Unlocking QCD dynamics using jet substructure

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Jefferson Lab Theory Seminar,
11/09/2020

Jet substructures and characteristic scales

Single prong observables

- Jet angularities
- Energy-energy correlations
- Jet shape

Multi-prong observables

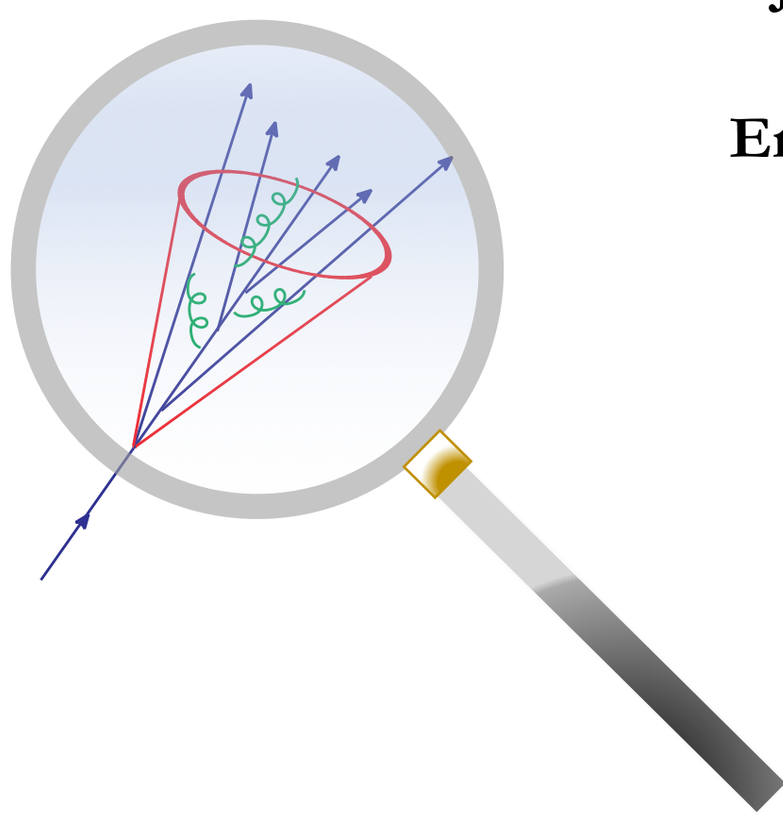
- N-subjettiness
- D_2

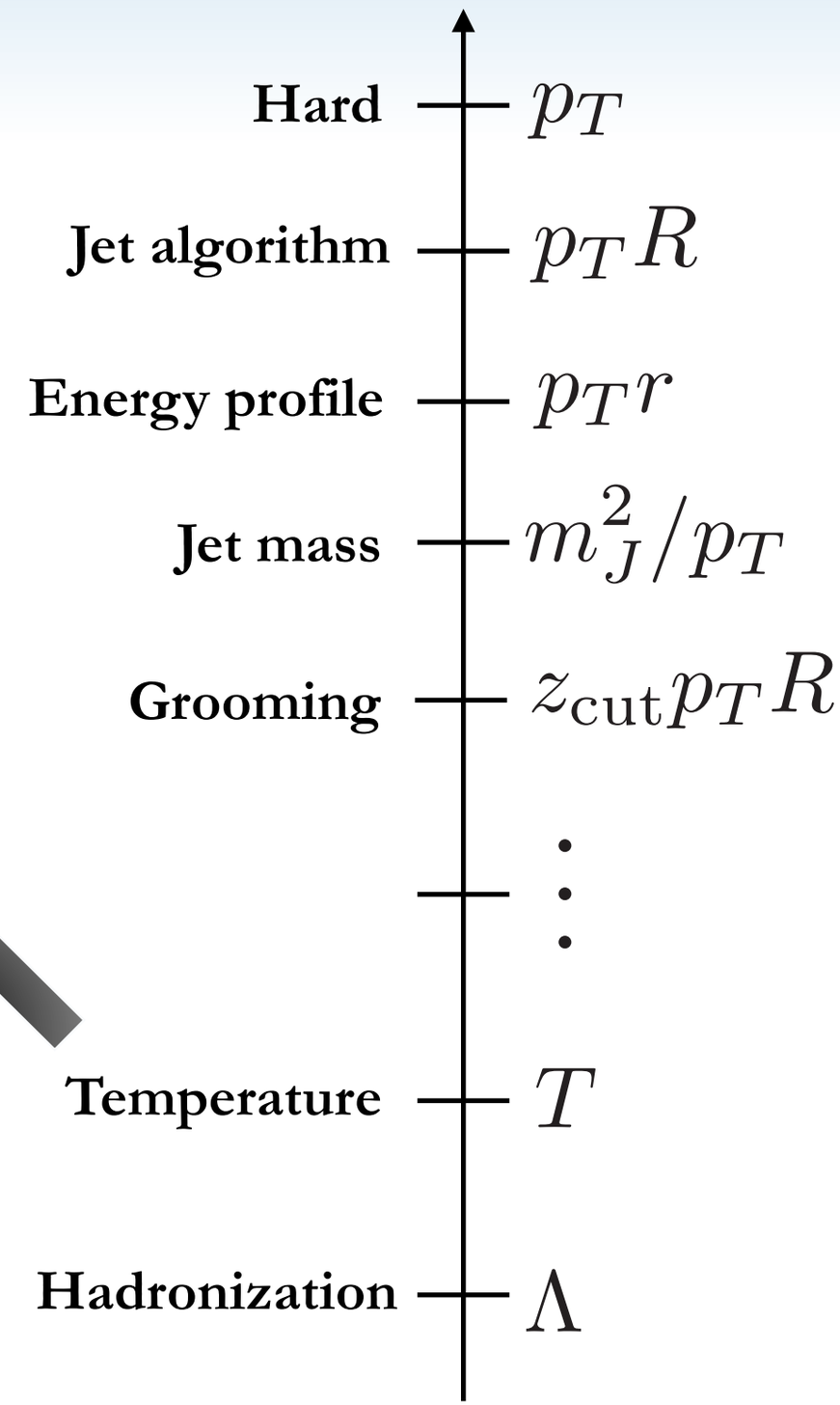
IRC unsafe /NP sensitive

- Hadron in jet
- Multiplicities
- Jet charge

Groomed observables

- All of the above
- Observables characterizing grooming (SD): $z_g, \quad \theta_g = R_g/R$





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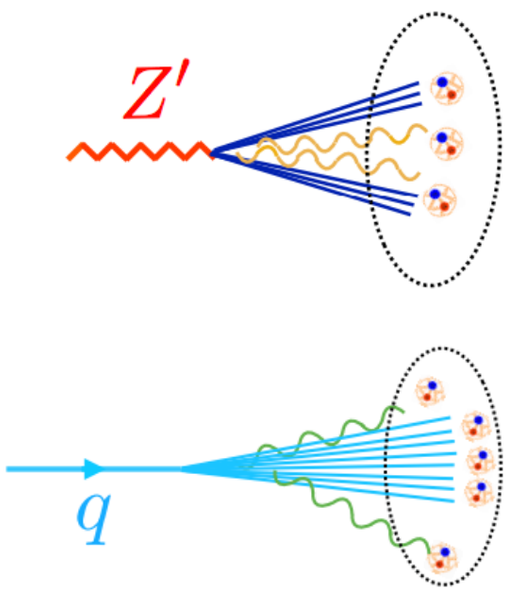
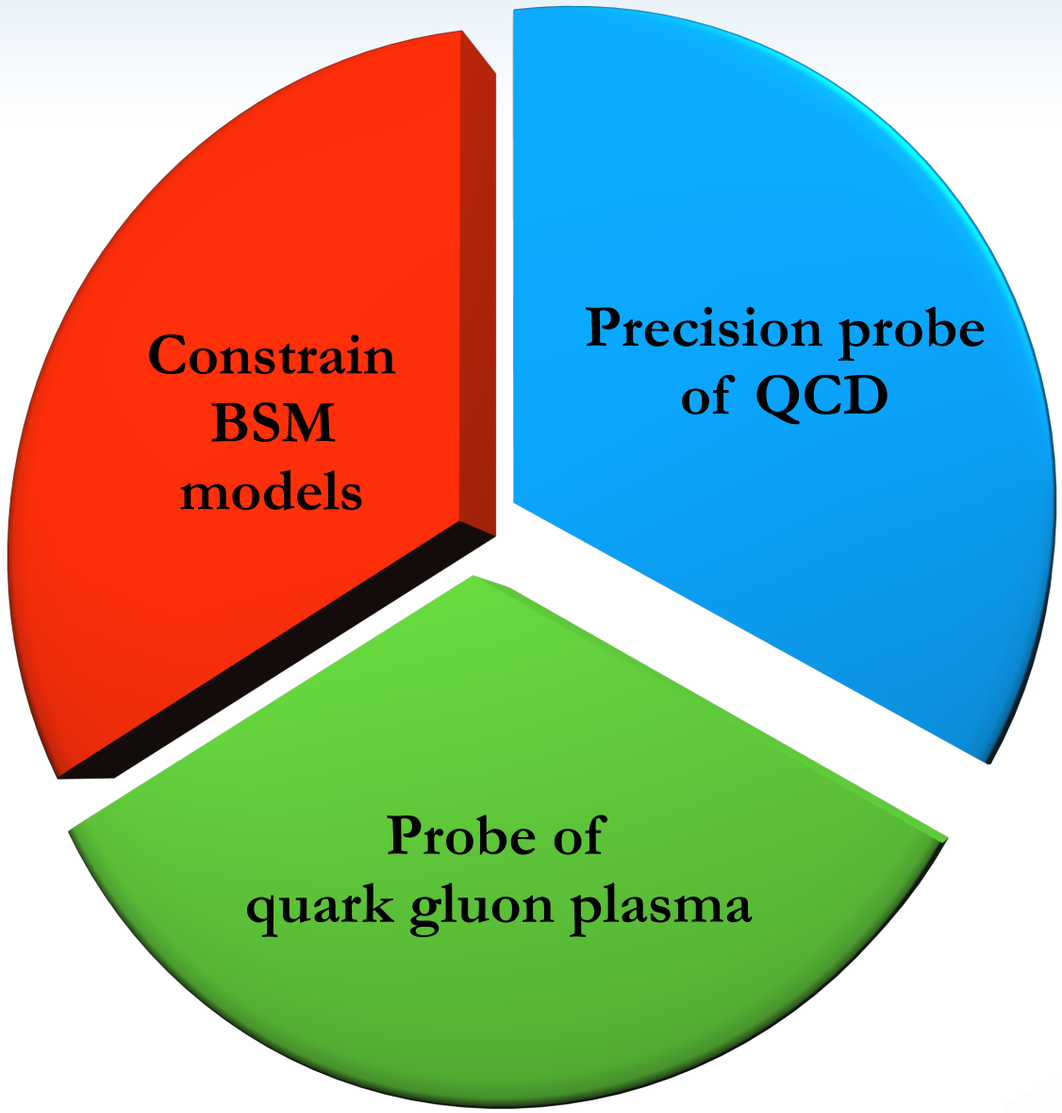
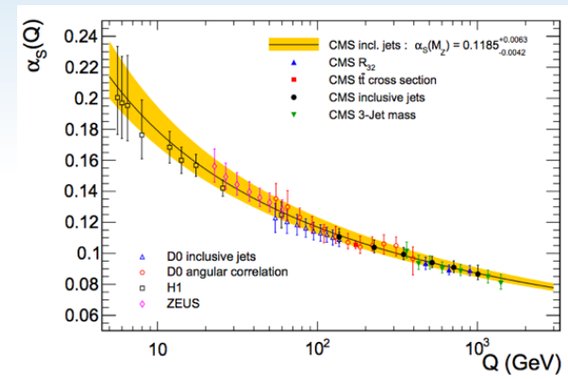
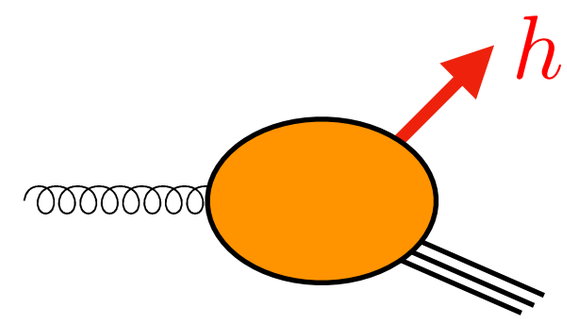
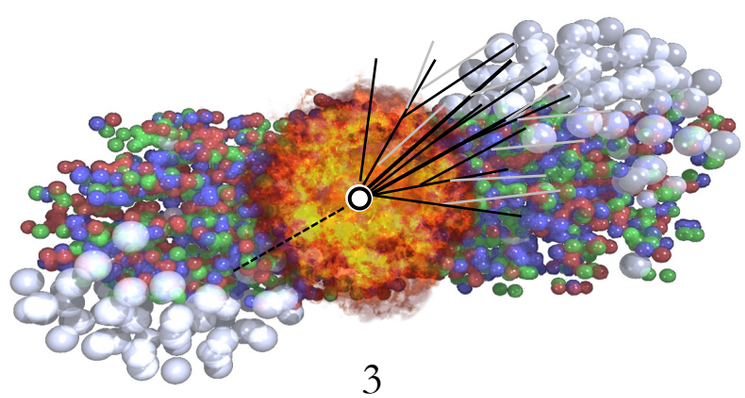










Lund diagram to map different splitting

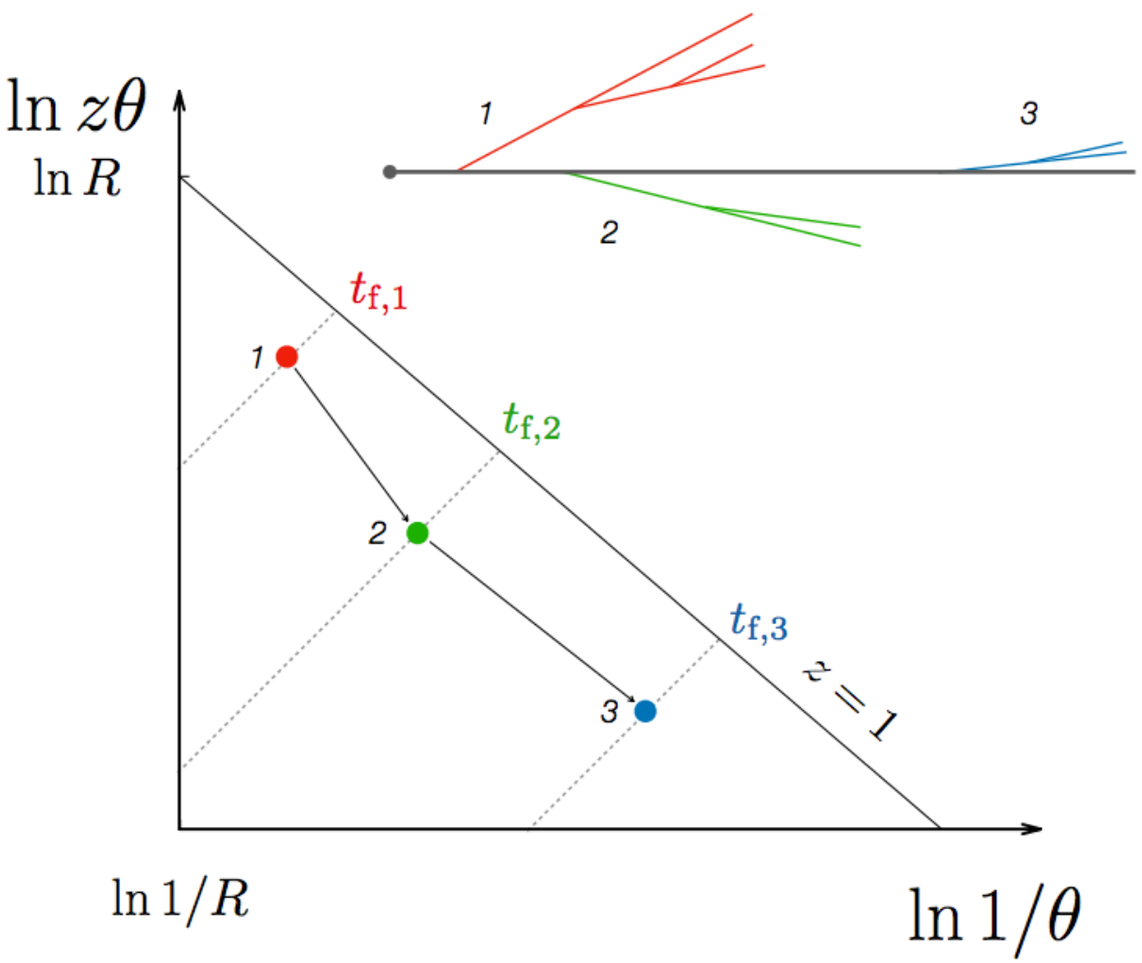
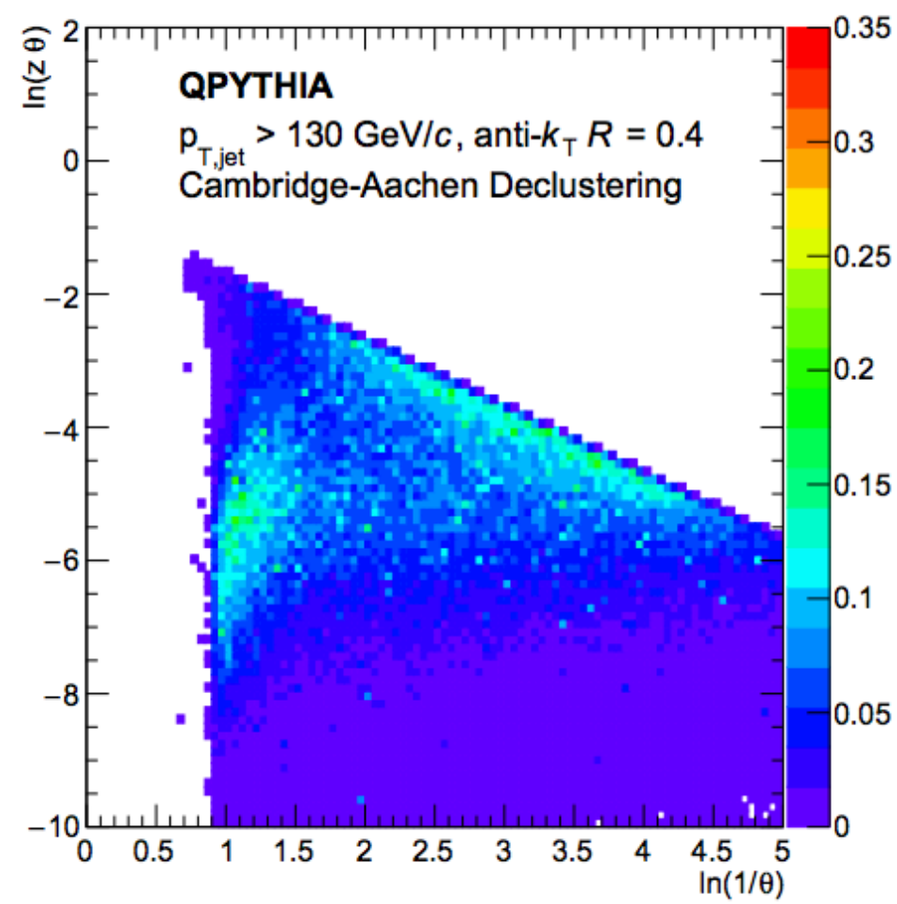


Fig. from 5th Heavy Ion Jet Workshop Report, 2018

- **Lund diagram is useful to visualize collinear and soft splittings.**
- **Map splitting history into Lund Plane.**



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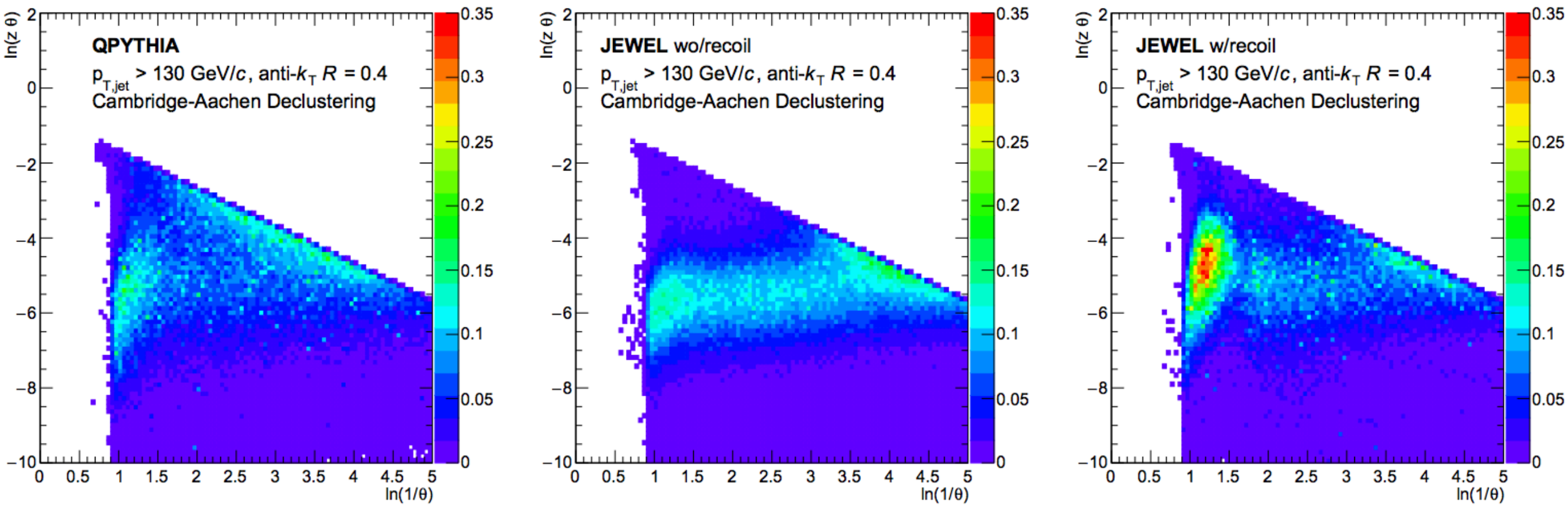
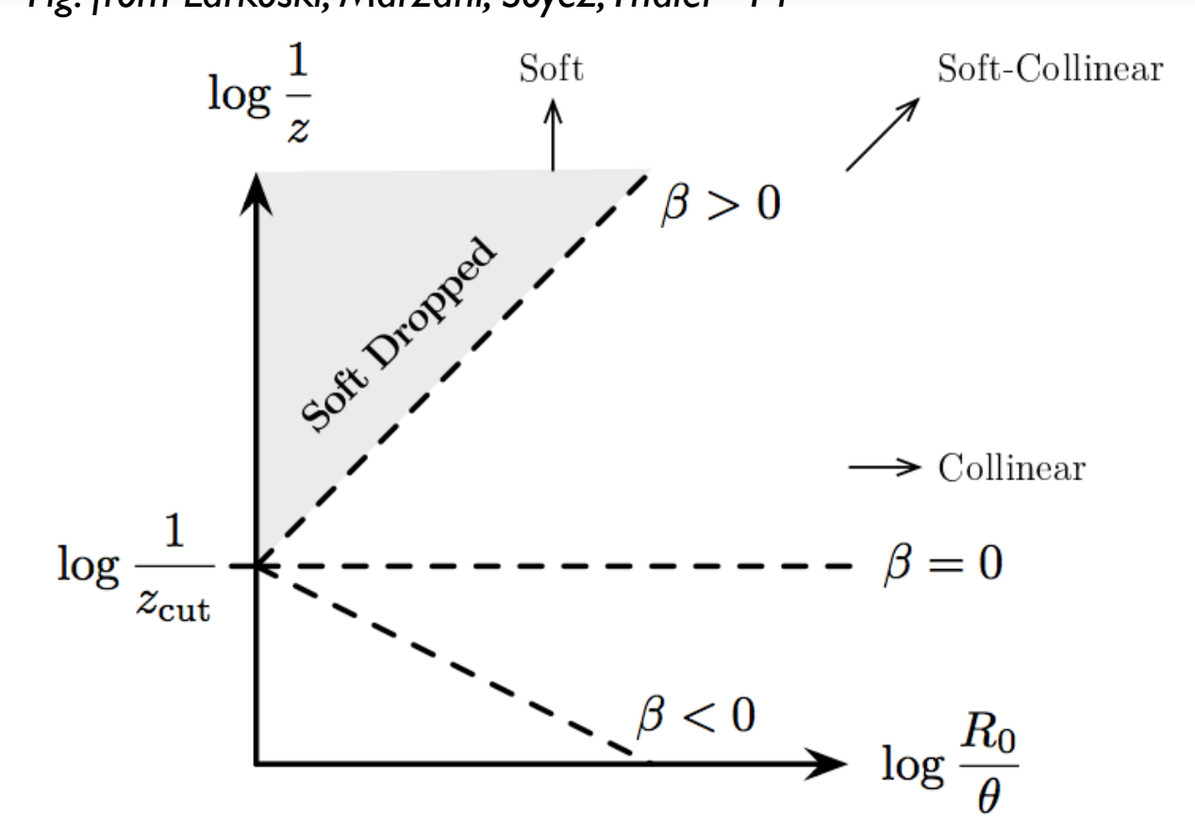


