Gluon TMDs in electroproduction

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JLab remote theory seminar
June 15th 2020

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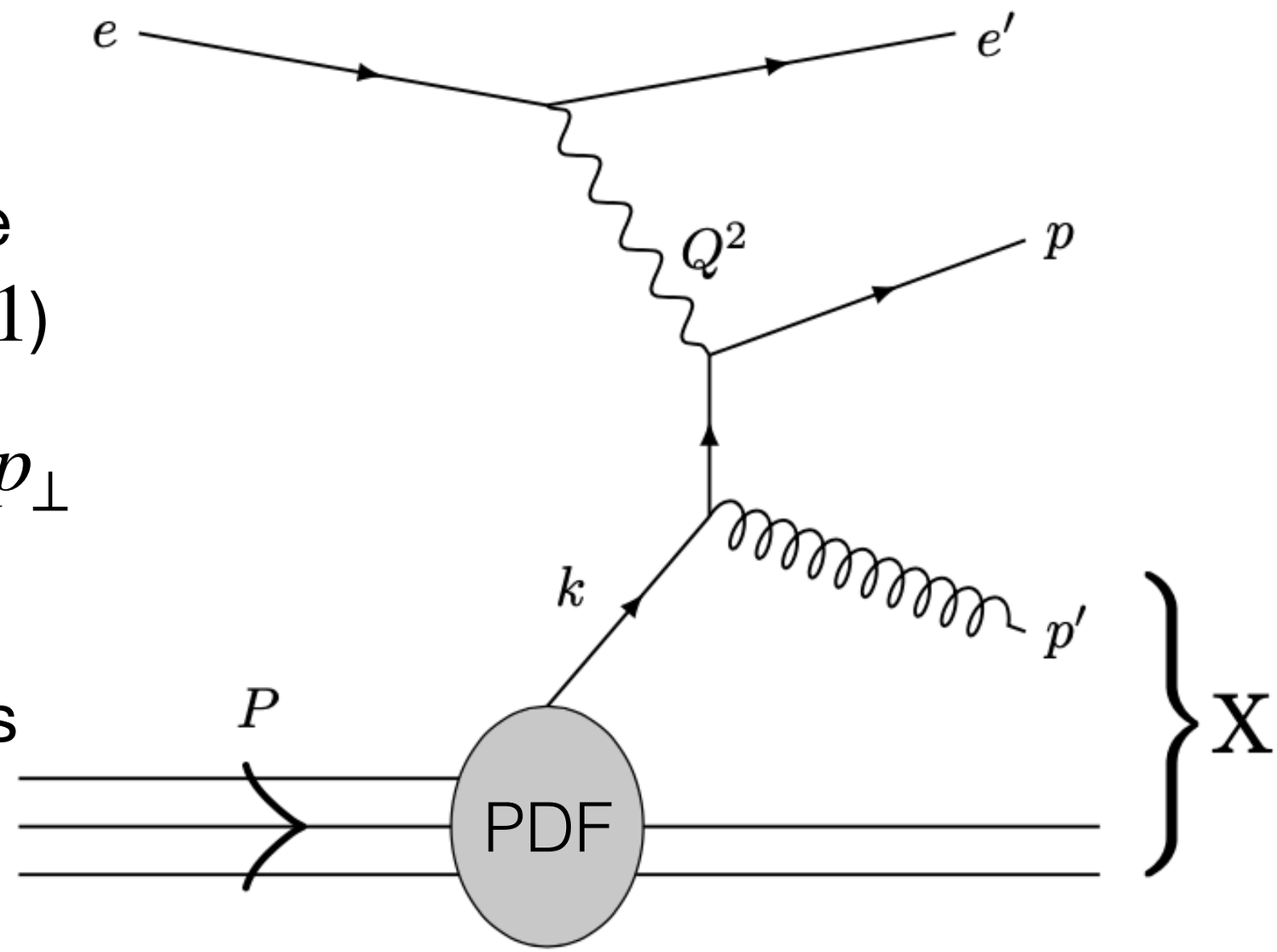
Reminder on transverse momentum dependent parton distribution functions (TMDs)

Primordial example: semi-inclusive DIS (SIDIS)

$Q^2 = -(e - e')^2$
is the hard scale which allows for a perturbative description ($\alpha_s(Q^2) \ll 1$)

Transverse momentum $p_\perp$ of the outgoing hadron (generated by recoil off unobserved parton) sets another scale



If $Q \sim p_\perp$, **collinear factorization** applies: $\sigma^{\gamma^* p \to hX} = \hat{\sigma}(Q^2) \otimes \sum_i f_i(x, Q^2) \otimes \sum_j D_j(z, Q^2)$

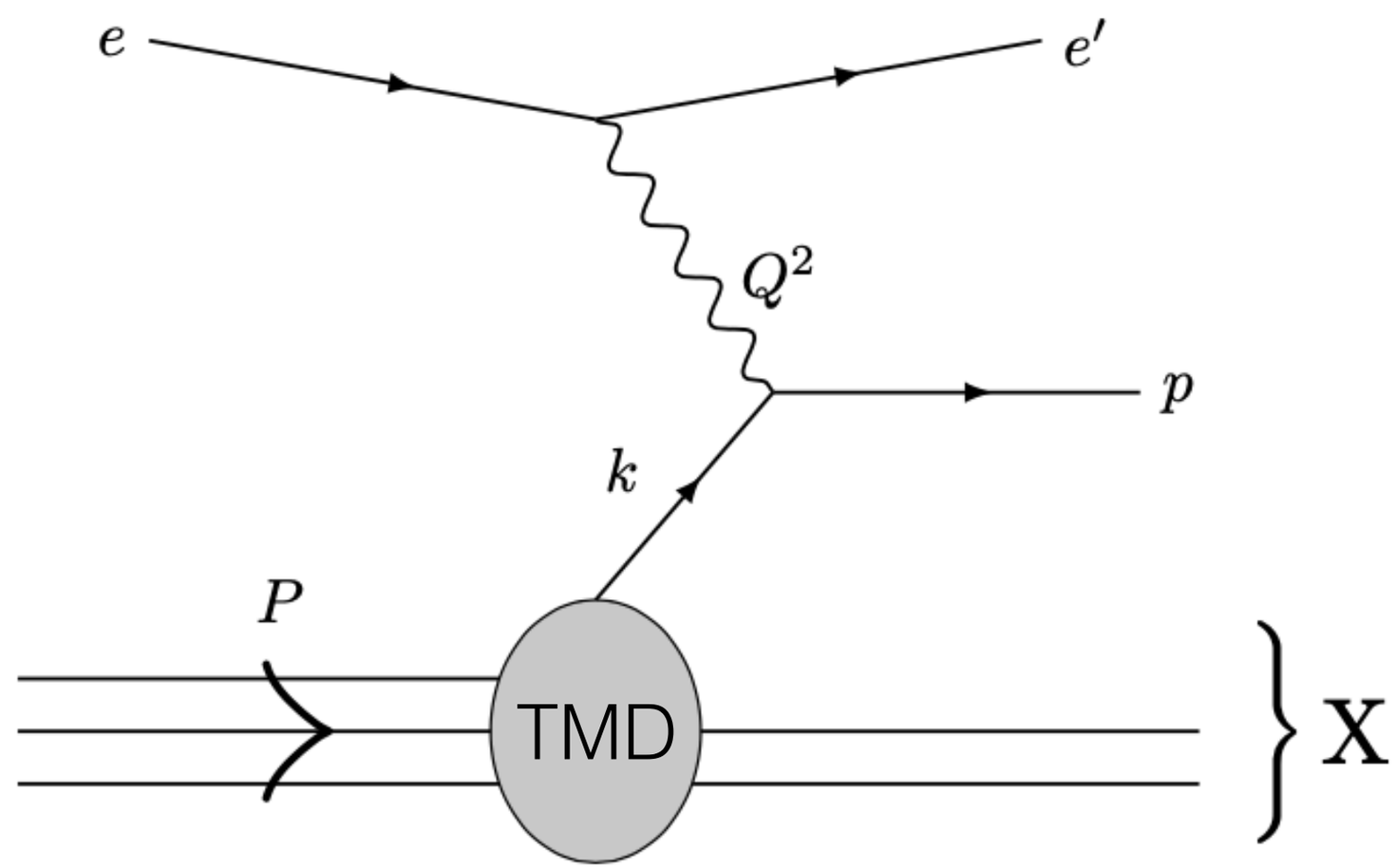
$$x = \frac{Q^2}{2P \cdot q} \simeq \frac{k^+}{P^+}$$

hard part **PDFs** **FFs**

Primordial example: semi-inclusive DIS (SIDIS)

What if $p_\perp \ll Q$? If transverse momentum is generated by soft emissions or stems from the primordial motion of the parton inside the hadron?

Need to resum large logs and introduce transverse momentum dependent PDFs (TMDs)



TMD factorization:

$$\sigma^{\gamma^* p \to hX} = \hat{\sigma}(Q^2) \otimes \int \mathrm{d}^2\mathbf{k}_\perp \sum_i f_i(x, k_\perp, Q^2) \otimes \sum_j D_j(z, p_\perp - k_\perp, Q^2)$$

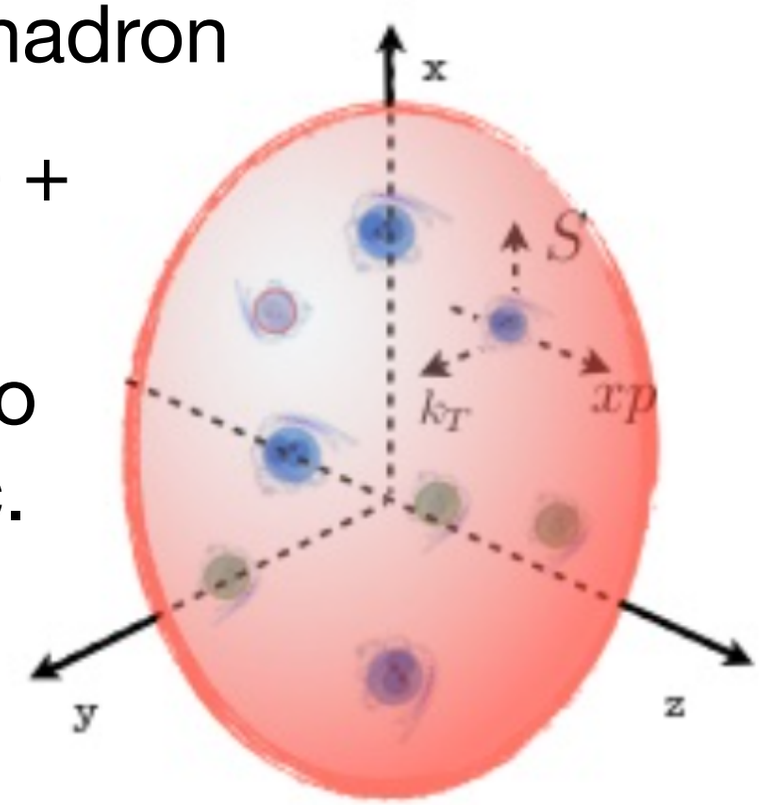
hard part **TMD PDFs** **TMD FFs**

Transverse momentum dependent PDFs (TMDs)

PDFs parameterize longitudinal structure of hadron

TMDs parameterize 3D momentum structure + spin correlations

Many intricacies: process dependence due to gauge invariance, complicated evolution, etc.



GLUONS	*unpolarized*	*circular*	*linear*
U	f_1^g		$h_1^{\perp g}$
L		g_{1L}^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	$h_{1T}^g,\ h_{1T}^{\perp g}$

Gluon TMDs are experimentally (almost) completely unknown!

Mulders & Rodrigues (2001)

Collins (2011)

Angelez-Martinez et al. (2015)

Operator definitions of (TMD) PDFs

Collins (2011)

Collinear gluon PDF:

$$\mathcal{G}(x,Q^2) = \int \frac{\mathrm{d}\xi^+}{\pi P^-} e^{ixP^-\xi^+} \mathrm{Tr}\langle P | F^{i-}(\xi^+) U^\dagger_{[\xi^+,0^+]} F^{i-}(0^+) U_{[\xi^+,0^+]} | P \rangle$$

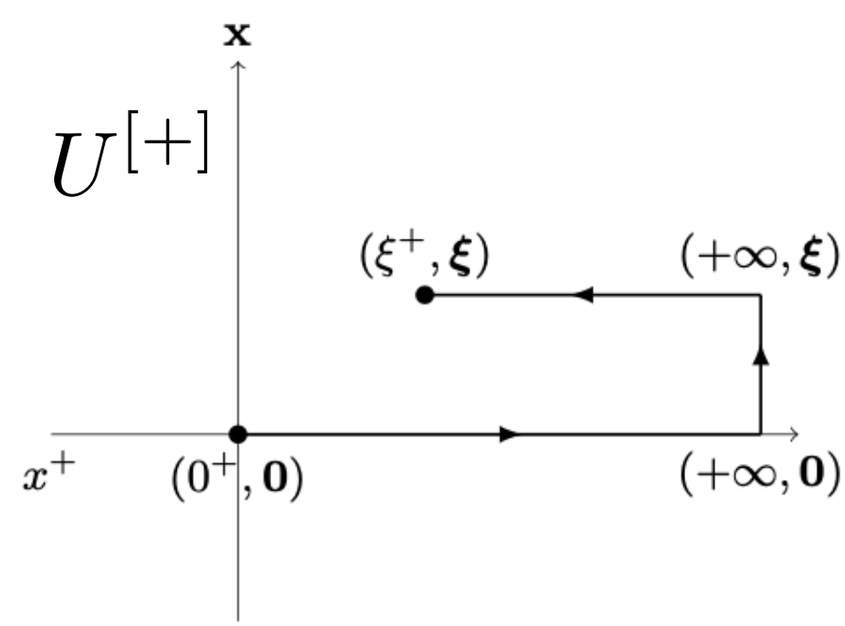
gauge links/Wilson lines to preserve gauge invariance

$$U_{[\xi^+,0^+]} = \mathcal{P} e^{ig_s \int \mathrm{d}x^+ A_c^-(x^+,\mathbf{0}) t^c}$$

Gluon TMD:

$$f^g_{1,WW}(x,k_\perp) = 2\int \frac{\mathrm{d}^3\vec{\xi}}{(2\pi)^3 P^-} e^{ixP^-\xi^+} e^{-i\mathbf{k}_\perp \boldsymbol{\xi}} \mathrm{Tr}\langle P | F^{i-}(\vec{\xi}) U^{[+]\dagger} F^{i-}(\vec{0}) U^{[+]} | P \rangle$$

different possible paths in $(x^+, \mathbf{x})$-plane to connect field strengths, TMDs come in different types



Towards CGC-TMD correspondence at NLO: gluon TMDs in trijet photoproduction at small x

