Status of first-principles lattice QCD+QED calculations of the hadronic contributions to the muon g-2

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December 20, 2021 - JLAB Seminar
What is a muon?

- Elementary point-like particle
- Same electric charge as an electron
- Approximately 200 times heavier than an electron
- Like the electron, behaves as if it was intrinsically spinning about a vector $\vec{S}$

These properties combine to give it a magnetic moment

$$\vec{\mu} = g \left( \frac{e}{2m} \right) \vec{S}$$

such that when put in a magnetic field, it exhibits precession similar to a spinning top.

We can measure this precession very precisely.
The magnetic moment and quantum corrections

The $g$-factor in $\vec{\mu} = g \left(\frac{e}{2m}\right) \vec{S}$ describes the strength of coupling to a magnetic field, which can be computed from theory also very precisely.

\begin{equation}
\text{Dirac: } g = 2
\end{equation}

\begin{equation}
\text{Anomalous magnetic moment } a = \frac{(g - 2)}{2}
\end{equation}

\begin{equation}
g > 2
\end{equation}

The quantum effects arise from virtual particle contributions from all known and unknown particles.

By comparing high-precision experiments and theory, we have the potential to learn about such contributions of new particles.
Contributions from known particles: The Standard Model

The Standard Model of Particle Physics

Unanswered Questions:

• "Why three generations?"
• Dark matter
• Dark energy
• Why (a lot) more matter than antimatter?
• Why?
• ...

The Higgs boson (discovered in 2012) completes the very successful SM

If new particles contribute to the muon’s anomalous magnetic moment, Standard Model theory will disagree with experiment.

➠ discovery potential of precision measurements

Open questions: dark matter, size of matter-antimatter asymmetry, origin of neutrino masses, . . . ⇒ Standard Model is incomplete
Contributions from known particles: The Standard Model

\[ a_\mu (\text{SM}) = a_\mu (\text{QED}) + a_\mu (\text{Weak}) + a_\mu (\text{Hadronic}) \]

QED

\[ + \ldots \]

\[ 116,584,718.9 (1) \times 10^{-11} \quad 0.001 \text{ ppm} \]

Weak

\[ + \ldots \]

\[ 153.6 (1.0) \times 10^{-11} \quad 0.01 \text{ ppm} \]

Hadronic…

…Vacuum Polarization (HVP)

\[ 6845 (40) \times 10^{-11} \quad 0.37 \text{ ppm} \]

\[ [0.6\%] \]

…Light-by-Light (HLbL)

\[ 92 (18) \times 10^{-11} \quad 0.15 \text{ ppm} \]

\[ [20\%] \]

Numbers from Theory Initiative Whitepaper

Uncertainty dominated by hadronic contributions
Systematically improvable methods are maturing; uncertainty to $a_\mu$ controlled at 0.15ppm; cross-checks detailed in Theory Initiative whitepaper.
Status and impact of hadronic vacuum polarization contribution

Ab-initio lattice QCD(+QED) calculations are maturing

Difficult problem: scales from $2m_\pi$ to several GeV enter; cross-checks needed at high precision

Hybrid window method restricts scales that enter from lattice/dispersive data

Dispersive, $e^+e^- \rightarrow$ hadrons (20+ years of experiments)

Now first published lattice result with sub-percent precision available (BMW20), cross-checks are crucial to establish or refute high-precision lattice methodology (same situation as for HLBhL)
Summary of HVP status:

- Decades of $e^+e^-$ dispersive results suggest a strong tension (4.2$\sigma$)

- A single lattice result (BMW20) suggests only minimal tension (1.5$\sigma$)

How can we move forward in our understanding? Main topic of this talk.

Two main questions:

- Consistency of BMW20 lattice result with previously known lattice results

- Consistency of lattice results with R-ratio
Consistency of BMW20 lattice result with previously known lattice results
Diagram shows the relevant diagrams for the QED correction to smaller than the lattice spacing uncertainty and its effective mass. We find statistical uncertainties. We use the finite-volume QED prescription and remove the universal 1 correction.

