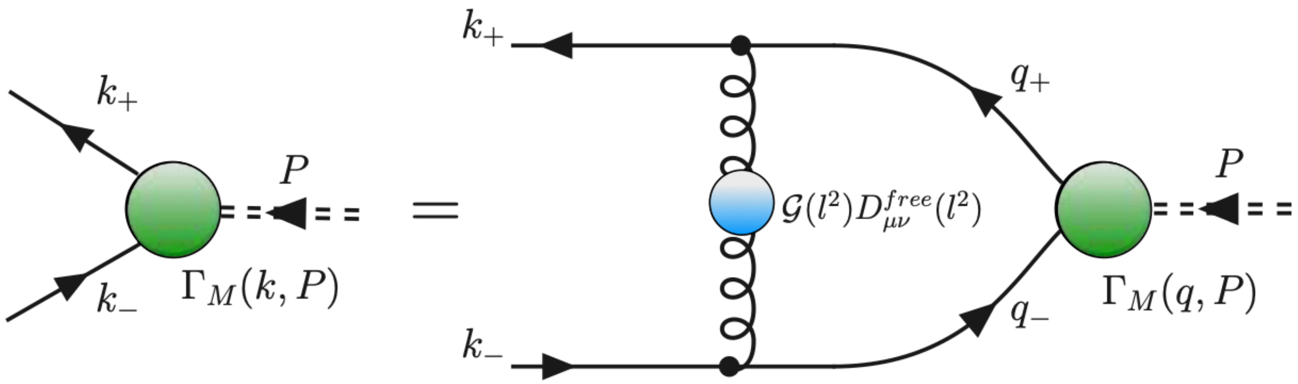
J. R. Lessa, F. E. Serna, B.E., A. Bashir, O. Oliveira, PRD 107 (2023)

Bethe-Salpeter equation for QCD bound states





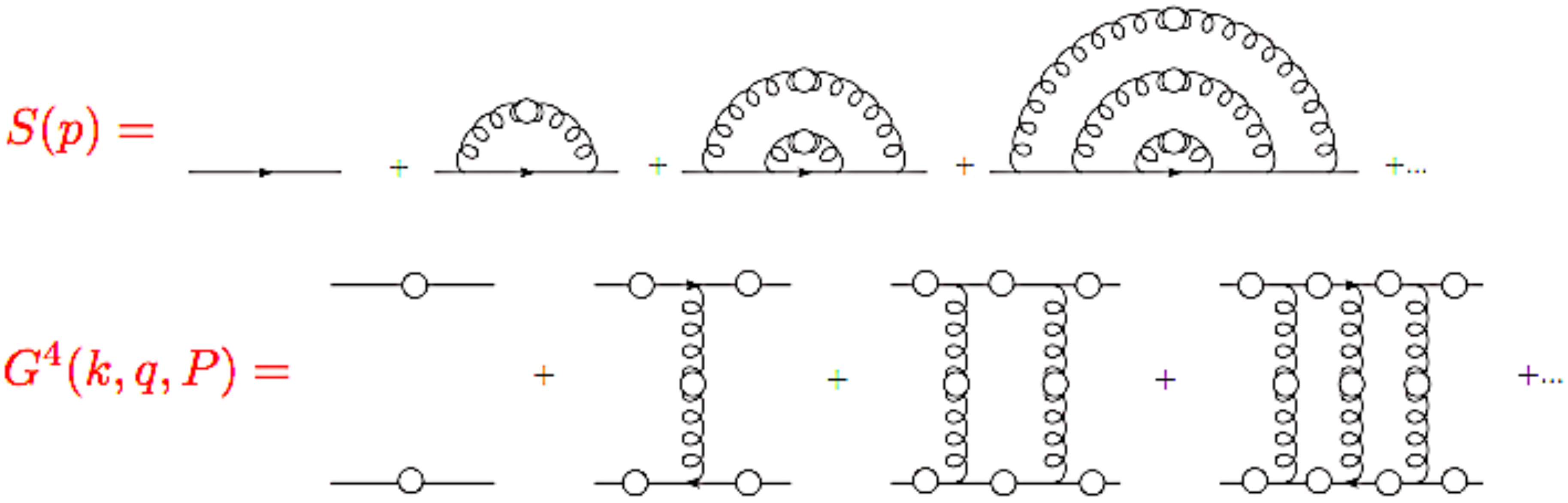
$$K(q,k;P) = -Z_2^2\,\mathcal{G}\left(l^2\right)\,D_{\mu\nu}^{\mathrm{free}}(l)\,\gamma_\mu\frac{\lambda^a}{2}\gamma_\nu\frac{\lambda^a}{2}$$



- Quark propagators are obtained by solving the gap equation (DSE) for space-like momenta.
- In solving the BSE in Euclidean space, the propagators are functions of $(k+P)^2$, $P=(0,0,0,iM)$.
- Extension to complex plane via Cauchy's integral theorem.

Bethe-Salpeter equation for QCD bound states

Rainbow-ladder truncation (leading symmetry-preserving approximation)



Model gluon propagator, solve quark propagator and 4-point Green function.

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- In solving the BSE in Euclidean space, the propagators are functions of $(k+P)^2$, $P=(0,0,0,iM)$.
- Extension to complex plane via Cauchy's integral theorem.

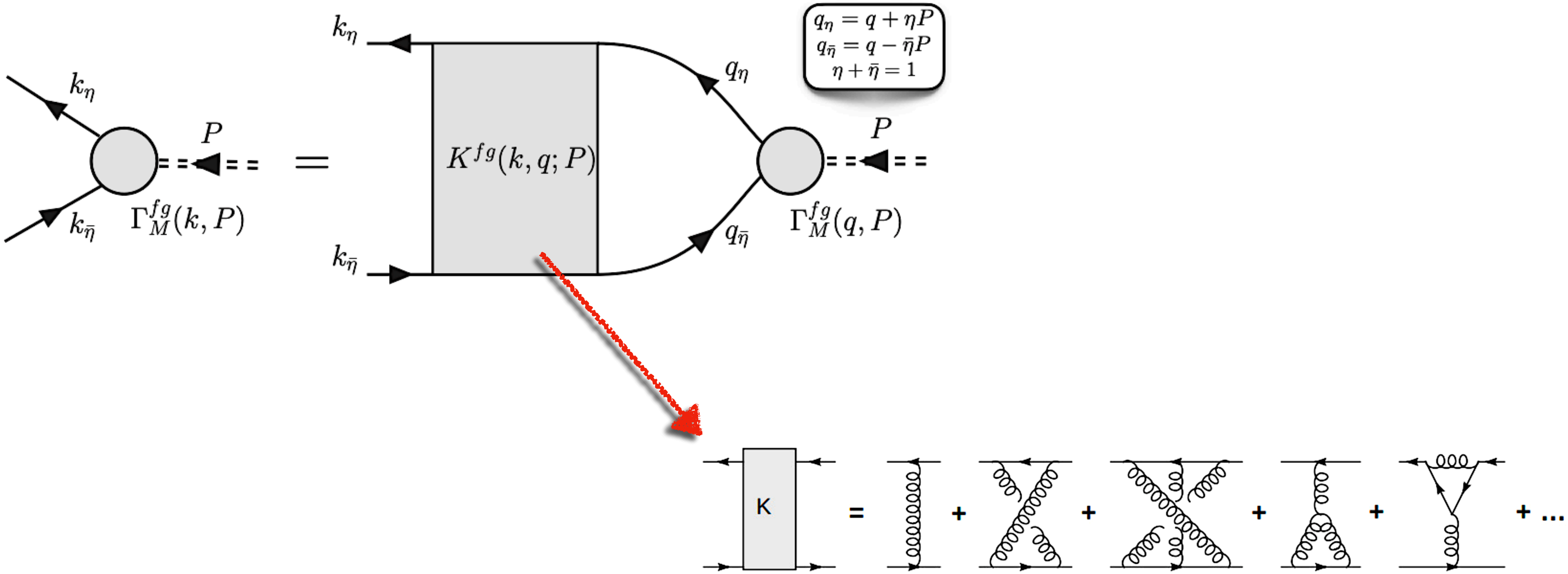
Bethe-Salpeter Equation for QCD Bound States

$$\Gamma_M^{fg}(k,P) = \int^{\Lambda} \frac{d^4q}{(2\pi)^4} K_{fg}(k,q;P) S_f(q_\eta) \Gamma_M^{fg}(q,P) S_g(q_{\bar\eta})$$

$K_{fg}(q,k;P)$ = Quark-antiquark scattering kernel

$S_f(q_\eta)$ = Dressed quark propagator

$\Gamma_M^{fg}(k,P)$ = Meson's Bethe-Salpeter Amplitude (BSA)



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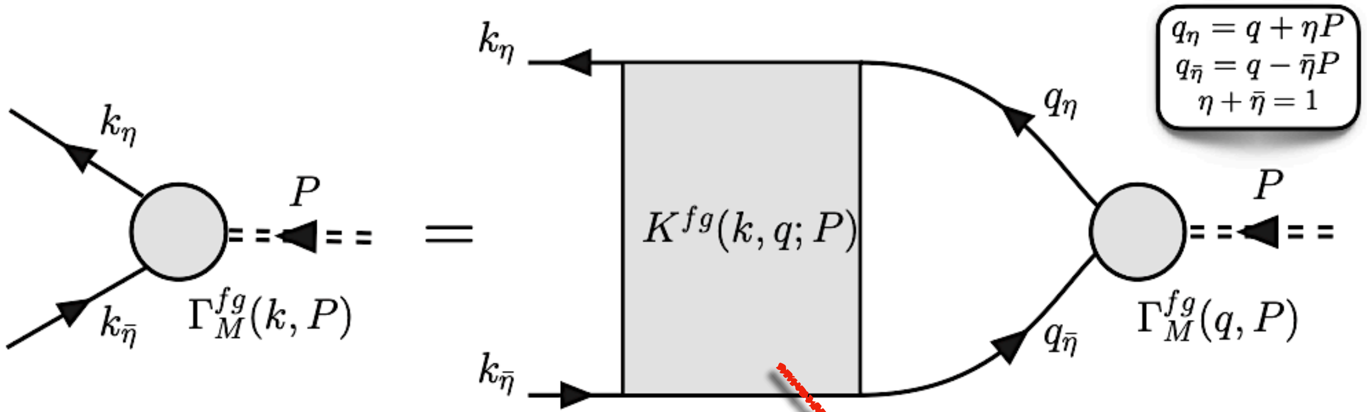
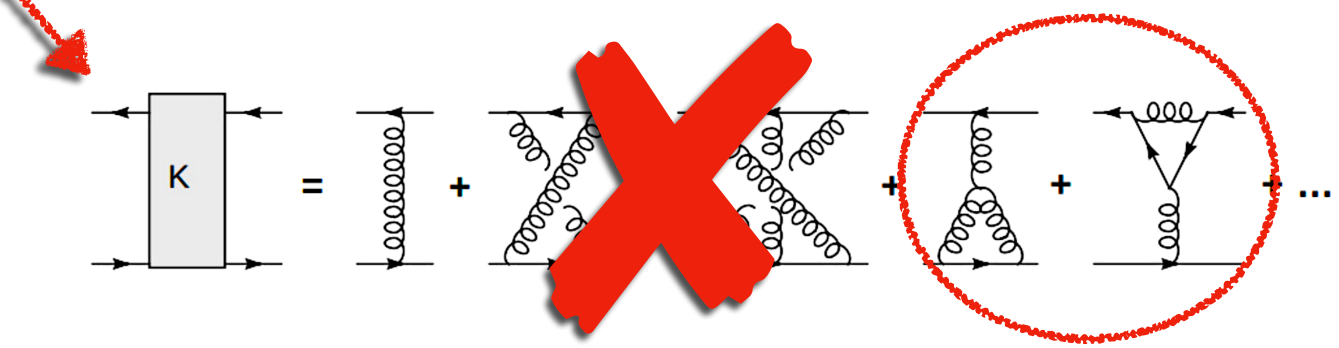