- **Devise jet substructure observables to probe different kinematical regions to discriminate different jet quenching models and tune Monte Carlo (with / without medium).**

5th Heavy Ion Jet Workshop Report, 2018

Soft Drop Groomed Jet Observables

Fig. from Larkoski, Marzani, Soyez, Thaler `14



- **Different grooming parameters removes different region of phase space**
- **Reduces NP effects by removing soft radiations.**

Soft Drop Condition

$$z > z_{\rm cut} \left(\frac{\Delta R_{12}}{R} \right)^{\beta}$$

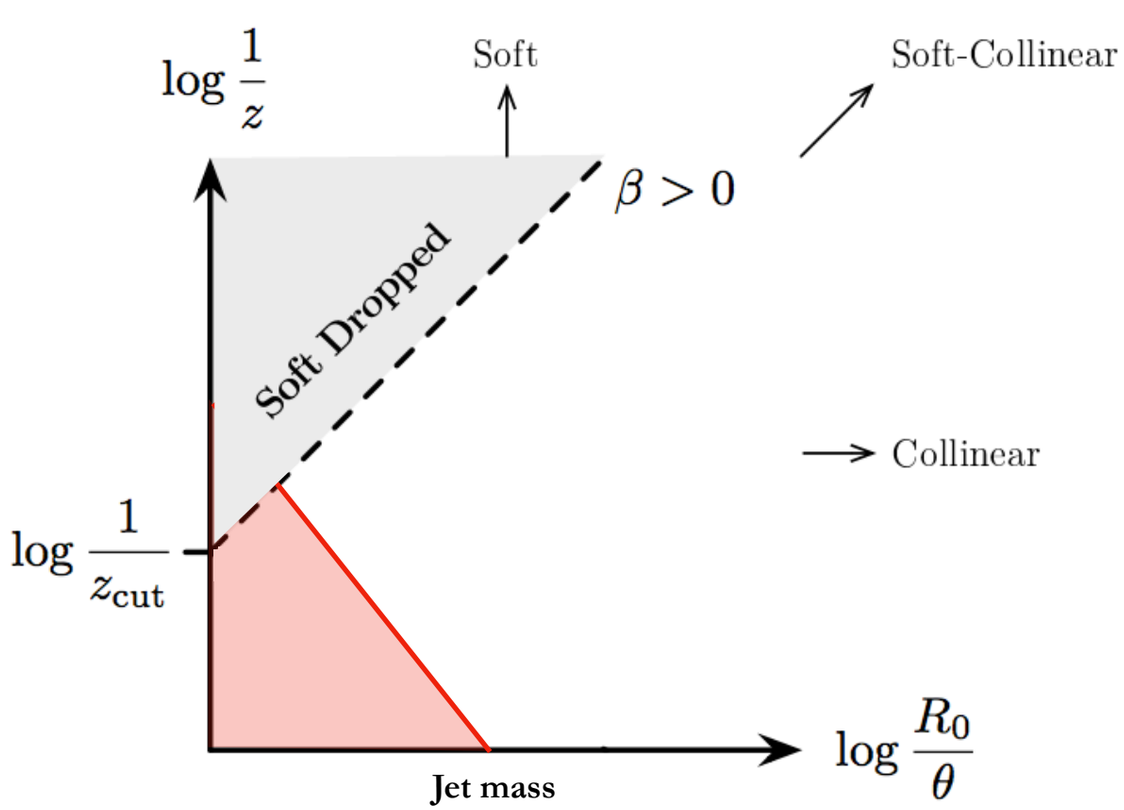
- **Soft drop grooming algorithms:**

1. Reorder emissions in the identified jet according to their relative angle using C/A jet algorithm.

2. Recursively remove soft branches until soft drop condition is met.

Larkoski, Marzani, Soyez, Thaler `14
Frye, Larkoski, Schwartz, Yan `16

Soft Drop Groomed Jet Observables

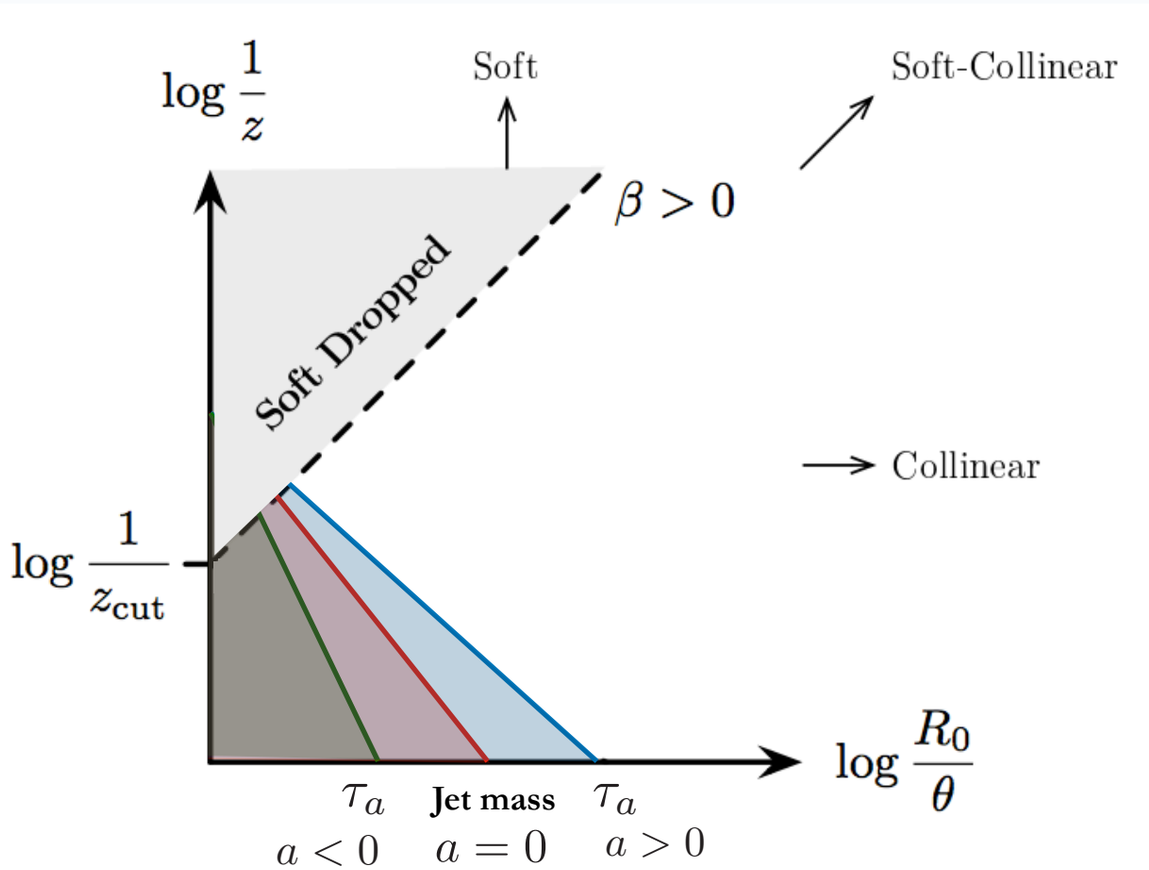


- Different grooming parameters removes different region of phase space
- Reduces NP effects by removing soft radiations.

I. Usual jet observables

- Jet mass

Soft Drop Groomed Jet Observables



- Different grooming parameters removes different region of phase space
- Reduces NP effects by removing soft radiations.

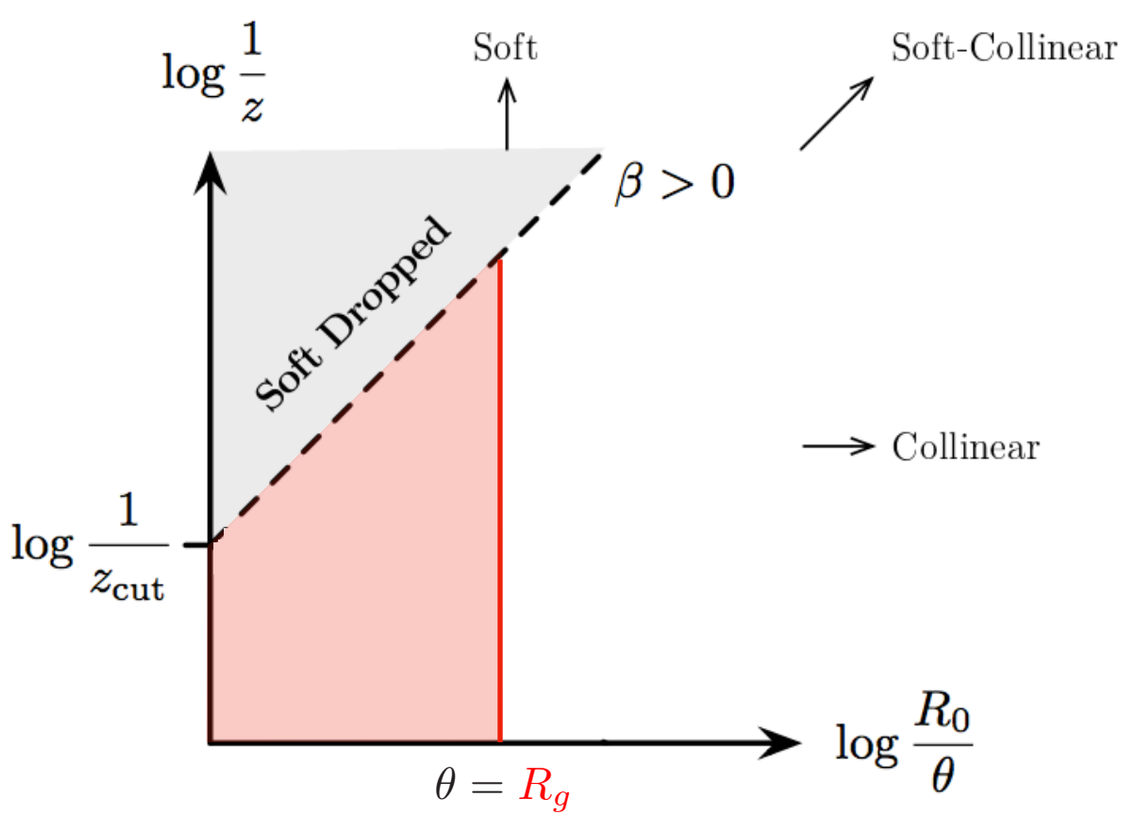
I. Usual jet observables

- Jet mass $(a = 0)$
- Jet angularity τ_a

$$\tau_a = \frac{1}{p_T} \sum_{i \in J} p_{T,i} \left(\Delta R_{iJ}\right)^{2-a}$$

- ...

Soft Drop Groomed Jet Observables



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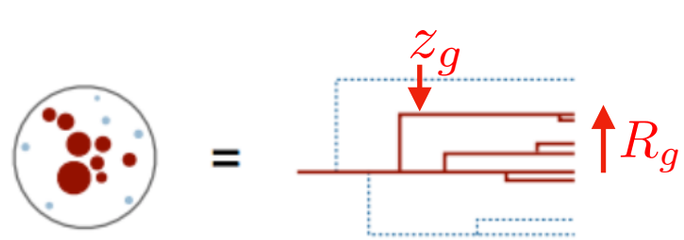
II. Observables unique to soft drop

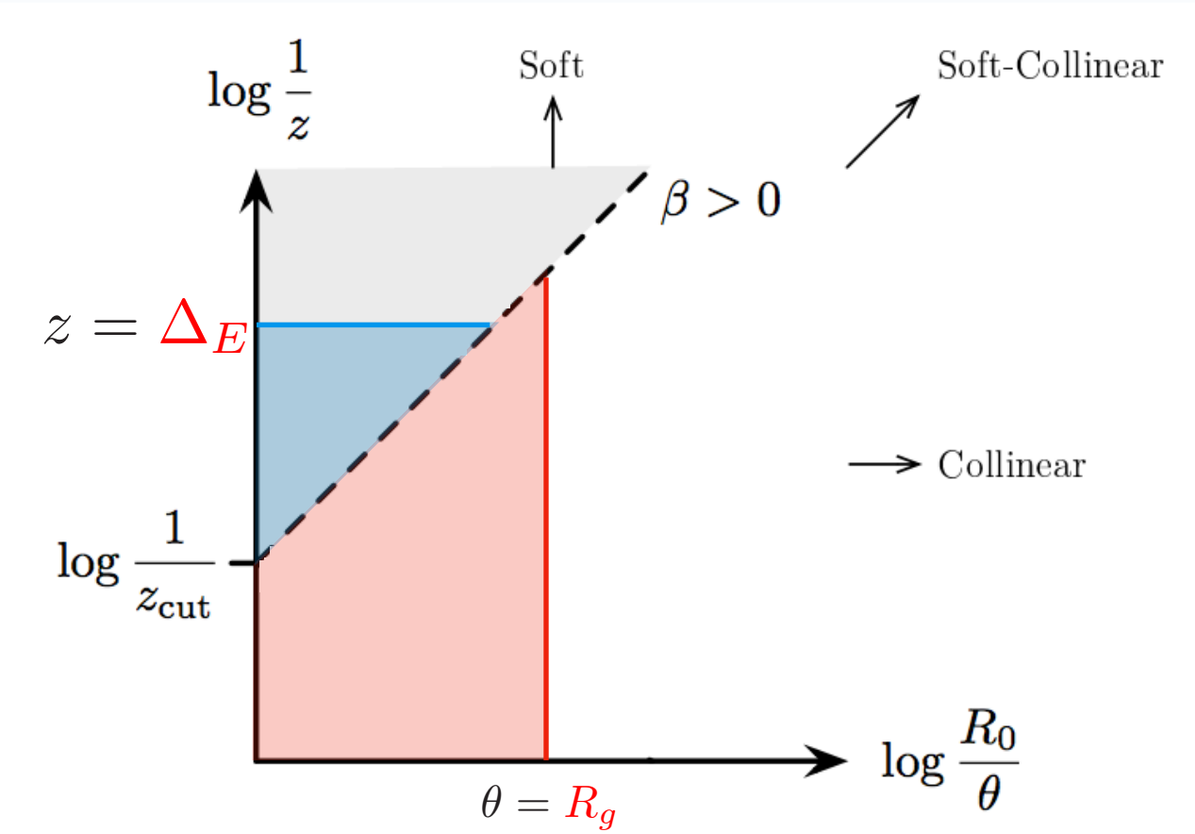
- At the time soft drop condition is met

$$z > z_{\rm cut} \left(\frac{\Delta R_{12}}{R}\right)^{\beta}$$



$$z_g = z \qquad R_g = \Delta R_{12}$$

Soft Drop Groomed Jet Observables



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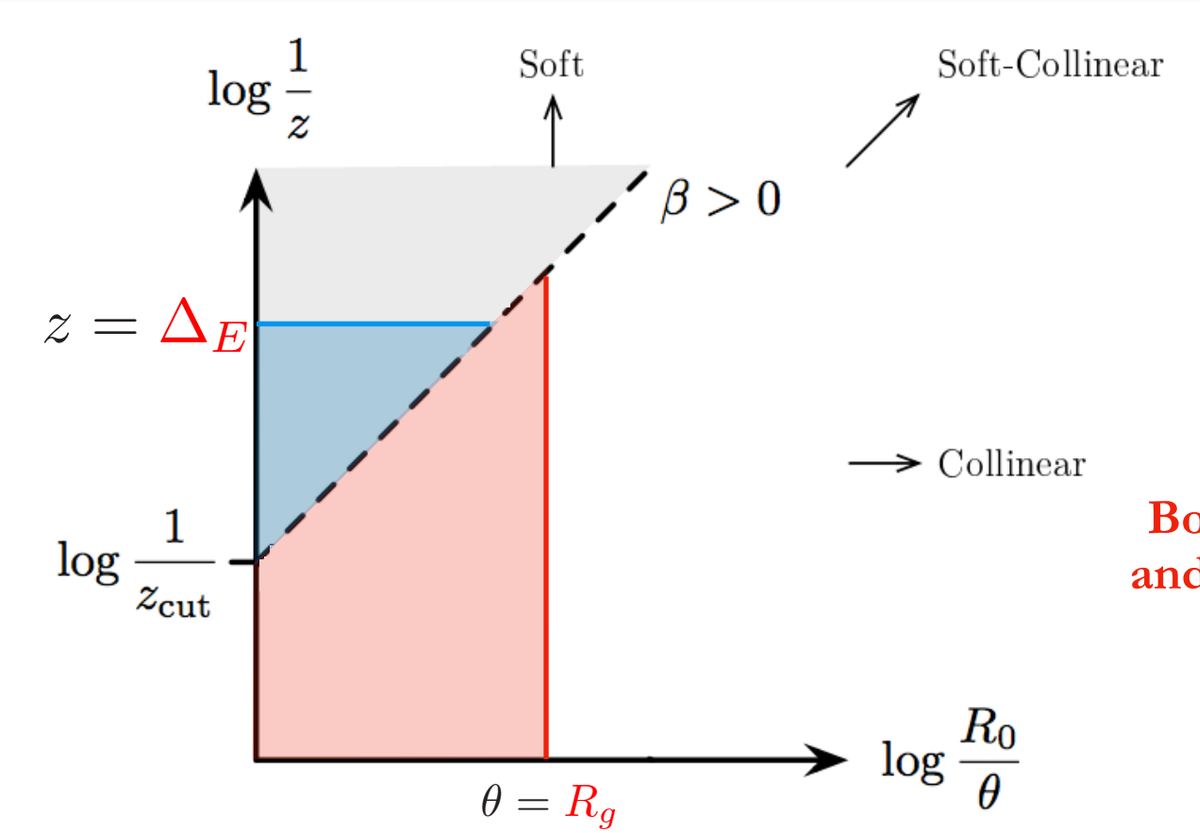
III. Soft-sensitive observables unique to groomed jets

- Energy drop $\Delta_E = \frac{p_T - p_T^{\rm gr}}{p_T}$
- Angle between standard and groomed jet axes
- …

Soft Drop Groomed Jet Observables



- Different grooming parameters removes different region of phase space
- Reduces NP effects by removing soft radiations.

I. Usual jet observables

- Jet mass $(a = 0)$
- Jet angularity τ_a

Both groomed and ungroomed

$$\tau_a = \frac{1}{p_T} \sum_{i \in J} p_{T,i} \left(\Delta R_{iJ}\right)^{2-a}$$

- …

II. Observables unique to soft drop

- At the time soft drop condition is met

$$z > z_{\rm cut} \left(\frac{\Delta R_{12}}{R}\right)^{\beta}$$

$$z_g = z \qquad R_g = \Delta R_{12}$$

III. Soft-sensitive observables unique to groomed jets

- Energy drop $\Delta_E = \frac{p_T - p_T^{\rm gr}}{p_T}$
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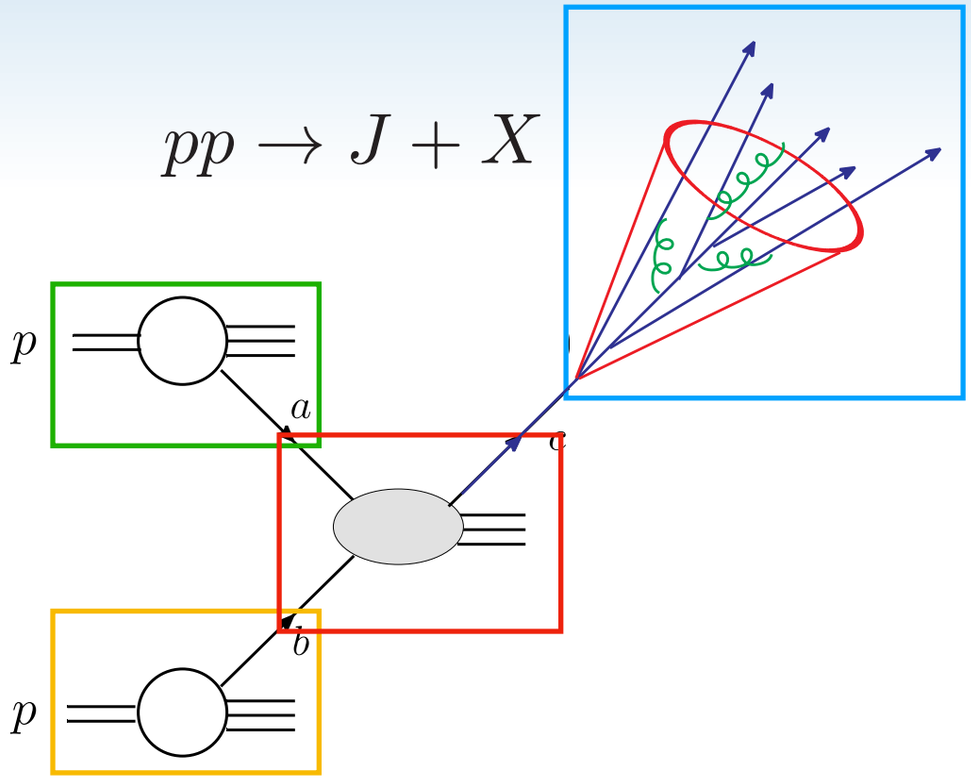
QCD factorization for inclusive jet



- Jet dynamics isolated to jet function J_c
- Similar factorization holds for AA collision?