The shift in the respective quark lines. The procedure used for SIB correction is computed by inserting scalar operators times the renormalization factor with photons coupled to local lattice vector currents multiplications, respectively. We keep only the leading corrections with dynamical up, down, and strange quarks and non-degenerate up and down quark masses. We compute the fine-structure constant perturbation around an isospin-symmetric lattice QCD limit.

We need to make sure not to double-count those, i.e., we need to include the heavy quark with mass 10; similarly, Fig. 2.

Figure 7: Mass-splitting and HVP 1-photon diagrams. In the former the dots and a heavy quark with mass from bottom quarks and from strange quarks and non-degenerate up and down quark masses. We compute the with dynamical up, down, and strange quarks and non-degenerate up and down quark masses. We will perform a full first-principles calculation of all O(1/\Lambda^2) corrections

Figure 6: Displacement probability for 48c run 1.
Overview of individual contributions
Diagrams – Isospin limit

FIG. 1. Quark-connected (left) and quark-disconnected (right) diagram for the calculation of $a_\mu^{\text{HVP LO}}$. We do not draw gluons but consider each diagram to represent all orders in QCD.
Up, down; isospin symmetric limit; $m_\pi = m_\pi^0$

Some tensions to be understood
Strange

\[ \text{with } C(t) = \frac{1}{3} \sum_{j=0,1,2} \mathcal{J}_j(\mathbf{x}, t) \mathcal{J}_j(0) \]. With appropriate definition of \( w_t \), we can therefore write an expression for the desired precision. 

**FIG. 3.** Strong isospin-breaking correction diagrams. The crosses denote the insertion of a scalar operator.

Strange

**FIG. 2.** QED-correction diagrams with external pseudo-scalar vertices. 

- **Diagram A**: Quark-connected diagrams.
- **Diagram B**: Quark-anti-quark diagrams.
- **Diagram C**: Diagram M gives the valence, diagram R the sea quark mass shift.
- **Diagram D**: Corrections to the meson masses. Diagram O would yield a correction to the HVP.

**In Fig. 3**, we show the SIB diagrams. In the calculation, we neglect diagrams T, D1, D2, and D3. This approximation is estimated to yield an 10% correction for isospin splittings. 

**Displacement probability for 48c run 1.** 

The external vertices are pseudo-scalar operators. For the current calculation, we neglect diagrams T, D1, D2, and D3. This approximation is estimated to yield an 10% correction for isospin splittings. 

**Experimental measurements**: 

- HPQCD 2014
- Mainz 2017
- ETMC 2017
- BMW 2017
- RBC/UKQCD 2018
- SK 2019
- Mainz 2019
- BMW 2020 v1
- LM 2020

**Graph**: 

- x-axis: 46 to 58
- y-axis: \( a_{\mu, s, \text{conn, isospin}} \times 10^{10} \)
with \( C(t) = 1 \) and \( P \). With appropriate definition of \( \omega \), we can therefore write

\[ a_\mu, c, \text{conn}, \text{isospin} \times 10^{10} \]

FIG. 3. Strong isospin-breaking correction diagrams. The crosses denote the insertion of a scalar operator.
with \( C(t) = 1 \)

\[
P \sim x P \sum_{j=0,1,2} h_j(x, t) J_j(0) i.\]

With appropriate definition of \( w_t \), we can therefore write

\[
a \mu, uds, disc, isospin \times 10^{10}
\]

Figure 3: Strong isospin-breaking correction diagrams. The crosses denote the insertion of a scalar operator.

Figure 1 shows the quark-connected and quark-disconnected contributions to the mass due to the QED correction is significantly smaller compared to our estimates.

Figure 7: Mass-splitting and HVP 1-photon diagrams. In the former the dots are external photon vertices. Note that only the parts of diagram F with additional gluons exchanged between the two quark loops contribute to the meson spectrum and the hadronic vacuum polarization. The external vertices are pseudo-scalar operators.