QCD at high energies or small x

The *energy* is the largest scale in the process

$$s \gg Q^2$$

Need to resum large logarithms

$$\ln \frac{s}{Q^2} \simeq \ln \frac{1}{x} \gg 1$$

Low-x evolution equation (BFKL) predicts very fast growth of the gluon distribution:

$$x\mathcal{G}(x, Q^2) \simeq \frac{1}{x^{2.77 \frac{\alpha_s N_c}{\pi}}}$$

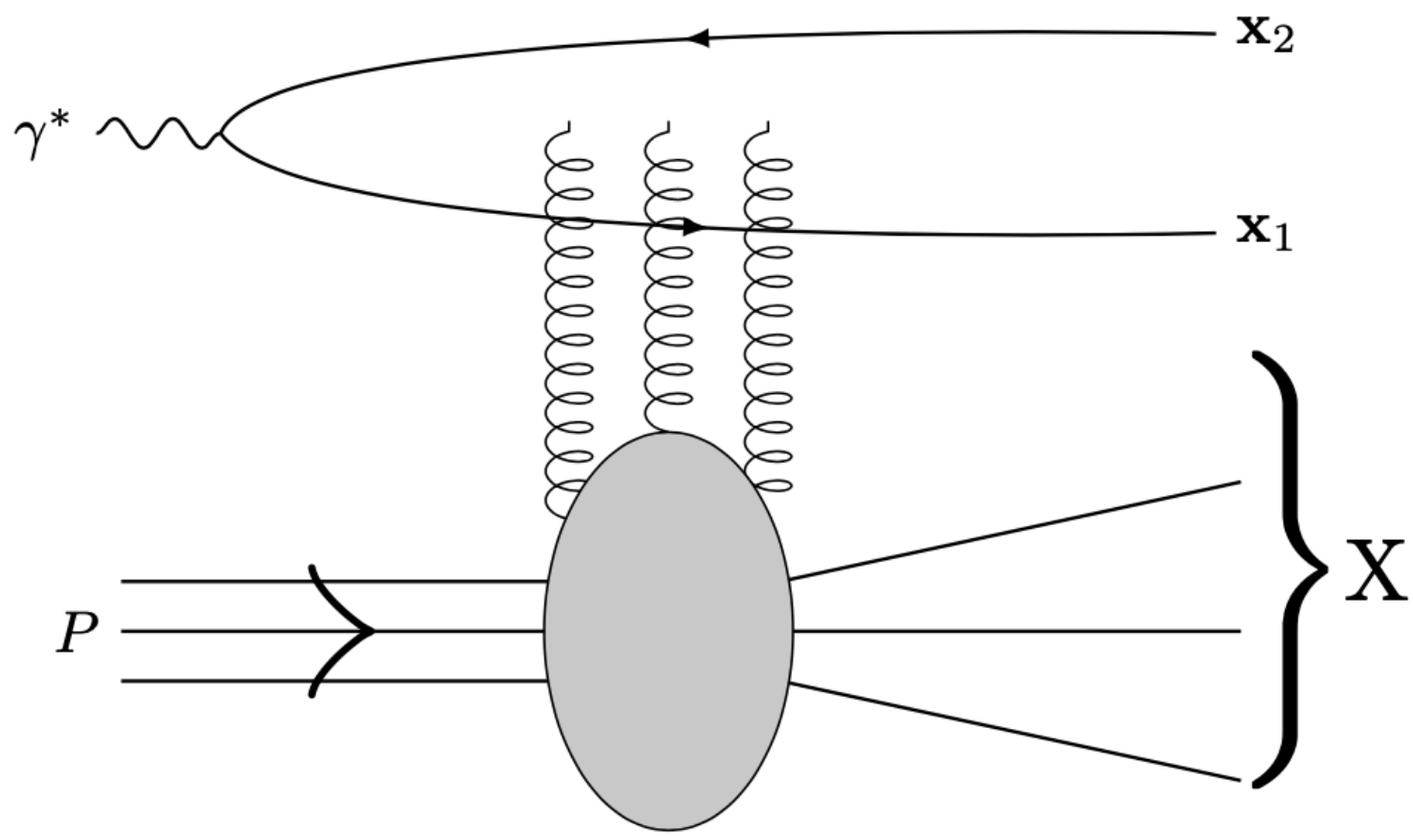
At high enough density, semiclassical regime is reached where

$$g_s A \sim 1$$

This regime is characterized by dynamically generated hard saturation scale Q_s, low-x evolution becomes nonlinear: BK-JIMWLK equations

McLerran, Venugopalan, Balitsky, Kovchegov, Iancu, Jalilian-Marian, Kovner, Leonidov, Weigert (1994-2002)

Dijet production in the CGC

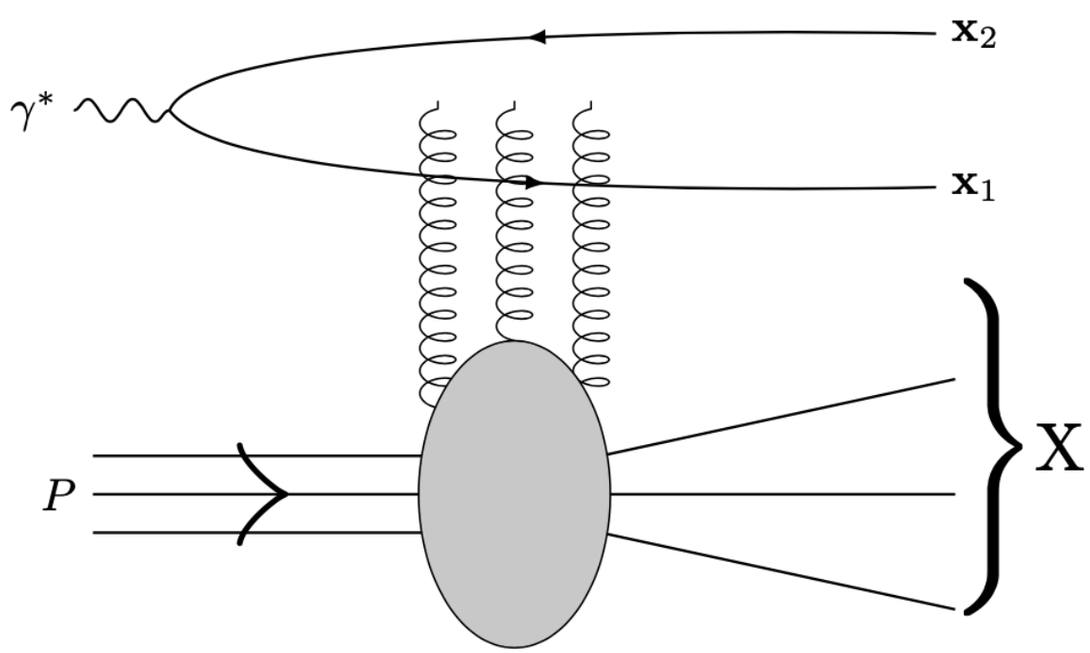


In Mueller's *dipole frame*, the virtual photon dissociates in a quark-antiquark pair long before the scattering off a highly boosted proton

Semiclassical approximation: frozen quark-antiquark dipole interacts with dense classical gluon fields through Wilson lines: $S_{F,A}(\mathbf{x}) = \mathcal{P} e^{ig_s \int \mathrm{d}x^+ A_c^-(\mathbf{x}^+,\mathbf{x}) t^c_{F,A}}$

Color Glass Condensate (CGC): nonperturbative model of initial state + perturbative evolution

Dijet production in the CGC



Scales: $s \gg Q^2 \gg Q_s^2$,
a priori no restrictions on the scales formed by the transverse momenta p_{1T} & p_{2T}

perturbative splitting functions

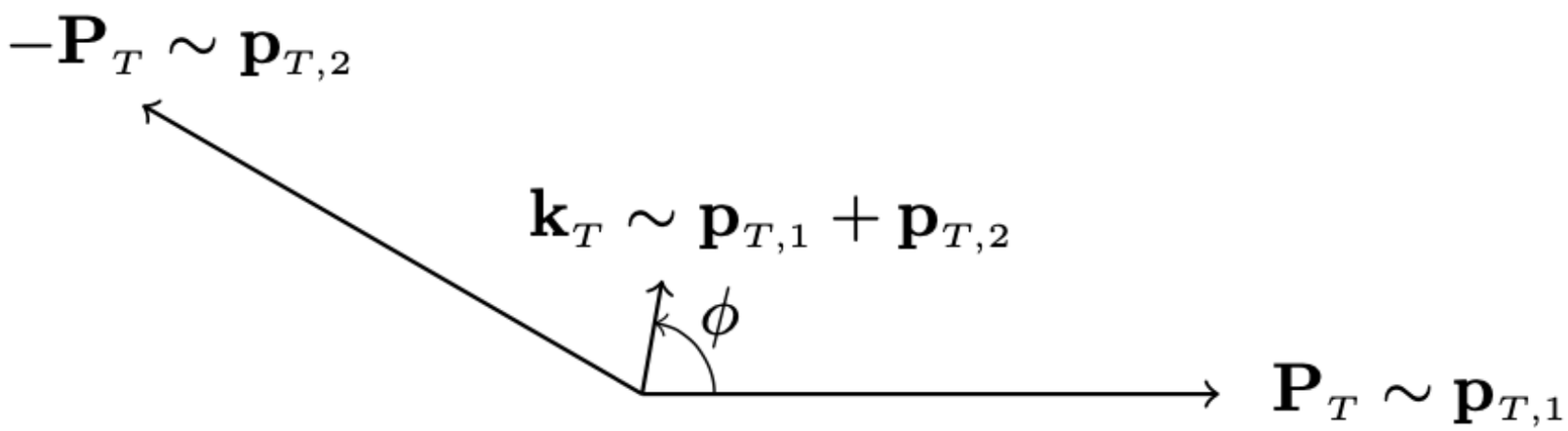
$$\frac{\mathrm{d}\sigma^{\gamma^* p \to q\bar{q}+X}}{\mathrm{d}^3\vec{p}_1 \mathrm{d}^3\vec{p}_2} \propto \int_{\mathbf{x}_1\mathbf{x}_2\mathbf{x}_1'\mathbf{x}_2'} e^{i\mathbf{p}_{1T}(\mathbf{x}_1-\mathbf{x}_1')} e^{i\mathbf{p}_{2T}(\mathbf{x}_2-\mathbf{x}_2')} \sum_{\lambda s_1 s_2} \psi^{L,T\lambda}_{s_1 s_2}(\mathbf{x}_1,\mathbf{x}_2)\psi^{L,T\lambda}_{s_1 s_2}(\mathbf{x}_1',\mathbf{x}_2')^*$$
$$\times \Big(Q(\mathbf{x}_1',\mathbf{x}_2',\mathbf{x}_2,\mathbf{x}_1) - s(\mathbf{x}_2,\mathbf{x}_1) - s(\mathbf{x}_2',\mathbf{x}_1') + 1\Big)$$

eikonal interaction of dipole with semiclassical gluon fields through Wilson lines:

$$s(\mathbf{x},\mathbf{y}) = \Big\langle \frac{1}{N_c}\mathrm{Tr}\big[S_F(\mathbf{x})S_F^\dagger(\mathbf{y})\big]\Big\rangle_{x_A},$$
$$Q(\mathbf{x},\mathbf{y},\mathbf{u},\mathbf{v}) = \Big\langle \frac{1}{N_c}\mathrm{Tr}\big[S_F(\mathbf{x})S_F^\dagger(\mathbf{y})S_F(\mathbf{u})S_F^\dagger(\mathbf{v})\big]\Big\rangle_{x_A}$$

$\langle \ldots \rangle_{x_A}$ = average over classical gluon fields of the target: nonperturbative info

Correlation limit



In the transverse back-to-back regime, one recovers an additional *small* scale given by the vector sum of the outgoing transverse momenta: $k_T \simeq |\mathbf{p}_{1T} + \mathbf{p}_{2T}|$

Scales: $s \gg Q^2 \sim p_{1T}^2 \sim p_{2T}^2 \gg k_T^2 \sim Q_s^2$
similar hierarchy as in TMD factorization!

Perform a twist expansion in k_T/Q = "correlation limit"

Wilson line structures involving derivatives:

$$\mathrm{Tr}\big\langle S_F^\dagger(\mathbf{u})\big[\partial_i S_F(\mathbf{u})\big] S_F^\dagger(\mathbf{v})\big[\partial_j S_F(\mathbf{v})\big]\big\rangle_x$$

Dominguez, Marquet, Xiao, Yuan (2011)

Emergence of gluon TMDs

Derivative on Wilson line → field strength

$$\partial_i S_F(\mathbf{x}) = \partial_i \mathcal{P} e^{ig_s \int_{-\infty}^{+\infty} \mathrm{d}x^+ A^-(x^+,\mathbf{x})}$$

$$= ig_s \int \mathrm{d}x^+ S_F(\mathbf{x}; -\infty, x^+) F^{i-}(x^+, \mathbf{x}) S_F(\mathbf{x}; x^+, +\infty)$$

After some algebra:

$$\mathrm{Tr}\big\langle S_F^\dagger(\mathbf{u})\big[\partial_i S_F(\mathbf{u})\big] S_F^\dagger(\mathbf{v})\big[\partial_j S_F(\mathbf{v})\big]\big\rangle_x$$

$$\overset{\lim x\to 0}{=} (ig_s)^2 \delta(0^+) \int \mathrm{d}\xi^+ e^{ixP^-\xi^+} \mathrm{Tr}\big\langle U^{[+]} F^{i-}(\xi^+, \mathbf{u}-\mathbf{v}) U^{[+]\dagger} F^{j-}(0^+, \mathbf{0})\big\rangle_x$$

…and finally:

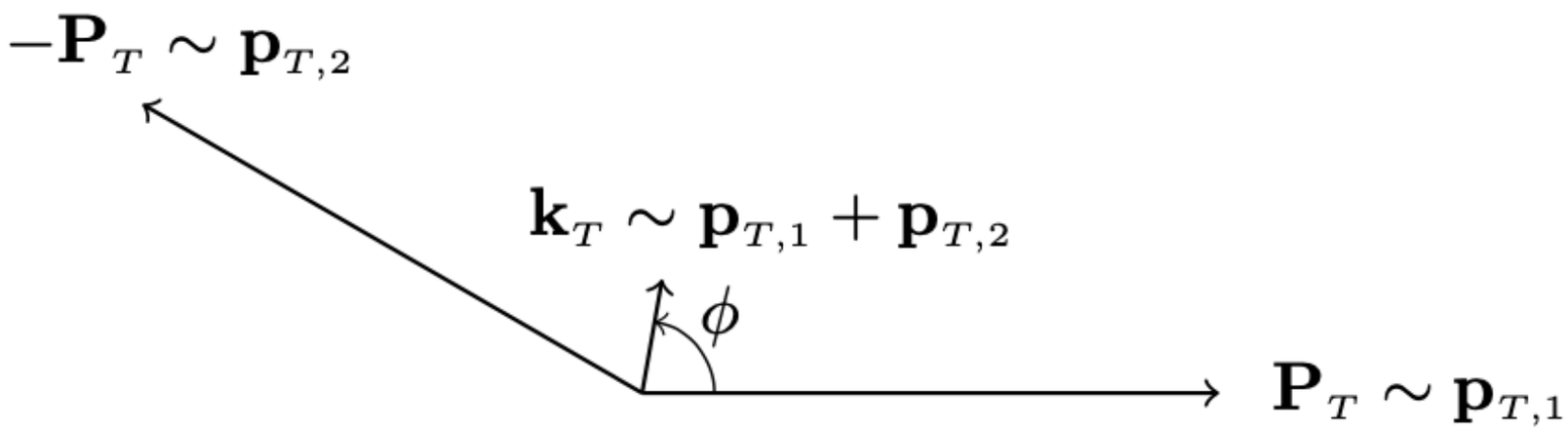
$$\int_{\mathbf{uv}} e^{-i\mathbf{k}_T(\mathbf{u}-\mathbf{v})} \mathrm{Tr}\big\langle S_F^\dagger(\mathbf{u})\big[\partial_i S_F(\mathbf{u})\big] S_F^\dagger(\mathbf{v})\big[\partial_j S_F(\mathbf{v})\big]\big\rangle_x$$

$$= -g_s^2 (2\pi)^3 \frac{1}{4}\left[\frac{1}{2}\delta^{ij} f^g_{1,WW}(x,k_T) + \frac{1}{2}\left(2\frac{\mathbf{k}_T^i \mathbf{k}_T^j}{k_T^2} - \delta^{ij}\right)\frac{k_T^2}{2M_p^2} h^{\perp g}_{1,WW}(x,k_T)\right]$$

unpolarized **linearly pol.**

Unpolarized and linearly polarized gluon TMDs of the Weizsäcker-Williams type

Correlation limit



TMD-factorized expression is recovered!

$$(2\pi)^6 \frac{\mathrm{d}\sigma^{\gamma^* p \to q\bar{q}+X}}{\mathrm{d}^3\vec{p}_1 \mathrm{d}^3\vec{p}_2}\bigg|_{\text{corr. limit}} = 2\pi\delta\left(p^+ - p_1^+ - p_2^+\right)\left[\mathrm{H}\right]_{ij}$$
$$\times\left[\frac{1}{2}\delta^{ij} f^g_{1,WW}(x,\mathbf{k}_T) + \frac{1}{2}\left(2\frac{k_T^i k_T^j}{\mathbf{k}_T^2} - \delta^{ij}\right)\frac{\mathbf{k}_T^2}{2M_p^2} h^{g\perp}_{1,WW}(x,\mathbf{k}_T)\right]$$

unpolarized **linearly polarized**

$$\sigma^{\gamma^* p \to q\bar{q}+X} = \hat{\sigma}(Q^2) \otimes \sum_i f_i(x, k_\perp, Q^2)$$

Dominguez, Marquet, Xiao, Yuan (2011)

Gluon TMDs at low x

For 2→2 processes in unpolarized collisions at low x, at LO the CGC and TMD frameworks yield *exactly* the same result.