Treated with effective couplings



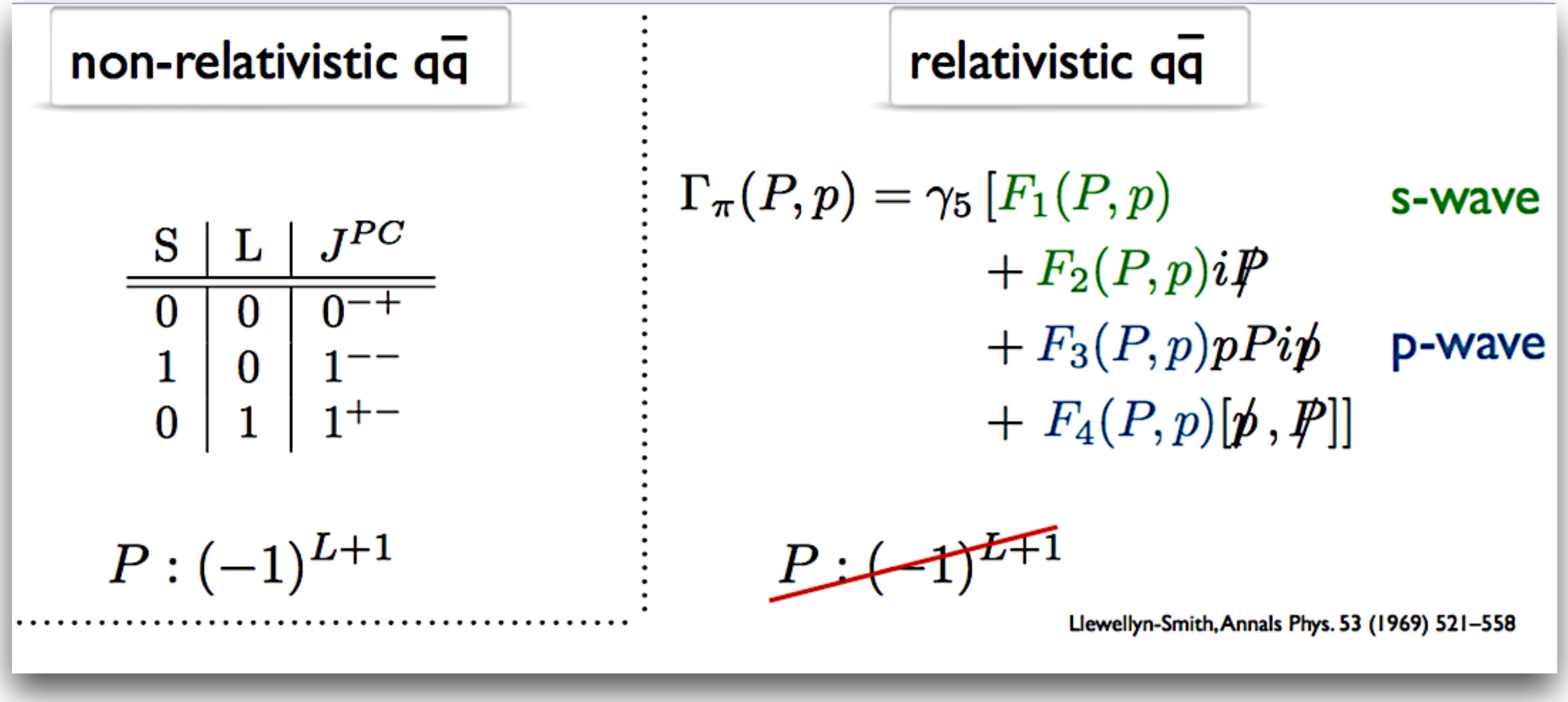
Bethe-Salpeter Amplitudes

- The general form of $\Gamma_M(k;P)$ is given by

$$\Gamma(k;P) = \sum_{i=1}^{N} \mathcal{T}^i(k,P)\mathcal{F}_i(k^2, z_k, P^2), \;\; z_k = k.P/|k||P|,$$

- where $\mathcal{T}^i$(k,P) are Dirac's covariants;
- $\mathcal{F}_i(k^2, z_k, P^2)$ are Lorentz invariant amplitudes;
- N denotes the number of covariants which are different for different meson's channel.
- For the case of pseudoscalar mesons we have $N = 4$ and for vector mesons one has $N = 8$

Beyond the Quark Model



non-relativistic $q\bar{q}$

S	L	J^{PC}
0	0	0^{-+}
1	0	1^{--}
0	1	1^{+-}

$$P : (-1)^{L+1}$$

relativistic $q\bar{q}$

$$\begin{aligned} \Gamma_\pi(P,p) = \gamma_5\,[F_1(P,p) & \\ + F_2(P,p)\, i\not{P} & \\ + F_3(P,p)\, pP i\not{p} & \\ + F_4(P,p)\, [\not{p}\,, \not{P}]] & \end{aligned}$$

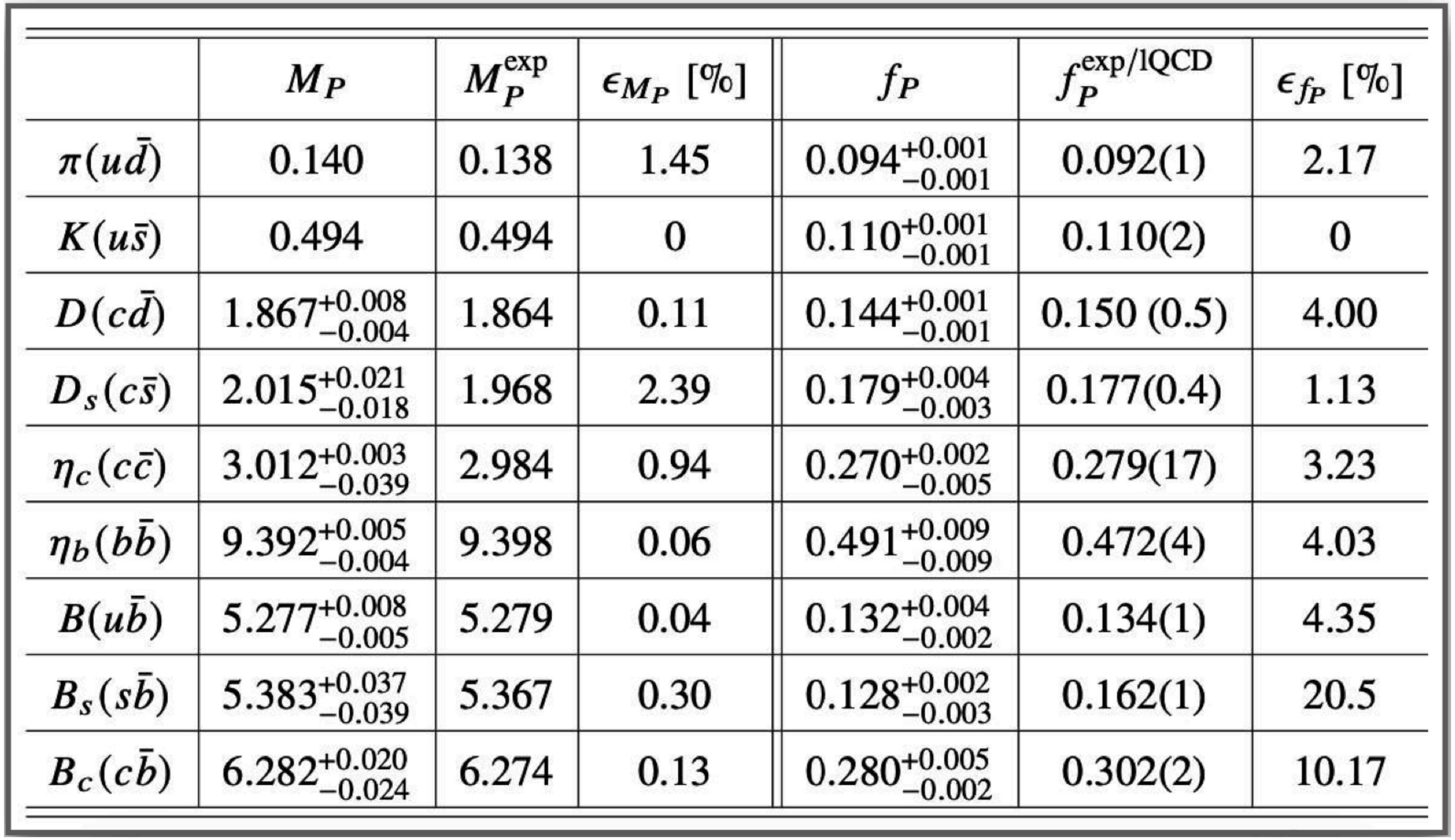
s-wave

p-wave

$$\cancel{P : (-1)^{L+1}}$$

Llewellyn-Smith, Annals Phys. 53 (1969) 521–558

Pseudoscalar meson spectrum



	M_P	M_P^{exp}	ϵ_{M_P} [%]	f_P	$f_P^{\text{exp/lQCD}}$	ϵ_{f_P} [%]
$\pi(u\bar{d})$	0.140	0.138	1.45	$0.094^{+0.001}_{-0.001}$	0.092(1)	2.17
$K(u\bar{s})$	0.494	0.494	0	$0.110^{+0.001}_{-0.001}$	0.110(2)	0
$D(c\bar{d})$	$1.867^{+0.008}_{-0.004}$	1.864	0.11	$0.144^{+0.001}_{-0.001}$	0.150 (0.5)	4.00
$D_s(c\bar{s})$	$2.015^{+0.021}_{-0.018}$	1.968	2.39	$0.179^{+0.004}_{-0.003}$	0.177(0.4)	1.13
$\eta_c(c\bar{c})$	$3.012^{+0.003}_{-0.039}$	2.984	0.94	$0.270^{+0.002}_{-0.005}$	0.279(17)	3.23
$\eta_b(b\bar{b})$	$9.392^{+0.005}_{-0.004}$	9.398	0.06	$0.491^{+0.009}_{-0.009}$	0.472(4)	4.03
$B(u\bar{b})$	$5.277^{+0.008}_{-0.005}$	5.279	0.04	$0.132^{+0.004}_{-0.002}$	0.134(1)	4.35
$B_s(s\bar{b})$	$5.383^{+0.037}_{-0.039}$	5.367	0.30	$0.128^{+0.002}_{-0.003}$	0.162(1)	20.5
$B_c(c\bar{b})$	$6.282^{+0.020}_{-0.024}$	6.274	0.13	$0.280^{+0.005}_{-0.002}$	0.302(2)	10.17

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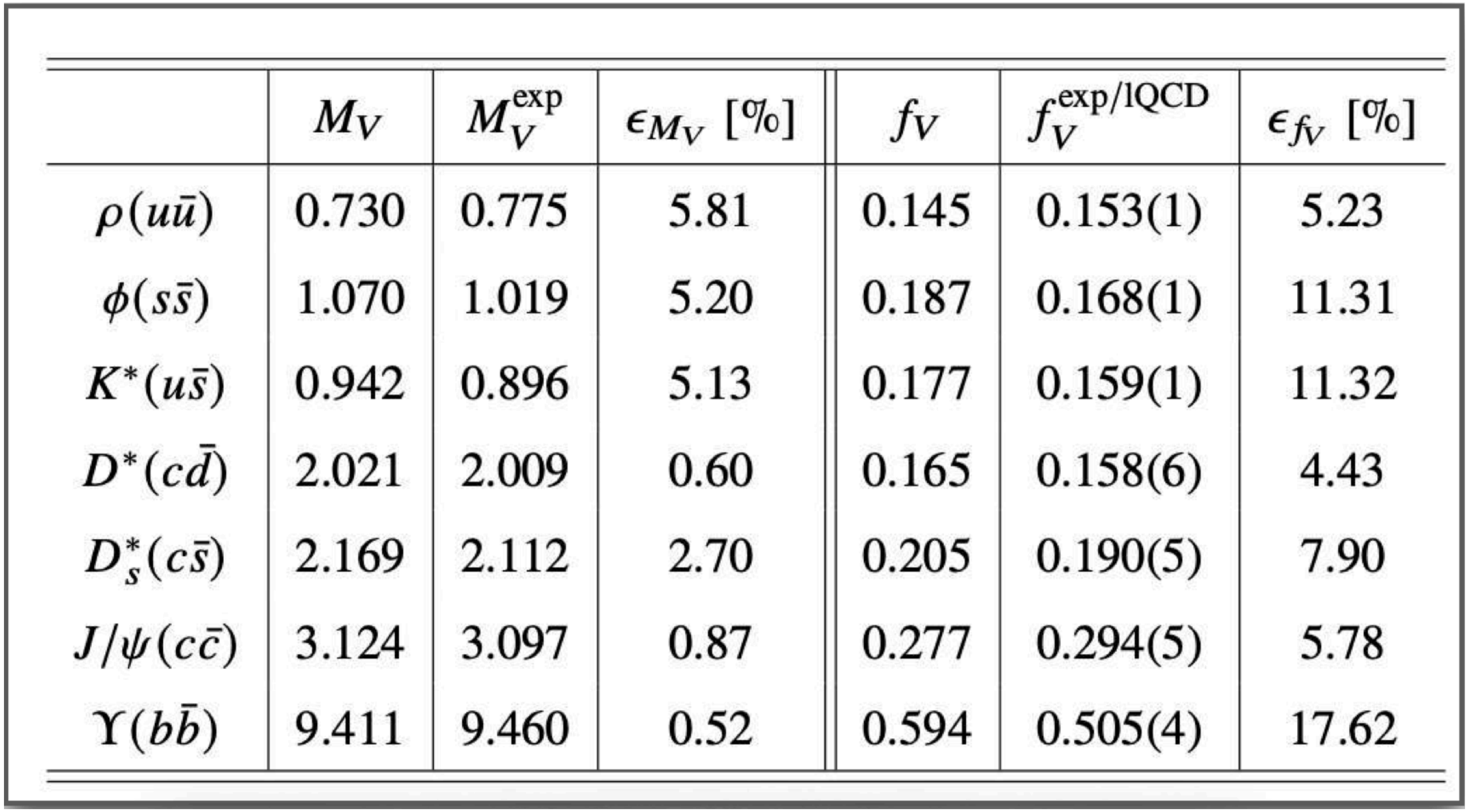
Pseudoscalar meson spectrum



	M_V	M_V^{exp}	ϵ_{M_V} [%]	f_V	$f_V^{\text{exp/lQCD}}$	ϵ_{f_V} [%]
$\rho(u\bar{u})$	0.730	0.775	5.81	0.145	0.153(1)	5.23
$\phi(s\bar{s})$	1.070	1.019	5.20	0.187	0.168(1)	11.31
$K^*(u\bar{s})$	0.942	0.896	5.13	0.177	0.159(1)	11.32
$D^*(c\bar{d})$	2.021	2.009	0.60	0.165	0.158(6)	4.43
$D_s^*(c\bar{s})$	2.169	2.112	2.70	0.205	0.190(5)	7.90
$J/\psi(c\bar{c})$	3.124	3.097	0.87	0.277	0.294(5)	5.78
$\Upsilon(b\bar{b})$	9.411	9.460	0.52	0.594	0.505(4)	17.62

