Parton Distributions from Boosted Fields in the Coulomb Gauge

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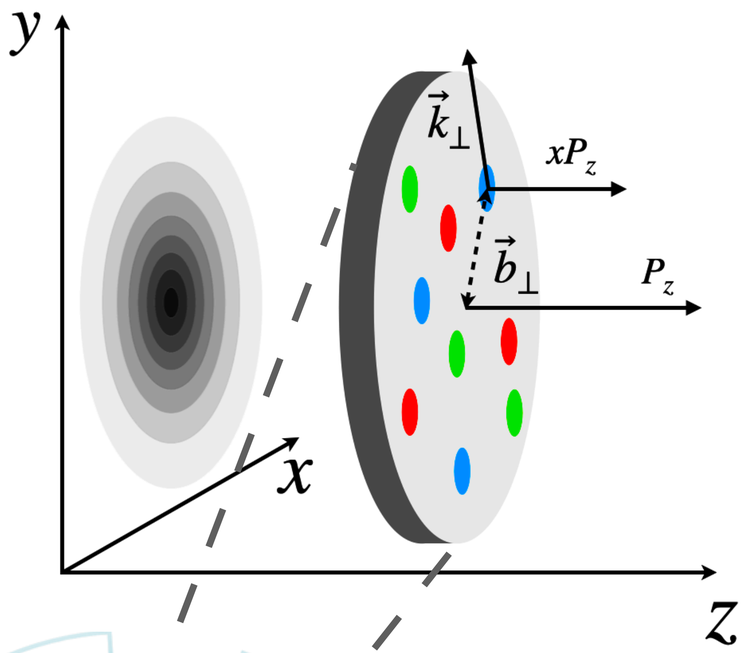
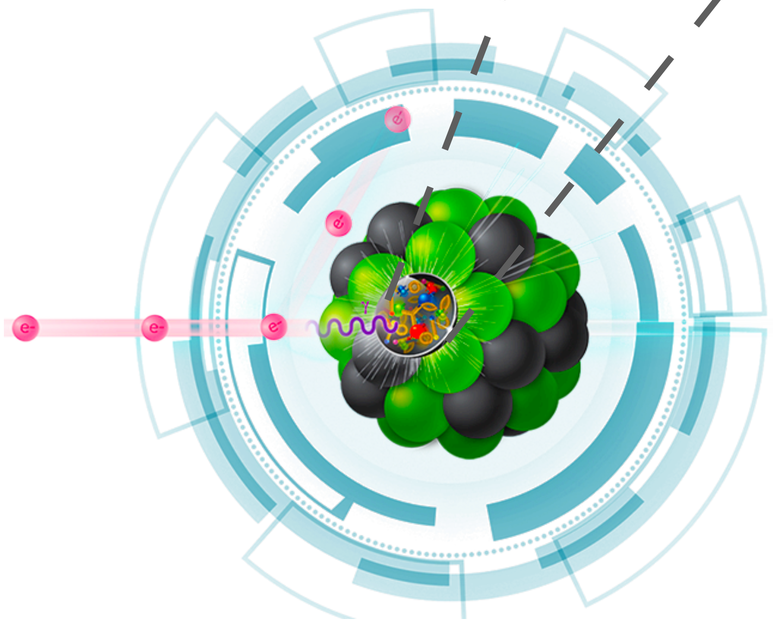
Based on arXiv: 2306.14960 with Wei-Yang Liu and Yong Zhao

Jefferson Lab Nov. 13, 2023

2 Hadron structure





- **The information of hadron structure are encoded in the parton distributions.**

$W(x, b_T, k_T)$

Wigner Distributions

$\int d^2b_T$ $\int d^2k_T$

$f(x, k_T)$ $f(x, b_T)$ $b_T \leftrightarrow \Delta$ $F(x, \xi, \Delta^2)$

Transverse momentum distributions (**TMDs**)

Impact parameter distributions (**IPDs**)

Generalized Parton distributions (**GPDs**)

$\int d^2k_T$ $\int d^2b_T$ $\int dx x^{n-1}$

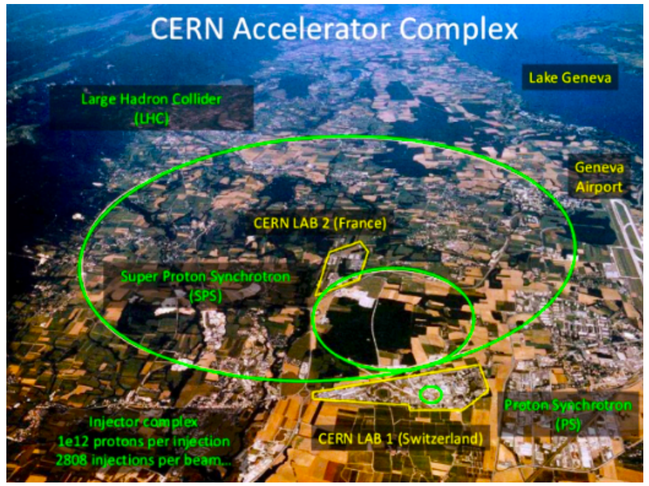
$f(x)$

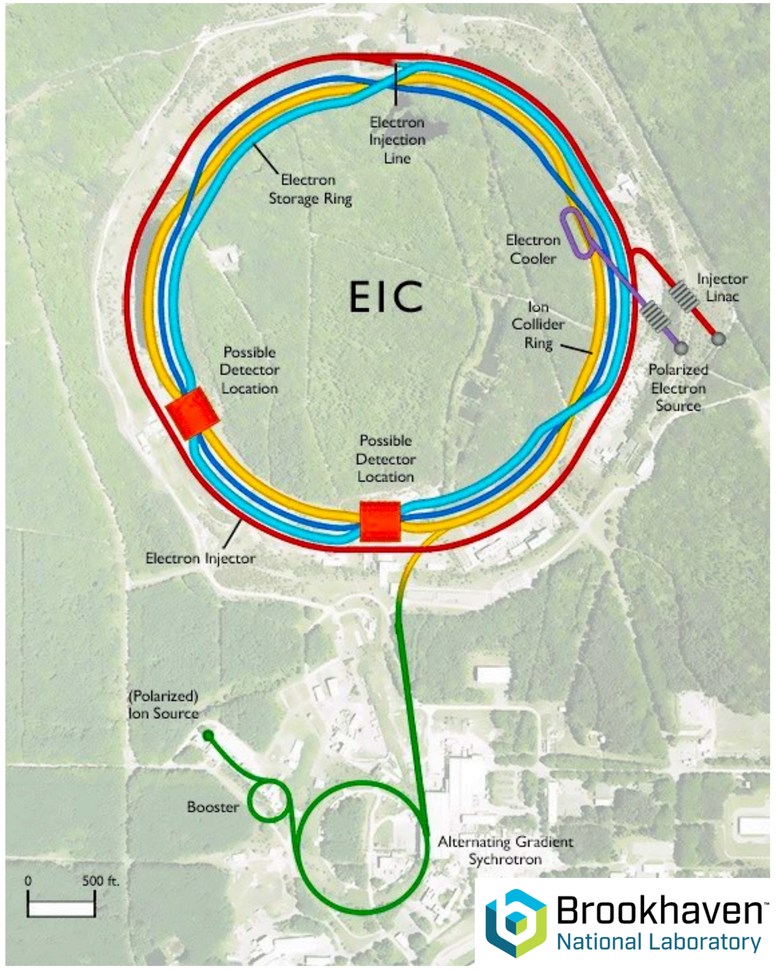
$A_{n,0}(t) + 4\xi A_{n,2}(t) + \ldots$

Parton distribution functions (**PDFs**)

Form factors and (**FFs**) and generalized FFs (**GFFs**)

3 Parton distributions from experiments



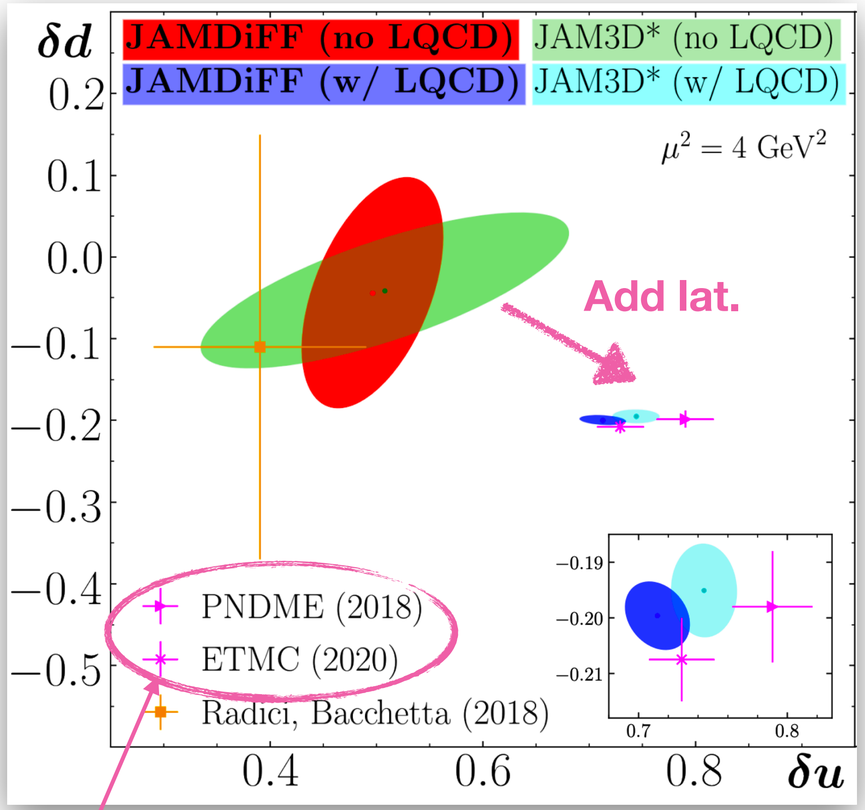


Ongoing and new experiments provide great opportunity for:

- Precision study of flavor and spin structure of nucleon.
- The three-dimensional structure of hadrons in coordinate and momentum spaces.