CGC framework (low-*x*)	TMD framework (Q>>k)
CGC calculation	Calculate collinear hard parts
Correlation limit	Insert gauge links (Bomhof, Mulders, Pijlman 2006)
	Take low-*x* limit
Low-*x* TMD expression	

CGC machinery can be applied to TMD physics and vice versa!

Does this also hold for more complicated reactions, or beyond leading order?

Towards NLO: three-jet photoproduction

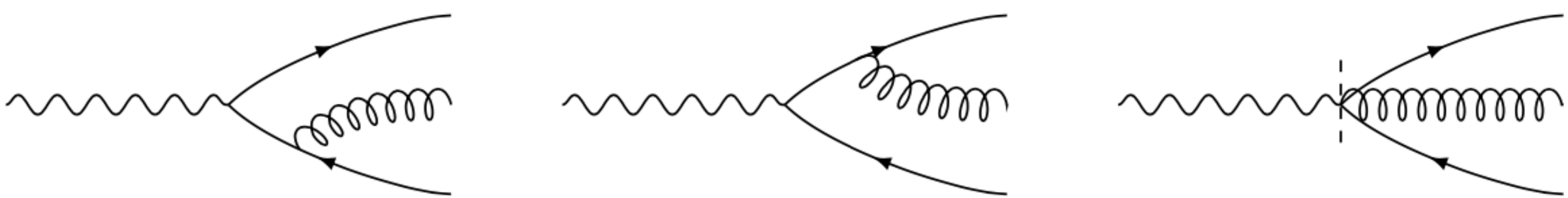
Cross section is given by:

$$2k_1^+ 2k_2^+ 2k_3^+ (2\pi)^9 2\pi\delta\big(p^+ - \sum_{j=1}^{3} k_j^+\big) 2p^+ \frac{\mathrm{d}\sigma^{\gamma A \to qg\bar{q}+X}}{\mathrm{d}^3\vec{k}_1\,\mathrm{d}^3\vec{k}_2\,\mathrm{d}^3\vec{k}_3}$$

$$= \frac{1}{2}\,{}_{\mathrm{out}}\big\langle(\boldsymbol{\gamma})\,[\vec{p}]_\lambda\big|N_q(\vec{k}_1)N_g(\vec{k}_2)N_{\bar{q}}(\vec{k}_3)\big|(\boldsymbol{\gamma})\,[\vec{p}]_\lambda\big\rangle_{\mathrm{out}}$$

Expansion of the photon Fock state

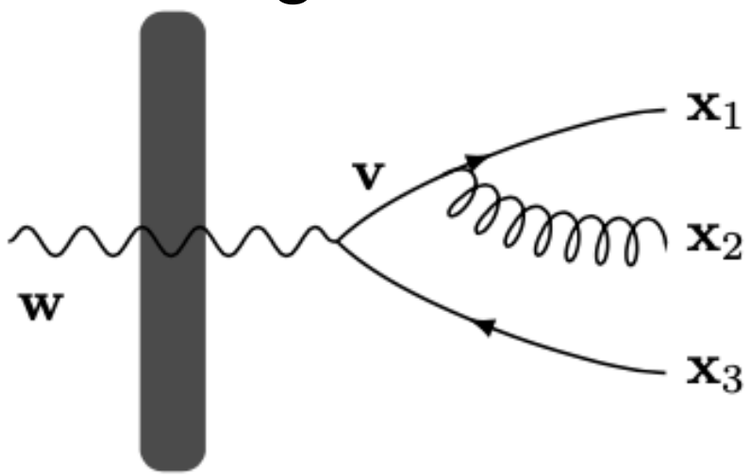
$$|\boldsymbol{\gamma}\rangle_{\mathrm{dressed}} = Z_0|\boldsymbol{\gamma}\rangle_0 + g_e Z_1|\mathbf{q}\bar{\mathbf{q}}\rangle_0 + g_e g_s Z_2|\mathbf{q}\bar{\mathbf{q}}\mathbf{g}\rangle_0 + \mathcal{O}$$



Altinoluk, Boussarie, Marquet, PT (2020)

Eikonal approximation

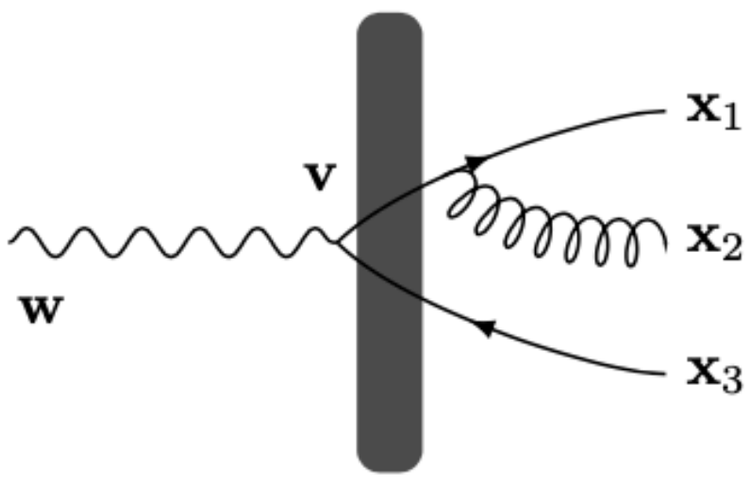
In our frame, dressed photon hits Lorentz contracted proton or nucleus target = ‘shockwave’



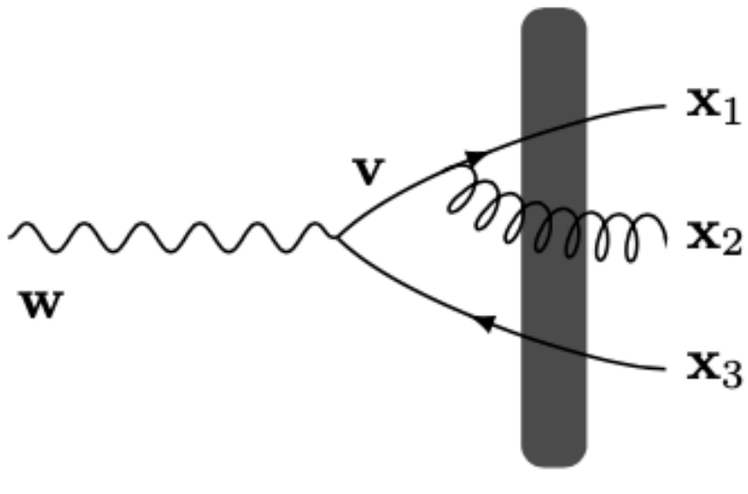
$\mathbb{1}$

Wilson line:

$$S_{F,A}(\mathbf{x}) = \mathcal{P} e^{ig_s \int \mathrm{d}x^+ A_c^-(\mathbf{x}^+,\mathbf{x}) t^c_{F,A}}$$



$$t^c S_F(\mathbf{v}) S_F^\dagger(\mathbf{x}_3)$$



$$S_F(\mathbf{x}_1) t^d S_A^{dc}(\mathbf{x}_2) S_F^\dagger(\mathbf{x}_3)$$

Outgoing photon state

$$
\begin{aligned}
|(\boldsymbol{\gamma})[\vec{p}]_\lambda\rangle_{\rm out} = g_e g_s \int \frac{{\rm d}k_1^+}{2\pi}\frac{{\rm d}k_2^+}{2\pi}\int_{\mathbf{w}\mathbf{x}_1\mathbf{x}_2\mathbf{x}_3} & |(\mathbf{q})[k_1^+,\mathbf{x}_1]_s^i;(\mathbf{g})[k_2^+,\mathbf{x}_2]_c^\eta;(\bar{\mathbf{q}})[k_3^+,\mathbf{x}_3]_{s'}^j\rangle_D \\
\times\Bigg\{\Big(\big[S_F(\mathbf{x}_1)t^d S_F^\dagger(\mathbf{x}_3)\big]_{ij} & S_A(\mathbf{x}_2)^{dc} - t_{ij}^c\Big) \\
& \times\left(F_q^{(2)}+F_{\bar{q}}^{(2)}+F_C^{(2)}\right)\Big[(\mathbf{q})[k_1^+,\mathbf{w}-\mathbf{x}_1];(\mathbf{g})[k_2^+,\mathbf{w}-\mathbf{x}_2];(\bar{\mathbf{q}})[k_3^+,\mathbf{w}-\mathbf{x}_3]\Big]_{s's}^{\eta\lambda} \\
-\int_{\mathbf{v}}\Big(\big[t^c S_F(\mathbf{v})S_F^\dagger(\mathbf{x}_3)\big]_{ij} & - t_{ij}^c\Big)F_\gamma^{(1)}\Big[(\mathbf{q})[k_1^+ + k_2^+,\mathbf{w}-\mathbf{v}];(\bar{\mathbf{q}})[k_3^+,\mathbf{w}-\mathbf{x}_3]\Big]_{\bar{s}s'}^{\lambda} \\
& \times F_q^{(1)}\Big[(\mathbf{q})[k_1^+,\mathbf{v}-\mathbf{x}_1];(\mathbf{g})[k_2^+,\mathbf{v}-\mathbf{x}_2]\Big]_{s\bar{s}}^{\eta} \\
-\int_{\mathbf{v}}\Big(\big[S_F(\mathbf{x}_1)S_F^\dagger(\mathbf{v})t^c\big]_{ij} & - t_{ij}^c\Big)F_\gamma^{(1)}\Big[(\mathbf{q})[k_1^+,\mathbf{w}-\mathbf{x}_1];(\bar{\mathbf{q}})[k_2^+ + k_3^+,\mathbf{w}-\mathbf{v}]\Big]_{s\tilde{s}}^{\lambda} \\
& \times F_{\bar{q}}^{(1)}\Big[(\mathbf{g})[k_2^+,\mathbf{v}-\mathbf{x}_2];(\bar{\mathbf{q}})[k_3^+,\mathbf{v}-\mathbf{x}_3]\Big]_{s'\tilde{s}}^{\eta}\Bigg\}
\end{aligned}
$$

Outgoing photon state

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\times\Big\{ & \Big(\big[S_F(\mathbf{x}_1)t^d S_F^\dagger(\mathbf{x}_3)\big]_{ij} S_A(\mathbf{x}_2)^{dc} - t_{ij}^c\Big) \\
& \times\left(F_q^{(2)}+F_{\bar{q}}^{(2)}+F_C^{(2)}\right)\Big[(\mathbf{q})[k_1^+,\mathbf{w}-\mathbf{x}_1];(\mathbf{g})[k_2^+,\mathbf{w}-\mathbf{x}_2];(\bar{\mathbf{q}})[k_3^+,\mathbf{w}-\mathbf{x}_3]\Big]_{s's}^{\eta\lambda} \\
& -\int_{\mathbf{v}}\Big(\big[t^c S_F(\mathbf{v})S_F^\dagger(\mathbf{x}_3)\big]_{ij}-t_{ij}^c\Big)F_\gamma^{(1)}\Big[(\mathbf{q})[k_1^++k_2^+,\mathbf{w}-\mathbf{v}];(\bar{\mathbf{q}})[k_3^+,\mathbf{w}-\mathbf{x}_3]\Big]_{\bar{s}s'}^{\lambda} \\
& \quad\times F_q^{(1)}\Big[(\mathbf{q})[k_1^+,\mathbf{v}-\mathbf{x}_1];(\mathbf{g})[k_2^+,\mathbf{v}-\mathbf{x}_2]\Big]_{s\bar{s}}^{\eta} \\
& -\int_{\mathbf{v}}\Big(\big[S_F(\mathbf{x}_1)S_F^\dagger(\mathbf{v})t^c\big]_{ij}-t_{ij}^c\Big)F_\gamma^{(1)}\Big[(\mathbf{q})[k_1^+,\mathbf{w}-\mathbf{x}_1];(\bar{\mathbf{q}})[k_2^++k_3^+,\mathbf{w}-\mathbf{v}]\Big]_{s\tilde{s}}^{\lambda} \\
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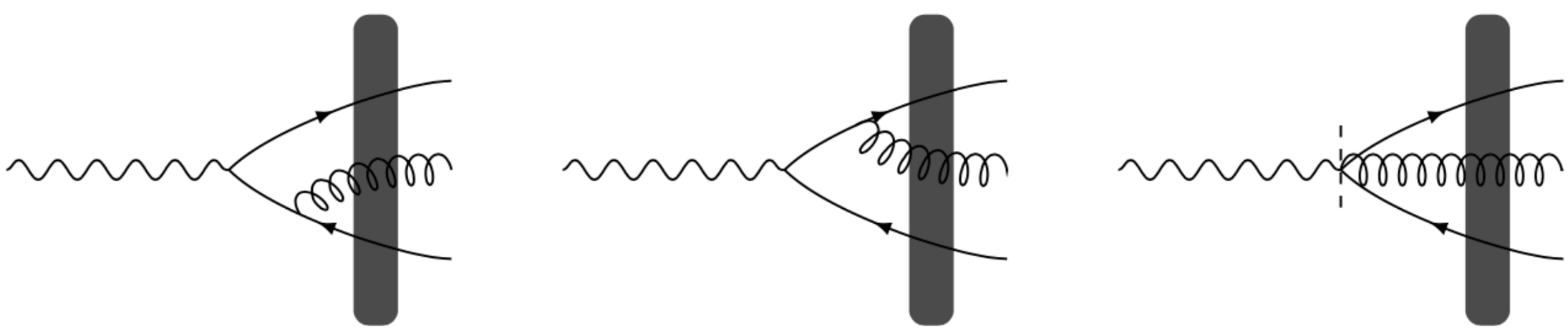