$J_c \to J_c^{\rm med}$ *Qiu, Ringer, Sato, Zurita `19*

Factorization for pp

Hadron $$\frac{d\sigma^{pp\to hX}}{dp_T d\eta} = \sum_{a,b,c} \underbrace{f_a}_{\Lambda_{\rm QCD}} \otimes \underbrace{f_b}_{p_T} \otimes \underbrace{H^c_{ab}}_{\Lambda_{\rm QCD}} \otimes D^h_c$$

Inclusive Jet $$\frac{d\sigma^{pp\to JX}}{dp_T d\eta} = \sum_{a,b,c} \underbrace{f_a}_{\Lambda_{\rm QCD}} \otimes \underbrace{f_b}_{p_T} \otimes \underbrace{H^c_{ab}}_{p_T} \otimes \underbrace{J_c}_{p_T R}$$

RG evolution

$$\mu \frac{d}{d\mu} J_i = \sum_j P_{ji} \otimes J_j$$

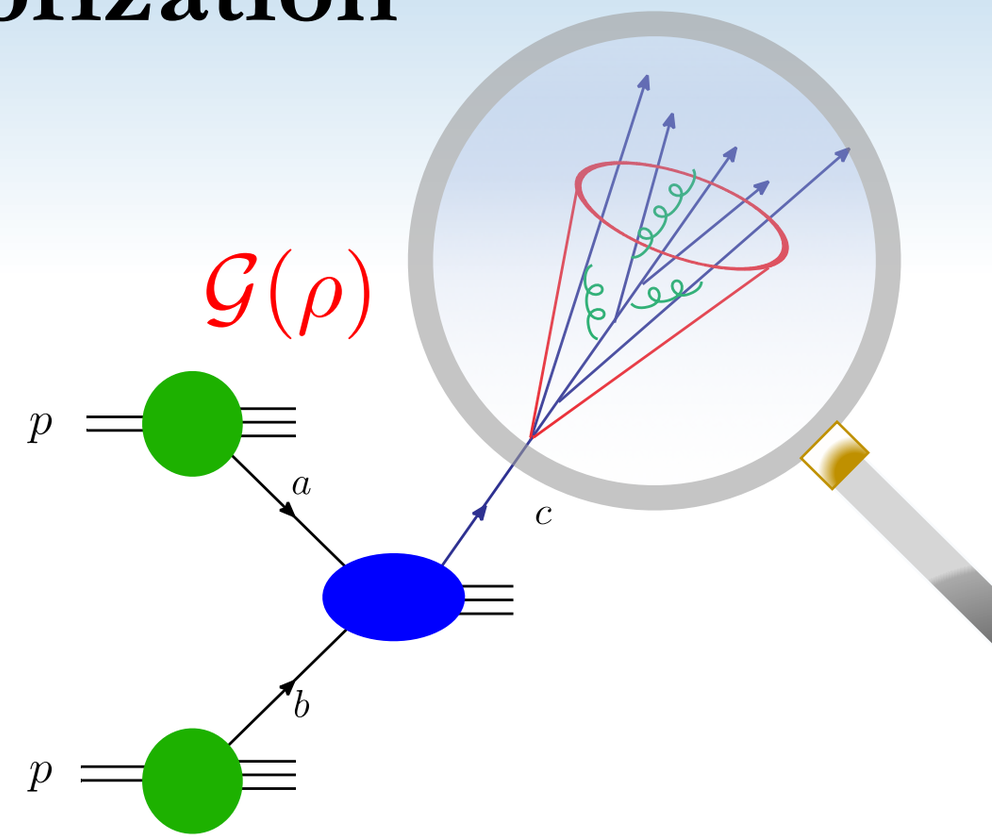
Dasgupta, Dreyer, Salam, Soyez `15
Kaufmann, Mukherjee, Vogelsang `15
Kang, Ringer, Vitev `16
Dai, Kim, Leibovich `16

Jet substructure factorization

Inclusive jet production $pp \to \text{jet} + X$

$$\frac{d\sigma^{pp\to \text{jet}X}}{dp_T d\eta} = \sum_{a,b,c} f_a \otimes f_b \otimes H^c_{ab} \otimes \boxed{J_c}$$

Λ_{QCD} p_T $p_T R$



Jet substructure ρ

$$\frac{d\sigma^{pp\to \text{jet}(\rho)X}}{dp_T d\eta\, d\rho} = \sum_{a,b,c} f_a \otimes f_b \otimes H^c_{ab} \otimes \boxed{\mathcal{G}_c(\rho)}$$

Λ_{QCD} p_T $p_T R$

and other scale(s) depending on ρ

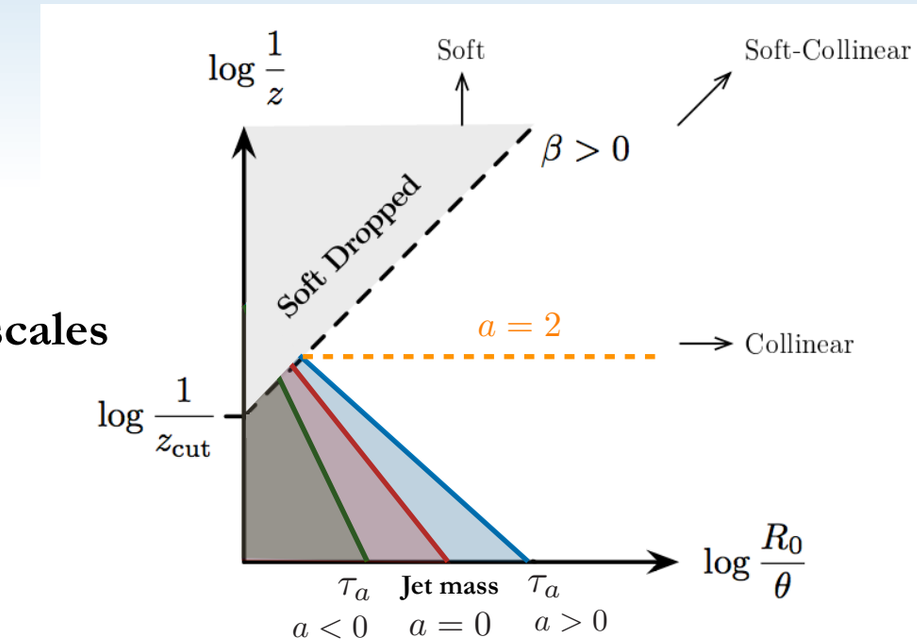
Jet angularity τ_a

$$\frac{d\sigma^{pp\to \mathrm{jet}(\tau_a)X}}{dp_T\, d\eta\, d\tau_a} = \sum_{a,b,c} f_a \otimes f_b \otimes H^c_{ab} \otimes \boxed{\mathcal{G}_c(\tau_a)}$$

Λ_{QCD} $\quad p_T \quad p_T R$

and τ_a dependent scales

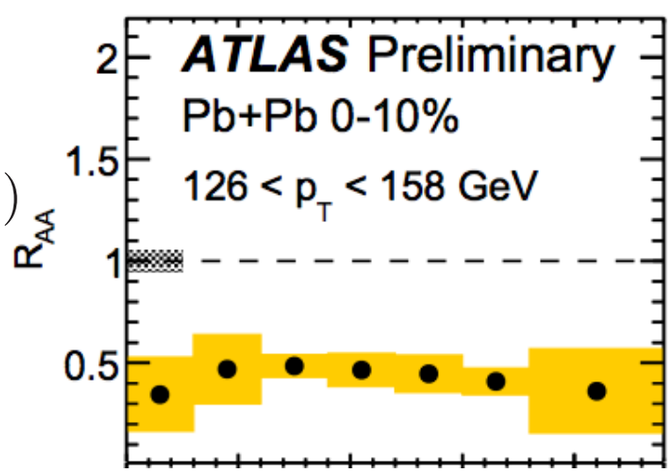
$$\tau_a = \frac{1}{p_T}\sum_{i\in J} p_{T,i}(\Delta R_{iJ})^{2-a}$$



- **A generalized class of IR safe observables for** $-\infty < a < 2$
- **Parameter** a **gives varying sensitivity to collinear radiations.**

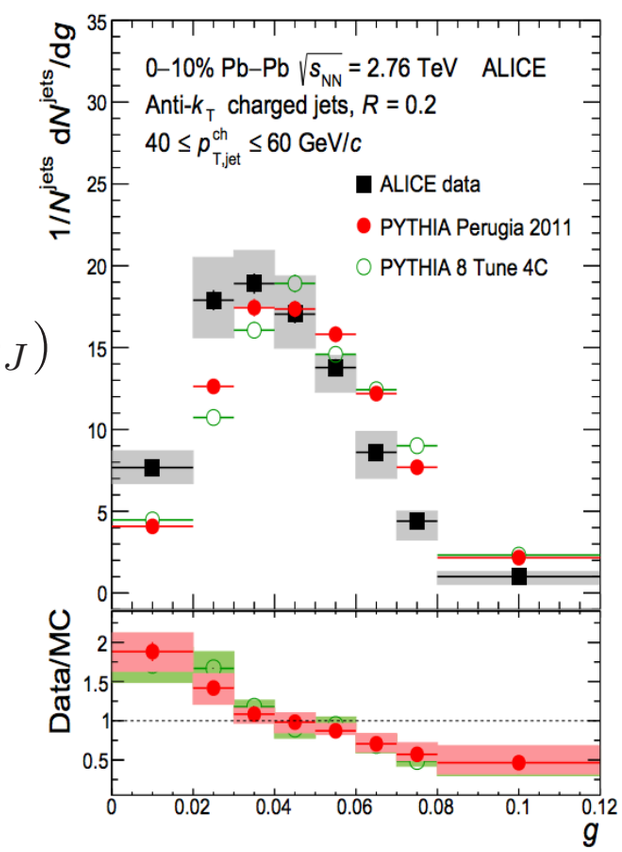
$a = 0$

$$\tau_0^{pp} = \frac{m_J^2}{p_T^2} + \mathcal{O}((\tau_0^{pp})^2)$$



$a = 1$

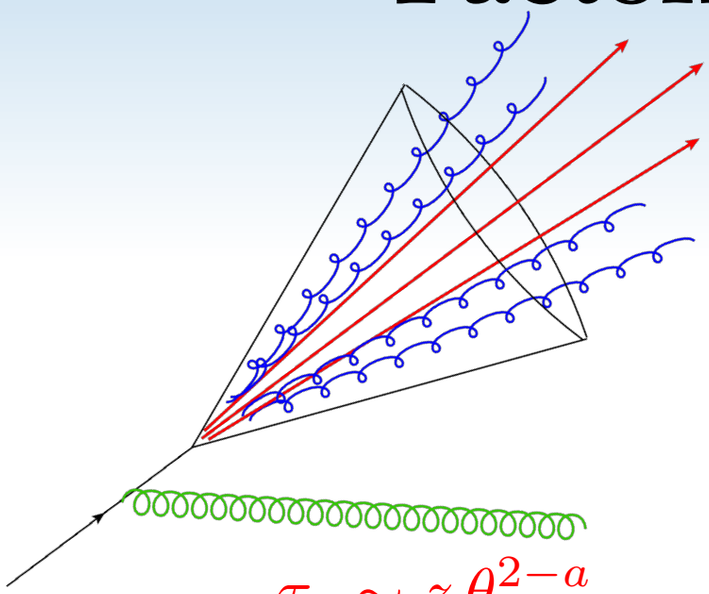
$$g(\text{broadening}) = \frac{1}{p_T}\sum_{i\in J} p_{T,i}\,(\Delta R_{iJ})$$



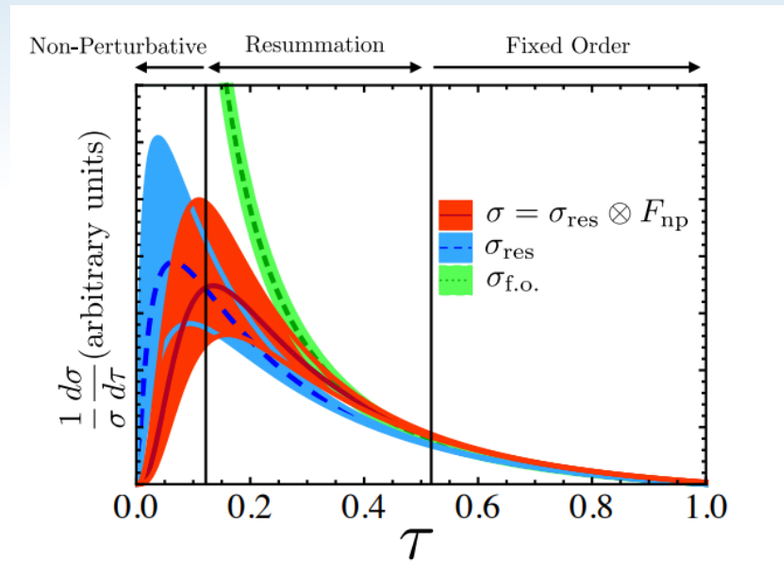
- **Analysis for different** a **currently in progress.**

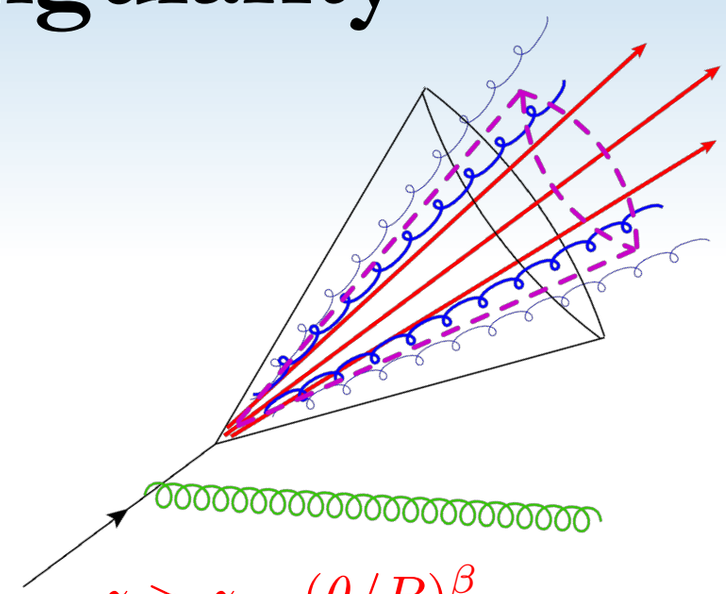
ALICE Preliminary

Factorization for the jet angularity



$\tau_a \sim z\,\theta^{2-a}$





$z > z_{\rm cut}(\theta/R)^{\beta}$

- **The ungroomed case ($\tau_a \ll R^{2-a}$)**

Hard-collinear

$$\theta_{\mathcal{H}} \sim R \qquad z_{\mathcal{H}} \sim 1$$

Collinear

$$z_c \sim 1 \qquad \theta_c \sim \tau_a^{\frac{1}{2-a}}$$

(Collinear-)soft

$$\theta_s \sim R \qquad z_{cs} \sim \frac{\tau_a}{R^{2-a}}$$

- **The groomed case ($\tau_{a,gr}/R^{2-a} \ll z_{\rm cut} \ll 1$)**

Hard-collinear

$$\theta_{\mathcal{H}} \sim R \qquad z_{\mathcal{H}} \sim 1$$

Collinear

$$z_c \sim 1 \qquad \theta_c \sim \tau_a^{\frac{1}{2-a}}$$

$\notin$ *gr* soft

$$\theta_{\notin \rm gr} \sim R \qquad z_{\notin \rm gr} \sim z_{\rm cut}\left(\frac{\theta}{R}\right)^{\beta} = z_{\rm cut}$$

$\in$ *gr* soft (collinear-soft)

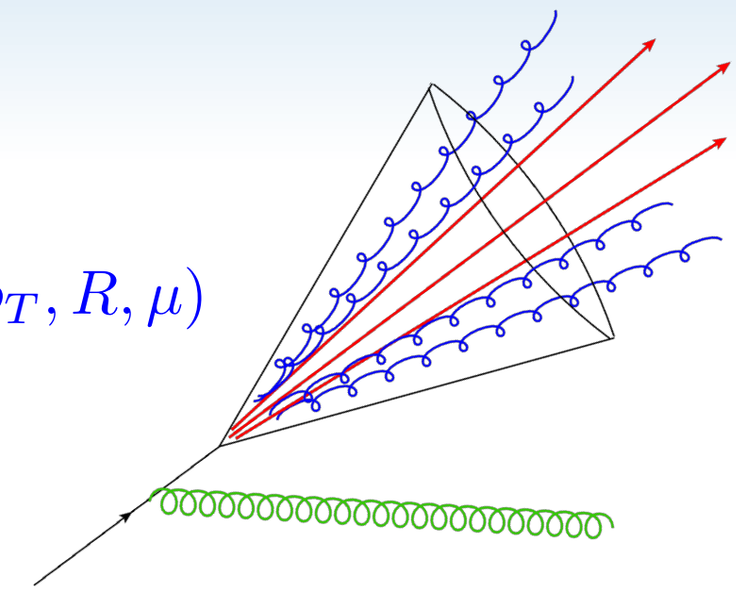
$$z_{\in \rm gr} \sim z_{\rm cut}\left(\frac{\theta}{R}\right)^{\beta} = z_{\rm cut}^{\frac{2-a}{2-a+\beta}}\left(\frac{\tau_a}{R^{2-a}}\right)^{\frac{\beta}{2-a+\beta}} \qquad \theta_{\in \rm gr} \sim \left(\frac{\tau_a R^{\beta}}{z_{\rm cut}}\right)^{\frac{1}{2-a+\beta}}$$

Factorization for the jet angularity

$$J_c \to \mathcal{G}_c$$

- **The ungroomed case (** $\tau_a \ll R^{2-a}$ **)**

$$\mathcal{G}_i(z, p_T R, \tau_a, \mu) = \sum_j \mathcal{H}_{i\to j}(z, p_T R, \mu) C_j(\tau_a, p_T, \mu) \otimes S_j(\tau_a, p_T, R, \mu)$$

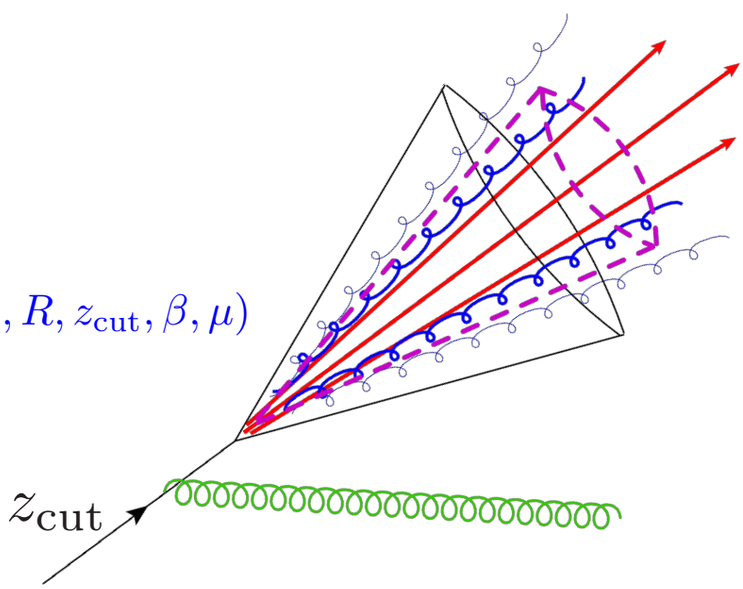


- **Jointly resums large logs** $\alpha_s^n \ln^n R$ **and** $\alpha_s^n \ln^{2n} \tau_a^{\frac{1}{2-a}}/R$

- **The groomed case (** $\tau_{a,gr}/R^{2-a} \ll z_{\rm cut} \ll 1$ **)**

$$\theta_{\mathcal{H}} \sim R$$
$$\theta_{\notin \rm gr} \sim R$$

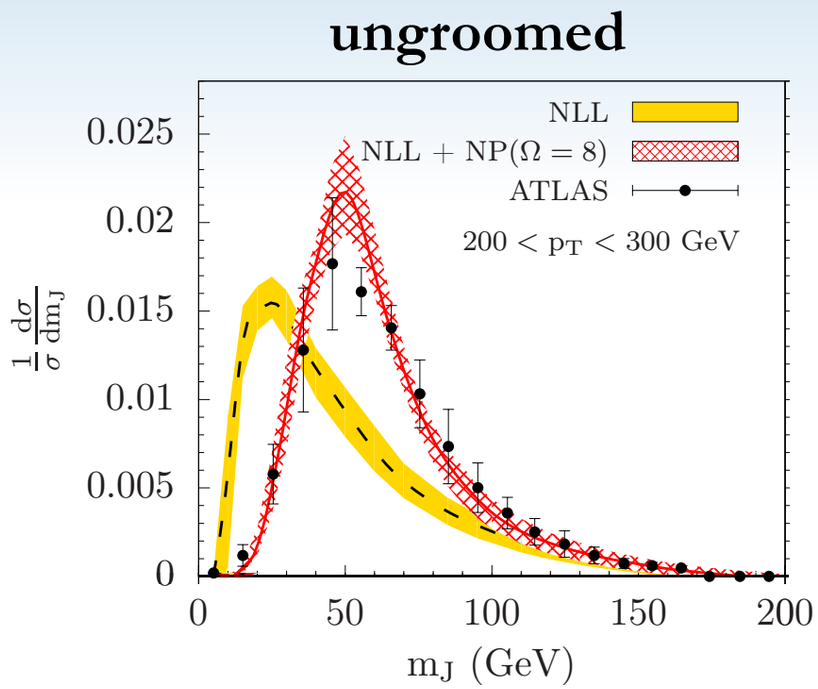
$$\mathcal{G}_i(z, p_T R, \tau_a, z_{\rm cut}, \beta, \mu) = \sum_j \mathcal{H}_{i\to j}(z, p_T R, \mu) S_j^{\notin \rm gr}(p_T, R, z_{\rm cut}, \beta, \mu) C_j(\tau_a, p_T, \mu) \otimes S_j^{\in \rm gr}(\tau_a, p_T, R, z_{\rm cut}, \beta, \mu)$$



- **Jointly resums large logs** $\alpha_s^n \ln^n R$**,** $\alpha_s^n \ln^{2n} \tau_a^{\frac{1}{2-a}}/R$**, and** $\alpha_s^n \ln^{2n} z_{\rm cut}$

Kang, KL, Liu, Ringer `18

Factorization for the groomed jet angularity



- **Ungroomed jet mass has large NP shifts**

$$\frac{d\sigma}{dp_T d\eta d\tau_a} = \int dk\, F_a(k) \frac{d\sigma^{\rm pert}}{dp_T d\eta d\tau_a}\left(\tau_a - \frac{R^{1-a}}{p_T}k\right)$$

Perturbative result $\otimes$ NP shape function

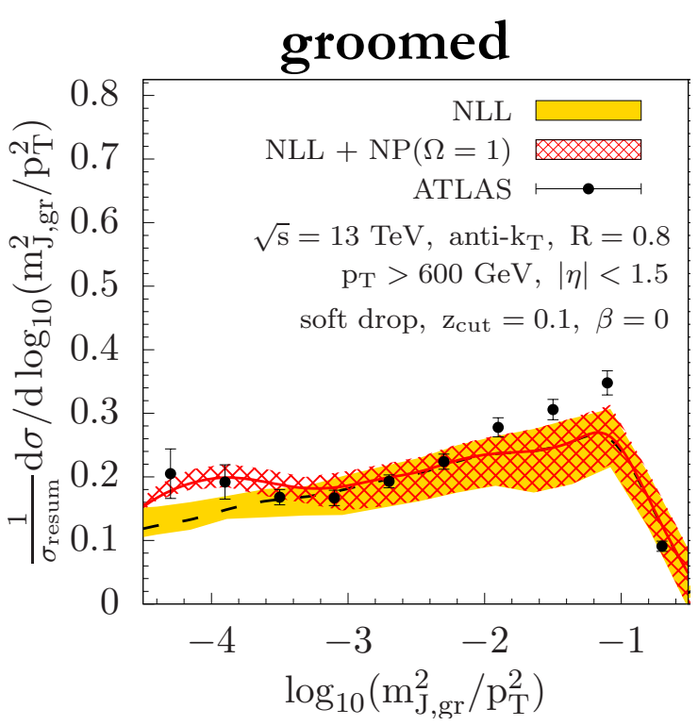
$$F_a(k) = \left(\frac{4k}{\Omega_a^2}\right)\exp\left(-\frac{2k}{\Omega_a}\right) \qquad \Omega_a = \int dk\, k\, F_a(k)$$

$$\Omega_a = \Omega_a^{\rm had} + \boxed{\Omega^{\rm MPI}}$$

$\Omega_{a=0}^{\rm had} = \langle 0|\mathcal{O}|0\rangle \sim 1\,{\rm GeV}$ **is universal.**

non-universal

Lee, Sterman `07
Stewart, Tackmann, Waalewijn `15



- **Groomed jet mass is more robust to NP effects**
- **Nonperturbative corrections to groomed jet mass**

Hoang, Mantry, Pathak, Stewart `19

Kang, KL, Liu, Ringer `18

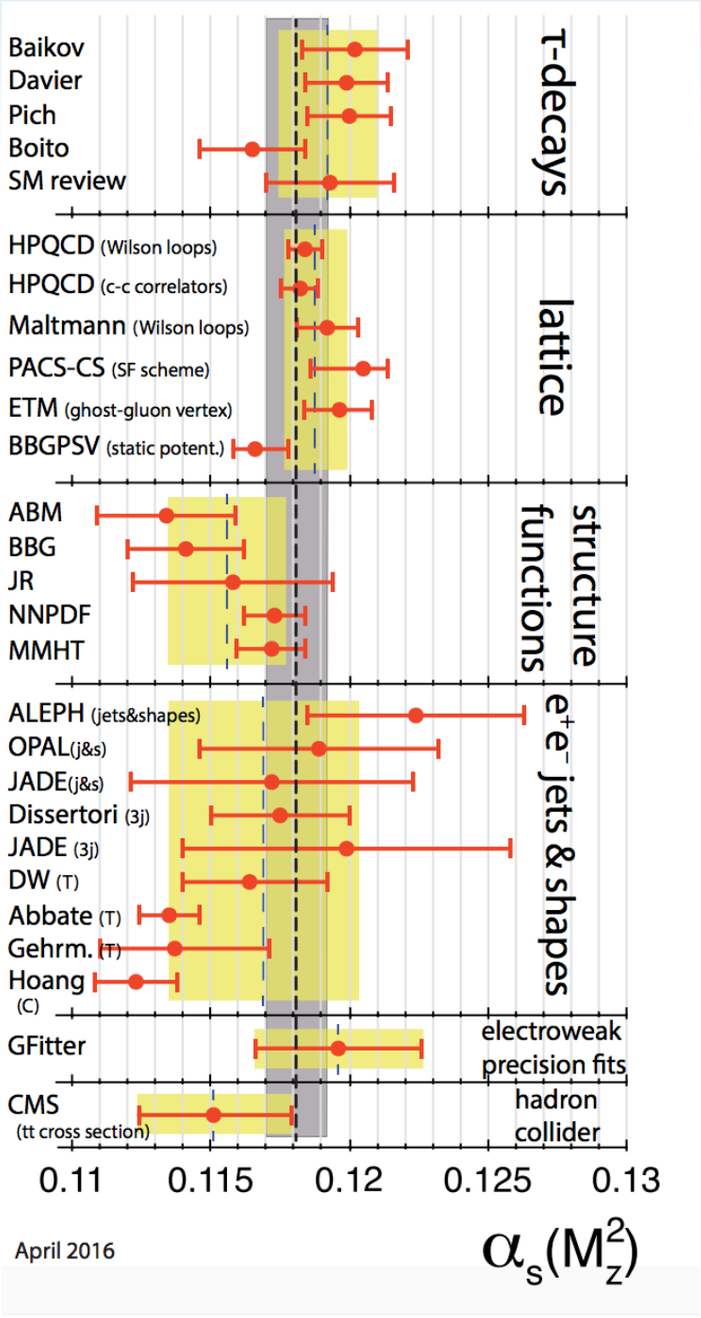
α_s extraction

- **World Average with 0.9% total uncertainty**

$$\alpha_s(m_Z) = 0.1181 \pm 0.0011$$

- **Most precise input: lattice determination**
- **Next precise input: e^+e^- event shape determination: thrust and C-parameter.**
  - $3 - 4\sigma$ **tension with lattice.**

