Unlocking QCD dynamics using jet substructure

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Jet substructures and characteristic scales

**Single prong observables**
- Jet angularities
- Energy-energy correlations
- Jet shape

**Multi-prong observables**
- N-subjettiness
- $D_2$

**IRC unsafe /NP sensitive**
- Hadron in jet
- Multiplicities
- Jet charge

**Groomed observables**
- All of the above
- Observables characterizing grooming (SD): $z_g, \ \theta_g = R_g/R$

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**Hard**
- $p_T$

**Jet algorithm**
- $p_T R$

**Energy profile**
- $p_T r$

**Jet mass**
- $m^2_J/p_T$

**Grooming**
- $z_{cut} p_TR$

**Temperature**
- $T$

**Hadronization**
- $\Lambda$
Jet substructures and characteristic scales

Constrain BSM models

Precision probe of QCD

Probe of quark gluon plasma

**Introduction**

Soft Drop Groomed Jets

Fragmenting Jet Function

Constrain BSM models

Precision probe of QCD

Probe of quark gluon plasma
Lund diagram to map different splitting

- Lund diagram is useful to visualize collinear and soft splittings.
- Map splitting history into Lund Plane.

Fig. from 5th Heavy Ion Jet Workshop Report, 2018
Lund diagram to map different splitting

- Devise jet substructure observables to probe different kinematical regions to discriminate different jet quenching models and tune Monte Carlo (with / without medium).
Soft Drop Groomed Jet Observables

- Different grooming parameters removes different region of phase space
- Reduces NP effects by removing soft radiations.

Soft Drop Condition
\[ z > z_{\text{cut}} \left( \frac{\Delta R_{12}}{R} \right)^\beta \]

- Soft drop grooming algorithms:
  1. Reorder emissions in the identified jet according to their relative angle using C/A jet algorithm.
  2. Recursively remove soft branches until soft drop condition is met.
Soft Drop Groomed Jet Observables

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I. Usual jet observables
- Jet mass
Soft Drop Groomed Jet Observables

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I. Usual jet observables

- Jet mass \( (a = 0) \)

- Jet angularity \( \tau_a \)

\[
\tau_a = \frac{1}{p_T} \sum_{i \in J} p_{T,i} (\Delta R_{i,J})^{2-a}
\]

- \( \ldots \)
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- …

II. Observables unique to soft drop

- At the time soft drop condition is met
  \[
  z > z_{cut} \left( \frac{\Delta R_{12}}{R} \right)^{\beta}
  \]
  \[\tau_g = z \quad R_g = \Delta R_{12}\]
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III. Soft-sensitive observables unique to groomed jets
- Energy drop \( \Delta_E = \frac{p_T - p_{T}^{gr}}{p_T} \)
- Angle between standard and groomed jet axes
- …
Soft Drop Groomed Jet Observables

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- Energy drop \(\Delta E = \frac{p_T - p_T^{gr}}{p_T}\)
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- …
QCD factorization for inclusive jet

\[ pp \rightarrow J + X \]

- Jet dynamics isolated to jet function \( J_c \)
- Similar factorization holds for AA collision? \( J_c \rightarrow J_c^{\text{med}} \)

**Factorization for pp**

\[
\frac{d\sigma^{pp\rightarrow hX}}{dp_T d\eta} = \sum_{a,b,c} f_a \otimes f_b \otimes H_{ab}^{c} \otimes D_{c}^{b} \\
\Lambda_{\text{QCD}} p_{T} \Lambda_{\text{QCD}}
\]

**Inclusive Jet**

\[
\frac{d\sigma^{pp\rightarrow JX}}{dp_T d\eta} = \sum_{a,b,c} f_a \otimes f_b \otimes H_{ab}^{c} \otimes J_{c} \\
\Lambda_{\text{QCD}} p_{T} p_{T} R
\]

**RG evolution**

\[
\mu \frac{d}{d\mu} J_i = \sum_{j} P_{ji} \otimes J_j
\]

Dasgupta, Dreyer, Salam, Soyez `15
Kaufmann, Mukherjee, Vogelsang `15
Kang, Ringer, Vitev `16
Dai, Kim, Leibovich `16
Jet substructure factorization

Inclusive jet production  \( pp \rightarrow \text{jet} + X \)

\[
\frac{d\sigma^{pp\rightarrow\text{jet}X}}{dp_T d\eta} = \sum_{a,b,c} f_a \otimes f_b \otimes H_{ab}^c \otimes J_c \Lambda_{QCD} p_T p_T R
\]