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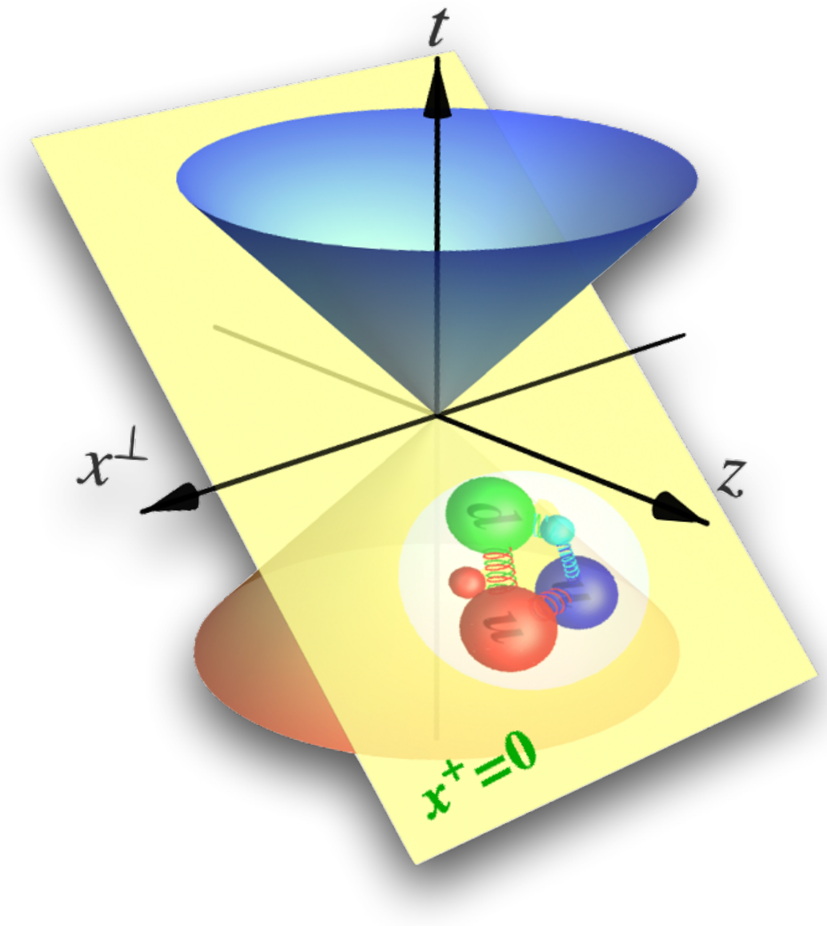


Light-Front Wave Functions

Light-Front Wave Functions

$$\chi(k,p) \;=\; \int d^4z\; e^{-ik\cdot z}\,\langle 0|\mathcal{T}\,u(z)\,\bar{d}(0)|\pi^+(p)\rangle$$

- The **LCWFs** are obtained from the **BSWF** via the light front projections:

$$\psi_M^{\uparrow\downarrow}(x,\mathbf{k}_\perp^2) \;=\; \sqrt{3}i\int\frac{dk^+dk^-}{\pi}\delta(xP^+-k^+)\mathrm{Tr}_D[\gamma^+\gamma_5\chi(k,P)],$$

$$\psi_M^{\uparrow\uparrow}(x,\mathbf{k}_\perp^2) \;=\; -\frac{\sqrt{3}i}{\mathbf{k}_\perp^2}\int\frac{dk^+dk^-}{\pi}\delta(xP^+-k^+)\mathrm{Tr}_D[i\sigma_{+i}k_T^i\gamma_5\chi(k,P)]$$

- With the **LCWF** one can readily derive two distributions:
  - The leading-twist **TMD** [B. Pasquini and P. Schweitzer (2014)]

$$f_M\left(x,\mathbf{k}_\perp^2,\mu\right) = \frac{1}{(2\pi)^3}\left|\psi_M^{\uparrow\downarrow}\left(x,\mathbf{k}_\perp^2,\mu\right)+\mathbf{k}_\perp^2\,\psi_M^{\uparrow\uparrow}\left(x,\mathbf{k}_\perp^2,\mu\right)\right|^2$$

  - The **PDF**

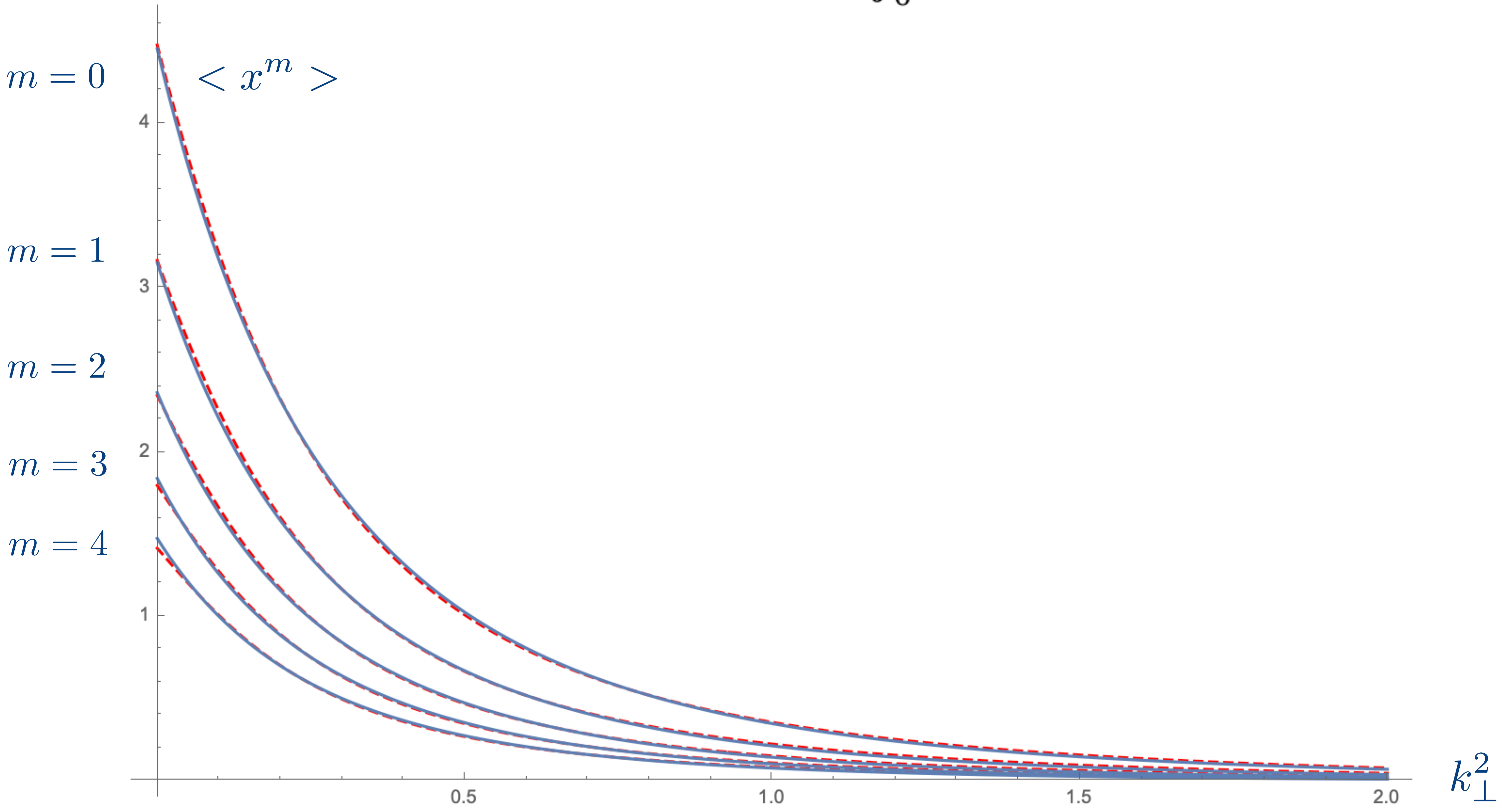
$$q_M(x,\mu) = \int d\mathbf{k}_\perp^2\,f_M(x,\mathbf{k}_\perp^2,\mu)$$

Normalization condition: $\displaystyle\int_0^1 dx\,q_M\left(x,\mu\right)=1.$

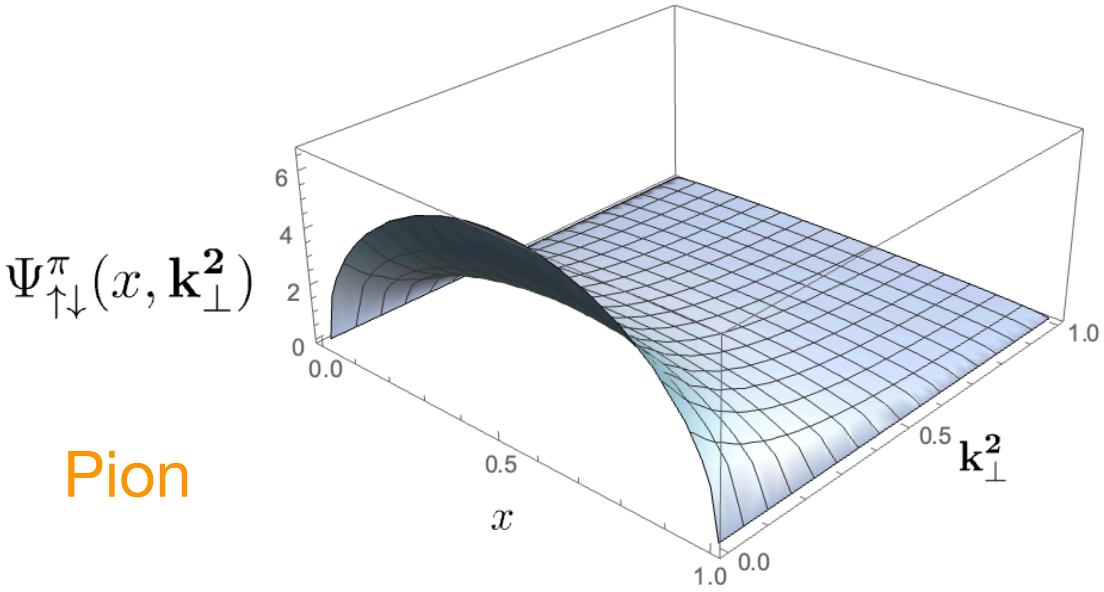
Light-Front Wave Functions

We calculate transverse momentum dependent moments:

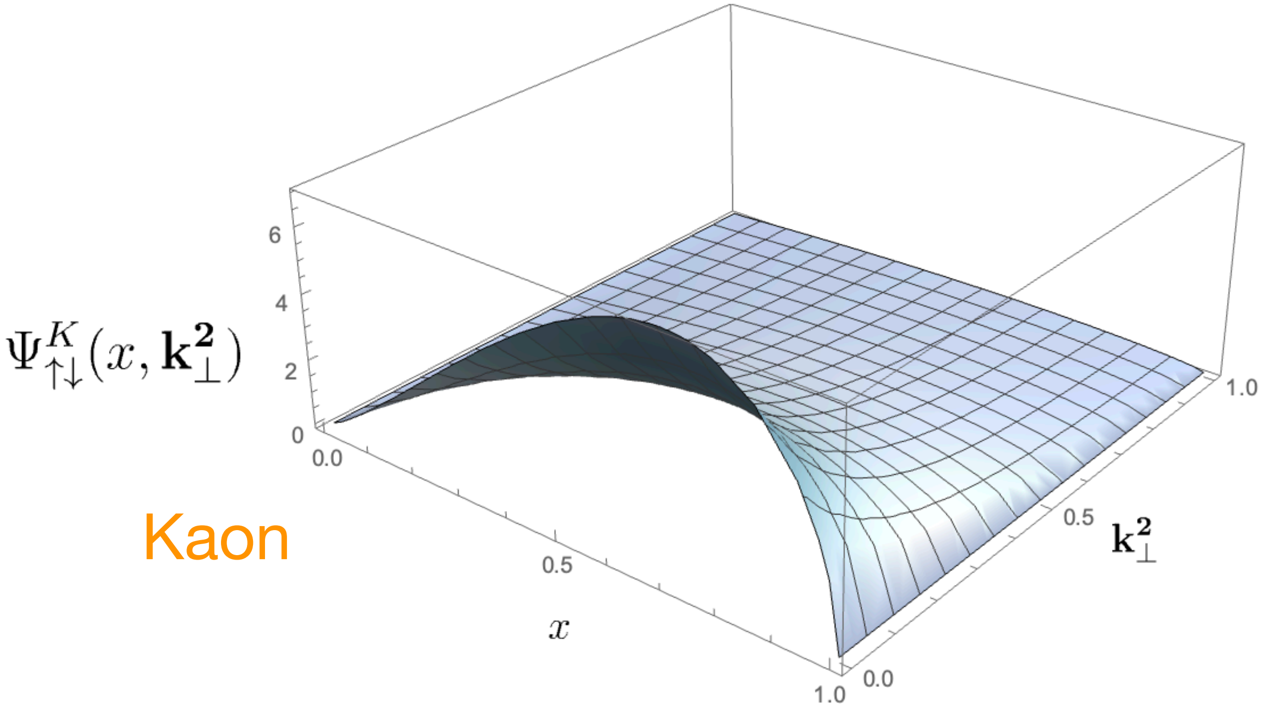
$$\langle x^m \rangle_{\uparrow\downarrow,\uparrow\uparrow}(\mathbf{k}_\perp^2, \mu) = \int_0^1 dx\, x^m\, \Psi_{\uparrow\downarrow,\uparrow\uparrow}(x, \mathbf{k}_\perp^2, \mu)$$