Figure 8: Mass-counterterm diagrams for mass-splitting and HVP 1-photon corrections. Also note that some diagrams are absent for flavor non-diagonal operators.

For the hadronic vacuum polarization the contribution of meson operators, in the latter the dots are external photon vertices. Note that some diagrams are absent for flavor non-diagonal operators.

In the current calculation, we neglect diagrams T, D1, D2, D3, and 10\( \times \) instead of HVP LO QED corrections.

We do not include separately.

We tune the bare up, down, and strange quark masses to produce a pion mass of 135.0 MeV and a kaon mass of 495.7 MeV. The correlator is expanded in perturbative QCD by integrating missing contributions to the meson masses. Diagram O would yield a correction to the HVP calculation.
For diagram F we enforce exchange of gluons between the quark loops as otherwise a cut through a single photon line would be possible. This single-photon contribution is counted as part of the HVP NLO and not included for the HVP LO.
For the finite-volume errors, the two-pion states in $d!$ are identical to the $I = 1$ contributions of $c$ and can be calculated using the GSL estimate which we use for $c$. For the omega-related finite-volume errors, I will take the fitted $d!$ and use this as the full result at finite-volume and compare it to a GS model with omega mass from the fitted $E!$ and width from the PDG in infinite-volume. I should also compare this to R-ratio results for the $I = 0$ channel.

Do this entire exercise for 24ID and 32ID to estimate discretization errors.

4 QED and SIB diagrams

We will perform a full first-principles calculation of all $O(\alpha)$ and $O(\alpha^2)$ corrections. The corresponding list of diagrams is given in Figs. 1 and 2.

(a) V
(b) S
(c) T
(d) $T_d$
(e) $D_1$
(f) $D_{1d}$
(g) $D_2$
(h) $D_{2d}$
(i) $F$
(j) $D_{3}$

Figure 1: QED corrections

(a) $M$
(b) $R$
(c) $R_d$
(d) $O$

Figure 2: SIB corrections

$-15 -10 -5 0 5 10 15$

$\mu$, QED, conn $\times 10^{10}$

RBC/UKQCD 2018
ETMC 2019
BMW 2020 v1
For the finite-volume errors, the two-pion states in $d$ are identical to the $I = 1$ contributions of $c$ and can be calculated using the GSL estimate which we use for $c$. For the omega-related finite-volume errors, I will take the fitted $d!$ and $E!$ and use this as the full result at finite-volume and compare it to a GS model with omega mass from the fitted $E!$ and width from the PDG in infinite-volume. I should also compare this to $R$-ratio results for the $I = 0$ channel.

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(j) $D_3$

Figure 1: QED corrections

(a) $M$
(b) $R$
(c) $R_d$
(d) $O$

Figure 2: SIB corrections

Attention needed
Diagrams – Strong isospin breaking

For the HVP $R$ is negligible since $\Delta m_u \approx -\Delta m_d$ and $O$ is SU(3) and $1/N_c$ suppressed.

Lehner, Meyer 2020: NLO PQChPT: FV effects in connected and disconnected cancel but are each significant $O(4 \times 10^{-10})$; PQChPT expects cancellation between connected and disconnected contribution $a_{\mu, \text{conn.}}^{\text{SIB}} = -a_{\mu, \text{disc.}}^{\text{SIB}} = 6.9 \times 10^{-10}$
For the finite-volume errors, the two-pion states in $d!$ are identical to the $I = 1$ contributions of $c$ and can be calculated using the GSL estimate which we use for $c$. For the omega-related finite-volume errors, I will take the fitted $d!$ and $E!$ and use this as the full result at finite-volume and compare it to a GS model with omega mass from the fitted $E!$ and width from the PDG in infinite-volume. I should also compare this to R-ratio results for the $I = 0$ channel.

Do this entire exercise for 24ID and 32ID to estimate discretization errors.

4 QED and SIB diagrams

We will perform a full first-principles calculation of all $O(\alpha)$ and $O(\mu^0_\mu)$ corrections. The corresponding list of diagrams is given in Figs. 1 and 2.

