Parton Distributions from Boosted Fields in the Coulomb Gauge

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Based on arXiv: 2306.14960 with Wei-Yang Liu and Yong Zhao

Jefferson Lab  Nov. 13, 2023
• The information of hadron structure are encoded in the parton distributions.

$W(x, b_T, k_T)$

Wigner Distributions

$\int d^2 b_T$

Transverse momentum distributions (TMDs)

$\int d^2 k_T$

Impact parameter distributions (IPDs)

$f(x, k_T)$

$f(x, b_T)$

$b_T \leftrightarrow \Delta$

$F(x, \xi, \Delta^2)$

Generalized Parton distributions (GPDs)

$\int d^2 k_T$

$\int d^2 b_T$

$f(x)$

Parton distribution functions (PDFs)

$\int dxx^{n-1}$

$A_{n,0}(t) + 4\xi A_{n,2}(t) + \ldots$

Form factors and (FFs) and generalized FFs (GFFs)
Parton distributions from experiments

Ongoing and new experiments provide great opportunity for:

- **Precision study** of flavor and spin structure of nucleon.
- The **three-dimensional** structure of hadrons in coordinate and momentum spaces.
Parton distributions from experiments

Nucleon transversity

\[ \delta d \]

- JAMDiFF (no LQCD)
- JAM3D* (no LQCD)
- JAMDiFF (w/ LQCD)
- JAM3D* (w/ LQCD)

\[ \mu^2 = 4 \text{ GeV}^2 \]

Add lat.

- PNDME (2018)
- ETMC (2020)
- Radici, Bacchetta (2018)

LQCD

C. Cocuzza et al., (JAM), arXiv: 2306.12998

Nucleon unpolarized GPD

GPD \( H_{R-d} \) at \( \xi = 1/3 \) and \( -t = 0.69 \text{ GeV}^2 \) tuned in DA–like region

- Original value
- Tuned with DA terms
- Lat. ref. value

Add Lat. constrain

Experimental constrain only

- Y. Guo, et al., JHEP 05 (2023) 150
Parton distributions from experiments

Nucleon transversity

Nucleon unpolarized GPD

Additional knowledge from lattice QCD is essential.
Parton distributions from lattice

- PDF from light-cone correlations

\[ \langle p_f | \bar{q}(-\frac{z^-}{2})\gamma^\mu \mathcal{W}(-\frac{z^-}{2}, \frac{z^-}{2})q(\frac{z^-}{2}) | p_i \rangle \]

- Moments from Local operator

\[ \bar{q} \gamma \{\mu_1 iD^\mu_2 \ldots iD^\mu_n \} q \]

- Limited up to \( \langle x^3 \rangle \) due to signal decay and power-divergent mixing under renormalization.

- Cannot be simulated on the Euclidean lattice

- Since the 1980s
Large momentum effective theory

The quasi distribution from equal-time correlators,

\[ \tilde{q}(x, P_z, \mu) = \int \frac{dz}{4\pi} e^{-ixP_zz} \langle P | \tilde{q}(0) \Gamma^{\mathcal{W}}(0,z)q(z) | P \rangle \]

\[ z + ct = 0, \quad z - ct \neq 0 \]

\[ t = 0, \quad z \neq 0 \]

\[ \langle P | \tilde{q}(0) \Gamma^{\mathcal{W}}(0,z^-)q(z^-) | P \rangle \]

\[ \langle P | \tilde{q}(0) \Gamma^{\mathcal{W}}(0,z)q(z) | P \rangle \]
Large momentum effective theory

Large $P_z$ expansion of quasi distribution:

$$\tilde{q}(x, P_z, \mu) = \int \frac{dy}{|y|} C(\xi, \frac{\mu}{yP_z}) q(y, \mu) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{x^2P_z^2} + \frac{\Lambda_{QCD}^2}{(1-x)^2P_z^2}\right)$$

Similar to DIS, DY, …, LaMET has direct sensitivity to the local $x \in [x_{\text{min}}, x_{\text{max}}]$ dependence of light-cone PDF $q(x, \mu)$.