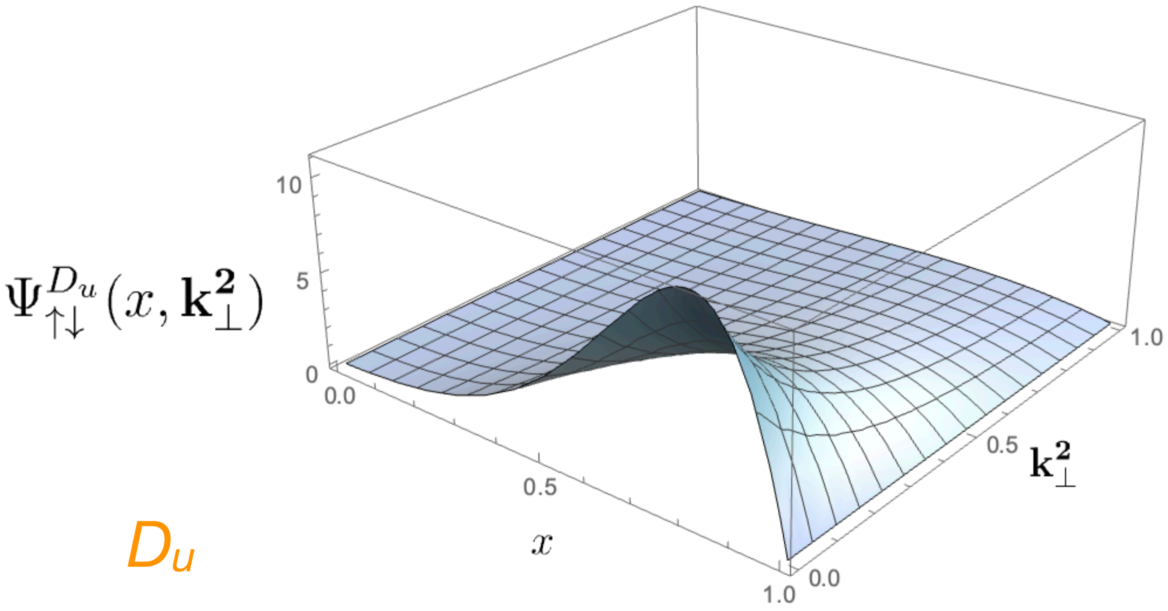
Light-Front Wave Functions



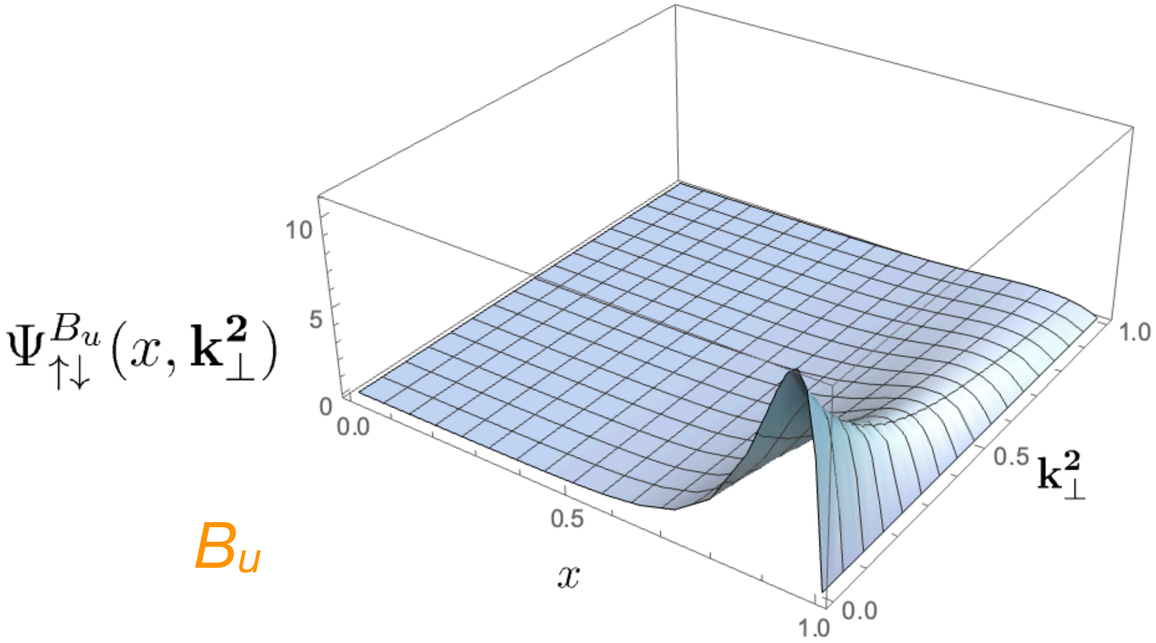
Pion



Kaon

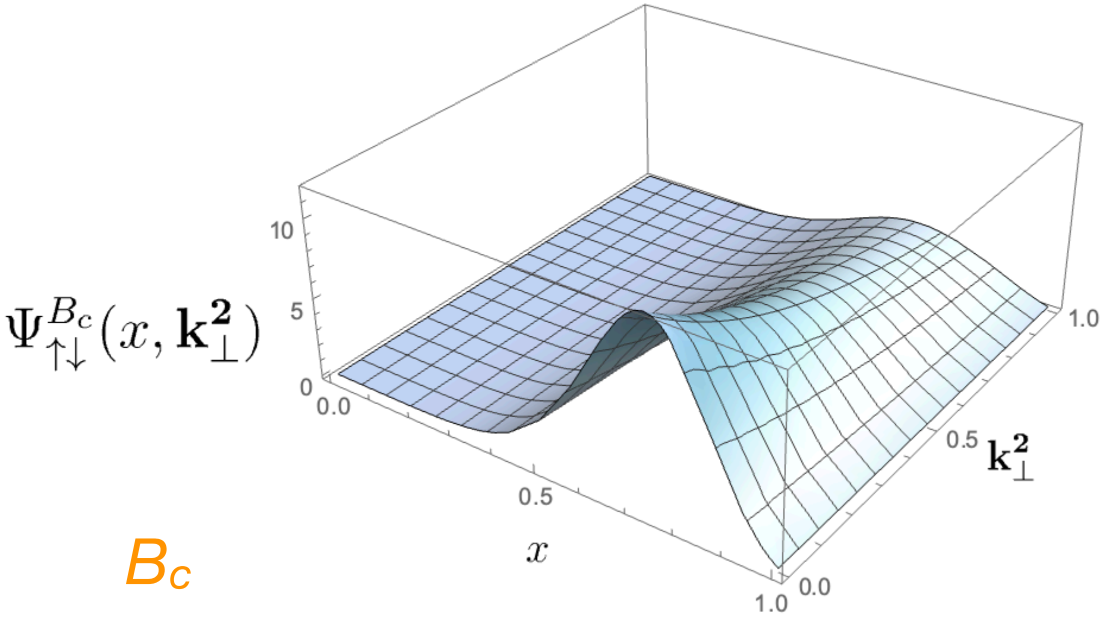


D_u

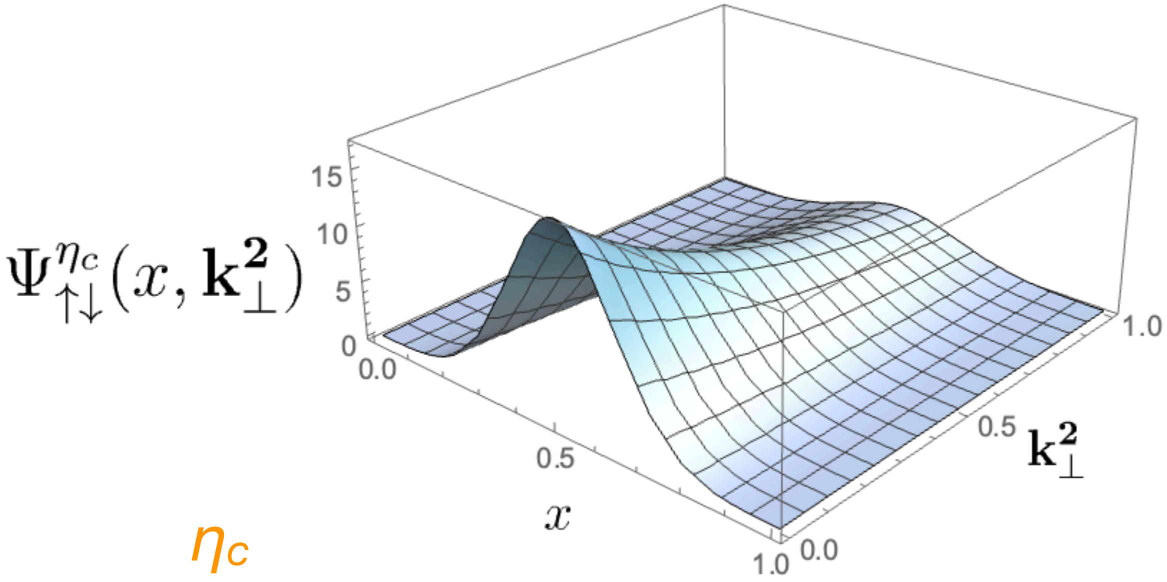


B_u

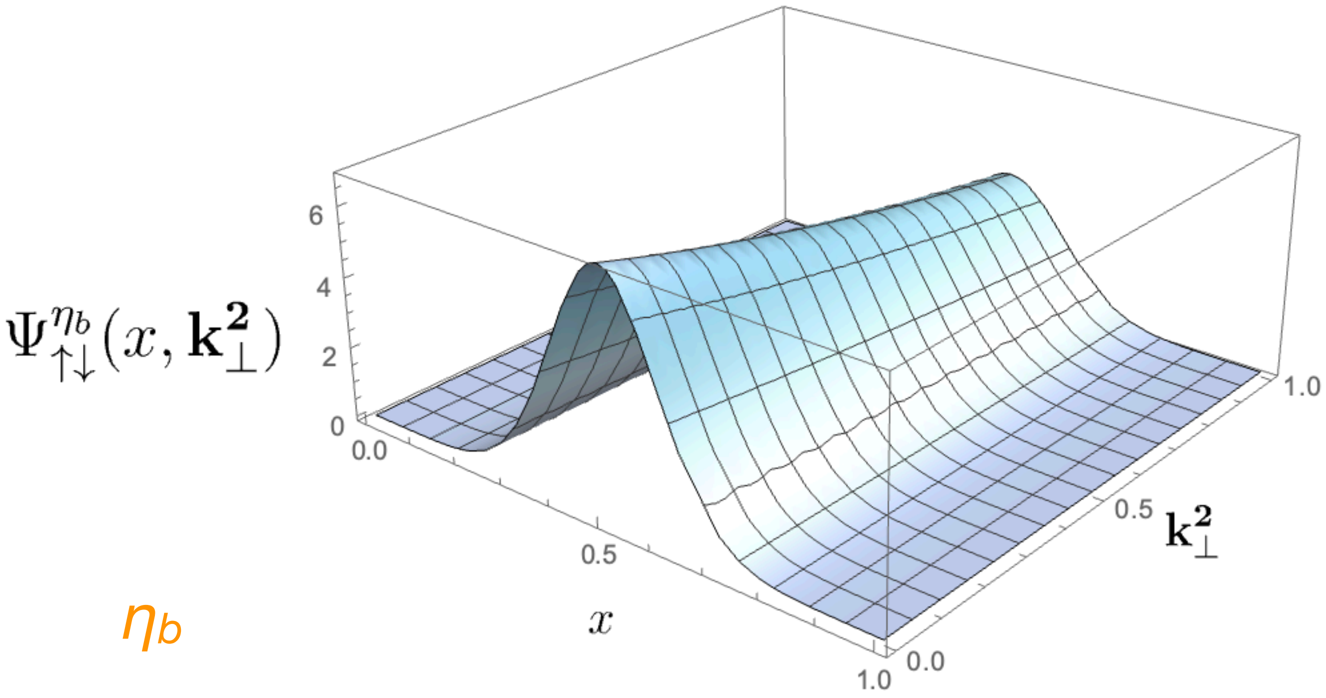
Light-Front Wave Functions



B_c

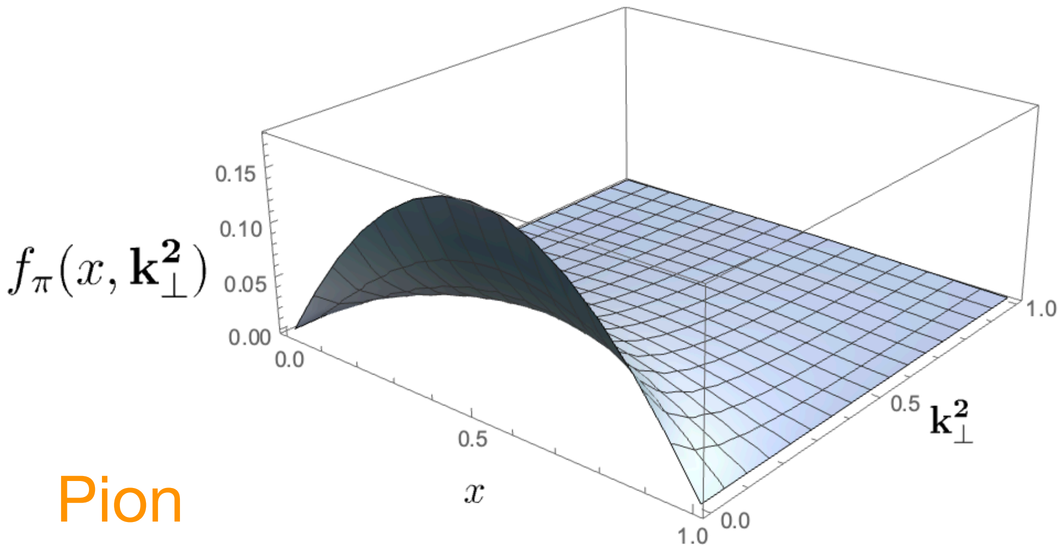