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&\times F_{\bar{q}}^{(1)}\Big[(\mathbf{g})[k_2^+,\mathbf{v}-\mathbf{x}_2];(\bar{\mathbf{q}})[k_3^+,\mathbf{v}-\mathbf{x}_3]\Big]_{s'\tilde{s}}^{\eta}\Big\}
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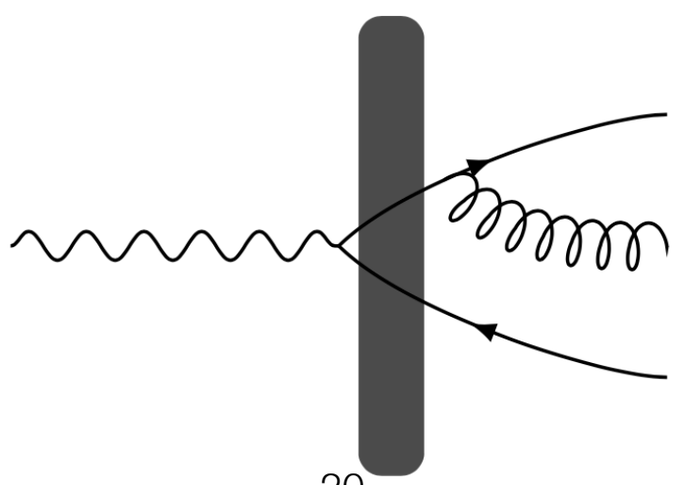

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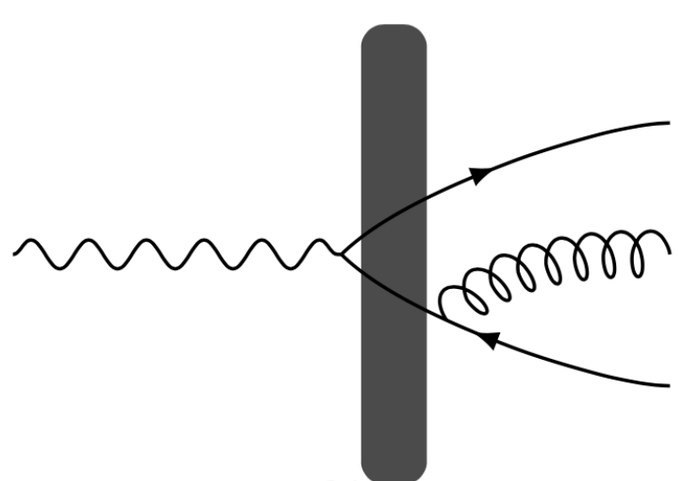
\times\Big\{\Big(&\big[S_F(\mathbf{x}_1)t^d S_F^\dagger(\mathbf{x}_3)\big]_{ij}S_A(\mathbf{x}_2)^{dc}-t_{ij}^c\Big) \\
&\times\left(F_q^{(2)}+F_{\bar{q}}^{(2)}+F_C^{(2)}\right)\Big[(\mathbf{q})[k_1^+,\mathbf{w}-\mathbf{x}_1];(\mathbf{g})[k_2^+,\mathbf{w}-\mathbf{x}_2];(\bar{\mathbf{q}})[k_3^+,\mathbf{w}-\mathbf{x}_3]\Big]_{s's}^{\eta\lambda} \\
-\int_{\mathbf{v}}\Big(&\big[t^c S_F(\mathbf{v})S_F^\dagger(\mathbf{x}_3)\big]_{ij}-t_{ij}^c\Big)F_\gamma^{(1)}\Big[(\mathbf{q})[k_1^++k_2^+,\mathbf{w}-\mathbf{v}];(\bar{\mathbf{q}})[k_3^+,\mathbf{w}-\mathbf{x}_3]\Big]_{\bar{s}s'}^{\lambda} \\
&\times F_q^{(1)}\Big[(\mathbf{q})[k_1^+,\mathbf{v}-\mathbf{x}_1];(\mathbf{g})[k_2^+,\mathbf{v}-\mathbf{x}_2]\Big]_{s\bar{s}}^{\eta} \\
-\int_{\mathbf{v}}\Big(&\big[S_F(\mathbf{x}_1)S_F^\dagger(\mathbf{v})t^c\big]_{ij}-t_{ij}^c\Big)F_\gamma^{(1)}\Big[(\mathbf{q})[k_1^+,\mathbf{w}-\mathbf{x}_1];(\bar{\mathbf{q}})[k_2^++k_3^+,\mathbf{w}-\mathbf{v}]\Big]_{s\tilde{s}}^{\lambda} \\
&\times F_{\bar{q}}^{(1)}\Big[(\mathbf{g})[k_2^+,\mathbf{v}-\mathbf{x}_2];(\bar{\mathbf{q}})[k_3^+,\mathbf{v}-\mathbf{x}_3]\Big]_{s'\tilde{s}}^{\eta}\Big\}
\end{aligned}
$$



Differential $\gamma A \to q\bar{q}g + X$ cross section

$$(2\pi)^9 \frac{\mathrm{d}\sigma^{\gamma A \to q\bar{q}g+X}}{\mathrm{d}^3\vec{k}_1 \mathrm{d}^3\vec{k}_2 \mathrm{d}^3\vec{k}_3} = g_e^2 g_s^2 \frac{1}{k_2^+ p^+} 2\pi\delta\big(p^+ - \sum_{i=1}^{3} k_i^+\big)$$
$$\times \big\langle I_{qq} + I_{\bar{q}\bar{q}} + I_{CC} + 2I_{q\bar{q}} + 2I_{Cq} + 2I_{C\bar{q}} \big\rangle_{x_A}$$

Differential $\gamma A \to q\bar{q}g + X$ cross section

$$
\begin{aligned}
\langle I_{qq} \rangle_{x_A} = \frac{N_c^2}{2} \mathcal{M}_{qq}^{\bar{\eta}\bar{\eta}';\bar{\lambda}\bar{\lambda}'}\left(\bar{\xi}_3, \frac{\xi_2}{\bar{\xi}_3}\right) \int_{\mathbf{v}\mathbf{v}'} \prod_{i=1}^{3} \int_{\mathbf{x}_i \mathbf{x}_i'} e^{i\mathbf{k}_i \cdot (\mathbf{x}_i' - \mathbf{x}_i)} \\
\times A^{\bar{\eta}}(\mathbf{x}_1 - \mathbf{x}_2) A^{\bar{\eta}'}(\mathbf{x}_1' - \mathbf{x}_2')\, \delta^{(2)}\left(\mathbf{v} - \frac{\xi_1}{\bar{\xi}_3}\mathbf{x}_1 - \frac{\xi_2}{\bar{\xi}_3}\mathbf{x}_2\right) \delta^{(2)}\left(\mathbf{v}' - \frac{\xi_1}{\bar{\xi}_3}\mathbf{x}_1' - \frac{\xi_2}{\bar{\xi}_3}\mathbf{x}_2'\right) \\
\times \Bigg\{ \left[W_1\left(\mathbf{x}_1.\mathbf{x}_2; \mathbf{x}_2', \mathbf{x}_1' \middle| \mathbf{x}_2, \mathbf{x}_3; \mathbf{x}_3', \mathbf{x}_2'\right) - \frac{1}{N_c^2} W_3\left(\mathbf{x}_1, \mathbf{x}_3; \mathbf{x}_3', \mathbf{x}_1'\right) \right] \\
\times \mathcal{A}^{\bar{\lambda}}\left(\xi_3, \mathbf{x}_3 - \mathbf{v}; \frac{\xi_1}{\bar{\xi}_3}, \mathbf{x}_1 - \mathbf{x}_2\right) \mathcal{A}^{\bar{\lambda}'}\left(\xi_3, \mathbf{x}_3' - \mathbf{v}'; \frac{\xi_1}{\bar{\xi}_3}, \mathbf{x}_1' - \mathbf{x}_2'\right) \\
- \left[W_2\left(\mathbf{x}_1, \mathbf{x}_2 \middle| \mathbf{x}_2, \mathbf{x}_3; \mathbf{x}_3', \mathbf{v}'\right) - \frac{1}{N_c^2} W_3\left(\mathbf{x}_1, \mathbf{x}_3; \mathbf{x}_3', \mathbf{v}'\right) \right] \\
\times \mathcal{A}^{\bar{\lambda}}\left(\xi_3, \mathbf{x}_3 - \mathbf{v}; \frac{\xi_1}{\bar{\xi}_3}, \mathbf{x}_1 - \mathbf{x}_2\right) A^{\bar{\lambda}'}(\mathbf{x}_3' - \mathbf{v}') \\
- \left[W_2\left(\mathbf{x}_2', \mathbf{x}_1' \middle| \mathbf{v}, \mathbf{x}_3; \mathbf{x}_3', \mathbf{x}_2'\right) - \frac{1}{N_c^2} W_3\left(\mathbf{v}, \mathbf{x}_3; \mathbf{x}_3', \mathbf{x}_1'\right) \right] \\
\times A^{\bar{\lambda}}(\mathbf{x}_3 - \mathbf{v}) \mathcal{A}^{\bar{\lambda}'}\left(\xi_3, \mathbf{x}_3' - \mathbf{v}'; \frac{\xi_1}{\bar{\xi}_3}, \mathbf{x}_1' - \mathbf{x}_2'\right) \\
- \left(1 - \frac{1}{N_c^2}\right) W_3\left(\mathbf{v}, \mathbf{x}_3; \mathbf{x}_3', \mathbf{v}'\right) A^{\bar{\lambda}}(\mathbf{x}_3 - \mathbf{v}) A^{\bar{\lambda}'}(\mathbf{x}_3' - \mathbf{v}') \Bigg\}
\end{aligned}
$$

Differential $\gamma A \to q\bar{q}g + X$ cross section

$$\langle I_{qq} \rangle_{x_A} = \frac{N_c^2}{2} \mathcal{M}_{qq}^{\bar{\eta}\bar{\eta}';\bar{\lambda}\bar{\lambda}'}\left(\bar{\xi}_3, \frac{\xi_2}{\bar{\xi}_3}\right) \int_{\mathbf{v}\mathbf{v}'} \prod_{i=1}^{3} \int_{\mathbf{x}_i \mathbf{x}_i'} e^{i\mathbf{k}_i \cdot (\mathbf{x}_i' - \mathbf{x}_i)}$$

$$\times A^{\bar{\eta}}(\mathbf{x}_1 - \mathbf{x}_2) A^{\bar{\eta}'}(\mathbf{x}_1' - \mathbf{x}_2')\, \delta^{(2)}\left(\mathbf{v} - \frac{\xi_1}{\bar{\xi}_3}\mathbf{x}_1 - \frac{\xi_2}{\bar{\xi}_3}\mathbf{x}_2\right) \delta^{(2)}\left(\mathbf{v}' - \frac{\xi_1}{\bar{\xi}_3}\mathbf{x}_1' - \frac{\xi_2}{\bar{\xi}_3}\mathbf{x}_2'\right)$$

$$\times \left\{ \left[W_1\left(\mathbf{x}_1.\mathbf{x}_2; \mathbf{x}_2', \mathbf{x}_1' \middle| \mathbf{x}_2, \mathbf{x}_3; \mathbf{x}_3', \mathbf{x}_2'\right) - \frac{1}{N_c^2} W_3\left(\mathbf{x}_1, \mathbf{x}_3; \mathbf{x}_3', \mathbf{x}_1'\right) \right] \right.$$

$$+ \ldots \qquad \times \mathcal{A}^{\bar{\lambda}}\left(\xi_3, \mathbf{x}_3 - \mathbf{v}; \frac{\xi_1}{\bar{\xi}_3}, \mathbf{x}_1 - \mathbf{x}_2\right) \mathcal{A}^{\bar{\lambda}'}\left(\xi_3, \mathbf{x}_3' - \mathbf{v}'; \frac{\xi_1}{\bar{\xi}_3}, \mathbf{x}_1' - \mathbf{x}_2'\right)$$

W's are combinations of dipoles and quadrupoles:

$$s(\mathbf{x}, \mathbf{y}) = \left\langle \frac{1}{N_c} \mathrm{Tr}\left[S_F(\mathbf{x}) S_F^\dagger(\mathbf{y})\right] \right\rangle_{x_A} ,$$

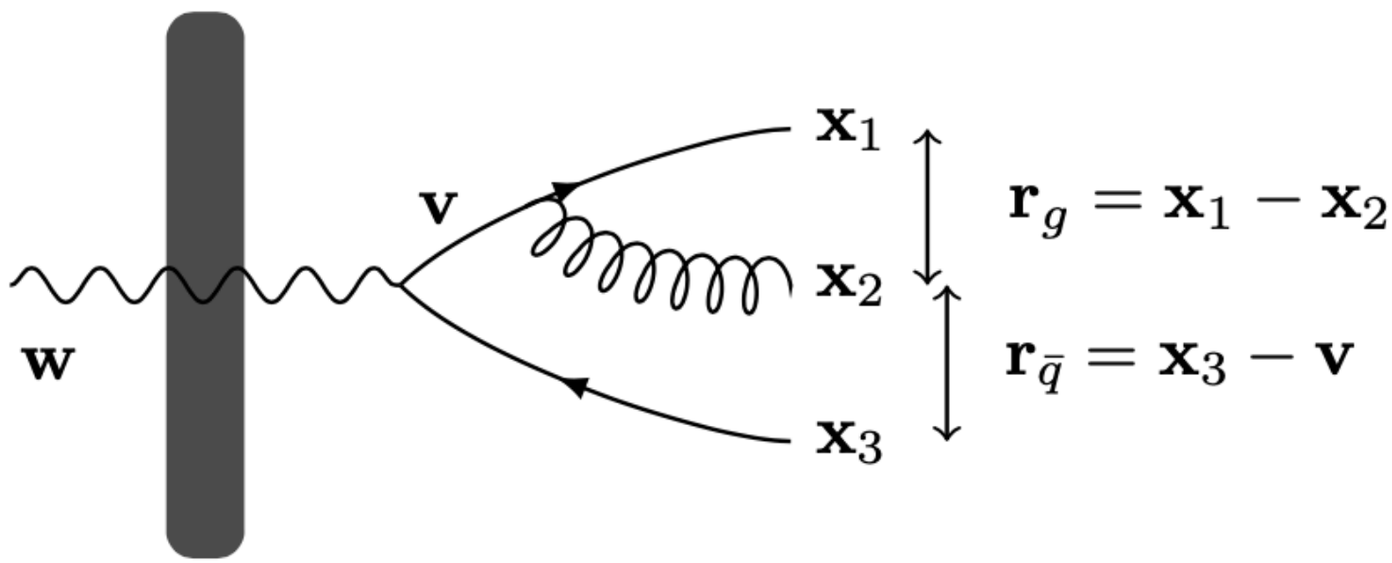
$$Q(\mathbf{x}, \mathbf{y}, \mathbf{u}, \mathbf{v}) = \left\langle \frac{1}{N_c} \mathrm{Tr}\left[S_F(\mathbf{x}) S_F^\dagger(\mathbf{y}) S_F(\mathbf{u}) S_F^\dagger(\mathbf{v})\right] \right\rangle_{x_A}$$

$\langle \ldots \rangle_{x_A}$ is the average over the semiclassical gluon fields of the target → contains the nonperturbative information on the hadron structure

Correlation limit

Defining the total transverse momentum: $\mathbf{q}_T \equiv \mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3$, study the limit $|\mathbf{q}_T| \ll |\mathbf{k}_1| \sim |\mathbf{k}_2| \sim |\mathbf{k}_3|$

$$|\mathbf{k}_i| \gg 1 \leftrightarrow |\mathbf{r}_i| \ll 1$$



Perform Taylor expansion around small dipole sizes $\mathbf{r}$

$\rightarrow$ derivatives enter Wilson line structures:

$$\mathrm{Tr}\langle S_F^\dagger(\mathbf{x})[\partial_i S_F(\mathbf{x})] S_F^\dagger(\mathbf{y})[\partial_j S_F(\mathbf{y})]\rangle_x$$