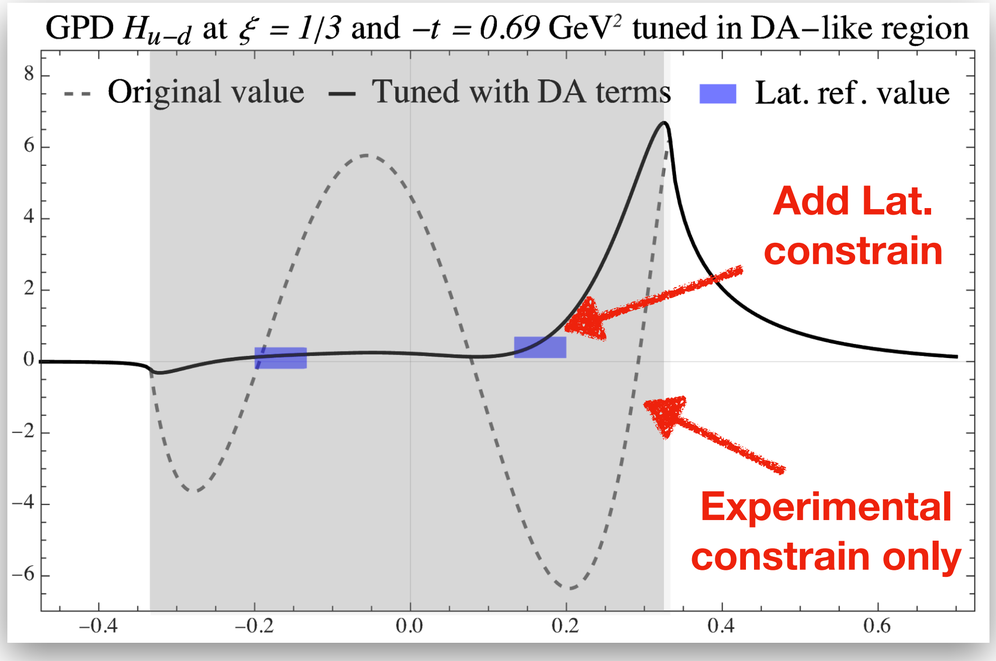
Parton distributions from experiments

Nucleon transversity



C. Cocuzza et al., (JAM), arXiv: 2306.12998

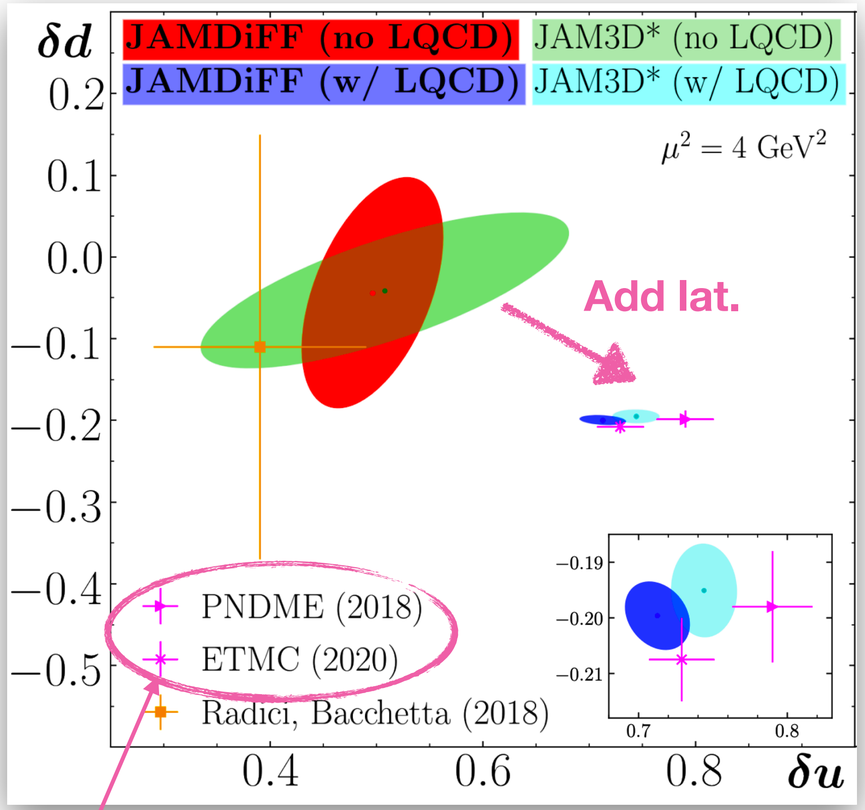
Nucleon unpolarized GPD



- Y. Guo, et al., JHEP 05 (2023) 150

5 Parton distributions from experiments

Nucleon transversity



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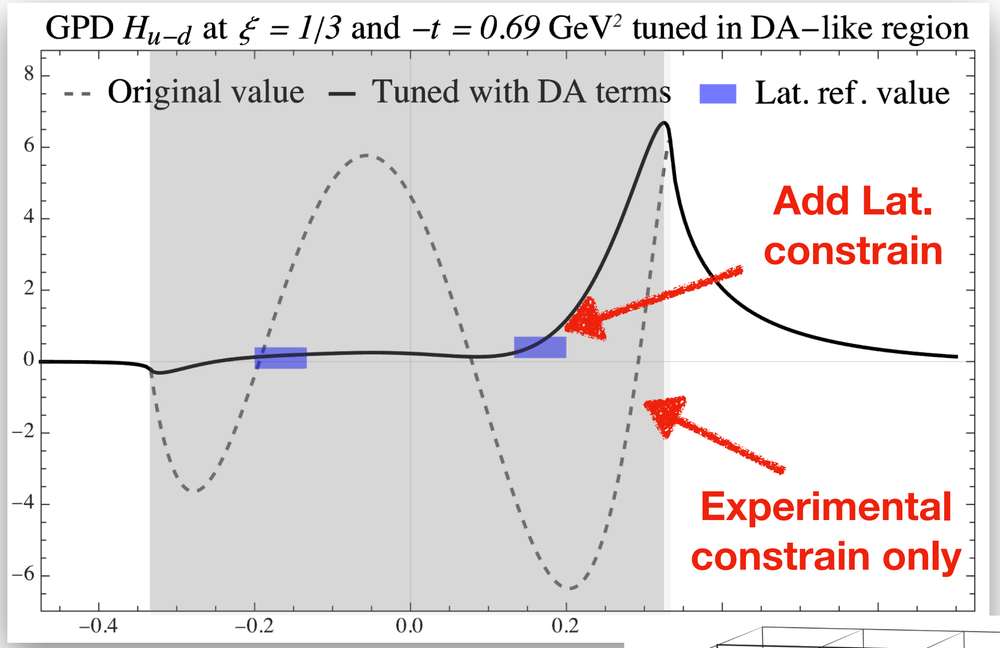
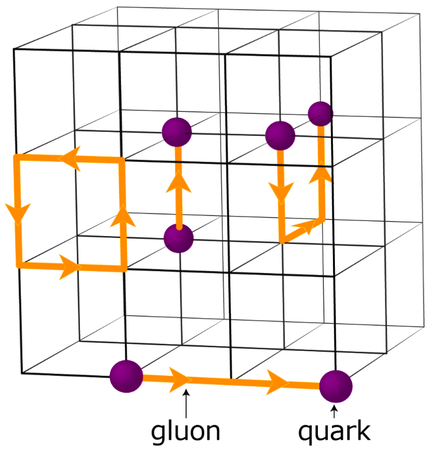
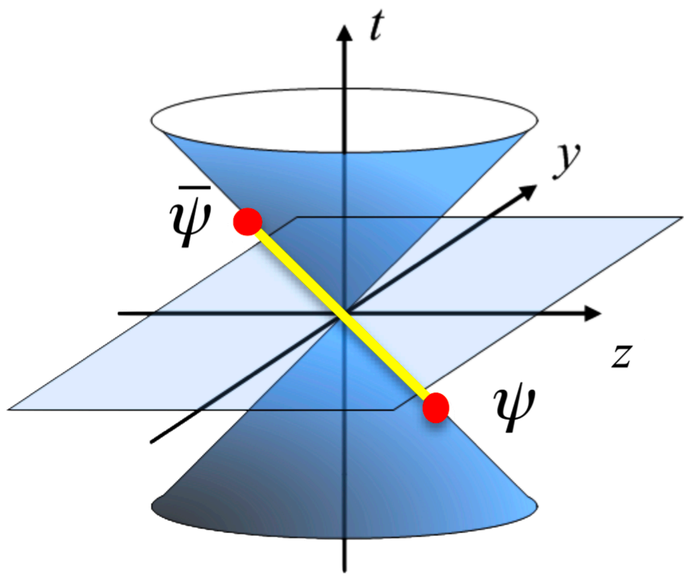


Complementary knowledge from lattice QCD is essential.



Parton distributions from lattice

- PDF from light-cone correlations



$$\langle p_f | \bar{q}(-\frac{z^-}{2}) \gamma^\mu \mathscr{W}(-\frac{z^-}{2}, \frac{z^-}{2}) q(\frac{z^-}{2}) | p_i \rangle$$

- Cannot be simulated on the Euclidean lattice

- Moments from Local operator

• Since the 1980s

$$\bar{q}\gamma^{\{\mu_1} iD^{\mu_2} \dots iD^{\mu_n\}} q$$

- Limited up to $\langle x^3 \rangle$ due to signal decay and power-divergent mixing under renormalization.

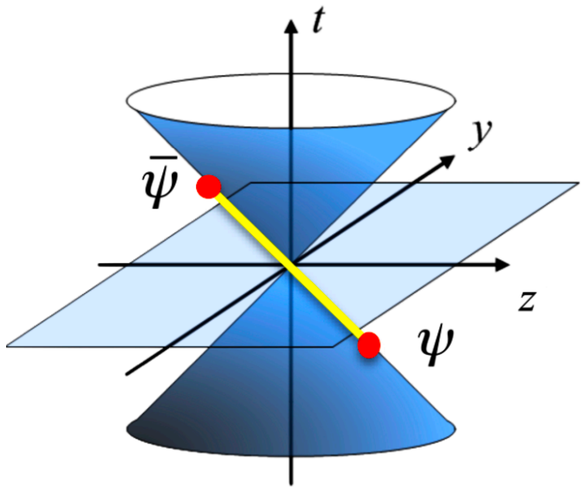
Large momentum effective theory

The quasi distribution from equal-time correlators,

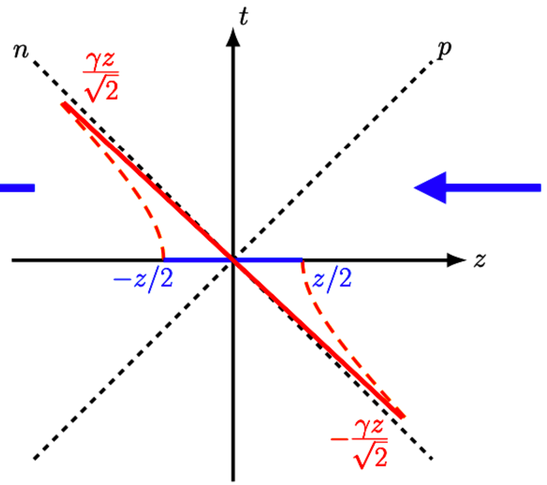
- X. Ji, PRL 110 (2013); SCPMA57 (2014);

$$\tilde{q}(x, P_z, \mu) = \int \frac{dz}{4\pi} e^{-ixP_z z} \langle P | \bar{q}(0) \Gamma \mathscr{W}(0,z) q(z) | P \rangle$$

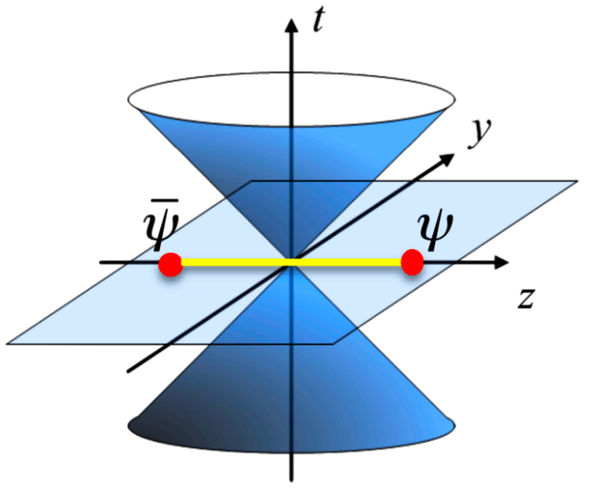
$z + ct = 0, \quad z - ct \neq 0$



Related by Lorentz boost



$t = 0, \quad z \neq 0$



Light-cone

$$\langle P | \bar{q}(0) \Gamma \mathscr{W}(0,z^-) q(z^-) | P \rangle$$

Quasi

$$\langle P | \bar{q}(0) \Gamma \mathscr{W}(0,z) q(z) | P \rangle$$

Large momentum effective theory

- X. Ji, PRL 110 (2013); SCPMA57 (2014);
- X. Xiong, X. Ji, et al, 90 PRD (2014);
- Y.-Q. Ma, et al, PRD98 (2018), PRL 120 (2018);
- T. Izubuchi, X. Ji, et al PRD98 (2018).
- X. Ji, Y. Zhao, et al, RMP 93 (2021).