Jet substructure  \( \rho \)

\[
\frac{d\sigma^{pp\rightarrow\text{jet}(\rho)X}}{dp_T d\eta d\rho} = \sum_{a,b,c} f_a \otimes f_b \otimes H_{ab}^c \otimes G_c(\rho) \Lambda_{QCD} p_T p_T R
\]

and other scale(s) depending on  \( \rho \)

\( \mathcal{G}(\rho) \)
Jet angularity $\tau_a$

\[
\frac{d\sigma_{pp\to jet(\tau_a)}X}{dp_T d\eta d\tau_a} = \sum_{a,b,c} f_a \otimes f_b \otimes H^c_{ab} \otimes G_c(\tau_a)
\]

and $\tau_a$ dependent scales

$\tau_a = \frac{1}{p_T} \sum_{i\in J} p_{T,i}(\Delta R_{i,j})^{2-a}$

- A generalized class of IR safe observables for $-\infty < a < 2$
- Parameter $a$ gives varying sensitivity to collinear radiations.

$\tau_{pp}^{0} = \frac{m_J^2}{p_T^2} + O((\tau_{pp}^{0})^2)$

ATLAS Preliminary
Pb+Pb 0-10%
126 < $p_T$ < 158 GeV

$g(\text{broadening}) = \frac{1}{p_T} \sum_{i\in J} p_{T,i}(\Delta R_{i,j})$

- Analysis for different $a$ currently in progress. ALICE Preliminary
Factorization for the jet angularity

- The ungroomed case ($\tau_a \ll R^{2-a}$)

\[
\tau_a \sim z \theta^{2-a}
\]

- The groomed case ($\tau_{a, gr} / R^{2-a} \ll z_{\text{cut}} \ll 1$)

\[
z > z_{\text{cut}} (\theta / R)^\beta
\]

Hard-collinear

\[
\theta_H \sim R \quad z_H \sim 1
\]

Collinear

\[
z_c \sim 1 \quad \theta_c \sim \tau_a \frac{1}{2-a}
\]

(Collinear-)soft

\[
\theta_s \sim R \quad z_{cs} \sim \frac{\tau_a}{R^{2-a}}
\]

\[\notin \text{ gr soft} \]

\[
\theta_{\notin \text{gr}} \sim R \quad z_{\notin \text{gr}} \sim z_{\text{cut}} \left(\frac{\theta}{R}\right)^\beta = z_{\text{cut}}
\]

\[\in \text{ gr soft (collinear-soft)}\]

\[
z_{\text{gr}} \sim z_{\text{cut}} \left(\frac{\theta}{R}\right)^\beta = z_{\text{cut}} \left(\frac{\frac{\tau_a}{R^{2-a}}}{2-a+\beta}\right) = z_{\text{cut}} \left(\frac{\tau a R^\beta}{z_{\text{cut}}}ight)^{\frac{1}{2-a+\beta}}
\]

\[
\theta_{\text{gr}} \sim \left(\frac{\tau a R^\beta}{z_{\text{cut}}}ight)^{\frac{1}{2-a+\beta}}
\]
Factorization for the jet angularity

\[ J_c \rightarrow G_c \]

- The ungroomed case ( \( \tau_a \ll R^{2-a} \) )
  \[ G_i(z, p_T R, \tau_a, \mu) = \sum_j H_{i \rightarrow j}(z, p_T R, \mu)C_j(\tau_a, p_T, \mu) \otimes S_j(\tau_a, p_T, R, \mu) \]

- Jointly resums large logs \( \alpha_s^n \ln^n R \) and \( \alpha_s^n \ln^{2n} \tau_a^{2-a} / R \)

- The groomed case ( \( \tau_{a,gr}/R^{2-a} \ll z_{cut} \ll 1 \) )
  \[ G_i(z, p_T R, \tau_a, z_{cut}, \beta, \mu) = \sum_j H_{i \rightarrow j}(z, p_T R, \mu)S_{j}^{gr}(p_T, R, z_{cut}, \beta, \mu)C_j(\tau_a, p_T, \mu) \otimes S_j^{gr}(\tau_a, p_T, R, z_{cut}, \beta, \mu) \]

- Jointly resums large logs \( \alpha_s^n \ln^n R \), \( \alpha_s^n \ln^{2n} \tau_a^{2-a} / R \), and \( \alpha_s^n \ln^{2n} z_{cut} \).
Factorization for the groomed jet angularity

- Ungroomed jet mass has large NP shifts

\[
\frac{d\sigma}{dp_T d\eta d\tau_a} = \int dk F_a(k) \frac{d\sigma_{\text{pert}}}{dp_T d\eta d\tau_a} \left( \tau_a - \frac{R^{1-a}}{p_T} k \right)
\]