Final result

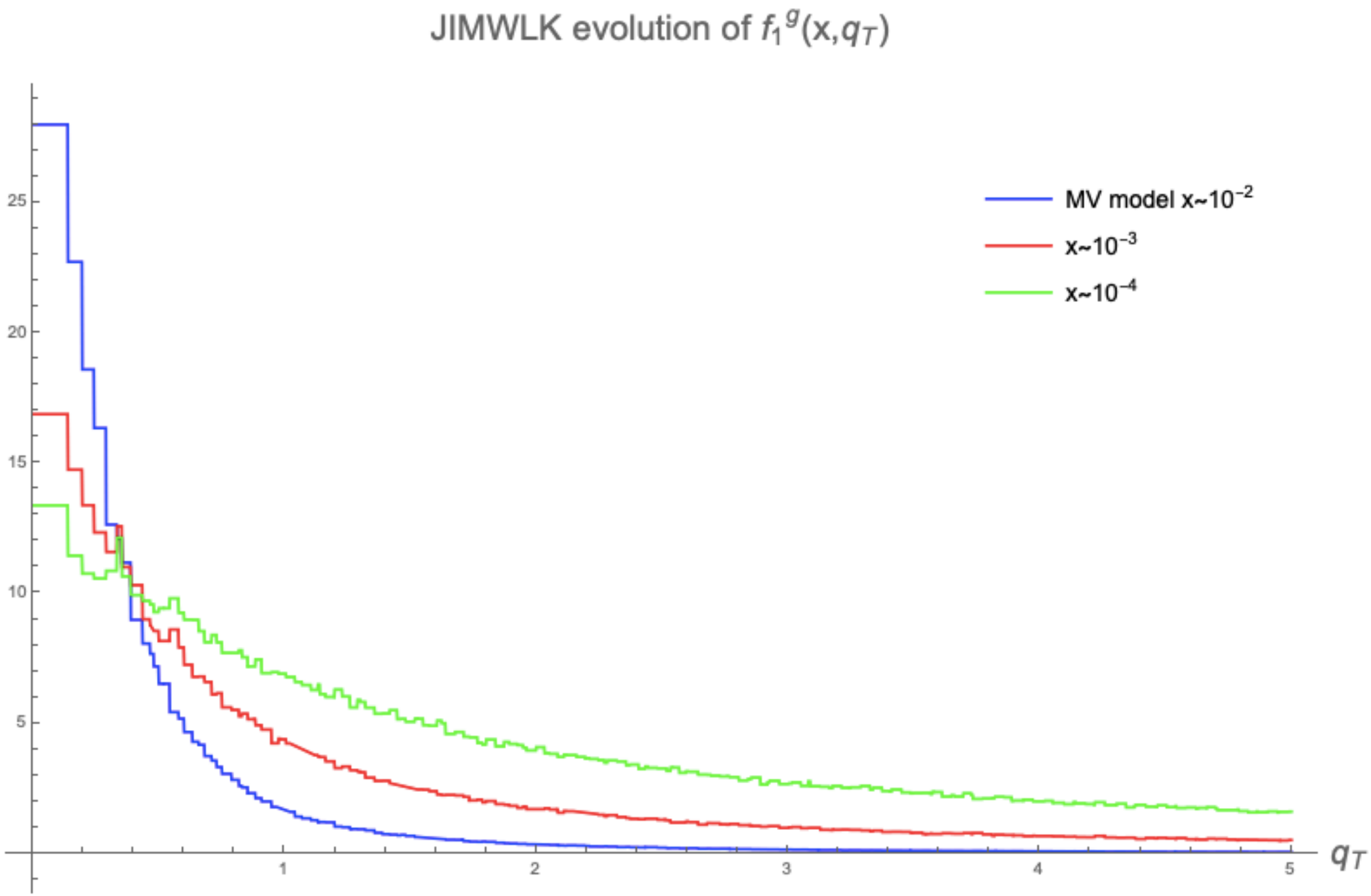
$$(2\pi)^9 \frac{\mathrm{d}\sigma^{\gamma A\to q\bar{q}g+X}}{\mathrm{d}^3\vec{k}_1\mathrm{d}^3\vec{k}_2\mathrm{d}^3\vec{k}_3}\bigg|_{\text{corr. limit}} = 2\pi\delta\big(p^+ - \sum_{i=1}^{3} k_i^+\big)\,[\mathrm{H}]_{ij}^{\text{total}}$$
$$\times\left[\frac{1}{2}\delta^{ij}\mathcal{F}_{gg}^{(3)}(x_A,\mathbf{q}_T) + \frac{1}{2}\Big(2\frac{\mathbf{q}_T^i\mathbf{q}_T^j}{\mathbf{q}_T^2} - \delta^{ij}\Big)\mathcal{H}_{gg}^{(3)}(x_A,\mathbf{q}_T)\right]$$

$$[\mathrm{H}]_{ij}^{\text{total}} = N_c\, g_e^2\, g_s^4\, \pi^3\, \frac{1}{k_2^+ p^+}\Big\{\mathcal{M}_{qq}^{\bar{\eta}\bar{\eta}';\bar{\lambda}\bar{\lambda}'}\Big(\bar{\xi}_3,\frac{\xi_2}{\bar{\xi}_3}\Big)[\mathrm{H}_{qq}]_{ij}^{\bar{\lambda}\bar{\lambda}';\bar{\eta}\bar{\eta}'} + \mathcal{M}_{\bar{q}\bar{q}}^{\bar{\eta}\bar{\eta}';\bar{\lambda}\bar{\lambda}'}\Big(\xi_1,\frac{\xi_2}{\bar{\xi}_1}\Big)[\mathrm{H}_{\bar{q}\bar{q}}]_{ij}^{\bar{\lambda}\bar{\lambda}';\bar{\eta}\bar{\eta}'}$$
$$+2\,\mathcal{M}_{q\bar{q}}^{\bar{\eta}\bar{\eta}';\bar{\lambda}\bar{\lambda}'}(\xi_1,\xi_2)\,[\mathrm{H}_{q\bar{q}}]_{ij}^{\bar{\lambda}\bar{\lambda}';\bar{\eta}\bar{\eta}'} + \mathcal{M}_{CC}(\xi_1,\xi_2)\frac{1}{\xi_3^2\bar{\xi}_3^2}[\mathrm{H}_{CC}]_{ij}$$
$$+2\,\mathcal{M}_{Cq}(\xi_1,\xi_2)\,\frac{2}{\xi_3\bar{\xi}_3}\,[\mathrm{H}_{Cq}]_{ij}^{\bar{\lambda}\bar{\eta}} + 2\,\mathcal{M}_{C\bar{q}}(\xi_1,\xi_2)\,\frac{2}{\xi_3\bar{\xi}_3}\,[\mathrm{H}_{C\bar{q}}]_{ij}^{\bar{\lambda}\bar{\eta}}\Big\}$$

Again the result is written in the form of a LO TMD factorization cross section.

Let us apply some CGC results to this cross section to do phenomenology.

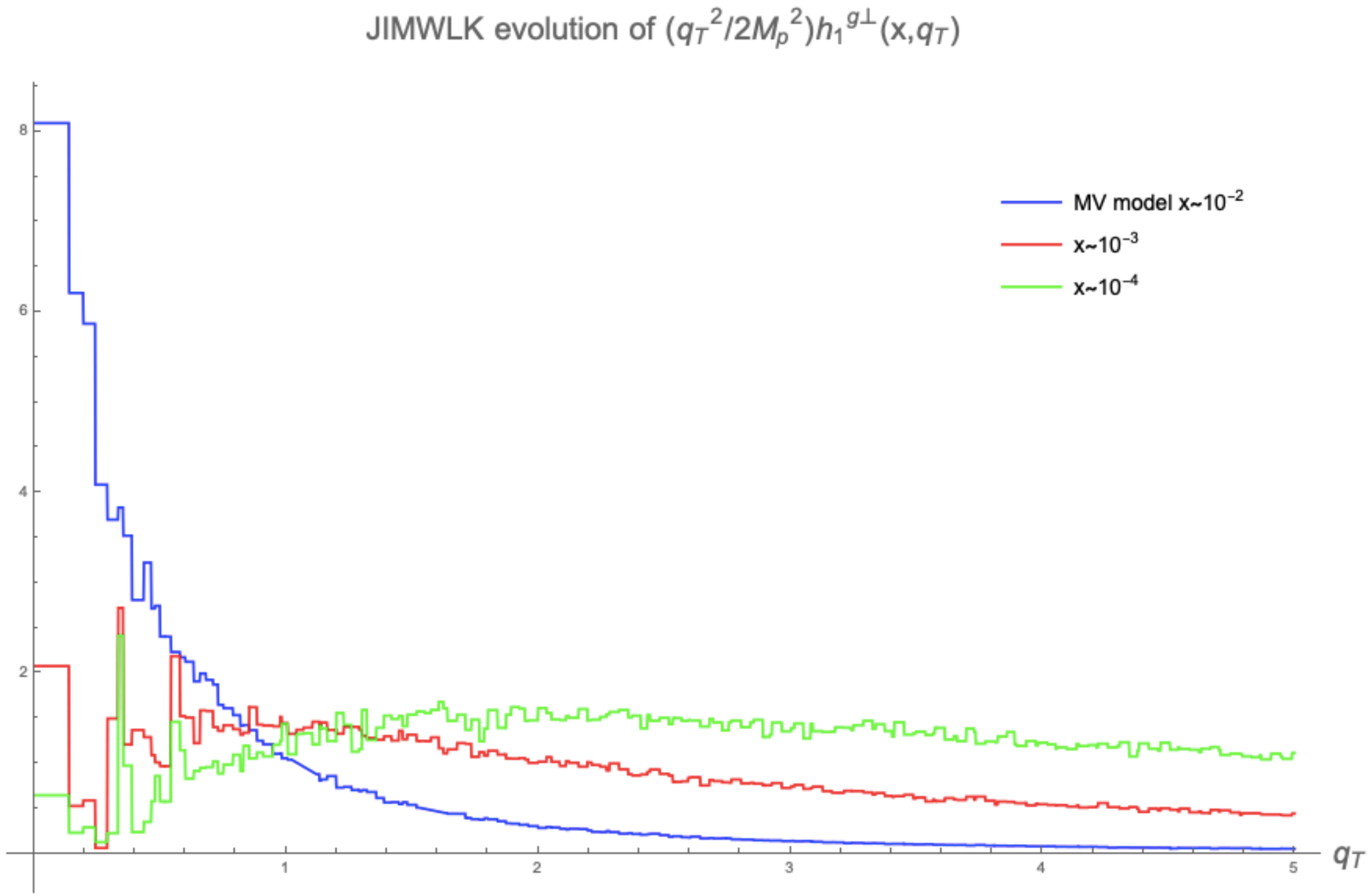
JIMWLK evolution of f_1^g and $h_1^{g\perp}$



Langevin formulation of JIMWLK: Weigert (2002), Rummukainen; Weigert (2004); Lappi (2008)

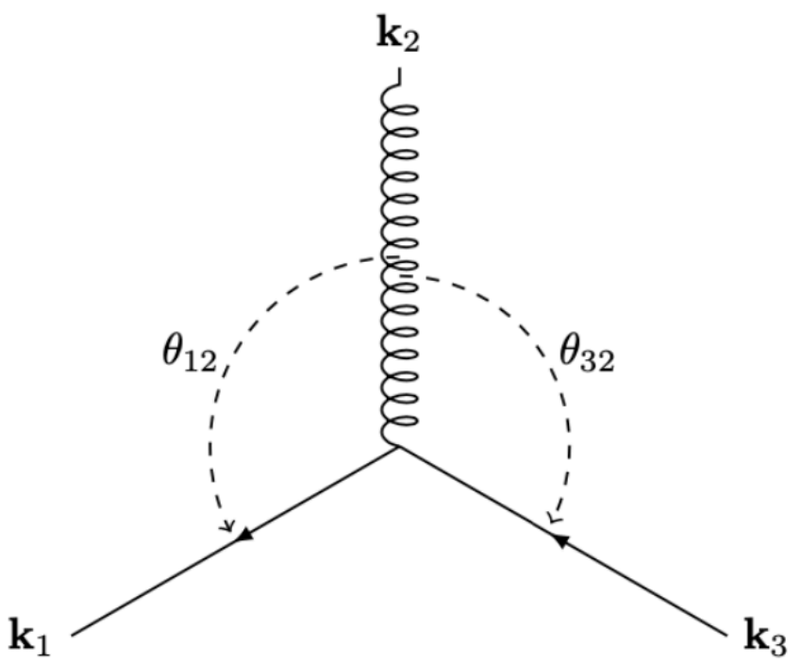
Marquet, Petreska, Roiesnel (2016); Marquet, Roiesnel, PT (2018)

JIMWLK evolution of f_1^g and $h_1^{g\perp}$



Final result in the MV model

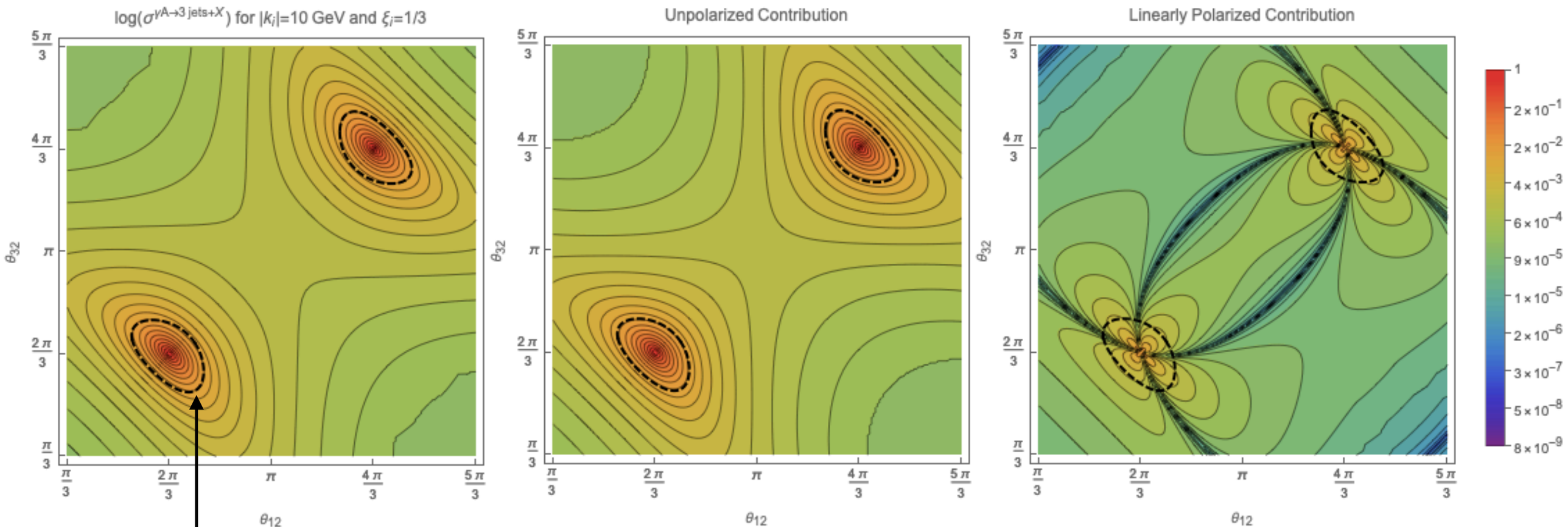
McLerran, Venugopalan (1994)



$$\mathcal{F}_{gg}^{(3)}(x,\mathbf{q}_T^2) = \frac{S_\perp C_F}{\alpha_s \pi^3}\int \mathrm{d}r \frac{J_0(|q_T r|)}{r}\left(1 - e^{-\frac{r^2}{4}Q_{sg}^2 \ln(1/r^2\Lambda^2)}\right)$$

$$\mathcal{H}_{gg}^{(3)}(x,\mathbf{q}_T^2) = \frac{S_\perp C_F}{\alpha_s \pi^3}\int \mathrm{d}r \frac{J_2(|q_T r|)}{r \ln\frac{1}{r^2\Lambda^2}}\left(1 - e^{-\frac{r^2}{4}Q_{sg}^2 \ln(1/r^2\Lambda^2)}\right)$$