Large P_z expansion of quasi distribution:

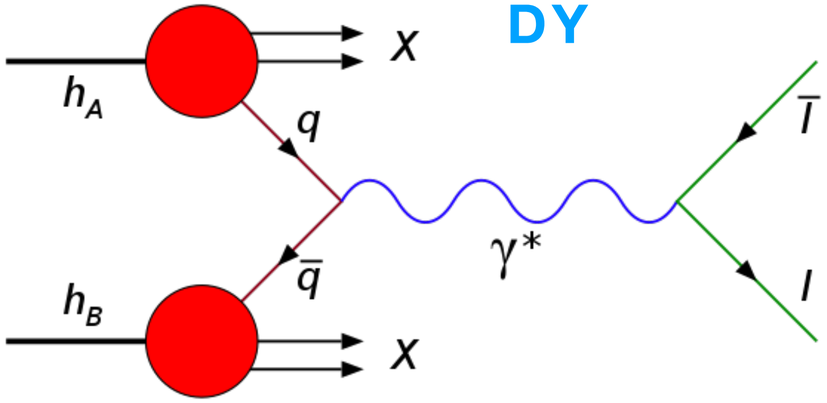
$$\tilde{q}(x,P_z,\mu)=\int\frac{dy}{|y|}C(\xi,\frac{\mu}{yP_z})q(y,\mu)+\mathcal{O}(\frac{\Lambda^2_{QCD}}{x^2P_z^2},\frac{\Lambda^2_{QCD}}{(1-x)^2P_z^2})$$

Perturbative kernel

large P_z is the key

$\xi = x/y$

- Similar to DIS, DY, ..., LaMET has direct sensitivity to the local $x \in [x_{\min}, x_{\max}]$ dependence of light-cone PDF $q(x,\mu)$.



$$\frac{d\sigma}{dQ^2}=\sigma_0\sum_{a,b}\int\frac{dx_1}{x_1}\int\frac{dx_2}{x_2}q_a^A(x_1,\mu^2)\ q_b^B(x_2,\mu^2)\omega_{ab}(z,\frac{\mu}{Q})$$

Short distance factorization

- SDF/OPE in coordinate space: Ioffe-time pseudo distributions

- A. V. Radyushkin et al., PRD 96 (2017).
- K. Orginos et al., PRD 96 (2017).
- T. Izubuchi et al., PRD 98 (2018).

$$\tilde{h}(z,P_z,\mu)=\tilde{h}(z^2,\lambda,\mu) \qquad \lambda=zP_z$$

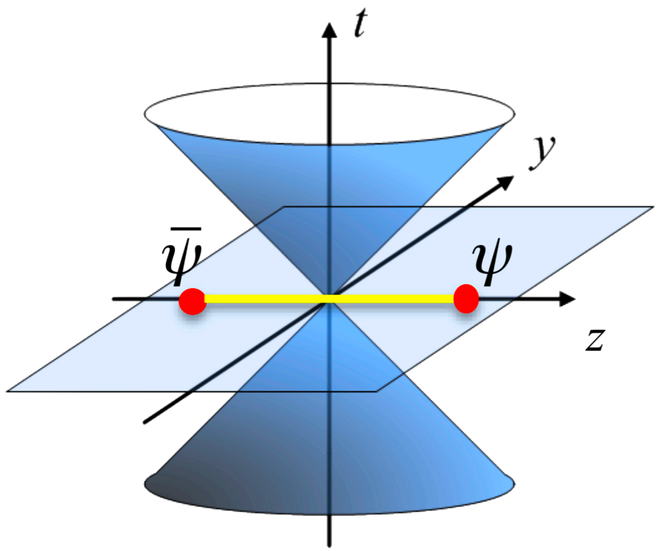
$$=\sum_{n=0}^{\infty}\frac{(-izP)^n}{n!}C_n(z^2\mu^2)\langle x^n\rangle(\mu)+\mathcal{O}(z^2\Lambda_{\rm QCD}^2)$$

Sum over the moments

$$=\int_{-1}^{1}d\alpha\mathscr{C}(\alpha,\mu^2z^2)\int_{-1}^{1}dye^{-iy\lambda}q(y,\mu)+\mathcal{O}(z^2\Lambda_{\rm QCD}^2)$$

- Extract moments $\langle x^n\rangle$ without power divergent mixing.
- The information is limited by the range of finite $\lambda=zP_z$, large P_z essential.

$$t=0,\quad z\neq 0$$

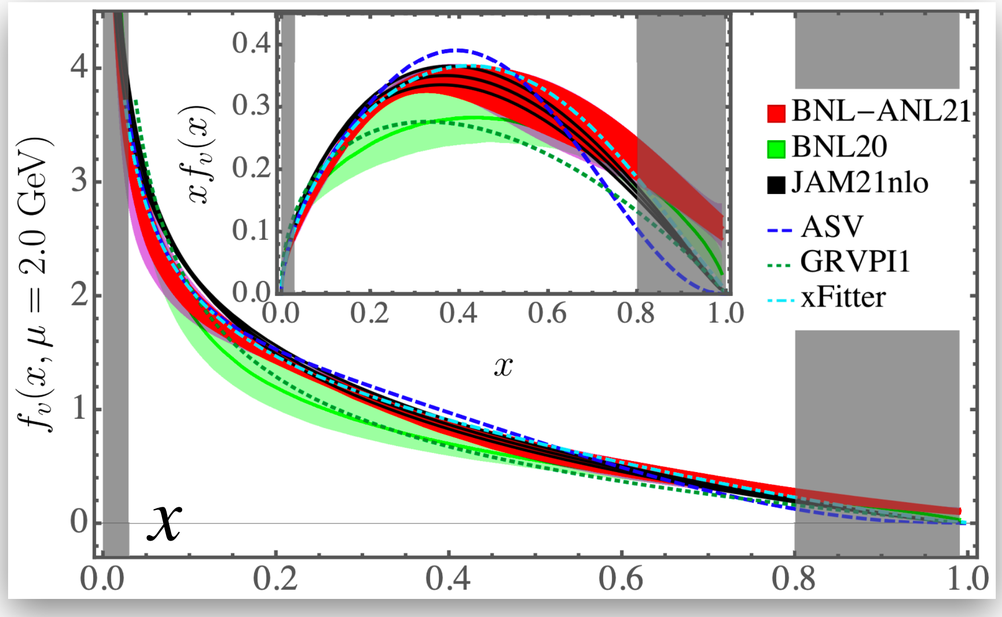


$$P^\mu\tilde{h}(z,P_z,\mu)=$$

$$\langle P\,|\,\bar{q}(-\tfrac{z}{2})\gamma^\mu\mathscr{W}(-\tfrac{z}{2},\tfrac{z}{2})q(\tfrac{z}{2})\,|\,P\rangle$$

Parton distributions from lattice

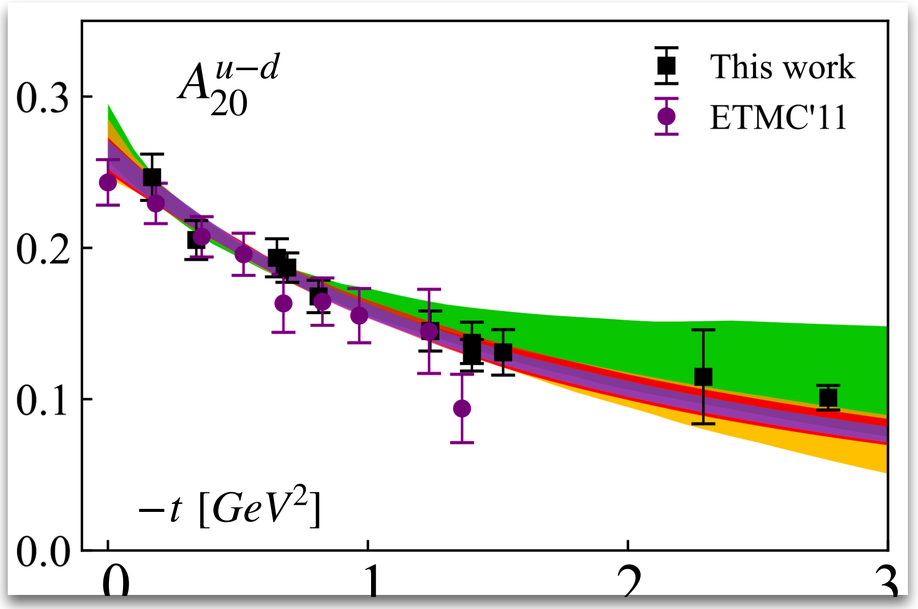
Pion valence quark PDF from LaMET



• XG, Y. Zhao et al., (BNL-ANL21), Phys.Rev.Lett. 128 (2022), 142003

- Agree with the global analysis at the moderate x region.
- End-point region can be improved by larger hadron momentum.

Moments of GPDs from SDF



• S. Bhattacharya, XG, et al., Phys.Rev.D 108 (2023) 1, 014507

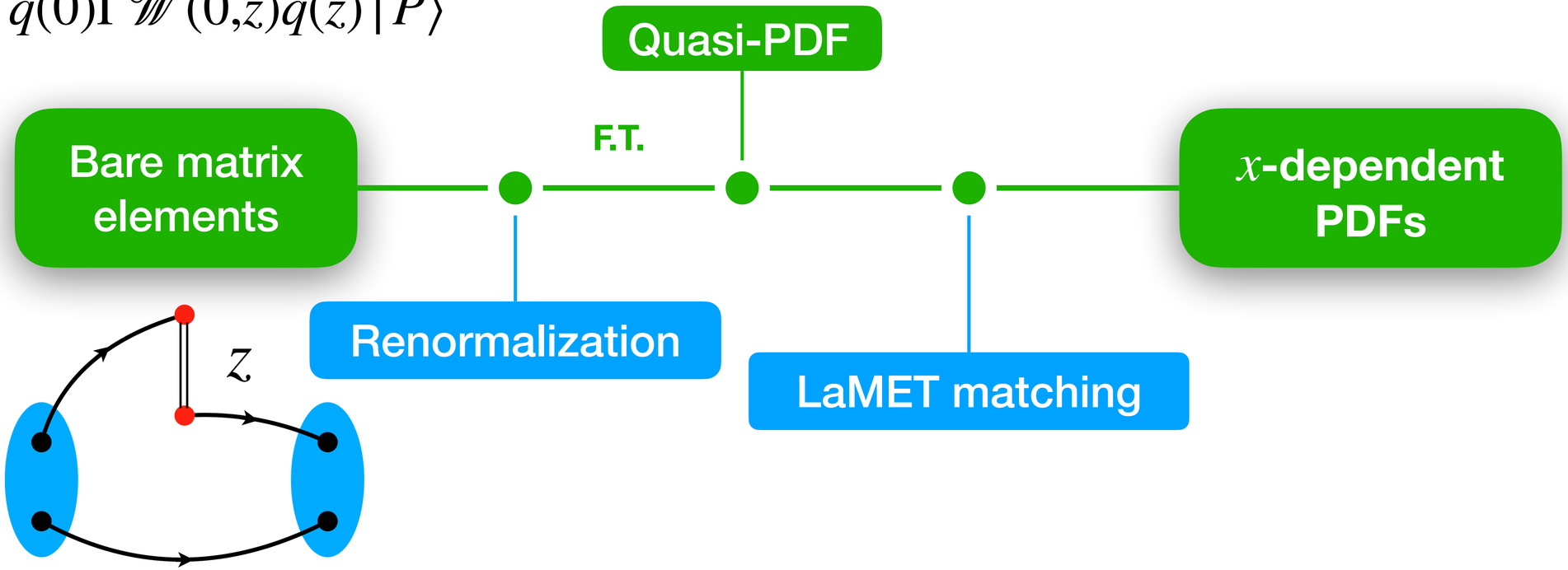
- Agree with traditional calculations using local operators (ETMC'11).
- Can systematically extract high moments with large momentum.