Perturbative result \( \otimes \) NP shape function

\[ F_a(k) = \left( \frac{4k}{Q_a^2} \right) \exp \left( -\frac{2k}{Q_a} \right) \]

\[ \Omega_a = \int dk k F_a(k) \]

\[ \Omega_a = \Omega_a^{\text{had}} + \Omega_a^{\text{MPI}} \]

\( \Omega_a^{\text{had}} = \langle 0 | \mathcal{O} | 0 \rangle \sim 1 \text{ GeV} \) is universal. non-universal

- Groomed jet mass is more robust to NP effects

- Nonperturbative corrections to groomed jet mass

\[ \text{Hoang, Mantry, Pathak, Stewart `19} \]

\[ \text{Lee, Sterman `07} \]

\[ \text{Stewart, Tackmann, Waalewijn `15} \]

\[ \text{Kang, KL, Liu, Ringer `18} \]

\[ \text{Kang, KL Liu, Ringer `18} \]
\( \alpha_s \) extraction

- World Average with 0.9\% total uncertainty
  \[ \alpha_s(m_Z) = 0.1181 \pm 0.0011 \]

- Most precise input: lattice determination

- Next precise input: \( e^+e^- \) event shape determination: thrust and C-parameter.
  - \( 3 - 4\sigma \) tension with lattice.

- High-quality of data pouring out of the LHC.
  - Can we carry out an \( \alpha_s \) extraction using \( pp \) data?
\( \alpha_s \) extraction

- **Key challenge:** (for \( e^+e^- \) event shape extraction or ungroomed jet substructure at \( pp \))

Degeneracy between the extractions of \( \alpha_s \) and fitting \( \Omega^{\text{had}} \)

Also, contaminations from MPI

Fig. from Abbatte, Fickinger, Hoang, Mateu, Stewart `10
\( \alpha_s \) extraction

- **Key challenge:** (for \( e^+e^- \) event shape extraction or ungroomed jet substructure at \( pp \))
  degeneracy between the extractions of \( \alpha_s \) and fitting \( \Omega^{\text{had}} \)

Also, contaminations from MPI

- **Desire:**
  - reduced sensitivity to non-perturbative effects (hadronizations and MPI)
  - additional handle(s) to lift degeneracy between extractions of \( \alpha_s \) and fitting \( \Omega^{\text{had}} \)

Groomed jet angularities!

- **Proof-of-principle Monte Carlo study shows groomed angularity to be an ideal observable.**
  - Currently feasible to determine with 10% uncertainty.

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Fig. from
Abbate, Fickinger, Hoang, Mateu, Stewart `10

Les Houches `17
Introduction

Soft Drop Groomed Jets

Fragmenting Jet Function

Groomed jet radius

• Dynamically determines the size of the hard-collinear splitting

$R_g = R_{12}$

- What is the transverse resolution length of the medium?

• Also gives the “active area” of the groomed jet

$\text{Active area } \sim \pi R_g^2$

- Can serve as a proxy for the sensitivity to pileup.
- Complicated by clustering effects

• Factorization formalism necessary for studying many groomed jet observables

- To regulate divergence of the Sudakov safe observables
- Sometimes the observables are invariably coupled to $R_g$ measurements.

ex: $\Delta E$, energy drop
Groomed jet radius

- Factorization derived to NLL
  Kang, KL, Liu, Neill, Ringer `19

- Data comparison
  ATLAS `19

• Small MPI effects

Kang, KL, Liu, Neill, Ringer `19
Larkoski, Marzani, Soyez, Thaler `14

\[ \log \frac{1}{z} \quad \beta > 0 \]

\[ \log \frac{1}{z_{cut}} \]

\[ \theta = R_g \]

\[ \log \frac{R_0}{\theta} \]

\[ \sqrt{s} = 13 \text{ TeV}, \ \text{anti-}k_T, \ R = 0.8 \]
\[ p_T > 600 \text{ GeV}, \ |\eta| < 1.5 \]
soft drop, \[ z_{cut} = 0.1 \]

\[ +\text{MPI+had} \]
\[ +\text{MPI} \]
\[ +\text{Partonic} \]
**Energy drop**

- Energy drop measurements in soft drop groomed jet is coupled with $R_g$ measurements.
  - Factorizations for trimmed jets, iterated soft drop groomed jets \( Cal, KL, Ringer, Waalewijn \ `20 \)
- All energy drop dependent “modes” are only sensitive to soft modes only.
  - Does the medium modify the soft and collinear radiations differently?