- **High-quality of data pouring out of the LHC.**
  - **Can we carry out an α_s extraction using pp data?**



α_s extraction

- **Key challenge:** (for e^+e^- event shape extraction or ungroomed jet substructure at pp)

  degeneracy between the extractions of α_s and fitting $\Omega^{\rm had}$

  Also, contaminations from MPI

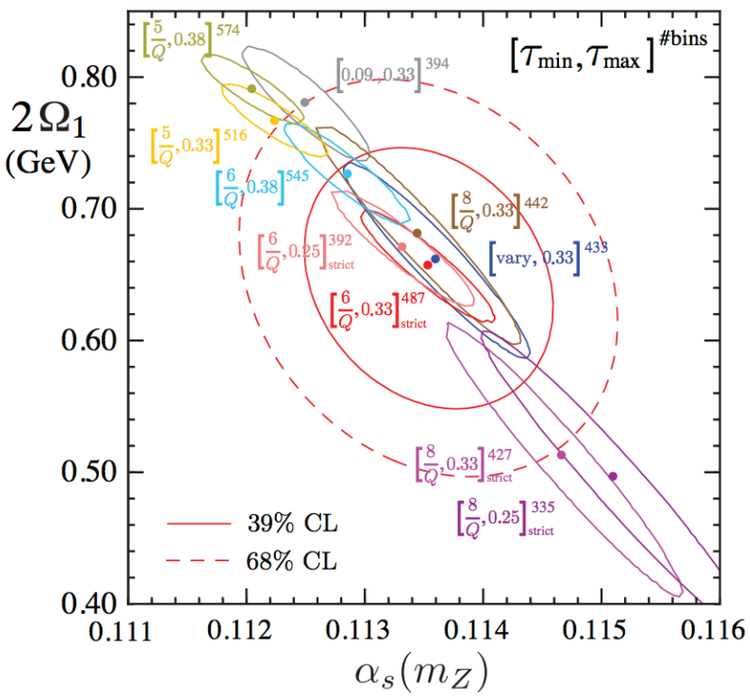
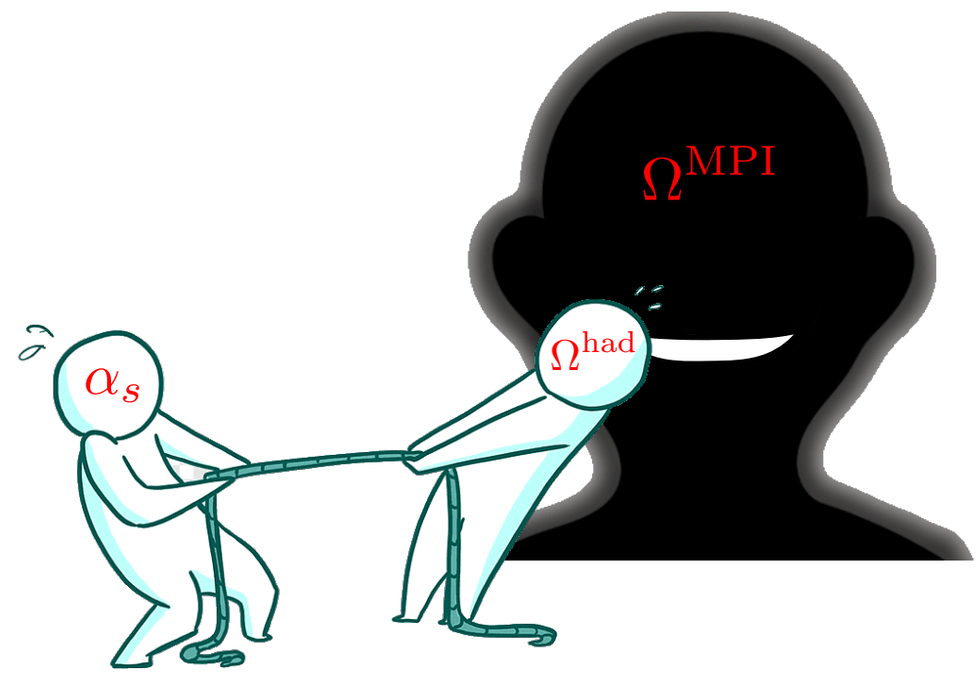


Fig. from
Abbate, Fickinger, Hoang, Mateu, Stewart `10



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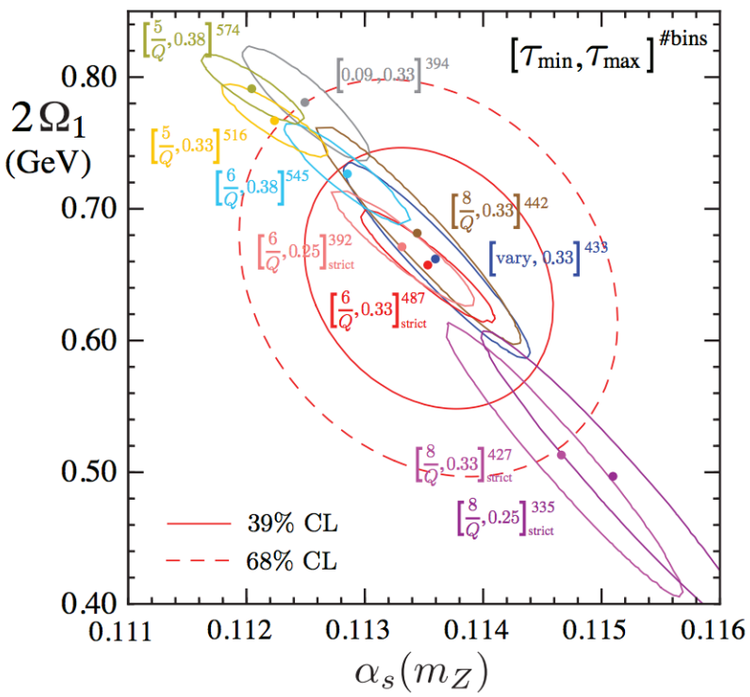

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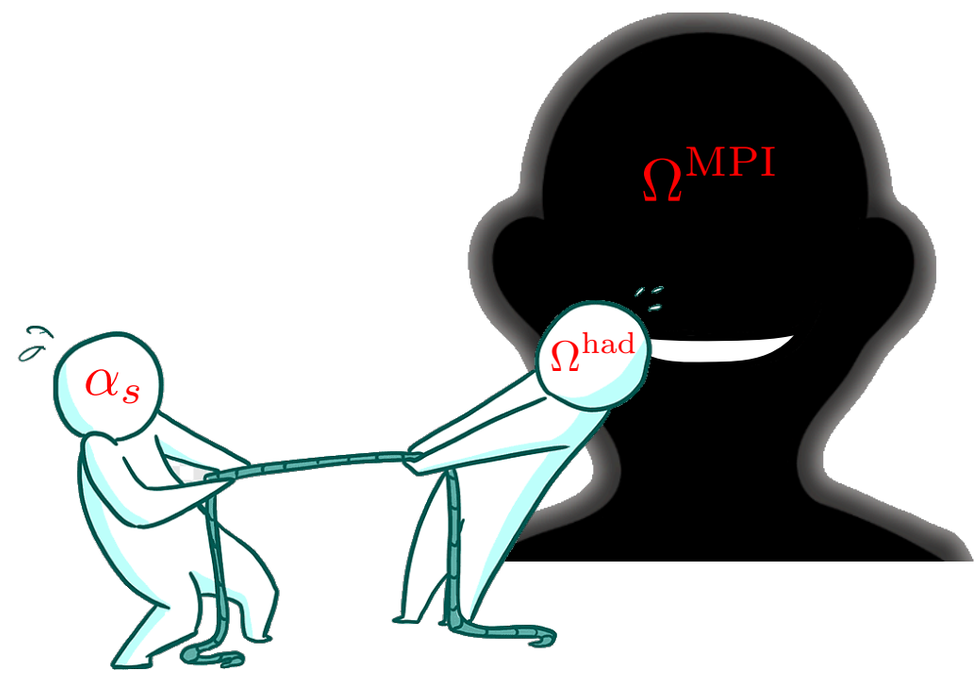
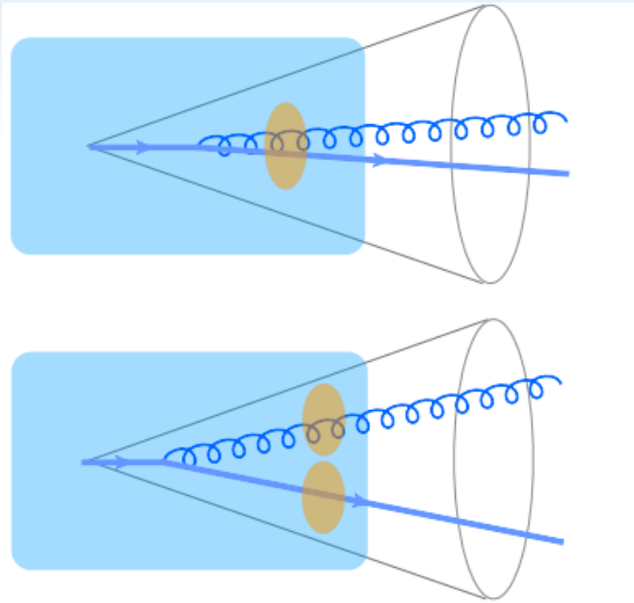


- **Desire:**
  - reduced sensitivity to non-perturbative effects (hadronizations and MPI)
  - additional handle(s) to lift degeneracy between extractions of α_s and fitting $\Omega^{\rm had}$

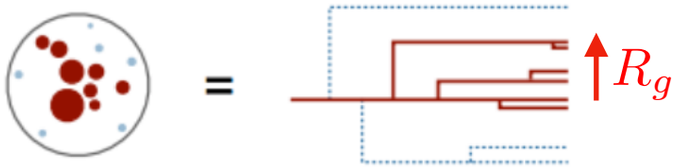
  Groomed jet angularities!

- Proof-of-principle Monte Carlo study shows groomed angularity to be an ideal observable.
  - Currently feasible to determine with 10% uncertainty. *Les Houches `17*

Groomed jet radius



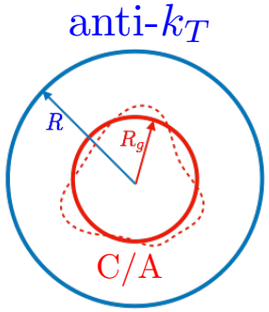
- Dynamically determines the size of the hard-collinear splitting



  - What is the transverse resolution length of the medium?

Y. Mehtar-Tani

- Also gives the "active area" of the groomed jet



$$\text{Active area} \sim \pi R_g^2$$

  - Can serve as a proxy for the sensitivity to pileup.
  - Complicated by clustering effects

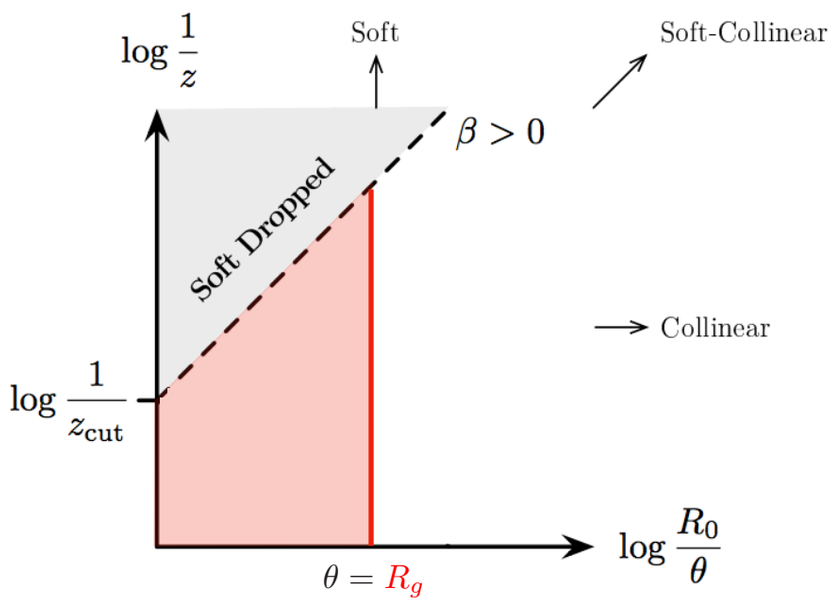
- Factorization formalism necessary for studying many groomed jet observables

  - To regulate divergence of the Sudakov safe observables
  - Sometimes the observables are invariably coupled to R_g measurements.

ex: Δ_E , energy drop

Kang, KL, Liu, Neill, Ringer `19
Larkoski, Marzani, Soyez, Thaler `14

Groomed jet radius

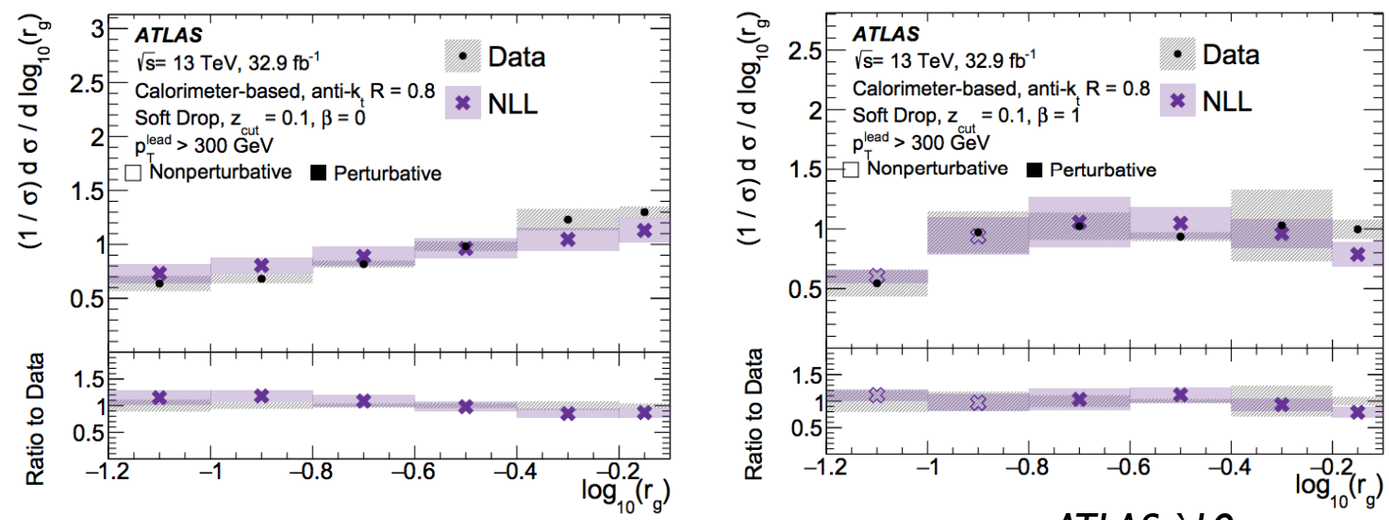


Kang, KL, Liu, Neill, Ringer `19
Larkoski, Marzani, Soyez, Thaler `14

- **Factorization derived to NLL**

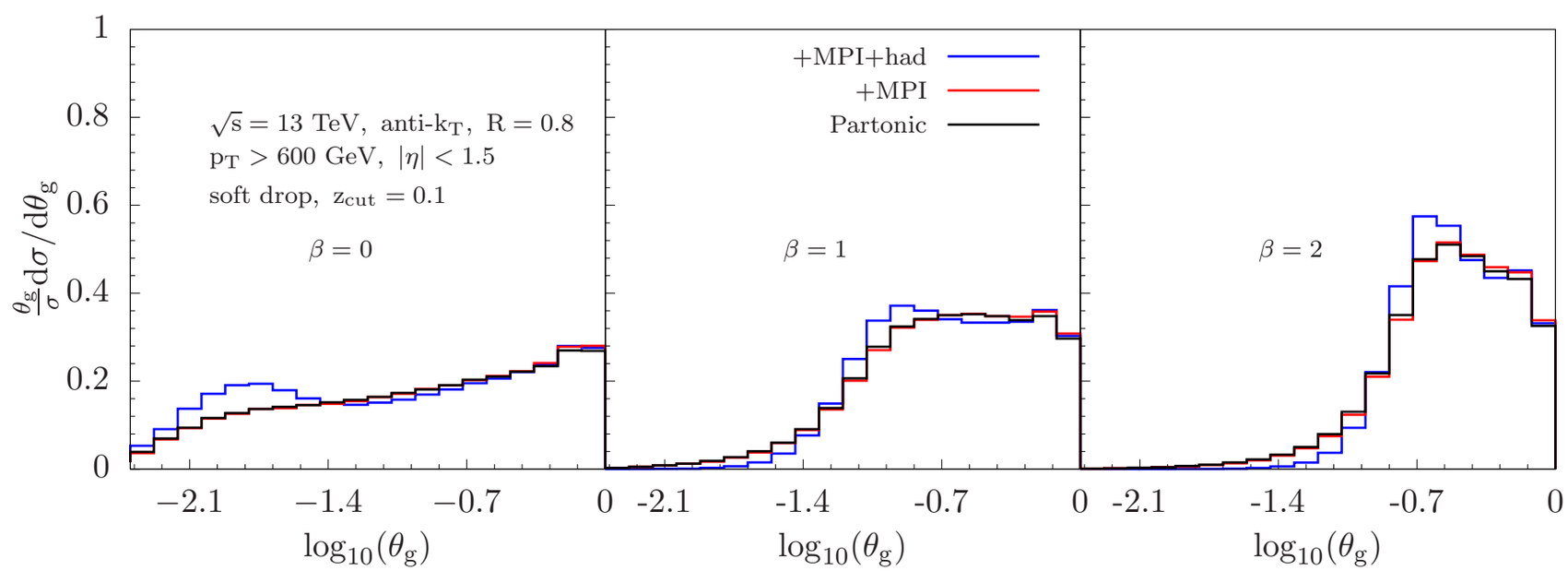
Kang, KL, Liu, Neill, Ringer `19

- **Data comparison**

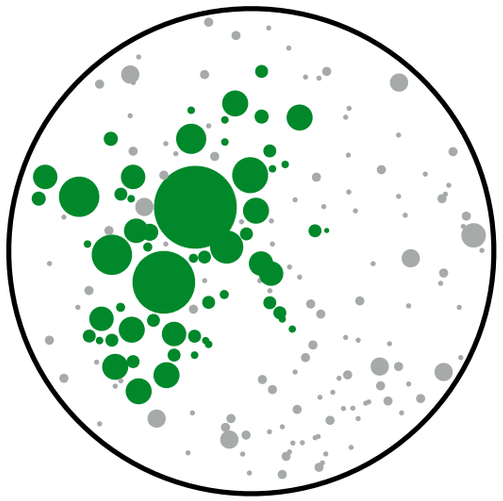


ATLAS `19

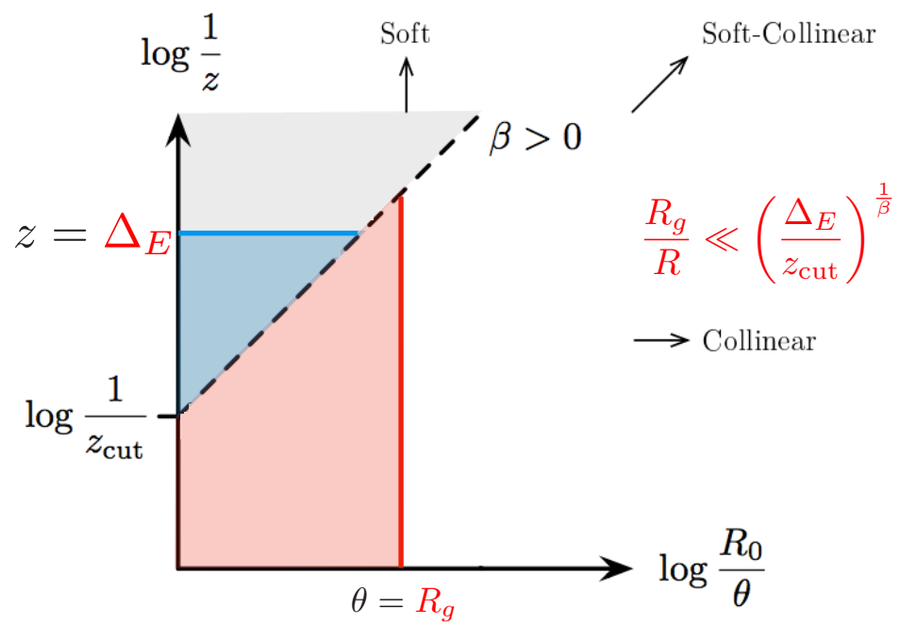
- **Small MPI effects**

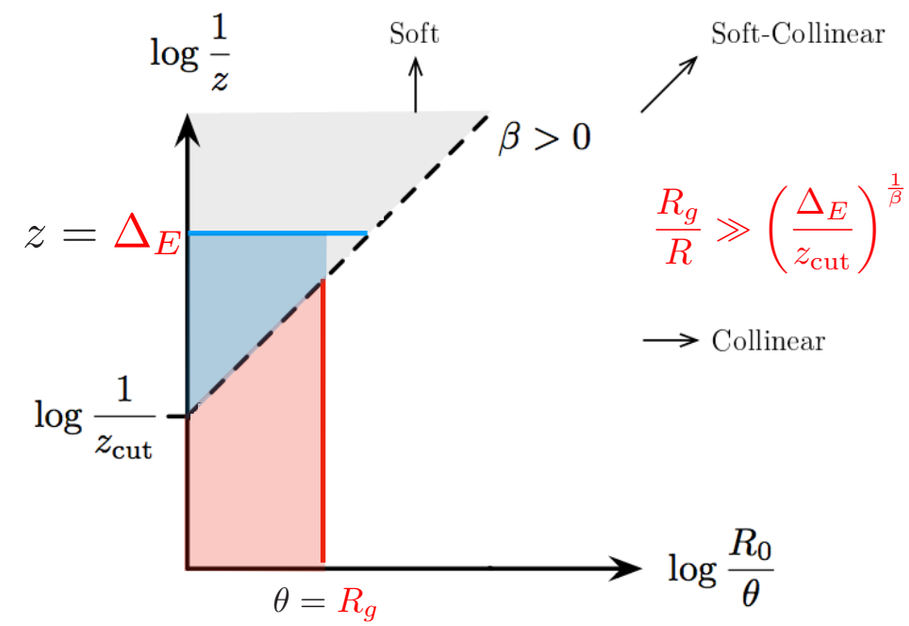


Energy drop



$$\Delta_E = \frac{p_T - p_T^{\rm gr}}{p_T}$$





Cal, KL, Ringer, Waalewijn `20
Larkoski, Marzani, Soyez, Thaler `14

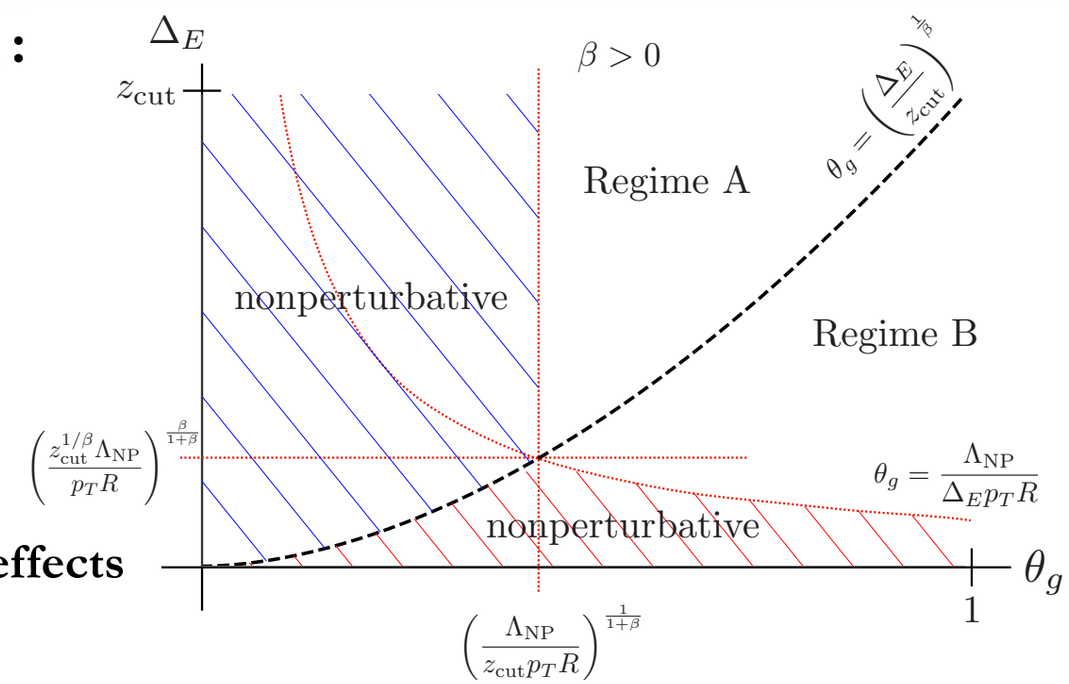
- **Energy drop measurements in soft drop groomed jet is coupled with R_g measurements.**
  - **Factorizations for trimmed jets, iterated soft drop groomed jets** *Cal, KL, Ringer, Waalewijn `20*
- **All energy drop dependent "modes" are only sensitive to soft modes only.**
  - **Does the medium modify the soft and collinear radiations differently?**