$$\xi = \frac{x}{y}$$

$$\frac{d\sigma}{dQ^2} = \sigma_0 \sum_{a,b} \int \frac{dx_1}{x_1} \int \frac{dx_2}{x_2} q^A_a(x_1, \mu^2) q^B_b(x_2, \mu^2) \omega_{ab}(z, \frac{\mu}{Q})$$
Short distance factorization

- SDF/OPE in coordinate space: Ioffe-time pseudo distributions

\[ \tilde{h}(z, P_z, \mu) = \tilde{h}(z^2, \lambda, \mu) \]

\[ = \sum_{n=0}^{\infty} \frac{(-izP)^n}{n!} C_n(z^2\mu^2) \langle x^n \rangle(\mu) + \mathcal{O}(z^2 \Lambda_{\text{QCD}}^2) \]

\[ \lambda = zP_z \]

\[ = \int_{-1}^{1} d\alpha \mathcal{C}(\alpha, \mu^2z^2) \int_{-1}^{1} dy e^{-iy\lambda} q(y, \mu) + \mathcal{O}(z^2 \Lambda_{\text{QCD}}^2) \]

- Extract moments \( \langle x^n \rangle \) without power divergent mixing.
- The information is limited by the range of finite \( \lambda = zP_z \), large \( P_z \) essential.


\[ t = 0, \quad z \neq 0 \]
Parton distributions from lattice

Pion valence quark PDF from LaMET

- Agree with the global analysis at the moderate $x$ region.
- End-point region can be improved by larger hadron momentum.

Moments of GPDs from SDF

- Agree with traditional calculations using local operators (ETMC’11).
- Can systematically extract high moments with large momentum.
Systematic control in LaMET

\[ \tilde{q}(x, P_z, \mu) = \int \frac{dy}{|y|} C(\xi, \frac{\mu}{yP_z}) q(y, \mu) + \mathcal{O}(\frac{\Lambda_{QCD}^2}{x^2P_z^2}, \frac{\Lambda_{QCD}^2}{(1 - x)^2P_z^2}) \]

\[ \xi = \frac{x}{y} \]

\[ \langle P | \tilde{q}(0) \Gamma W(0, z) q(z) | P \rangle \]

- Bare matrix elements
- Renormalization
- FT.
- LaMET matching
- Quasi-PDF
- \( x \)-dependent PDFs
Renormalization

The operator can be multiplicatively renormalized:

\[ \left[ \bar{\psi}(0) \Gamma W_{\tilde{z}}(0, z) \psi(z) \right]_B = e^{-\delta m(z)} \left[ \bar{\psi}(0) \Gamma W_{\tilde{z}}(0, z) \psi(z) \right]_R \]

\[ \delta m(a) = m_1(a)/a + m_0 \]

Wilson-line self energy + renomalon ambiguity

\[ h^R(z, P_z) = e^{\delta m|z-z_s|} \frac{\tilde{h}(z, P_z, a)}{Z_X(z_s, P_0^z, a)} \]

- Subtract the linear divergence from Wilson-line self energy:
  - Self-renormalization.
  - Static potential.
  - ...

- Subtracted from the lattice Wilson line self energy:

\[ h^R, \text{lattice}(z, P_z) \rightarrow h^R, \overline{\text{MS}}(z, P_z) \]

- J. Green, K. Jansen and F. Steffens, PRL121 (2018)
Renormalization

The operator can be multiplicatively renormalized:

\[ \begin{align*}
[\bar{\psi}(0)\Gamma W_{\xi}(0,z)\psi(z)]_B &= e^{-\delta m|z|}Z(a)[\bar{\psi}(0)\Gamma W_{\xi}(0,z)\psi(z)]_R \\
\delta m(a) &= m_{-1}(a)/a + m_0
\end{align*} \]

The number of diagrams grows, \( r_n \sim n! \): diverge in perturbation theory at any \( \alpha_s \).

\[ \delta m = \frac{1}{a} \sum \alpha_s^{n+1}(a)r_n \]

- Subtract the linear divergence from Wilson-line self energy:
  - Self-renormalization.
  - Static potential.
  - ...

