Higher-order QCD corrections to SIDIS

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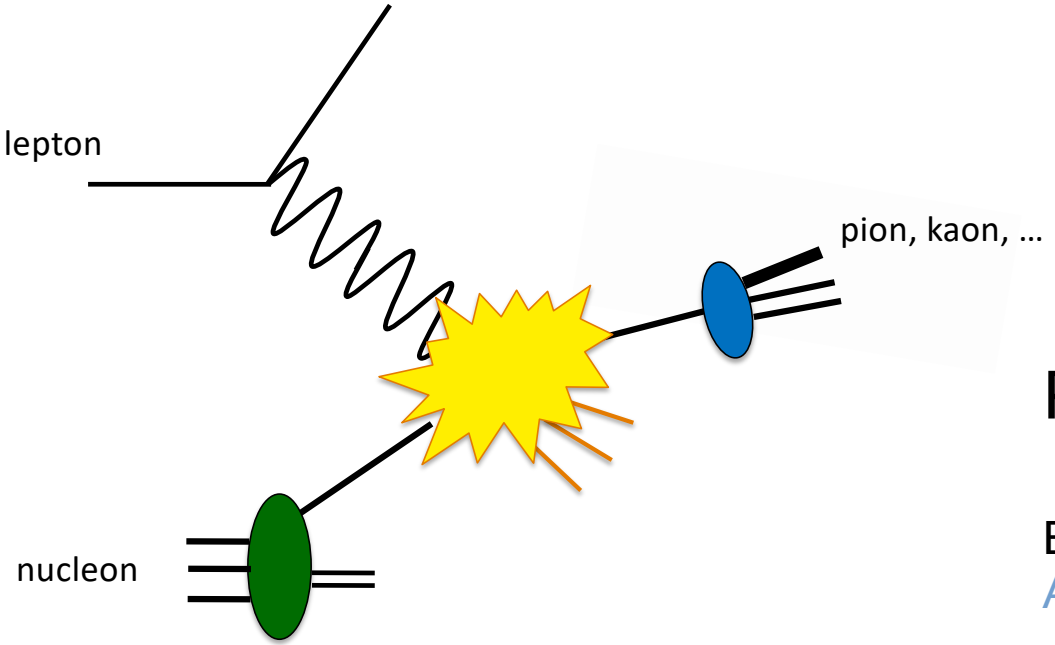
Remote Theory Seminar @ Jefferson Lab

DFG Deutsche Forschungsgemeinschaft
German Research Foundation

Semi-inclusive Deep Inelastic Scattering (SIDIS)



Framework: basic collinear SIDIS

Earlier work: NLL Resummation
Anderle, Ringer, Vogelsang: PRD87 (2013)

Approximate NNLO
de Florian, Vogelsang, MA: 2109.00847

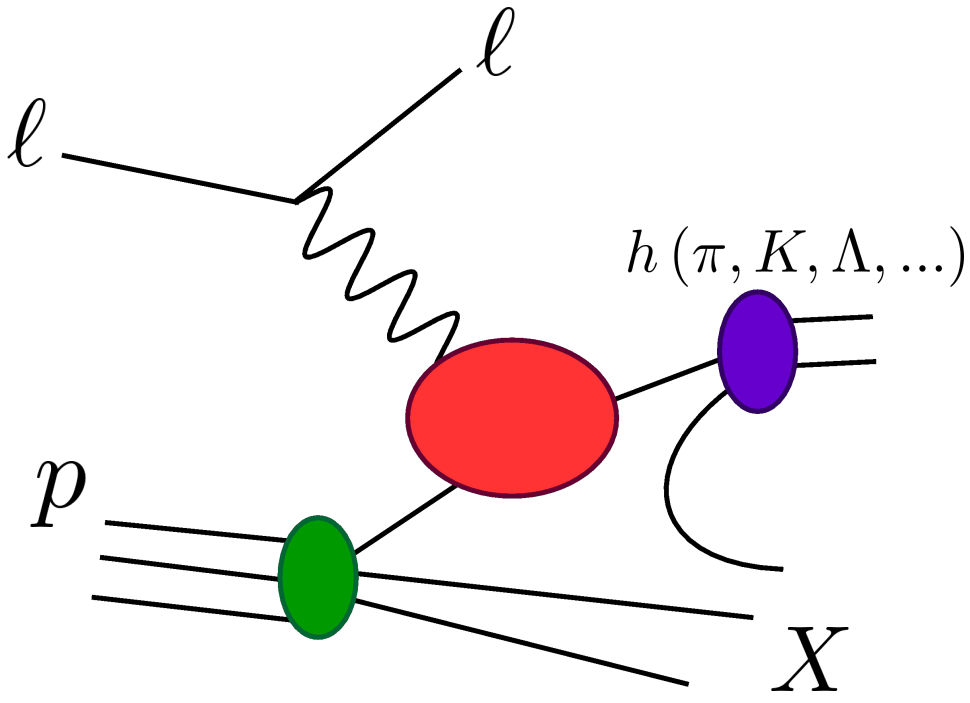
In preparation: N3LL Resummation

Outline

- Semi-inclusive DIS
- Threshold Resummation
- Approximate higher-order corrections
- Phenomenology
- Conclusions

Semi-inclusive DIS

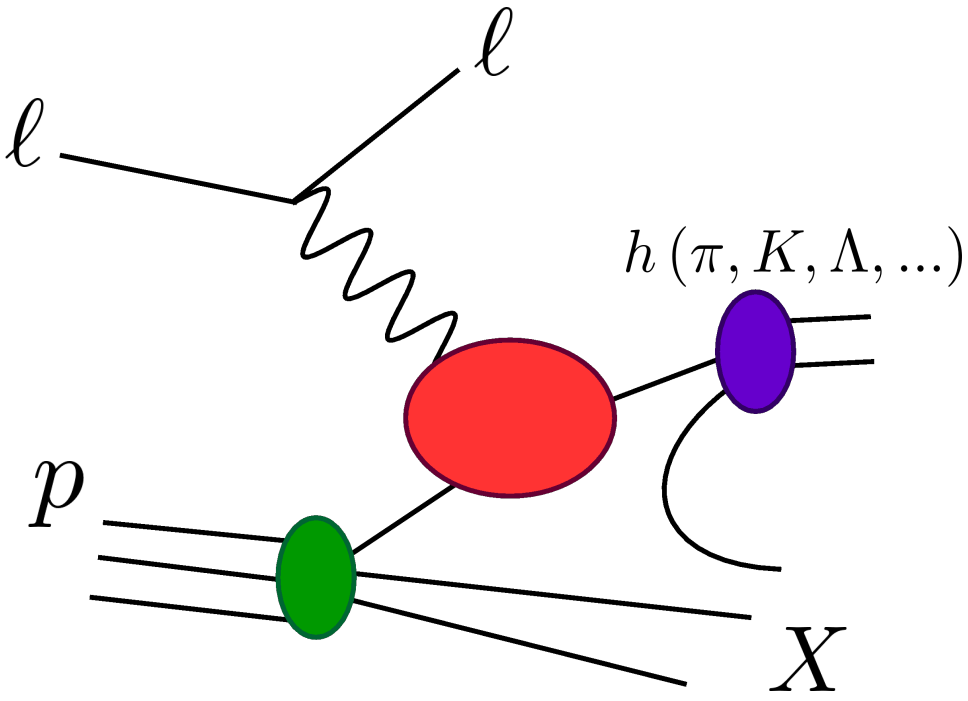
$$\ell\, p \;\rightarrow\; \ell\, h\, X$$



SIDIS established probe of nucleons

- DESY: HERMES
- CERN: COMPASS
- Jefferson Lab
- EIC: future experiments ⇨ need for higher precision

$$\ell\, p \;\rightarrow\; \ell\, h\, X$$



SIDIS used for extraction of

de Florian, Sassot, Stratmann, Vogelsang;...

- (helicity) parton distributions PDF (ΔPDF)
- fragmentation functions FF

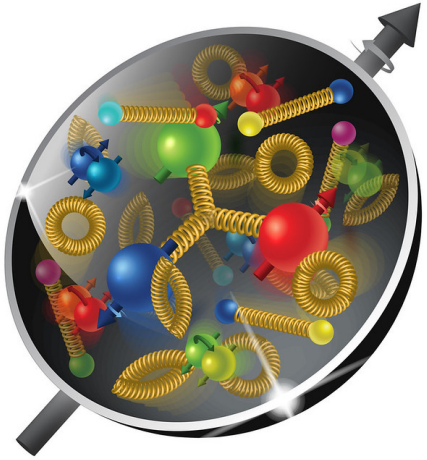
Anderle, Kaufmann, Stratmann, Ringer;
Borsa, de Florian, Sassot, Stratmann 2110.14015

- combinations of ΔPDF and FF

Ethier, Sato, Melnitchouk;
Moffat, Melnitchouk, Rogers, Sato

these distributions are non-perturbative

insights into the hadronic structure



@ NNLO ?

- flavour separation
- spin contributions of quarks & gluons (ΔPDF)
- transverse momentum distribution (TMD)

} nucleon structure

- global analysis of fragmentation (FF): hadronization
- testbed for QCD calculations

