New Framework for the Analysis of Dihadron Fragmentation

Daniel Pitonyak
Lebanon Valley College, Annville, PA, USA

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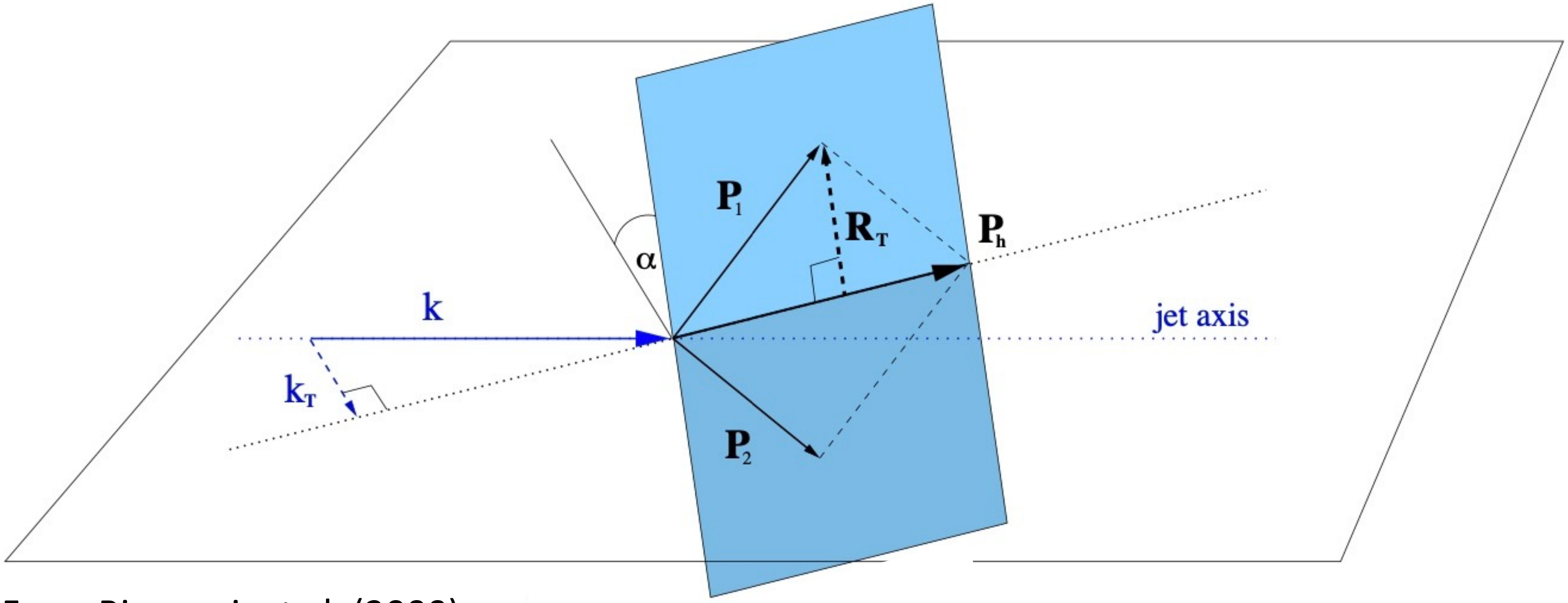
Outline

- Background and motivation: why dihadron fragmentation?
- New DiFF theory developments: number density interpretation, sum rules, and evolution equations
- New DiFF phenomenology from the JAM Collaboration
- Summary and outlook

Background and Motivation



From Bianconi, et al. (2000)

Bianconi, et al. (2000); Bacchetta, Radici (2003, 2004), …

$$P_h = P_1 + P_2 \qquad R = (P_1 - P_2)/2 \qquad z = z_1 + z_2 \qquad \zeta = (z_1 - z_2)/z$$

$$P_1 = \left(\frac{M_1^2 + \vec{R}_T^2}{(1+\zeta)P_h^-}, \frac{1+\zeta}{2} P_h^-, \vec{R}_T \right) \qquad P_2 = \left(\frac{M_2^2 + \vec{R}_T^2}{(1-\zeta)P_h^-}, \frac{1-\zeta}{2} P_h^-, -\vec{R}_T \right)$$

$$\vec{R}_T^2 = \frac{1-\zeta^2}{4} M_h^2 - \frac{1-\zeta}{2} M_1^2 - \frac{1+\zeta}{2} M_2^2$$

- Dihadron fragmentation involves more structures than single-hadron fragmentation (only unpolarized hadron FFs are shown below)

Single-hadron FFs

$$\Delta^{h/q}(z,\vec{k}_T) \longrightarrow D_1^{h/q}(z,z^2\vec{k}_T^2),\ -\frac{\epsilon_T^{ij}k_T^j}{M_h}H_1^{\perp h/q}(z,z^2\vec{k}_T^2)$$

Dihadron FFs

(Bianconi, et al. (2000);
Bacchetta, Radici (2003, 2004))

$$\Delta^{h_1h_2/q}(z,\zeta,\vec{k}_T,\vec{R}_T) \longrightarrow \left\{ \begin{array}{l} D_1^{h_1h_2/q}(z,\zeta,\vec{k}_T^2,\vec{R}_T^2,\vec{k}_T\cdot\vec{R}_T), \\ \dfrac{\epsilon_T^{ij}R_T^ik_T^j}{M_h^2}G_1^{\perp\,h_1h_2/q}(z,\zeta,\vec{k}_T^2,\vec{R}_T^2,\vec{k}_T\cdot\vec{R}_T), \\ -\dfrac{\epsilon_T^{ij}R_T^j}{M_h}H_1^{\sphericalangle'\,h_1h_2/q}(z,\zeta,\vec{k}_T^2,\vec{R}_T^2,\vec{k}_T\cdot\vec{R}_T), \\ -\dfrac{\epsilon_T^{ij}k_T^j}{M_h}H_1^{\perp'\,h_1h_2/q}(z,\zeta,\vec{k}_T^2,\vec{R}_T^2,\vec{k}_T\cdot\vec{R}_T) \end{array} \right.$$

- Dihadron fragmentation involves more structures than single-hadron fragmentation (only unpolarized hadron FFs are shown below)

Single-hadron FFs

$$\int d^2\vec{k}_T\,\Delta^{h/q}(z,\vec{k}_T) \longrightarrow D_1^{h/q}(z)$$

Dihadron FFs

$$\int d^2\vec{k}_T\,\Delta^{h_1h_2/q}(z,\zeta,\vec{k}_T,\vec{R}_T) \longrightarrow \begin{cases} D_1^{h_1h_2/q}(z,\zeta,\vec{R}_T^2), \\ -\dfrac{\epsilon_T^{ij}R_T^j}{M_h}H_1^{\sphericalangle h_1h_2/q}(z,\zeta,\vec{R}_T^2), \end{cases}$$

- Dihadron fragmentation involves more structures than single-hadron fragmentation (only unpolarized hadron FFs are shown below)

Single-hadron FFs

$$\int d^2\vec{k}_T\,\Delta^{h/q}(z,\vec{k}_T) \longrightarrow D_1^{h/q}(z)$$

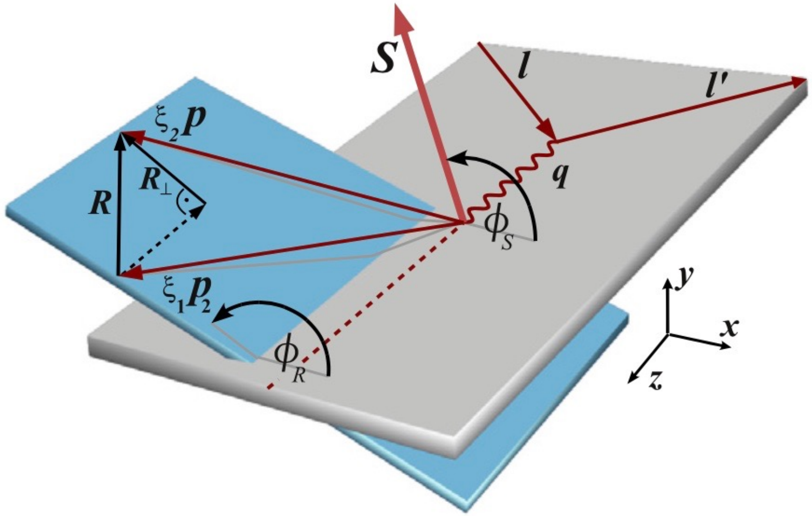
Dihadron FFs

$$\int d^2\vec{k}_T\,\Delta^{h_1h_2/q}(z,\zeta,\vec{k}_T,\vec{R}_T) \longrightarrow \left\{\begin{array}{l} D_1^{h_1h_2/q}(z,\zeta,\vec{R}_T^2), \\ -\dfrac{\epsilon_T^{ij}R_T^j}{M_h}H_1^{\sphericalangle h_1h_2/q}(z,\zeta,\vec{R}_T^2), \end{array}\right.$$

chiral-odd “interference” FF (IFF)

(Collins, et al. (1994); Bianconi, et al. (2000); Bacchetta, Radici (2003, 2004); Courtoy, et al. (2012); Matevosyan, et al. (2018); Radici, et al. (2013, 2015, 2018); Benel, et al. (2020), Courtoy, et al. (2014, 2022))

$$\ell\, N \to \ell\,(h_1 h_2)\, X$$



$$A_{UT}^{\sin(\phi_R+\phi_S)} = \frac{\sum_q e_q^2\, \boldsymbol{h_1^q(x)}\, \boldsymbol{H_1^{\sphericalangle,q}(z, M_h)}}{\sum_q e_q^2\, f_1^q(x)\, \boldsymbol{D_1^q(z, M_h)}}$$

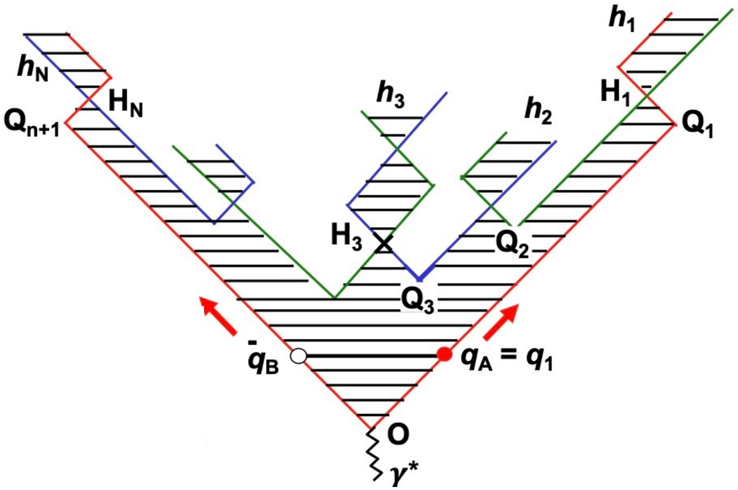
$h_1(x)$ = transversity PDF - connection to the tensor charges of the nucleon

$$A_{LU}^{\sin\phi_R} \sim \frac{\sum_q e_q^2 \left[x\boldsymbol{e^q(x)}\, \boldsymbol{H_1^{\sphericalangle,q}(z, M_h)} + \frac{M_h}{zM}\, f_1^q(x) \tilde{G}^{\sphericalangle,q}(z, M_h) \right]}{\sum_q e_q^2\, f_1^q(x)\, \boldsymbol{D_1^q(z, M_h)}}$$

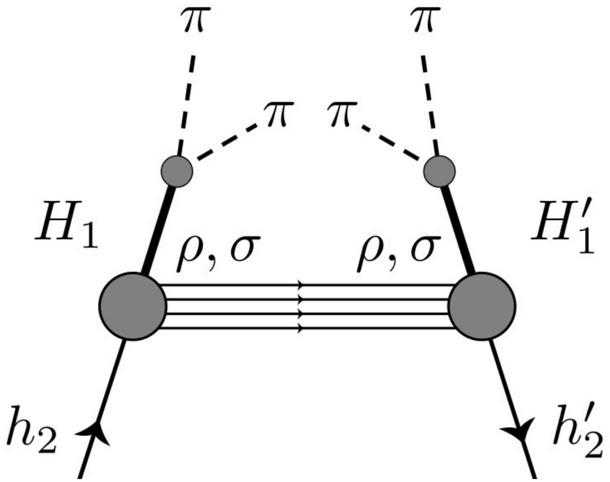
e(x) = twist-3 PDF - connection to the decomposition of the nucleon mass

(Collins, et al. (1994); Bianconi, et al. (2000); Bacchetta, Radici (2003, 2004); Courtoy, et al. (2012); Matevosyan, et al. (2018); Radici, et al. (2013, 2015, 2018); Benel, et al. (2020), Courtoy, et al. (2014, 2022))