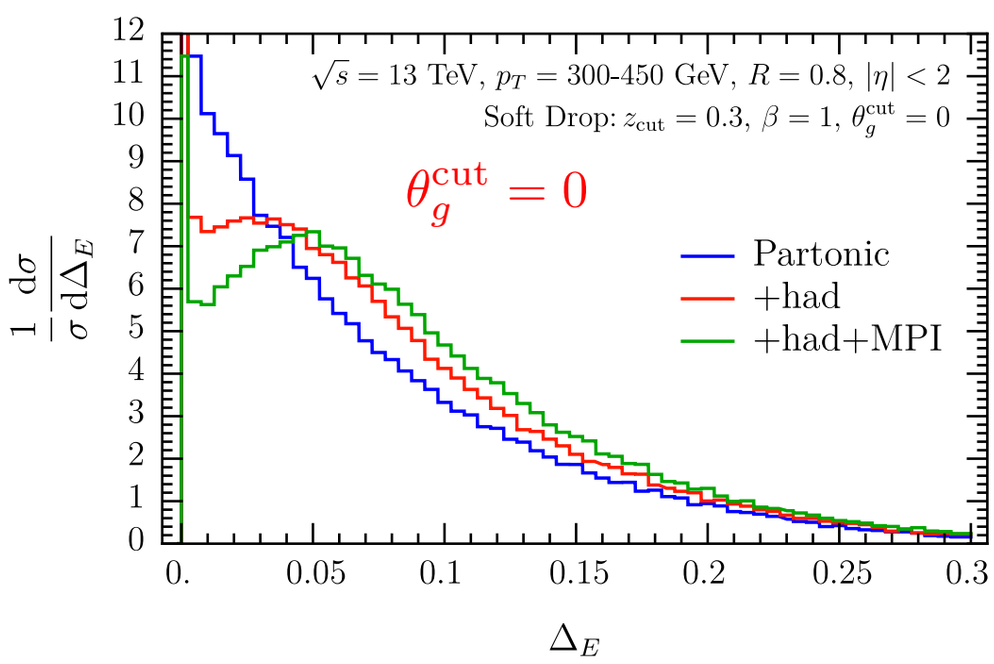
Energy drop

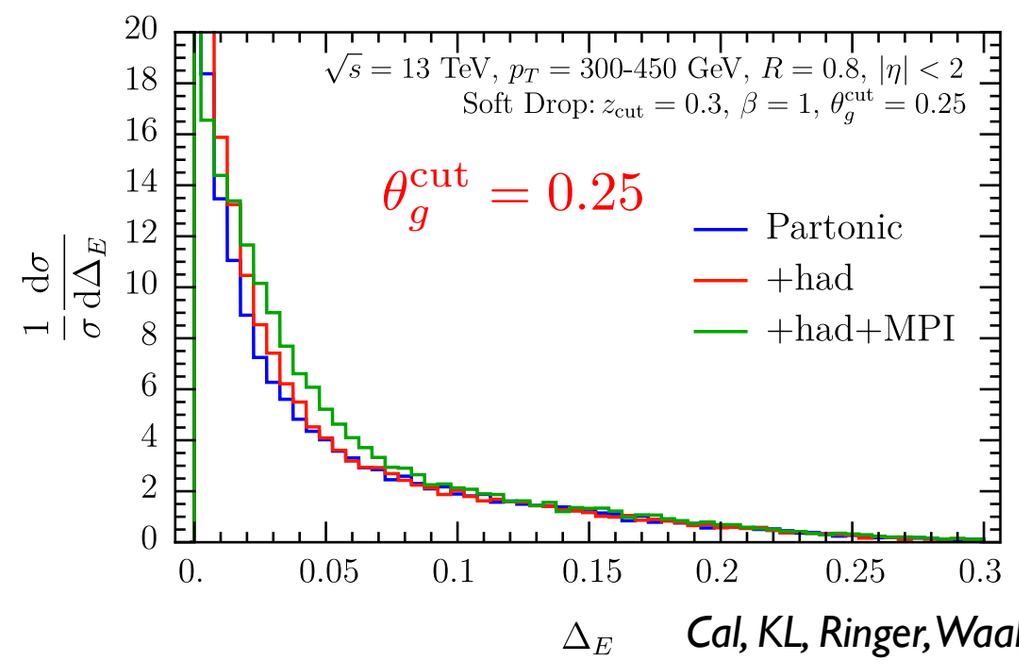
- **Factorization framework differential in both Δ_E, R_g :**

$$\frac{d\sigma}{dp_T d\eta d\Delta_E dR_g}$$

- **Integrate over regions of R_g :** $\theta_g^{\rm cut} < \theta_g < 1 \quad \left(\theta_g = \frac{R_g}{R}\right)$
  - **Experimental tracking efficiency may require $\theta_g^{\rm cut}$**
  - **Different $\theta_g^{\rm cut}$ gives varying sensitivity to non-perturbative effects**
    - **May be useful for MC tuning**







Cal, KL, Ringer, Waalewijn `20

Energy drop

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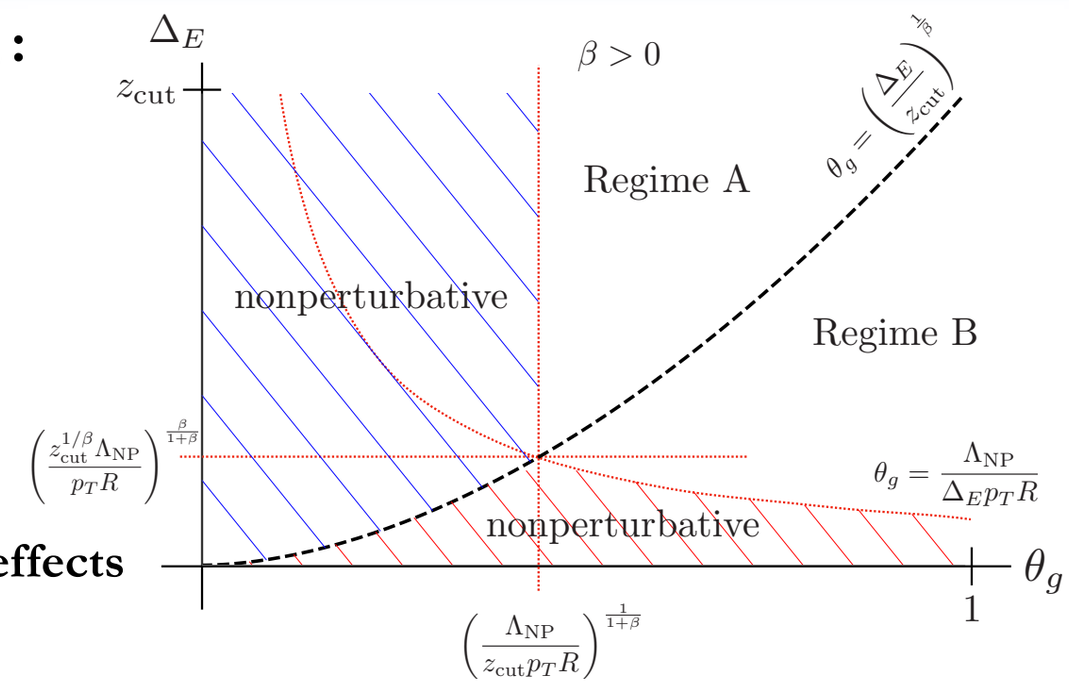
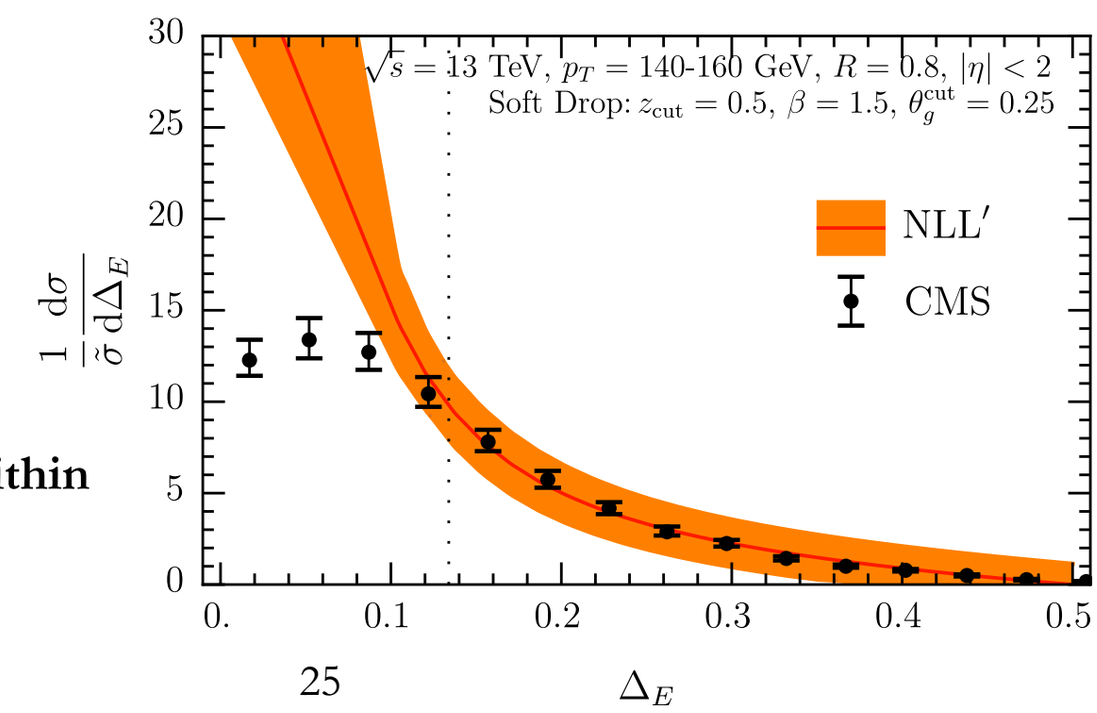


- **Good agreement within perturbative region**

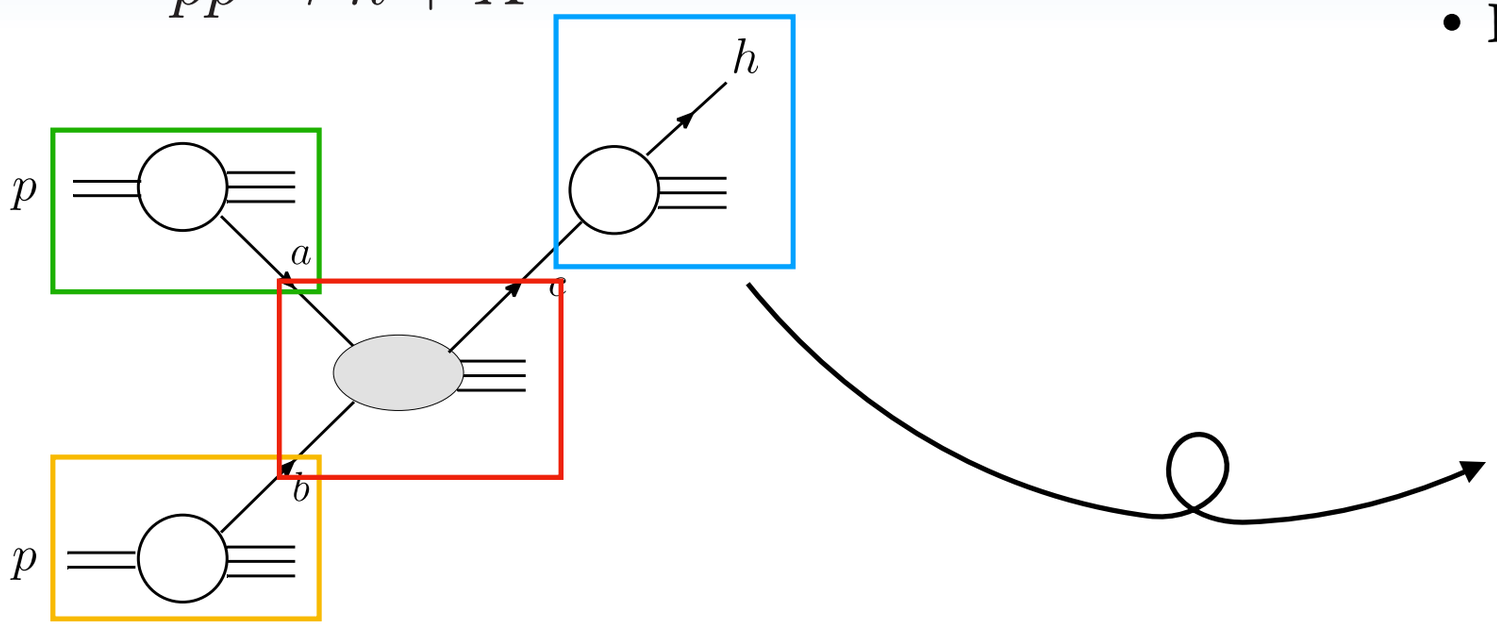


CMS `17

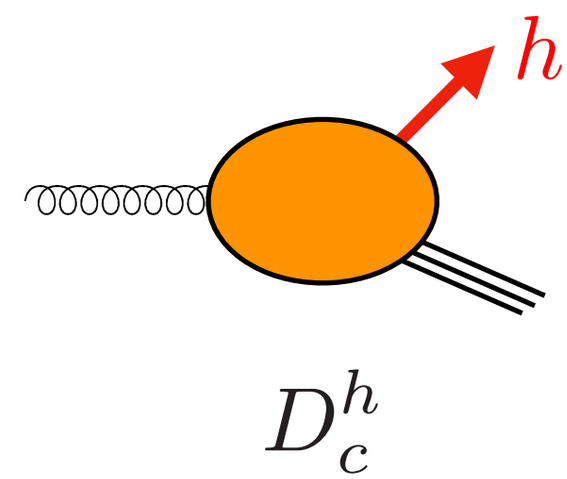
Cal, KL, Ringer, Waalewijn `20

Fragmenting Jet Function (FJF)

$pp \to h + X$



- **Fragmentation Functions**



Factorization: separation of dynamics

Hadron $$\frac{d\sigma^{pp\to hX}}{dp_T d\eta} = \sum_{a,b,c} \boxed{f_a} \otimes \boxed{f_b} \otimes \boxed{H_{ab}^c} \otimes \boxed{D_c^h}$$

$\Lambda_{\rm QCD}$ p_T $\Lambda_{\rm QCD}$

Parton polarization

Hadron polarization	U	L/circ	T/lin
U	$D^{h/c}$		$H^{\perp h/c}$
L		$G^{h/c}$	$H_L^{\perp h/c}$
T	$D_T^{\perp h/c}$	$G_T^{h/c}$	$H^{h/c}$ $H_T^{\perp h/c}$

Fragmenting Jet Function (FJF)

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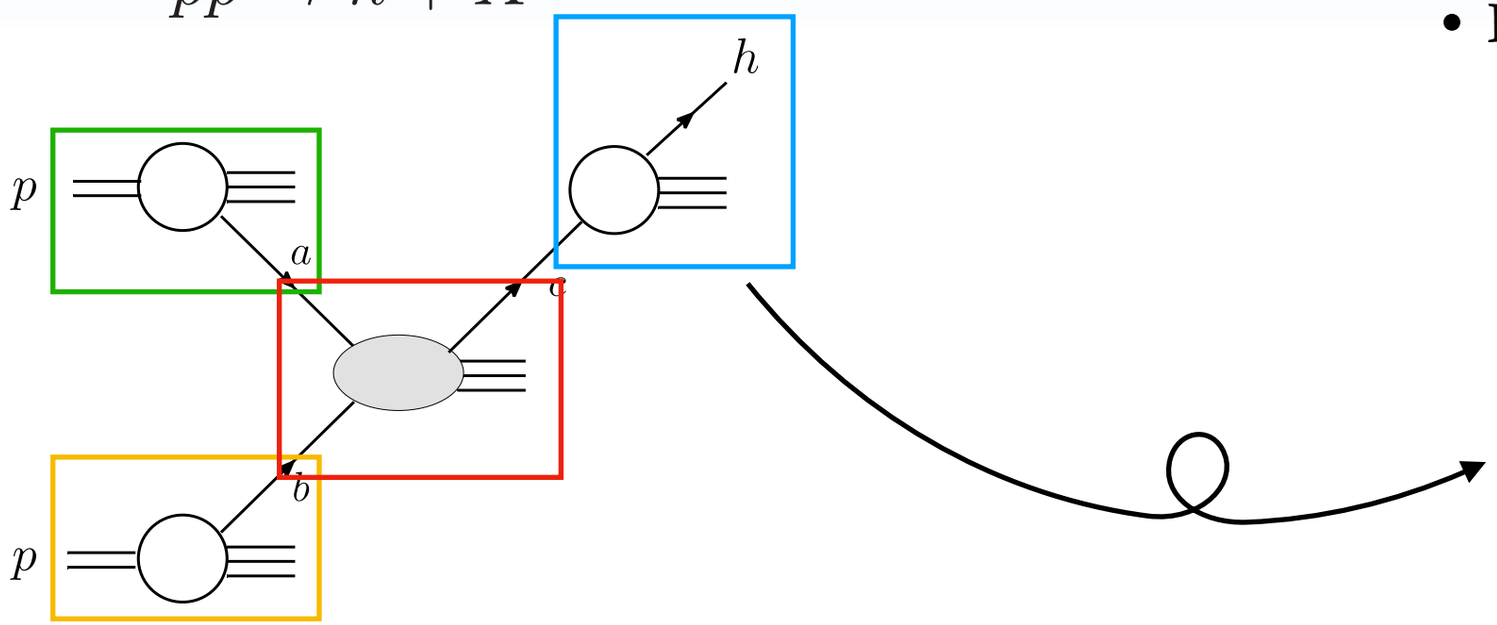
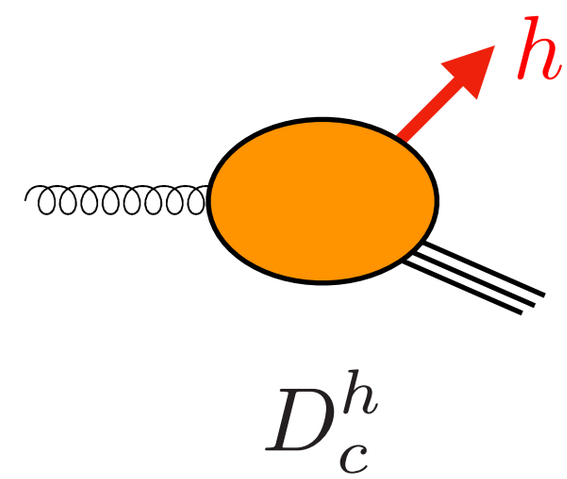


- **Fragmentation Functions**



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$\Lambda_{\rm QCD}$ p_T $\Lambda_{\rm QCD}$

- **Can jet substructure measurements help us to constrain fragmentation functions?**

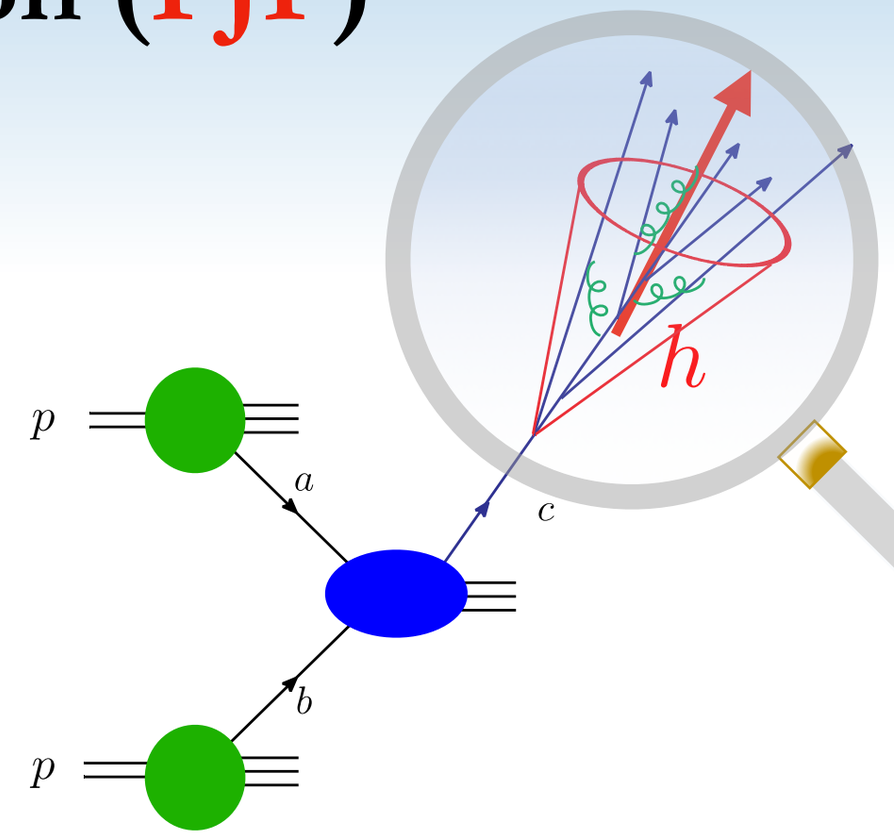
Fragmenting Jet Function (FJF)

Unpolarized case:

$$\frac{d\sigma^{pp\to \mathrm{jet}(h)X}}{dp_T d\eta dz_h} = \sum_{a,b,c} \underset{\Lambda_{\mathrm{QCD}}}{f_{a/A}} \otimes \underset{\Lambda_{\mathrm{QCD}}}{f_{b/B}} \otimes \underset{p_T}{H^c_{ab}} \otimes \underset{\substack{p_T R \\ \Lambda_{\mathrm{QCD}}}}{\mathcal{G}^h_c(z_h)}$$

where $z = p_T^J / p_T^c$

$z_h = p_T^h / p_T^J$



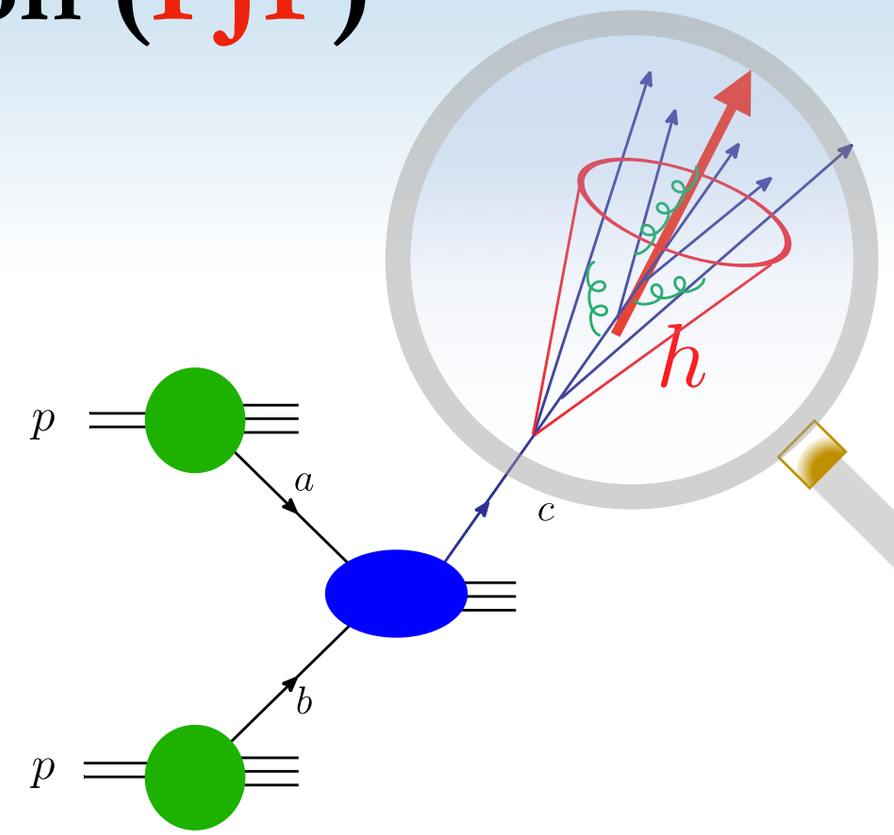
Procura, Stewart `10
Arleo, Fontannaz, Guillet, Nguyen `14
Kaufmann, Mukherjee, Vogelsang `15
Kang, Ringer, Vitev `16
Dai, Kim, Leibovich `16