Systematic control in LaMET

$$\tilde{q}(x,P_z,\mu)=\int\frac{dy}{|y|}C(\xi,\frac{\mu}{yP_z})q(y,\mu)+\mathcal{O}(\frac{\Lambda^2_{QCD}}{x^2P_z^2},\frac{\Lambda^2_{QCD}}{(1-x)^2P_z^2})$$

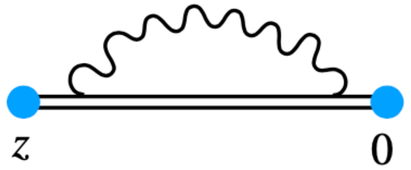
$$\xi=x/y$$

$$\langle P|\bar{q}(0)\Gamma\mathscr{W}(0,z)q(z)|P\rangle$$



Renormalization

- X. Ji, J. H. Zhang and Y. Zhao, PRL120 (2018)
- J. Green, K. Jansen and F. Steffens, PRL121 (2018)
- T. Ishikawa, et al, PRD 96 (2017)



$$= \delta m(a)\,|z| \propto \frac{|z|}{a}$$

The operator can be multiplicatively renormalized:

$$[\overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_B$$

$$= e^{-\delta m|z|} Z(a) [\overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_R$$

$$\delta m(a) = m_{-1}(a)/a + m_0$$

Wilson-line self energy + renormalon ambiguity

- Subtract the linear divergence from Wilson-line self energy:
  - Self-renormalization.
  - Static potential.
  - …

- Y. Huo, et al. (LPC), NPB 969 (2021).
- XG, Y. Zhao et al., (BNL/ ANL) PRL 128 (2022).

$$h^R(z, P_z) = e^{\delta m|z-z_s|} \frac{\tilde{h}(z, P_z, a)}{Z_X(z_s, P_z^0, a)}$$

$$h^{R,\ \text{lattice}}(z, P_z) \to h^{R,\ \overline{\text{MS}}}(z, P_z)$$

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- X. Ji, J. H. Zhang and Y. Zhao, PRL120 (2018)
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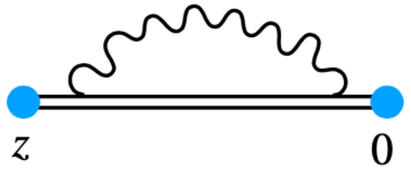

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Wilson-line self energy + renormalon ambiguity

linear power correction $\mathcal{O}(\Lambda_{\rm QCD}/P_z)$ in qPDF

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- XG, Y. Zhao et al., (BNL/ANL) PRL 128 (2022).

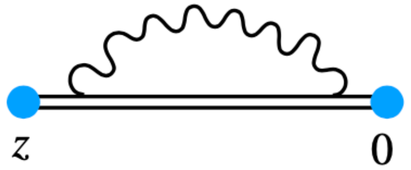
$$h^R(z, P_z) = e^{\delta m|z-z_s|} \frac{\tilde{h}(z, P_z, a)}{Z_X(z_s, P_z^0, a)}$$

◉ The number of diagrams grows, $r_n \sim n!$: diverge in perturbation theory at any α_s.

$$\delta m = \frac{1}{a} \sum \alpha_s^{n+1}(a) r_n$$

Renormalization

- X. Ji, J. H. Zhang and Y. Zhao, PRL120 (2018)
- J. Green, K. Jansen and F. Steffens, PRL121 (2018)
- T. Ishikawa, et al, PRD 96 (2017)



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  - …

- Y. Huo, et al. (LPC), NPB 969 (2021).
- XG, Y. Zhao et al., (BNL/ANL) PRL 128 (2022).

- Subtract the renormalon ambiguity from a resummation prescription:

$$h^{R,\,\text{lattice}}(z, P_z = 0)$$

$$= C_0^{\overline{\text{MS}},\,\text{LRR}}(z, P_z = 0) e^{-m_0|z|}$$

LRR: leading renormalon resummation

$$\mathcal{O}(\Lambda_{\text{QCD}}/P_z) + \mathcal{O}(\Lambda^2_{\text{QCD}}/P_z^2)$$

- J. Holligan, et al., (UMD) NPB 993 (2023).
- R. Zhang, et al., (UMD) PLB 844 (2023).

LaMET matching

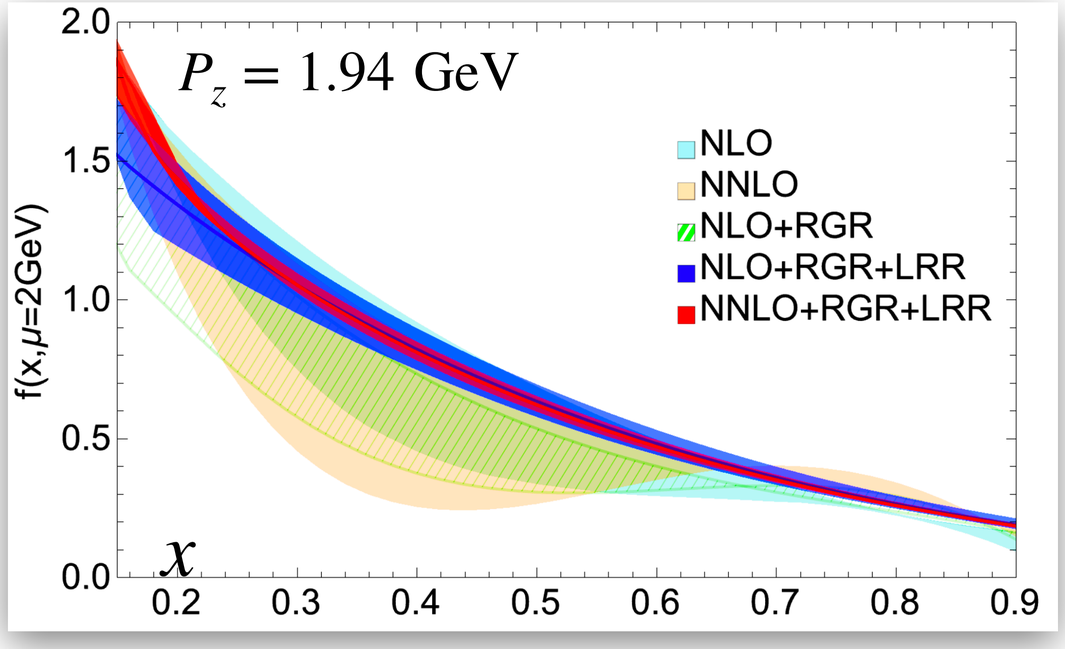
$$\lim_{\xi\to 1} C\left(\xi, \frac{\mu}{P^z}\right) = 1 + \frac{\alpha_s(\mu) C_F}{2\pi}\left[2\frac{\ln|1-\xi|}{|1-\xi|} - \frac{2}{1-\xi}\ln\frac{\mu^2}{(2xP_z)^2}\theta(1-\xi) + \frac{3}{2|1-\xi|}\right] + \mathcal{O}(\alpha_s^2)$$

$$\xi = x/y$$

- Next-to-next-to-leading order (NNLO) kernel.
  - L.-B. Chen, et al., Phys. Rev. Lett. 126 (2021) 072002
  - Z.-Y. Li, et al., Phys. Rev. Lett. 126 (2021) 072001
- Resummation of DGLAP (RG) logs $\ln(\mu^2/(2xP_z)^2)$.
  - XG, Y. Zhao et al. (BNL), Phys.Rev.D 103 (2021), 094504
  - Y.S. Su, et al., Nucl.Phys.B 991 (2023) 116201
- Resummation of large-x (threshold) logarithms $\ln|1-\xi|/|1-\xi|$.
  - XG, Y. Zhao et al. (BNL), Phys.Rev.D 103 (2021), 094504
  - X. Ji, et al., JHEP 08 (2023) 037
- Resummation/subtraction of leading renormalon.
  - J. Holligan, et al., (UMD) NPB 993 (2023).
  - R. Zhang, et al., (UMD) PLB 844 (2023).

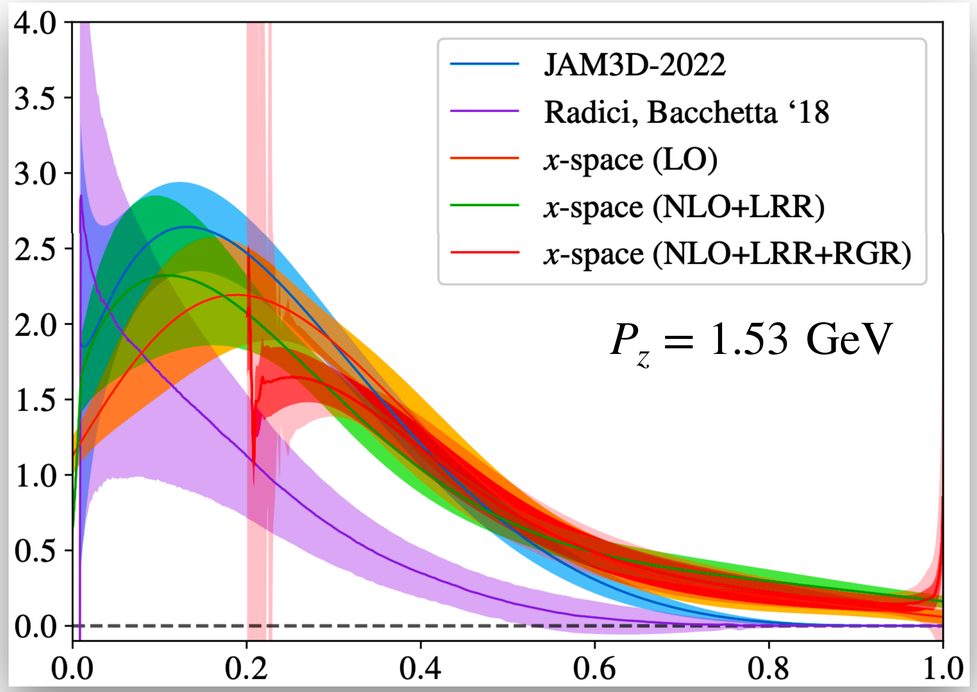
PDFs from LaMET with improved precision

- Pion valence quark PDF



- R. Zhang, et al., (UMD) PLB 844 (2023).

- Nucleon transversity PDF



- XG, A. Hanlon, et al., (BNL+ANL), arXiv: 2310.19047

- Leading renormalon resummation (LRR): remove the linear power correction $\mathcal{O}(\Lambda_{\rm QCD}/P_z)$, reduces the (theoretical) uncertainty.
- DGLAP (RGR): enhances the accuracy of the scale evolution at moderate x, also indicate the perturbative matching is unreliable at small x ($2xP_z \sim \Lambda_{\rm QCD}$).

17 Universality in LaMET

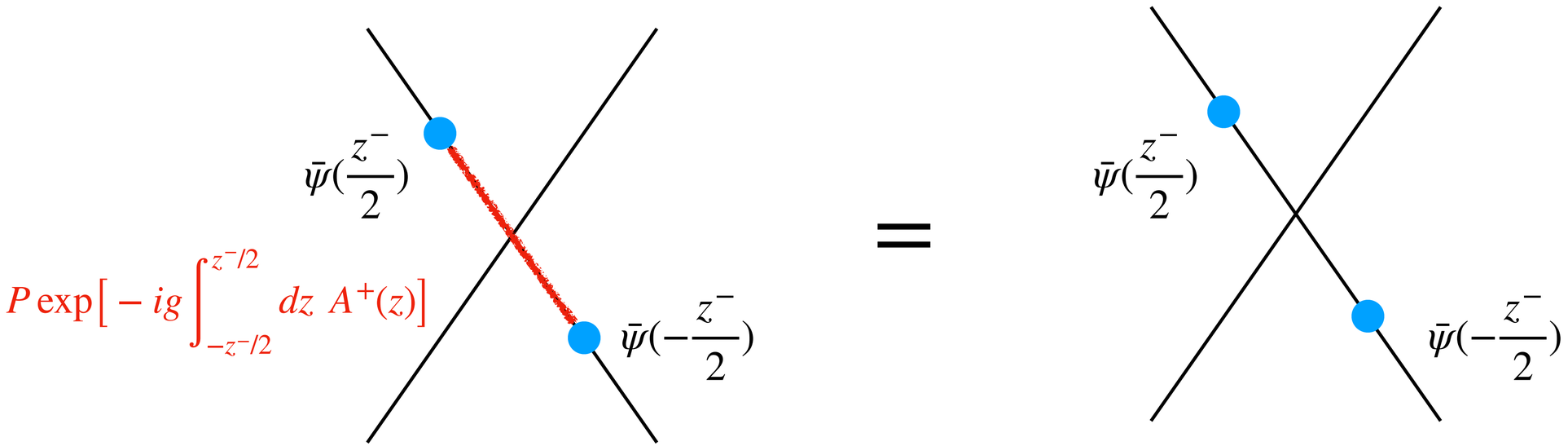
- The non-local operator in gauge theory:

$$\bar{\psi}^*(-\frac{z}{2})\Gamma\psi^*(\frac{z}{2})$$

with $\psi^*(z) = \psi(z)e^{iC(z)}$ and $C(z)$ is a linear function of A_μ .