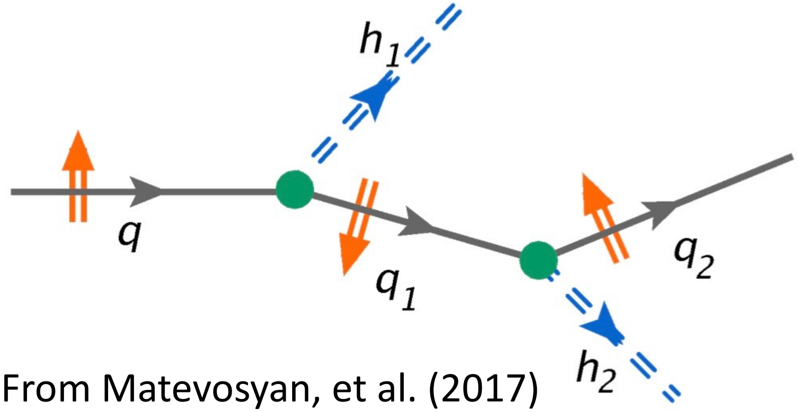
- DiFFs are interesting in their own right, e.g., one can test models for (un)polarized parton fragmentation/hadronization (Collins, Ladinsky (1994); Jaffe, et al. (1998); Bianconi, et al. (2000); Bacchetta, Radici (2006); Matevosyan, et al. (2017, 2018); Kerbizi, et al. (2019))



From Kerbizi, et al. (2019)



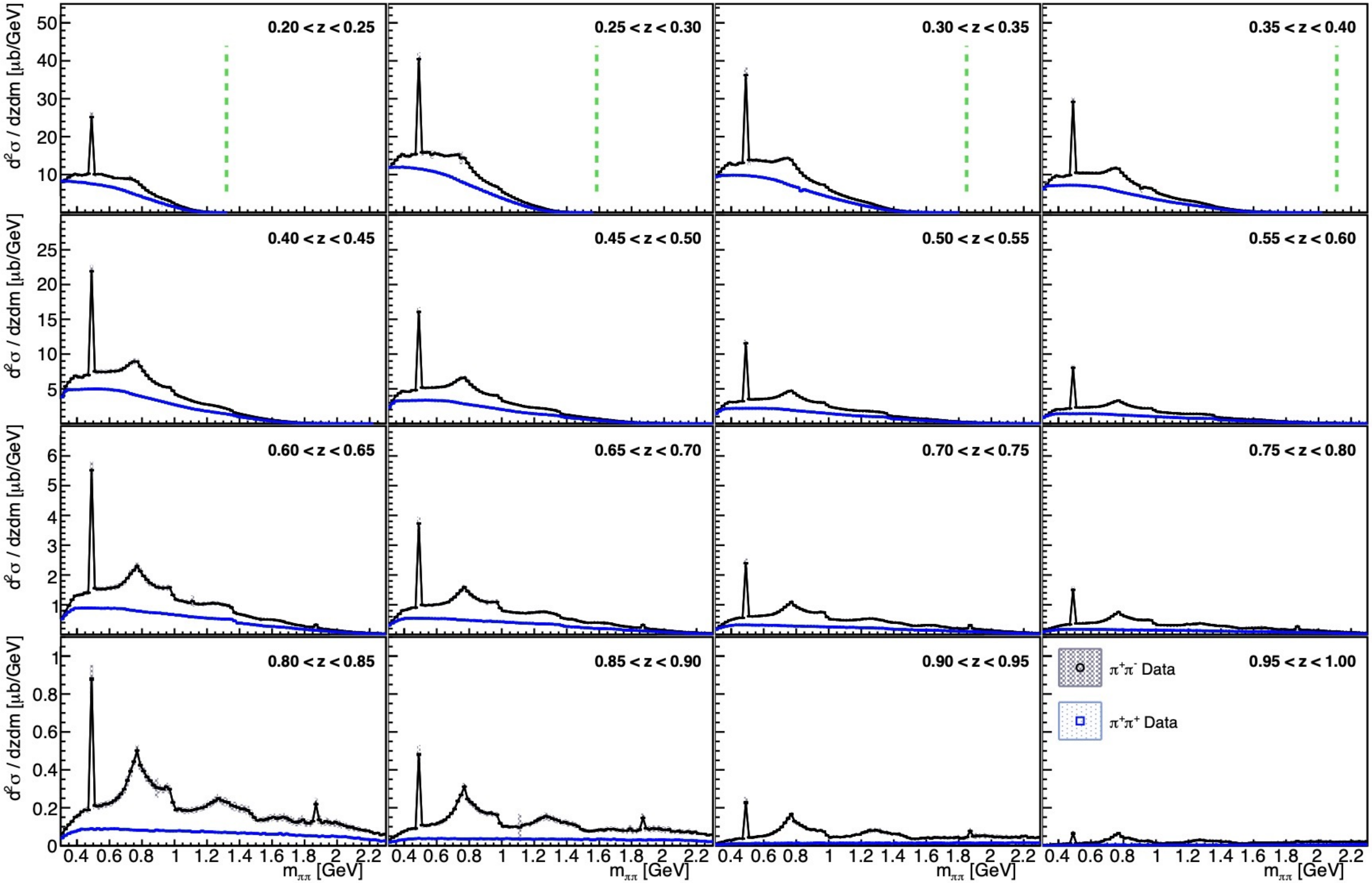
From Jaffe, et al. (1998)



From Matevosyan, et al. (2017)

- There is also a complicated/interesting resonance structure that can/must be analyzed

Belle (2017)



New DiFF Theory Developments

DP, Cocuzza, Metz, Prokudin, Sato, arXiv:2305.11995

- (TMD) PDFs and (single-hadron) FFs are defined in a way so that they are number densities (before renormalization)

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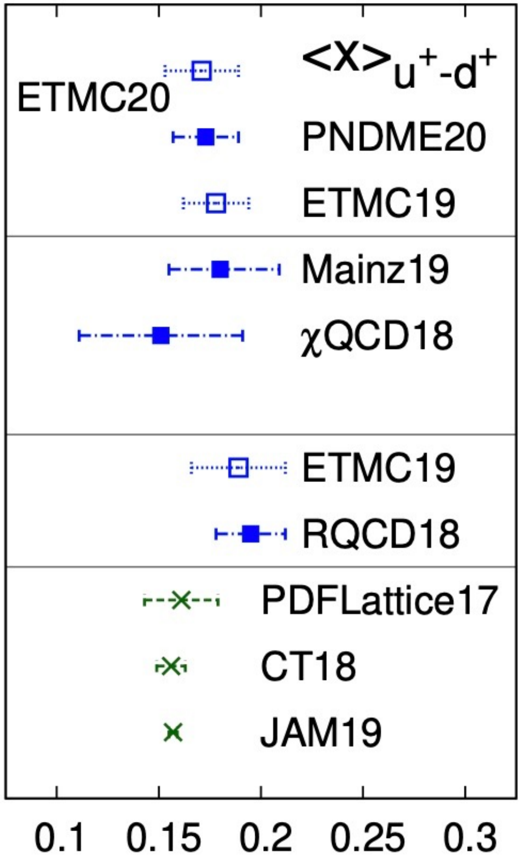
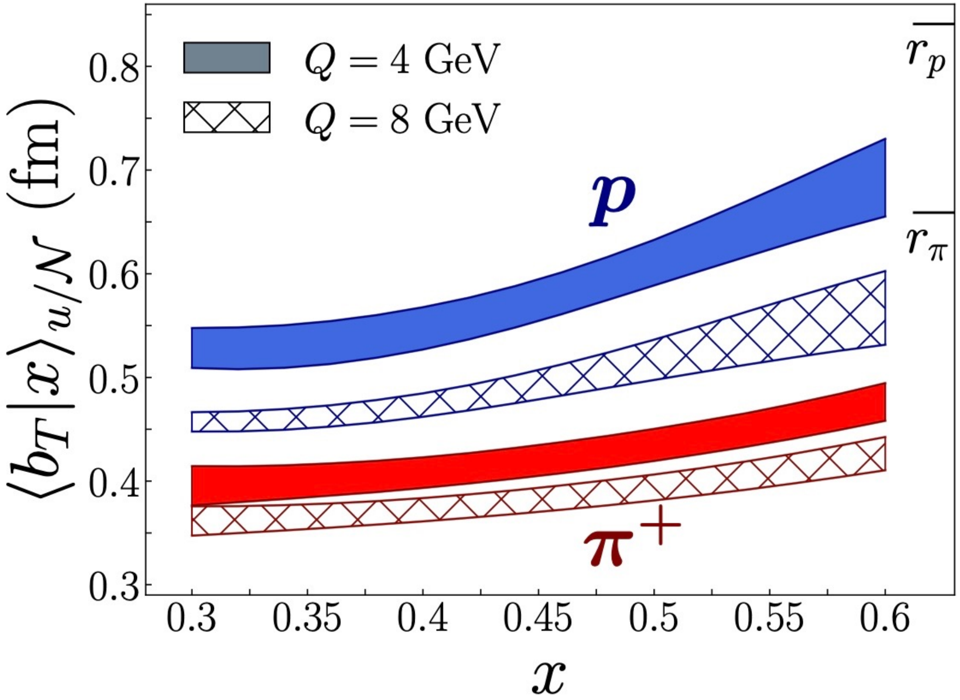
  - *crucial property for our ability to use them to understand hadronic structure (e.g., calculating expectation values)*

Constantinou, et al., PDFLattice Report (2020)



Barry, et al. (2023) - JAM Collaboration



- (TMD) PDFs and (single-hadron) FFs are defined in a way so that they are number densities (before renormalization)
  - *certain sum rules are satisfied that can be used to constrain/cross-check phenomenological extractions of PDFs/FFs and model calculations*

Number sum rules

$$\sum_{i=u,d,s,..}\int_0^1 dx\,[f_1^{i/N}(x) - f_1^{\bar{i}/N}(x)] = \mathcal{B}$$

($\mathcal{B}$ is the baryon number, e.g.,= 3 for a proton)

$$\sum_h \int_0^1 dz\, D_1^{h/i}(z) = \mathcal{N}$$

($\mathcal{N}$ is the total number of hadrons produced when the parton fragments)

Momentum sum rules

$$\sum_{\text{all } i}\int_0^1 dx\, x\, f_1^{i/N}(x) = 1 \qquad \sum_h \int_0^1 dz\, z\, D_1^{h/i}(z) = 1$$

NB: sum rules hold under renormalization

- An aside: reference frames (Collins (2011))
  - "Parton frame" (p): parton has no transverse momentum, hadron has transverse momentum $P_\perp$ - useful in the formulation of FFs as number densities and proofs of sum rules
  - "Hadron frame" (h): hadron has no transverse momentum, parton has transverse momentum k_T - more practical for phenomenology

$$V_{\mathrm{p}}^- = V_{\mathrm{h}}^- \equiv V^-$$

$$V_{\mathrm{p}}^+ = (\vec{k}_T/k^-)^2\, V^-/2 + V_{\mathrm{h}}^+ - \vec{k}_T \cdot \vec{V}_T/k^-$$

$$\vec{V}_\perp = -(\vec{k}_T/k^-)V^- + \vec{V}_T$$

- *NB***:** All proofs discussed follow closely the work of Meissner, Metz, DP, Phys. Lett. **B690**, 296 (2010)

$$D_1^{h/q}(z,\vec{P}_\perp^2) = \frac{1}{N_c}\,\frac{\mathbf{1}}{\boldsymbol{4z}}\sum_X\!\!\!\!\!\!\int \frac{d\xi^+ d^2\vec{\xi}_\perp}{(2\pi)^3}\, e^{ik^-\xi^+}\,\mathrm{Tr}\Big[\langle 0|\mathcal{W}(\infty,\xi)\psi_q(\xi^+,0^-,\vec{\xi}_\perp)|P;X\rangle \\ \times\,\langle P;X|\bar{\psi}_q(0^+,0^-,\vec{0}_\perp)\mathcal{W}(0,\infty)|0\rangle\,\gamma^-\Big]$$

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This ***prefactor is key to the number density interpretation*** of single-hadron FFs → allows us to introduce the number operator when deriving the number sum rule

$$\hat{N} \equiv \sum_h \int \frac{dP^- d^2\vec{P}_\perp}{(2\pi)^3\,2P^-}\,\hat{a}_h^\dagger \hat{a}_h = \sum_h \int \frac{dz\,d^2\vec{P}_\perp}{(2\pi)^3\,2z}\,\hat{a}_h^\dagger \hat{a}_h$$

$$D_1^{h/q}(z,\vec{P}_\perp^2) = \frac{1}{N_c}\,\boldsymbol{\frac{1}{4z}}\sum_X \int \frac{d\xi^+ d^2\vec{\xi}_\perp}{(2\pi)^3}\, e^{ik^-\xi^+}\,\mathrm{Tr}\Big[\langle 0|\mathcal{W}(\infty,\xi)\psi_q(\xi^+,0^-,\vec{\xi}_\perp)|P;X\rangle \times \langle P;X|\bar{\psi}_q(0^+,0^-,\vec{0}_\perp)\mathcal{W}(0,\infty)|0\rangle\,\gamma^-\Big]$$

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⋮

$$\sum_h \int_0^1 dz \int d^2\vec{P}_\perp\, D_1^{h/q}(z,\vec{P}_\perp^2) = \mathcal{N}$$

$$\Delta_{\alpha\beta}^{h_1 h_2/i}(z_1, z_2, \vec{P}_{1\perp}, \vec{P}_{2\perp}) = \frac{1}{N_i} \sum_X \int \frac{d\xi^+ d^2\vec{\xi}_\perp}{(2\pi)^3}\, e^{ik\cdot\xi}\, \mathcal{O}_{\alpha\beta}^{h_1 h_2/i}(\xi)\Big|_{\xi^-=0}$$