\[
\Delta E = \frac{p_T - p_T^{gr}}{p_T}
\]
Energy drop

- Factorization framework differential in both $\Delta E$, $R_g$:
  \[ \frac{d\sigma}{dp_T d\eta d\Delta E dR_g} \]

- Integrate over regions of $R_g$: $\theta_g^{\text{cut}} < \theta_g < 1$ ($\theta_g = \frac{R_g}{R}$)
  
  - Experimental tracking efficiency may require $\theta_g^{\text{cut}}$
  
  - Different $\theta_g^{\text{cut}}$ gives varying sensitivity to non-perturbative effects
    
    - May be useful for MC tuning

---

Experimental tracking efficiency may require $\theta_g^{\text{cut}}$

Different $\theta_g^{\text{cut}}$ gives varying sensitivity to non-perturbative effects

- May be useful for MC tuning
Energy drop

- Factorization framework differential in both $\Delta_E, R_g$:
  \[
  \frac{d\sigma}{dp_T d\eta d\Delta_E dR_g}
  \]

- Integrate over regions of $R_g$: $\theta_g^{\text{cut}} < \theta_g < 1 \ (\theta_g = \frac{R_g}{R})$
  - Experimental tracking efficiency may require $\theta_g^{\text{cut}}$
  - Different $\theta_g^{\text{cut}}$ gives varying sensitivity to non-perturbative effects
    - May be useful for MC tuning

- Good agreement within perturbative region

\[\sqrt{s} = 13 \text{ TeV}, p_T = 140-160 \text{ GeV}, R = 0.8, |\eta| < 2\]
Soft Drop: $z_{\text{cut}} = 0.5, \beta = 1.5, \theta_g^{\text{cut}} = 0.25$

Cal, KL, Ringer, Waalewijn '20

CMS '17
Fragmenting Jet Function (FJJF)

\[ pp \rightarrow h + X \]

Factorization: separation of dynamics

\[
\frac{d\sigma_{pp\rightarrow hX}}{dp_T d\eta} = \sum_{a,b,c} f_a \otimes f_b \otimes H_{ab}^c \otimes D_h^c
\]

\[ \Lambda_{QCD} \quad p_T \quad \Lambda_{QCD} \]

- Fragmentation Functions

\[ D_c^h \]

Parton polarization

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<thead>
<tr>
<th>Hadron polarization</th>
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Fragmenting Jet Function (FJF)

\[ pp \rightarrow h + X \]

Factorization: separation of dynamics

\[
\frac{d\sigma_{pp\rightarrow hX}}{d^3p_T d\eta} = \sum_{a,b,c} f_a \otimes f_b \otimes H_{ab}^c \otimes D_c^h
\]

\[ \Lambda_{\text{QCD}} \quad p_T \quad \Lambda_{\text{QCD}} \]

• Fragmentation Functions

• Can jet substructure measurements help us to constrain fragmentation functions?
Fragmenting Jet Function (FJJ)

Unpolarized case:

\[
\frac{d\sigma^{pp\rightarrow \text{jet}(h)}_X}{dp_T d\eta d z_h} = \sum_{a,b,c} f_{a/A} \otimes f_{b/B} \otimes H_{ab}^{c} \otimes g_{c}^{h}(z_h) \otimes p_T R \Lambda_{QCD}
\]

where

\[
z = \frac{p_T^J}{p_T^c}
\]

\[
z_h = \frac{p_T^h}{p_T^J}
\]

Procura, Stewart `10
Arleo, Fontannaz, Guillet, Nguyen `14
Kaufmann, Mukherjee, Vogelsang `15
Kang, Ringer, Vitev `16
Dai, Kim, Leibovich `16
Fragmenting Jet Function (FJJ)

Unpolarized case:

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\frac{d\sigma^{pp\rightarrow jet(h)}X}{dp_Td\eta dz_h} = \sum_{a,b,c} f_{a/A} \otimes f_{b/B} \otimes H_{ab}^c \otimes g_c^h(z_h) \otimes \frac{p_T R}{\Lambda_{QCD}}
\]

where \[z = \frac{p_T}{p_T}
\]

\[
z_h = \frac{p^{h}_T}{p^{J}_T}
\]

where \[z = \frac{p^{h}_T}{p^{c}_T}
\]

Procura, Stewart `10
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Fragmenting Jet Function (FJF)

Unpolarized case:

\[
\frac{d\sigma^{pp\rightarrow\text{jet}(h)}X}{dp_T d\eta d\zeta_h} = \sum_{a,b,c} f_{a/A} \otimes f_{b/B} \otimes H_{ab}^c \otimes g_{c}^h(\zeta_h) \otimes \frac{p_T}{\Lambda_{\text{QCD}}} \\
\text{where } \zeta = \frac{p_T^J}{p_T^C} \\
\zeta_h = \frac{p_T^h}{p_T^J}
\]