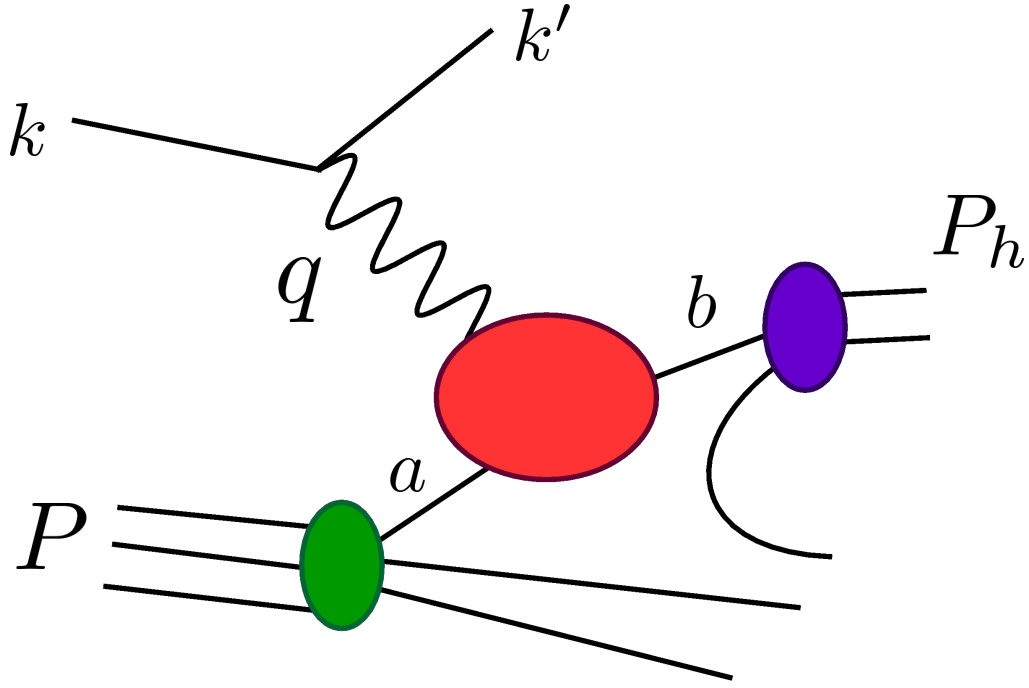
to gain such information we need the perturbatively calculable parts

perturbative SIDIS cross section

$$Q^2 = -q^2 = -(k - k')^2$$

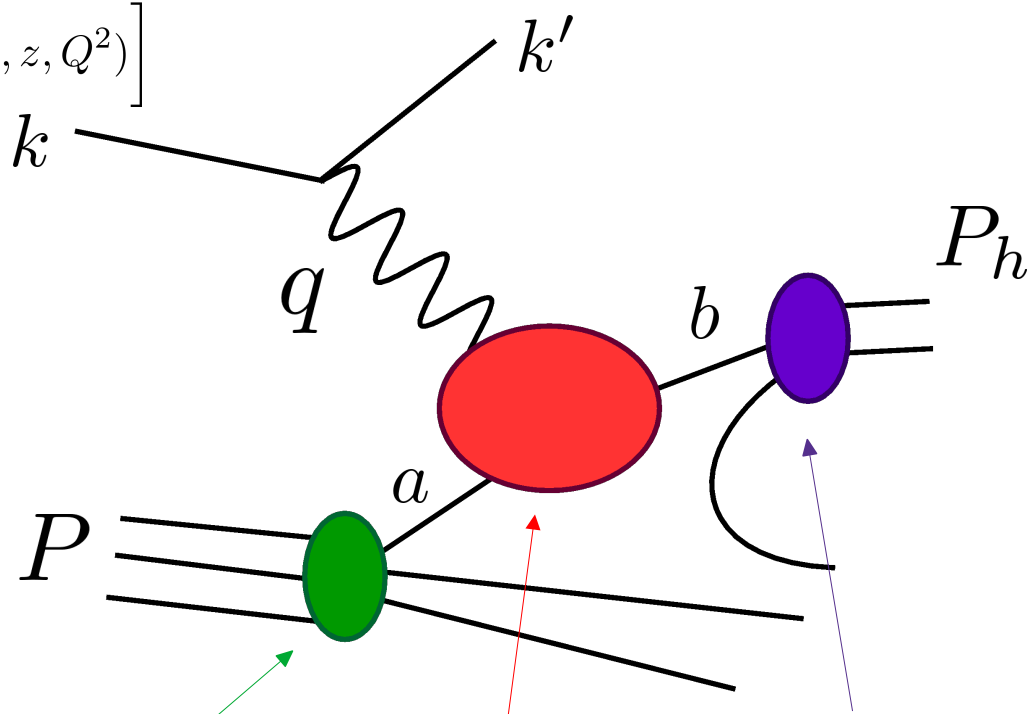
$$x = \frac{Q^2}{2P \cdot q}$$

$$z = \frac{P \cdot P_h}{P \cdot q}$$



$$\frac{d^3\sigma^h}{dxdydz} = \frac{4\pi\alpha^2}{Q^2}\left[\frac{1+(1-y)^2}{2y}\mathcal{F}_T^h(x,z,Q^2) + \frac{1-y}{y}\mathcal{F}_L^h(x,z,Q^2)\right]$$

$$\frac{d^3\sigma^h}{dxdydz} = \frac{4\pi\alpha^2}{Q^2}\left[\frac{1+(1-y)^2}{2y}\mathcal{F}_T^h(x,z,Q^2) + \frac{1-y}{y}\mathcal{F}_L^h(x,z,Q^2)\right]$$



partonic variables:

$$\hat{x} = \frac{Q^2}{2p_a \cdot q} \qquad \hat{z} = \frac{p_a \cdot p_b}{p_a \cdot q}$$

$$\mathcal{F}_T^h(x,z,Q^2) = \sum_{a,b}\int_x^1 \frac{d\hat{x}}{\hat{x}}\int_z^1 \frac{d\hat{z}}{\hat{z}}\, f_a\left(\frac{x}{\hat{x}},Q^2\right)\omega_{ab}\left(\hat{x},\hat{z},\alpha_s\right)D_b^h\left(\frac{z}{\hat{z}},Q^2\right)$$

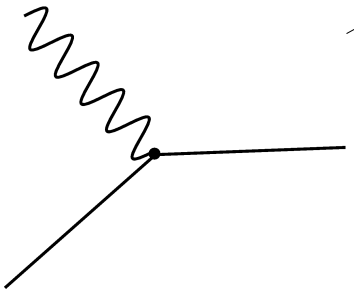
parton distribution

perturbative hard scattering coefficient function

fragmentation function

+ P.C.

$$\omega_{ab} = \omega_{ab}^{(0)} + \frac{\alpha_s}{\pi}\,\omega_{ab}^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \omega_{ab}^{(2)} + \mathcal{O}(\alpha_s^3)$$

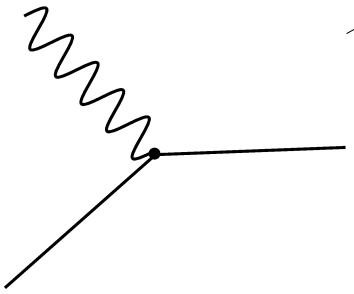


LO: $\omega_{qq}^{(0)}(\hat{x},\hat{z}) = e_q^2 \delta(1-\hat{x})\delta(1-\hat{z})$

$$\Longrightarrow \quad \mathcal{F}_T^h(x,z,Q^2) = \sum_q e_q^2 f_q\left(x,Q^2\right) D_q^h\left(z,Q^2\right)$$

flavour separation

$$\omega_{ab} = \omega_{ab}^{(0)} + \frac{\alpha_s}{\pi}\,\omega_{ab}^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \omega_{ab}^{(2)} + \mathcal{O}(\alpha_s^3)$$



LO: $\omega_{qq}^{(0)}(\hat{x},\hat{z}) = e_q^2 \delta(1-\hat{x})\delta(1-\hat{z})$

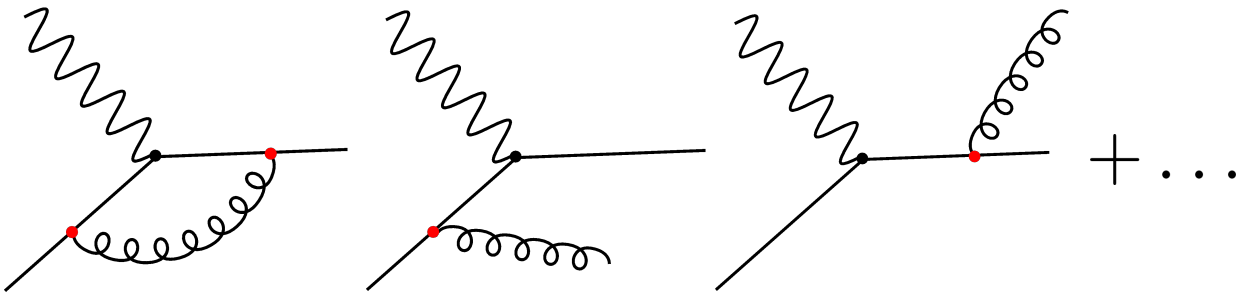
$$\Longrightarrow \quad \mathcal{F}_T^h(x,z,Q^2) = \sum_q e_q^2 f_q\left(x,Q^2\right) D_q^h\left(z,Q^2\right)$$

$\omega_{qq}^{(0)}(\hat{x},\hat{z}) = \Delta\omega_{qq}^{(0)}(\hat{x},\hat{z})$

$$\Longrightarrow \quad g_1^h(x,z,Q^2) = \frac{1}{2}\sum_q e_q^2 \Delta f_q\left(x,Q^2\right) D_q^h\left(z,Q^2\right)$$

$$\omega_{ab} = \omega_{ab}^{(0)} + \frac{\alpha_s}{\pi}\,\omega_{ab}^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \omega_{ab}^{(2)} + \mathcal{O}(\alpha_s^3)$$

NLO:



NNLO?

well known for long time

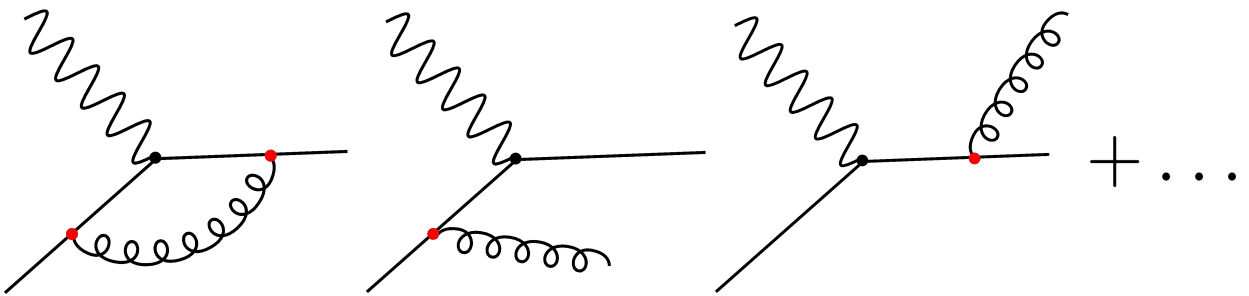
Altarelli et al;...
de Florian, Stratmann, Vogelsang

- exact corrections up to now unknown
- can we approximate NNLO ?

$$\omega_{ab} = \omega_{ab}^{(0)} + \frac{\alpha_s}{\pi}\omega_{ab}^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \omega_{ab}^{(2)} + \mathcal{O}(\alpha_s^3)$$

NLO:

NNLO?



