SHEDDING LIGHT ON SHADOW GENERALIZED PARTON DISTRIBUTIONS (GPDS)

JLAB THEORY SEMINAR 10-16-23

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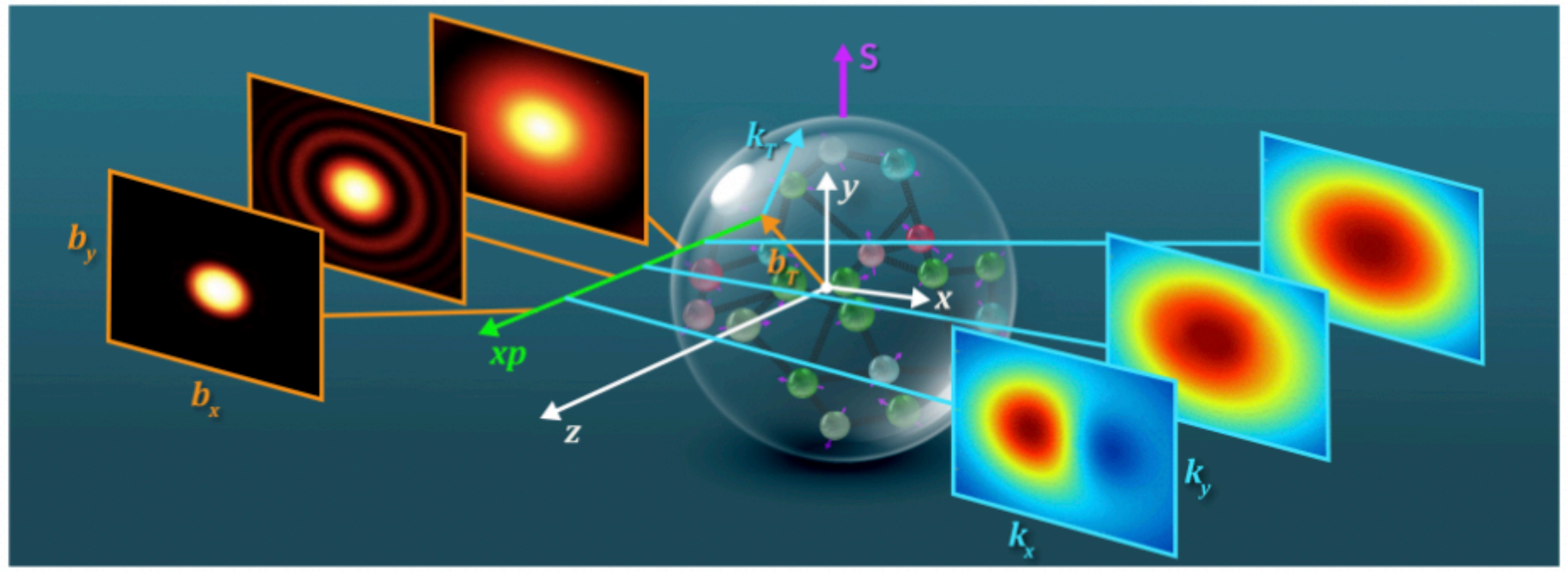
Motivation



- Generalized Parton Distributions (GPDs) contain information about many hadron properties:
  - 3D structure
  - Spin sum
  - Pressure and shear force distributions
- Goal:
  - Perform a global extraction of GPDs from available data
- Obstacle:
  - Shadow GPDs (SGPDS) (Bertone, et. al. Phys.Rev.D 103 (2021) 11, 114019):
    - There is an infinite number of functions that can give the same observable.

QUARK-GLUON TOMOGRAPHY COLLABORATION

QuantOm Collaboration

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GPDs

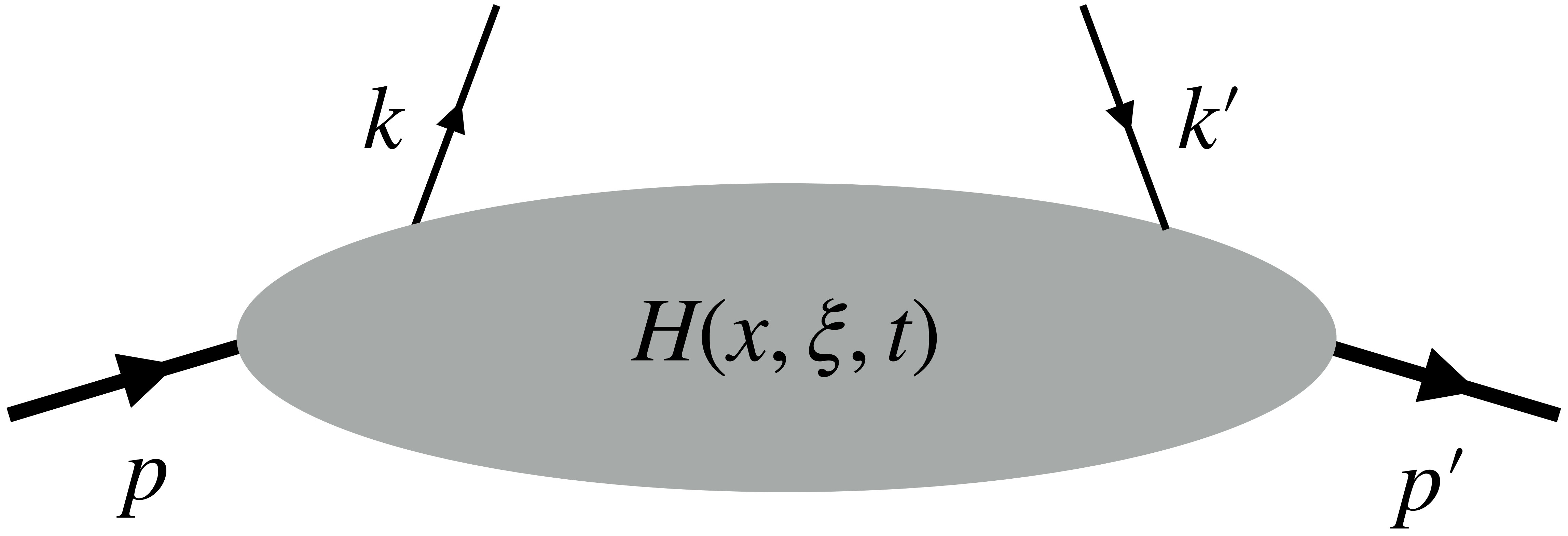
- Definition:

$$P\cdot n\int\frac{d\lambda}{2\pi}\,e^{ixP\cdot n\lambda}\left\langle p'\left|\bar{\psi}^q(-\tfrac{1}{2}\lambda n)\,\not{n}\,\psi^q(\tfrac{1}{2}\lambda n)\right|p\right\rangle=\bar{u}(p')\left[\;H^q(x,\xi,t;\mu^2)\,\not{n}+\;E^q(x,\xi,t;\mu^2)\,\frac{i\sigma^{n\Delta}}{2M}\right]u(p),$$

$$n_\mu n_\nu\int\frac{d\lambda}{2\pi}\,e^{ixP\cdot n\lambda}\left\langle p'\left|G^{\mu\alpha}(-\tfrac{1}{2}\lambda n)\,G_\alpha{}^\nu(\tfrac{1}{2}\lambda n)\right|p\right\rangle=\bar{u}(p')\left[x\,H^g(x,\xi,t;\mu^2)\,\not{n}+x\,E^g(x,\xi,t;\mu^2)\,\frac{i\sigma^{n\Delta}}{2M}\right]u(p),$$

- Functions of x, ξ, and t:

$$x=\frac{k^++k'^+}{p^++p'^+}\quad \xi=\frac{p'^+-p^+}{p^++p'^+}\quad t=(p'-p)^2$$



GPDs

- Forward Limit ($\xi, t \to 0$):
  - H GPD:

$$H^q(x,0,0) = q(x)\,\Theta(x) - \bar{q}(-x)\,\Theta(-x),$$
$$2\,H^g(x,0,0) = g(x)\,\Theta(x) - g(-x)\,\Theta(-x),$$

  - Forward limit of E does not map to known functions

GPDs

- Polynomiality:

$$\int_{-1}^{1} \mathrm{d}x\, x^s H^a(x,\xi,t;\mu^2) = \sum_{i=0\ (\mathrm{even})}^{s} (2\xi)^i\, A^a_{s+1,i}(t,\mu^2) + \mathrm{mod}(s,2)\, (2\xi)^{s+1}\, C^a_{s+1}(t,\mu^2),$$

$$\int_{-1}^{1} \mathrm{d}x\, x^s E^a(x,\xi,t;\mu^2) = \sum_{i=0\ (\mathrm{even})}^{s} (2\xi)^i\, B^a_{s+1,i}(t,\mu^2) - \mathrm{mod}(s,2)\, (2\xi)^{s+1}\, C^a_{s+1}(t,\mu^2),$$

GPDs

- First moments give electromagnetic form factors:

$$\int_{-1}^{1} dx\, H^q(x, \xi, t; \mu^2) = F_1^q(t; \mu^2)$$

$$\int_{-1}^{1} dx\, E^q(x, \xi, t; \mu^2) = F_2^q(t; \mu^2)$$

GPDs

- Second moments give gravitational form factors:
  - Ji sum rule:

$$2J^a(\mu^2) = A^a_{01}(0,\mu^2) + B^a_{01}(0,\mu^2) = \int_{-1}^{1} dx\, x\left[H^a(x,\xi,0;\mu^2) + E^a(x,\xi,0;\mu^2)\right]$$

  - C_1^a is related to internal stresses

GPDs

- Evolution:
  - GPDs change with the energy scale in accordance with evolution equations of the general form:

$$\frac{\mathrm{d}H^a(x,\xi,t)}{\mathrm{d}\ln Q^2} = \int \mathrm{d}x P^a(x,\xi) H^a(x,\xi,t;Q_0^2)$$

The Inverse Problem

- Deeply virtual Compton scattering:
  - Compton Form Factors:

$$\mathscr{H}(\xi,t,Q^2) = \int_{-1}^{1} dx \sum_a C^a(x,\xi,Q^2,\mu^2)\, H^a(x,\xi,t;\mu^2) \quad \mathscr{E}(\xi,t,Q^2) = \int_{-1}^{1} dx \sum_a C^a(x,\xi,Q^2,\mu^2)\, E^a(x,\xi,t;\mu^2)$$

- x-dependence is lost in the integration
- While a fit could obtain a GPD: Does the x-dependence represent the true GPD?
  - There is an infinite number of functions that can give the same CFF.

Shadow GPDs

- The difference between one of the multiple solutions to the inverse problem and the true GPD:

$$F_S^a(x, \xi; \mu^2) = F_F^a(x, \xi; \mu^2) - F_T^a(x, \xi; \mu^2)$$

- Can rule out any F_F^a that do not satisfy the properties of GPDs, therefore SGPDs:
  - Must satisfy polynomiality
  - Zero contribution to CFF:

$$\sum_a C^a(x, \xi, Q^2, \mu^2) \otimes F_S^a(x, \xi; \mu^2) = 0$$

  - Forward Limit: $H_S^a(x,0,0) = 0$

Evolution and SGPDs

- Example SGPDs explored in Bertone, et. al. Phys.Rev.D 103 (2021) 11, 114019 give very small but non-zero CFF after evolution to a different energy scale.
- SGPDs can be multiplied by any factor and the result would still be a SGPD at the input scale
- Non-zero CFF after evolution would be multiplied by this factor
- Data spanning a range of energy scales would give a limit to the possible scaling factors