IR sensitivity and require matching:

\[
g_{c}^h(\zeta, \zeta_h, p_T R, \mu) = \sum_j j_{ij}(\zeta, \zeta_h, p_T R, \mu) \otimes D_{ij}^h(\zeta_h, \mu)
\]

• Collinear FJFs can be related to collinear FFs

Procura, Stewart ¹⁰
Arleo, Fontannaz, Guillet, Nguyen ¹⁴
Kaufmann, Mukherjee, Vogelsang ¹⁵
Kang, Ringer, Vitev ¹⁶
Dai, Kim, Leibovich ¹⁶
Polarized Fragmenting Jet Function (FJF)

Polarized case:

$$\frac{d\sigma_{\bar{p}p \rightarrow \text{jet}(\bar{h}) X}}{dp_T\,dn\,dz_h} = \sum_{a,b,c} \Lambda_A \, g_{a/A} \otimes f_{b/B} \otimes \Delta_{LL} H_{\bar{a}\bar{b}}^{\bar{c}} \otimes \Delta_{L} G_{c}^{h}(z_h) \, \Lambda_h$$

$$\frac{d\sigma_{\bar{p}p \rightarrow \text{jet}(\bar{h}) X}}{dp_T\,dn\,dz_h} = \sum_{a,b,c} S_{A\perp}^i \, h_{a/A} \otimes f_{b/B} \otimes \Delta_{TT} (H_{ij})_{\bar{a}\bar{b}}^{\bar{c}} \otimes \Delta_{T} G_{c}^{h}(z_h) \, S_{h\perp}^j$$

where

$$z = \frac{p_T^J}{p_T^c}$$

$$z_h = \frac{p_h^J}{p_T^J}$$

$$d\sigma_{LL} = \frac{d\sigma(\Lambda_A, \Lambda_h) - d\sigma(\Lambda_A, -\Lambda_h)}{2}$$

$$d\sigma_{TT} = \frac{d\sigma(\bar{S}_{A\perp}, \bar{S}_{h\perp}) - d\sigma(\bar{S}_{A\perp}, -\bar{S}_{h\perp})}{2}$$

Similar definitions for

$$\Delta_{LL} H_{\bar{a}\bar{b}}^{\bar{c}}$$ and $$\Delta_{TT} (H_{ij})$$

Kang, KL, Zhao, `20
Polarized Fragmenting Jet Function (FJF)

Polarized case:

\[
\frac{d\sigma_{\bar{p}p\to\text{jet}(\vec{h})X}}{dp_T d\eta d\varepsilon} = \sum_{a,b,c} \Lambda_A g_{a/A} \otimes f_{b/B} \otimes \Delta_{LL} H^c_{ab} \otimes \Delta_L G^h_c(z_h) \Lambda_h
\]

\[
\frac{d\sigma_{\bar{p}p\to\text{jet}(\vec{h})X}}{dp_T d\eta d\varepsilon} = \sum_{a,b,c} S_{A\perp}^i h_{a/A} \otimes f_{b/B} \otimes \Delta_{TT}(H_{ij})^\varepsilon_{ab} \otimes \Delta_T G^h_c(z_h) S_{h\perp}^j
\]

where \( z = \frac{p_T^J}{p_T^C} \)

\( z_h = \frac{p_T^h}{p_T^J} \)

Quark polarization

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Polarized Fragmenting Jet Function (FJF)

Polarized case:

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\]

\[
\frac{d\sigma_{\overline{p}p \to \text{jet}(\vec{h})X}}{dp_T d\eta d\vec{z}_h} = \sum_{a,b,c} S_{A\perp}^i h_{a/A} \otimes f_{b/B} \otimes \Delta_{TT}(H_{ij})_{\vec{c}ab} \otimes \Delta_T G_c^h(z_h) S_h^i
\]

where

\[
z = \frac{p_T^J}{p_T^C}
\]

\[
z_h = \frac{p_T^h}{p_T^J}
\]

IR sensitivity and require matching:

\[
G_c^h(z, z_h, p_TR, \mu) = \sum_j J_{ij}(z, z_h, p_TR, \mu) \otimes D_j^h(z_h, \mu)
\]

\[
\Delta_L G_c^h(z, z_h, p_TR, \mu) = \sum_j \Delta_L J_{ij}(z, z_h, p_TR, \mu) \otimes G_j^h(z_h, \mu)
\]

\[
\Delta_T G_c^h(z, z_h, p_TR, \mu) = \sum_j \Delta_T J_{ij}(z, z_h, p_TR, \mu) \otimes H_j^h(z_h, \mu)
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Hadron polarization