η_c

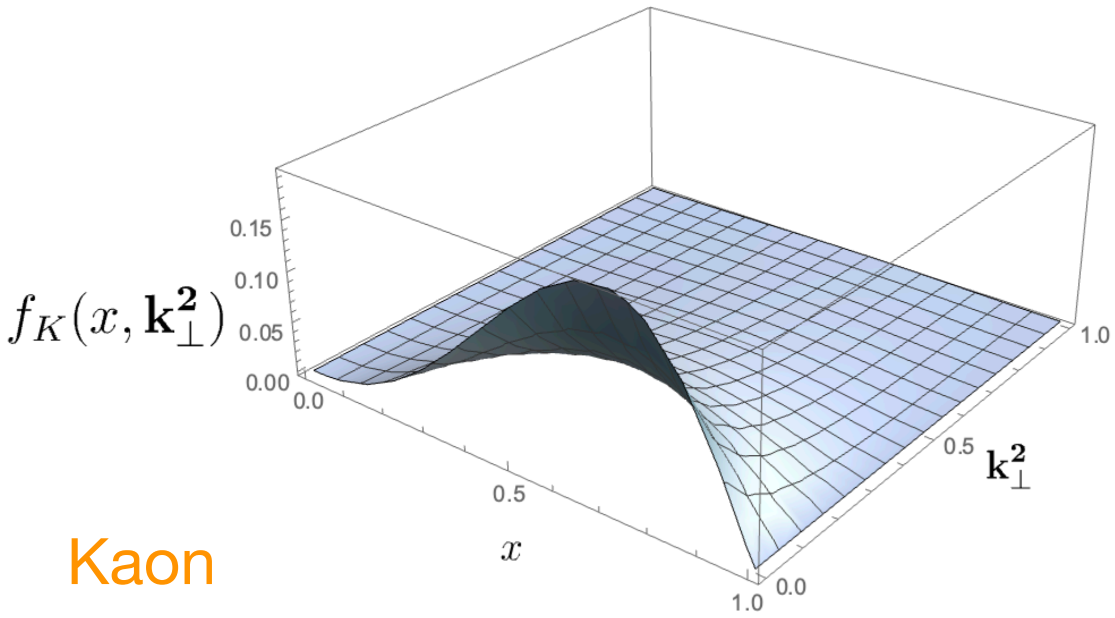


η_b

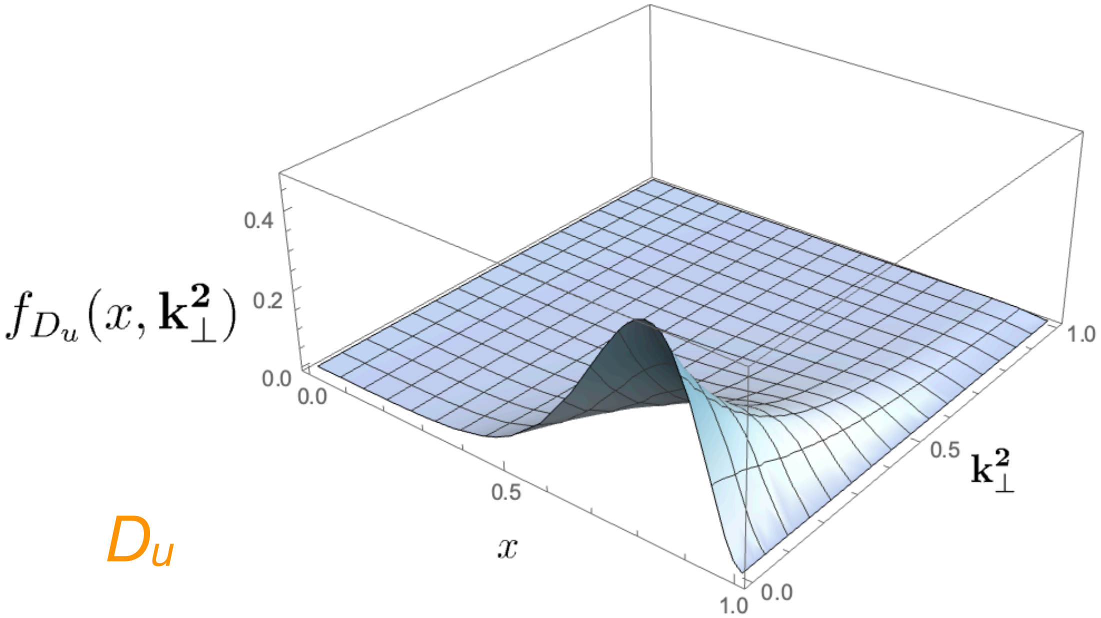
Transverse Distribution Functions



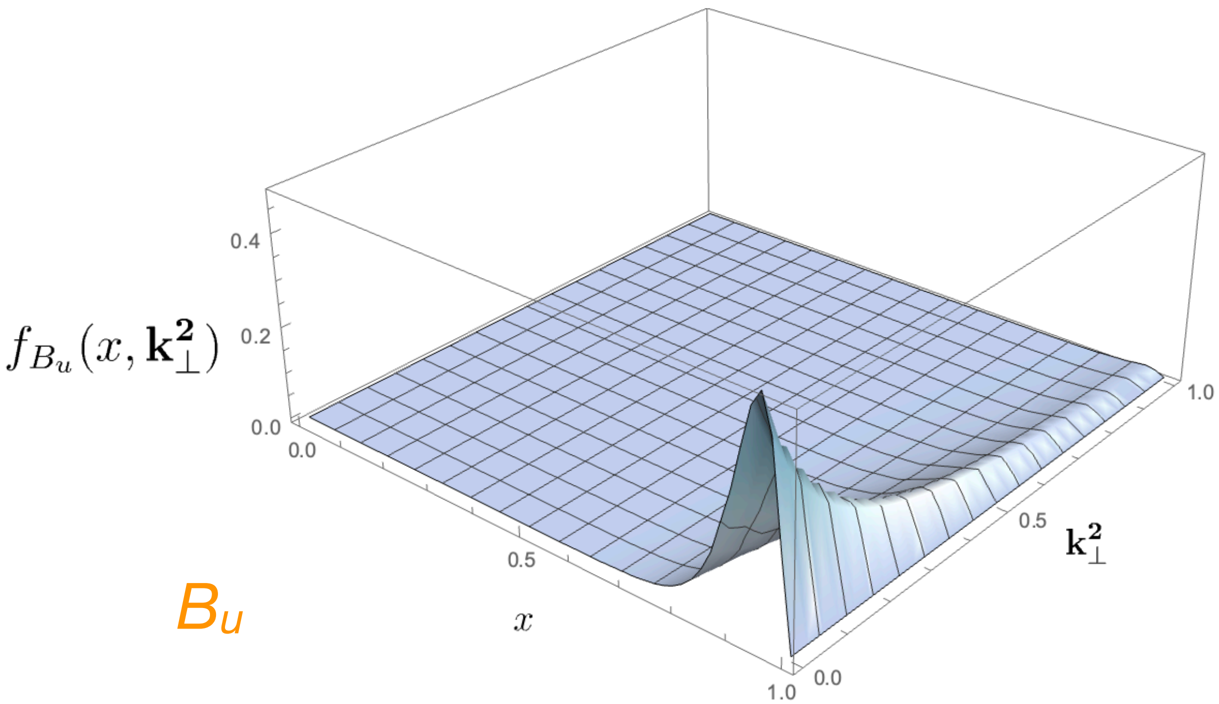
Pion



Kaon

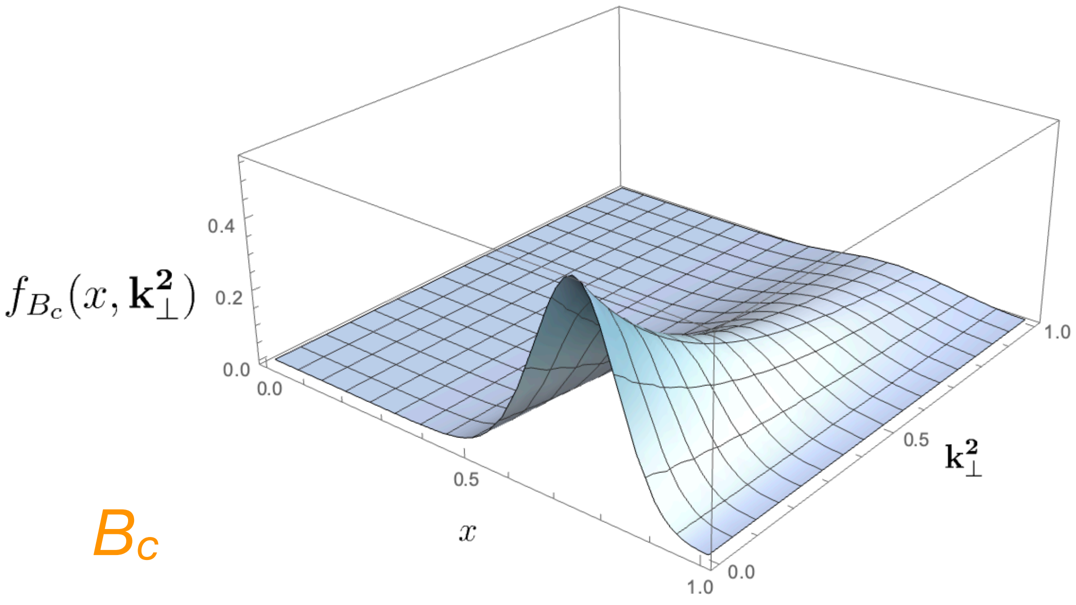


D_u

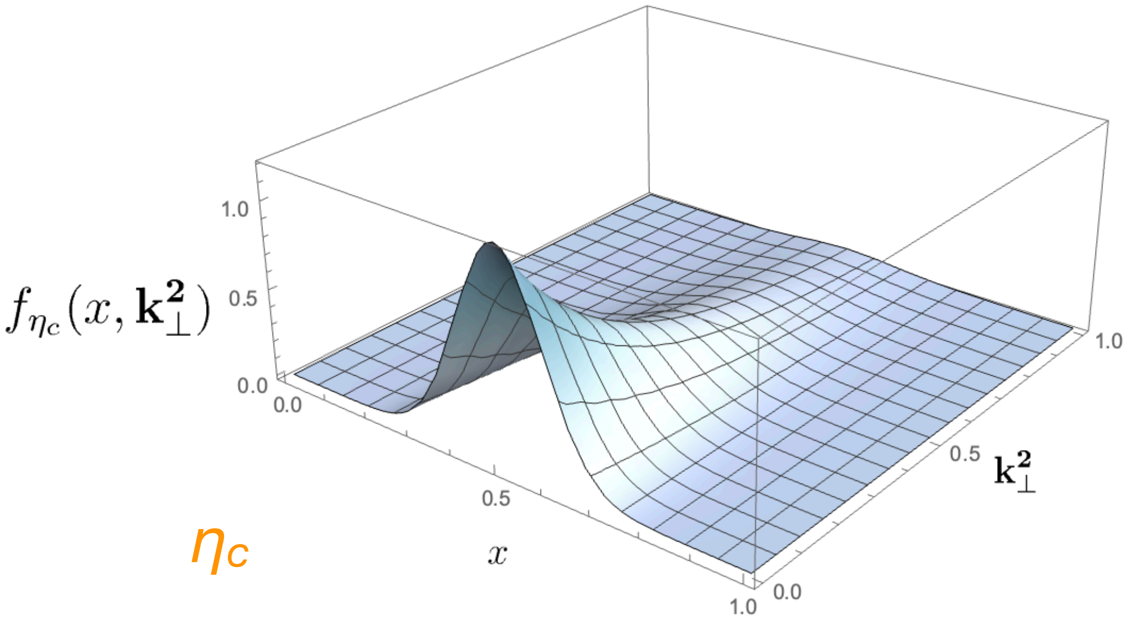


B_u

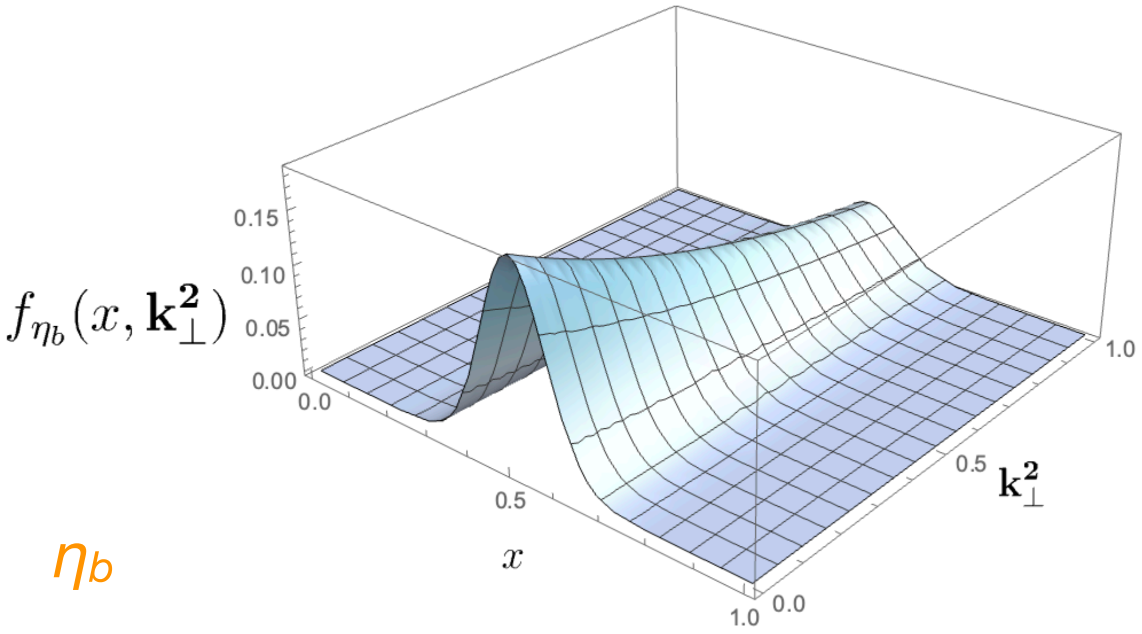