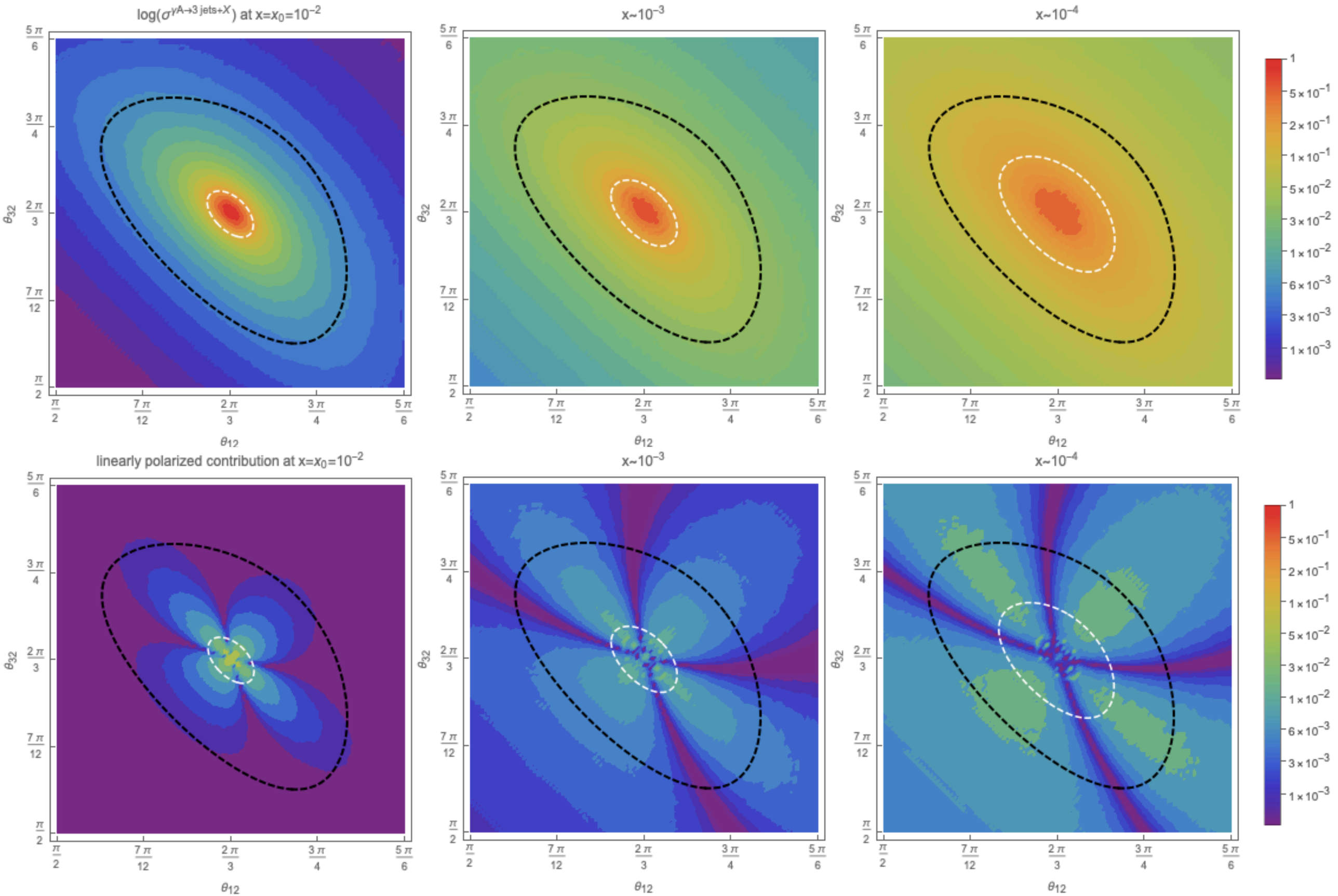
Dominguez, Xiao, Yuan (2011)
Metz, Zhou (2011)



$q_T = \sqrt{10}\,\mathrm{GeV}$

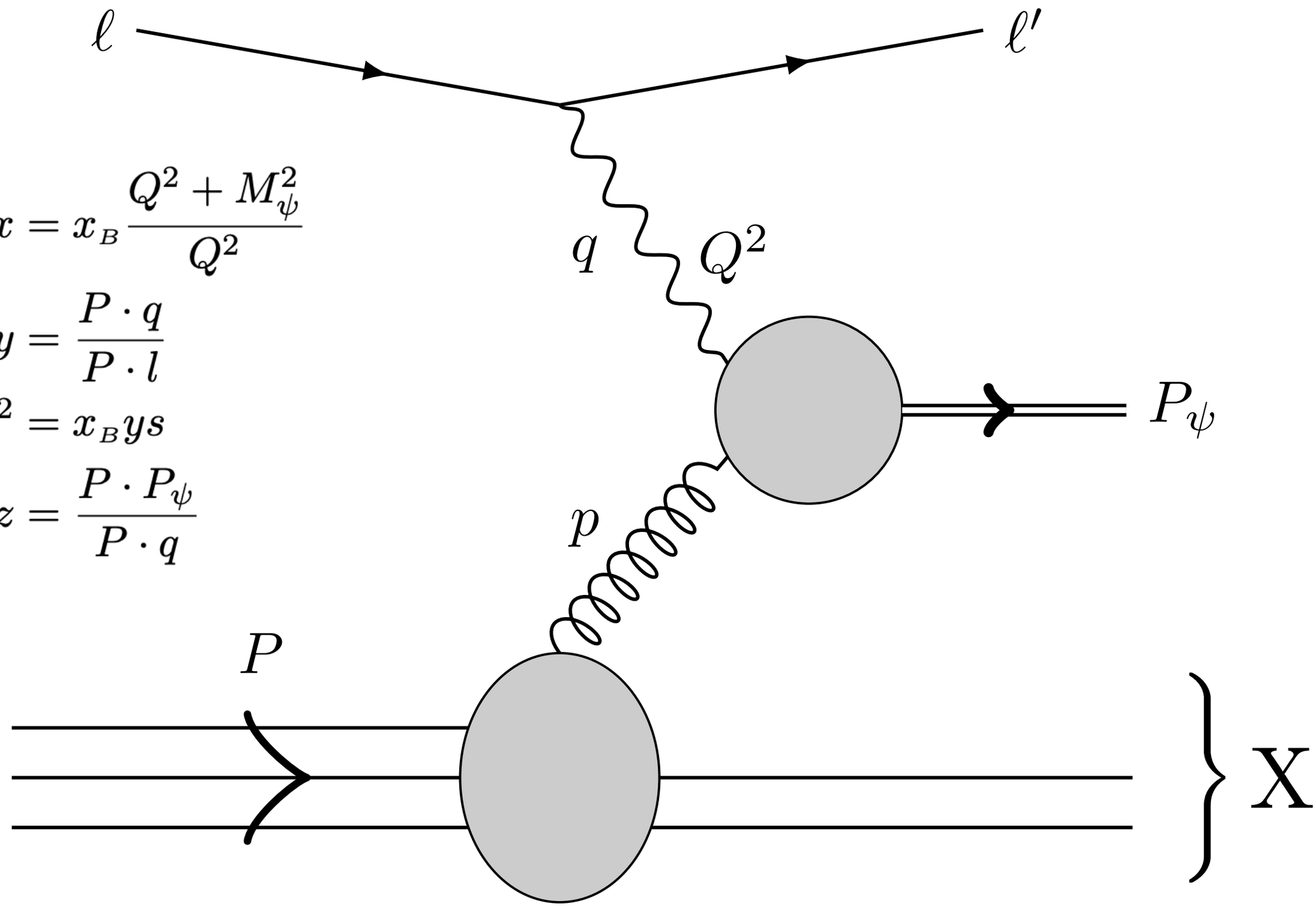
$\xi_i = 1/3$ and $|\mathbf{k}_i| = 10\,\mathrm{GeV}$

JIMWLK evolution of final result



Gluon TMDs in electroproduction of quarkonium

$$\ell + p \to \ell + J/\psi + X$$



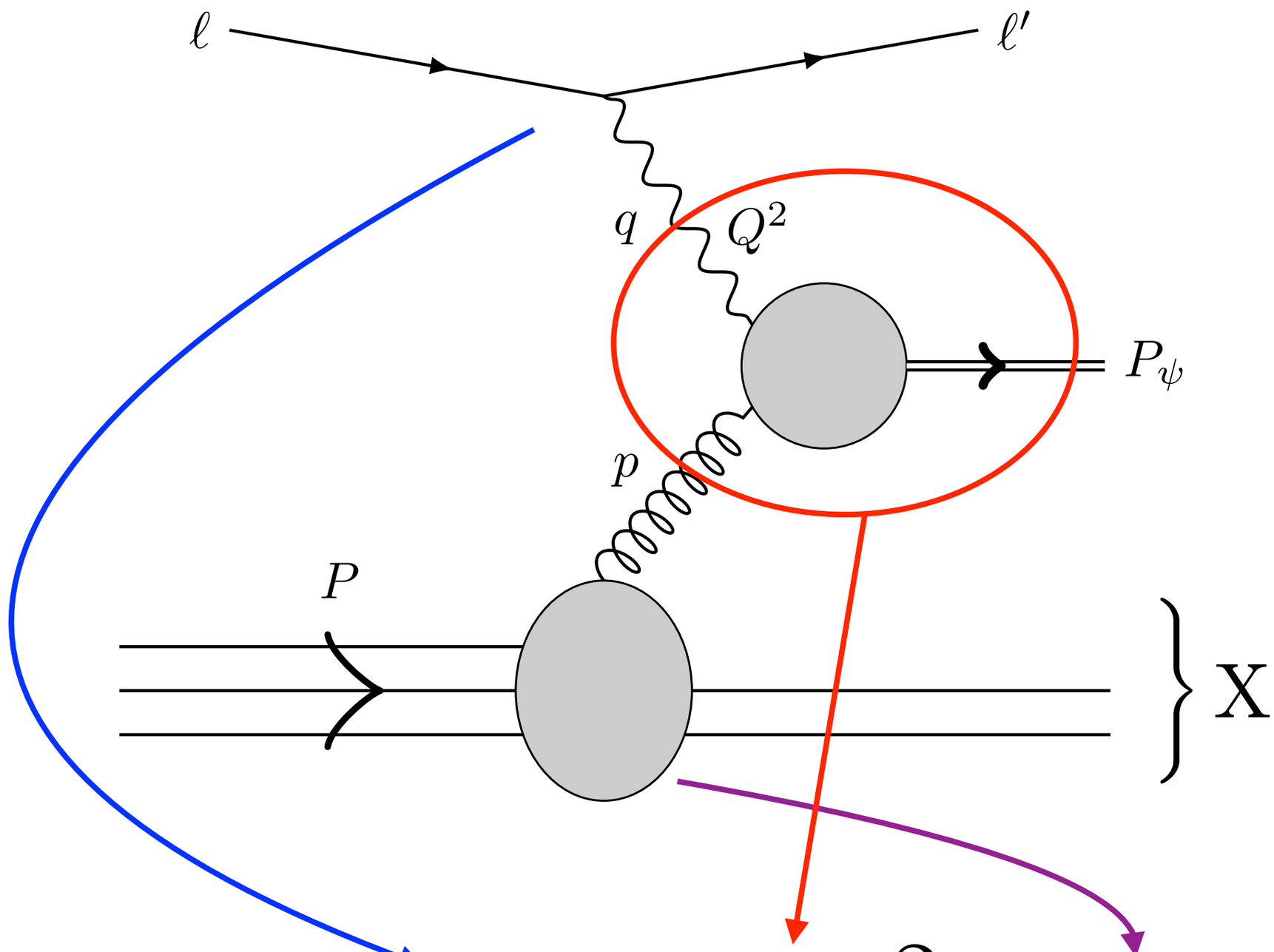
$$x = x_B \frac{Q^2 + M_\psi^2}{Q^2}$$

$$y = \frac{P \cdot q}{P \cdot l}$$

$$Q^2 = x_B y s$$

$$z = \frac{P \cdot P_\psi}{P \cdot q}$$

$$\ell + p \to \ell + J/\psi + X$$



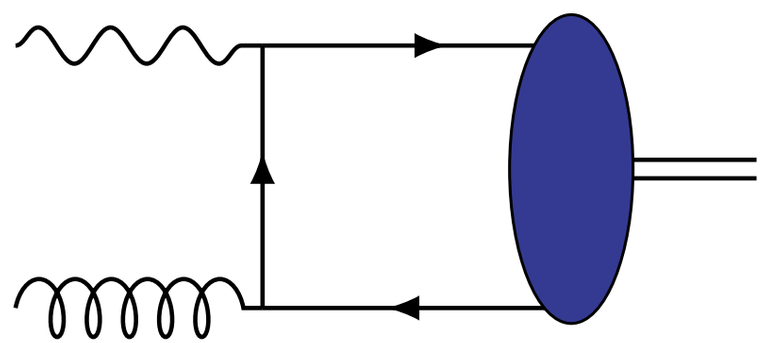
$$\mathrm{d}\sigma = \Gamma \otimes |M|^2 \otimes \Phi$$ Ansatz!