<u>quark fragmentation</u> ($N_i = N_c$)

$$\begin{aligned}\mathcal{O}_{\alpha\beta}^{h_1 h_2/q}(\xi) =& \langle 0|\mathcal{W}(\infty,\xi)\psi_{q,\alpha}(\xi^+, 0^-, \vec{\xi}_\perp)|P_1, P_2; X\rangle \\ & \times \langle P_1, P_2; X|\bar{\psi}_{q,\beta}(0^+, 0^-, \vec{0}_\perp)\mathcal{W}(0,\infty)|0\rangle\end{aligned}$$

<u>gluon fragmentation</u> ($N_i = N_c^2 - 1$)

$$\begin{aligned}\mathcal{O}_{\alpha\beta}^{h_1 h_2/g}(\xi) =& \langle 0|\mathcal{W}^{ba}(\infty,\xi)F_{+\alpha}^a(\xi^+, 0^-, \vec{\xi}_\perp)|P_1, P_2; X\rangle \\ & \times \langle P_1, P_2; X|F_{+\beta}^c(0^+, 0^-, \vec{0}_\perp)\mathcal{W}^{cb}(0,\infty)|0\rangle\end{aligned}$$

NB: we will focus on quark fragmentation, but similar results hold for gluon fragmentation

$$\frac{1}{64\pi^3 z_1 z_2}\mathrm{Tr}\left[\Delta^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}, \vec{P}_{2\perp})\gamma^-\right] = D_1^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}^2, \vec{P}_{2\perp}^2, \vec{P}_{1\perp}\cdot\vec{P}_{2\perp})$$

This ***prefactor is key to the number density interpretation*** of dihadron FFs (see also Majumder, Wang (2004)) because in order to prove a number sum rule we need to introduce the number operator separately for each hadron (*j* = 1, 2)

$$\hat{N}_{h_j} \equiv \int \frac{dP_j^- d^2\vec{P}_{j\perp}}{(2\pi)^3\, 2P_j^-}\,\hat{a}_{h_j}^\dagger \hat{a}_{h_j} = \int \frac{dz_j d^2\vec{P}_{j\perp}}{(2\pi)^3\, 2z_j}\,\hat{a}_{h_j}^\dagger \hat{a}_{h_j}$$

$$\frac{1}{64\pi^3 z_1 z_2}\mathrm{Tr}\left[\Delta^{h_1h_2/q}(z_1,z_2,\vec{P}_{1\perp},\vec{P}_{2\perp})\gamma^-\right]=D_1^{h_1h_2/q}(z_1,z_2,\vec{P}_{1\perp}^2,\vec{P}_{2\perp}^2,\vec{P}_{1\perp}\cdot\vec{P}_{2\perp})$$

$$\vdots$$

$$\sum_{h_1}\int_0^1 dz_1\int d^2\vec{P}_{1\perp}D_1^{h_1h_2/q}(z_1,z_2,\vec{P}_{1\perp}^2,\vec{P}_{2\perp}^2,\vec{P}_{1\perp}\cdot\vec{P}_{2\perp})=(\mathcal{N}-1)\,D_1^{h_2/q}(z_2,\vec{P}_{2\perp}^2)$$

$$\sum_{h_1}\sum_{h_2}\int_0^1 dz_2\int_0^{1-z_2}dz_1\int d^2\vec{P}_{1\perp}\int d^2\vec{P}_{2\perp}D_1^{h_1h_2/q}(z_1,z_2,\vec{P}_{1\perp}^2,\vec{P}_{2\perp}^2,\vec{P}_{1\perp}\cdot\vec{P}_{2\perp})=\mathcal{N}(\mathcal{N}-1)$$

Total number of *hadron pairs* produced when the parton fragments

$$\frac{1}{64\pi^3 z_1 z_2}\mathrm{Tr}\left[\Delta^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}, \vec{P}_{2\perp})\gamma^-\right] = D_1^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}^2, \vec{P}_{2\perp}^2, \vec{P}_{1\perp}\cdot\vec{P}_{2\perp})$$

.
.
.

$$\sum_{h_1}\int_0^1 dz_1 \int d^2\vec{P}_{1\perp}\, D_1^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}^2, \vec{P}_{2\perp}^2, \vec{P}_{1\perp}\cdot\vec{P}_{2\perp}) = (\mathcal{N}-1)\, D_1^{h_2/q}(z_2, \vec{P}_{2\perp}^2)$$

$$\sum_{h_1}\sum_{h_2}\int_0^1 dz_2 \int_0^{1-z_2} dz_1 \int d^2\vec{P}_{1\perp}\int d^2\vec{P}_{2\perp}\, D_1^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}^2, \vec{P}_{2\perp}^2, \vec{P}_{1\perp}\cdot\vec{P}_{2\perp}) = \mathcal{N}(\mathcal{N}-1)$$

Total number of *hadron pairs*
produced when the parton fragments

The proof is not possible if a prefactor of $1/(4z) = 1/(4(z_1+z_2))$ is used!

$$\frac{1}{64\pi^3 z_1 z_2}\mathrm{Tr}\Big[\Delta^{h_1h_2/q}(z_1,z_2,\vec{P}_{1\perp},\vec{P}_{2\perp})\gamma^-\Big] = D_1^{h_1h_2/q}(z_1,z_2,\vec{P}_{1\perp}^2,\vec{P}_{2\perp}^2,\vec{P}_{1\perp}\cdot\vec{P}_{2\perp})$$

$$\frac{1}{64\pi^3 z_1 z_2}\mathrm{Tr}\Big[\Delta^{h_1h_2/q}(z_1,z_2,\vec{P}_{1\perp},\vec{P}_{2\perp})\gamma^-\gamma_5\Big] = -\frac{\epsilon_\perp^{ij}R_\perp^i P_{h\perp}^j}{zM_h^2}\,G_1^{\perp\,h_1h_2/q}(z_1,z_2,\vec{P}_{1\perp}^2,\vec{P}_{2\perp}^2,\vec{P}_{1\perp}\cdot\vec{P}_{2\perp})$$

$$\begin{aligned}\frac{1}{64\pi^3 z_1 z_2}\mathrm{Tr}\Big[\Delta^{h_1h_2/q}(z_1,z_2,\vec{P}_{1\perp},\vec{P}_{2\perp})i\sigma^{i-}\gamma_5\Big] &= -\frac{\epsilon_\perp^{ij}R_\perp^j}{M_h}H_1^{\sphericalangle'\,h_1h_2/q}(z_1,z_2,\vec{P}_{1\perp}^2,\vec{P}_{2\perp}^2,\vec{P}_{1\perp}\cdot\vec{P}_{2\perp})\\ &\quad+\frac{\epsilon_\perp^{ij}P_{h\perp}^j}{zM_h}H_1^{\perp'\,h_1h_2/q}(z_1,z_2,\vec{P}_{1\perp}^2,\vec{P}_{2\perp}^2,\vec{P}_{1\perp}\cdot\vec{P}_{2\perp})\end{aligned}$$

***NB*: number density interpretation holds not only for unpolarized quarks (γ^- projection) but also for longitudinally ($\gamma^-\gamma^5$ projection) and transversely ($i\sigma^{i-}\gamma^5$ projection) polarized quarks**

Number sum rule

$$\sum_{h_1}\sum_{h_2}\int_0^1 dz_2\int_0^{1-z_2} dz_1\int d^2\vec{P}_{1\perp}\int d^2\vec{P}_{2\perp}\,D_1^{h_1h_2/i}(z_1,z_2,\vec{P}_{1\perp}^2,\vec{P}_{2\perp}^2,\vec{P}_{1\perp}\cdot\vec{P}_{2\perp}) = \mathcal{N}(\mathcal{N}-1)$$

$$\Longrightarrow \boldsymbol{D_1^{h_1h_2/i}(w,x,\vec{Y}^2,\vec{Z}^2,\vec{Y}\cdot\vec{Z})} \equiv \mathcal{J}\cdot D_1^{h_1h_2/i}(z_1,z_2,\vec{P}_{1\perp}^2,\vec{P}_{2\perp}^2,\vec{P}_{1\perp}\cdot\vec{P}_{2\perp})$$

is a number density

Jacobian for the variable transformation

Number sum rule

$$\sum_{h_1}\sum_{h_2}\int_0^1 dz_2 \int_0^{1-z_2} dz_1 \int d^2\vec{P}_{1\perp}\int d^2\vec{P}_{2\perp}\, D_1^{h_1 h_2/i}(z_1, z_2, \vec{P}_{1\perp}^2, \vec{P}_{2\perp}^2, \vec{P}_{1\perp}\cdot\vec{P}_{2\perp}) = \mathcal{N}(\mathcal{N}-1)$$

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is a number density

Jacobian for the variable transformation

Using our new definition, DiFFs can now be interpreted as densities in any momentum variables of choice for the number of hadron pairs ($h_1\, h_2$) fragmenting from the parton

Number sum rule

$$\sum_{h_1}\sum_{h_2}\int_0^1 dz_2\int_0^{1-z_2} dz_1\int d^2\vec{P}_{1\perp}\int d^2\vec{P}_{2\perp}\,D_1^{h_1h_2/i}(z_1,z_2,\vec{P}_{1\perp}^2,\vec{P}_{2\perp}^2,\vec{P}_{1\perp}\cdot\vec{P}_{2\perp}) = \mathcal{N}(\mathcal{N}-1)$$

$$\Longrightarrow \boldsymbol{D_1^{h_1h_2/i}(w,x,\vec{Y}^2,\vec{Z}^2,\vec{Y}\cdot\vec{Z})} \equiv \mathcal{J}\cdot D_1^{h_1h_2/i}(z_1,z_2,\vec{P}_{1\perp}^2,\vec{P}_{2\perp}^2,\vec{P}_{1\perp}\cdot\vec{P}_{2\perp})$$

is a number density

Momentum sum rule

$$\sum_{h_1}\int_0^{1-z_2} dz_1\int d^2\vec{P}_{1\perp}\;z_1\,D_1^{h_1h_2/i}(z_1,z_2,\vec{P}_{1\perp}^{\,2},\vec{P}_{2\perp}^{\,2},\vec{P}_{1\perp}\!\cdot\!\vec{P}_{2\perp}) = (1-z_2)\,D_1^{h_2/i}(z_2,\vec{P}_{2\perp}^{\,2})$$

NB: $D_1^{h_1h_2/i}(z_1,z_2,\vec{P}_{1\perp}^{\,2},\vec{P}_{2\perp}^{\,2},\vec{P}_{1\perp}\!\cdot\!\vec{P}_{2\perp})/D_1^{h_2/i}(z_2,\vec{P}_{2\perp}^{\,2})$ is a conditional number density

Generalization to n-hadron fragmentation

$$\frac{\mathbf{1}}{\mathbf{4(16\pi^3)^{n-1}z_1\cdots z_n}}\mathrm{Tr}\Big[\Delta^{\{h_i\}_n/q}(\{z_i\}_n,\{\vec{P}_{i\perp}\}_n)\gamma^-\Big] = D_1^{\{h_i\}_n/q}(\{z_i\}_n,\{\vec{P}_{i\perp}^2\}_n,\{\vec{P}_{i\perp}\cdot\vec{P}_{j\perp}\}_n)$$

$$\sum_{h_1,\ldots,h_n}\int dz_n\cdots dz_1\int d^2\vec{P}_{1\perp}\cdots d^2\vec{P}_{n\perp}\,D_1^{\{h_i\}_n/i}(\{z_i\}_n,\{\vec{P}_{i\perp}^2\}_n,\{\vec{P}_{i\perp}\cdot\vec{P}_{j\perp}\}_n) = \prod_{k=0}^{n-1}(\mathcal{N}-k)$$

- Connection to phenomenology - work in a frame where the dihadron has no transverse momentum

$$P_h = P_1 + P_2 \qquad R = (P_1 - P_2)/2 \qquad z = z_1 + z_2 \qquad \zeta = (z_1 - z_2)/z$$

$$(z_1, z_2, \vec{P}_{1\perp}, \vec{P}_{2\perp}) \longrightarrow (z, \zeta, \vec{k}_T, \vec{R}_T) : \quad \mathcal{J} = z^3/2$$

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$$D_1^{h_1 h_2/q}(z, \zeta, \vec{k}_T^2, \vec{R}_T^2, \vec{k}_T \cdot \vec{R}_T) = \frac{\boldsymbol{z}}{\boldsymbol{32\pi^3(1-\zeta^2)}} \mathrm{Tr}\Big[\Delta^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}, \vec{P}_{2\perp})\gamma^-\Big]$$

is a number density in $(z, \zeta, \vec{k}_T, \vec{R}_T)$

- Connection to phenomenology - work in a frame where the dihadron has no transverse momentum

$$P_h = P_1 + P_2 \qquad R = (P_1 - P_2)/2 \qquad z = z_1 + z_2 \qquad \zeta = (z_1 - z_2)/z$$

$$(z_1, z_2, \vec{P}_{1\perp}, \vec{P}_{2\perp}) \longrightarrow (z, \zeta, \vec{k}_T, \vec{R}_T) : \quad \mathcal{J} = z^3/2$$

$$D_1^{h_1 h_2/q}(z, \zeta, \vec{k}_T^2, \vec{R}_T^2, \vec{k}_T \cdot \vec{R}_T) = \frac{\boldsymbol{z}}{\boldsymbol{32\pi^3(1-\zeta^2)}} \mathrm{Tr}\Big[\Delta^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}, \vec{P}_{2\perp})\gamma^-\Big]$$

is a number density in $(z, \zeta, \vec{k}_T, \vec{R}_T)$

Compare this to the original definition of Bianconi, Boffi, Jakob, Radici (2000) that has been the basis for all dihadron research (sensitive to R_T) for the last 20+ years

$$D_1^{h_1 h_2/q,\mathrm{BBJR}}(z, \zeta, \vec{k}_T^2, \vec{R}_T^2, \vec{k}_T \cdot \vec{R}_T) = \frac{\boldsymbol{1}}{\boldsymbol{4z}} \mathrm{Tr}\Big[\Delta^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}, \vec{P}_{2\perp})\gamma^-\Big]$$