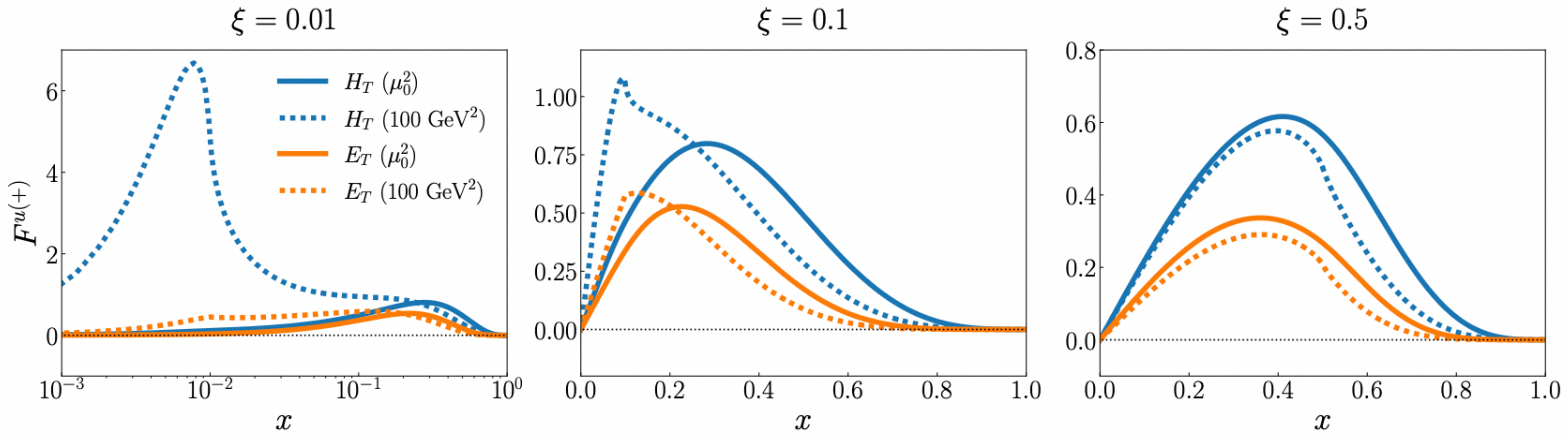
Evolution and SGPDs

- In this work:
  - Generate simulated CFF data spanning a range of energy scales and skewness using a model
  - Calculate how this data constrains a Monte Carlo sampling of SGPDs
  - Explore how these SGPDs would impact:
    - Spin sum
    - Pressure and shear stresses
    - Tomography

“True” GPDs

- Use VGG model as a proxy for the “true” GPD:
  - Vanderhaeghen, et. al., Phys. Rev. Lett. 80, 5064 (1998)
  - Vanderhaeghen, et. al., Phys. Rev. D 60, 094017 (1999)
  - Goeke, et. al., Prog. Part. Nucl. Phys. 47, 401 (2001)
  - Guidal, et. al., Phys. Rev. D 72, 054013 (2005)
- Use PDFs from JAM20-SIDIS (EM, et. al., Phys. Rev. D 104, 016015 (2021))

“True” GPDs



Calculating Shadow GPDs

- Start from a double distribution (DD):

$$F_{DD}(\alpha,\beta) = \sum_{m \text{ even}, n \text{ odd}}^{m+n \leq N} c_{mn}\alpha^m \beta^n$$

- SGPD is a Radon transform of the DD:

$$H_S(x,\xi) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \delta(x-\beta-\alpha\xi) F_{DD}(\alpha,\beta)$$

- This guarantees the SGPDs satisfy polynomiality

Calculating Shadow GPDs

- SGPD conditions give a set of equations that can be solved for the unknowns (c_{mn})
  - For a given N there are more unknown coefficients than constraining equations:
    - Assign random values to enough randomly selected coefficients to reduce the number of unknowns so that the equations can be solved
  - Use $N = 27$
  - SGPDs give zero contribution to the CFF at next-to-leading order

Calculating Shadow GPDs

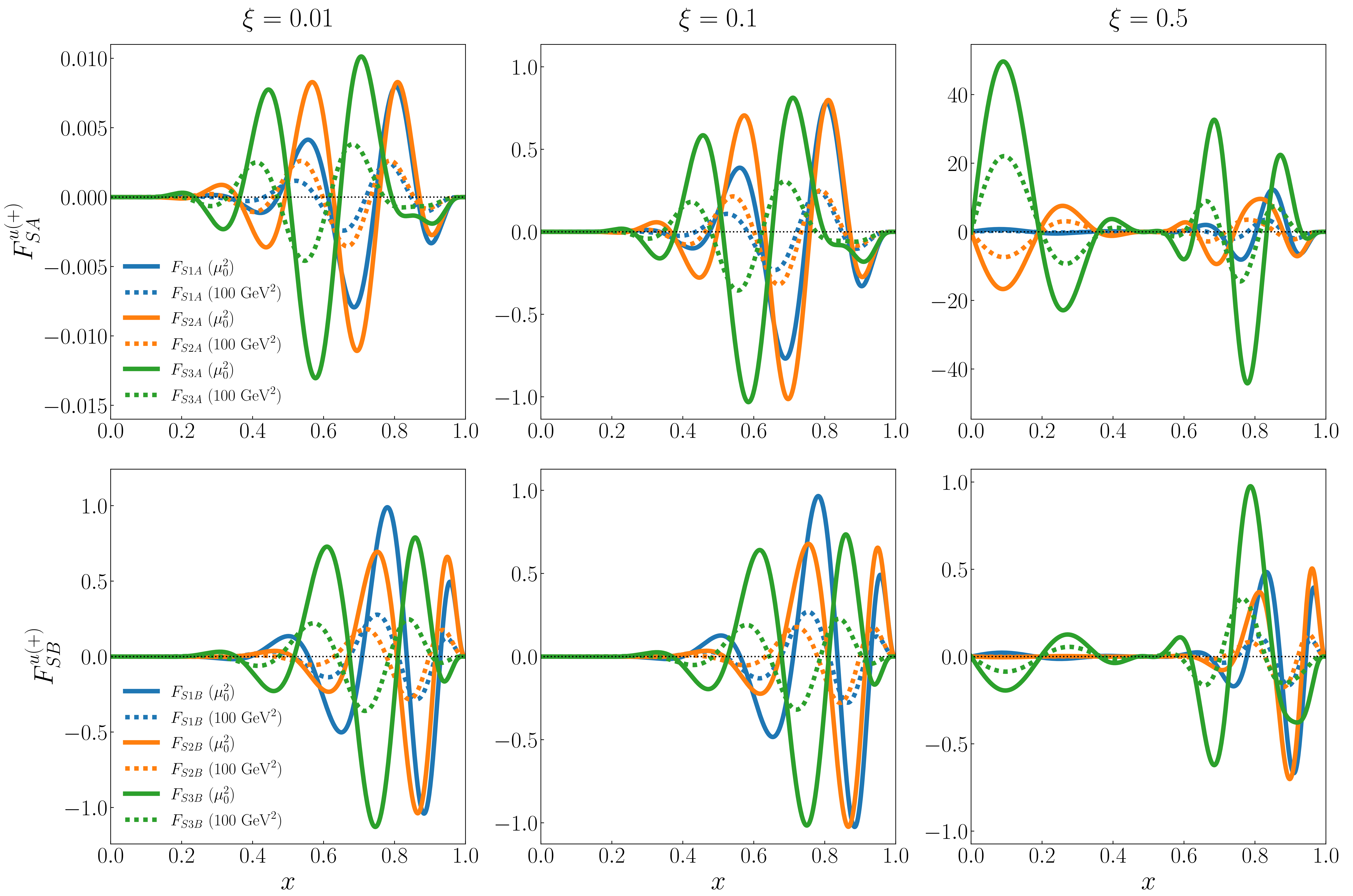
- For SGPDs derived this way we can impose the forward limit in two ways:
  - Type A:
    - Consistent with Bertone, et. al. Phys.Rev.D 103 (2021) 11, 114019:

$$H_S^{u(+)}(x,0;\mu_0)) = 0$$

  - Type B:
    - Could also multiply F_{DD} by a function of t that is zero when $t = 0$

$$H_S^{u(+)}(x,0;\mu_0)) \neq 0$$

Example Shadow GPDs



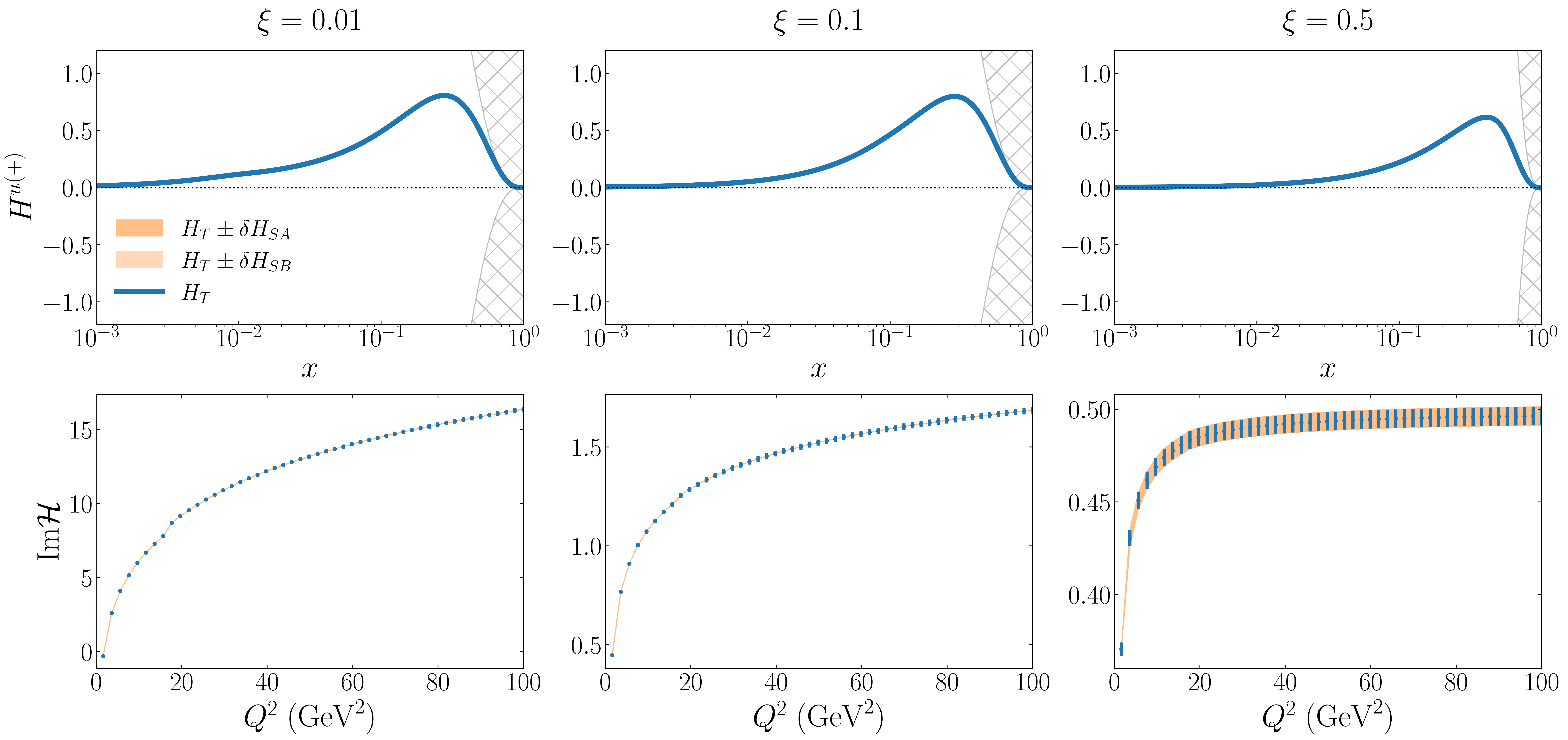
Exploring SGPDs and Evolution

- Use Monte Carlo sampling to generate replicas that are linear combinations of three SGPDs:

$$H^{u(+)}(x,\xi;\mu^2,\lambda) = H_T^{u(+)}(x,\xi;\mu^2) + \lambda_1 H_{S1}^{u(+)}(x,\xi;\mu^2) + \lambda_2 H_{S2}^{u(+)}(x,\xi;\mu^2) + \lambda_3 H_{S3}^{u(+)}(x,\xi;\mu^2)$$