$\hat{x}, \hat{z} \to 1$

$$\omega_{qq}^{(1)} = e_q^2 C_F \left[\delta(1-\hat{x}) \left(\frac{\ln(1-\hat{z})}{1-\hat{z}} \right)_+ + \delta(1-\hat{z}) \left(\frac{\ln(1-\hat{x})}{1-\hat{x}} \right)_+ \right.$$

$$\left. + \frac{1}{(1-\hat{x})_+(1-\hat{z})_+} - 4\delta(1-\hat{x})\delta(1-\hat{z}) + \ldots \right]$$

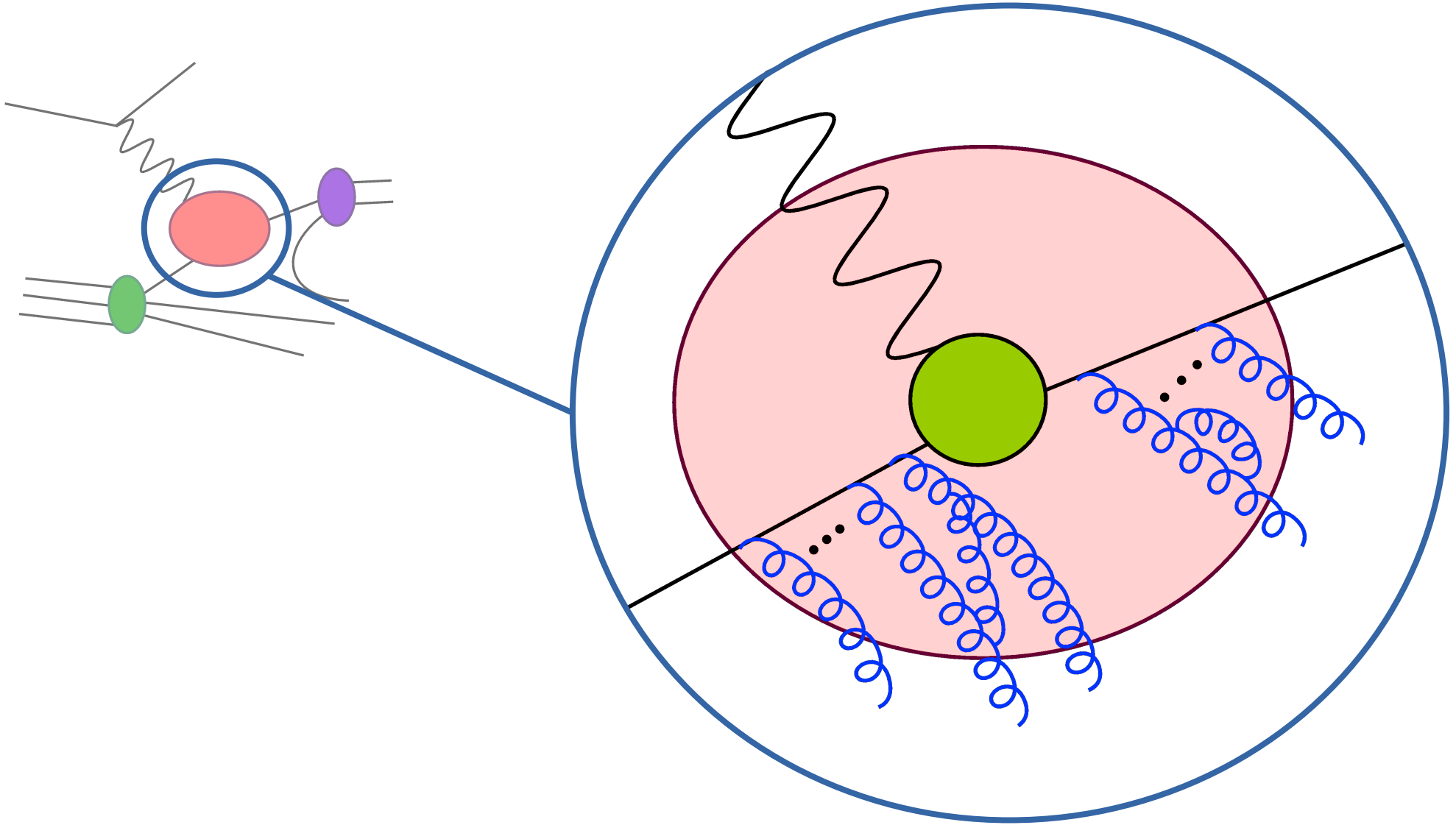
$$\hat{x}, \hat{z} \to 1$$

kth order of perturbation theory

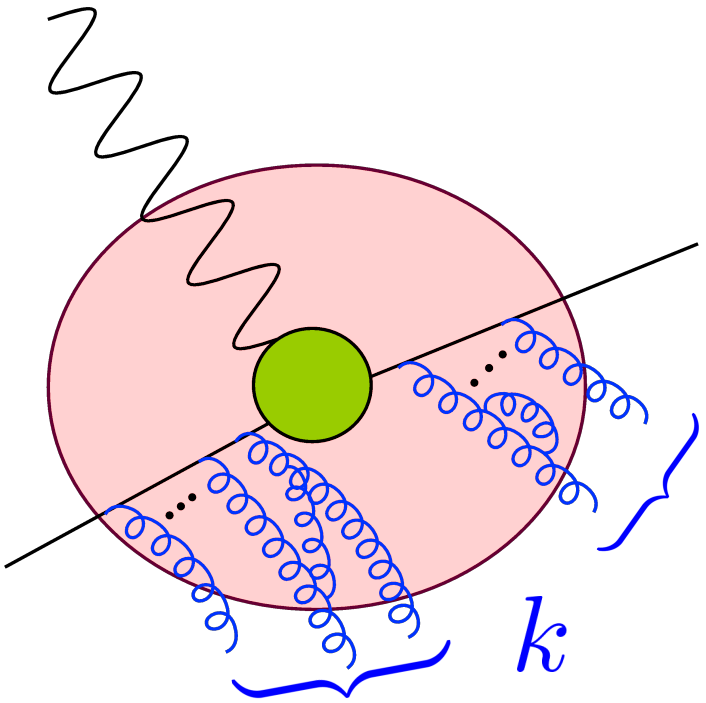
$$\alpha_s^k \omega_{qq}^{(k)}(\hat{x}, \hat{z}) \sim \alpha_s^k \left[\delta(1-\hat{x}) \left(\frac{\ln^{2k-1}(1-\hat{z})}{1-\hat{z}} \right)_+ + \delta(1-\hat{z}) \left(\frac{\ln^{2k-1}(1-\hat{x})}{1-\hat{x}} \right)_+ + \dots \right]$$

threshold logarithms

- appear in all QCD sensitive processes
- are well understood today in the case of DIS and DY (not yet for SIDIS)



soft radiation with total momentum k



$$\hat{x} = \hat{z} = 1$$

limitation on total energy of partonic radiation

$$(1-\hat{x}) + (1-\hat{z}) \approx \frac{2k_0}{Q}$$

real and virtual corrections imbalanced

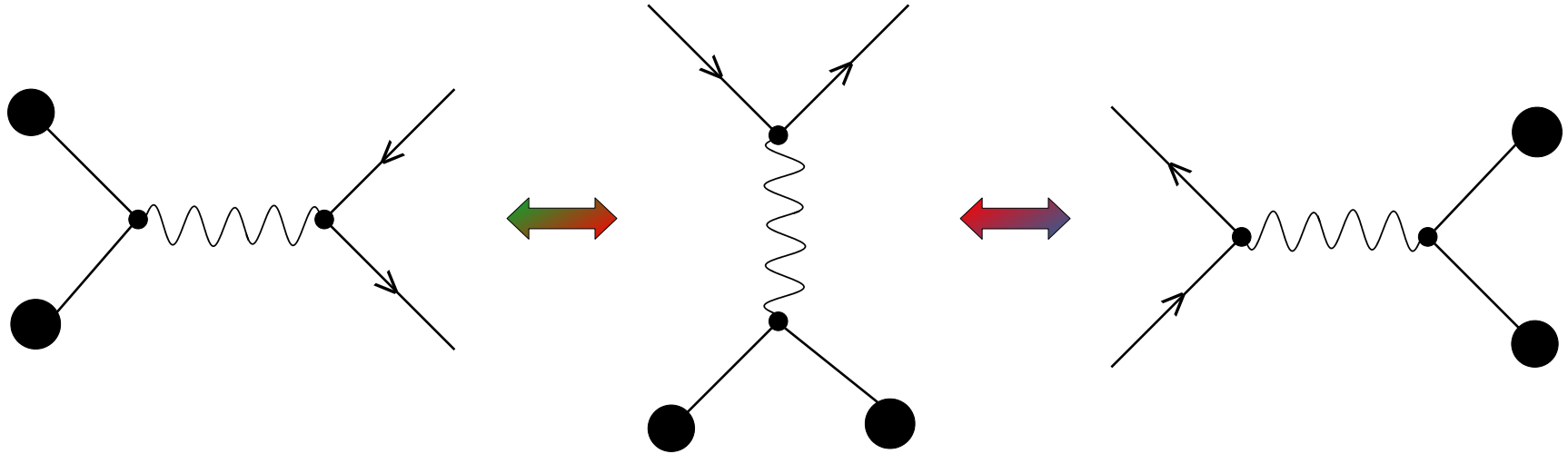
Sterman;
Catani, Trentadue; ...

- logarithms can be resummed to all orders: threshold resummation
- can be used to approximate dominant parts of NNLO corrections

crossed threshold resummation

Sterman, Vogelsang

universal structure of resummed logarithmic corrections



$h_1 h_2 \to \ell^+ \ell^- X$

Drell-Yan

$h_1 \ell \to \ell h_2 X$

SIDIS

$\ell^+ \ell^- \to h_1 h_2 X$

double-inclusive
leptonic annihilation

Threshold Resummation

Mellin transformation

$$\tilde{\mathcal{F}}_T^h(N,M,Q^2) \equiv \int_0^1 dx\, x^{N-1} \int_0^1 dz\, z^{M-1} \mathcal{F}_T^h(x,z,Q^2)$$

$$\sum_{a,b} \int_x^1 \frac{d\hat{x}}{\hat{x}} \int_z^1 \frac{d\hat{z}}{\hat{z}}\, f_a\left(\frac{x}{\hat{x}},Q^2\right) \omega_{ab}\left(\hat{x},\hat{z},\alpha_s\right) D_b^h\left(\frac{z}{\hat{z}},Q^2\right)$$

$$\Longrightarrow \quad \tilde{\mathcal{F}}_T^h(N,M,Q^2) = \sum_{a,b} \tilde{f}_a\left(N,Q^2\right) \tilde{\omega}_{ab}\left(N,M,\alpha_s\right) \tilde{D}_b^h\left(M,Q^2\right)$$

$$\tilde{\mathcal{F}}_T^h(N,M,Q^2) = \sum_{a,b} \tilde{f}_a\left(N,Q^2\right)\tilde{\omega}_{ab}\left(N,M,\alpha_s\right)\tilde{D}_b^h\left(M,Q^2\right)$$