Does ***not*** allow sum rules to be derived that justify a number density interpretation

- Experimental measurements are sensitive to the so-called "extended" DiFFs where k_T (and usually ζ) is integrated out

$$\frac{z}{32\pi^3(1-\zeta^2)}\int d^2\vec{k}_T\,\mathrm{Tr}\Big[\Delta^{h_1h_2/q}(z_1,z_2,\vec{P}_{1\perp},\vec{P}_{2\perp})\gamma^-\Big] = D_1^{h_1h_2/q}(z,\zeta,\vec{R}_T^2)$$

$$\frac{z}{32\pi^3(1-\zeta^2)}\int d^2\vec{k}_T\,\mathrm{Tr}\Big[\Delta^{h_1h_2/q}(z_1,z_2,\vec{P}_{1\perp},\vec{P}_{2\perp})i\sigma^{i-}\gamma_5\Big] = -\frac{\epsilon_T^{ij}R_T^j}{M_h}H_1^{\sphericalangle\, h_1h_2/q}(z,\zeta,\vec{R}_T^2)$$

are number densities in $(z,\zeta,\vec{R}_T)$

- Experimental measurements are sensitive to the so-called "extended" DiFFs where k_T (and usually $\boldsymbol{\zeta}$) is integrated out

$$\frac{z}{32\pi^3(1-\zeta^2)}\int d^2\vec{k}_T\,\mathrm{Tr}\Big[\Delta^{h_1h_2/q}(z_1,z_2,\vec{P}_{1\perp},\vec{P}_{2\perp})\gamma^-\Big] = D_1^{h_1h_2/q}(z,\zeta,\vec{R}_T^2)$$

$$\frac{z}{32\pi^3(1-\zeta^2)}\int d^2\vec{k}_T\,\mathrm{Tr}\Big[\Delta^{h_1h_2/q}(z_1,z_2,\vec{P}_{1\perp},\vec{P}_{2\perp})i\sigma^{i-}\gamma_5\Big] = -\frac{\epsilon_T^{ij}R_T^j}{M_h}H_1^{\sphericalangle\, h_1h_2/q}(z,\zeta,\vec{R}_T^2)$$

are number densities in $(z,\zeta,\vec{R}_T)$

NB: Experiments report measurements in terms of M_h

$$\vec{R}_T^2 = \frac{1-\zeta^2}{4}M_h^2 - \frac{1-\zeta}{2}M_1^2 - \frac{1+\zeta}{2}M_2^2$$

One *cannot* simply replace the R_T dependence in the DiFF with an M_h and still maintain a number density interpretation in M_h

$$(z_1,z_2,\vec{P}_{1\perp},\vec{P}_{2\perp}) \longrightarrow (z,\zeta,\vec{k}_T,M_h,\phi_{R_T}): \quad \mathcal{J} = z^3(1-\zeta^2)/8$$

- Experimental measurements are sensitive to the so-called "extended" DiFFs where k_T (and usually $\boldsymbol{\zeta}$) is integrated out

$$D_1^{h_1 h_2/i}(z, M_h) \equiv \frac{\pi}{2} M_h \int_{-1}^{1} d\zeta \, (1-\zeta^2) D_1^{h_1 h_2/i}(z, \zeta, \vec{R}_T^2)$$

is a number density in (z, M_h)

- Experimental measurements are sensitive to the so-called "extended" DiFFs where k_T (and usually $\boldsymbol{\zeta}$) is integrated out

$$D_1^{h_1 h_2/i}(z, M_h) \equiv \frac{\pi}{2} M_h \int_{-1}^{1} d\zeta\, (1-\zeta^2) D_1^{h_1 h_2/i}(z, \zeta, \vec{R}_T^2)$$

is a number density in (z, M_h)

$$\frac{d\sigma}{dz\, dM_h} = \frac{4\pi N_c \alpha_{\rm em}^2}{3Q^2} \sum_q e_q^2\, D_1^{h_1 h_2/q}(z, M_h)$$

Measured by Belle (2017) for $\pi^+\pi^-$ and $\pi^+\pi^+$

New definition will allow us to calculate, e.g., the average value of *z* or of M_h for the ensemble of all $\pi^+\pi^-$ (or $\pi^+\pi^+$) pairs from the fragmentation of an unpolarized quark

$$\langle \mathcal{O}(z, M_h) \rangle^{h_1 h_2/i} = \int dz\, dM_h\, \mathcal{O}(z, M_h)\, D_1^{h_1 h_2/i}(z, M_h)$$

- Experimental measurements are sensitive to the so-called "extended" DiFFs where k_T (and usually **ζ**) is integrated out

$$D_1^{h_1 h_2/i}(z, M_h) \equiv \frac{\pi}{2} M_h \int_{-1}^{1} d\zeta\,(1-\zeta^2)\,D_1^{h_1 h_2/i}(z, \zeta, \vec{R}_T^2)$$

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partonic cross section for $e^+e^- \to \gamma \to q\bar{q}$

This is exactly the structure *dσ* should have if *D_1* has a number density interpretation

- Experimental measurements are sensitive to the so-called "extended" DiFFs where k_T (and usually $\boldsymbol{\zeta}$) is integrated out

$$D_1^{h_1 h_2/i}(z, M_h) \equiv \frac{\pi}{2} M_h \int_{-1}^{1} d\zeta \, (1-\zeta^2) D_1^{h_1 h_2/i}(z, \zeta, \vec{R}_T^2)$$

is a number density in (z, M_h)

$$\frac{d\sigma}{dz\, d\zeta\, d^2\vec{R}_T} = \boxed{\frac{4\pi N_c \alpha_{\rm em}^2}{3Q^2} \sum_q e_q^2} D_1^{h_1 h_2/q}(z, \zeta, \vec{R}_T^2)$$

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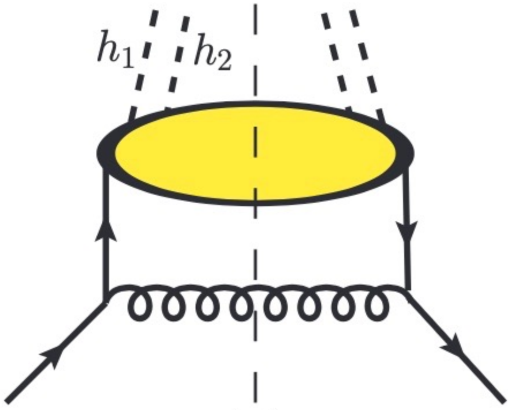
- Experimental measurements are sensitive to the so-called "extended" DiFFs where k_T (and usually $\boldsymbol{\zeta}$) is integrated out

$$D_1^{h_1 h_2/i}(z, M_h) \equiv \frac{\pi}{2} M_h \int_{-1}^{1} d\zeta \, (1-\zeta^2) D_1^{h_1 h_2/i}(z, \zeta, \vec{R}_T^2)$$

$$H_1^{\sphericalangle\, h_1 h_2/i}(z, M_h) \equiv \frac{\pi}{2} M_h \int_{-1}^{1} d\zeta \, \frac{|\vec{R}_T|}{M_h} \, (1-\zeta^2) H_1^{\sphericalangle\, h_1 h_2/i}(z, \zeta, \vec{R}_T^2)$$

The analytical formulas in the literature need to be modified to account for these new definitions

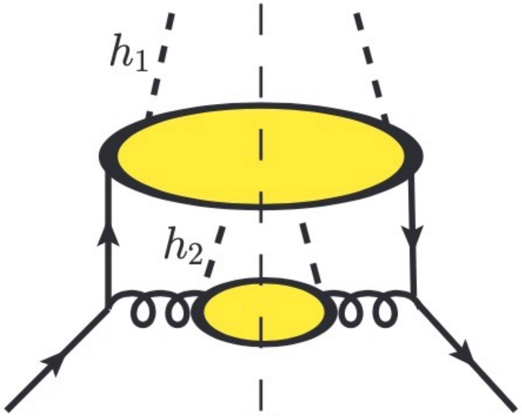
- Evolution equations for extended DiFFs



(a)

$$D_1^{h_1 h_2/i} \to D_1^{h_1 h_2/j}$$

"Homogeneous term"

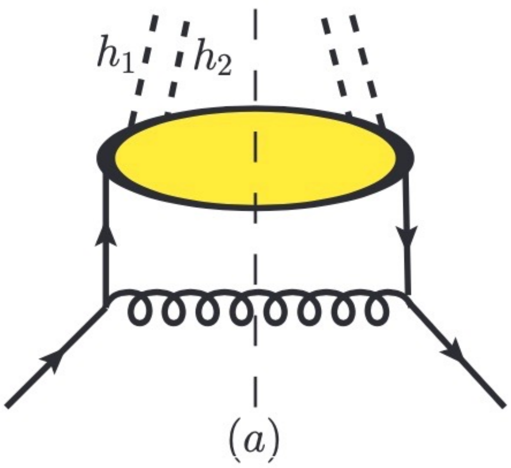


(b)

$$D_1^{h_1 h_2/i} \to D_1^{h_1/j} \otimes D_1^{h_2/k}$$

"Inhomogeneous term"

- Evolution equations for extended DiFFs



$$D_1^{h_1h_2/i} \to D_1^{h_1h_2/j}$$

"Homogeneous term"

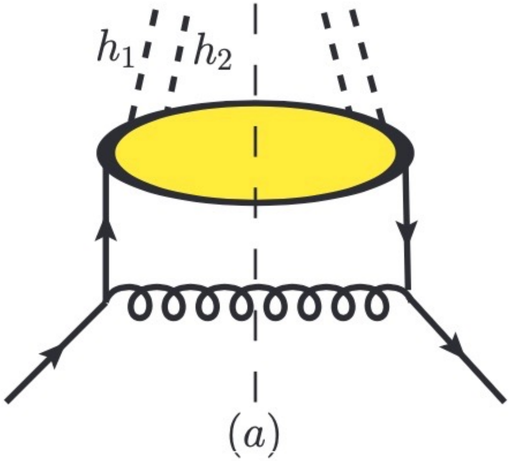
$$D_1^{h_1h_2/i} \to D_1^{h_1/j} \otimes D_1^{h_2/k}$$

"Inhomogeneous term"

$$D_1^{h_1h_2/q,(\mathrm{b})}(z_1, z_2, \vec{R}_T^2; \mu) = \frac{1}{\vec{R}_T^2}\frac{C_F\alpha_s}{2\pi^2}\mu^{2\epsilon}\int_{z_1}^{1-z_2}\frac{dw}{w(1-w)}D_1^{h_1/q}(z_1/w)D_1^{h_2/g}(z_2/(1-w))\frac{1+w^2}{1-w}$$

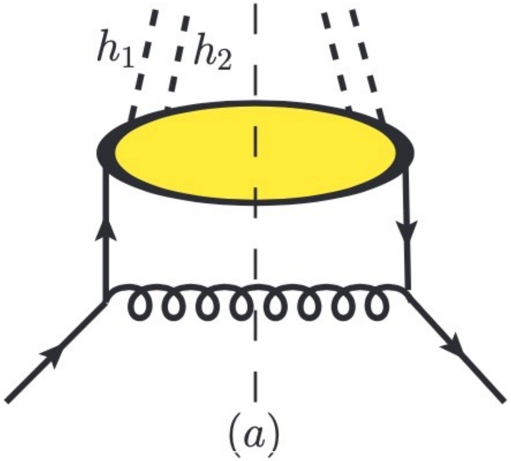
The inhomogeneous terms are *not* UV divergent when one keeps the dependence on R_T
(see also Ceccopieri, et al. (2007))

- Evolution equations for extended DiFFs



$$D_1^{h_1 h_2/q}(z,\zeta,\vec{R}_T^2) = \frac{z}{32\pi^3(1-\zeta^2)}\int d^2\vec{k}_T\,\mathrm{Tr}\Big[\Delta^{h_1 h_2/q}(z_1,z_2,\vec{P}_{1\perp},\vec{P}_{2\perp})\gamma^-\Big]$$