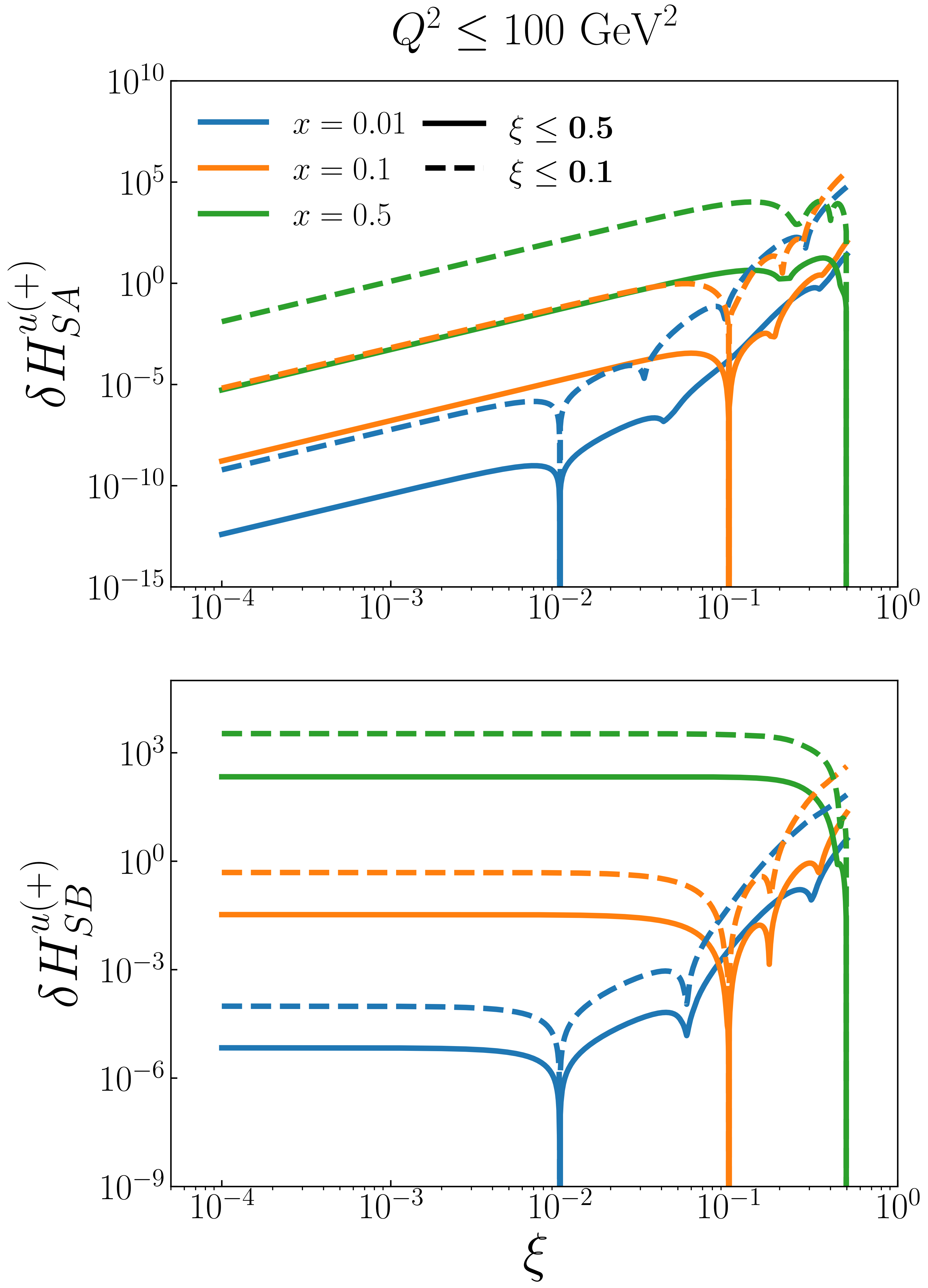
  - Randomly select the scaling factors until we get 10000 replicas that all give CFFs that are within 1% of the simulated data from the model.
  - Plot the region δH_S: Outer boundary of all 10000 replicas