\[ h^R(z, P_z) = e^{\delta m|z-z_s|} \frac{\tilde{h}(z, P_z, a)}{Z_X(z_s, P_0, a)} \]

Wilson-line self energy + renormalon ambiguity

linear power correction \( \mathcal{O}(\Lambda_{\text{QCD}}/P_z) \) in qPDF

- J. Green, K. Jansen and F. Steffens, PRL121 (2018)
Renormalization

- J. Green, K. Jansen and F. Steffens, PRL 121 (2018)

The operator can be multiplicatively renormalized:

\[ [\overline{\psi}(0)\Gamma W(z, 0, z)]_B = e^{-\delta m |z|} Z(a) [\overline{\psi}(0)\Gamma W(z, 0, z)]_R \]

\[ \delta m(a) = m_{-1}(a)/a + m_0 \]

Wilson-line self energy + renormalon ambiguity

- Subtract the linear divergence from Wilson-line self energy:
  - Self-renormalization.
  - Static potential.
  - ...

- Subtract the renormalon ambiguity from a resummation prescription:

\[ h^R, \text{lattice} (z, P_z = 0) \]
\[ = C_{0}^{\overline{\text{MS}}, \text{LRR}} (z, P_z = 0) e^{-m_0 |z|} \]

LRR: leading renormalon resummation

\[ O(\Lambda_{QCD}/P_z) + O(\Lambda_{QCD}^2/P_z^2) \]

LaMET matching

$$\lim_{\xi \to 1} C \left( \xi, \frac{\mu}{P^z} \right) = 1 + \frac{\alpha_s(\mu) C_F}{2\pi} \left[ 2 \frac{\ln |1 - \xi|}{|1 - \xi|} - \frac{2}{1 - \xi} \ln \frac{\mu^2}{(2xP_z)^2} \theta(1 - \xi) + \frac{3}{2 |1 - \xi|} \right] + \mathcal{O}(\alpha_s^2)$$

$$\xi = x/y$$

- **Next-to-next-to-leading order (NNLO) kernel.**

- **Resummation of DGLAP (RG) logs $\ln(\mu^2/(2xP_z)^2)$**.
  - XG, Y. Zhao et al. (BNL), Phys.Rev.D 103 (2021), 094504

- **Resummation of large-$x$ (threshold) logarithms $\ln |1 - \xi|/|1 - \xi|$**.
  - XG, Y. Zhao et al. (BNL), Phys.Rev.D 103 (2021), 094504
  - X. Ji, et al., JHEP 08 (2023) 037

- **Resummation/subtraction of leading renormalon**.
PDFs from LaMET with improved precision

- Pion valence quark PDF
  \[ P_z = 1.94 \text{ GeV} \]

- Nucleon transversity PDF
  \[ P_z = 1.53 \text{ GeV} \]


- Leading renormalon resummation (LRR): remove the linear power correction \( \mathcal{O}(\Lambda_{\text{QCD}}/P_z) \), reduces the (theoretical) uncertainty.

- DGLAP (RGR): enhances the accuracy of the scale evolution at moderate \( x \), also indicate the perturbative matching is unreliable at small \( x \) \( (2xP_z \sim \Lambda_{\text{QCD}}) \).
Universality in LaMET

- The non-local operator in gauge theory:
  \[ \bar{\psi}^*(-\frac{z}{2}) \Gamma \psi^*(\frac{z}{2}) \]
  with \( \psi^*(z) = \psi(z)e^{iC(z)} \) and \( C(z) \) is a linear function of \( A_\mu \).