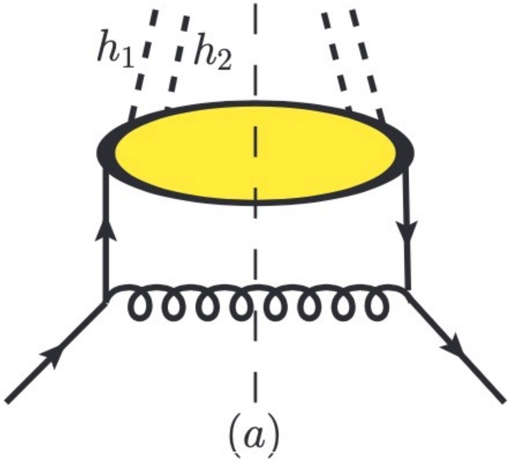
- Evolution equations for extended DiFFs



$$D_1^{h_1 h_2/q}(z,\zeta,\vec{R}_T^2) = \frac{z}{32\pi^3(1-\zeta^2)}\int d^2\vec{k}_T\,\mathrm{Tr}\Big[\Delta^{h_1 h_2/q}(z_1,z_2,\vec{P}_{1\perp},\vec{P}_{2\perp})\gamma^-\Big]$$

$$D_1^{h/q}(z) = \frac{z}{4}\int d^2\vec{k}_T\,\Delta^{h/q}(z,\vec{k}_T)$$

- Evolution equations for extended DiFFs

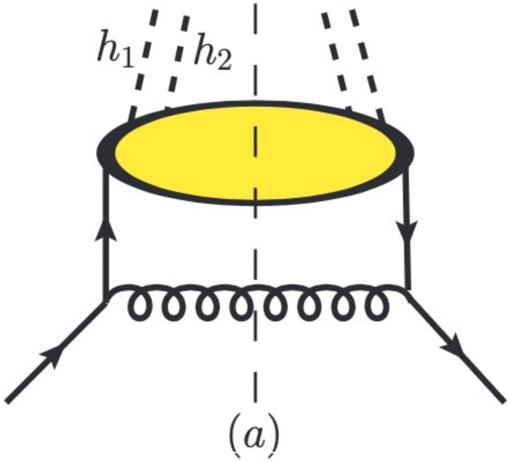


$$D_1^{h_1 h_2/q}(z,\zeta,\vec{R}_T^2) = \frac{z}{32\pi^3(1-\zeta^2)}\boxed{\int d^2\vec{k}_T}\,\mathrm{Tr}\left[\Delta^{h_1 h_2/q}(z_1,z_2,\vec{P}_{1\perp},\vec{P}_{2\perp})\gamma^-\right]$$

→ ζ dependence is not altered by evolution

$$D_1^{h/q}(z) = \frac{z}{4}\boxed{\int d^2\vec{k}_T}\,\Delta^{h/q}(z,\vec{k}_T)$$

- Evolution equations for extended DiFFs



$$D_1^{h_1 h_2/q}(z,\zeta,\vec{R}_T^2) = \frac{\boxed{z}}{32\pi^3(1-\boxed{\zeta^2})}\boxed{\int d^2\vec{k}_T}\,\mathrm{Tr}\left[\Delta^{h_1 h_2/q}(z_1,z_2,\vec{P}_{1\perp},\vec{P}_{2\perp})\gamma^-\right]$$

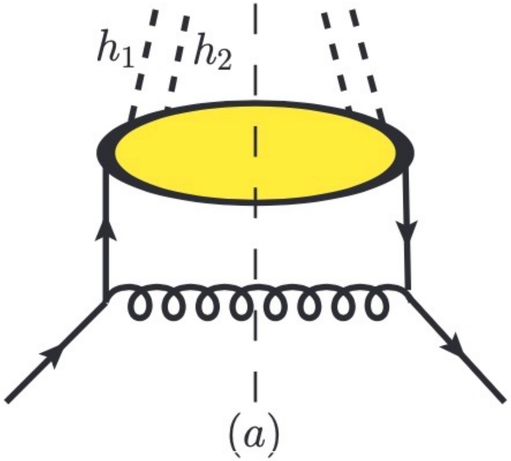
→ ζ dependence is not altered by evolution

$$D_1^{h/q}(z) = \frac{\boxed{z}}{4}\boxed{\int d^2\vec{k}_T}\,\Delta^{h/q}(z,\vec{k}_T)$$

Evolution is independent of the target (in the case of PDFs) or final (in the case of FFs) state (Collins (2011))

➡ The evolution equations of the extended DiFFs are the same as single-hadron collinear FFs

- Evolution equations for extended DiFFs



$$\frac{\partial \mathcal{D}^{h_1 h_2/i}(z,\zeta,\vec{R}_T^2;\mu)}{\partial \ln \mu^2} = \sum_{i'} \int_z^1 \frac{dw}{w} \mathcal{D}^{h_1 h_2/i'}\Big(\frac{z}{w},\zeta,\vec{R}_T^2;\mu\Big) P_{i\to i'}(w)$$

where $\mathcal{D} = D_1$ or $H_1^{\sphericalangle}$

use unpolarized splitting kernels

use transversely polarized splitting kernels

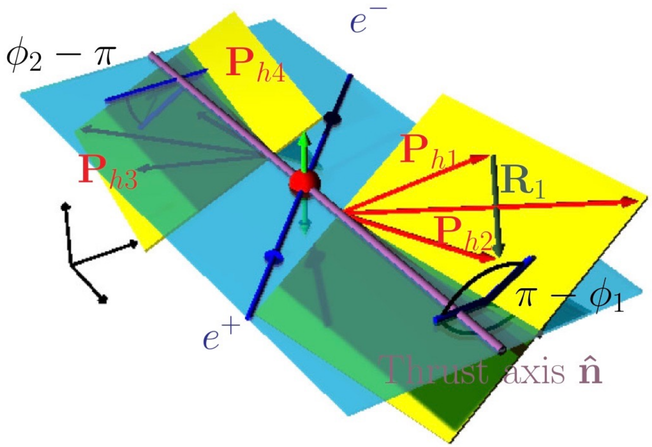
New DiFF Phenomenology

Cocuzza, Metz, DP, Prokudin, Sato, Seidl, arXiv:2306.XXXXX

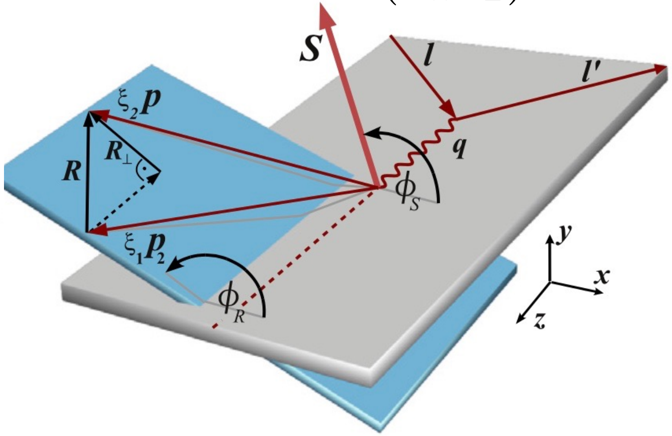
Note: all phenomenological results are PRELIMINARY

Jefferson Lab Angular Momentum Collaboration

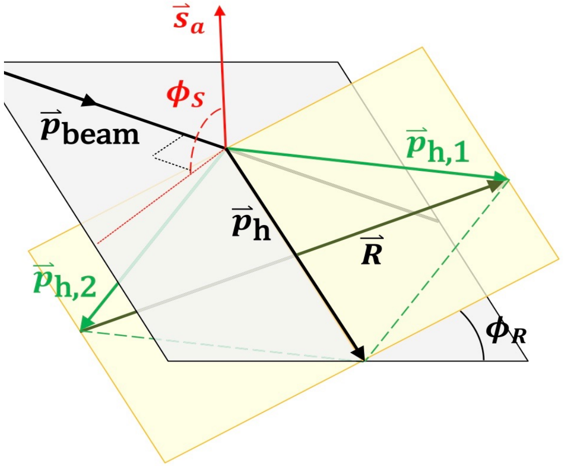
$e^+e^- \to (h_1 h_2)(\bar{h}_1 \bar{h}_2)\, X$



$\ell\, N^{\uparrow} \to \ell\, (h_1 h_2)\, X$



$p^{\uparrow} p \to (h_1 h_2)\, X$



(Collins, et al. (1994); Bianconi, et al. (2000); Bacchetta, Radici (2003, 2004); Courtoy, et al. (2012); Matevosyan, et al. (2018); Radici, et al. (2013, 2015, 2018); Benel, et al. (2020), …)

$$a_{12R} = \frac{\sin^2\theta_2 \sum_q e_q^2\, H_1^{\sphericalangle,q}(z, M_h)\, H_1^{\sphericalangle,\bar{q}}(\bar{z}, \overline{M}_h)}{(1+\cos^2\theta_2) \sum_q e_q^2\, D_1^q(z, M_h)\, D_1^{\bar{q}}(\bar{z}, \overline{M}_h)}$$

Artru-Collins asymmetry

<u>Note</u>: *D₁* can be constrained using data on dσ/dzdM$_h$ from BELLE (2017)

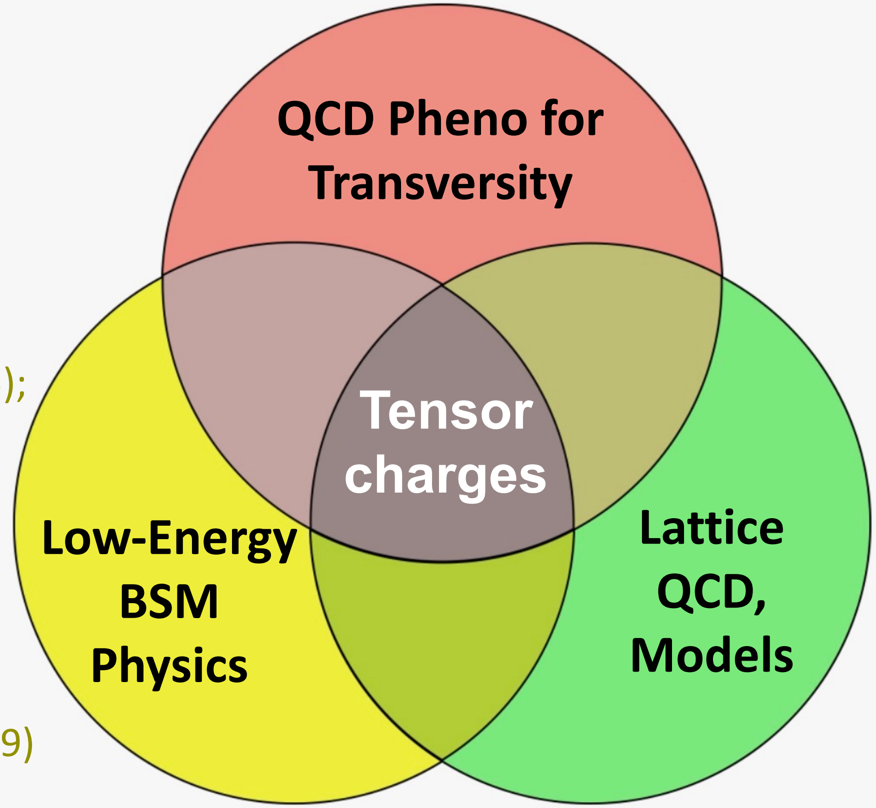
$$\frac{d\sigma}{dz\, dM_h} = \frac{4\pi N_c \alpha_{\text{em}}^2}{3Q^2} \sum_q e_q^2\, D_1^q(z, M_h)$$

$$A_{UT}^{\sin(\phi_R+\phi_S)} = \frac{\sum_q e_q^2\, h_1^q(x)\, H_1^{\sphericalangle,q}(z, M_h)}{\sum_q e_q^2\, f_1^q(x)\, D_1^q(z, M_h)}$$

$$A_{UT}^{\sin(\phi_R-\phi_S)} \sim \frac{\frac{d\Delta\hat{\sigma}_{a^\uparrow b \to c^\uparrow}}{d\hat{t}} \otimes h_1^a(x_a) \otimes f_1^b(x_b) \otimes H_1^{\sphericalangle,c}(z, M_h)}{\frac{d\hat{\sigma}_{ab \to c}}{d\hat{t}} \otimes f_1^a(x_a) \otimes f_1^b(x_b) \otimes D_1^c(z, M_h)}$$

$$\delta q \equiv \int_0^1 dx\,[h_1^q(x) - h_1^{\bar{q}}(x)] \qquad g_T \equiv \delta u - \delta d$$

Anselmino, et al. (2007, 2009, 2013, 2015);
Goldstein, et al. (2014); Kang, et al. (2016); Radici, et al. (2013, 2015, 2018);
Benel, et al. (2020); D'Alesio, et al. (2020); Cammarota, et al. (2020);
Gamberg, et al. (2022); Cocuzza, et al. (2023)



QCD Pheno for Transversity

Tensor charges

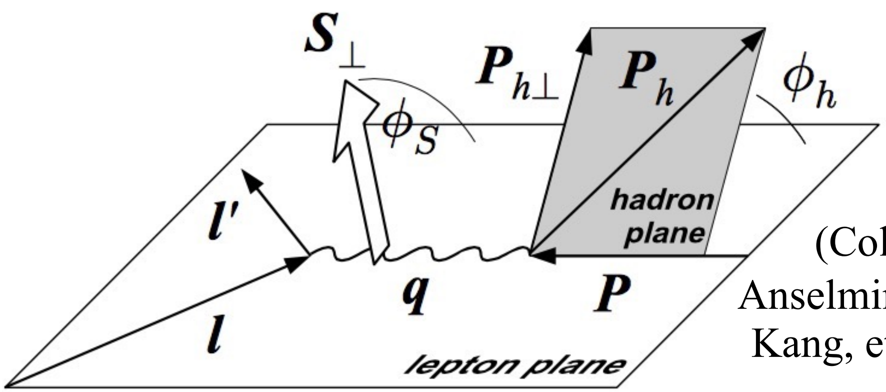
Low-Energy BSM Physics

Lattice QCD, Models

Herczeg (2001);
Erler, Ramsey-Musolf (2005);
Pospelov, Ritz (2005);
Severijns, et al. (2006);
Cirigliano, et al. (2013);
Courtoy, et al. (2015);
Yamanaka, et al. (2017);
Liu, et al. (2018);
Gonzalez-Alonso, et al. (2019)