Fragmenting Jet Function (FJF)

Unpolarized case:

$$\frac{d\sigma^{pp\to \mathrm{jet}(h)X}}{dp_T d\eta dz_h} = \sum_{a,b,c} f_{a/A} \otimes f_{b/B} \otimes H^c_{ab} \otimes \mathcal{G}^h_c(z_h)$$

Λ_{QCD} p_T $p_T R$ Λ_{QCD}



where $z = p_T^J / p_T^c$

$z_h = p_T^h / p_T^J$

$$\frac{d\sigma^{pp\to hX}}{dp_T d\eta} = \sum_{a,b,c} f_{a/A} \otimes f_{b/B} \otimes H^c_{ab} \otimes \boxed{D^h_c}$$

Λ_{QCD} p_T Λ_{QCD}

where $z = p_T^h / p_T^c$

Procura, Stewart `10
Arleo, Fontannaz, Guillet, Nguyen `14
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Fragmenting Jet Function (FJF)

Unpolarized case:

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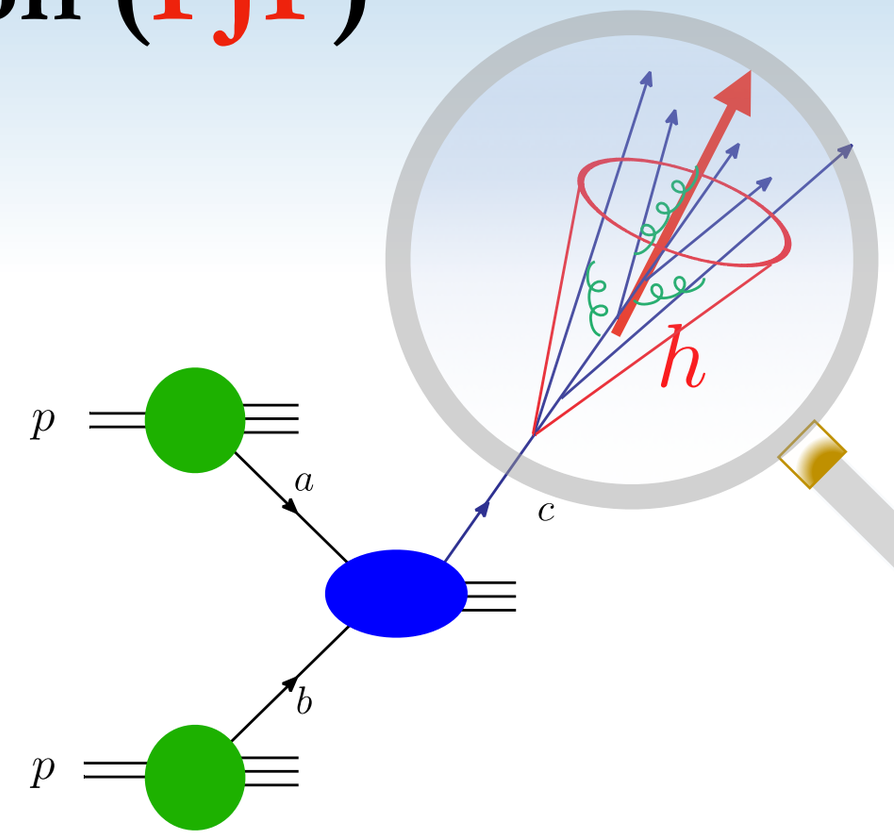
Λ_{QCD} p_T $p_T R$ Λ_{QCD}



where $z = p_T^J / p_T^c$

$z_h = p_T^h / p_T^J$

IR sensitivity and require matching:

$$\mathcal{G}^h_c(z, z_h, p_T R, \mu) = \sum_j \mathcal{J}_{ij}(z, z_h, p_T R, \mu) \otimes D^h_j(z_h, \mu)$$

$p_T R$ Λ_{QCD}

matching coefficients **collinear FFs**

- **Collinear FJFs can be related to collinear FFs**

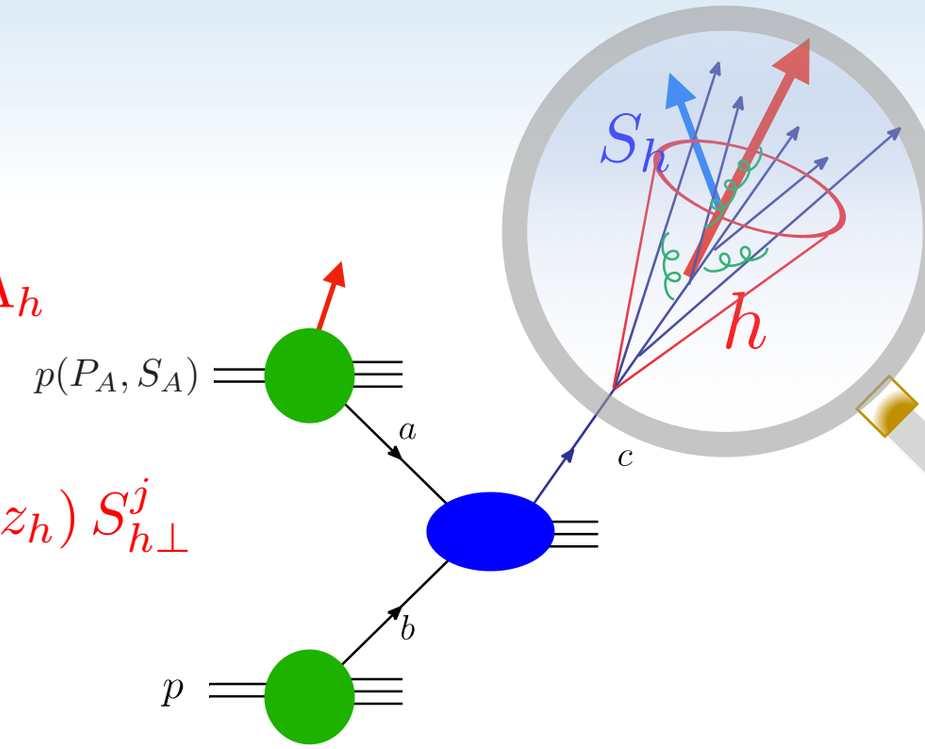
Procura, Stewart `10
Arleo, Fontannaz, Guillet, Nguyen `14
Kaufmann, Mukherjee, Vogelsang `15
Kang, Ringer, Vitev `16
Dai, Kim, Leibovich `16

Polarized Fragmenting Jet Function (FJF)

Polarized case:

$$\frac{d\sigma_{LL}^{\bar{p}p\to \mathrm{jet}(\bar{h})X}}{dp_T d\eta dz_h} = \sum_{a,b,c} \Lambda_A\, g_{a/A} \otimes f_{b/B} \otimes \Delta_{LL} H_{\bar{a}b}^{\bar{c}} \otimes \Delta_L \mathcal{G}_c^h(z_h)\, \Lambda_h$$

$$\frac{d\sigma_{TT}^{\bar{p}p\to \mathrm{jet}(\bar{h})X}}{dp_T d\eta dz_h} = \sum_{a,b,c} S_{A\perp}^i\, h_{a/A} \otimes f_{b/B} \otimes \Delta_{TT} (H_{ij})_{\bar{a}b}^{\bar{c}} \otimes \Delta_T \mathcal{G}_c^h(z_h)\, S_{h\perp}^j$$



where $z = p_T^J / p_T^c$

$$z_h = p_T^h / p_T^J$$

$$d\sigma_{LL} \equiv \frac{d\sigma(\Lambda_A, \Lambda_h) - d\sigma(\Lambda_A, -\Lambda_h)}{2}$$

$$d\sigma_{TT} \equiv \frac{d\sigma(\vec{S}_{A\perp}, \vec{S}_{h\perp}) - d\sigma(\vec{S}_{A\perp}, -\vec{S}_{h\perp})}{2}$$

Similar definitions for $\Delta_{LL} H_{\bar{a}b}^{\bar{c}}$ **and** $\Delta_{TT}\,(H_{ij})$

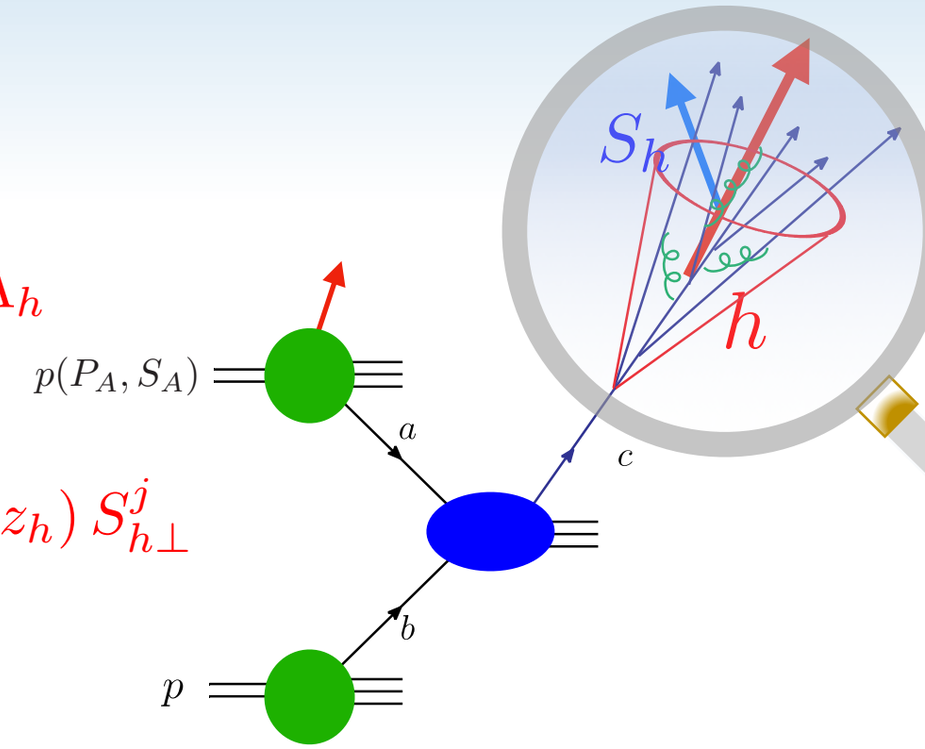
Kang, KL, Zhao, `20

Polarized Fragmenting Jet Function (FJF)

Polarized case:

$$\frac{d\sigma_{LL}^{\bar{p}p\to \mathrm{jet}(\bar{h})X}}{dp_T d\eta dz_h} = \sum_{a,b,c} \Lambda_A\, g_{a/A} \otimes f_{b/B} \otimes \Delta_{LL} H_{\bar{a}b}^{\bar{c}} \otimes \Delta_L \mathcal{G}_c^h(z_h)\, \Lambda_h$$

$$\frac{d\sigma_{TT}^{\bar{p}p\to \mathrm{jet}(\bar{h})X}}{dp_T d\eta dz_h} = \sum_{a,b,c} S_{A\perp}^i\, h_{a/A} \otimes f_{b/B} \otimes \Delta_{TT} (H_{ij})_{\bar{a}b}^{\bar{c}} \otimes \Delta_T \mathcal{G}_c^h(z_h)\, S_{h\perp}^j$$



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$$\frac{d\sigma_{TT}^{\bar{p}p\to \bar{h}X}}{dp_T d\eta} = \sum_{a,b,c} S_{A\perp}^i\, h_{a/A} \otimes f_{b/B} \otimes \Delta_{TT} (H_{ij})_{\bar{a}b}^{\bar{c}} \otimes \boxed{H_c^h\, S_{h\perp}^j}$$

Quark polarization

Hadron polarization	U	L	T
U	$D^{h/q}$		
L		$G^{h/q}$	
T			$H^{h/q}$

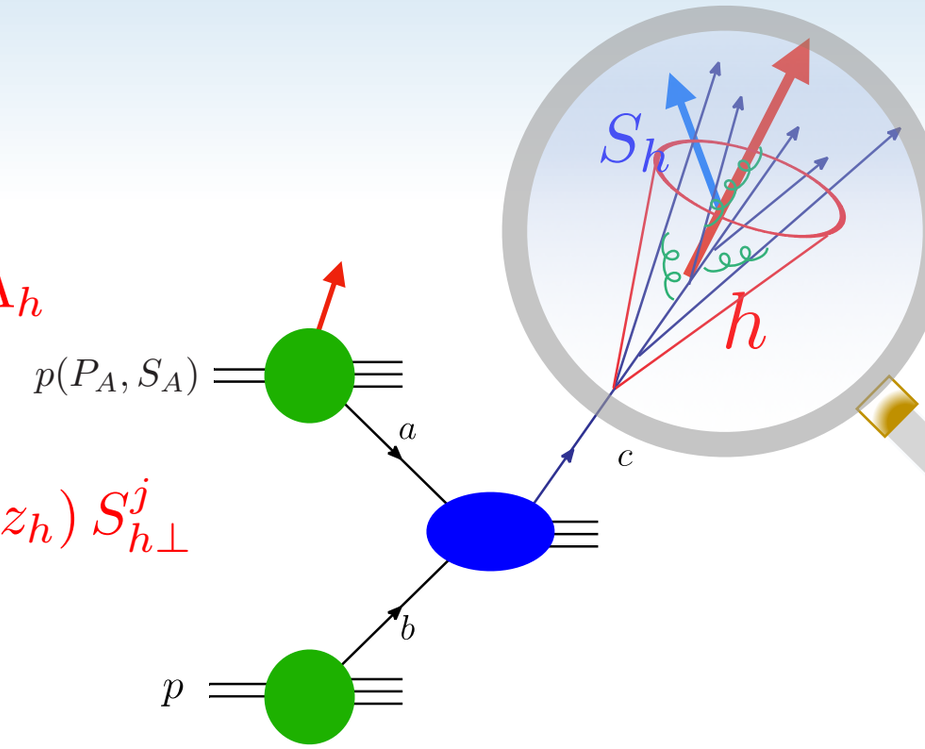
Kang, KL, Zhao, `20

Polarized Fragmenting Jet Function (FJF)

Polarized case:

$$\frac{d\sigma_{LL}^{\bar{p}p\to\text{jet}(\bar{h})X}}{dp_T d\eta dz_h} = \sum_{a,b,c} \Lambda_A\, g_{a/A} \otimes f_{b/B} \otimes \Delta_{LL} H_{\bar{a}b}^{\bar{c}} \otimes \Delta_L \mathcal{G}_c^h(z_h)\, \Lambda_h$$

$$\frac{d\sigma_{TT}^{\bar{p}p\to\text{jet}(\bar{h})X}}{dp_T d\eta dz_h} = \sum_{a,b,c} S_{A\perp}^i\, h_{a/A} \otimes f_{b/B} \otimes \Delta_{TT} (H_{ij})_{\bar{a}b}^{\bar{c}} \otimes \Delta_T \mathcal{G}_c^h(z_h)\, S_{h\perp}^j$$



where $z = p_T^J / p_T^c$

$$z_h = p_T^h / p_T^J$$

IR sensitivity and require matching:

$$\mathcal{G}_c^h(z, z_h, p_T R, \mu) = \sum_j \mathcal{J}_{ij}(z, z_h, p_T R, \mu) \otimes D_j^h(z_h, \mu)$$

$p_T R$ Λ_{QCD}

$$\Delta_L \mathcal{G}_c^h(z, z_h, p_T R, \mu) = \sum_j \Delta_L \mathcal{J}_{ij}(z, z_h, p_T R, \mu) \otimes G_j^h(z_h, \mu)$$

$$\Delta_T \mathcal{G}_c^h(z, z_h, p_T R, \mu) = \sum_j \Delta_T \mathcal{J}_{ij}(z, z_h, p_T R, \mu) \otimes H_j^h(z_h, \mu)$$

Quark polarization

Hadron polarization	U	L	T
U	$D^{h/q}$		
L		$G^{h/q}$	
T			$H^{h/q}$

Kang, KL, Zhao, `20

Study of FJF

	$e^+e^- \to hX$	$pp \to hX$	$pp \to \mathrm{jet}(h)X$
Gluon sensitivity	✗	✓	✓
Differential in z_h	✓	✗	✓

Also large amounts of data available for pp!

Fragmenting Jet Function (FJF)

- **Light charged hadrons**

  Arleo, Fontannaz, Guillet, Nguyen `14
  Kaufmann, Mukherjee, Vogelsang `15
  Kang, Ringer, Vitev `16
  Neill, Scimemi, Waalewijn `16

- **Photons**

  Kaufmann, Mukherjee, Vogelsang `16

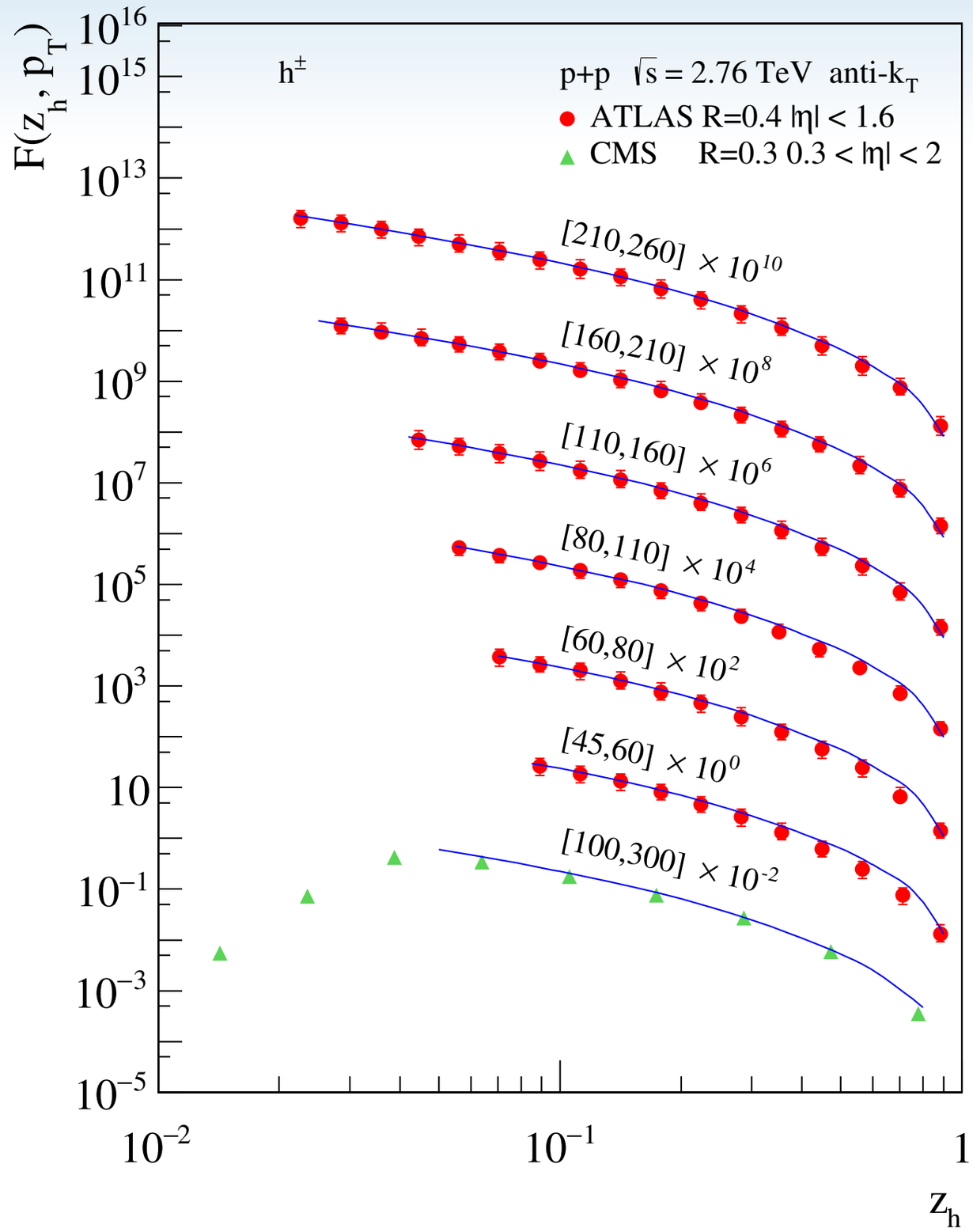
- **Heavy flavor mesons**

  Chien, Kang, Ringer, Vitev, Xing `15
  Bain, Dai, Hornig, Leibovich, Makris, Mehen `16
  Anderle, Kaufmann, Stratmann, Ringer, Vitev `17

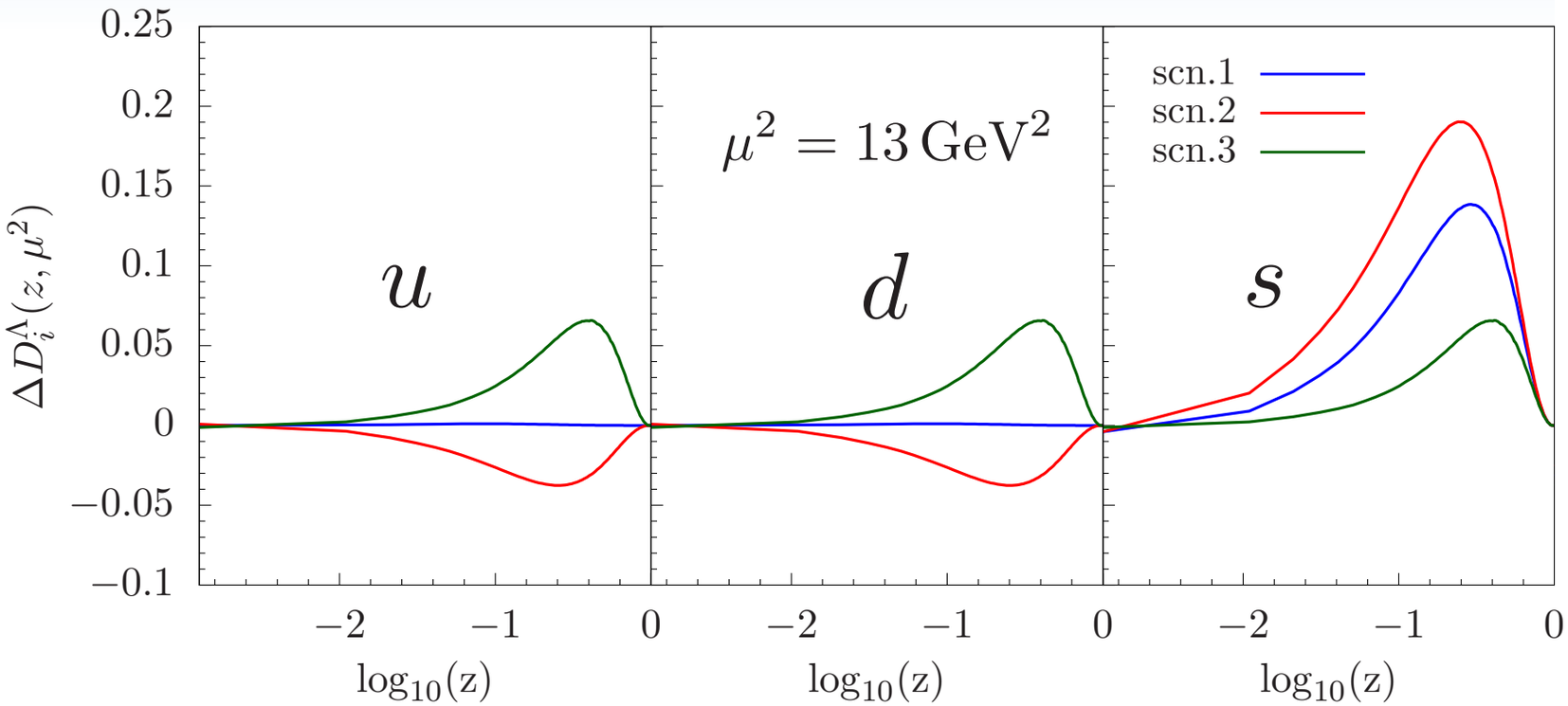
- **Quarkonia**

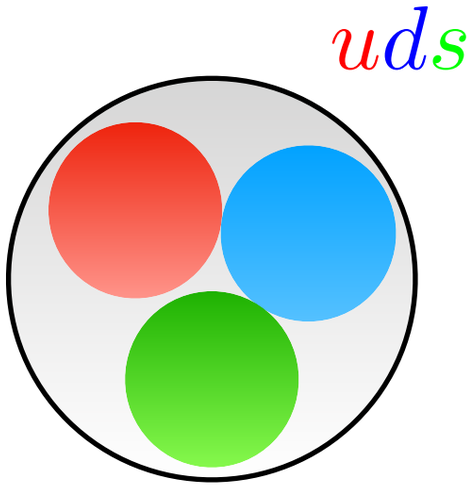
  Baumgart, Leibovich, Mehen, Rothstein `14
  Bain, Dai, Hornig, Leibovich, Makris, Mehen `16
  Kang, Qiu, Ringer, Xing, Zhang `17
  Bain, Dai, Leibovich, Makris, Mehen `17

$$F(z_h, p_T) = \frac{d\sigma^{pp\to(\mathrm{jet}h)X}}{dp_T d\eta dz_h} \Big/ \frac{d\sigma^{pp\to\mathrm{jet}X}}{dp_T d\eta}$$