• P. A. M. Dirac, Can.J.Phys. 33 (1955) 650

- In DIS, the physical quark $\psi(z)e^{iC(z)}$ represents a gauge-invariant object with a gauge link extended to infinity along the light-cone direction:

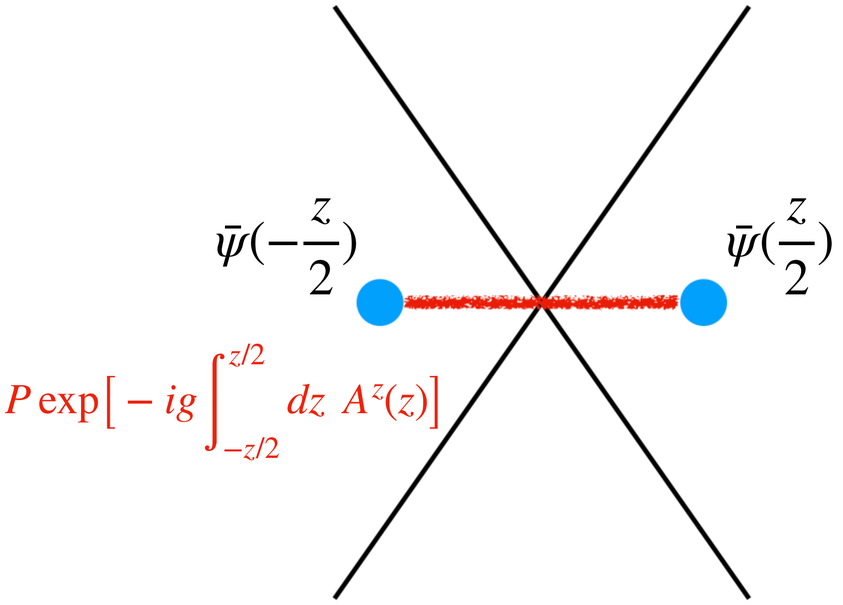


Gauge invariant

$A^+ = 0$ gauge

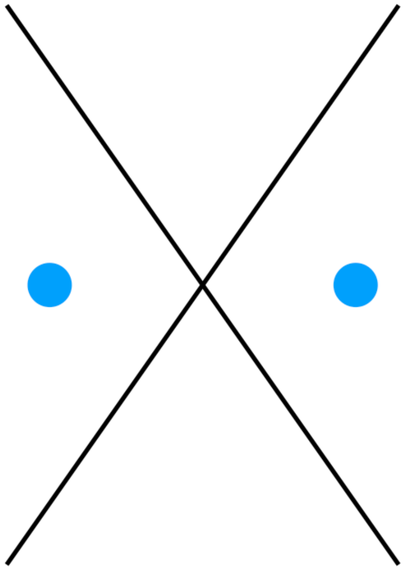
Universality in LaMET

Gauge invariant (GI) qPDF



$$\bar{\psi}(-\frac{z}{2})\Gamma W(-\frac{z}{2},\frac{z}{2})\psi(\frac{z}{2})$$

=

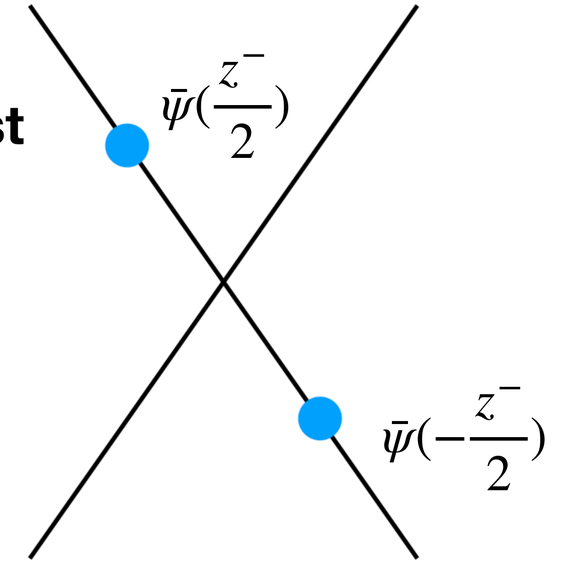


$$\bar{\psi}(-\frac{z}{2})\Gamma\psi(\frac{z}{2})\,|_{A^z=0}$$

$P \to \infty$ **boost**

LaMET

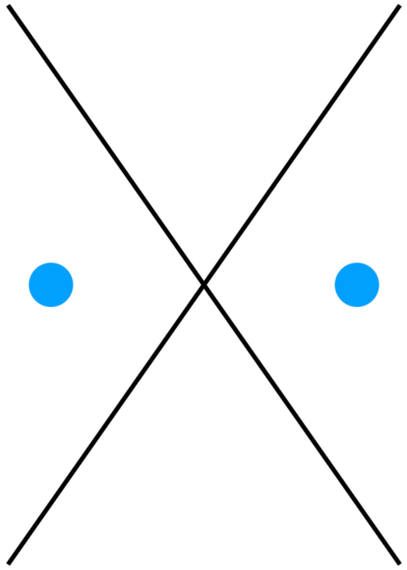
Light-cone PDF



$$\bar{\psi}(-\frac{z^-}{2})\Gamma W(-\frac{z^-}{2},\frac{z^-}{2})\psi(\frac{z^-}{2})$$

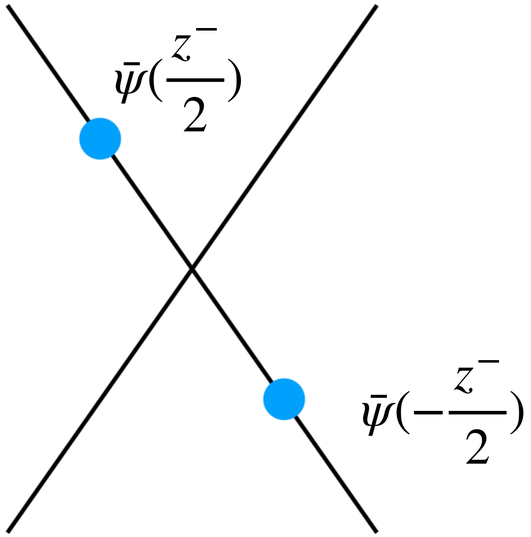
or $\bar{\psi}(-\frac{z^-}{2})\Gamma\psi(\frac{z^-}{2})\,|_{A^+=0}$

Universality in LaMET



$P \to \infty$ **boost**

LaMET



Physical gauge like: $A^z, A^0,$
Coulomb gauge $\vec{\nabla} \cdot \vec{A} = 0$

$$\bar{\psi}(-\frac{z^-}{2})\Gamma W(-\frac{z^-}{2}, \frac{z^-}{2})\psi(\frac{z^-}{2})$$

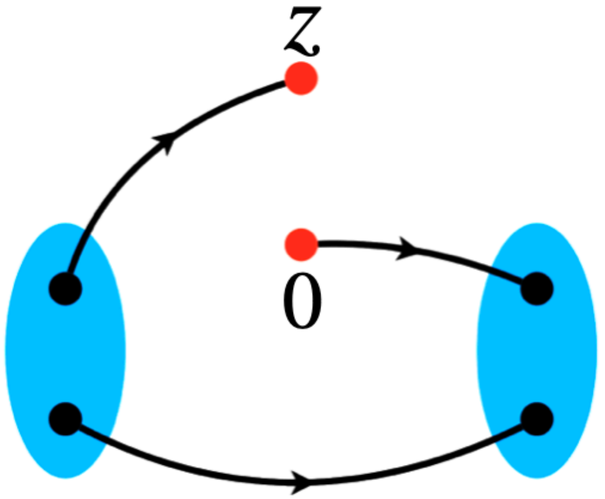
or $\bar{\psi}(-\frac{z^-}{2})\Gamma\psi(\frac{z^-}{2})\,|_{A^+=0}$

LaMET in Coulomb gauge

- **Quasi-PDF in Coulomb gauge (CG):**

$$\tilde{f}(x, P^z, \mu) = P^z \int_{-\infty}^{\infty} \frac{dz}{2\pi} e^{ixP^z z} \tilde{h}(z, P^z, \mu)$$

$$\tilde{h}(z, P^z, \mu) = \frac{1}{2P^t} \langle P | \bar{\psi}(z) \gamma^t \psi(0) \Big|_{\vec{\nabla}\cdot\vec{A}=0} | P \rangle$$

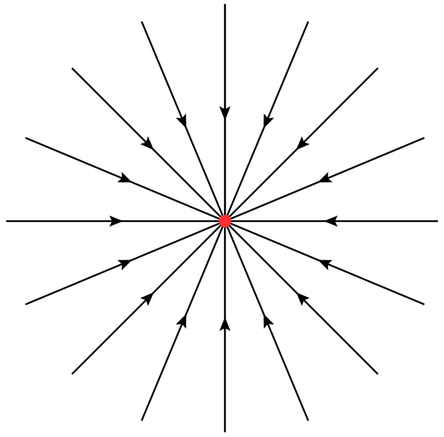


- **First proposed in the lattice calculation of gluon helicity**

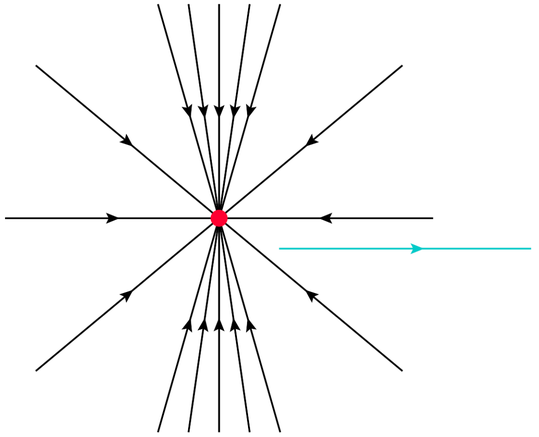
$$\Delta G = \langle P_\infty | (\mathbf{E} \times \mathbf{A})^3 |_{\nabla\cdot\mathbf{A}=0} | P_\infty \rangle$$

- X. Ji, J.-H. Zhang and Y. Zhao, PRL 111 (2013);
- Y. Hatta, X. Ji, and Y. Zhao, PRD 89 (2014);
- X. Ji, J.-H. Zhang and Y. Zhao, PLB 743 (2015);
- Y.-B. Yang, R. Sufian, Y. Zhao, et al. PRL 118 (2017)

Static charge



Moving charge



LaMET factorization

$$\tilde{q}(x,P_z,\mu)=\int\frac{dy}{|y|}C(\xi,\frac{\mu}{yP_z})q(y,\mu)+\mathcal{O}(\frac{\Lambda^2_{QCD}}{x^2P_z^2},\frac{\Lambda^2_{QCD}}{(1-x)^2P_z^2})$$

$$\xi=x/y$$

- **One-loop matching:**

$$C\Big(\xi,\frac{\mu}{p^z}\Big)=\delta(\xi-1)+\frac{\alpha_s C_F}{2\pi}C^{(1)}\Big(\xi,\frac{\mu}{p^z}\Big)+\mathcal{O}(\alpha_s^2)$$

$$C^{(1)}\Big(\xi,\frac{\mu}{p^z}\Big)=C^{(1)}_{\text{ratio}}\Big(\xi,\frac{\mu}{p^z}\Big)+\frac{1}{2|1-\xi|}+\delta(1-\xi)\left[-\frac{1}{2}\ln\frac{\mu^2}{4p_z^2}+\frac{1}{2}-\int_0^1 d\xi'\frac{1}{1-\xi'}\right]$$

$$C^{(1)}_{\text{ratio}}\Big(\xi,\frac{\mu}{p^z}\Big)=\left[P_{qq}(\xi)\ln\frac{4p_z^2}{\mu^2}+\xi-1\right]^{[0,1]}_{+(1)}+\left\{P_{qq}(\xi)\Big[\mathbf{sgn}(\xi)\ln|\xi|+\mathbf{sgn}(1-\xi)\ln|1-\xi|\Big]\right.$$

$$\left.+\mathbf{sgn}(\xi)+\frac{3\xi-1}{\xi-1}\frac{\tan^{-1}\left(\sqrt{1-2\xi}/|\xi|\right)}{\sqrt{1-2\xi}}-\frac{3}{2|1-\xi|}\right\}^{(-\infty,\infty)}_{+(1)}$$

- **Collinear divergences are identical between CG qPDF and light-cone PDF.**
- **Satisfies particle number conservation.**

Short-distance factorization

$$\tilde{h}(z, P_z, \mu) = \int_{-1}^{1} d\alpha \mathcal{C}(\alpha, \mu^2 z^2) \int_{-1}^{1} dy e^{-iy\lambda} q(y, \mu) + \mathcal{O}(z^2 \Lambda_{\mathrm{QCD}}^2)$$

- A. V. Radyushkin et al., PRD 96 (2017).