![Diagram with labels](image)

(a) V  (b) S  (c) T  (d) T 
(e) D1  (f) D1 
(g) D2  (h) D2 
(i) F  (j) D3

Figure 1: QED corrections

![Diagram with labels](image)

(a) M  (b) R  (c) R 
(d) O

Figure 2: SIB corrections

LM 2020
LM 2020 FV
BMW 2020 v1
ETMC 2019
RBC/UKQCD 2018
FHM 2017

\[ a_\mu, \text{SIB, conn} \times 10^{10} \]
For the finite-volume errors, the two-pion states in \( d \) are identical to the \( I = 1 \) contributions of \( c \) and can be calculated using the GSL estimate which we use for \( c \). For the omega-related finite-volume errors, I will take the fitted \( d \) and \( E \) and use this as the full result at finite-volume and compare it to a GS model with omega mass from the fitted \( E \) and width from the PDG in infinite-volume. I should also compare this to R-ratio results for the \( I = 0 \) channel.

Do this entire exercise for 24ID and 32ID to estimate discretization errors.

**4 QED and SIB diagrams**

We will perform a full first-principles calculation of all \( O(\alpha) \) and \( O(\mu^m_{ud}) \) corrections. The corresponding list of diagrams is given in Figs. 1 and 2.

(a) V  
(b) S  
(c) T  
(d) T 
(e) D1  
(f) D1  
(g) D2  
(h) D2  
(i) F  
(j) D3

**Figure 1: QED corrections**

(a) M  
(b) R  
(c) R_d  
(d) O

**Figure 2: SIB corrections**

\[ a_\mu, \text{SIB, disc} \times 10^{10} \]
Attention on light-quark isospin-symmetric contribution and QED disconnected contribution
Lattice QCD – Time-Moment Representation

Starting from the vector current $J_\mu(x) = i \sum_f Q_f \bar{\Psi}_f(x) \gamma_\mu \Psi_f(x)$ we may write

$$a_\mu^{\text{HVP LO}} = \sum_{t=0}^{\infty} w_t C(t)$$

with

$$C(t) = \frac{1}{3} \sum_{\vec{x}} \sum_{j=0,1,2} \langle J_j(\vec{x}, t) J_j(0) \rangle$$

and $w_t$ capturing the photon and muon part of the HVP diagrams (Bernecker-Meyer 2011).

The correlator $C(t)$ is computed in lattice QCD+QED at physical pion mass with non-degenerate up and down quark masses including up, down, strange, and charm quark contributions. The missing bottom quark contributions are computed in pQCD.
Lattice QCD – Example of correlation function $C(t)$ (RBC/UKQCD18)

Large discretization errors at short distance, large finite-volume errors and statistical errors at large distance
Window method (introduced in RBC/UKQCD 2018)

We therefore also consider a window method. Following Meyer-Bernecker 2011 and smearing over $t$ to define the continuum limit we write

$$a_\mu = a^{\text{SD}}_\mu + a^{\text{W}}_\mu + a^{\text{LD}}_\mu$$

with

$$a^{\text{SD}}_\mu = \sum_t C(t) w_t [1 - \Theta(t, t_0, \Delta)] ,$$

$$a^{\text{W}}_\mu = \sum_t C(t) w_t [\Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta)] ,$$

$$a^{\text{LD}}_\mu = \sum_t C(t) w_t \Theta(t, t_1, \Delta) ,$$

$$\Theta(t, t', \Delta) = \frac{1}{2} [1 + \tanh \left( \frac{(t - t')}{\Delta} \right) ] .$$

All contributions are well-defined individually and can be computed from lattice or R-ratio via $C(t) = \frac{1}{12 \pi^2} \int_0^\infty d(\sqrt{s}) R(s) s e^{-\sqrt{s}t}$ with $R(s) = \frac{3s}{4\pi \alpha^2} \sigma(s, e^+ e^- \to \text{had})$.