Mellin moments

$$\tilde{f}_a(N,Q^2) \equiv \int_0^1 dx\, x^{N-1} f(x,Q^2),$$

$$\tilde{D}_b^h(M,Q^2) \equiv \int_0^1 dz\, z^{M-1} D_b^h(z,Q^2),$$

$$\tilde{\omega}_{ab}\left(N,M,\alpha_s\right) \equiv \int_0^1 d\hat{x}\, \hat{x}^{N-1} \int_0^1 d\hat{z}\, \hat{z}^{M-1}\, \omega_{ab}\left(\hat{x},\hat{z},\alpha_s\right)$$

$$\tilde{\mathcal{F}}_T^h(N, M, Q^2) = \sum_{a,b} \tilde{f}_a\left(N, Q^2\right) \tilde{\omega}_{ab}\left(N, M, \alpha_s\right) \tilde{D}_b^h\left(M, Q^2\right)$$

$\hat{x}, \hat{z} \to 1 \quad \Longrightarrow \quad$ large N, M

spin-dependent case:

$$\tilde{\mathcal{G}}^h(N, M, Q^2) = \sum_{a,b} \Delta\tilde{f}_a\left(N, Q^2\right) \Delta\tilde{\omega}_{ab}\left(N, M, \alpha_s\right) \tilde{D}_b^h\left(M, Q^2\right)$$

$$\tilde{\mathcal{F}}_T^h(N,M,Q^2) = \sum_{a,b} \tilde{f}_a\left(N,Q^2\right)\underbrace{\tilde{\omega}_{ab}\left(N,M,\alpha_s\right)}\tilde{D}_b^h\left(M,Q^2\right)$$

for large N,M
NLO:

$$\tilde{\omega}_{ab} = \tilde{\omega}_{ab}^{(0)} + \frac{\alpha_s}{\pi}\tilde{\omega}_{ab}^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \tilde{\omega}_{ab}^{(2)} + \mathcal{O}(\alpha_s^3)$$

$$\omega_{qq}^{(1)} = e_q^2 C_F \left[\delta(1-\hat{x})\left(\frac{\ln(1-\hat{z})}{1-\hat{z}}\right)_+ + \delta(1-\hat{z})\left(\frac{\ln(1-\hat{x})}{1-\hat{x}}\right)_+ + \frac{1}{(1-\hat{x})_+(1-\hat{z})_+} - 4\delta(1-\hat{x})\delta(1-\hat{z}) + \ldots\right]$$

$$\bar{N} = N\mathrm{e}^{\gamma_E}$$
$$\bar{M} = M\mathrm{e}^{\gamma_E}$$

$$\Longrightarrow \quad \tilde{\omega}_{qq}^{(1)} = e_q^2 C_F \left[\frac{1}{2}\left(\ln\bar{N} + \ln\bar{M}\right)^2 - 4 + \frac{\pi^2}{6} + \ldots\right]$$

leading logarithms (**LL**)

$$\int_0^1 d\hat{x}\,\hat{x}^{N-1}\delta(1-\hat{x}) = 1 \qquad \int_0^1 d\hat{z}\,\hat{z}^{M-1}\left(\frac{\ln(1-\hat{z})}{1-\hat{z}}\right)_+ = \frac{1}{2}\ln^2\bar{M} + \frac{\pi^2}{12} + \ldots$$

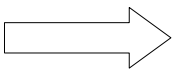
threshold resummation

at kth order: $\alpha_s^k \left(\ln \bar{N} + \ln \bar{M}\right)^{2k} + \ldots$

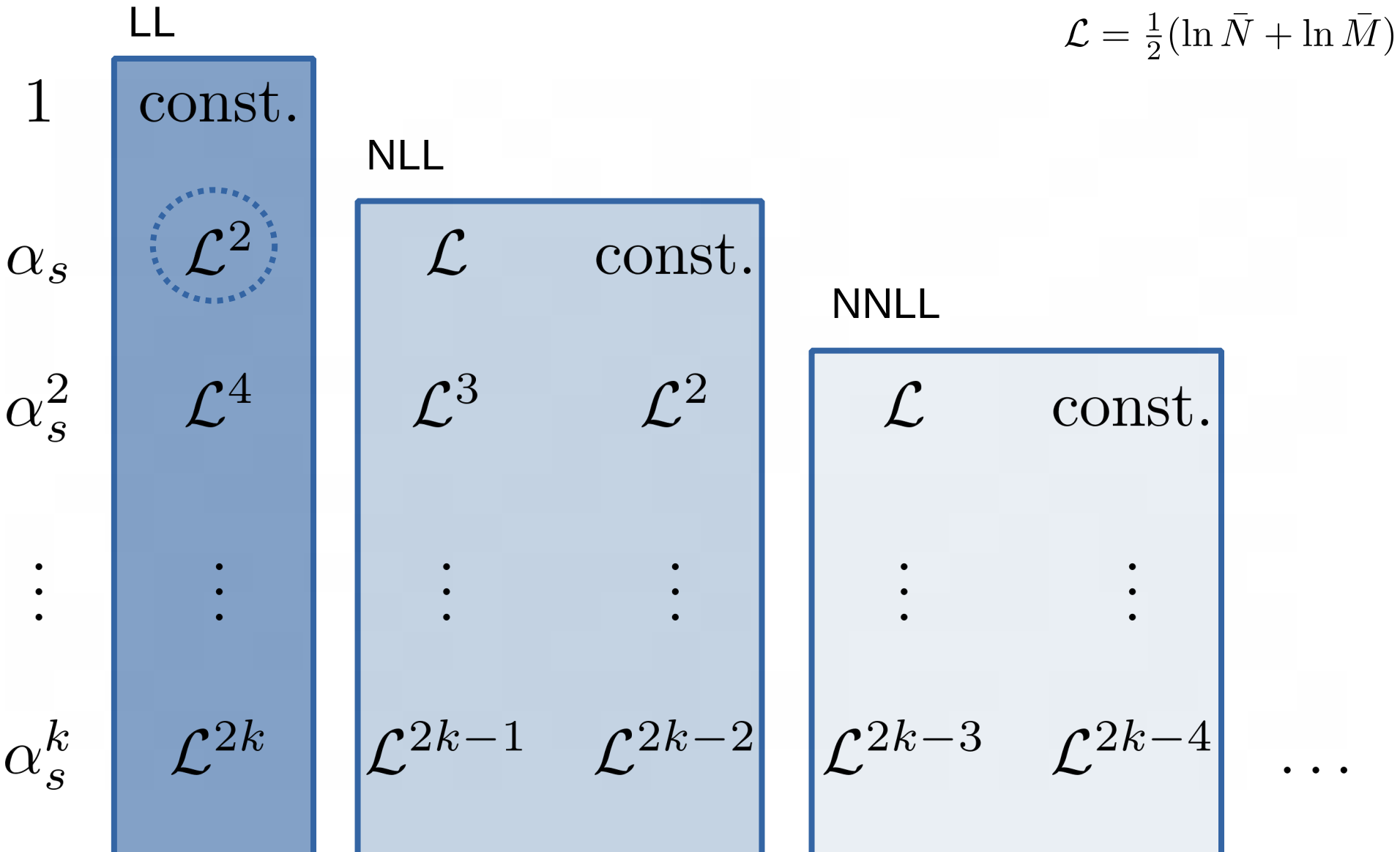
- for large N,M we obtain large double-logarithmic corrections
- they can be resummed to all orders
- these corrections then exponentiate

(eikonal exponentiation)

Sterman;
Catani, Trentadue; ...



may be used for the
NNLO approximation


$\mathcal{L} = \frac{1}{2}(\ln \bar{N} + \ln \bar{M})$
LL
NLL
NNLL
1
const.
α_s
$\mathcal{L}^2$
$\mathcal{L}$
const.
α_s^2
$\mathcal{L}^4$
$\mathcal{L}^3$
$\mathcal{L}^2$
$\mathcal{L}$
const.
α_s^k
$\mathcal{L}^{2k}$
$\mathcal{L}^{2k-1}$
$\mathcal{L}^{2k-2}$
$\mathcal{L}^{2k-3}$
$\mathcal{L}^{2k-4}$
...

timelike inclusive Drell-Yan

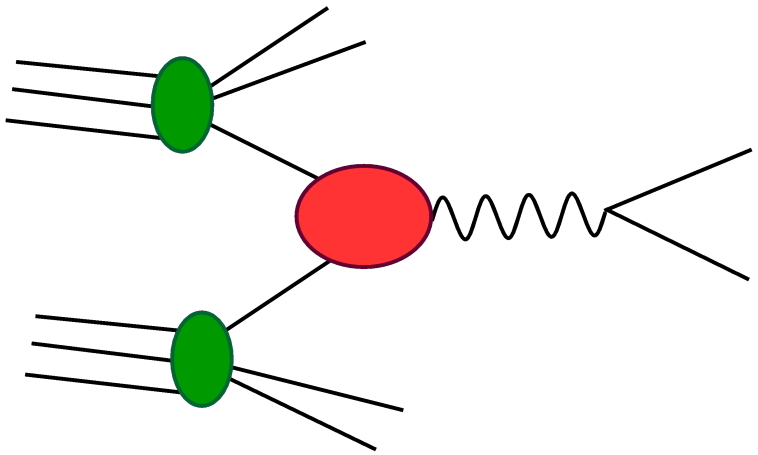
$$\tilde{\omega}_{q\bar{q}}^{\mathrm{DY,res}}(N)$$