Kang, KL, Zhao, `20
## Study of FJJ

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<td>✓</td>
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<tr>
<td><strong>Differential in $z_h$</strong></td>
<td>✓</td>
<td>X</td>
<td>✓</td>
</tr>
</tbody>
</table>

Also large amounts of data available for $pp$!
Fragmenting Jet Function (FJF)

- Light charged hadrons
  
  Arleo, Fontannaz, Guillet, Nguyen `14  
  Kaufmann, Mukherjee, Vogelsang `15  
  Kang, Ringer, Vitev `16  
  Neill, Scimemi, Waalewijn `16

- Photons

  Kaufmann, Mukherjee, Vogelsang `16

- Heavy flavor mesons

  Chien, Kang, Ringer, Vitev, Xing `15  
  Bain, Dai, Hornig, Leibovich, Makris, Mehen `16  
  Anderle, Kaufmann, Stratmann, Ringer, Vitev `17

- Quarkonia

  Baumgart, Leibovich, Mehen, Rothstein `14  
  Bain, Dai, Hornig, Leibovich, Makris, Mehen `16  
  Kang, Qiu, Ringer, Xing, Zhang `17  
  Bain, Dai, Leibovich, Makris, Mehen `17

\[
F(z_h, p_T) = \frac{d\sigma^{pp\rightarrow(jet,h)X}}{dp_T d\eta dz_h} / \frac{d\sigma^{pp\rightarrow jet X}}{dp_T d\eta}
\]
Longitudinally polarized $\Lambda$

Three different scenarios

- **Scenario 1**: Only strange quarks have contribution to the fragmentation.
- **Scenario 2**: Negative distributions of up and down quarks are assumed.
- **Scenario 3**: Same fragmentation for up, down, and strange quarks.
**FJF to study longitudinally polarized Λ**

\[ D_{\text{jet} \Lambda}^{\text{LL}} = \frac{d\Delta_{\text{LL}} \sigma}{d\sigma} \]

- **Scenario 1**: Only strange quarks have contribution to the fragmentation.
- **Scenario 2**: Negative distributions of up and down quarks are assumed.
- **Scenario 3**: Same fragmentation for up, down, and strange quarks.

**RHIC kinematics**
- \( \sqrt{s} = 200 \text{ GeV}, \ R = 0.4 \)
- \(|\eta_J| < 1.2, \ 10 < p_{JT} < 15 \text{ GeV}\)

\[ p(P_A, S_A) \]

\[ \Lambda_{\text{LL}}(\%) \]

\[ z_\Lambda \]

\[ D_{\text{LL}} \]

\[ h \]

\[ a, b, c \]

Gives shape expected from the scenarios
TMD hadron in jet (TMDFJF)

- Still hard-collinear factorization structure other than jet function modified.

\[
\frac{d\sigma_{pp\to \text{jet}(h)}X}{dp_T d\eta dz_h} = \sum_{a,b,c} f_{a/A} \otimes f_{b/B} \otimes H_{ab}^c \otimes G_c^h(z_h)
\]

\[
\frac{d\sigma_{pp\to \text{jet}(h)}X}{dp_T d\eta dz_h d^2 j_\perp} = \sum_{a,b,c} f_{a/A} \otimes f_{b/B} \otimes H_{ab}^c \otimes G_c^h(z_h, j_\perp)
\]
TMD hadron in jet (TMDFJF)

• Still hard-collinear factorization structure other than jet function modified.

\[
\frac{d\sigma_{pp\rightarrow \text{jet}(h)X}}{d\mathbf{p}_T d\eta dz_h d^2\mathbf{j}_\perp} = \sum_{a,b,c} f_{a/A} \otimes f_{b/B} \otimes H^c_{ab} \otimes G^h_c(z_h, \mathbf{j}_\perp)
\]

When \( \Lambda_{QCD} \lesssim j_\perp \ll p_TR, \quad \lambda \sim j_\perp/p_T \)

\[
\begin{align*}
collinear & \quad k_c \sim p_T(\lambda^2, 1, \lambda) \\
soft & \quad k_s \sim p_T(\lambda R, \lambda/R, \lambda)
\end{align*}
\]

\[
G^h_c(z, z_h, \omega, R, \mathbf{j}_\perp, \mu) = \mathcal{H}_{c\rightarrow i}(z, \omega, R, \mu) \int d^2k_\perp d^2\lambda_\perp \delta^2(z_h\lambda_\perp + k_\perp - \mathbf{j}_\perp) \\
& \quad \times D_{h/i}(z_h, \mathbf{k}_\perp, \mu, \nu) S_i(\lambda_\perp, \mu, \nu R) \\
& = C_{c\rightarrow i}(z, p_TR, \mu) \hat{D}_{h/i}(z_h, \mathbf{j}_\perp; \mu = p_TR)
\]