Exploring SGPDs and Evolution



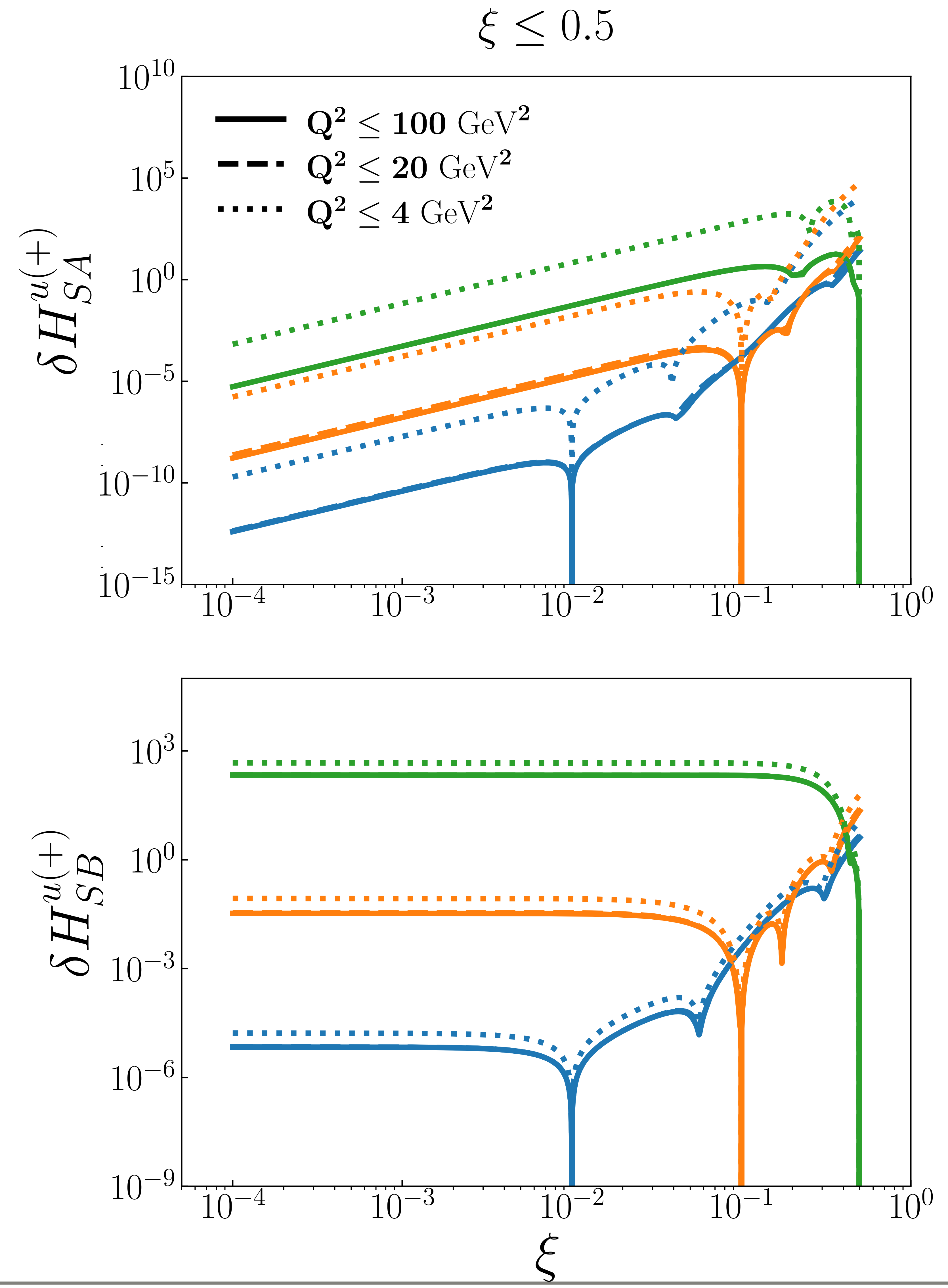
Exploring SGPDs and Evolution

- Inclusion of higher ξ data leads to better constraint of SGPDs at smaller ξ
  - True over the full range of x when $H_S^{u(+)}(x,0;\mu_0)) = 0$
  - Only true for low x when $H_S^{u(+)}(x,0;\mu_0)) \neq 0$

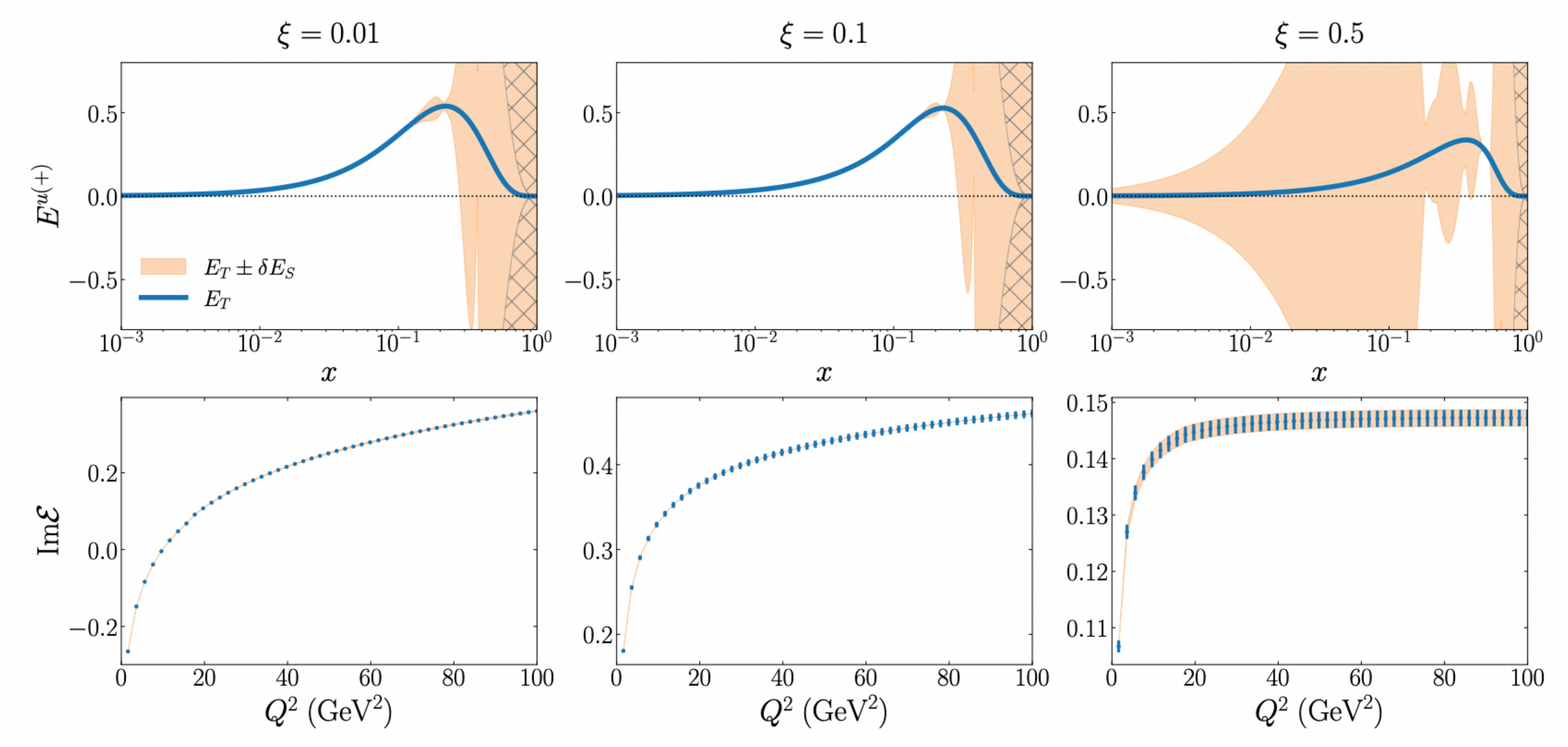


Exploring SGPDs and Evolution

* Some range of Q^2 is necessary for evolution to constrain the SGPDs but a large range is not as necessary as having large ξ data.