- **One-loop matching:**

$$\mathcal{C}\left(u, \frac{\mu}{p^z}\right) = \delta(u-1) + \frac{\alpha_s C_F}{2\pi} \mathcal{C}^{(1)}\left(u, \frac{\mu}{p^z}\right) + \mathcal{O}(\alpha_s^2)$$

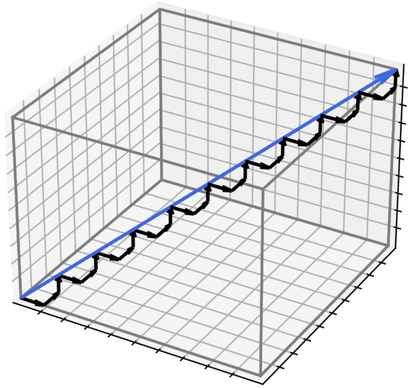
$$\mathcal{C}^{(1)}(u, z^2\mu^2) = \mathcal{C}^{(1)}_{\mathrm{ratio}}(u, z^2\mu^2) + \frac{1}{2}\delta(1-u)\left(1 - \ln\frac{z^2\mu^2 e^{2\gamma_E}}{4}\right)$$

$$\mathcal{C}^{(1)}_{\mathrm{ratio}}(u, z^2\mu^2) = \left[-P_{qq}(u)\ln\frac{z^2\mu^2 e^{2\gamma_E}}{4} - \frac{4\ln(1-u)}{1-u} + 1 - u\right]_{+(1)}^{[0,1]}$$

$$+ \left[\frac{3u-1}{u-1}\frac{\tan^{-1}\left(\frac{\sqrt{1-2u}}{|u|}\right)}{\sqrt{1-2u}} - \frac{3}{|1-u|}\right]_{+(1)}^{(-\infty,\infty)}$$

Lattice setup

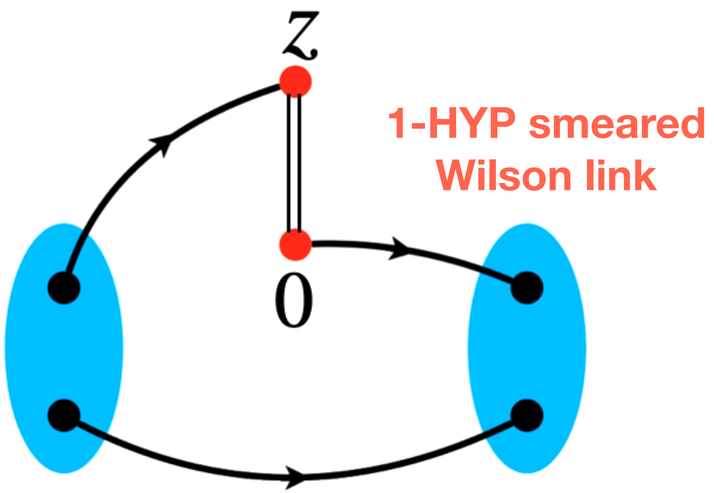
- Clover-fermion on 2+1f HISQ gauge ensembles, 1-step HYP smearing.
- $48^3 \times 64$, a = 0.06 fm, $m_\pi = 300$ MeV.
- Momentum $\vec{p} = \frac{2\pi}{L_s}\vec{n}$.
- 109 configurations & AMA.



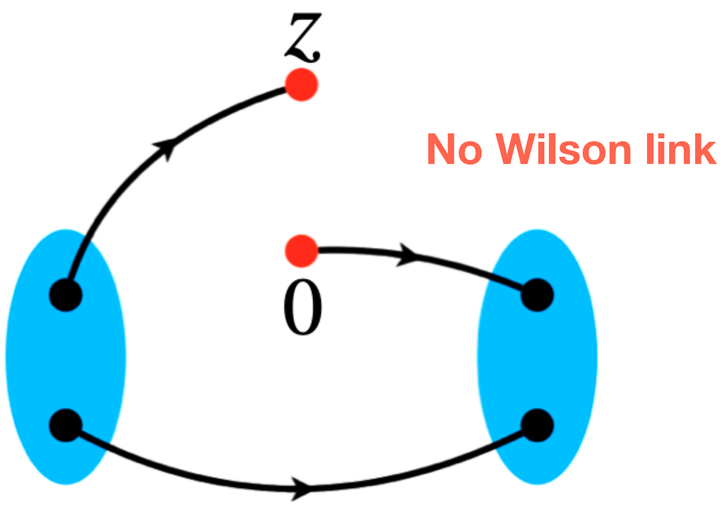
$\lvert\vec{p}\rvert$ (GeV)	$\vec{n}$	$\vec{k}$	t_s/a	(#ex,#sl)
0	(0,0,0)	(0,0,0)	8,10,12	(1, 16)
1.72	(0,0,4)	(0,0,3)	8	(1, 32)
			10	(3, 96)
			12	(8, 256)
2.15	(0,0,5)	(0,0,3)	8	(2, 64)
			10	(8, 256)
			12	(8, 256)
2.24	(3,3,3)	(2,2,2)	8	(2, 64)
			10	(8, 256)
			12	(8, 256)

- In CG without Wilson line, the quark separation can be off-axis.

Lattice setup



Gauge invariant (GI)



Coulomb gauge (CG)

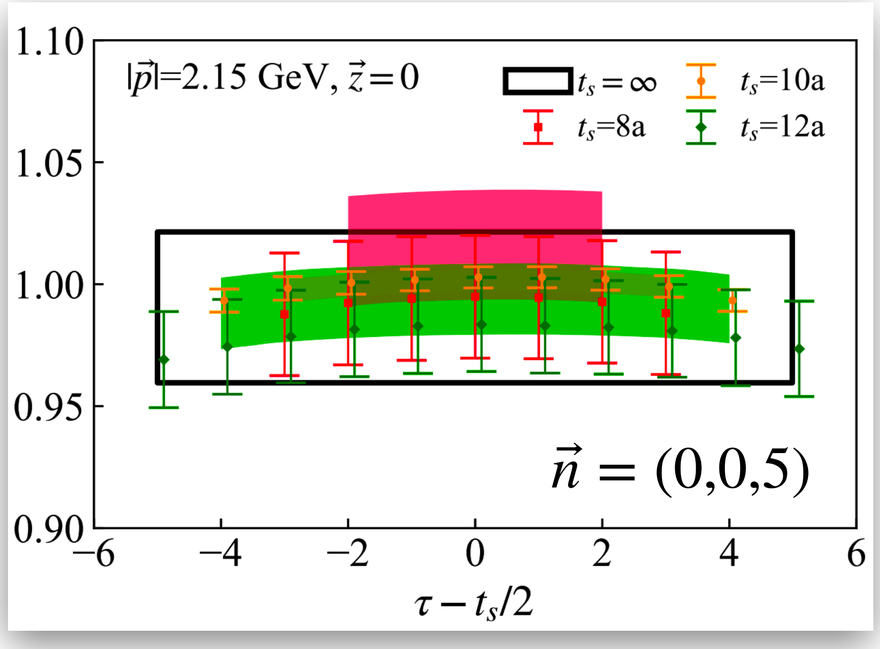
- The contraction of both GI and CG cases can be done simultaneously using the same propagators in Coulomb gauge.

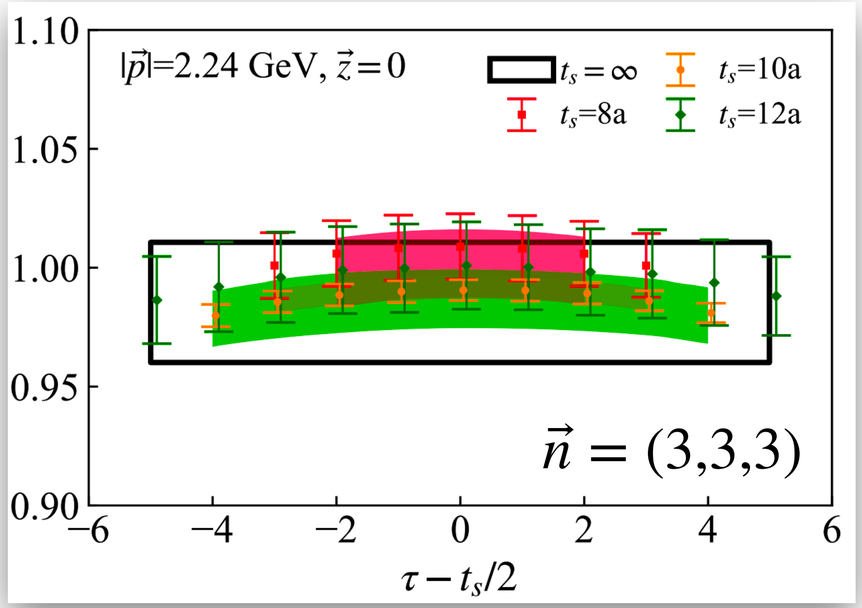
Bare matrix elements: data precision

$$R(t_s,\tau)=\frac{\langle N(t_s)\mathcal{O}^f_\Gamma(\vec{z},\tau)\overline{N}(0)\rangle}{\langle N(t_s)\overline{N}(0)\rangle}$$

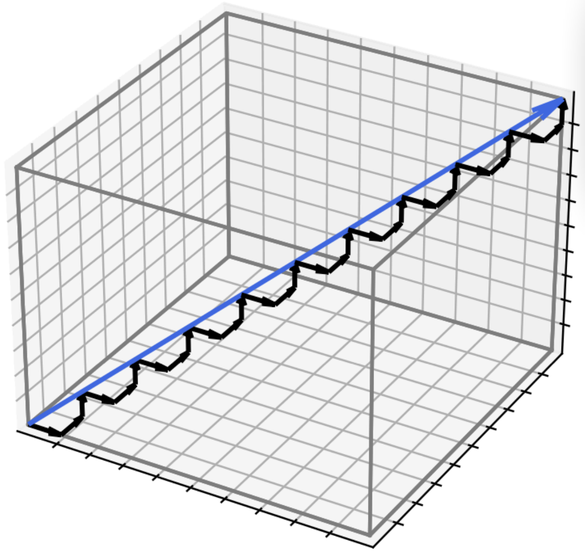
$$\xrightarrow{t_s\to\infty}\tilde{h}^B(z,\,P_z)$$

- Off-axis momentum shows some signal improvement .