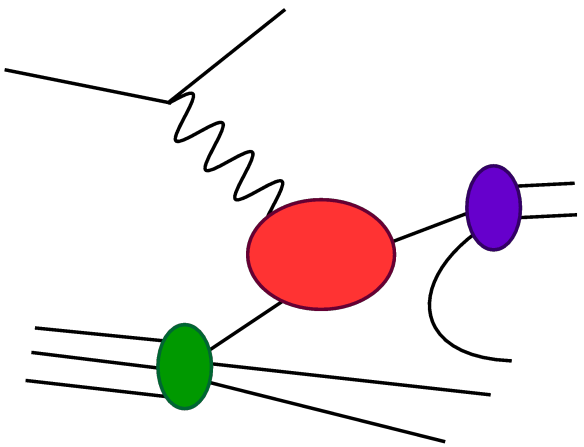
to NLL

$$N \to \sqrt{NM}$$

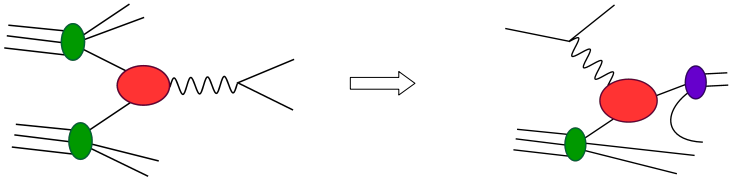
spacelike SIDIS

$$\tilde{\omega}_{qq}^{\mathrm{res}}(N,M)$$





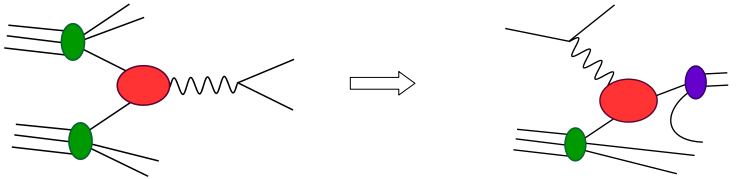
Sterman, Vogelsang;
Anderle, Ringer, Vogelsang



inclusive Drell-Yan

$$\tilde{\omega}_{qq}^{\mathrm{DY,res}}(N) \propto \exp\left[\int_0^{Q^2} \frac{dk_\perp^2}{k_\perp^2} A_q\left(\alpha_s(k_\perp^2)\right) \left\{\int_{k_\perp^2/Q^2}^1 \frac{d\xi}{\xi}\left(\mathrm{e}^{-N\left(\xi+\frac{k_\perp^2}{\xi Q^2}\right)}-1\right)+2\ln\bar{N}\right\}\right]$$

$$\approx 2\left[K_0\left(N\frac{2k_\perp}{Q}\right)+\ln\left(\frac{k_\perp}{Q}\bar{N}\right)\right]$$



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$$N \to \sqrt{NM}$$

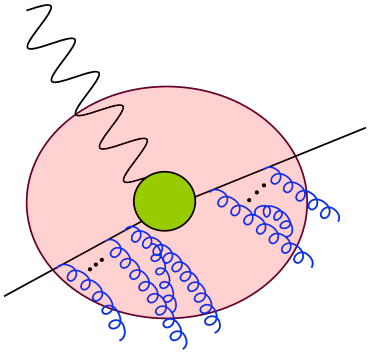
$$\approx 2\left[K_0\left(N\frac{2k_\perp}{Q}\right) + \ln\left(\frac{k_\perp}{Q}\bar{N}\right)\right]$$

SIDIS

$$\tilde{\omega}_{qq}^{\mathrm{res}}(N,M) \propto \exp\left[\int_0^{Q^2} \frac{dk_\perp^2}{k_\perp^2} A_q\left(\alpha_s(k_\perp^2)\right) \left\{\int_{k_\perp^2/Q^2}^1 \frac{d\xi}{\xi}\left(\mathrm{e}^{-N\xi - M\frac{k_\perp^2}{\xi Q^2}}-1\right) + \ln\bar{N} + \ln\bar{M}\right\}\right]$$

$$\approx 2\left[K_0\left(\sqrt{NM}\frac{2k_\perp}{Q}\right) + \ln\left(\frac{k_\perp}{Q}\sqrt{\bar{N}\bar{M}}\right)\right]$$

fully resummed coefficient function



Catani, de Florian, Grazzini, Nason;
Hinderer, Ringer, Sterman, Vogelsang

N,M independent
correction in resummation

$$\tilde{\omega}_{qq}^{\mathrm{res}}\left(N,M,\alpha_s\right) = e_q^2\, H_{qq}^{\mathrm{SIDIS}}\left(\alpha_s\right) \widehat{C}_{qq}\left(\alpha_s\right)$$

$$\times\ \exp\left\{\int_{Q^2/(\bar{N}\bar{M})}^{Q^2} \frac{d\mu^2}{\mu^2}\left[A_q\left(\alpha_s(\mu)\right)\ln\left(\frac{\mu^2\bar{N}\bar{M}}{Q^2}\right) - \frac{1}{2}\widehat{D}_q\left(\alpha_s(\mu)\right)\right]\right\}$$

hard virtual corrections

Kodaira, Trentadue;
Moch, Vermaseren, Vogt;…

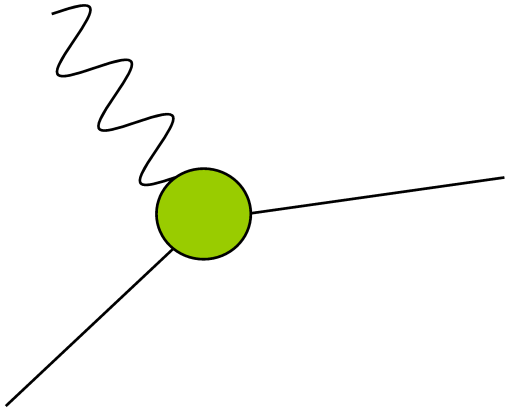
Vogt;
Catani, de Florian, Grazzini;

complete logarithmic contribution is in the exponent
i.e. to all log order (LL, NLL, NNLL, …)

hard virtual corrections

spacelike quark form factor

$$H_{qq}^{\mathrm{SIDIS}}(\alpha_s) = \left| \left[1 - \tilde{I}(\epsilon, \alpha_s) \right] F_q \right|^2$$



subtraction of collinear and soft divergences

Catani, Cieri, de Florian, Ferrera, Grazzini;

@ 2 loops:
Gehrmann, Huber, Maitre;
Moch, Vermaseren, Vogt;

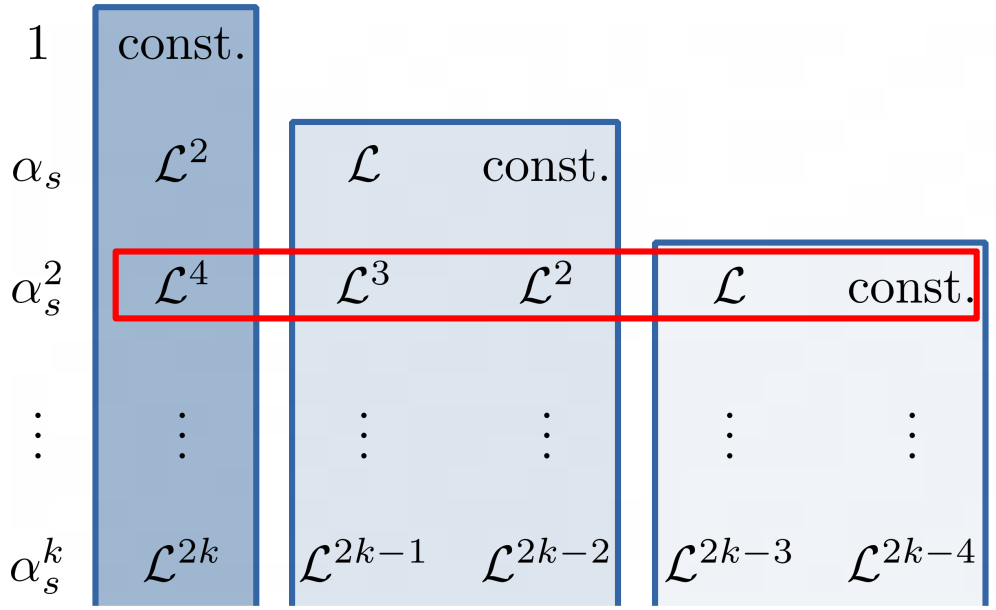
@ 3 loops:
Gehrmann, Glover, Huber, Ikizlerli, Studerus

$$H_{qq}^{\mathrm{SIDIS}}(\alpha_s) = 1 + \frac{\alpha_s}{\pi} H_{qq}^{\mathrm{SIDIS},(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 H_{qq}^{\mathrm{SIDIS},(2)} + \mathcal{O}(\alpha_s^3)\,,$$

only contains constant contributions i.e. no logarithmic terms

$$\mathcal{L} = \tfrac{1}{2}(\ln \bar{N} + \ln \bar{M})$$

NNLO

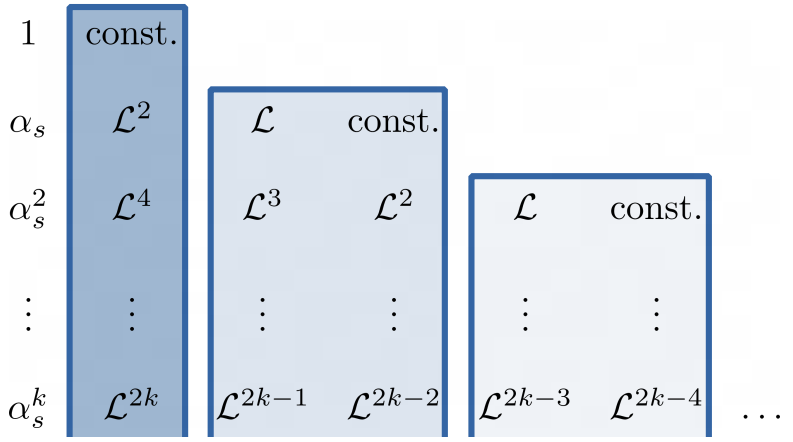


expansion to NNLL

$$\tilde{\omega}_{qq}^{\mathrm{res}}(N, M, \alpha_s) = e_q^2 H_{qq}^{\mathrm{SIDIS}}(\alpha_s)\, \widehat{C}_{qq}(\alpha_s) \exp\left\{ \frac{\lambda}{\alpha_s b_0} h_q^{(1)}(\lambda) + h_q^{(2)}(\lambda) + \alpha_s h_q^{(3)}(\lambda) + \ldots \right\}$$

$$\lambda = b_0 \alpha_s \mathcal{L} \qquad \text{LL} \qquad \text{NLL} \qquad \text{NNLL}$$

$$\mathcal{L} = \tfrac{1}{2}(\ln \bar{N} + \ln \bar{M})$$



$$\tilde{\omega}_{qq}^{\rm res}(N, M, \alpha_s) = e_q^2 H_{qq}^{\rm SIDIS}(\alpha_s)\, \widehat{C}_{qq}(\alpha_s) \exp\left\{ \frac{\lambda}{\alpha_s b_0} h_q^{(1)}(\lambda) \; + \; h_q^{(2)}(\lambda) \; + \; \alpha_s h_q^{(3)}(\lambda) + \ldots \right\}$$

$$\lambda = b_0 \alpha_s \mathcal{L} \qquad\qquad h_q^{(1)}(\lambda) = \frac{A_q^{(1)}}{2\pi b_0 \lambda} \left[2\lambda + (1 - 2\lambda) \ln(1 - 2\lambda) \right]$$

Approximate higher-order corrections

subleading contributions near threshold $\frac{1}{N}$, $\frac{1}{M}$

- leading power (**LP**) corrections $\alpha_s^n L^m \quad m \in \{2n, \ldots, 2n-4\}$
- next-to-leading power (**NLP**) corrections