Lepton tensor **'Hard' part** **Hadron tensor**

Non-relativistic QCD (NRQCD)

Color octet (CO) mechanism: heavy-quark pair is produced with *all allowed quantum numbers* and then hadronizes as encoded in the non-perturbative long-distance matrix elements (LDMEs)

$$\langle|\mathcal{O}_8^{J/\psi}(^3P_J)|0\rangle \qquad \langle|\mathcal{O}_8^{J/\psi}(^1S_0)|0\rangle$$



$$Q\bar{Q}[^{2S+1}L_J^{(8)}] \qquad \mathcal{O}(\alpha_{\rm em}\alpha_{\rm s})$$

NRQCD also encompasses color singlet (CS) mechanism where J/ψ is directly produced with correct quantum numbers

Bodwin, Braaten and Lepage (1995)

Hadron tensor

$$\Gamma_U^{\mu\nu}(x,\boldsymbol{p}_T) = \frac{x}{2}\left\{ -g_T^{\mu\nu} f_1^g(x,\boldsymbol{p}_T^2) + \left(\frac{p_T^\mu p_T^\nu}{M_p^2} + g_T^{\mu\nu}\frac{\boldsymbol{p}_T^2}{2M_p^2}\right) h_1^{\perp g}(x,\boldsymbol{p}_T^2)\right\}$$

unpolarized gluons

linearly polarized gluons

Sivers function

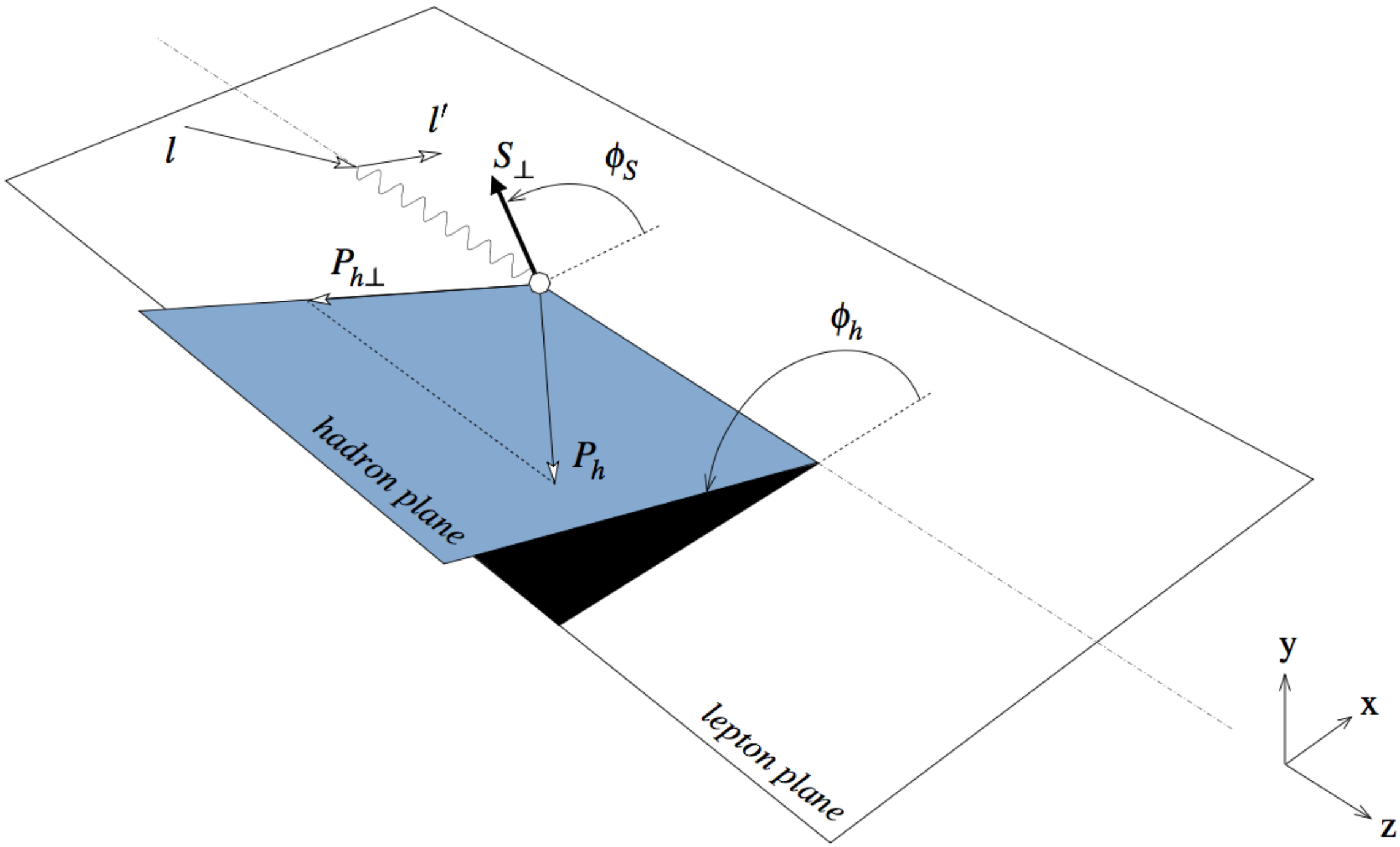
circularly polarized gluons

$$\Gamma_T^{\mu\nu}(x,\mathbf{p}_T^2) = \frac{x}{2}\left\{ g_T^{\mu\nu}\frac{\epsilon_T^{\rho\sigma} p_{\perp\rho} S_{\perp\sigma}}{M_p} f_{1T}^{\perp g}(x,\mathbf{p}_T^2) + i\epsilon_T^{\mu\nu}\frac{p_\perp\cdot S_\perp}{M_p} g_{1T}^g(x,\mathbf{p}_T^2) \right.$$
$$\left. + \frac{p_{\perp\rho}\epsilon_T^{\rho\{\mu} p_\perp^{\nu\}}}{2M_p^2}\frac{p_\perp\cdot S_\perp}{M_p} h_{1T}^{\perp g}(x,\mathbf{p}_T^2) - \frac{p_{\perp\rho}\epsilon_T^{\rho\{\mu} S_\perp^{\nu\}} + S_{\perp\rho}\epsilon_T^{\rho\{\mu} p_\perp^{\nu\}}}{4M_p} h_{1T}^g(x,\mathbf{p}_T^2)\right\}$$

linearly polarized gluons

Mulders & Rodrigues (2001); Meissner, Metz & Goeke (2007)

Reference frame



Bacchetta, D'Alesio, Diehl & Miller (2004)

Cross section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}y\mathrm{d}x_B\mathrm{d}^2\mathbf{q}_T} \equiv \mathrm{d}\sigma^U(\phi_T) + \mathrm{d}\sigma^T(\phi_T, \phi_S)$$

$$\mathrm{d}\sigma^U = \mathcal{N}\left[A^U f_1^g(x, \boldsymbol{q}_T^2) + \frac{\boldsymbol{q}_T^2}{M_p^2} B^U h_1^{\perp g}(x, \boldsymbol{q}_T^2)\cos 2\phi_T\right]$$

unpolarized

linearly polarized

T even

Sivers function

T odd -> only CO!

$$\mathrm{d}\sigma^T = \mathcal{N}|\mathbf{S}_T|\frac{|\mathbf{q}_T|}{M_p}\left\{A^T f_{1T}^{\perp g}(x, \mathbf{q}_T^2)\sin(\phi_S - \phi_T)\right.$$

$$\left. + B^T\left[h_1^g(x, \mathbf{q}_T^2)\sin(\phi_S + \phi_T) - \frac{\mathbf{q}_T^2}{2M_p^2} h_{1T}^{\perp g}(x, \mathbf{q}_T^2)\sin(\phi_S - 3\phi_T)\right]\right\}$$

linearly polarized

Azimuthal asymmetries

probe *ratios* of gluon TMDs

$$A^{W(\phi_S,\phi_T)} \equiv 2\,\frac{\int \mathrm{d}\phi_S\,\mathrm{d}\phi_T\,W(\phi_S,\phi_T)\,\mathrm{d}\sigma(\phi_S,\,\phi_T)}{\int \mathrm{d}\phi_S\,\mathrm{d}\phi_T\,\mathrm{d}\sigma(\phi_S,\phi_T)}$$

…we have:

$$\int \mathrm{d}\phi_S\,\mathrm{d}\phi_T\,\mathrm{d}\sigma(\phi_S,\phi_T) = (2\pi)^2\,\mathcal{N}\,A^U\,f_1^g(x,\boldsymbol{q}_T^2)$$

$$A^{\cos 2\phi_T} = H(y,M_\psi,Q)\,\frac{\boldsymbol{q}_T^2}{M_p^2}\,\frac{h_1^{\perp\,g}(x,\boldsymbol{q}_T^2)}{f_1^g(x,\boldsymbol{q}_T^2)}$$

$$A^{\sin(\phi_S-\phi_T)} = \frac{|\boldsymbol{q}_T|}{M_p}\,\frac{f_{1T}^{\perp\,g}(x,\boldsymbol{q}_T^2)}{f_1^g(x,\boldsymbol{q}_T^2)}$$

$$A^{\sin(\phi_S+\phi_T)} = H(y,M_\psi,Q)\,\frac{|\boldsymbol{q}_T|}{M_p}\,\frac{h_1^g(x,\boldsymbol{q}_T^2)}{f_1^g(x,\boldsymbol{q}_T^2)}$$

$$A^{\sin(\phi_S-3\phi_T)} = -H(y,M_\psi,Q)\,\frac{|\boldsymbol{q}_T|^3}{2M_p^3}\,\frac{h_{1T}^{\perp\,g}(x,\boldsymbol{q}_T^2)}{f_1^g(x,\boldsymbol{q}_T^2)}$$

Bachetta, Boer, Pisano, PT (2018)

Upper bounds

Polarized gluon TMDs satisfy the following positivity bounds:

$$\frac{|\boldsymbol{p}_T|}{M_p}\,|f_{1T}^{\perp\,g}(x,\boldsymbol{p}_T^2)| \leq f_1^g(x,\boldsymbol{p}_T^2)$$

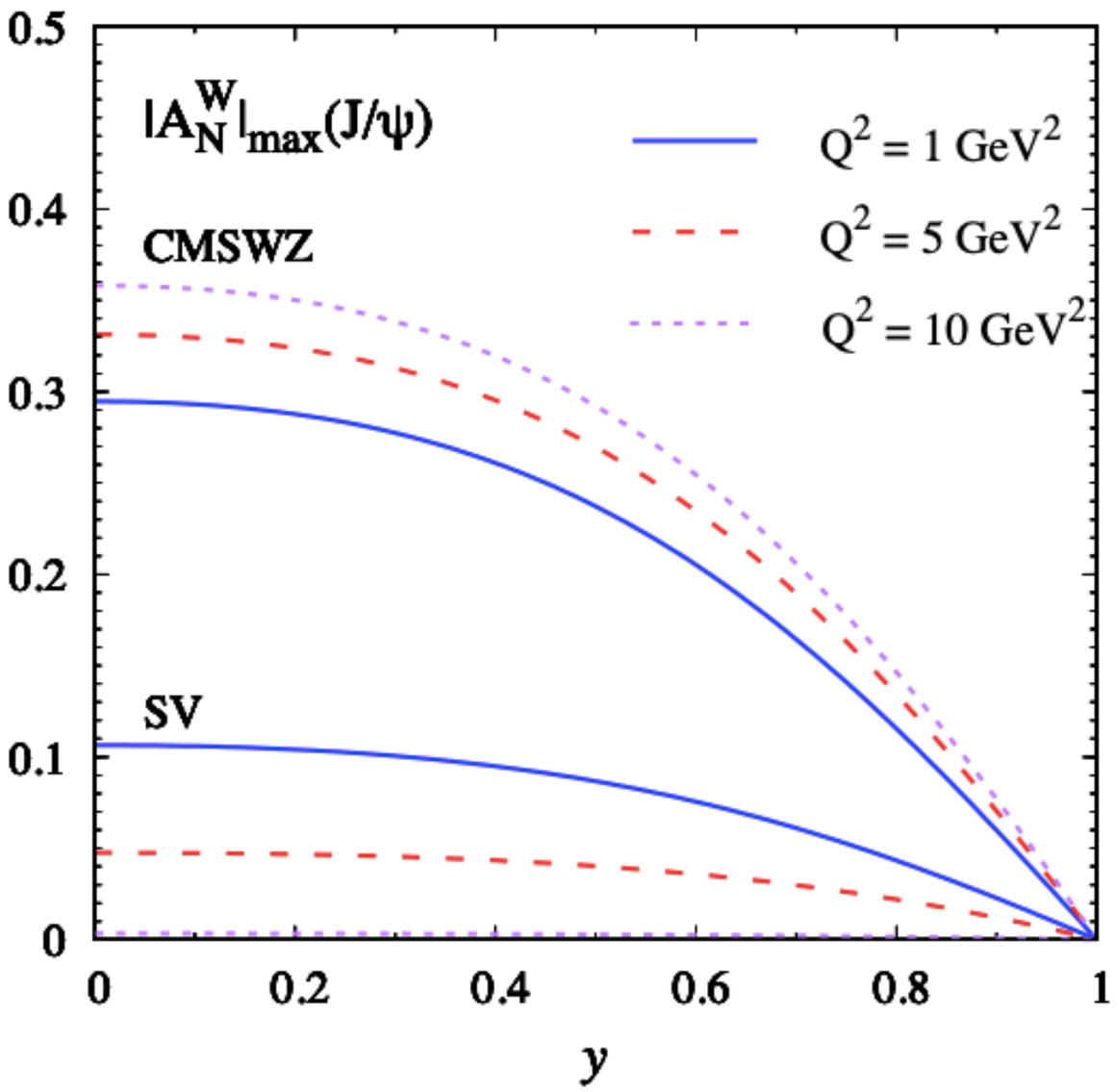
$$\frac{|\boldsymbol{p}_T|}{M_p}\,|h_1^g(x,\boldsymbol{p}_T^2)| \leq f_1^g(x,\boldsymbol{p}_T^2)$$

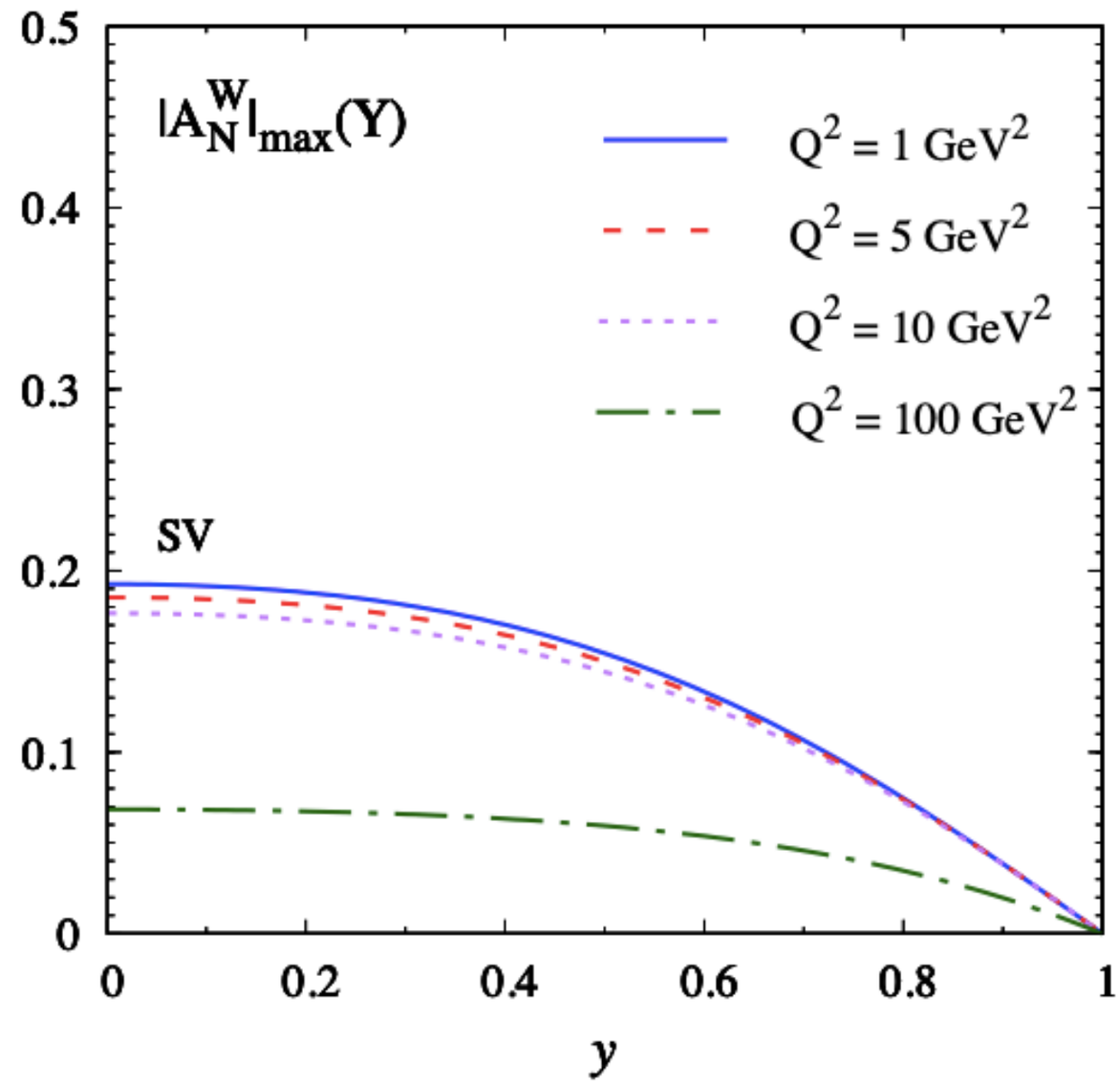
$$\frac{\boldsymbol{p}_T^2}{2M_p^2}\,|h_1^{\perp\,g}(x,\boldsymbol{p}_T^2)| \leq f_1^g(x,\boldsymbol{p}_T^2)$$

$$\frac{|\boldsymbol{p}_T|^3}{2M_p^3}\,|h_{1T}^{\perp\,g}(x,\boldsymbol{p}_T^2)| \leq f_1^g(x,\boldsymbol{p}_T^2)$$

We can use the maximum allowed values of the gluon TMDs to illustrate the sensitivity of our inclusive charmonium electroproduction to the gluon content of the proton:

Upper bounds





Bachetta, Boer, Pisano, PT (2018)

$\ell + p \to \ell + J/\psi + X$: matching of collinear and TMD calculation

Notion of LDMEs needs to be extended to small transverse momenta (cfr. fragmentation functions).