$a^{\text{W}}_\mu$ has small statistical and systematic errors on lattice!
Use these windows as a lattice internal cross-check

\[ (t_0, t_1, \Delta) = (0.4, 1.0, 0.15) \text{ fm} \]

Aubin et al. 19
Aubin et al. 19 - finest as
LM 20
BMW 20
FHM 20 (prelim., stat only)
RBC/UKQCD 18
ETMC 20 (prelim.)
Mainz/CLS 20 (prelim.)

\[ a_\mu^W (\text{ud, conn, iso}) \times 10^{10} \]

Plot by D. Giusti

Plot from recent theory initiative workshop (https://indico.cern.ch/event/956699/)
Status of consistency of lattice results

Significant difference between published high-precision LQCD results (BMW20 and RBC/UKQCD18) for window with $t_0 = 0.4\text{fm}$ and $t_1 = 1.0\text{fm}$:

\[
\begin{align*}
    a_W^{\text{BMW20}} &= 207.3(1.4) \times 10^{-10}, \\
    a_W^{\text{RBC/UKQCD18}} &= 202.9(1.4)(0.4) \times 10^{-10}
\end{align*}
\]  

and therefore there is a $2.2\sigma$ tension

\[
    a_W^{\text{BMW20}} - a_W^{\text{RBC/UKQCD18}} = 4.4(2.0) \times 10^{-10}.
\]

Scaled to the total $a_\mu^{\text{HVP}}$ this corresponds to $15 \times 10^{-10}$ uncertainty on the lattice HVP compared to current $5.5 \times 10^{-10}$ uncertainty of BMW20.

Urgently need new results for this and other windows. Update by RBC/UKQCD 2018 is in preparation. Hopefully available within two months. More groups to join. Important: different regulators!
Continuum extrapolation - What lattice spacing is fine enough?


BMW 20 - light quark window

3.7σ tension between BMW20 and R-ratio for Window! Discuss in second part of talk.

Red line for comparison with next slide
Continuum extrapolation - What lattice spacing is fine enough?

HPQCD 20 charm quark full $a_\mu$


Restricting to fixed lattice spacing range can lead to different discretization errors for different UV regulators; systematically independent calculations very desirable!
What if we include the $R$-ratio in the fits? Would the lattice result be consistent?

Simple quadratic fit gives mixed results. If we include the coarsest data point, the agreement with the $R$-ratio is not great (better than the linear fit).

RESULTS

What about a “perturbation-theory-inspired” fit? Polynomials are simple but not physically motivated [compare “old-school” chiral fits].

Expansions of quantities in $a^2$ would have $\log(a^2)$ terms which we could in principle calculate exactly [e.g., Husung, Marquard, Sommer, EPJC 80, 200 (2020)].

As a simple first step:
Consistency of lattice result with $R$-ratio
\[ R(s) = \frac{3s}{4\pi\alpha^2} \sigma(s, e^+ e^- \to \text{had}), \quad C(t) = \frac{1}{12\pi^2} \int_{0}^{\infty} d(\sqrt{s}) R(s) s e^{-\sqrt{s} t} \]
Tensions in input data, however, already taken into account in WP20 merger of KNT19 and DHMZ19:

![Graph showing experimental data and SM predictions](image1.png)

![Graph showing experimental data and SM predictions](image2.png)

![Graph showing experimental data and SM predictions](image3.png)

![Graph showing experimental data and SM predictions](image4.png)
What does tension in windows mean for R-ratio?

If there is a shift in R-ratio, it crucially depends on which energy to understand what the impact on $\Delta \alpha$ and EW precision physics is.