$\alpha_s^n L^m/N$ or $\alpha_s^n L^m/M$

L^m

any product of m logarithms in N or M

$m \in \{2n-1, \ldots, 2n-3\}$

dominant subleading term $\alpha_s^n L^{2n-1}/N$ or $\alpha_s^n L^{2n-1}/M$

subleading contributions near threshold $\frac{1}{N}$, $\frac{1}{M}$

dominant NLP terms $\alpha_s^n L^{2n-1}/N$ or $\alpha_s^n L^{2n-1}/M$

i.e. @ NNLO $\alpha_s^2 L^3/N$ or $\alpha_s^2 L^3/M$

where

$$L \in \{\ln^3(N), \ln^2(N)\ln(M), \ln(N)\ln^2(M), \ln^3(M)\}$$

subleading contributions near threshold $\frac{1}{N}$, $\frac{1}{M}$

$$\alpha_s^n L^m/N \quad \text{or} \quad \alpha_s^n L^m/M$$

NLP terms:

- mostly studied in color-singlet hard-scattering processes
- knowledge stems from different approaches
  - NLP corrections to eikonal approximation
  - resummation of NLP terms
  - generalized factorization theorems at NLP

van Beekveld et al. ;
Vogt, Moch;
Ravindran et al.

extensive list of literature in 2109.00847

Catani, de Florian, Grazzini, Nason;
Kramer, Laenen, Spira;
Kulesza, Sterman, Vogelsang;

How do we get dominant NLP terms?

rewrite the resummed cross section

factorization scale

$$\tilde{f}_q\left(N,\mu_F\right)\tilde{\omega}_{qq}^{\mathrm{res}}\left(N,M,\alpha_s,\frac{\mu_F}{Q}\right)\tilde{D}_q^h\left(M,\mu_F\right)$$

$$=\tilde{f}_q\left(N,Q/\sqrt{\bar{N}\bar{M}}\right)\tilde{\omega}_{qq}^{\mathrm{res,evol}}\left(N,M,\alpha_s\right)\tilde{D}_q^h\left(M,Q/\sqrt{\bar{N}\bar{M}}\right)$$

new scale

Catani, de Florian, Grazzini, Nason;
Kramer, Laenen, Spira;
Kulesza, Sterman, Vogelsang;

How do we get dominant NLP terms?

evolve the PDF and FF **back** via the DGLAP equations including NLP corrections

$$Q/\sqrt{\bar{N}\bar{M}} \;\to\; \mu_F$$

$$\tilde{f}_q\left(N, \frac{Q}{\sqrt{\bar{N}\bar{M}}}\right) = \exp\left\{\int_{\mu_F^2}^{Q^2/(\bar{N}\bar{M})} \frac{d\mu^2}{\mu^2}\left[-A_q\big(\alpha_s(\mu)\big)\ln\bar{N} + P_{q,\delta}\big(\alpha_s(\mu)\big) + \frac{\alpha_s(\mu)}{\pi}\,\frac{Q_q^{(1)}}{N}\right]\right\}\tilde{f}_q(N,\mu_F)$$

analogous for FF

NLP term

same cross section as before

we then obtain

$$\tilde{f}_q\left(N,\mu_F\right)\,\tilde{\omega}_{qq}^{\rm res}\left(N,M,\alpha_s,\frac{\mu_F}{Q}\right)\,\tilde{D}_q^h\left(M,\mu_F\right)$$
$$\times\exp\left\{-\int_{\mu_F^2}^{Q^2/(\bar{N}\bar{M})}\frac{d\mu^2}{\mu^2}\frac{\alpha_s(\mu)}{\pi}\frac{C_F}{2N}\right\}\exp\left\{-\int_{\mu_F^2}^{Q^2/(\bar{N}\bar{M})}\frac{d\mu^2}{\mu^2}\frac{\alpha_s(\mu)}{\pi}\frac{C_F}{2M}\right\}$$

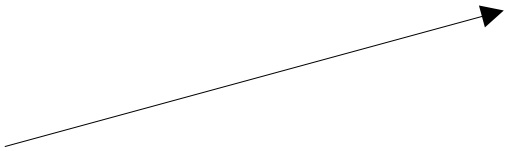
dominant NLP terms from evolution

approximate NNLO

expansion to NNLO

$$\tilde{\omega}_{qq}(N, M, \alpha_s) = 1 + \frac{\alpha_s}{\pi}\,\tilde{\omega}_{qq}^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \tilde{\omega}_{qq}^{(2)} + \mathcal{O}(\alpha_s^3)$$

$\mathcal{L} = \frac{1}{2}(\ln \bar{N} + \ln \bar{M})$



approximate NLO

$$\tilde{\omega}_{qq}^{(1)}\,(N, M) = e_q^2 C_F \left\{ 2\mathcal{L}^2 - 4 + \frac{\pi^2}{6} + \mathcal{L}\left(\frac{1}{N} + \frac{1}{M}\right) \right\}$$

dominant subleading term

approximate NNLO

expansion to NNLO

$$\tilde{\omega}_{qq}(N,M,\alpha_s) = 1 + \frac{\alpha_s}{\pi}\,\tilde{\omega}_{qq}^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \tilde{\omega}_{qq}^{(2)} + \mathcal{O}(\alpha_s^3)$$

$\mathcal{L} = \frac{1}{2}(\ln \bar{N} + \ln \bar{M})$

approximate NNLO

$$\frac{1}{e_q^2}\,\tilde{\omega}_{qq}^{(2)}\,(N,M) = 2C_F^2\mathcal{L}^4 + \frac{4\pi}{3}\,b_0 C_F \mathcal{L}^3 + C_F\mathcal{L}^2\left[C_F\left(-8+\frac{\pi^2}{3}\right) + \left(\frac{67}{18}-\frac{\pi^2}{6}\right)C_A - \frac{5}{9}N_f\right]$$

$$+ C_F\mathcal{L}\left[\left(\frac{101}{27}-\frac{7}{2}\,\zeta(3)\right)C_A - \frac{14}{27}\,N_f\right] + C_F^2\left[\frac{511}{64}-\frac{\pi^2}{16}-\frac{\pi^4}{60}-\frac{15}{4}\,\zeta(3)\right]$$

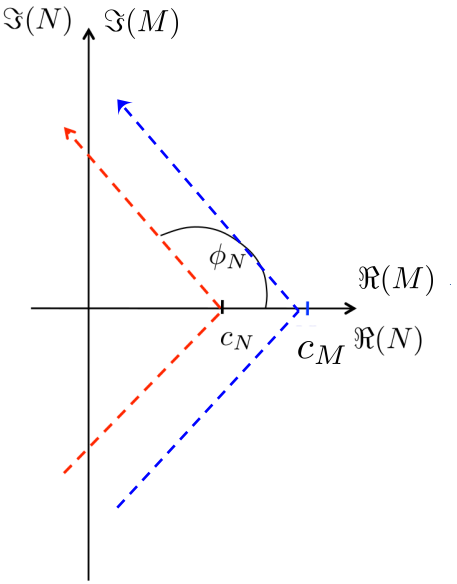
$$+ C_F C_A\left[-\frac{1535}{192}-\frac{5\pi^2}{16}+\frac{7\pi^4}{720}+\frac{151}{36}\,\zeta(3)\right] + C_F N_f\left[\frac{127}{96}+\frac{\pi^2}{24}+\frac{\zeta(3)}{18}\right]$$

$$+ 2\,C_F^2\,\mathcal{L}^3\left(\frac{1}{N}+\frac{1}{M}\right)$$

same corrections are obtained in the spin dependent case

Phenomenology

numerical analysis



- NNLO PDF CT18
- NNLO FF Anderle, Kaufmann, Ringer, Stratmann

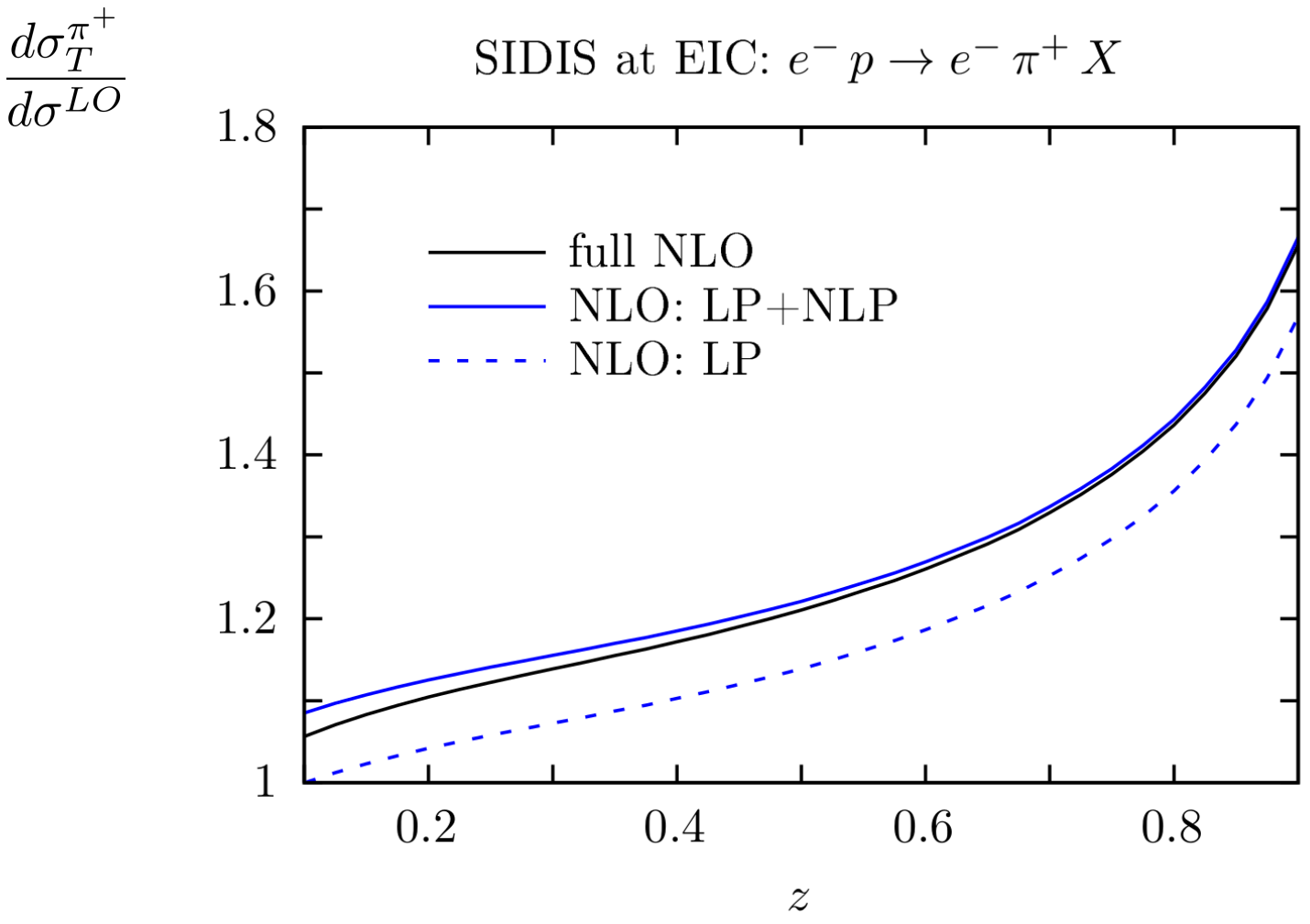
  both fitted to get Mellin moments of distributions