Transverse Distribution Functions



B_c

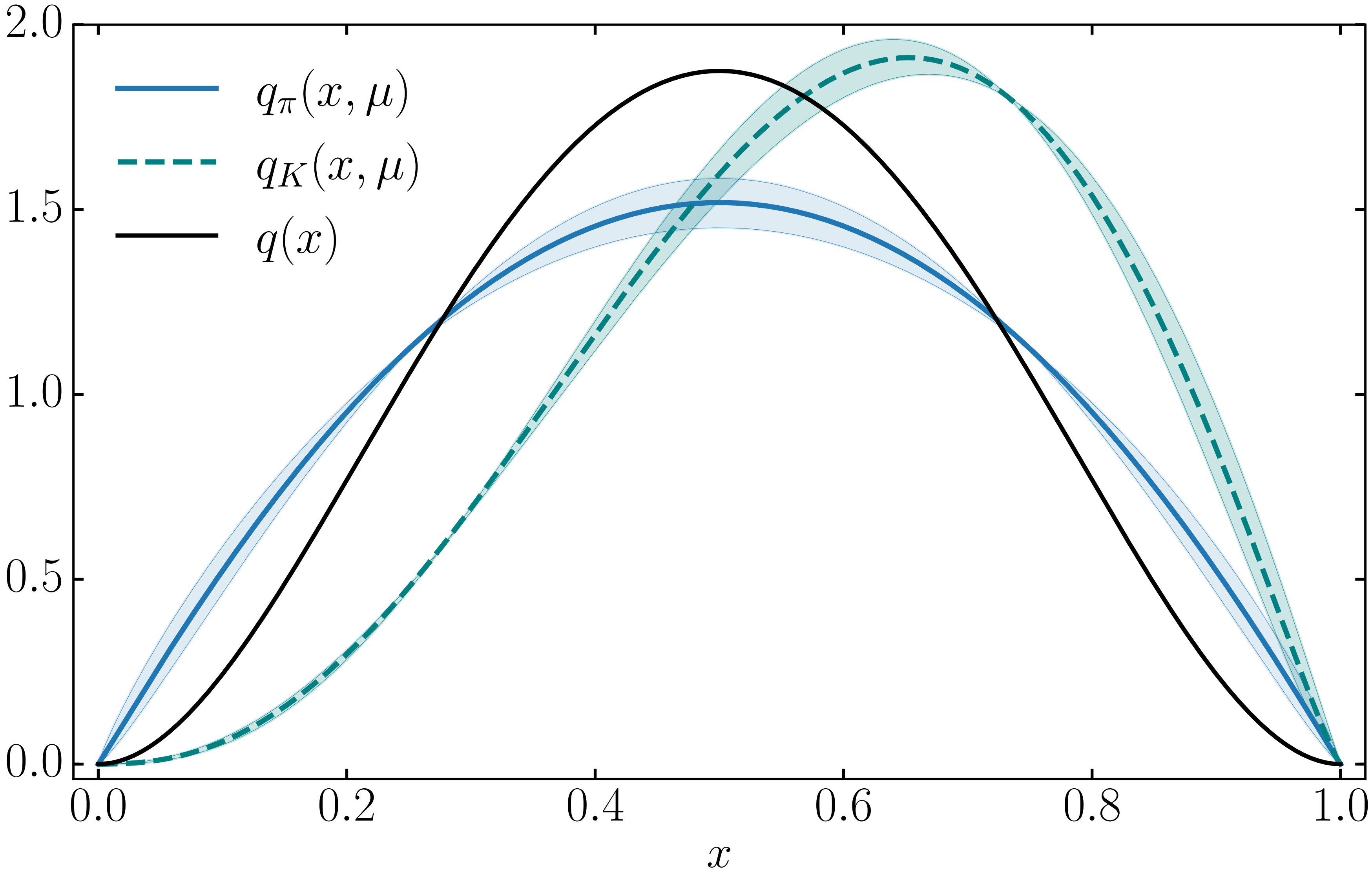


η_c



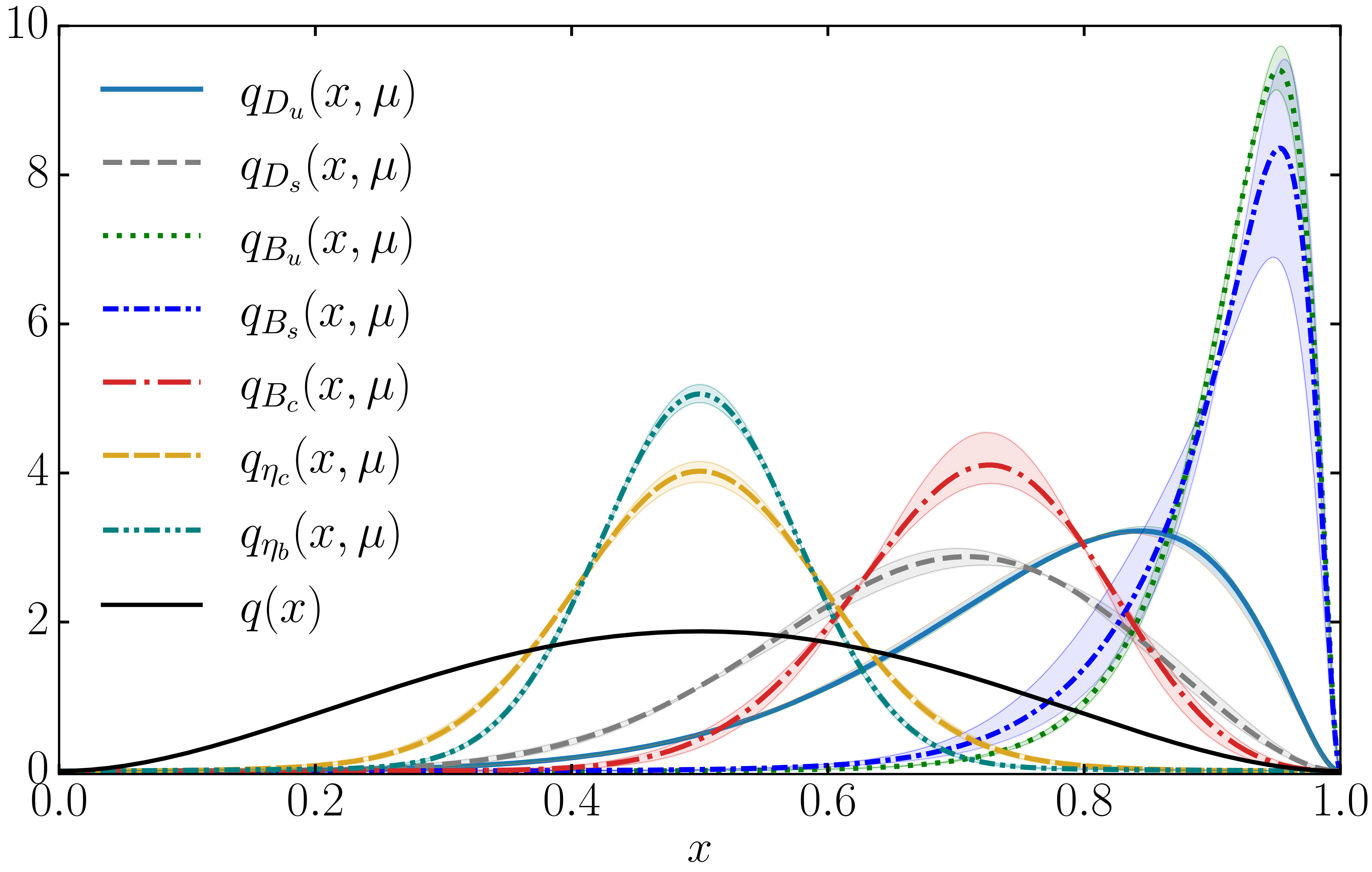
η_b

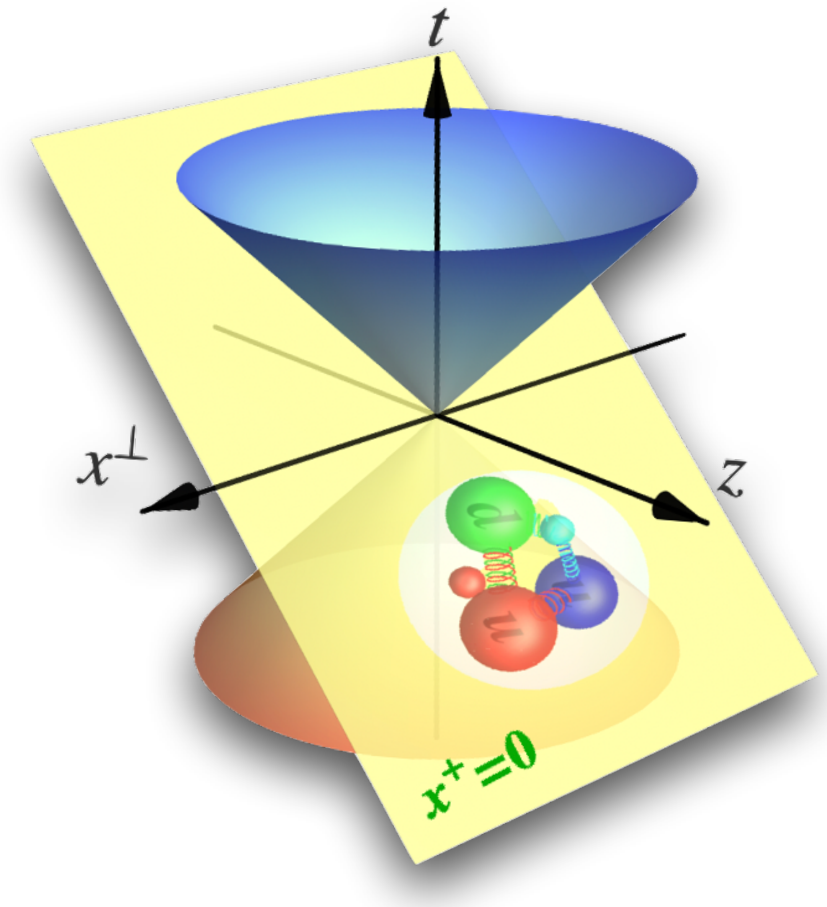
Parton Distribution Functions



The PDFs can be parametrized by: $q(x;\mu_0) = 30[x(1-x)]^2 \left[1 + \sum_{j=1}^{j_m} a_j C_j^{5/2}(2x-1)\right]$