Standard subtracted TMDFFs, say in SIDIS

Shown to be true perturbatively at NLO

• TMDFJFs can be related to TMDFFs
Polarized **TMDFJF**

- Still hard-collinear factorization structure other than jet function modified.

\[
\frac{d\sigma_{LL}^{\bar{p}p \to \text{jet}(h)X}}{dp_T d\eta dz_h} = \sum_{a,b,c} \Lambda_A \ g_{a/A} \otimes f_{b/B} \otimes \Delta_{LL} H_{ab}^{c} \otimes \Delta_L G_{c}^{h}(z_h) \Lambda_h
\]

\[
\frac{d\sigma_{TT}^{\bar{p}p \to \text{jet}(h)X}}{dp_T d\eta dz_h} = \sum_{a,b,c} S_{A \perp}^{i} \ h_{a/A} \otimes f_{b/B} \otimes \Delta_{TT} \left( H_{ij} \right)_{ab}^{c} \otimes \Delta_T G_{c}^{h}(z_h) \ S_{h \perp}^{j}
\]
Introduction

Soft Drop Groomed Jets

Fragmenting Jet Function

Polarized TMDFJF

- Still hard-collinear factorization structure other than jet function modified.

\[
\frac{d\sigma_{\bar{p}p \to \text{jet}(\bar{h})}}{dp_T d\eta dz_h} = \sum_{a,b,c} \Lambda_A g_{a/A} \otimes f_{b/B} \otimes \Delta_{LL} H_{\bar{a}b}^c \otimes \Delta_L G_c^h(z_h) \Lambda_h
\]

\[
\frac{d\sigma_{T\bar{T} \to \text{jet}(\bar{h})}}{dp_T d\eta dz_h} = \sum_{a,b,c} S_{A \perp}^i h_{a/A} \otimes f_{b/B} \otimes \Delta_{TT} (H_{ij})_{\bar{a}b}^c \otimes \Delta_T G_c^h(z_h) S_{h \perp}^j
\]

Polarized TMD hadron in jet production

when fragmenting parton is unpolarized,

\[
G_c^h(z_h) \rightarrow G_c^h(z_h, j_\perp) F_U
\]

// longitudinally polarized,

\[
\Delta_L G_c^h(z_h) S_L \rightarrow \Delta_L G_c^h(z_h, j_\perp) F_L
\]

// transversely polarized,

\[
\Delta_T G_c^h(z_h) S_{h_T}^i \rightarrow \Delta_T G_c^h(z_h, j_\perp) F_T^i
\]

\( F_U, F_L \) and \( F_T^i \) can depend on \( j_\perp, S_{h \perp}, \Lambda_h \) depending on the polarization of the final hadron.
Polarized TMDFJF

TMD Fragmentation Functions (TMDFF)

<table>
<thead>
<tr>
<th>Quark polarization</th>
</tr>
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<tbody>
<tr>
<td>U</td>
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<td>L</td>
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<td>T</td>
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Polarized TMD hadron in jet production

- when fragmenting parton is unpolarized,
- longitudinally polarized,
- transversely polarized,

$F_U, F_L$ and $F_T^{i}$ can depend on $j_\perp, S_{h\perp}, \Lambda_h$ depending on the polarization of the final hadron.

TMD Fragmenting Jet Functions (TMDFJF)

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Actually not really good notations:

- $G_c^h(z_h) \rightarrow G_c^h(z_h, j_\perp) F_U$
- $\Delta_L G_c^h(z_h) S_L \rightarrow \Delta_L G_c^h(z_h, j_\perp) F_L$
- $\Delta_T G_c^h(z_h) S_{hT}^i \rightarrow \Delta_T G_c^h(z_h, j_\perp) F_T^{i}$
Z-tagged jet

- LHCb collaboration measured collinear FFs and \( \hat{j}_\perp \)

  Agrees well in the \( 0.1 < z_h < 0.5 \) region
Z-tagged jet

- LHCb collaboration measured collinear FFs and $j_\perp$
  
  Agrees well in the $0.1 < z_h < 0.5$ region

- LHCb collaboration measured $j_\perp$ recently.
  but with entire $0 < z_h < 1$
Polarizing FF

TMD Fragmentation Functions

<table>
<thead>
<tr>
<th></th>
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<th>L</th>
<th>T</th>
</tr>
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<tbody>
<tr>
<td>Quark polarization</td>
<td>(D^{h/q})</td>
<td>(G^{h/q})</td>
<td>(H_{L}^{h/q})</td>
</tr>
<tr>
<td>Hadron polarization</td>
<td>(D_{T}^{h/q})</td>
<td>(G_{T}^{h/q})</td>
<td>(H^{h/q}) (H_{T}^{h/q})</td>
</tr>
</tbody>
</table>

• Describes transversely polarized hadron inside unpolarized parton.