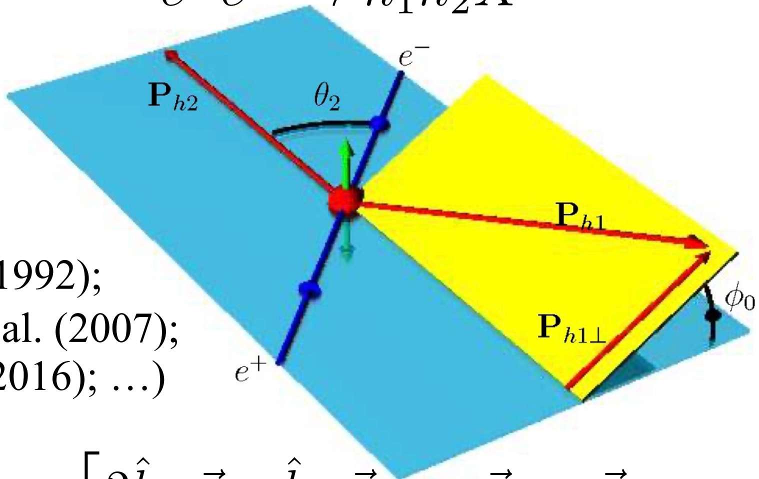
He, Ji (1995);
Barone, et al. (1997);
Schweitzer, et al. (2001);
Gamberg, Goldstein (2001);
Pasquini, et al. (2005);
Wakamatsu (2007);
Lorce (2009);
Gupta, et al. (2018);
Yamanaka, et al. (2018);
Hasan, et al. (2019);
Alexandrou, et al. (2019, 2023);
Yamanaka, et al. (2013);
Pitschmann, et al. (2015);
Xu, et al. (2015);
Wang, et al. (2018);
Liu, et al. (2019)

$$\ell\, N^\uparrow \to \ell\, h\, X$$



$$e^+e^- \to h_1 h_2 X$$

(Collins (1992);
Anselmino, et al. (2007);
Kang, et al. (2016); …)



$$F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}\left[-\frac{\hat{h}\cdot\vec{p}_\perp}{M_h}\boldsymbol{h_1 H_1^\perp}\right] \qquad F_{UU}^{\cos(2\phi_0)} = \mathcal{C}\left[\frac{2\hat{h}\cdot\vec{p}_{a\perp}\,\hat{h}\cdot\vec{p}_{b\perp}-\vec{p}_{a\perp}\cdot\vec{p}_{b\perp}}{M_a M_b}\boldsymbol{H_1^\perp\,\bar{H}_1^\perp}\right]$$

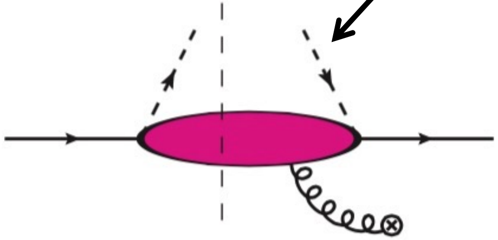
TMD/Collins-Soper-Sterman (CSS) Evolution

OPE

Sudakov exponentials (gluon radiation)

$$\boldsymbol{\tilde{h}_1(x, b_T; Q^2, \mu_Q)} \quad \sim \quad \boldsymbol{h_1(x;\mu_{b_*})}\exp\left[-S_{pert}(b_*(b_T);\mu_{b_*},Q,\mu_Q) - S_{NP}^{h_1}(b_T,Q)\right]$$

$$\boldsymbol{\tilde{H}_1^{\perp(1)}(z, b_T; Q^2, \mu_Q)} \quad \sim \quad \boldsymbol{H_1^{\perp(1)}(z;\mu_{b_*})}\exp\left[-S_{pert}(b_*(b_T);\mu_{b_*},Q,\mu_Q) - S_{NP}^{H_1^\perp}(b_T,Q)\right]$$



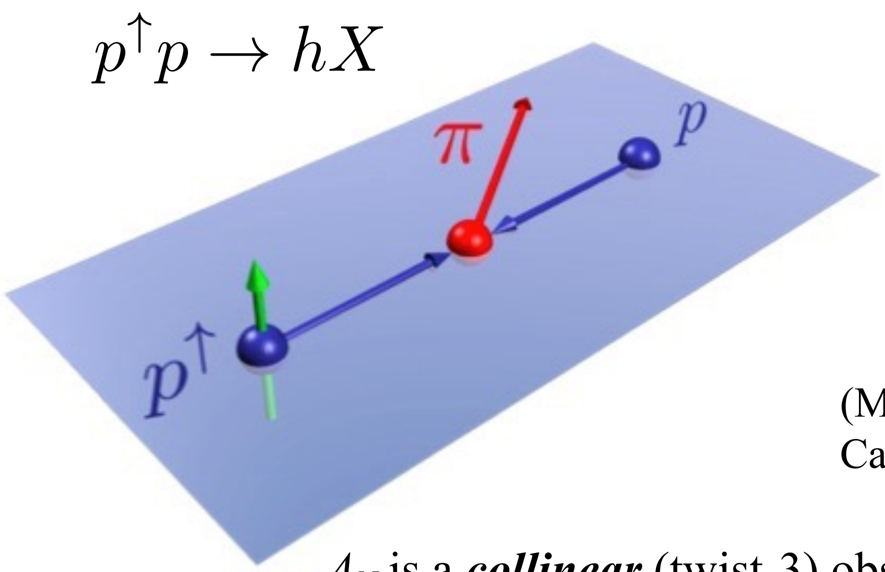
qgq
fragmentation
function

Parton model

$$h_1(x) = \int d^2\vec{k}_T\, h_1(x,\vec{k}_T^2)$$

$$H_1^{\perp(1)}(z) = z^2\int d^2\vec{p}_\perp\,\frac{p_\perp^2}{2M_h^2}H_1^\perp(z,z^2p_\perp^2)$$

$$p^{\uparrow}p \to hX$$

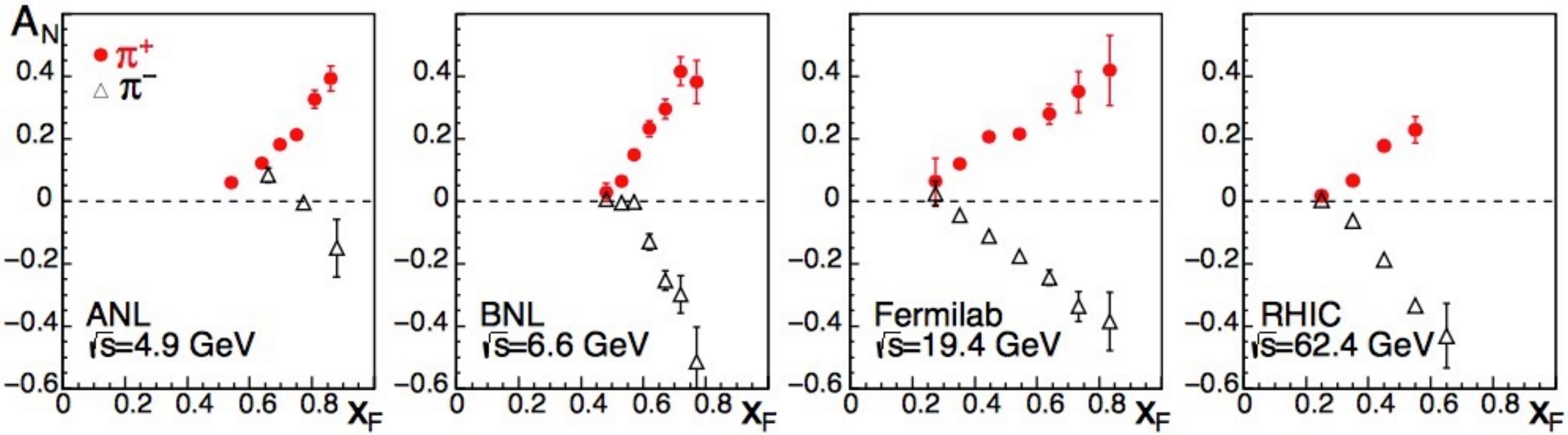


$$d\Delta\sigma(S_T) \sim \underbrace{H_{QS} \otimes f_1 \otimes \boldsymbol{F_{FT}} \otimes D_1}_{\text{Qiu-Sterman term}}$$

$$+\boxed{\underbrace{H_F \otimes f_1 \otimes \boldsymbol{h_1} \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{\tilde{H}}\right)}_{\text{Fragmentation term}}}$$

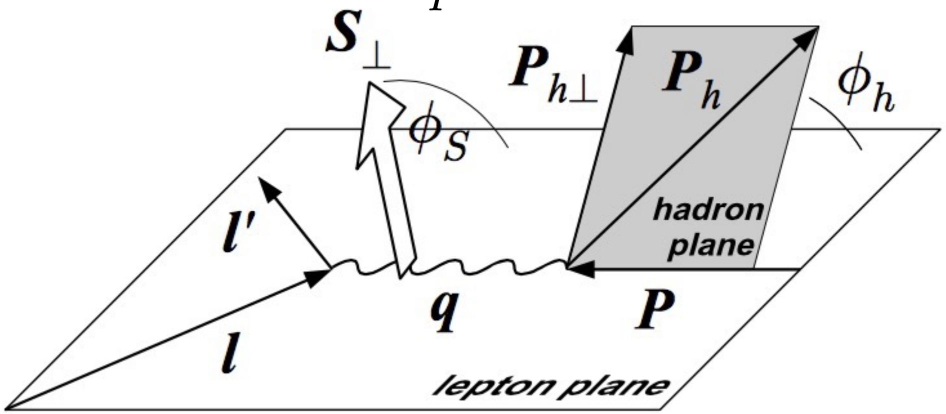
(Metz, DP (2012); Kanazawa, et al. (2014);
Cammarota, et al. (2020); Gamberg, et al. (2017, 2022))

A_N is a ***collinear*** (twist-3) observable

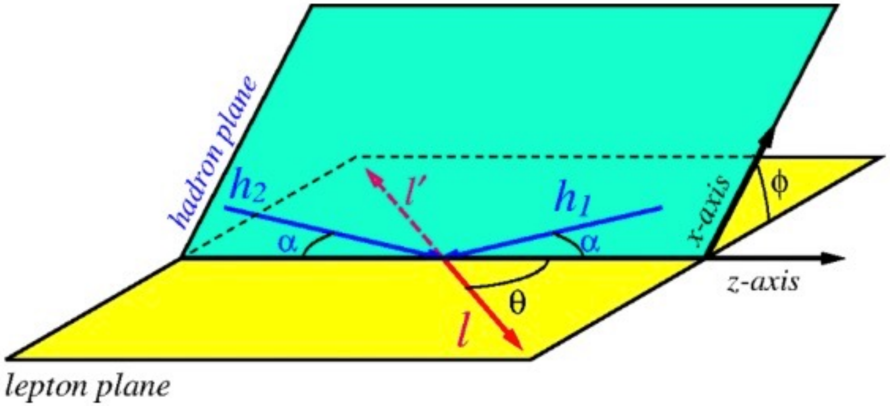


1976 →

$\ell p^{\uparrow} \to \ell h X$



$\{\pi, p\}p^{\uparrow} \to \{\ell^+\ell^-, W^{\pm}, Z\}X$



$$F_{UT}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{h}\cdot\vec{k}_T}{M}\, \boldsymbol{f_{1T}^{\perp}} D_1\right]$$

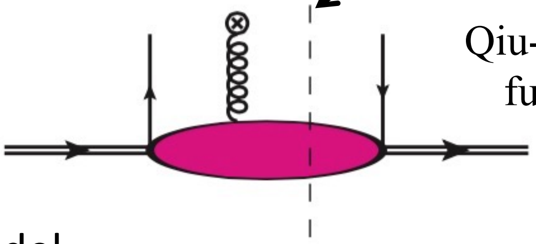
$$F_{TU}^{\sin\phi} = \mathcal{C}\left[-\frac{\hat{h}\cdot\vec{k}_{aT}}{M_a}\, \boldsymbol{f_{1T}^{\perp}}\, \bar{f}_1\right]$$

TMD/Collins-Soper-Sterman (CSS) Evolution

OPE

Sudakov exponentials (gluon radiation)

$$\boldsymbol{\tilde{f}_{1T}^{\perp(1)}(x, b_T; Q^2, \mu_Q)} \sim \boldsymbol{F_{FT}(x, x; \mu_{b_*})} \exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{f_{1T}^{\perp}}(b_T, Q)\right]$$