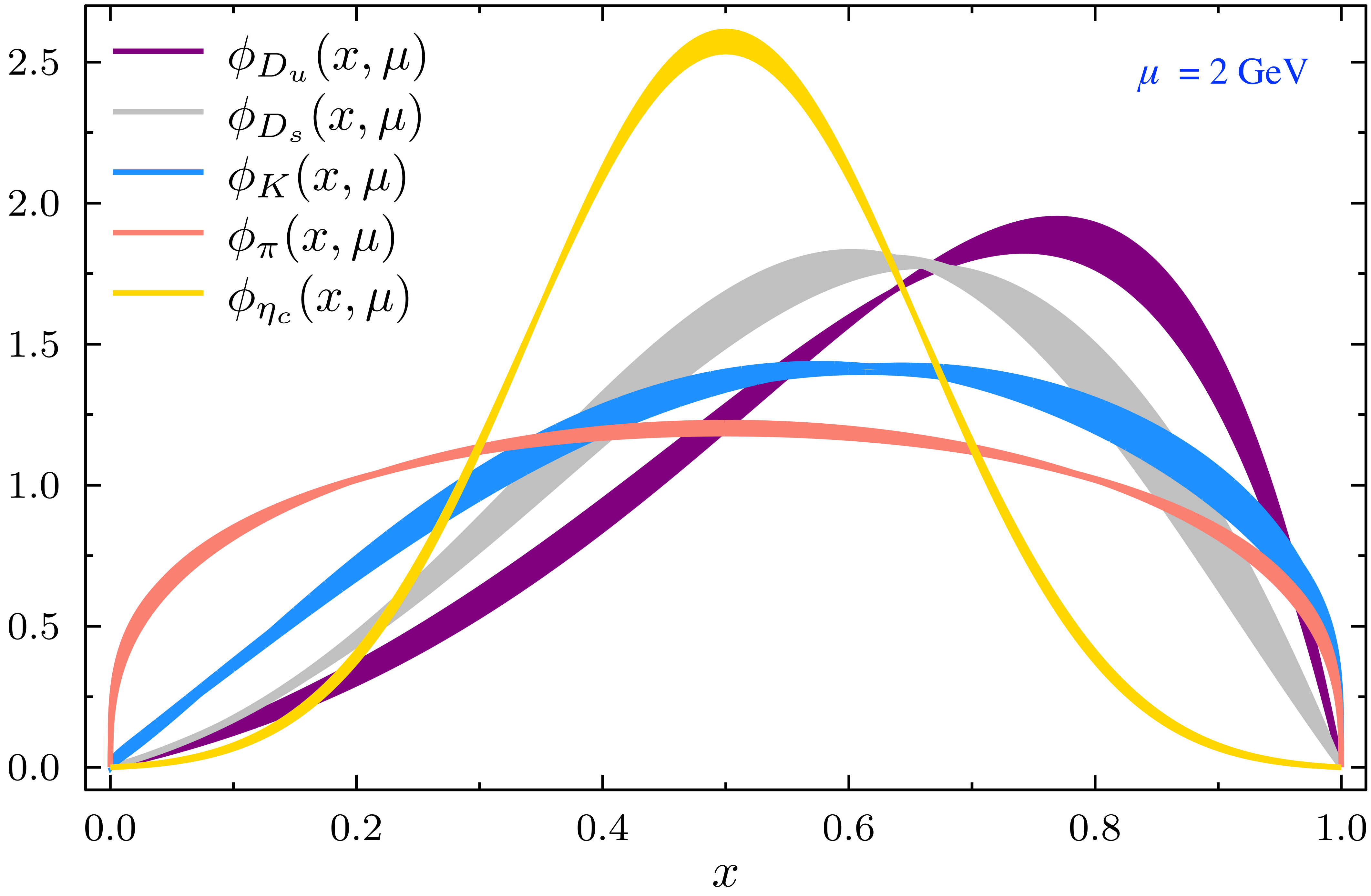
Parton Distribution Functions





Meson Distribution Amplitudes on the Light Front

Light-Cone Distribution Amplitudes



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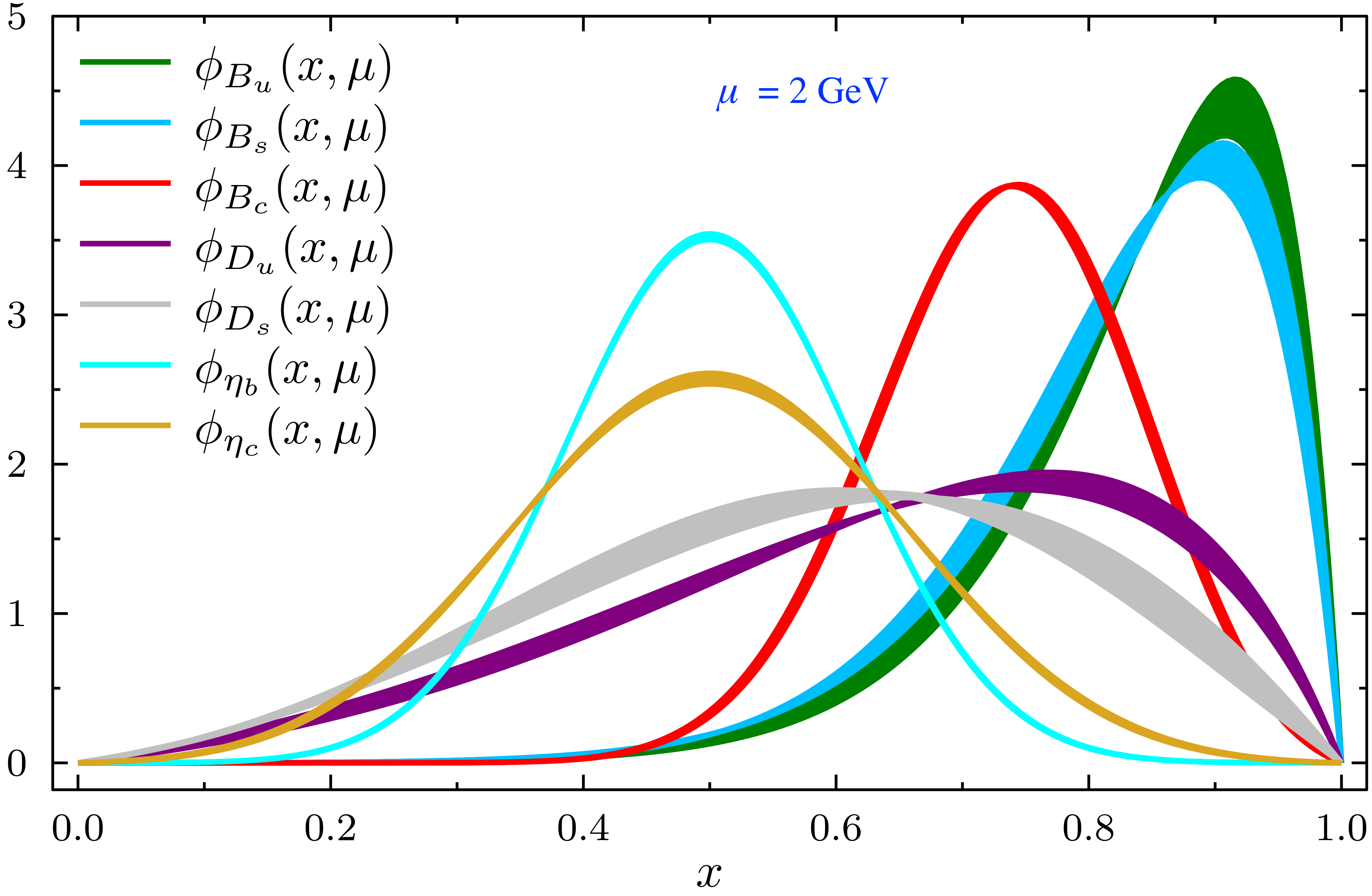
Light-Cone Distribution Amplitudes



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Vector Meson LCDAs

- Vector mesons are described by two distribution amplitudes $\phi_V^{\parallel}(x;\mu)$ and $\phi_V^{\perp}(x;\mu)$
- The distributions are obtained via the projections :

$$(n\cdot P) f_V \phi_V^{\parallel}(x;\mu) = \frac{m_V N_c \mathcal{Z}_2}{\sqrt{2}} \mathrm{Tr}_D \int^{\Lambda} \frac{d^4k}{(2\pi)^4} \delta(n\cdot k_\eta - x\, n\cdot P)\gamma\cdot n\, n_\nu \chi^{fg}_{V\nu}(k;P),$$

$$f_V^{\perp} \phi_V^{\perp}(x;\mu) = -\frac{N_c \mathcal{Z}_T}{2\sqrt{2}} \mathrm{Tr}_D \int^{\Lambda} \frac{d^4k}{(2\pi)^4} \delta(n\cdot k_\eta - x\, n\cdot P) n_\mu \sigma_{\mu\rho} \mathcal{O}^{\perp}_{\rho\nu} \chi^{fg}_{V\nu}(k;P),$$

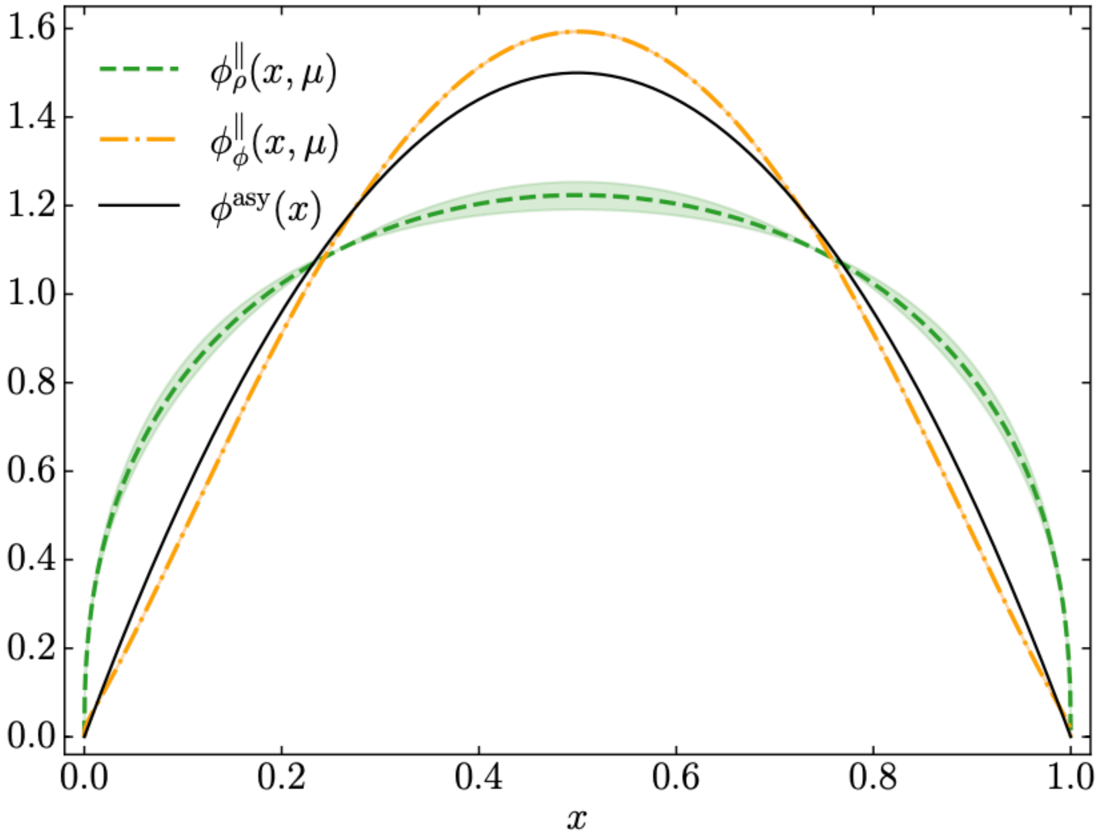
with

$$\chi^{fg}_{V\nu}(k;P) = S_f(k_\eta)\Gamma^{fg}_{V\nu}(k;P)S_g(k_{\bar\eta}), \quad \text{and} \quad \mathcal{O}^{\perp}_{\rho\nu} = \delta_{\rho\nu} + n_\rho \bar{n}_\nu + \bar{n}_\rho n_\nu,$$

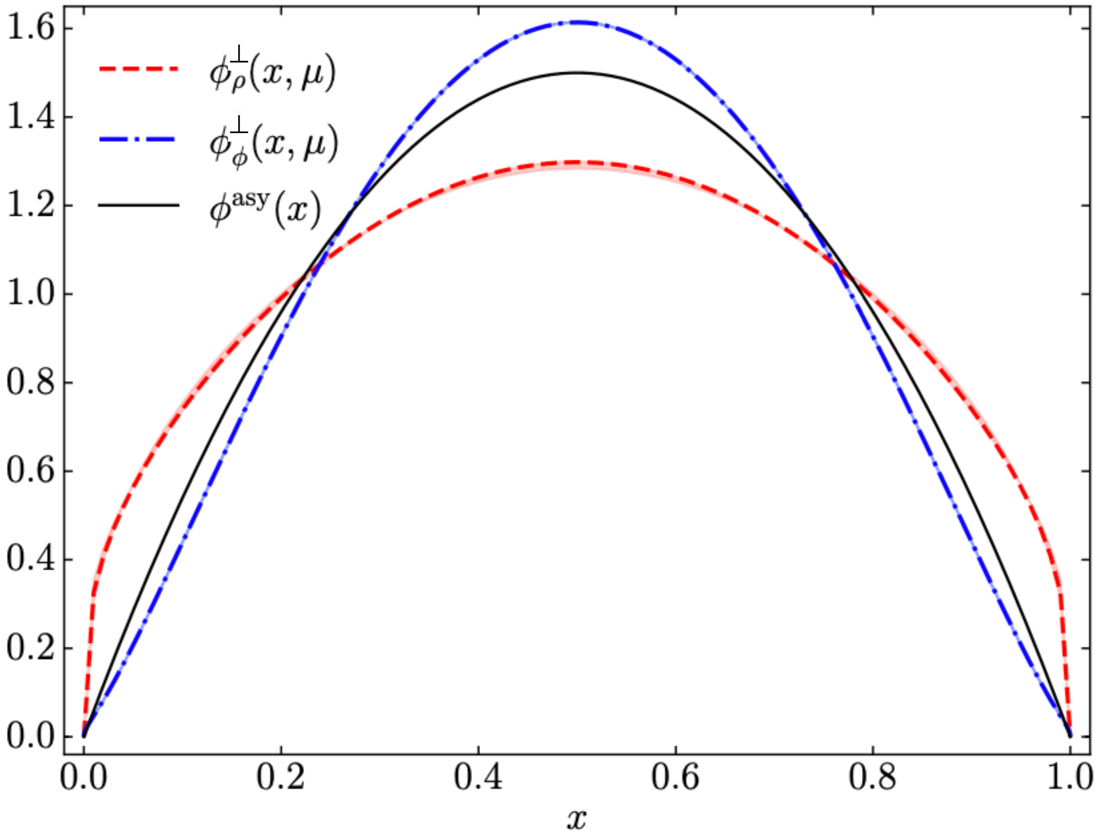
$n^2 = 0$; $P^2 = -m_V^2$ and $n\cdot P = -m_V$; $\bar{n}$ is a conjugate light-like four-vector, $\bar{n}^2 = 0$, $n\cdot \bar{n} = -1$, $\bar{n}\cdot P = -m_V/2$