Express Euclidean Windows in time-like region:

$$a_\mu = \int_0^\infty ds \, R(s)K(s)$$

and window

$$a_\mu^W = \int_0^\infty ds \, R(s)K(s)P(s).$$
Study of windows for different $t_0$ and $t_1$ can give some energy resolution!
Study of windows for different $t_0$ and $t_1$ can give some energy resolution!
Study of windows for different $t_0$ and $t_1$ can give some energy resolution!
Below black line, we can use Lellouche-Lüscher-Meyer formalism to get \( R(s) \) from lattice directly! Programs for this by Mainz and RBC/UKQCD.
First results for more windows already available - Lehner & Meyer 2020

Here: $t_0 = t$, $t_1 = t + 0.1\text{fm}$

No results for QED, SIB, and charm contribution yet available.
First results for more windows already available - Lehner & Meyer 2020

<table>
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<th>$t_0$/fm</th>
<th>$t_1$/fm</th>
<th>$\Delta$/fm</th>
<th>$a_{\mu}^{ud,conn.,isospin \times 10^{10}}$</th>
<th>$a_{\mu}^{s,conn.,isospin \times 10^{10}}$</th>
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<td>52.83(22)(65)</td>
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<td>0.81(00)(12)</td>
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<td>4.666(10)(59)</td>
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<td>4.866(13)(74)</td>
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<td>4.505(17)(44)</td>
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<td>3.527(19)(76)</td>
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<td>2.973(19)(75)</td>
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<td>2.441(18)(77)</td>
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<tr>
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<td>1.534(15)(60)</td>
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<td>1.181(13)(52)</td>
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<td>0.894(12)(44)</td>
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<td>0.667(10)(37)</td>
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<td>1.9  2.0 0.15</td>
<td>16.59(89)(75)</td>
<td>0.357(07)(24)</td>
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</tr>
</tbody>
</table>

More results expected by other collaborations soon!
What can we expect from LQCD in the coming years?

- More published results with high precision with different regulators for the standard window $t_0 = 0.4\,\text{fm}$, $t_1 = 1.0\,\text{fm}$, $\Delta = 0.15\,\text{fm}$. This will clarify the $2.2\sigma$ tension between BMW20 and RBC/UKQCD18 for this quantity.

- More results for different windows, which will give energy resolution to locate possible remaining tension with R-ratio in time-like energy. After this: any impact on $\Delta\alpha$ and EW precision physics?

- More results of complete high-precision HVP results from major lattice collaborations. RBC/UKQCD18 is conducting a blind analysis with several independent analysis groups.
Outlook

- Expect more lattice HVP calculations at few per-mille level precision which allows for proper scrutiny at high precision; For total $a_\mu$ as well as windows!

- Data-driven dispersive results will improve with expected experimental results from Belle II, BESIII, CMD-3, and SND

- MUonE at CERN will provide complementary measurements for the HVP

- Theory Initiative will publish updated SM predictions as experiment and theory improves; provides platform for cross-checks and establishing new methodology

Thank You!
Papers that directly enter the WP20 SM prediction

▶ M. Hoferichter, B. L. Hoid and B. Kubis, JHEP 08, 137 (2019)
▶ M. Hoferichter, B. L. Hoid, B. Kubis, S. Leupold and S. P. Schneider, JHEP 10, 141 (2018)

Results in plots that have appeared after the WP deadline

Backup
The anomalous magnetic moment of the muon in the Standard Model

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\[ a_{\mu}^{\text{SM}} = a_{\mu}^{\text{QED}} + a_{\mu}^{\text{Weak}} + a_{\mu}^{\text{HVP}} + a_{\mu}^{\text{HLbL}} = 116591810 (43) \times 10^{-11} \]
The large discrepancy between BABAR and KLOE in the SM predictions is applied, we quote this discrepancy as an additional uncertainty in our new evaluation.

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Summary

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A. El-Khadra

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M. Davier et al, arXiv:1908.00921

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input, and the isospin-symmetry-breaking effects were included only as an estimate.
The HVP should have to explain the experimental measurement of $(g\mu - 2)$, assuming no new physics.

The systematic error is dominated by the continuum extrapolation from the noisy, large-distance region of the current–current correlator. The statistical error comes mainly from the limited number of lattice spacings in the fit. Our result is shown with the results of DHMZ'19 and KNT'19.