- In DIS, the physical quark \( \psi(z)e^{iC(z)} \) represents a gauge-invariant object with a gauge link extended to infinity along the light-cone direction:

\[ P \exp\left[-ig \int_{-z^-/2}^{z^-/2} dz\ A^+(z) \right] \geq \bar{\psi}(\frac{z^-}{2}) \psi(-\frac{z^-}{2}) \]

Gauge invariant

\[ A^+ = 0 \text{ gauge} \]
Universality in LaMET

Gauge invariant (GI) qPDF

\[ P \text{ exp} \left[ - i g \int_{-z/2}^{z/2} dz \, A^z(z) \right] \]

\[ \tilde{\psi}(-\frac{z}{2}) \Gamma W(-\frac{z}{2}, \frac{z}{2}) \psi(\frac{z}{2}) \]

Light-cone PDF

\[ \tilde{\psi}(\frac{z^-}{2}) \]

\[ \tilde{\psi}(-\frac{z^-}{2}) \]

\[ P \rightarrow \infty \text{ boost} \]

\[ \tilde{\psi}(-\frac{z^-}{2}) \Gamma W(-\frac{z^-}{2}, \frac{z^-}{2}) \psi(\frac{z^-}{2}) \]

or

\[ \tilde{\psi}(-\frac{z^-}{2}) \Gamma \psi(\frac{z^-}{2}) \bigg|_{A^z=0} \]
Universality in LaMET

Physical gauge like: $A^z, A^0$,
Coulomb gauge $\vec{\nabla} \cdot \vec{A} = 0$

$P \rightarrow \infty$ boost

LaMET

$\bar{\psi}(\frac{-z^-}{2})\Gamma W(\frac{-z^-}{2}, \frac{z^-}{2})\psi(\frac{z^-}{2})$

or

$\bar{\psi}(\frac{-z^-}{2})\Gamma \psi(\frac{z^-}{2}) |_{A^+ = 0}$
LaMET in Coulomb gauge

- Quasi-PDF in Coulomb gauge (CG):

\[ \tilde{f}(x, P^z, \mu) = P^z \int_{-\infty}^{\infty} \frac{dz}{2\pi} e^{ixP^zz} \tilde{h}(z, P^z, \mu) \]

\[ \tilde{h}(z, P^z, \mu) = \frac{1}{2P^t} \langle P | \overline{\psi}(z) \gamma^t \psi(0) | \vec{\nabla} \cdot A = 0 | P \rangle \]

- First proposed in the lattice calculation of gluon helicity

\[ \Delta G = \langle P_\infty | (E \times A)^3 | \vec{\nabla} \cdot A = 0 | P_\infty \rangle \]

- X. Ji, J.-H. Zhang and Y. Zhao, PRL 111 (2013);
- Y. Hatta, X. Ji, and Y. Zhao, PRD 89 (2014);
- X. Ji, J.-H. Zhang and Y. Zhao, PLB 743 (2015);
- Y.-B. Yang, R. Sufian, Y. Zhao, et al. PRL 118 (2017)
LaMET factorization

\[ \tilde{q}(x, P_z, \mu) = \int \frac{dy}{|y|} C(\xi, \frac{\mu}{yP_z}) q(y, \mu) + \mathcal{O}(\frac{\Lambda_{QCD}^2}{x^2 P_z^2}, \frac{\Lambda_{QCD}^2}{(1-x)^2 P_z^2}) \]

\[ \xi = x/y \]

- One-loop matching:

\[ C(\xi, \frac{\mu}{p^z}) = \delta(\xi - 1) + \frac{\alpha_s C_F}{2\pi} C^{(1)}(\xi, \frac{\mu}{p^z}) + \mathcal{O}(\alpha_s^2) \]

\[ C^{(1)}(\xi, \frac{\mu}{p^z}) = C^{(1)}_{\text{ratio}}(\xi, \frac{\mu}{p^z}) + \frac{1}{2|1-\xi|} + \delta(1-\xi) \left[ -\frac{1}{2} \ln \frac{\mu^2}{4p_z^2} + \frac{1}{2} - \int_0^1 d\xi' \frac{1}{1-\xi'} \right] \]

\[ C^{(1)}_{\text{ratio}}(\xi, \frac{\mu}{p^z}) = \left[ P_{qq}(\xi) \ln \frac{4p_z^2}{\mu^2} + \xi - 1 \right]_{[0,1]}^{[0,1]} + \left\{ P_{qq}(\xi) \left[ \text{sgn}(\xi) \ln |\xi| + \text{sgn}(1-\xi) \ln |1-\xi| \right] \right\}_{(+1)} \]

\[ + \text{sgn}(\xi) + \frac{3\xi - 1}{\xi - 1} \tan^{-1} \frac{\sqrt{1-2\xi} / |\xi|}{\sqrt{1-2\xi}} - \frac{3}{2|1-\xi|} \right\}_{(+1)}^{(-\infty,\infty)} \]

- Collinear divergences are identical between CG qPDF and light-cone PDF.