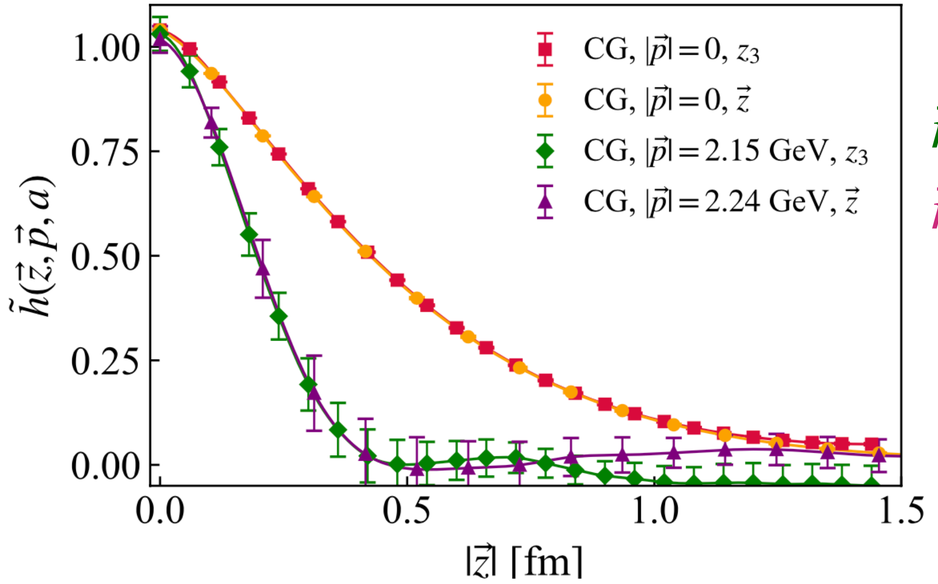


Bare matrix elements: Rotational symmetry



- CG
- GI: Zig-zag

- B. Musch et al., PRD 83 (2011).



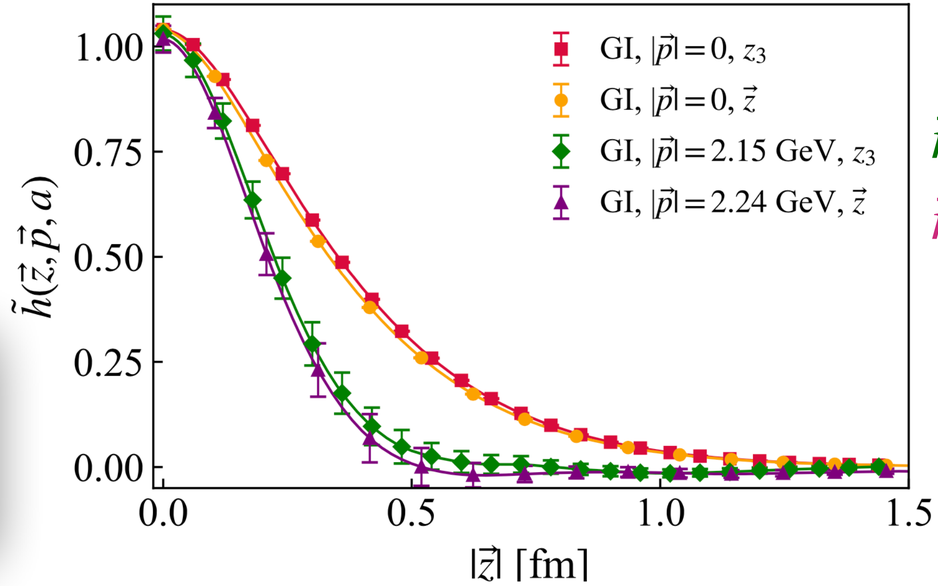
$\vec{n} = (0,0,5)$
$\vec{n} = (3,3,3)$

- On-axis

$\vec{n} = (0,0,n_3)$
$\vec{z} = (0,0,z_3)$

- Off-axis

$\vec{n} = (n_1, n_2, n_3)$
$\vec{z} = (z_1, z_2, z_3)$



$\vec{n} = (0,0,5)$
$\vec{n} = (3,3,3)$

- CG matrix elements precisely preserve the 3D rotational symmetry, which is broken for GI with a zig-zagged Wilson line.

Bare matrix elements: Renormalizability

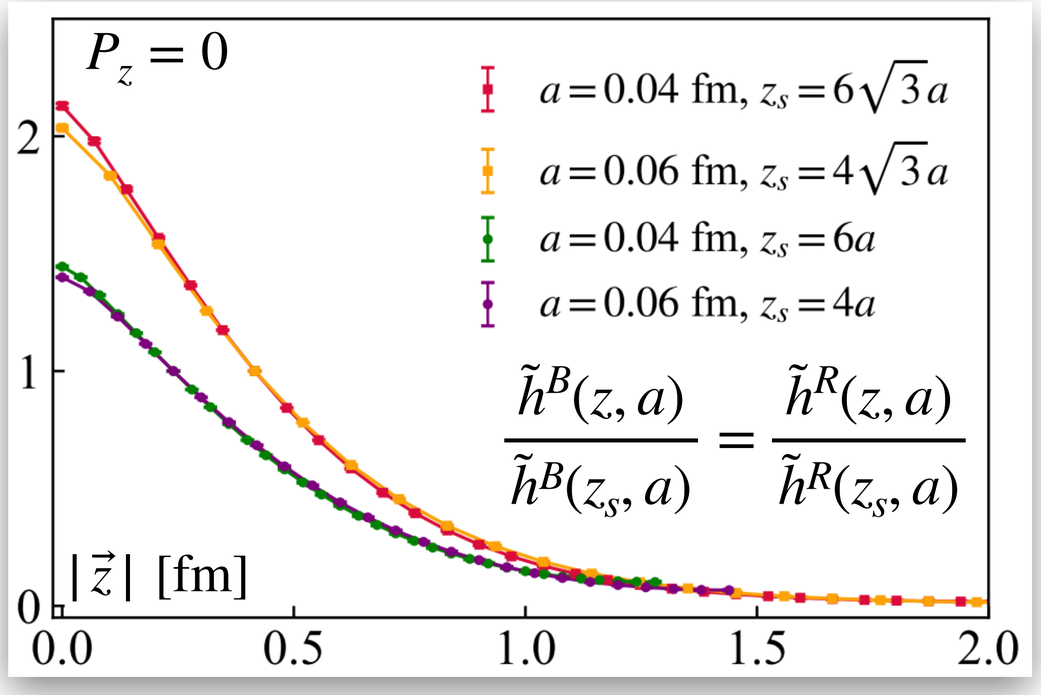
- The lattice quark propagator in Coulomb gauge are multiplicatively renormalizable.
- No Wilson-line and linear divergence renormalization needed.

- D. Zwanziger, NPB 518 (1998);
- Baulieu and Zwanziger, NPB 548 (1999);
- A. Niegawa, PRD 74 (2006);
- Niegawa, Inui and Kohyama, PRD 74 (2006
- G. Burgio, et al., PRD 86 (2012)

Overall renormalization

$$[\bar{\psi}(0)\Gamma\psi(z)]_B = Z_\psi(a)[\bar{\psi}(0)\Gamma\psi(z)]_R$$

$$\frac{\tilde{h}^B(z,a)}{\tilde{h}^B(z_s,a)} = \frac{\tilde{h}^R(z,a)}{\tilde{h}^R(z_s,a)}$$



Another lattice is used to check the renormalization:

$64^3 \times 64$, a = 0.04 fm, $m_\pi = 300$ MeV.

Excellent continuum limit!

Renormalized matrix elements in short distance

Ratio scheme renormalization:

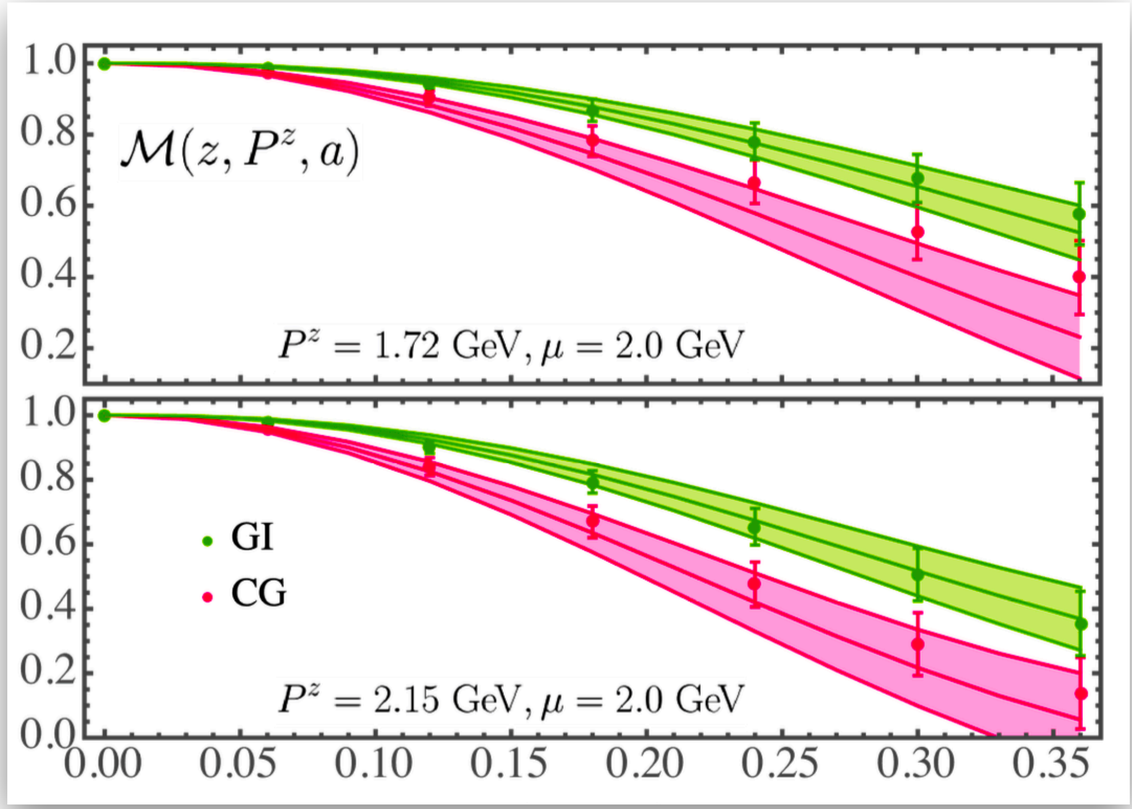
$$\mathcal{M}(z, P^z, a) = \frac{\tilde{h}(z, P^z, a)}{\tilde{h}(z,0,a)} \frac{\tilde{h}(0,0,a)}{\tilde{h}(0,P^z, a)}$$

- K. Orginos et al., PRD 96 (2017).