- perform double Mellin inverse: we give results in x, z space

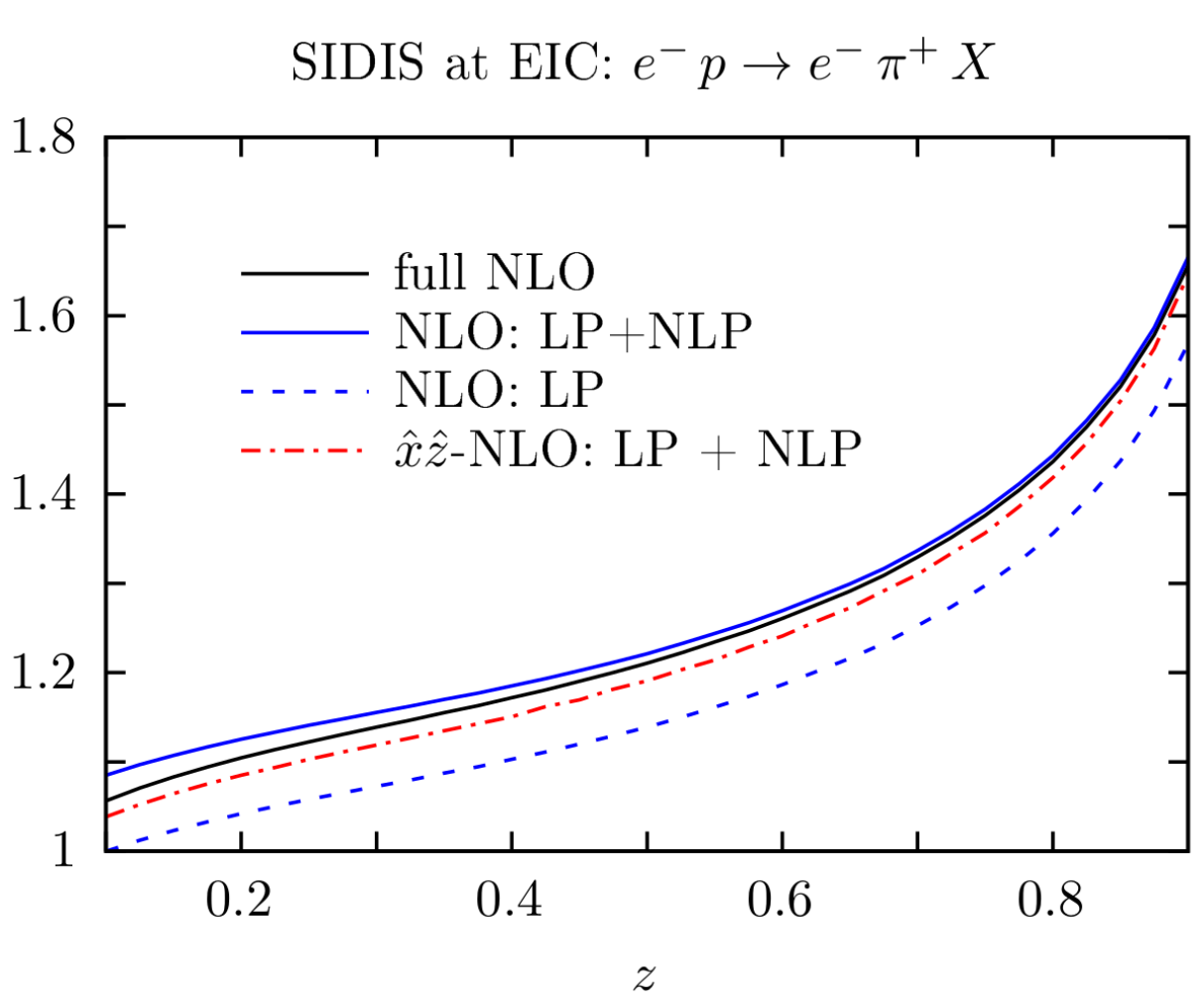
$$\mathcal{F}_T^h(x,z,Q^2) = \int_{\mathcal{C}_N} \frac{dN}{2\pi i}\, x^{-N} \int_{\mathcal{C}_M} \frac{dM}{2\pi i}\, z^{-M}\, \tilde{\mathcal{F}}_T^h(N,M,Q^2)$$

- results for the EIC $\sqrt{s} = 100\ \mathrm{GeV} \quad e^- p \to e^- \pi^+ X$

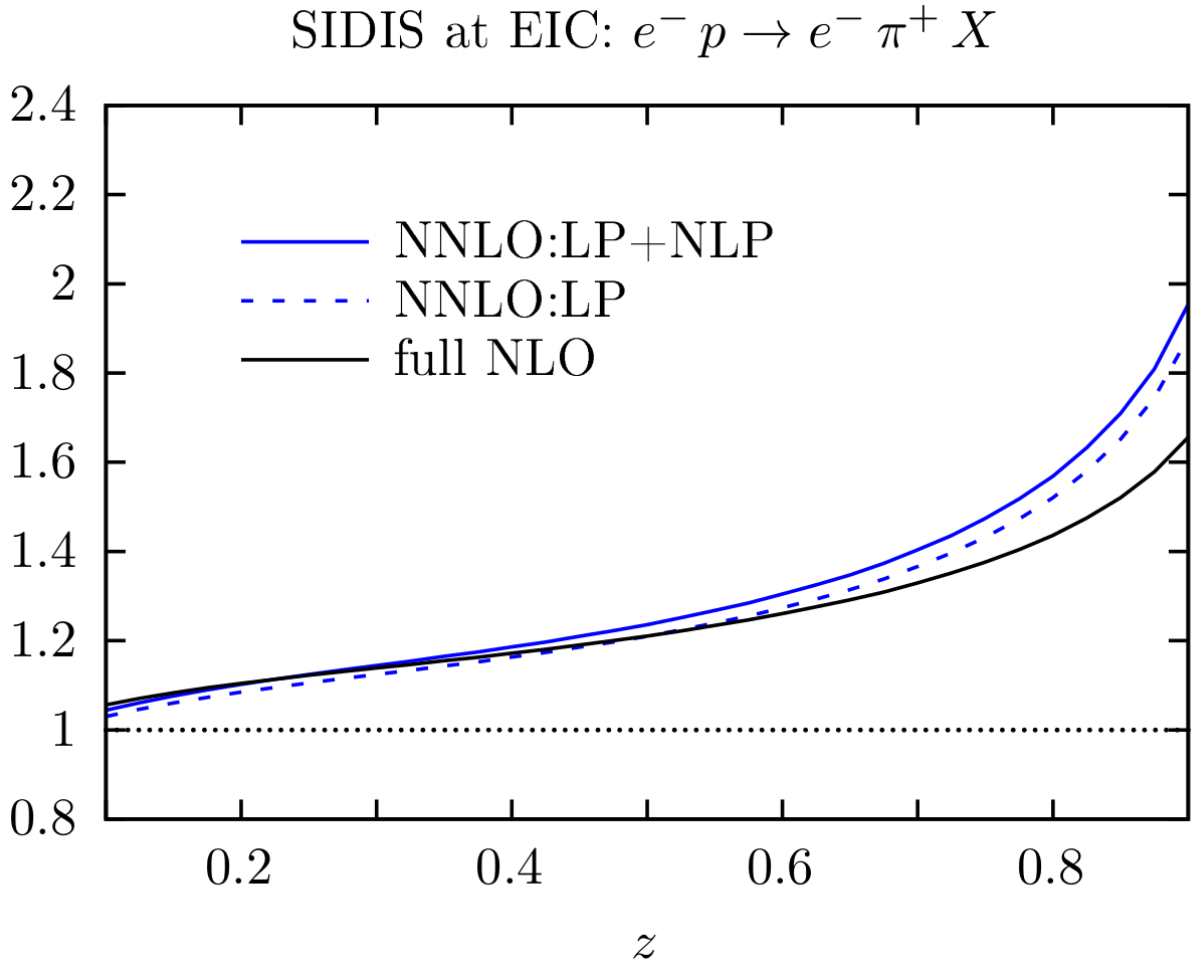
$$y \in [0.1, 0.9] \qquad x \in [0.1, 0.8] \qquad z \in [0.1, 0.9]$$