If μ is the hard scale, collinear factorization holds when $\mu \sim P_{\psi T} \gg M_p$.

TMD factorization is applicable when $\mu \gg P_{\psi T} \gtrsim M_p$.

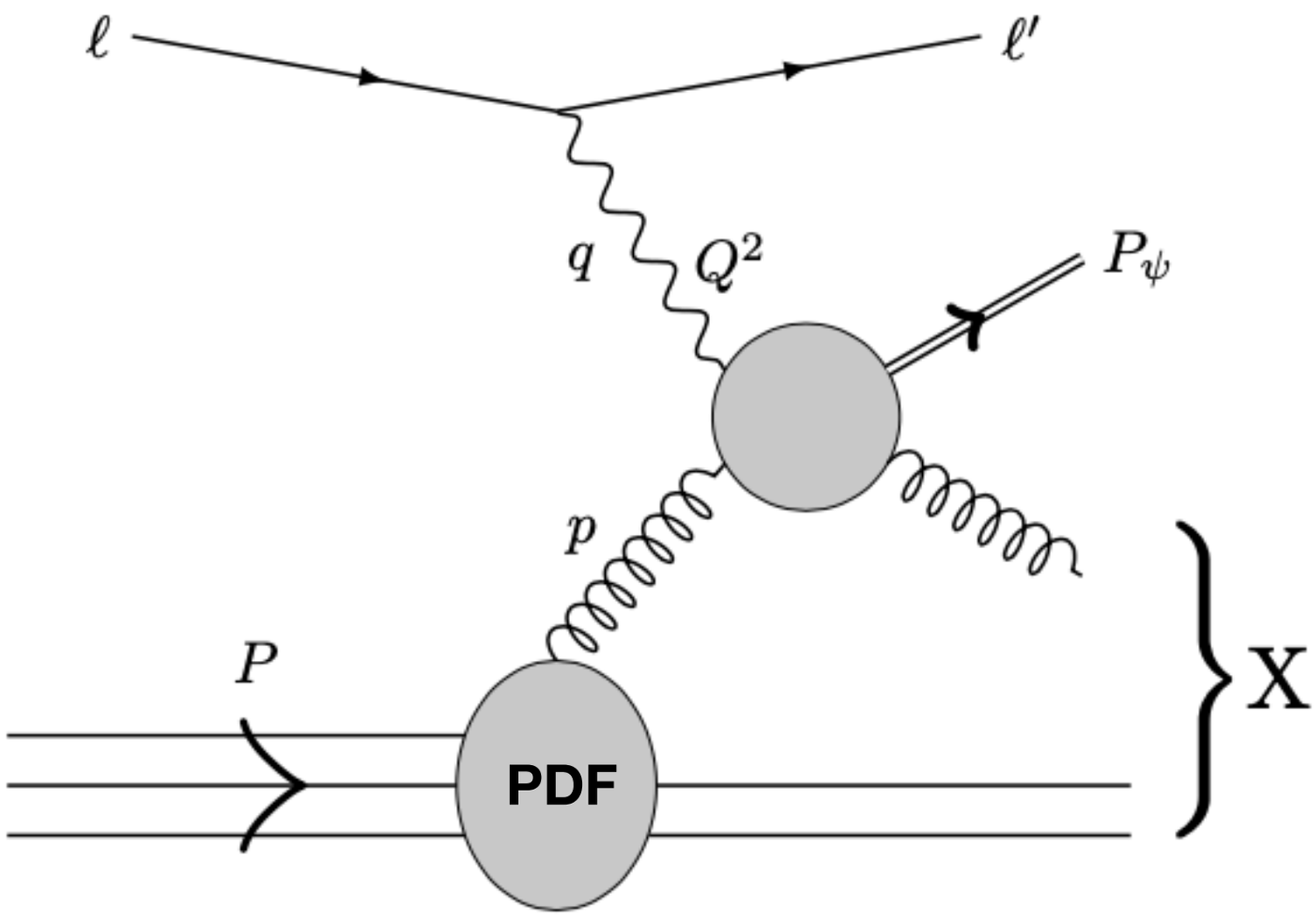
The two calculations should match in the region where both frameworks are valid, i.e. when $\mu \gg P_{\psi T} \gg M_p$.

Strategy: assume TMD factorization, and study which conditions follow from the matching requirement.

$\ell + p \to \ell + J/\psi + X$:
matching of collinear and TMD calculation

In the collinear regime: $\mu \sim P_{\psi T} \gg M_p$ with $\mu = \sqrt{Q^2 + M_\psi^2}$

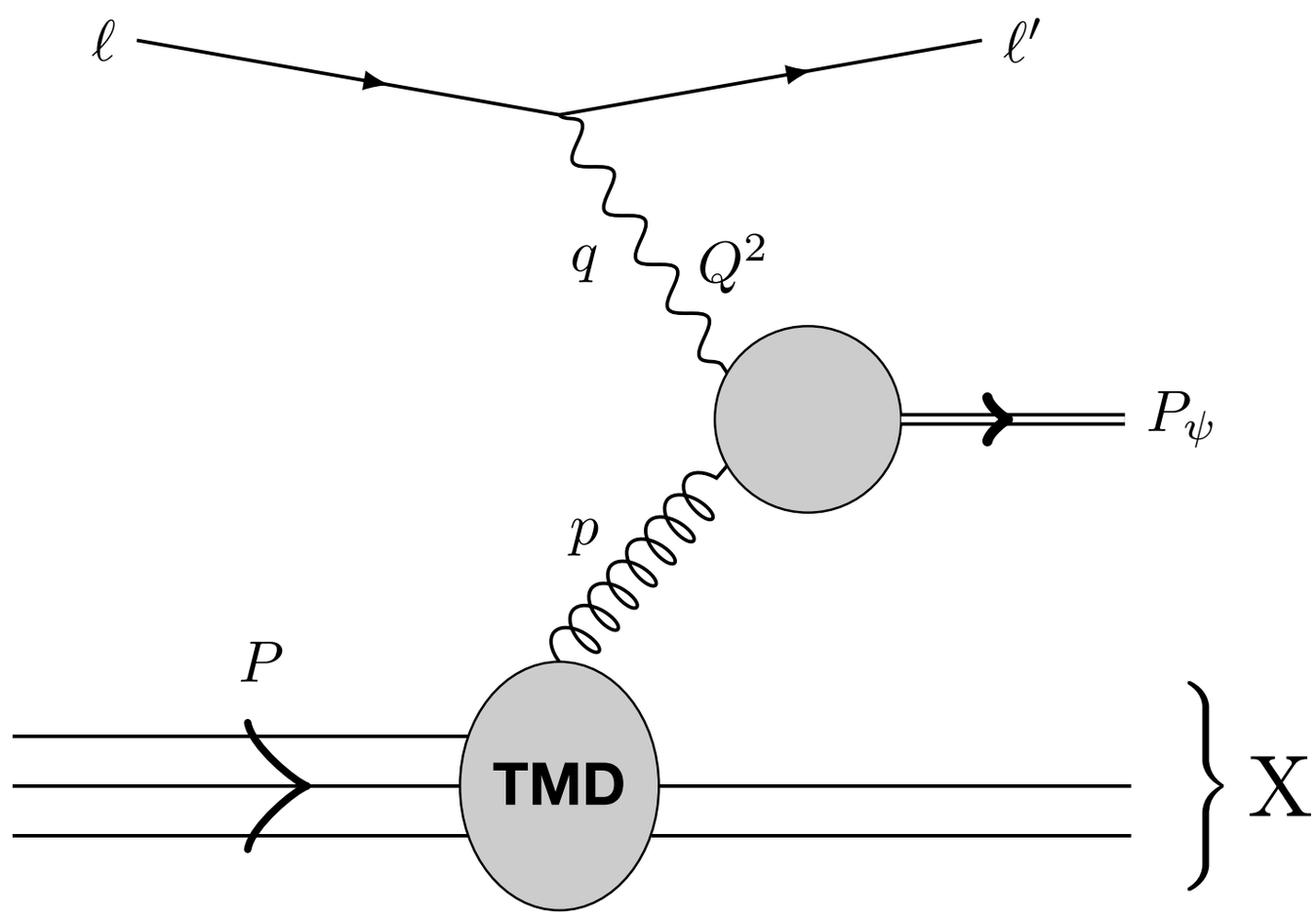
$P_{\psi T}$ is generated by recoil off hard parton



$\ell + p \to \ell + J/\psi + X$:
matching of collinear and TMD calculation

In the TMD regime: $\mu \gg P_{\psi T} \gtrsim M_p$

$P_{\psi T}$ stems from intrinsic transverse momentum in target, or from soft emissions



$\ell + p \to \ell + J/\psi + X$:
matching of collinear and TMD calculation

If TMD factorization holds, both the collinear and the TMD calculation should match in overlapping kinematic region

$\mu \gg P_{\psi T} \gg M_p$

From collinear calculation: hard emission is absorbed into DGLAP

From TMD calculation: high-$p_\perp$ tail also matches to DGLAP

Bacchetta, Boer, Diehl & Mulders (2008)
Bacchetta, Bozzi, Echevarria, Pisano, Prokudin & Radici (2019)

$\ell + p \to \ell + J/\psi + X$:
matching of collinear and TMD calculation

Collinear calculation at small q_T:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}y\,\mathrm{d}x_B\,\mathrm{d}z\,\mathrm{d}q_T^2\,\mathrm{d}\phi_\psi} = \frac{\alpha^2}{y}\left[\frac{1+(1-y)^2}{Q^2}F_{UU,T} + 4(1-y)\,F_{UU,L} + (1-y)\cos 2\phi_\psi\,F_{UU}^{\cos 2\phi_\psi}\right]$$

transverse γ^* **longitudinal γ^*** **lin. pol. γ^***

TMD calculation at high q_T:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}y\,\mathrm{d}x_B\,\mathrm{d}z\,\mathrm{d}\boldsymbol{q}_T^2\,\mathrm{d}\phi_\psi} = \frac{\alpha^2}{y}\left[\frac{1+(1-y)^2}{Q^2}\mathcal{F}_{UU,T} + 4(1-y)\,\mathcal{F}_{UU,L} + (1-y)\,\cos 2\phi_\psi\,\mathcal{F}_{UU}^{\cos 2\phi_\psi}\right]\delta(1-z)$$

In kinematic region of overlap:

$$\mathcal{F}_{UU,T} = F_{UU,T} - \sigma_{UU,T}\,C_A \ln\left(\frac{Q^2+M_\psi^2}{q_T^2}\right)$$

$$\mathcal{F}_{UU,L} = F_{UU,L} - \sigma_{UU,L}\,C_A \ln\left(\frac{Q^2+M_\psi^2}{q_T^2}\right)$$

$$\mathcal{F}_{UU}^{\cos 2\phi_\psi} = F_{UU}^{\cos 2\phi_\psi}$$

$\ell + p \to \ell + J/\psi + X$: matching of collinear and TMD calculation

Therefore, assuming both TMD factorization and enforcing the matching:

$$\mathcal{F}_{UU,T} = \sum_n \mathcal{H}^{[n]}_{UU,T} \mathcal{C}\left[f_1^g \Delta^{[n]}\right](x, \mathbf{q}_T^2)$$

$$\mathcal{F}_{UU,L} = \sum_n \mathcal{H}^{[n]}_{UU,L} \mathcal{C}\left[f_1^g \Delta^{[n]}\right](x, \mathbf{q}_T^2)$$

$$\mathcal{F}_{UU}^{\cos 2\phi_\psi} = \sum_n \mathcal{H}^{[n],\cos 2\phi_\psi}_{UU,} \mathcal{C}\left[w h_1^{\perp g} \Delta^{[n]}\right](x, \mathbf{q}_T^2)$$

we have to introduce the following ‘smearing’ function:

$$\Delta^{[n]}(\boldsymbol{k}_T^2, \mu^2) = \frac{\alpha_s}{2\pi^2 \boldsymbol{k}_T^2} C_A \langle 0|\mathcal{O}_c(n)|0\rangle \ln \frac{\mu^2}{\boldsymbol{k}_T^2}$$

Boer, D’Alesio, Murgia, Pisano, PT (2020)
Echevarria (2019); Fleming, Makris, Mehen (2019)

$\ell + p \to \ell + J/\psi + X$: matching of collinear and TMD calculation

Matching perfectly works (at LO) if we expand the LDMEs as follows:

$$\Delta^{[n]}(\boldsymbol{k}_T^2, \mu^2) = \frac{\alpha_s}{2\pi^2 \boldsymbol{k}_T^2} C_A \langle 0|\mathcal{O}_c(n)|0\rangle \ln \frac{\mu^2}{\boldsymbol{k}_T^2}$$

This is exactly the perturbative part of the LDME *shape functions* introduced by our colleagues from SCET !

Important step towards proof of TMD factorization of single-inclusive quarkonium production.

At least for ${}^1S_0^{[8]}$ and ${}^3P_J^{[8]}$ states, the transverse momentum dependence of the shape function is the same.

Boer, D'Alesio, Murgia, Pisano, PT (2020)
Echevarria (2019); Fleming, Makris, Mehen (2019)

Conclusions and outlook

Conclusions & outlook

Gluon TMDs provide valuable insight in three-dimensional proton structure, unfortunately for the moment no fits available !

Leptoproduction of J/ψ (+jet) at the Electron-Ion Collider seems a very promising process to probe gluon TMDs. Factorization Ansatz is reasonable and corroborated by SCET studies and matching. We need dedicated fit at low transverse momentum of the shape functions.

TMDs in the CGC at NLO in progress, hopeful that the correspondence at LO for 2->2 processes is not coincidental.

Thanks for your attention !