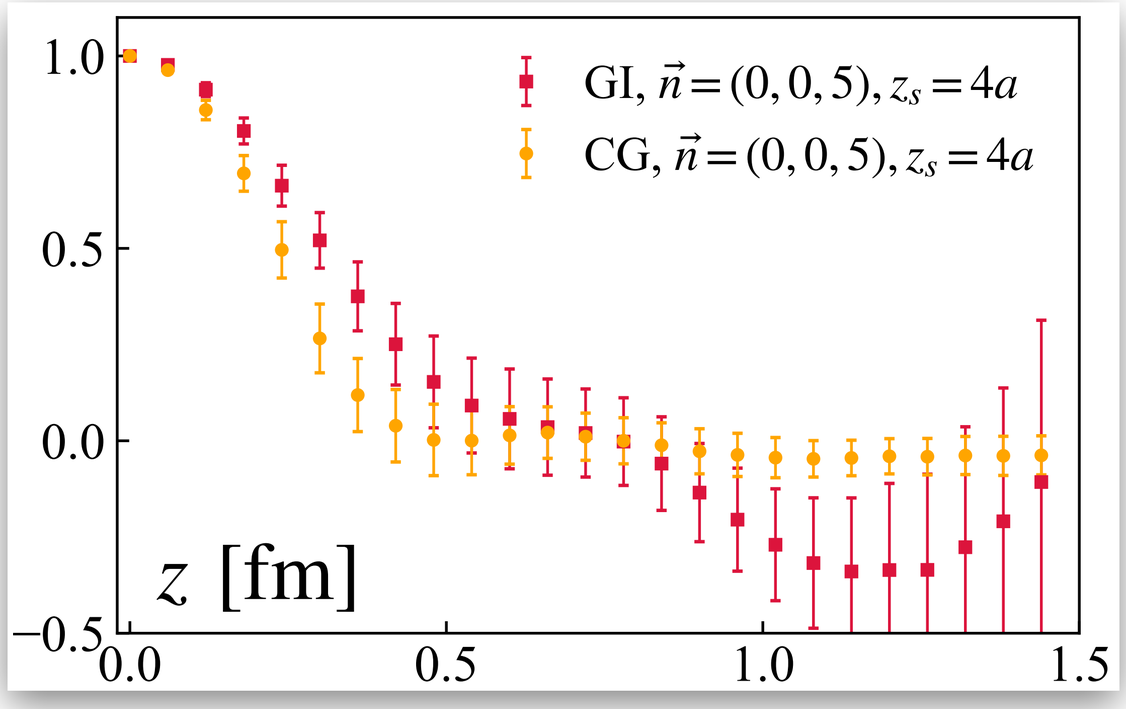
- The prediction from GI agree well with the CG matrix elements:

$$\text{qPDF}_{\text{GI}} \rightarrow \text{PDF} \rightarrow \text{qPDF}_{\text{CG}}$$

NLO **NLO**



x-dependent PDF: hybrid renormalization



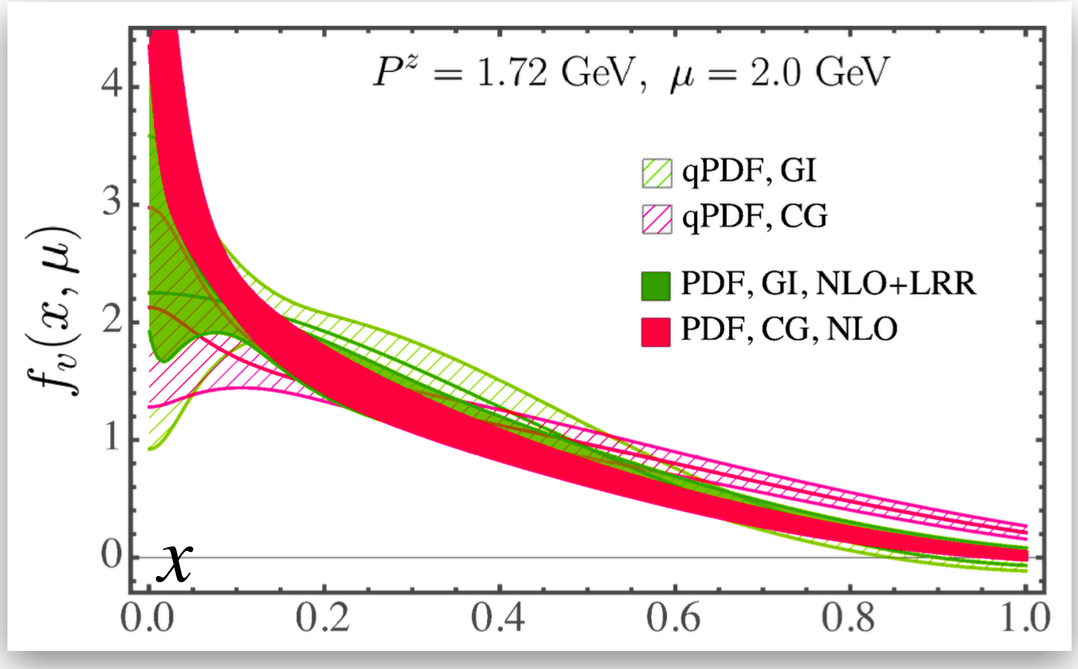
$$|z| \le z_s, \quad \frac{\tilde{h}(z, P_z, a)}{\tilde{h}(z,0,a)}$$

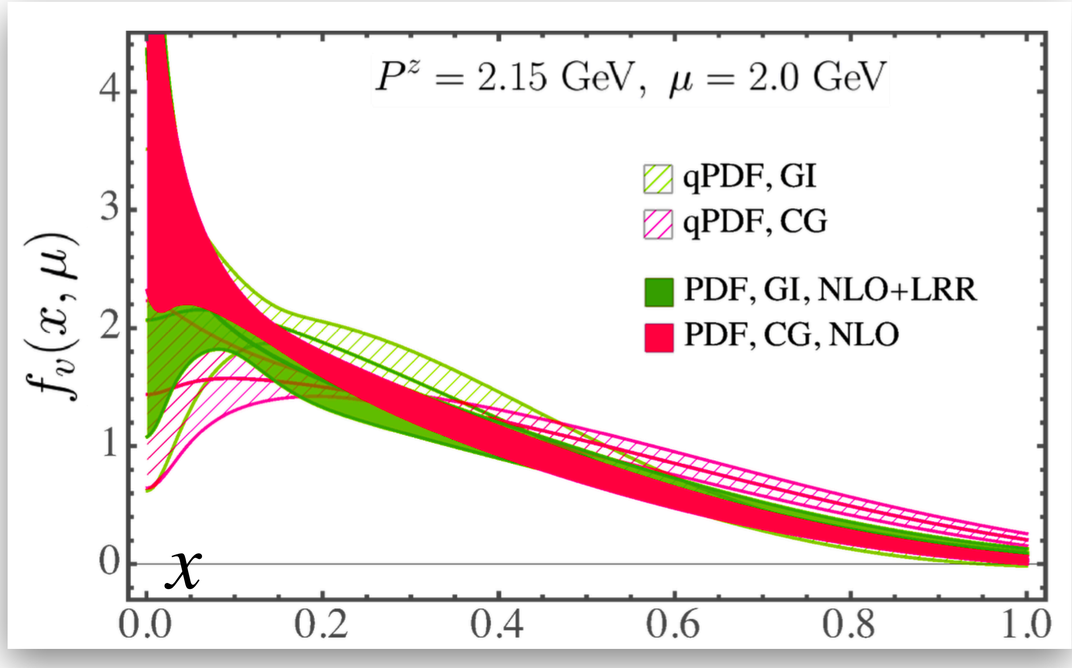
$$|z| > z_s, \quad \frac{\tilde{h}(z, P_z, a)}{\tilde{h}(z_s,0,a)} e^{(\delta m(a)+\bar{m}_0)|z-z_s|}$$

• X. Ji, et al., Nucl.Phys.B 964 (2021) 115311

- No linear divergence and renormalon need to be removed in CG case.
- Better controlled long-range precision in CG case.

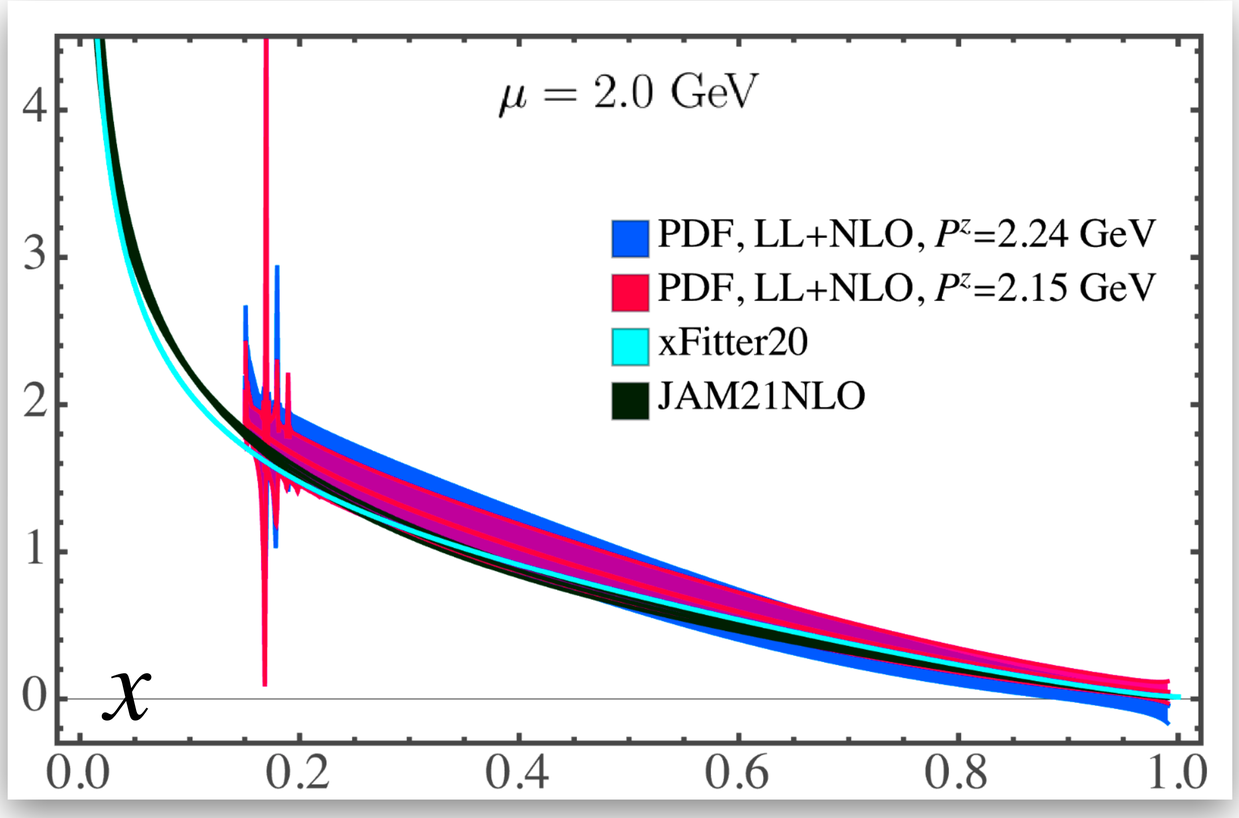
x-dependent PDF: F.T. and matching





- The quasi-PDF are different at finite momentum between CG and GI.
- The PDF after matching show good agreement beyond $x > 0.2$, demonstrating the universality in LaMET.

x-dependent PDF: CG results



- LL: leading logarithm DGLAP resummation:

$$C(\xi, \frac{\mu}{P_z}) \sim \left(\frac{\alpha_s(2xP_z)}{\alpha_s(\mu)} \right)^{\frac{\gamma_n}{\beta_0}}$$

- Agree with global fits from JAM and xFitter for $x \gtrsim 0.2$ within the errors.
- Precision can be considerably improved with more statistics in the future.

Comparison between GI and CG quasi-PDFs

	Momentum direction	Renormalization	Power corrections	Gribov copies	Mixing	Higher-order corrections
Gauge invariant (GI)	$(0,0,n_z)$	Linear divergence + vertex and wave function	$\Lambda^2_{\rm QCD}/P_z^2$ w. renormalon subtraction	N/A	Lorentz symmetry	Available up to NNLO
Coulomb gauge (CG)	(n_x, n_y, n_z)	Wave function	$\Lambda^2_{\rm QCD}/\vec{p}^2$	May affect long range region*	3D rotational symmetry	May be harder to go beyond NLO

*: Currently negligible otherwise the errors will not reduce as the number of configuration increasing.

Summary

- We demonstrate the universality in LaMET through the equivalence of CG and GI quasi-PDF methods at NLO.
- The CG correlations have the advantages of access to larger off-axis momenta, absence of linear divergence, and enhanced long-range precision.
- It is almost free to compute the GI and CG matrix elements simultaneously.
- The CG methods could have broader use in the future particularly in the TMD physics, for which the Wilson-line geometry and renormalization can be significantly simplified.

Factorization of quasi-TMDs in CG

- Y. Zhao, arXiv: 2311.01391

Thanks for your attention!