The corresponding data are shown with grey points, from the continuum-extrapolated result is shown with the results of ref. [3]. The blue shaded region is the value that would have to have to explain the experimental measurement of $(g\mu - 2)$, assuming no new physics.

In principle, one can reduce the uncertainty of our results for them.

We correct for finite-volume effects on $a\mu$ light$^3$ and in $a^2$ [fm$^2$]. The corresponding data are shown with grey points, from the continuum-extrapolated result is shown with the results of DHMZ'19 and KNT'19.

The blue shaded region is the value that would have to have to explain the experimental measurement of $(g\mu - 2)$, assuming no new physics.

The precision needed to explain the experimental measurement of $(g\mu - 2)$, assuming no new physics.

Any methods, additional references, Nature Research reporting summary; and (iii). the input, and the isospin-symmetry-breaking effects were included only as an estimate.

The isospin-symmetry breaking is implemented by taking derivative of isospin-symmetric and isospin-symmetry-breaking contributions of $\pi$ masses. These effects are included both in the scale determination and the continuum extrapolation error from 8.0 to 4.1 by assuming no new physics.

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μ-e elastic scattering to measure $a_\mu^{\text{HVP}}$


- use CERN M2 muon beam (150 GeV)
- Physics beyond colliders program @ CERN
- LOI June 2019
- pilot run in 2021
- full apparatus in 2023-2024
### Comparison of two frequently used compilations for HLbL in units of 10⁻¹¹

<table>
<thead>
<tr>
<th>Contribution</th>
<th>PdRV(09) [471]</th>
<th>N/JN(09) [472, 573]</th>
<th>J(17) [27]</th>
<th>Our estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\pi^0, \eta, \eta')-poles</td>
<td>114(13)</td>
<td>99(16)</td>
<td>95.45(12.40)</td>
<td>93.8(4.0)</td>
</tr>
<tr>
<td>(\pi, K)-loops/boxes</td>
<td>-19(19)</td>
<td>-19(13)</td>
<td>-20(5)</td>
<td>-16.4(2)</td>
</tr>
<tr>
<td>S-wave (\pi\pi) rescattering</td>
<td>-7(7)</td>
<td>-7(2)</td>
<td>-5.98(1.20)</td>
<td>-8(1)</td>
</tr>
<tr>
<td>subtotal</td>
<td>88(24)</td>
<td>73(21)</td>
<td>69.5(13.4)</td>
<td>69.4(4.1)</td>
</tr>
<tr>
<td>scalars</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-1(3)</td>
</tr>
<tr>
<td>tensors</td>
<td>-</td>
<td>-</td>
<td>1.1(1)</td>
<td></td>
</tr>
<tr>
<td>axial vectors</td>
<td>15(10)</td>
<td>22(5)</td>
<td>7.55(2.71)</td>
<td>6(6)</td>
</tr>
<tr>
<td>(u, d, s)-loops / short-distance</td>
<td>-</td>
<td>21(3)</td>
<td>20(4)</td>
<td>15(10)</td>
</tr>
<tr>
<td>(c)-loop</td>
<td>2.3</td>
<td>-</td>
<td>2.3(2)</td>
<td>3(1)</td>
</tr>
<tr>
<td>total</td>
<td>105(26)</td>
<td>116(39)</td>
<td>100.4(28.2)</td>
<td>92(19)</td>
</tr>
</tbody>
</table>
Dispersive method - Overview

\[ e^+ e^- \rightarrow \text{hadrons}(\gamma) \]
\[ J_\mu = V_{\mu}^{l=1, l_3=0} + V_{\mu}^{l=0, l_3=0} \]

\[ \tau \rightarrow \nu \text{hadrons}(\gamma) \]
\[ J_\mu = V_{\mu}^{l=1, l_3=\pm 1} - A_{\mu}^{l=1, l_3=\pm 1} \]

Knowledge of isospin-breaking corrections and separation of vector and axial-vector components needed to use \( \tau \) decay data.