Qiu-Sterman function

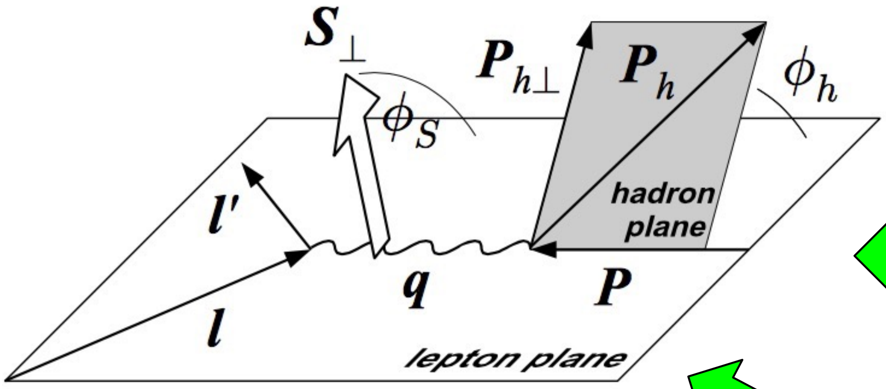
$$g_{f_{1T}^{\perp}}(x, b_T) + g_K(b_T)\ln(Q/Q_0)$$

(Aybat, et al. (2012); Bury, et al. (2021); Echevarria, et al. (2014, 2021))

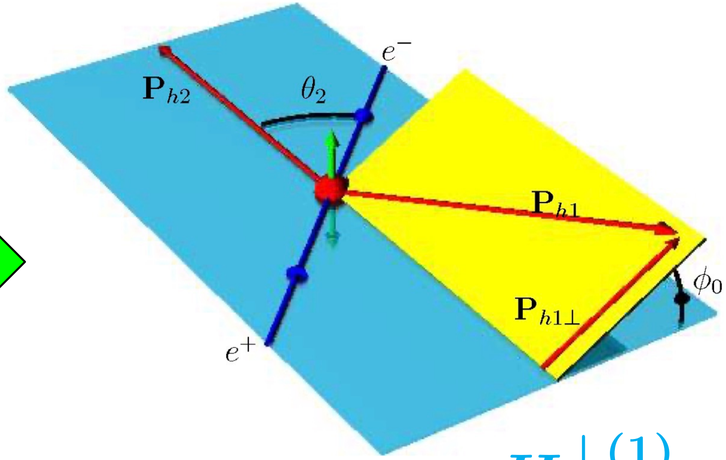
Parton model

$$\pi F_{FT}(x,x) = \int d^2\vec{k}_T\, \frac{k_T^2}{2M^2} f_{1T}^{\perp}(x, k_T^2) \equiv f_{1T}^{\perp(1)}(x)$$

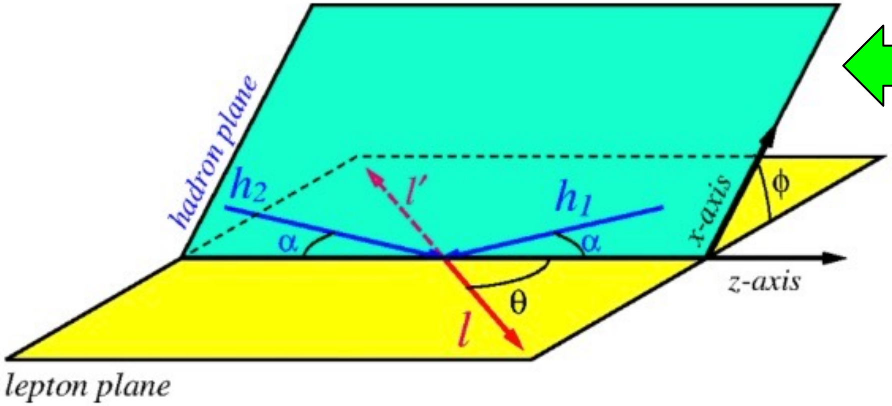
(Boer, Mulders, Pijlman (2003))



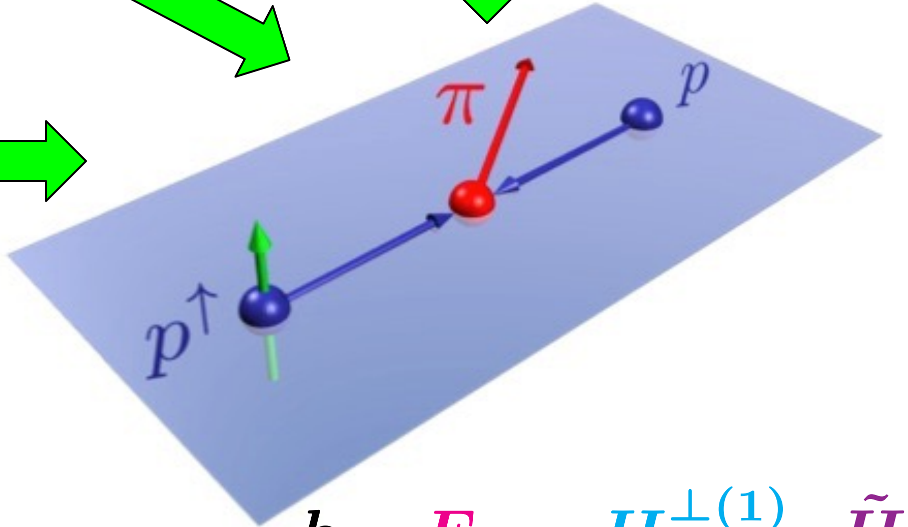
h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$



$H_1^{\perp(1)}$



F_{FT}



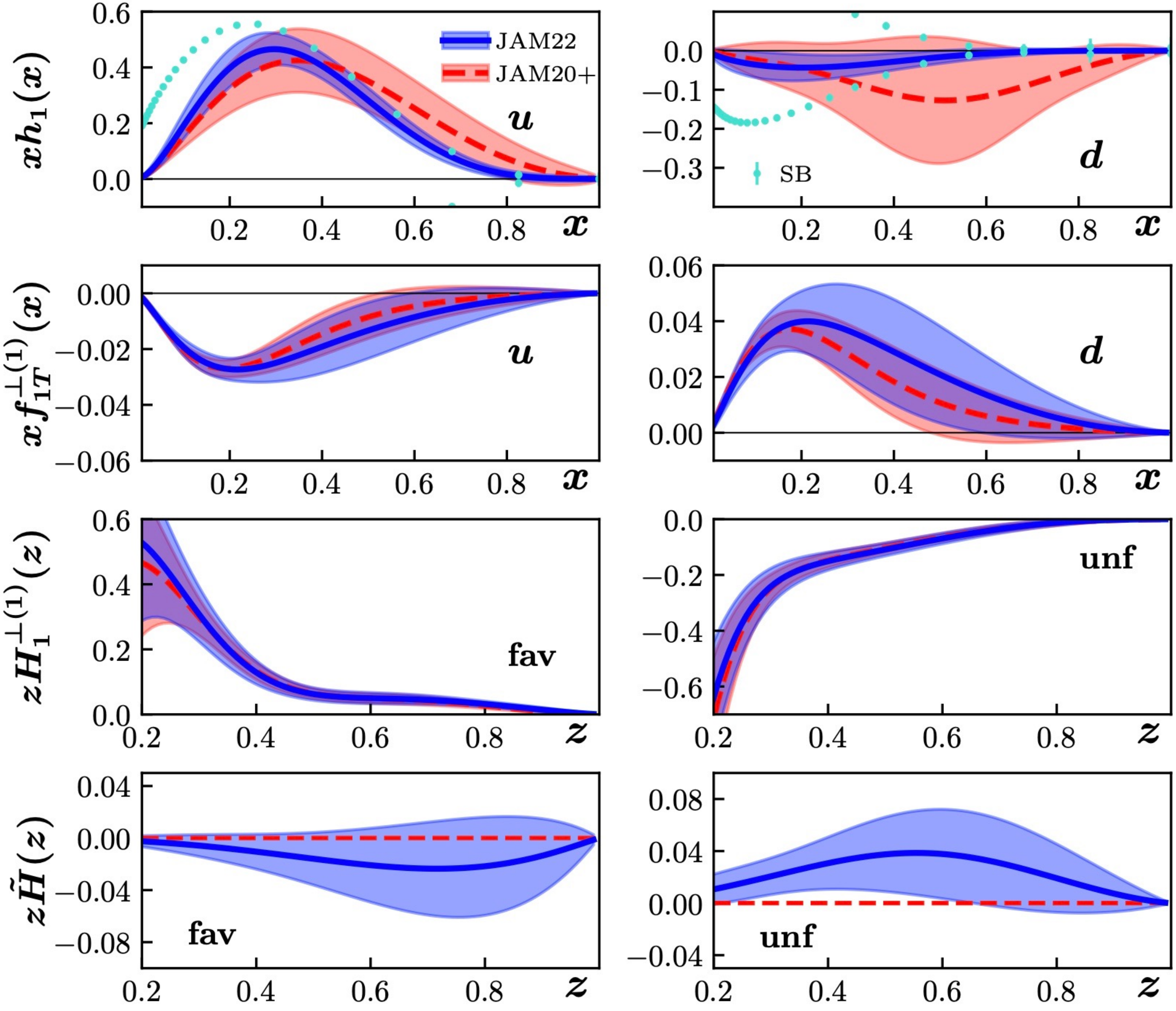
h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$

Gamberg, Malda, Miller, DP, Prokudin, Sato, PRD **106**, 034014 (2022)

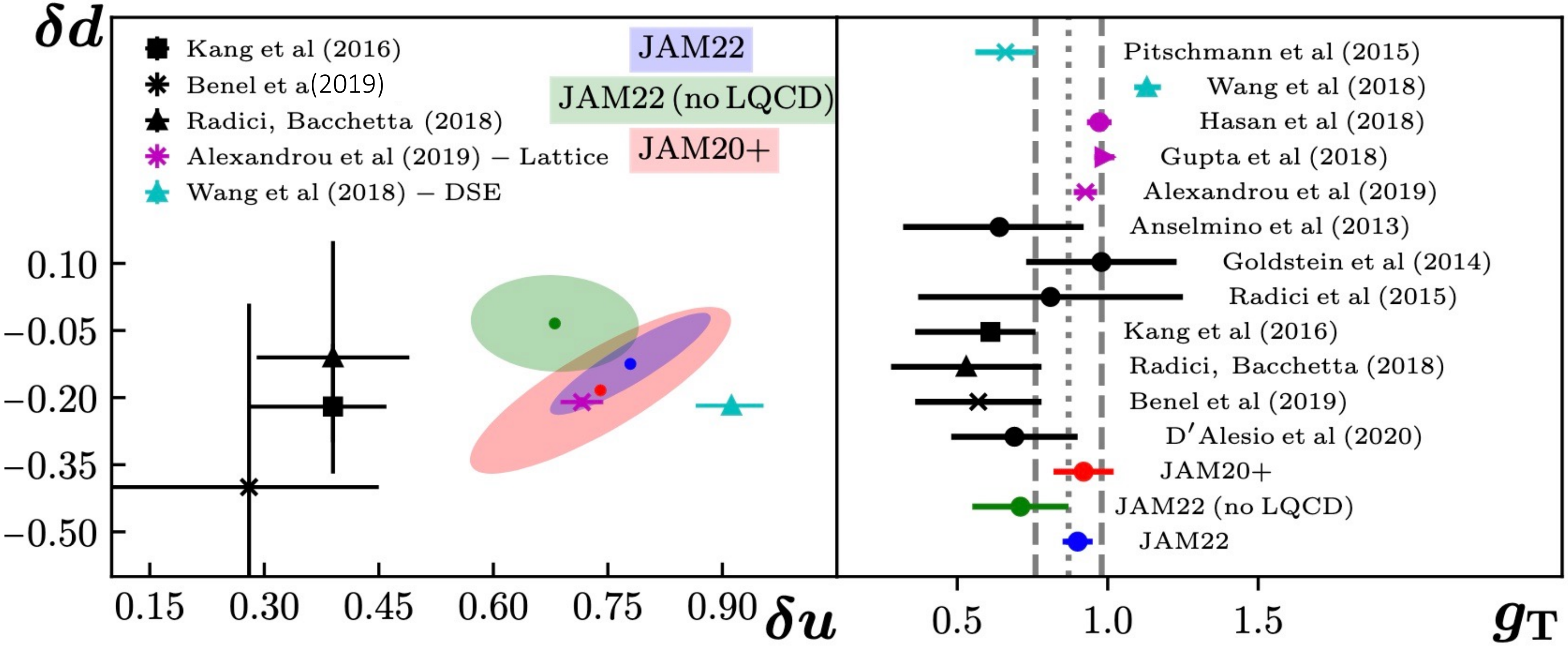
User-friendly jupyter notebook to calculate functions and asymmetries:
https://colab.research.google.com/github/pitonyak25/jam3d_dev_lib/blob/main/JAM3D_Library.ipynb

LHAPDF tables available (thanks to C. Cocuzza):
https://github.com/pitonyak25/jam3d_dev_lib/tree/main/LHAPDF_tables