Longitudinally polarized Λ



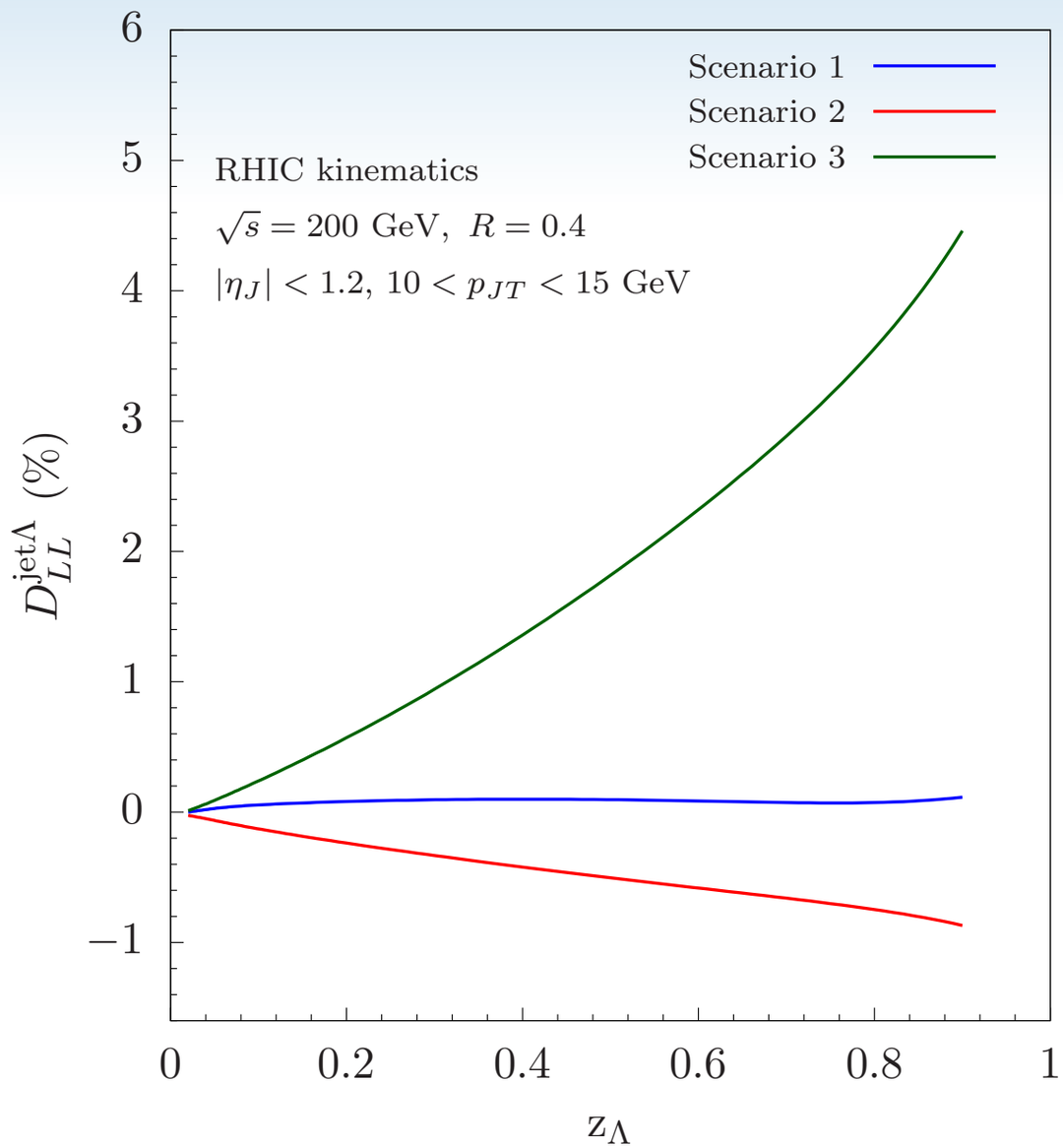


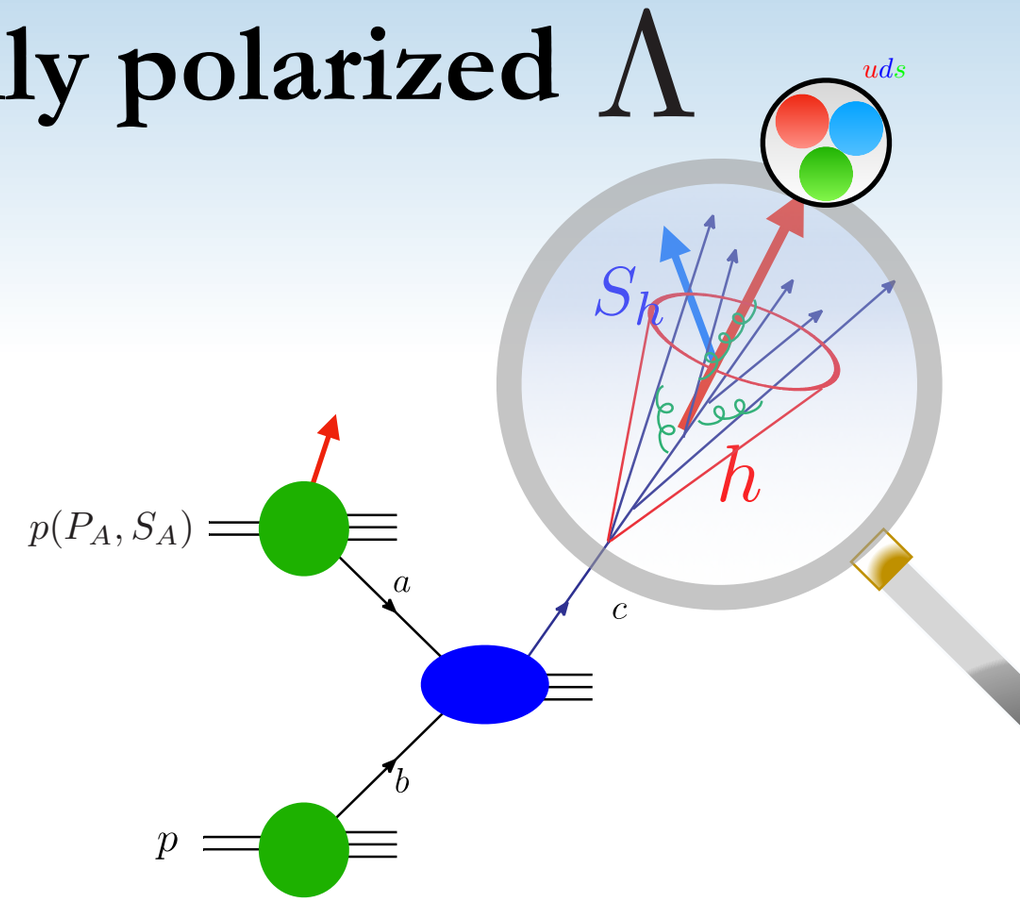
Three different scenarios

- **Scenario 1:** Only strange quarks have contribution to the fragmentation.
- **Scenario 2:** Negative distributions of up and down quarks are assumed.
- **Scenario 3:** Same fragmentation for up, down, and strange quarks.

Burkardt, Jaffe, `93
De Florian, Stratmann, Vogelsang, `98

FJF to study longitudinally polarized Λ





Gives shape expected from the scenarios

$$D_{LL}^{\mathrm{jet}\Lambda} = \frac{d\Delta_{LL}\sigma}{d\sigma}$$

- **Scenario 1:** Only strange quarks have contribution to the fragmentation.
- **Scenario 2:** Negative distributions of up and down quarks are assumed.
- **Scenario 3:** Same fragmentation for up, down, and strange quarks.

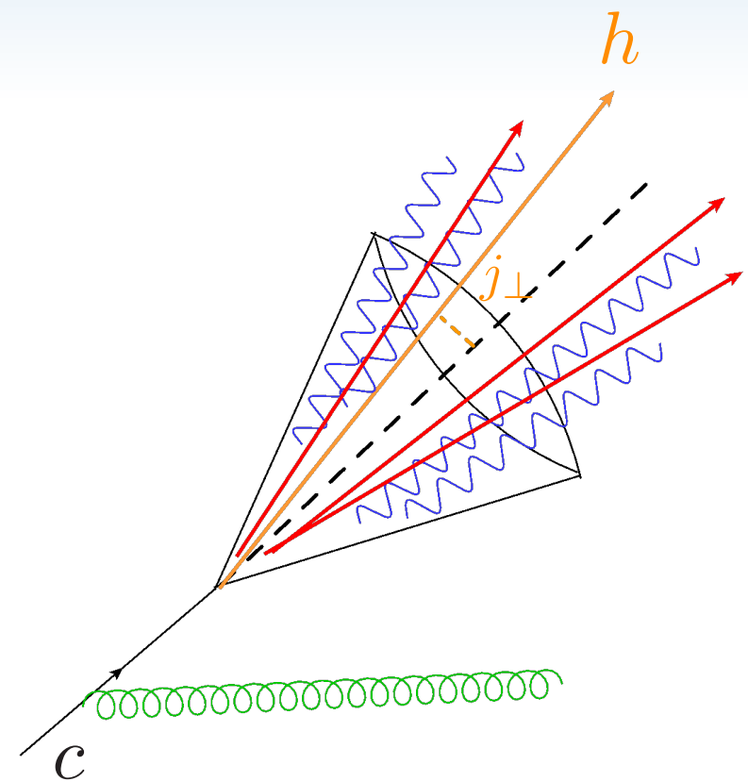
Kang, KL, Zhao, `20

TMD hadron in jet (TMDFJF)

- **Still hard-collinear factorization structure other than jet function modified.**

$$\frac{d\sigma^{pp\to\mathrm{jet}(h)X}}{dp_T d\eta dz_h} = \sum_{a,b,c} f_{a/A} \otimes f_{b/B} \otimes H^c_{ab} \otimes \boxed{\mathcal{G}^h_c(z_h)}$$

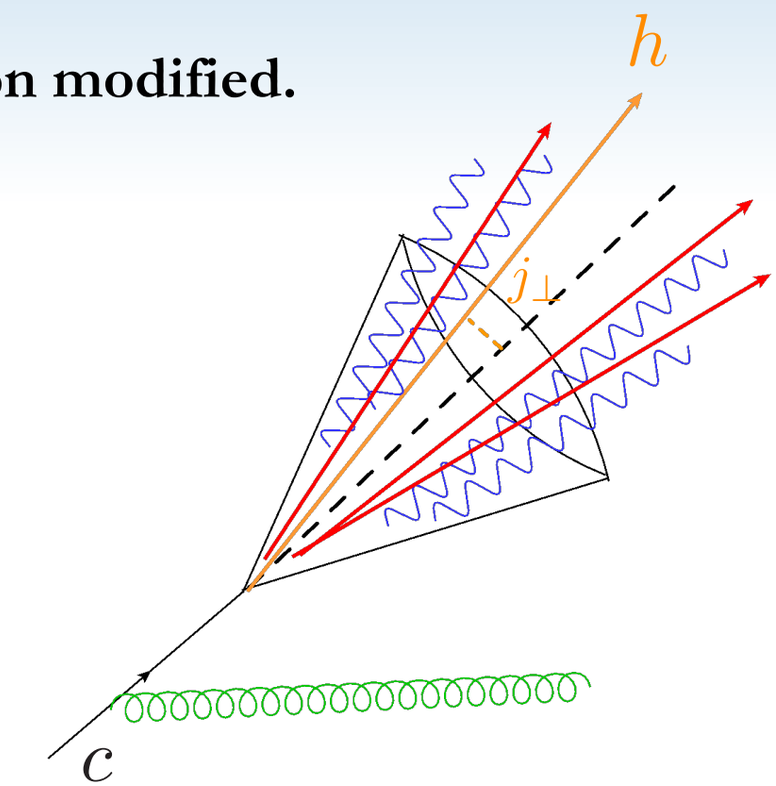
$$\frac{d\sigma^{pp\to\mathrm{jet}(h)X}}{dp_T d\eta dz_h d^2\boldsymbol{j}_\perp} = \sum_{a,b,c} f_{a/A} \otimes f_{b/B} \otimes H^c_{ab} \otimes \boxed{\mathcal{G}^h_c(z_h, \boldsymbol{j}_\perp)}$$



TMD hadron in jet (TMDFJF)

- **Still hard-collinear factorization structure other than jet function modified.**

$$\frac{d\sigma^{pp\to \mathrm{jet}(h)X}}{dp_T d\eta dz_h d^2\boldsymbol{j}_\perp} = \sum_{a,b,c} f_{a/A}\otimes f_{b/B}\otimes H^c_{ab}\otimes \mathcal{G}^h_c(z_h,\boldsymbol{j}_\perp)$$



When $\Lambda_{\mathrm{QCD}} \lesssim j_\perp \ll p_T R, \qquad \lambda \sim j_\perp/p_T$

collinear $\quad k_c \sim p_T(\lambda^2, 1, \lambda)$

soft $\quad k_s \sim p_T(\lambda R, \lambda/R, \lambda)$

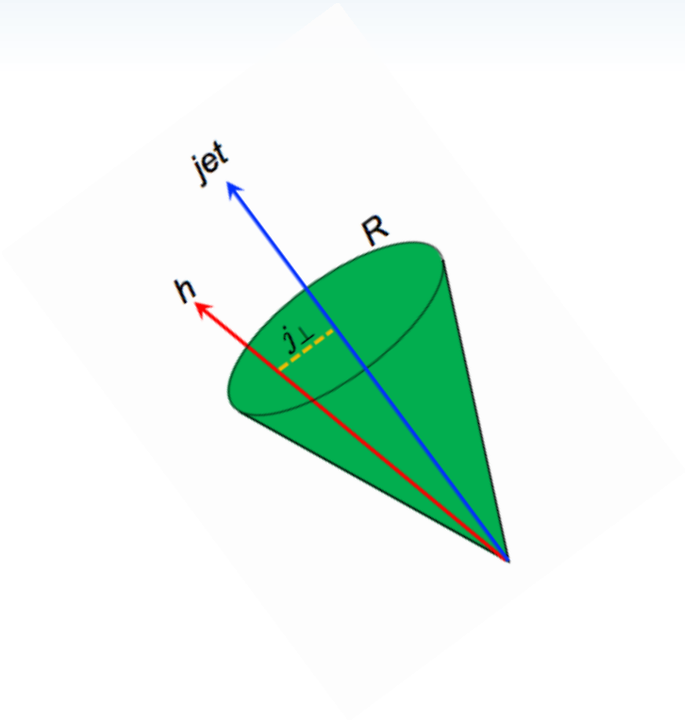
$$\mathcal{G}^h_c(z, z_h, \omega_J R, \boldsymbol{j}_\perp, \mu) = \mathcal{H}_{c\to i}(z,\omega_J R,\mu)\int d^2\boldsymbol{k}_\perp d^2\boldsymbol{\lambda}_\perp \delta^2\left(z_h\boldsymbol{\lambda}_\perp + \boldsymbol{k}_\perp - \boldsymbol{j}_\perp\right)$$
$$\times D_{h/i}(z_h,\boldsymbol{k}_\perp,\mu,\nu) S_i(\boldsymbol{\lambda}_\perp,\mu,\nu R)$$
$$= \mathcal{C}_{c\to i}(z,p_T R,\mu)\underbrace{\hat{D}_{h/i}(z_h,\boldsymbol{j}_\perp;\mu=p_T R)}$$

Standard subtracted TMDFFs, say in SIDIS
Shown to be true perturbatively at NLO

- **TMDFJFs can be related to TMDFFs**

Polarized TMDFJF

- Still hard-collinear factorization structure other than jet function modified.



$$\frac{d\sigma_{LL}^{\bar{p}p\to \mathrm{jet}(\bar{h})X}}{dp_T d\eta dz_h} = \sum_{a,b,c} \Lambda_A\, g_{a/A} \otimes f_{b/B} \otimes \Delta_{LL} H_{\bar{a}b}^{\bar{c}} \otimes \boxed{\Delta_L \mathcal{G}_c^h(z_h)\, \Lambda_h}$$

$$\frac{d\sigma_{TT}^{\bar{p}p\to \mathrm{jet}(\bar{h})X}}{dp_T d\eta dz_h} = \sum_{a,b,c} S_{A\perp}^i\, h_{a/A} \otimes f_{b/B} \otimes \Delta_{TT} (H_{ij})_{\bar{a}b}^{\bar{c}} \otimes \boxed{\Delta_T \mathcal{G}_c^h(z_h)\, S_{h\perp}^j}$$

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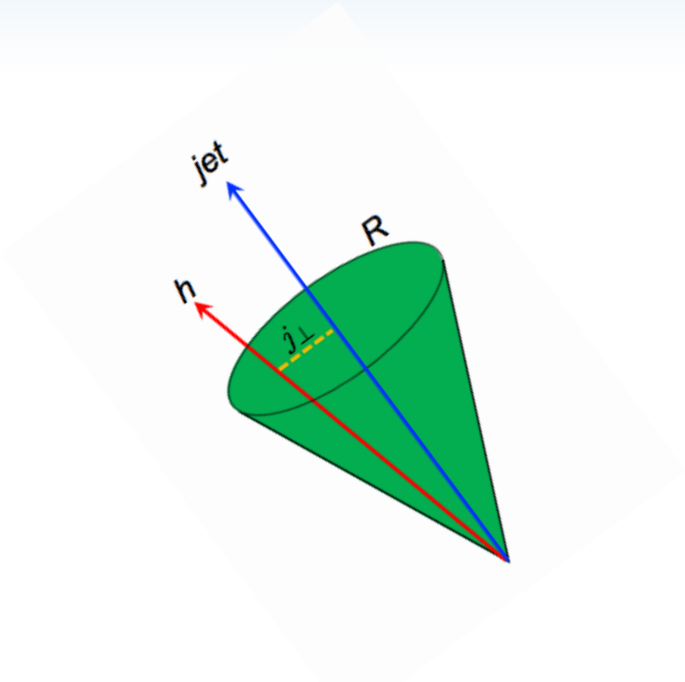
Polarized TMDFJF

- Still hard-collinear factorization structure other than jet function modified.



$$\frac{d\sigma_{LL}^{\bar{p}p\to \mathrm{jet}(\bar{h})X}}{dp_T d\eta dz_h} = \sum_{a,b,c} \Lambda_A\, g_{a/A} \otimes f_{b/B} \otimes \Delta_{LL} H_{\bar{a}b}^{\bar{c}} \otimes \boxed{\Delta_L \mathcal{G}_c^h(z_h)\, \Lambda_h}$$

$$\frac{d\sigma_{TT}^{\bar{p}p\to \mathrm{jet}(\bar{h})X}}{dp_T d\eta dz_h} = \sum_{a,b,c} S_{A\perp}^i\, h_{a/A} \otimes f_{b/B} \otimes \Delta_{TT} (H_{ij})_{\bar{a}b}^{\bar{c}} \otimes \boxed{\Delta_T \mathcal{G}_c^h(z_h)\, S_{h\perp}^j}$$

<u>Polarized TMD hadron in jet production</u>

when fragmenting parton is unpolarized, $\mathcal{G}_c^h(z_h) \to \mathcal{G}_c^h(z_h, j_\perp) F_U$

" longitudinally polarized, $\Delta_L \mathcal{G}_c^h(z_h) S_L \to \Delta_L \mathcal{G}_c^h(z_h, j_\perp) F_L$

" transversely polarized, $\Delta_T \mathcal{G}_c^h(z_h) S_{hT}^i \to \Delta_T \mathcal{G}_c^h(z_h, j_\perp) F_T^i$

F_U, F_L and F_T^i can depend on $j_\perp, S_{h\perp}, \Lambda_h$ depending on the polarization of the final hadron.

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Polarized TMDFJF

TMD Fragmentation Functions (TMDFF)

Quark polarization

Hadron polarization	U	L	T
U	$D^{h/q}$		$H^{\perp h/q}$
L		$G^{h/q}$	$H_L^{\perp h/q}$
T	$D_T^{\perp h/q}$	$G_T^{h/q}$	$H^{h/q}$ $H_T^{\perp h/q}$

TMD Fragmenting Jet Functions (TMDFJF)

Quark polarization

Hadron polarization	U	L	T
U	$\mathcal{D}^{h/q}$		$\mathcal{H}^{\perp h/q}$
L		$\mathcal{G}^{h/q}$	$\mathcal{H}_L^{\perp h/q}$
T	$\mathcal{D}_T^{\perp h/q}$	$\mathcal{G}_T^{h/q}$	$\mathcal{H}^{h/q}$ $\mathcal{H}_T^{\perp h/q}$

Polarized TMD hadron in jet production

when fragmenting parton is unpolarized,

" longitudinally polarized,

" transversely polarized,

Actually not really good notations:

$$\mathcal{G}_c^h(z_h) \to \mathcal{G}_c^h(z_h, j_\perp) F_U$$

$$\Delta_L \mathcal{G}_c^h(z_h) S_L \to \Delta_L \mathcal{G}_c^h(z_h, j_\perp) F_L$$

$$\Delta_T \mathcal{G}_c^h(z_h) S_{hT}^i \to \Delta_T \mathcal{G}_c^h(z_h, j_\perp) F_T^i$$

F_U, F_L and F_T^i can depend on $j_\perp, S_{h\perp}, \Lambda_h$ depending on the polarization of the final hadron.