Can have both energy-scan and ISR setup.
<table>
<thead>
<tr>
<th>Experiment</th>
<th>( 2m_{\pi^\pm} - 0.36 \text{ GeV} )</th>
<th>( a_{\mu}^{\text{had,LO}}[\pi\pi, \tau] ) (10(^{-10}))</th>
<th>( 0.36 - 1.8 \text{ GeV} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALEPH</td>
<td>9.80 ± 0.40 ± 0.05 ± 0.07</td>
<td>501.2 ± 4.5 ± 2.7 ± 1.9</td>
<td></td>
</tr>
<tr>
<td>CLEO</td>
<td>9.65 ± 0.42 ± 0.17 ± 0.07</td>
<td>504.5 ± 5.4 ± 8.8 ± 1.9</td>
<td></td>
</tr>
<tr>
<td>OPAL</td>
<td>11.31 ± 0.76 ± 0.15 ± 0.07</td>
<td>515.6 ± 9.9 ± 6.9 ± 1.9</td>
<td></td>
</tr>
<tr>
<td>Belle</td>
<td>9.74 ± 0.28 ± 0.15 ± 0.07</td>
<td>503.9 ± 1.9 ± 7.8 ± 1.9</td>
<td></td>
</tr>
<tr>
<td>Combined</td>
<td>9.82 ± 0.13 ± 0.04 ± 0.07</td>
<td>506.4 ± 1.9 ± 2.2 ± 1.9</td>
<td></td>
</tr>
</tbody>
</table>

**Davier et al. 2013:** \( a_{\mu}^{\text{had,LO}}[\pi\pi, \tau] = 516.2(3.5) \times 10^{-10} \) (2\(m_{\pi}^\pm\) – 1.8 GeV)

Compare to \( e^+e^- \):
- \( a_{\mu}^{\text{had,LO}}[\pi\pi, e^+e^-] = 507.1(2.6) \times 10^{-10} \) (DHMZ17, 2\(m_{\pi}^\pm\) – 1.8 GeV)
- \( a_{\mu}^{\text{had,LO}}[\pi\pi, e^+e^-] = 503.7(2.0) \times 10^{-10} \) (KNT18, 2\(m_{\pi}^\pm\) – 1.937 GeV)

Here treatment of isospin-breaking to relate matrix elements of \( V_{\mu}^{l=1, l_3=1} \) to \( V_{\mu}^{l=1, l_3=0} \) crucial. Progress towards a first-principles calculation from LQCD+QED (arXiv:1811.00508).
Analysis of the Hadronic Light-by-Light Contributions to the Muon $g - 2$

Johan Bijnens, Elisabetta Pallante, Joaquim Prades

We calculate the hadronic light-by-light contributions to the muon $g - 2$. We use both $1/N_c$ and chiral counting to organize the calculation. Then we calculate the leading and next-to-leading order in the $1/N_c$ expansion low energy contributions using the Extended Nambu–Jona–Lasinio model as hadronic model. We do that to all orders in the external momenta and quark masses expansion. Although the hadronic light-by-light contributions to muon $g - 2$ are not saturated by these low energy contributions we estimate them conservatively. A detailed analysis of the different hadronic light-by-light contributions to muon $g - 2$ is done. The dominant contribution is the twice anomalous pseudoscalar exchange diagram. The final result we get is $\alpha_\mu^{\text{light-by-light}} = (-9.2 \pm 3.2) \cdot 10^{-10}$. This is between two and three times the expected experimental uncertainty at the forthcoming BNL muon $g - 2$ experiment.
Add \( a^{-1} = 2.77 \) GeV lattice spacing

- Third lattice spacing for strange data (\( a^{-1} = 2.77 \) GeV with \( m_\pi = 234 \) MeV with sea light-quark mass corrected from global fit):

For light quark need new ensemble at physical pion mass. Data still being generated on Summit in USA and Booster in Germany (\( a^{-1} = 2.77 \) GeV with \( m_\pi = 139 \) MeV)