• Belle data shows different trend in \(p_T\) with different range of \(z_{\Lambda}\)

*Belle Collaboration `19*

• FJF at hadron colliders would be a great complimentary results!
Polarizing FJF

\[ \mathcal{D}_{T}^{h/c} \]

- Describes transversely polarized hadron inside the jet produced from an unpolarized parton.

\[
\frac{d\Delta \sigma_{pp \rightarrow \text{jet}(h) X}}{d \mathbf{p}_{T} d\eta d z_h d^2 \mathbf{j}_\perp} = \sum_{a,b,c} f_a \otimes f_b \otimes H_{ab}^c \otimes \mathcal{D}_{T}^{h/c}(z_h, \mathbf{j}_\perp)
\]

When \( \Lambda_{\text{QCD}} \lesssim j_\perp \ll p_T R \),

\[
\mathcal{D}_{T}^{h/c}(z, z_h, \omega_j R, j_\perp, \mu) = \mathcal{H}_{c \rightarrow i}(z, \omega_j R, \mu) \frac{\epsilon_{\mu \nu}^i S_{h \perp}^\mu}{z_h M_h} \int d^2 \mathbf{k}_\perp d^2 \mathbf{\lambda}_\perp \delta^2 (z_h \mathbf{\lambda}_\perp + \mathbf{k}_\perp - \mathbf{j}_\perp) \\
\times k_\perp^\nu D_{T}^{h/i}(z_h, \mathbf{k}_\perp, \mu, \nu) S_{i}(\mathbf{\lambda}_\perp, \mu, \nu R) \\
\sim S_{h \perp} \sin(\phi_s - \phi_j)
\]
Polarizing FJF

• Used PFF fits from Belle data

\[ \text{Callos, Kang, Terry, '20} \]

\[ D_{1T,M}(z_{\Lambda}, Q) \]

• Predictions at the LHC kinematics
• Positive from up quark PFF at small \( z_{\Lambda} \)
• Negative from down quark PFF at \( z_{\Lambda} \gtrsim 0.3 \)

LHC kinematics
\( \sqrt{s} = 8 \text{ TeV}, \ R = 0.5 \)
\( 2.5 < \eta_J < 4 \)
\( 0 < j_{\perp} < 1 \text{ GeV} \)
Azimuthal angular dependence

\[
\frac{d\sigma^{p(S_A)+p/e\rightarrow (\text{jet } h(S_h))}}{dp_T d\eta_J dz_h d^2 j_\perp} = F_{UU,U} + |S_T| \sin(\phi_{S_A} - \hat{\phi}_h) F_{TU,U}^{\sin(\phi_{S_A} - \hat{\phi}_h)} \\
+ \Lambda_h \left[ \lambda F_{LU,L} + |S_T| \cos(\phi_{S_A} - \hat{\phi}_h) F_{TU,L}^{\cos(\phi_{S_A} - \hat{\phi}_h)} \right] \\
+ |S_{h\perp}| \left\{ \sin(\hat{\phi}_h - \hat{\phi}_{S_h}) F_{UU,T}^{\sin(\hat{\phi}_h - \hat{\phi}_{S_h})} + \lambda \cos(\hat{\phi}_h - \hat{\phi}_{S_h}) F_{LU,T}^{\cos(\hat{\phi}_h - \hat{\phi}_{S_h})} \\
+ |S_T| \left( \cos(\phi_{S_A} - \hat{\phi}_{S_h}) F_{TU,T}^{\cos(\phi_{S_A} - \hat{\phi}_{S_h})} + \cos(2\hat{\phi}_h - \hat{\phi}_{S_h} - \hat{\phi}_{S_A}) F_{TU,T}^{\cos(2\hat{\phi}_h - \hat{\phi}_{S_h} - \hat{\phi}_{S_A})} \right) \right\},
\]

\[F_{S_A S_B, S_h}\]

Polarization of \( A, B, h \)

- Different structures come with different characteristic angular dependence.
Thank you!