Vector Meson LCDAs

ρ and ϕ mesons: longitudinal LCDA



ρ and ϕ mesons: transverse LCDA



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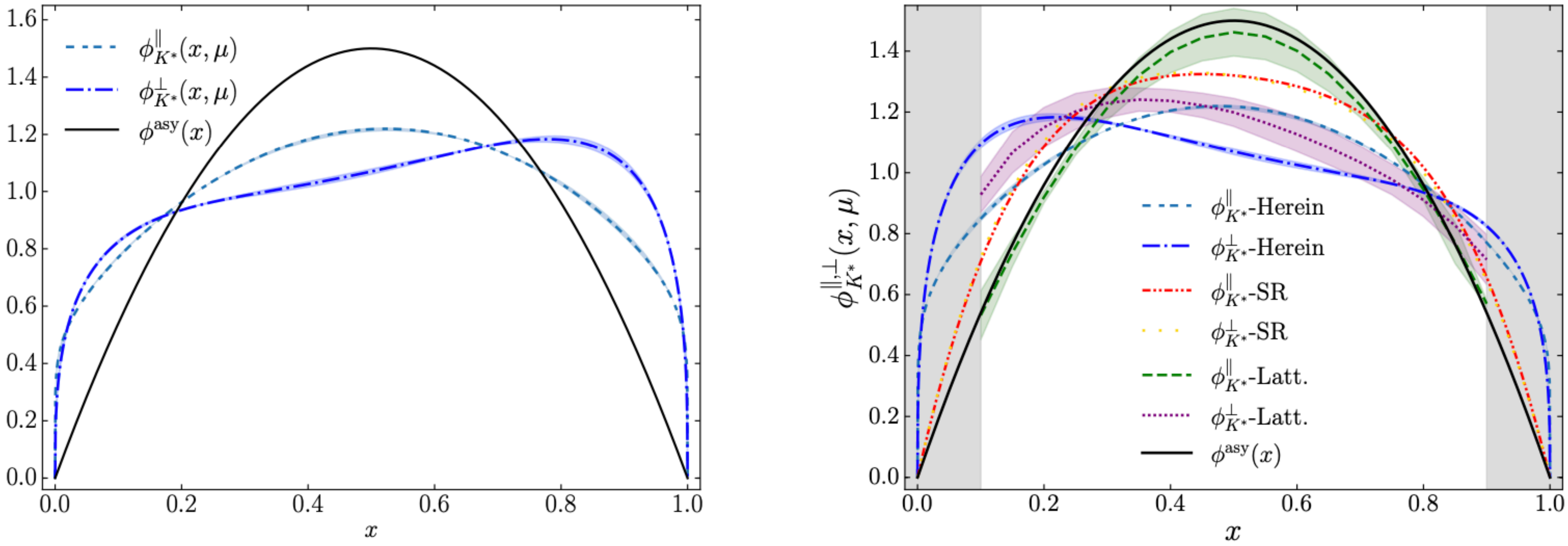
We find: $\phi_\phi^{\parallel}(x,\mu) \approx \phi_\phi^{\perp}(x,\mu)$

Light-Cone Distribution Amplitudes

K^* meson longitudinal and transverse LCDA



Comparison between our predictions for the K^*-LCDAs and Lattice QCD calculations [20]. **Left panel:** Calculated longitudinal and transverse LCDAs: $\phi^{\parallel}_{K^*}(x,\mu)$ and $\phi^{\perp}_{K^*}(x,\mu)$. **Right panel:** Comparison with SR and lattice QCD calculations [16, 20], to compare our results we just replacing $x \to 1-x$ in Eq. (3.37).

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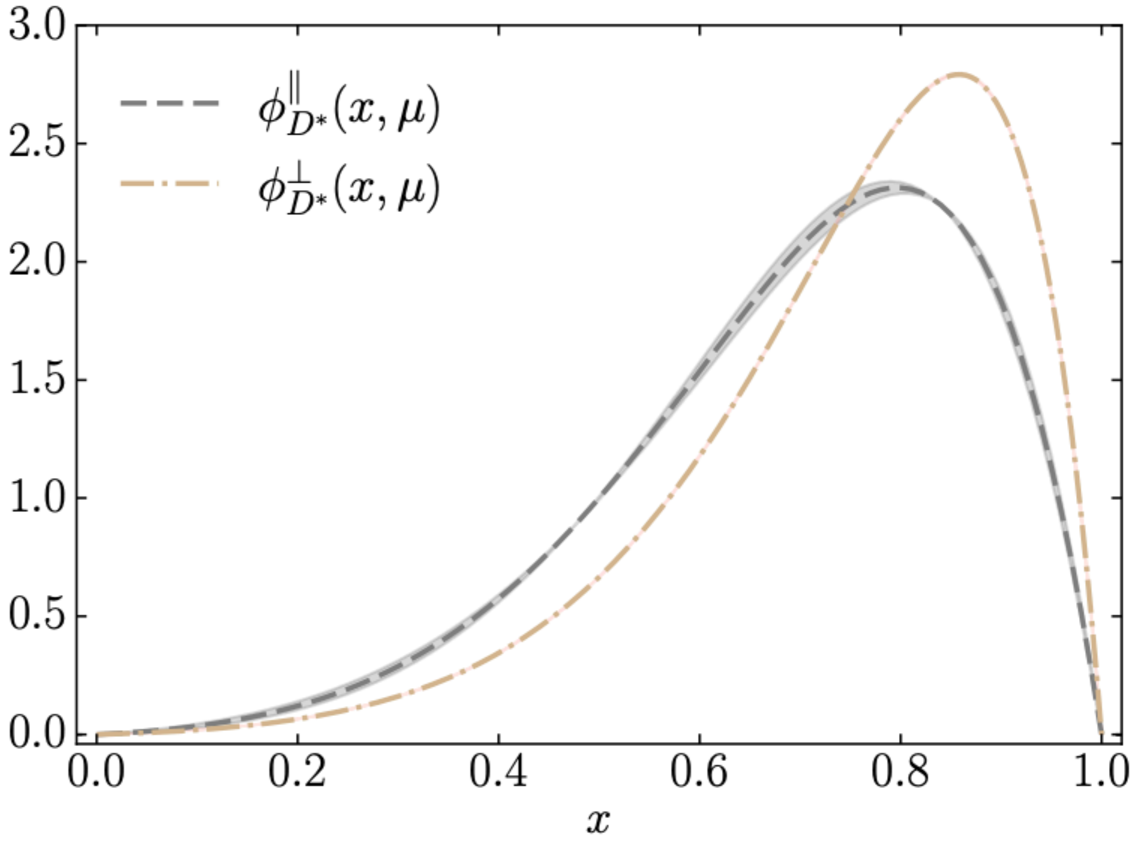
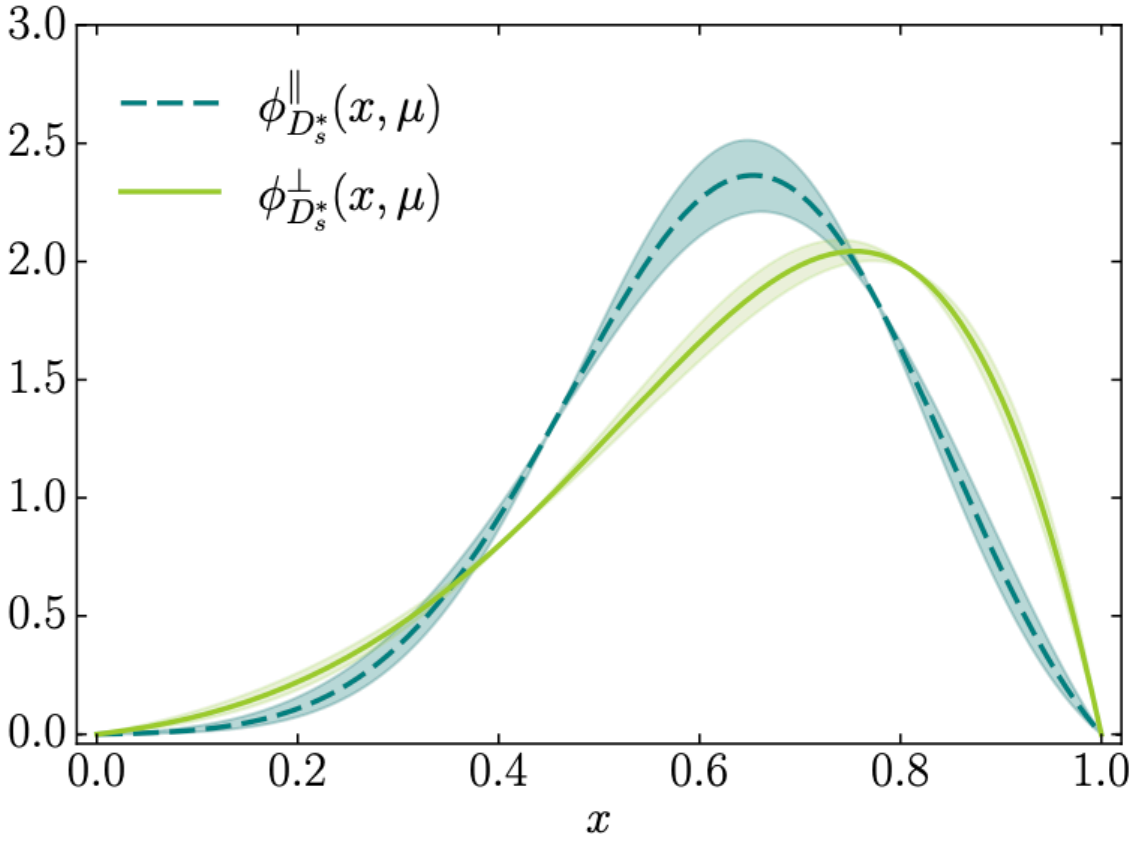
Light-Cone Distribution Amplitudes

D^* and D_s^* mesons longitudinal and transverse LCDA





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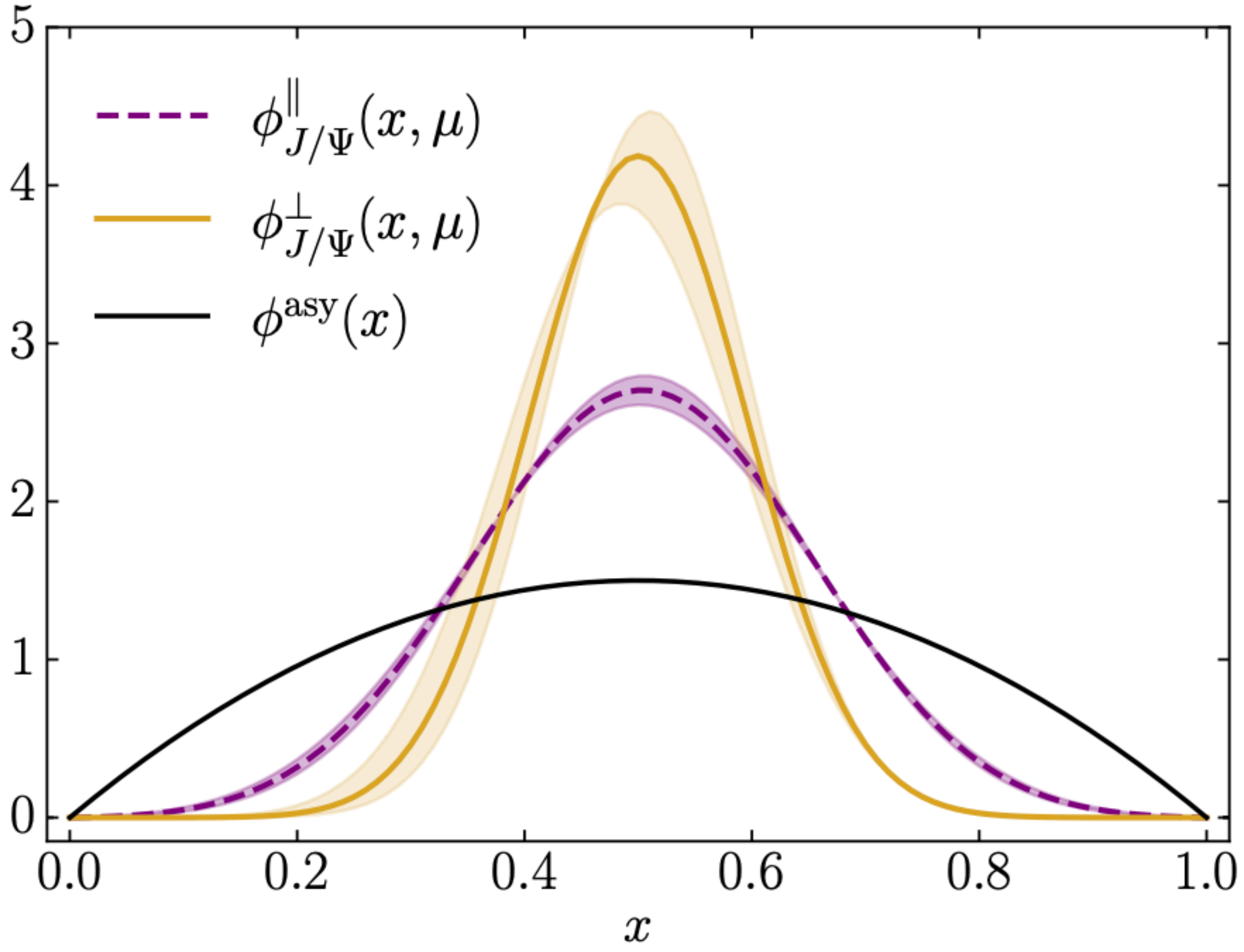
Light-Cone Distribution Amplitudes

J/ψ meson longitudinal and transverse LCDA



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Conclusions & Progress

- Much progress was made from QCD-based modeling toward nonperturbative numerical solutions of quark propagators and quark-antiquark bound states for flavored mesons satisfying chiral symmetry and Poincaré covariance.

- This approach reproduces very well the charmonium and bottonium as well as D and B meson mass spectrum and their weak decay constants.

- The three-dimensional momentum landscape of light and heavy mesons is obtained from different light-front projection of their Bethe-Salpeter wave function and don't involve the calculation of diagrams.

- For all LCDA, PDF and TMD we can readily provide functional parametrized expressions.

- Current progress: *improve beyond-leading corrections in BSE kernel for heavy-light mesons; computing GPDs and gravitational form factors; extension to nucleons.*