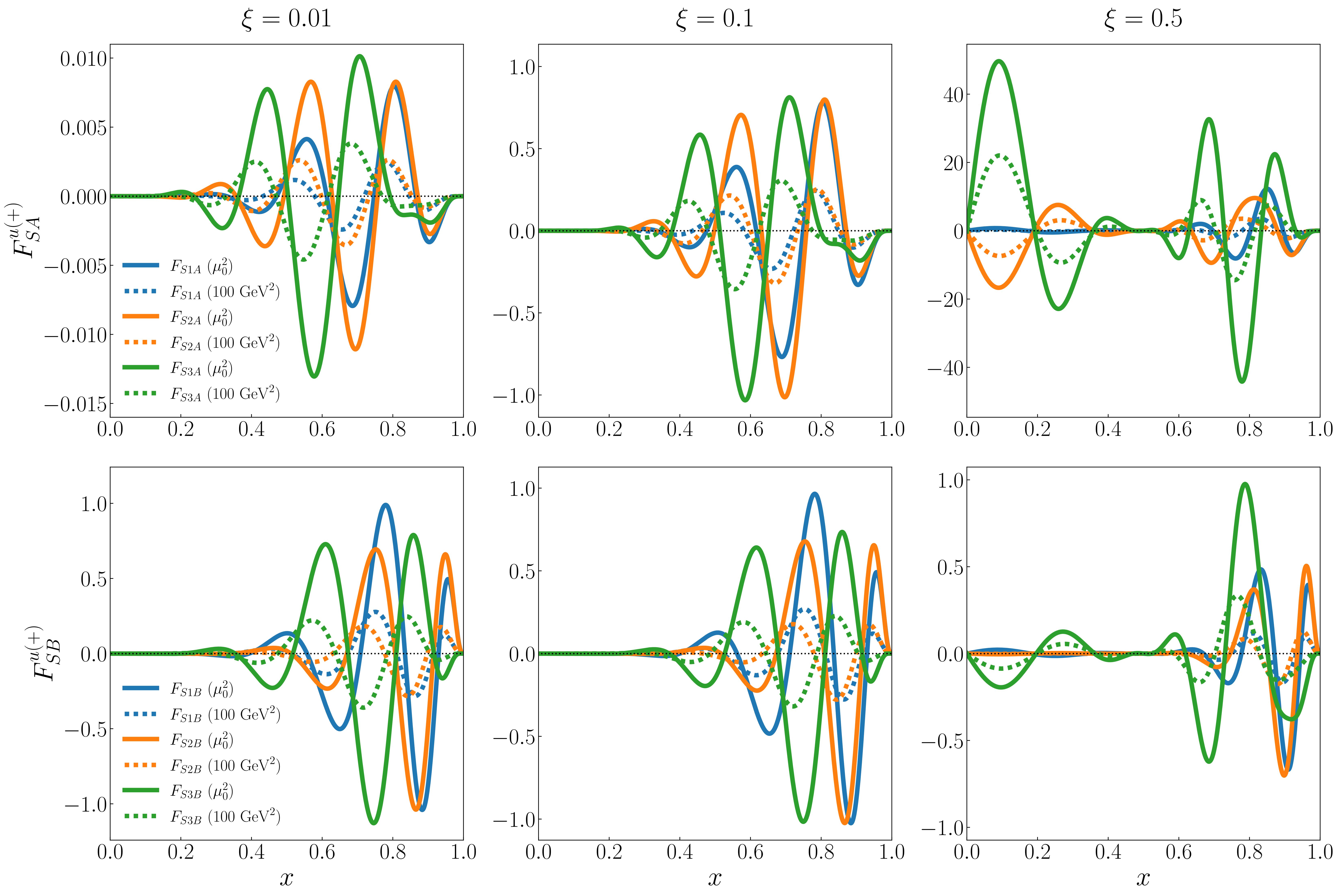
Exploring SGPDs and Evolution



Exploring SGPDs and Evolution

- The trend of larger ξ data leading to better constrained SGPDs at smaller ξ is a direct result of the ξ dependence of the SGPDs
  - Independent of the model used as a proxy for the “true” GPD
  - Independent of the chosen uncertainty