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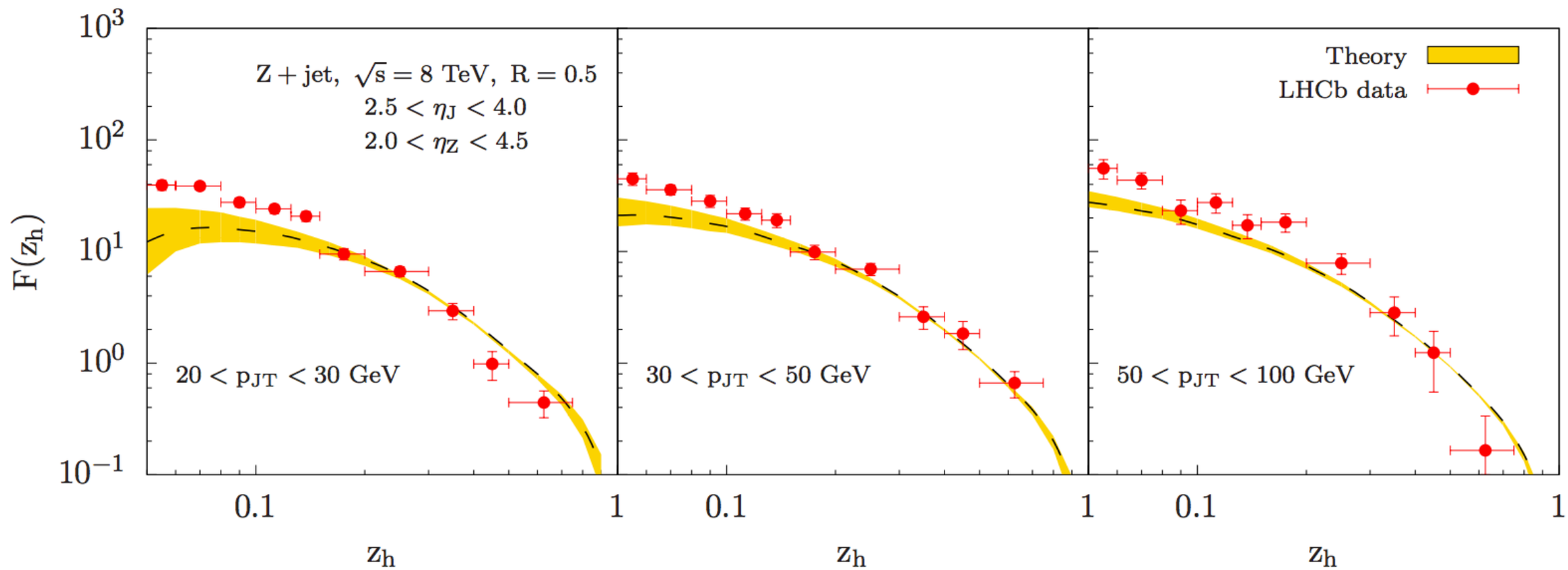
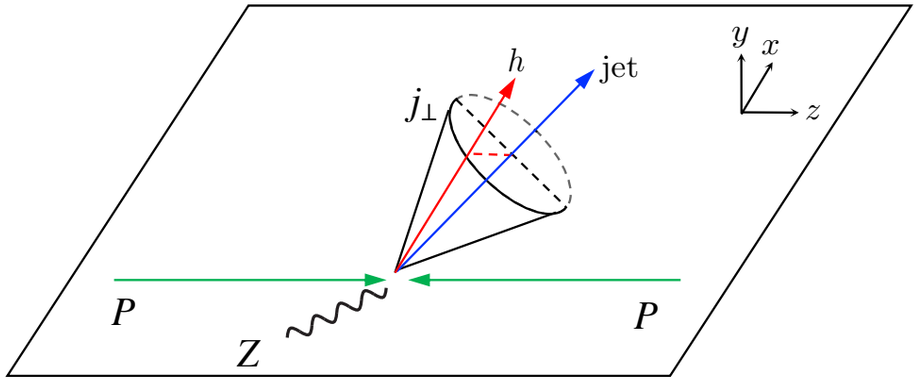
Z-tagged jet



- **LHCb collaboration measured collinear FFs and** $j_\perp$

  Agrees well in the $0.1 < z_h < 0.5$ **region**



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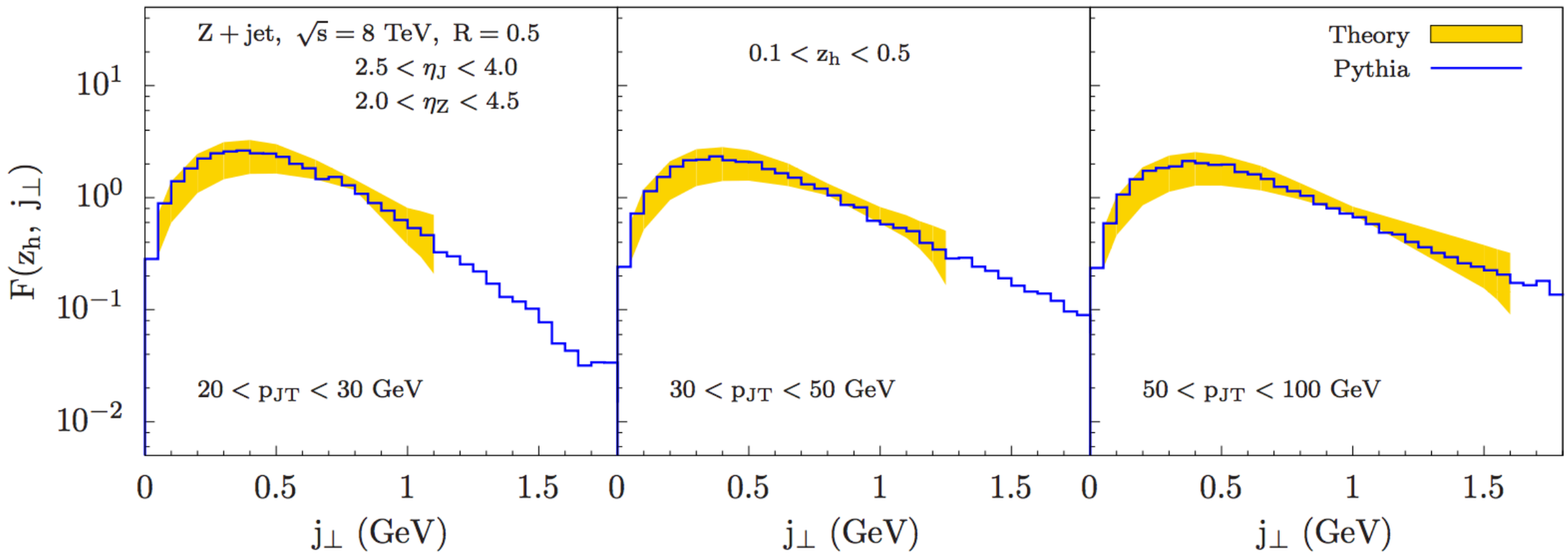
Z-tagged jet

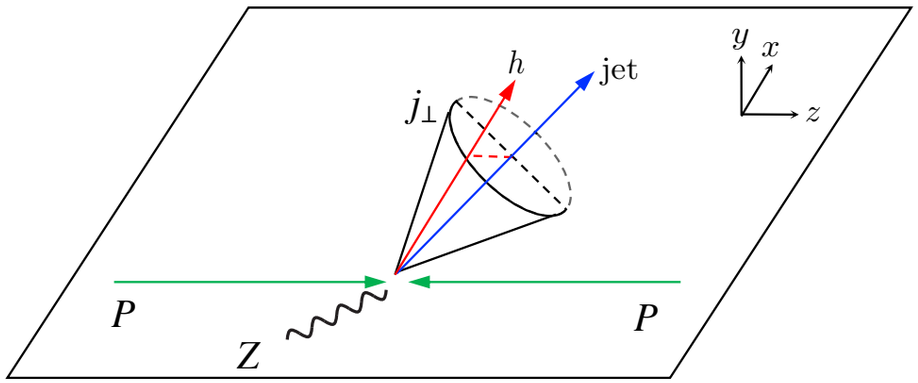


- **LHCb collaboration measured collinear FFs and** $j_\perp$

  Agrees well in the $0.1 < z_h < 0.5$ **region**

- **LHCb collaboration measured** $j_\perp$ **recently.**

  but with entire $0 < z_h < 1$



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Polarizing FF

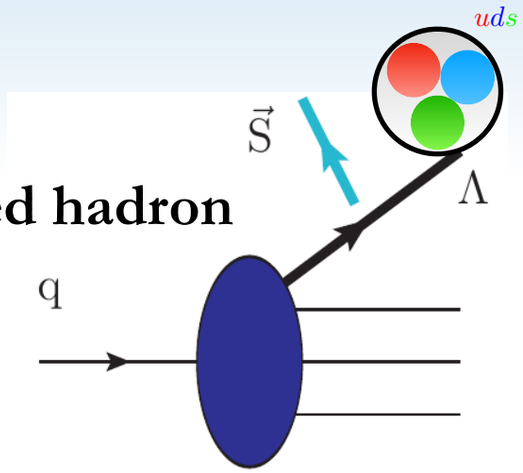
TMD Fragmentation Functions

Quark polarization

Hadron polarization

	U	L	T
U	$D^{h/q}$		$H^{\perp h/q}$
L		$G^{h/q}$	$H_L^{\perp h/q}$
T	$D_T^{\perp h/q}$	$G_T^{h/q}$	$H^{h/q}$ $H_T^{\perp h/q}$

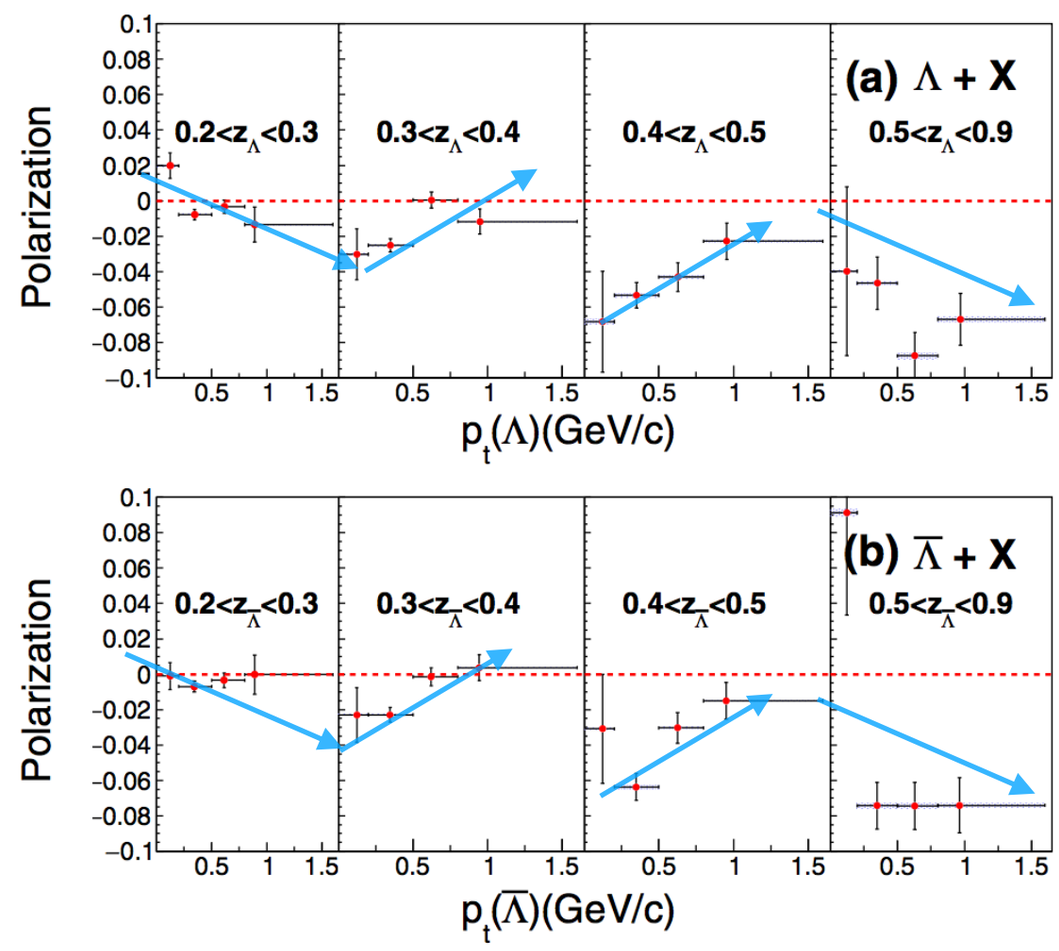
- Describes transversely polarized hadron inside unpolarized parton.



- Belle data shows different trend in p_t with different range of z_Λ

Belle Collaboration `19

- FJF at hadron colliders would be a great complimentary results!

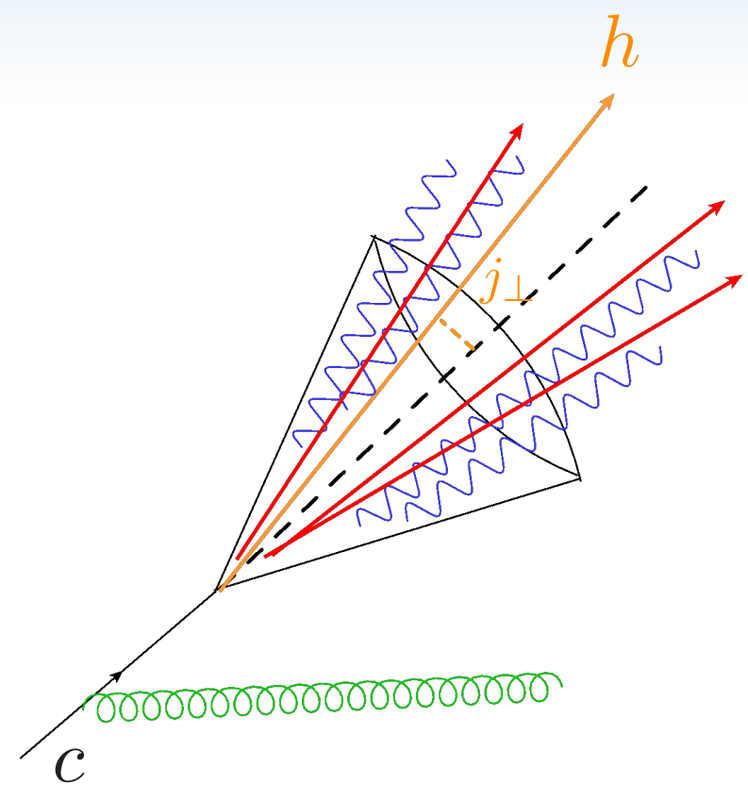


Polarizing FJF

$\mathcal{D}_T^{\perp h/c}$

- **Describes transversely polarized hadron inside the jet produced from an unpolarized parton.**

$$\frac{d\Delta\sigma^{pp\to \mathrm{jet}(\vec{h})X}}{dp_T d\eta dz_h d^2\boldsymbol{j}_\perp} = \sum_{a,b,c} f_a \otimes f_b \otimes H_{ab}^c \otimes \mathcal{D}_T^{\perp h/c}(z_h, \boldsymbol{j}_\perp)$$



When $\Lambda_{\mathrm{QCD}} \lesssim j_\perp \ll p_T R$,

$$\mathcal{D}_T^{\perp h/c}(z, z_h, \omega_J R, \boldsymbol{j}_\perp, \mu) = \mathcal{H}_{c\to i}(z, \omega_J R, \mu) \frac{\epsilon_\perp^{\mu\nu} S_{h\perp}^\mu}{z_h M_h} \int d^2\boldsymbol{k}_\perp d^2\boldsymbol{\lambda}_\perp \delta^2\left(z_h \boldsymbol{\lambda}_\perp + \boldsymbol{k}_\perp - \boldsymbol{j}_\perp\right)$$
$$\times\, k_\perp^\nu D_T^{h/i}(z_h, \boldsymbol{k}_\perp, \mu, \nu) S_i(\boldsymbol{\lambda}_\perp, \mu, \nu R)$$

$$\sim S_{h\perp} \sin(\phi_s - \phi_j)$$

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Polarizing FJF

- Used PFF fits from Belle data

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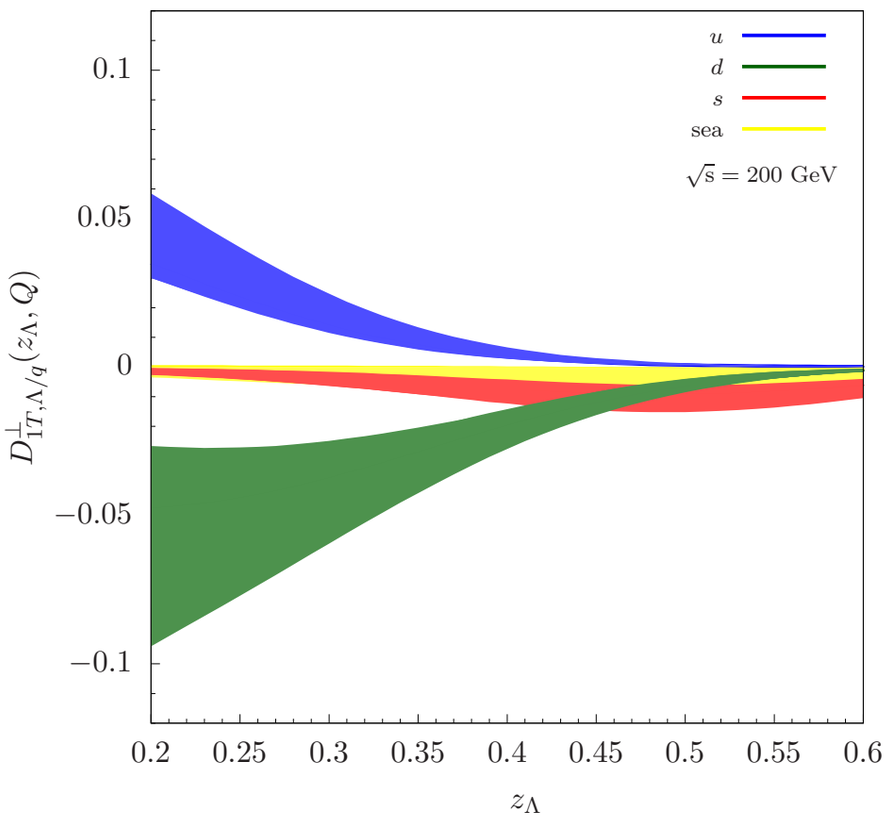

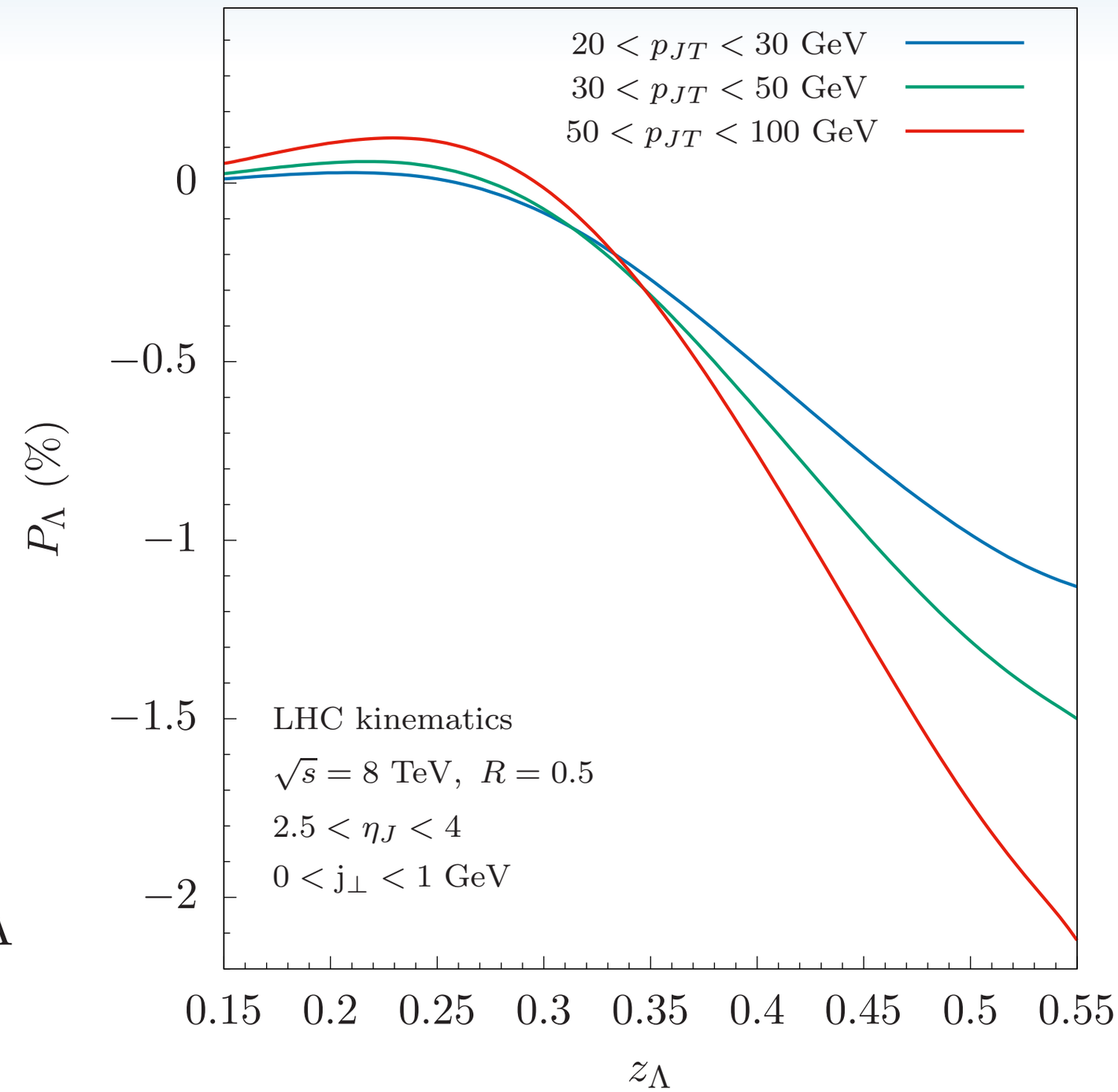

- Predictions at the LHC kinematics
- Positive from up quark PFF at small z_Λ
- Negative from down quark PFF at $z_\Lambda \gtrsim 0.3$

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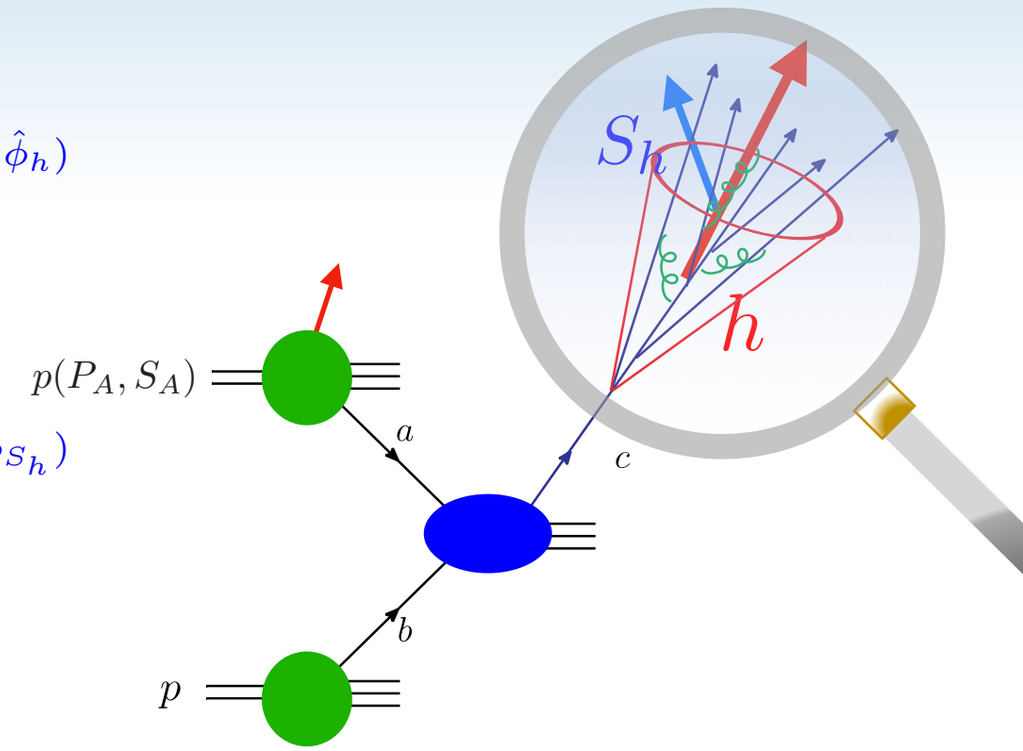
Azimuthal angular dependence

$$\frac{d\sigma^{p(S_A)+p/e\to(\text{jet } h(S_h))X}}{dp_{JT}d\eta_J dz_h d^2\boldsymbol{j}_\perp} = F_{UU,U} + |\boldsymbol{S}_T|\sin(\phi_{S_A}-\hat{\phi}_h)F_{TU,U}^{\sin(\phi_{S_A}-\hat{\phi}_h)}$$
$$+\Lambda_h\left[\lambda F_{LU,L} + |\boldsymbol{S}_T|\cos(\phi_{S_A}-\hat{\phi}_h)F_{TU,L}^{\cos(\phi_{S_A}-\hat{\phi}_h)}\right]$$
$$+|\boldsymbol{S}_{h\perp}|\Big\{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})F_{UU,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})} + \lambda\cos(\hat{\phi}_h-\hat{\phi}_{S_h})F_{LU,T}^{\cos(\hat{\phi}_h-\hat{\phi}_{S_h})}$$
$$+|\boldsymbol{S}_T|\big(\cos(\phi_{S_A}-\hat{\phi}_{S_h})F_{TU,T}^{\cos(\phi_{S_A}-\hat{\phi}_{S_h})}$$
$$+\cos(2\hat{\phi}_h-\hat{\phi}_{S_h}-\phi_{S_A})F_{TU,T}^{\cos(2\hat{\phi}_h-\hat{\phi}_{S_h}-\phi_{S_A})}\big)\Big\},$$



$$F_{S_A S_B, S_h}$$

Polarization of A, B, h

- **Different structures come with different characteristic angular dependence.**

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Thank you!