- Satisfies particle number conservation.
Short-distance factorization

\[ \tilde{h}(z, P_z, \mu) = \int_{-1}^{1} d\alpha \mathcal{C}(\alpha, \mu^2 z^2) \int_{-1}^{1} dy e^{-iy\lambda} q(y, \mu) + \mathcal{O}(z^2 \Lambda_{QCD}^2) \]

- **One-loop matching:**

\[ \mathcal{C}(u, \frac{\mu}{p^z}) = \delta(u - 1) + \frac{\alpha_s C_F}{2\pi} \mathcal{C}^{(1)}(u, \frac{\mu}{p^z}) + \mathcal{O}(\alpha_s^2) \]

\[ \mathcal{C}^{(1)}(u, z^2 \mu^2) = \mathcal{C}^{(1)}_{\text{ratio}}(u, z^2 \mu^2) + \frac{1}{2} \delta(1 - u) \left( 1 - \ln \frac{z^2 \mu^2 e^{2\gamma_E}}{4} \right) \]

\[ \mathcal{C}^{(1)}_{\text{ratio}}(u, z^2 \mu^2) = \left[ -P_{qq}(u) \ln \frac{z^2 \mu^2 e^{2\gamma_E}}{4} - \frac{4 \ln(1 - u)}{1 - u} + 1 - u \right] \quad [0,1] \]

\[ + \left[ \frac{3u - 1}{u - 1} \tan^{-1} \left( \frac{\sqrt{1 - 2u}}{|u|} \right) - \frac{3}{|1 - u|} \right] \quad (-\infty, \infty) \]

\[ + \quad (1) \]

Lattice setup

- Clover-fermion on 2+1f HISQ gauge ensembles, 1-step HYP smearing.
- $48^3 \times 64$, $a = 0.06$ fm, $m_\pi = 300$ MeV.
- Momentum $\vec{p} = \frac{2\pi}{L_s} \vec{n}$.
- 109 configurations & AMA.

| $|\vec{p}|$ (GeV) | $\vec{n}$ | $\vec{k}$ | $t_s/a$ | (#ex, #sl) |
|------------------|-----------|-----------|---------|------------|
| 0                | (0,0,0)   | (0,0,0)   | 8,10,12 | (1, 16)    |
| 1.72             | (0,0,4)   | (0,0,3)   | 8       | (1, 32)    |
|                  |           |           | 10      | (3, 96)    |
|                  |           |           | 12      | (8, 256)   |
| 2.15             | (0,0,5)   | (0,0,3)   | 8       | (2, 64)    |
|                  |           |           | 10      | (8, 256)   |
|                  |           |           | 12      | (8, 256)   |
| 2.24             | (3,3,3)   | (2,2,2)   | 8       | (2, 64)    |
|                  |           |           | 10      | (8, 256)   |
|                  |           |           | 12      | (8, 256)   |

- In CG without Wilson line, the quark separation can be off-axis.
The contraction of both GI and CG cases can be done simultaneously using the same propagators in Coulomb gauge.
Bare matrix elements: data precision

\[ R(t_s, \tau) = \frac{\langle N(t_s) \sigma^f_\Gamma(\vec{z}, \tau) N(0) \rangle}{\langle N(t_s) N(0) \rangle} \]

\[ t_s \to \infty \implies \tilde{h}^B(\vec{z}, P_z) \]

- Off-axis momentum shows some signal improvement.
Bare matrix elements: Rotational symmetry

- CG
- GI: Zig-zag

- On-axis
  \( \vec{n} = (0,0,n_3) \)
  \( \vec{z} = (0,0,z_3) \)

- Off-axis
  \( \vec{n} = (n_1, n_2, n_3) \)
  \( \vec{z} = (z_1, z_2, z_3) \)

CG matrix elements precisely preserve the 3D rotational symmetry, which is broken for GI with a zig-zagged Wilson line.