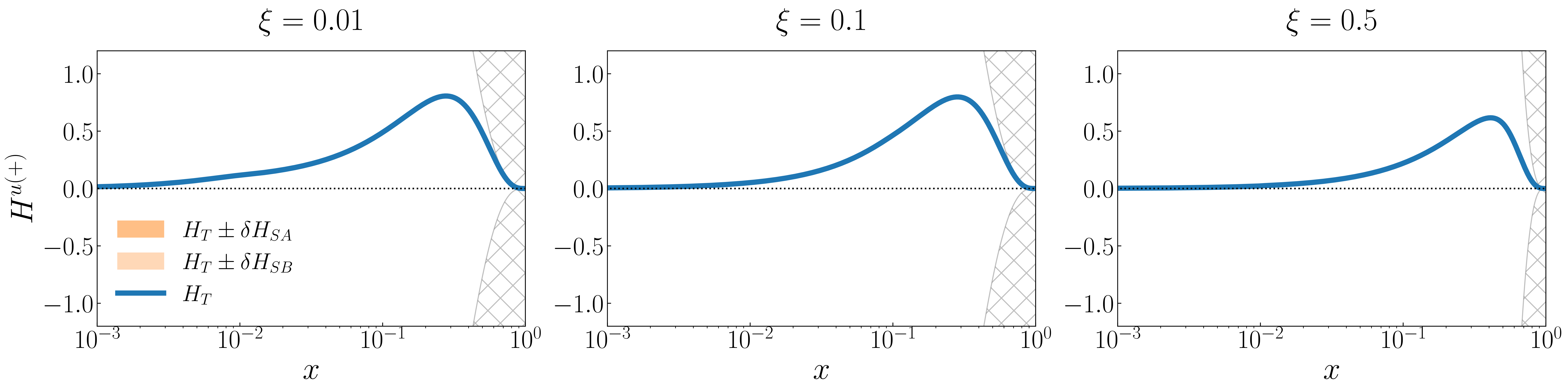
Example Shadow GPDs



Positivity constraints and SGPDs

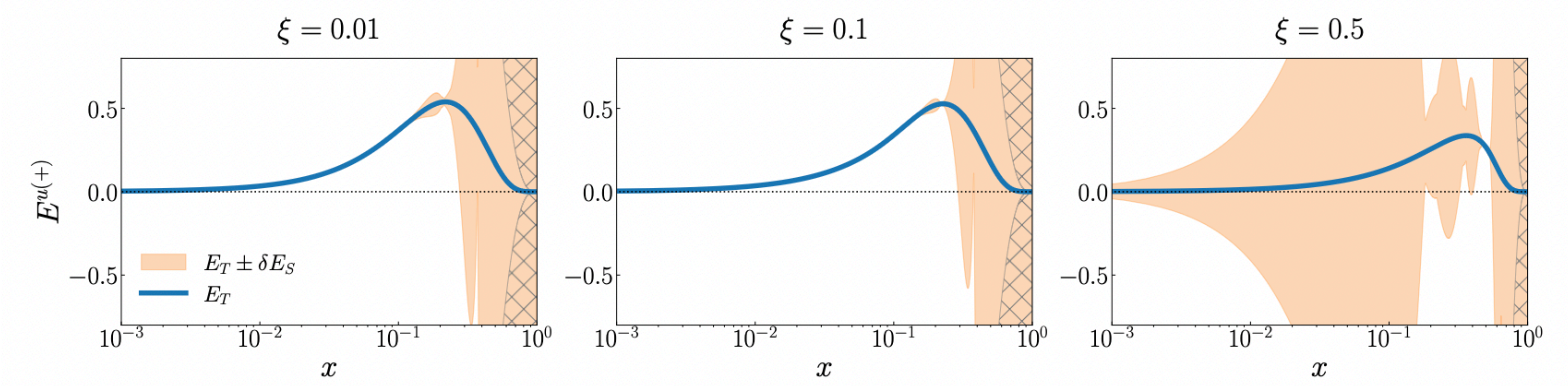
- Positivity constraints can help to better constrain SGPDs (Dutrieux, et al., Eur. Phys. J. C 82, 252 (2022))
- Care must be taken since these inequalities can be violated by regularization and renormalization effects in QCD (Collins, Rogers, Sato, Phys. Rev. D 105, 076010 (2022))

Positivity constraints and SGPDs



$$\left|H^q(x,\xi,t;\mu^2)\right| \leqslant \sqrt{\left(1 - \frac{t_{\min}\,\xi^2}{t_{\min} - t}\right) \frac{q(x_{\rm in};\mu^2)\, q(x_{\rm out};\mu^2)}{1-\xi^2}},$$

Positivity constraints and SGPDs



$$\left|E^q(x,\xi,t;\mu^2)\right| \leqslant \frac{2M}{\sqrt{t_{\min}-t}}\sqrt{q(x_{\mathrm{in}};\mu^2)\,q(x_{\mathrm{out}};\mu^2)}.$$

SGPDs and Spin

- Ji sum rule:

$$2J^a(\mu^2) = A^a_{01}(0,\mu^2) + B^a_{01}(0,\mu^2) = \int_{-1}^{1} dx\, x\left[H^a(x,\xi,0;\mu^2) + E^a(x,\xi,0;\mu^2)\right]$$

- For H:

$$A^q_{01}(0) = \int_{-1}^{1} dxxH^q(x,0,0;\mu^2) = \int_{0}^{1} dxx(q(x;\mu^2) + \bar{q}(x;\mu^2))$$

$$A^g_{01}(0) = \int_{-1}^{1} dxxH^g(x,0,0;\mu^2) = 2\int_{0}^{1} dxxg(x;\mu^2)$$

  - Since this contribution can be determined from the PDFs, H SGPDs would not contribute.

- For E:

  - E SGPDs can contribute to the spin because the forward limit is not known

SGPDs and Spin

- Calculating the spin contributions:
  - H_T: $J^{u+} = 0.389$
  - E_T: $J^{u+} = 0.219$
  - δE_S: $J^{u+} = 0.009$
- The contribution of E SGPDs to the spin is ~4%.
- Knowledge of the forward limit of the E GPD from lattice would reduce the possible E SGPDs to those for which the forward limit gives zero.

SGPDs and Internal Stresses

- Internal stresses in the hadron are connected to C_1^a in the second moment of the GPD.
- This contribution comes from the D-term portion of the GPD
- The SGPDs explored here have no D-term and so would not affect internal stress calculations
- Dutrieux, et. al., Eur. Phys. J. C 81, 300 (2021):
    - Found different D-terms that fit data equally well (shadow D-terms)
    - Result in significantly different internal stresses

SGPDs and Tomography

- Transverse spatial distribution can be obtained from a transverse Fourier transform of the H GPD at $\xi = 0$.
  - Requires accurate knowledge of t-dependence at $\xi = 0$.
  - Not accessible experimentally.
  - Must extrapolate from t-dependence at non-zero ξ.
- Impact of Type A SGPDs would be minimal since they get smaller as $\xi \to 0$
- Impact of Type B SGPDs would be minimal at small x but could be substantial at large x
- Quantitative analysis of the impact SGPDs could have on tomography requires a thorough exploration of possible t-dependent SGPDs which we leave for future work.

Conclusions

- For the SGPDs that have been explored here:
  - Data at larger ξ leads to the SGPDs being better constrained at lower ξ at least in the range of low x
  - These findings are independent of the model used as the proxy for the “true” GPD.
  - Positivity constraints could help to constrain SGPDs even further.
- The contribution of H SGPDs to spin would be zero because of the forward limit constraint.
- The contribution of E SGPDs to spin is not negligible. Determination of the forward limit by other means such as lattice QCD could help reduce this.
- Though not explored in this work, shadow D-terms could have a significant impact on internal stress calculations (Dutrieux, et. al., Eur. Phys. J. C 81, 300 (2021))
- Quantitative analysis of the impact SGPDs could have on tomography requires a thorough exploration of possible t-dependent SGPDs which we leave for future work.

Conclusions

- The SGPDs explored are only a small sampling of all possible SGPDs:
  - Though we have not yet found examples of SGPDs that do not exhibit the behavior found in this work, we cannot generalize these results to all SGPDs
  - Data spanning a range of Q^2 at larger ξ is a necessary but possibly not sufficient condition for extracting GPDs from DVCS data.