$\frac{d\sigma_T^{\pi^+}}{d\sigma^{LO}}$
SIDIS at EIC: $e^- p \to e^- \pi^+ X$
full NLO
NLO: LP+NLP
NLO: LP
1.8
1.6
1.4
1.2
1
0.2
0.4
0.6
0.8
z

$$\frac{d\sigma_T^{\pi^+}}{d\sigma^{LO}}$$



approximate NNLO



scale dependence of μ_R, μ_F for the EIC

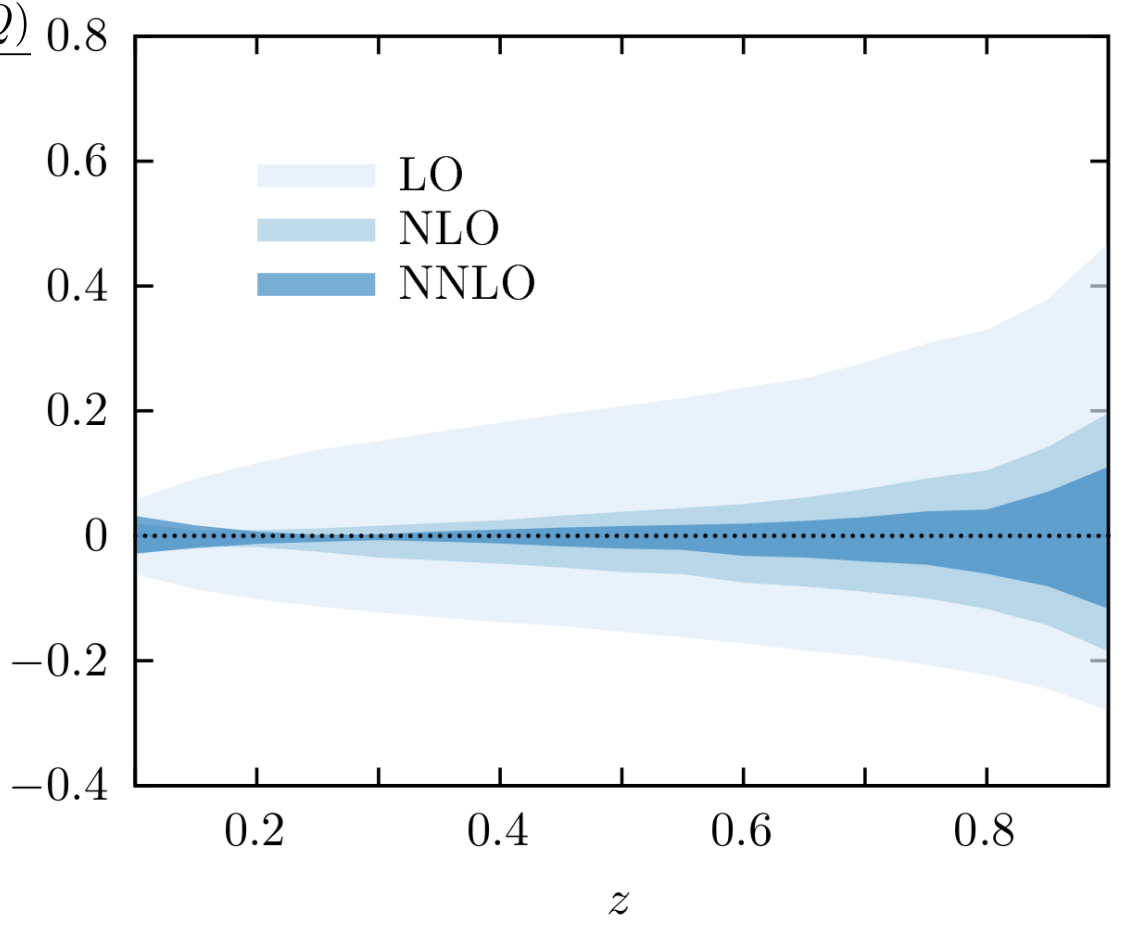
$$\frac{d\sigma(\mu_R, \mu_F) - d\sigma(Q, Q)}{d\sigma(Q, Q)}$$

we vary independently

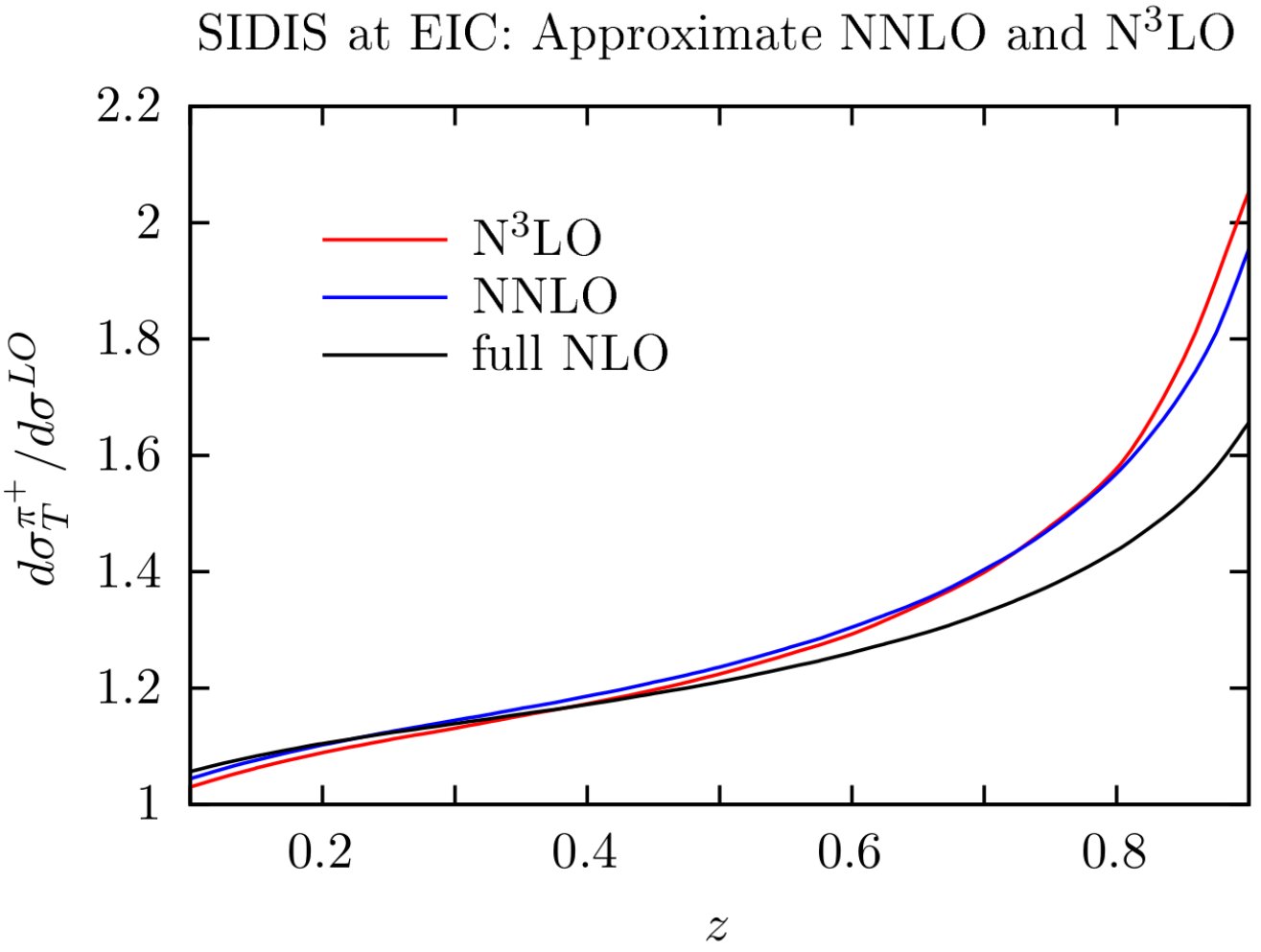
$$\mu_F = Q/2, Q, 2Q$$

$$\mu_R = Q/2, Q, 2Q$$

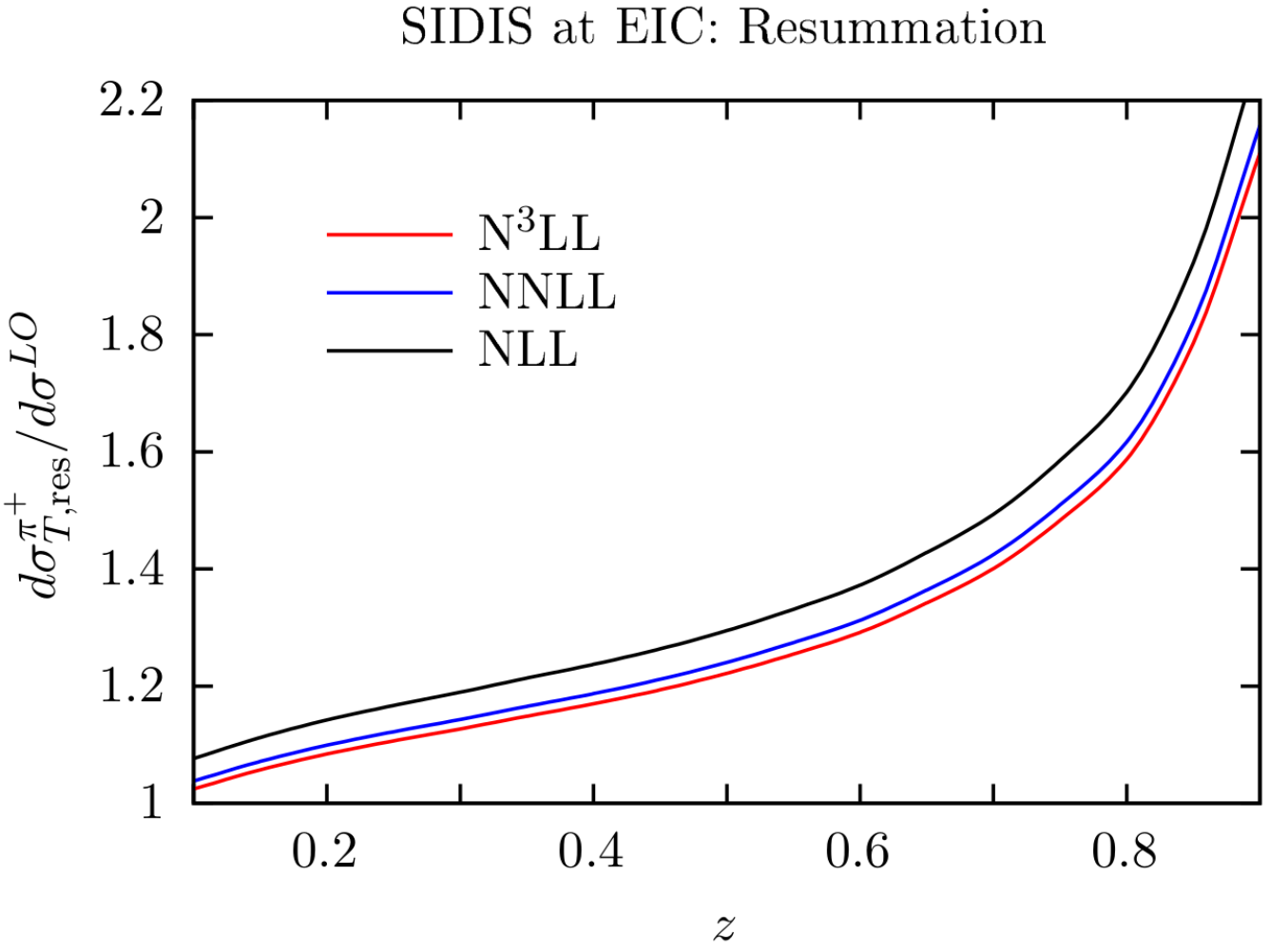
and take the envelope of the results



approximate N3LO



full resummation



Conclusions

conclusions

- generally modest beyond-NLO effects
- at large z we obtain a significant increase by NNLO
- reduction in scale dependence
- benchmark for future full NNLO calculations
- outlook: longitudinal structure function and qg contributions