- B. Musch et al., PRD 83 (2011).

\[ \vec{n} = (0,0,5) \quad \vec{n} = (3,3,3) \]
**Bare matrix elements: Renormalizability**

- The lattice quark propagator in Coulomb gauge are multiplicatively renormalizable.
- No Wilson-line and linear divergence renormalization needed.

- D. Zwanziger, NPB 518 (1998);
- Baulieu and Zwanziger, NPB 548 (1999);
- A. Niegawa, PRD 74 (2006);

**Overall renormalization**

\[
\begin{aligned}
\frac{\bar{\psi}(0) \Gamma \psi(z)}{\bar{\psi}(z_s, a)} &= \frac{Z_\psi(a)}{Z_\psi(z_s, a)} \frac{\bar{\psi}(0) \Gamma \psi(z)}{\bar{\psi}(0) \Gamma \psi(z_s)}
\end{aligned}
\]

Another lattice is used to check the renormalization:

64\(^3\) × 64, \(a = 0.04\) fm, \(m_\pi = 300\) MeV.

**Excellent continuum limit!**
Renormalized matrix elements in short distance

\[ \mathcal{M}(z, P^z, a) = \frac{\tilde{h}(z, P^z, a)}{\tilde{h}(z, 0, a)} \cdot \frac{\tilde{h}(0, 0, a)}{\tilde{h}(0, P^z, a)} \]


The prediction from GI agree well with the CG matrix elements:

\[ qPDF_{GI} \rightarrow PDF \rightarrow qPDF_{CG} \]
\(x\)-dependent PDF: hybrid renormalization

- No linear divergence and renormalon need to be removed in CG case.
- Better controlled long-range precision in CG case.

\[
|z| \leq z_s, \quad \frac{\tilde{h}(z, P_z, a)}{\tilde{h}(z, 0, a)}
\]

\[
|z| > z_s, \quad e^{(\delta m(a) + \bar{m}_0)|z - z_s|}
\]

The quasi-PDF are different at finite momentum between CG and GI.

The PDF after matching show good agreement beyond $x > 0.2$, demonstrating the universality in LaMET.
\( x \)-dependent PDF: CG results

- LL: leading logarithm
- DGLAP resummation:

\[
C(\xi, \frac{\mu}{P_z}) \sim \left( \frac{\alpha_s(2xP_z)}{\alpha_s(\mu)} \right)^{\frac{\gamma_n}{\beta_0}}
\]

- Agree with global fits from JAM and xFitter for \( x \gtrsim 0.2 \) within the errors.
- Precision can be considerably improved with more statistics in the future.
### Comparison between GI and CG quasi-PDFs

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<th>Power corrections</th>
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<td>$(0,0,n_z)$</td>
<td>Linear divergence + vertex and wave function</td>
<td>$\frac{\Lambda_{QCD}^2}{P_z^2}$ w. renormalon subtraction</td>
<td>N/A</td>
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<td>Available up to NNLO</td>
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<tr>
<td><strong>Coulomb gauge (CG)</strong></td>
<td>$(n_x, n_y, n_z)$</td>
<td>Wave function</td>
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<td>May affect long range region*</td>
<td>3D rotational symmetry</td>
<td>May be harder to go beyond NLO</td>
</tr>
</tbody>
</table>

*: Currently negligible otherwise the errors will not reduce as the number of configuration increasing.
Summary

- We demonstrate the universality in LaMET through the equivalence of CG and GI quasi-PDF methods at NLO.

- The CG correlations have the advantages of access to larger off-axis momenta, absence of linear divergence, and enhanced long-range precision.

- It is almost free to compute the GI and CG matrix elements simultaneously.

- The CG methods could have broader use in the future particularly in the TMD physics, for which the Wilson-line geometry and renormalization can be significantly simplified.

Factorization of quasi-TMDs in CG

- Y. Zhao, arXiv: 2311.01391

Thanks for your attention!