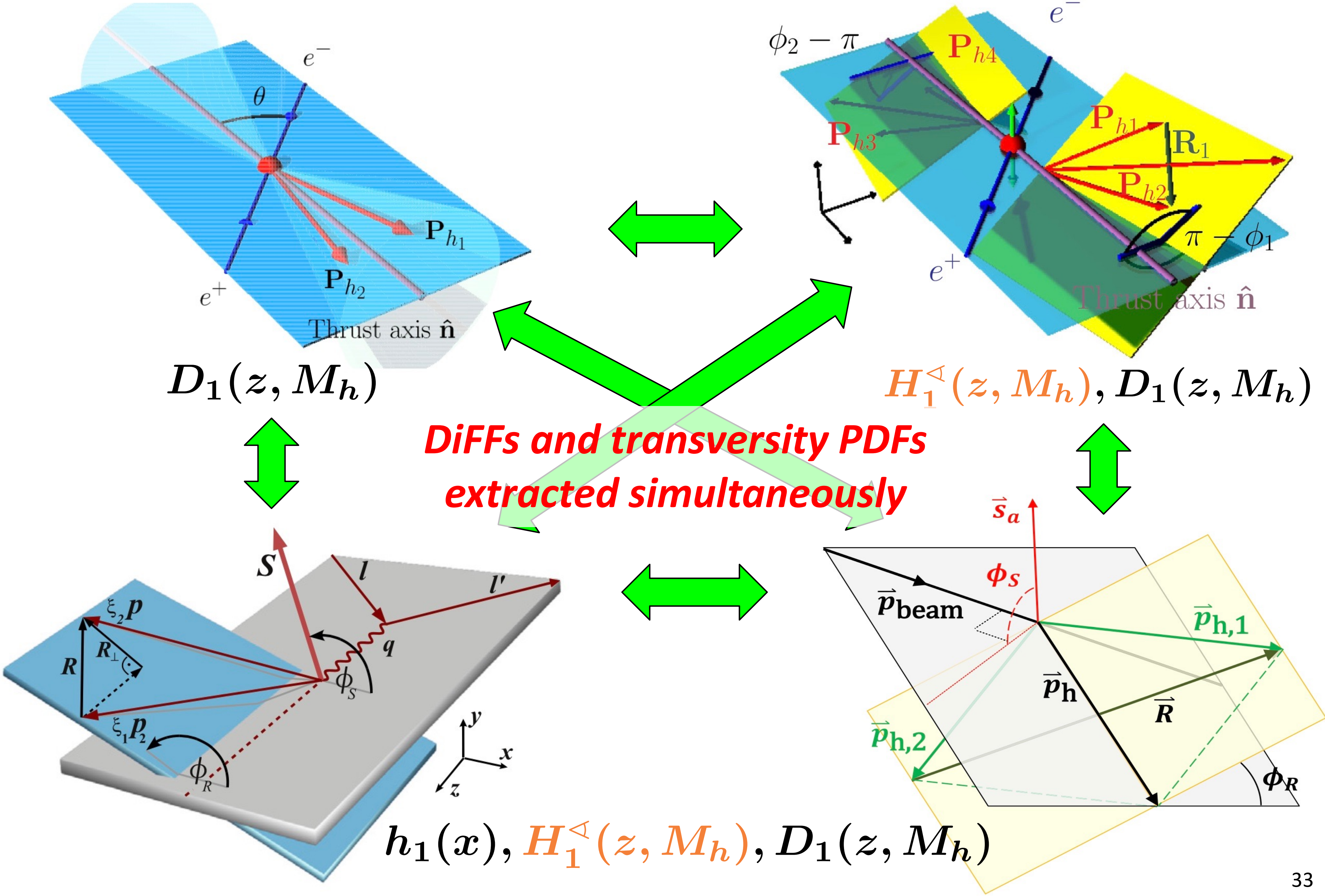
JAM Jefferson Lab Angular Momentum Collaboration



First direct information from experiment on $\tilde{H}(z)$



- ➢ Dihadron (e.g., Radici, Bacchetta (2018); Benel, Courtoy, Ferro-Hernandez (2019)) and TMD analyses that only include e^+e^- and SIDIS Collins effect data (e.g., Kang, et al. (2016)), are generally below the lattice values for g_T and δu
- ➢ Because of the Soffer bound on h_1, one initially finds JAM3D-22 has more tension with lattice, but this does *not* imply phenomenology and lattice are incompatible – one can only fully answer this by including lattice data in the analysis
- ➢ **Once the the lattice g_T data point is included, we find the non-perturbative functions can accommodate it *and still describe the experimental data well***



$D_1(z, M_h)$

$H_1^{\sphericalangle}(z, M_h), D_1(z, M_h)$

DiFFs and transversity PDFs extracted simultaneously

$h_1(x), H_1^{\sphericalangle}(z, M_h), D_1(z, M_h)$

- Parameterization of the $\pi^+\pi^-$ DiFFs $\boldsymbol{D_1(z, M_h)}$, $\boldsymbol{H_1^{\sphericalangle}(z, M_h)}$

  - <u>Symmetry relations</u> (Courtoy, et al. (2012))

$$D_1^u = D_1^d = D_1^{\bar{u}} = D_1^{\bar{d}}, \qquad H_1^{\sphericalangle,u} = -H_1^{\sphericalangle,d} = -H_1^{\sphericalangle,\bar{u}} = H_1^{\sphericalangle,\bar{d}},$$

$$D_1^s = D_1^{\bar{s}}, \quad D_1^c = D_1^{\bar{c}}, \quad D_1^b = D_1^{\bar{b}} \qquad H_1^{\sphericalangle,s} = -H_1^{\sphericalangle,\bar{s}} = H_1^{\sphericalangle,c} = -H_1^{\sphericalangle,\bar{c}} = 0$$

also have D_1^g

  - <u>Positivity bounds</u> (Bacchetta, Radici (2003))

$$D_1(z, M_h) > 0 \qquad |H_1^{\sphericalangle}(z, M_h)| < D_1(z, M_h)$$

- Parameterization of the $\pi^+\pi^-$ DiFFs $\boldsymbol{D_1(z, M_h)}$, $\boldsymbol{H_1^{\sphericalangle}(z, M_h)}$
  - Because of the resonance structure of the e^+e^- cross section, we use a grid in M_h whose density depends on the flavor and DiFF and then at each grid point we have the functional form $\sim N z^a (1-z)^b$. The grid is then interpolated to give a general M_h dependence. E.g., for the up quark for D_1,

$$\mathbf{M}_h^u = [2m_\pi, 0.40, 0.50, 0.70, 0.75, 0.80, 0.90, 1.00, 1.20, 1.30, 1.40, 1.60, 1.80, 2.00]\ \text{GeV}$$

$$D_1^u(z, \mathbf{M}_h^{u,i}) = \sum_{j=1,2,3} \frac{N_{ij}^u z^{\alpha_{ij}^u} (1-z)^{\beta_{ij}^u}}{\mathrm{B}[\alpha_{ij}^u + 1, \beta_{ij}^u + 1]}$$

$\rightarrow$ 204 parameters for D_1 and 48 parameters for $H_1^{\sphericalangle}$

➢ Parameterization of the $\pi^+\pi^-$ DiFFs $\boldsymbol{D_1(z, M_h)}$, $\boldsymbol{H_1^{\sphericalangle}(z, M_h)}$

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$\rightarrow$ 204 parameters for D_1 and 48 parameters for $H_1^{\sphericalangle}$

- The Belle data is not sufficient to perform a flavor separation for D_1, so we supplement with data from Pythia for σ^q/σ^{tot} for $q = s, c, b$ at

$$\sqrt{s} = [10.58, 30.73, 50.88, 71.04, 91.19]\ \text{GeV}$$

using different tunes to quantify systematic uncertainties

→ constrain D_1 for s, c, b as well as the gluon through scaling violations

➢ Parameterization of $\boldsymbol{h_1(x)}$

- We use the following functional form for the transversity PDFs u_v, d_v, and $\bar{u} = -\bar{d}$ (from large-N_c limit (Pobylitsa (2003))) and impose the Soffer bound ($2|h_1^q(x)| \leq (f_1^q(x) + g_1^q(x))$)

$$F(x) = \frac{Nx^\alpha(1-x)^\beta(1+\gamma\sqrt{x}+\delta x)}{\mathrm{B}[\alpha+1,\beta+1]+\gamma\mathrm{B}[\alpha+\frac{3}{2},\beta+1]+\delta\mathrm{B}[\alpha+2,\beta+1]}$$

$\rightarrow$ 15 parameters for h_1

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$$F(x) = \frac{N x^{\enclose{circle}{\alpha}} (1-x)^{\beta} (1 + \gamma\sqrt{x} + \delta x)}{\mathrm{B}[\alpha+1, \beta+1] + \gamma \mathrm{B}[\alpha + \frac{3}{2}, \beta+1] + \delta \mathrm{B}[\alpha+2, \beta+1]}$$

Use constraint from small-*x* asymptotics (Kovchegov, Sievert (2019))

$$\alpha \xrightarrow{x\to 0} 1 - 2\sqrt{\frac{\alpha_s N_c}{2\pi}} \quad \Longrightarrow \quad \alpha = 0.170 \pm 0.085$$

50% uncertainty due to unaccounted for $1/N_c$ and NLO corrections

- Parameterization of $\boldsymbol{h_1(x)}$

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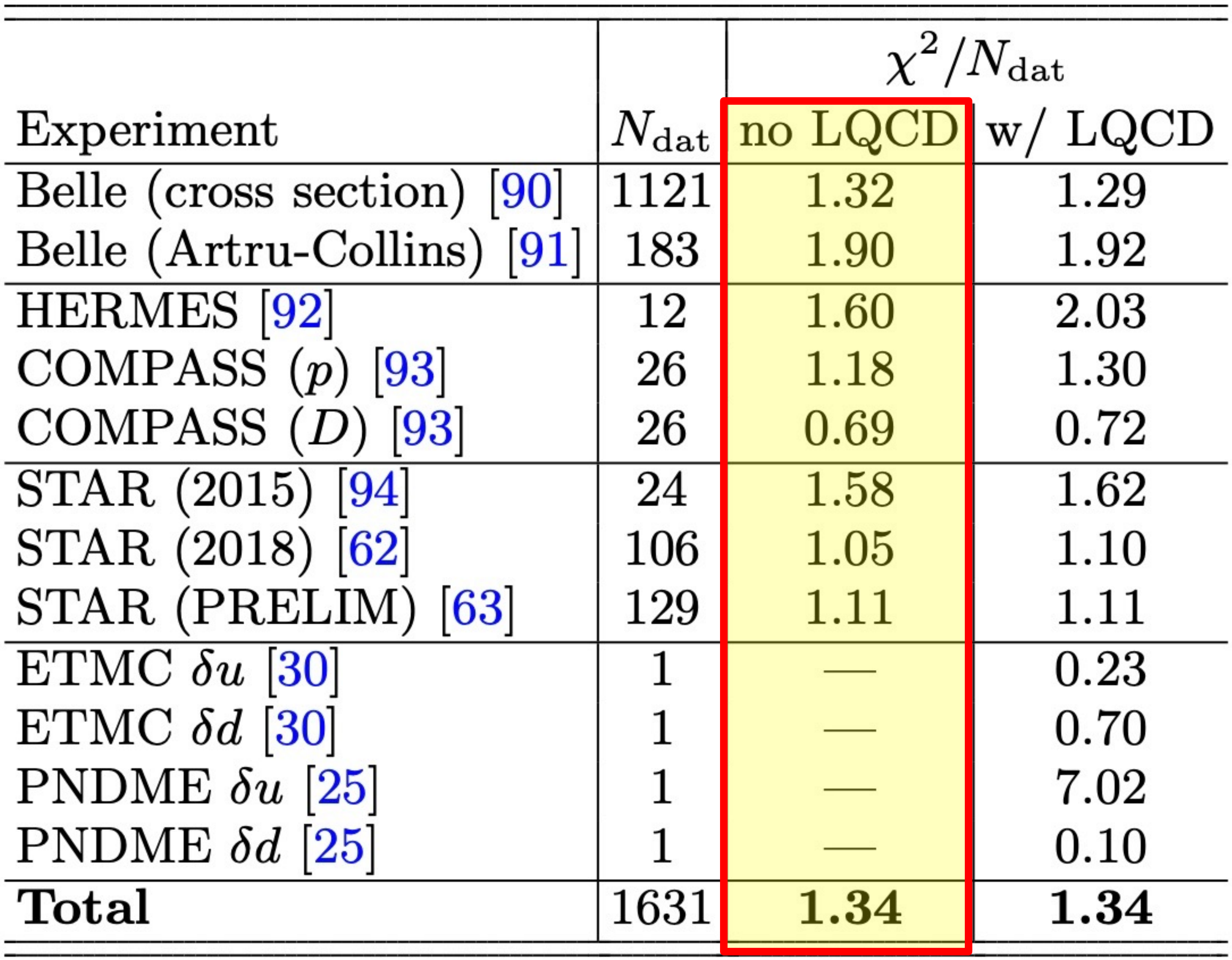
$\rightarrow$ 15 parameters for h_1

- Perform the analysis with and without LQCD data for the tensor charges $\boldsymbol{\delta u}, \boldsymbol{\delta d}$ from ETMC (Alexandrou, et al. (2019)) and PNDME (Gupta, et al. (2018)) (physical pion mass and 2+1+1 flavors)



Experiment	N_{dat}	χ^2/N_{dat}	
		no LQCD	w/ LQCD
Belle (cross section) [90]	1121	1.32	1.29
Belle (Artru-Collins) [91]	183	1.90	1.92
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COMPASS (p) [93]	26	1.18	1.30
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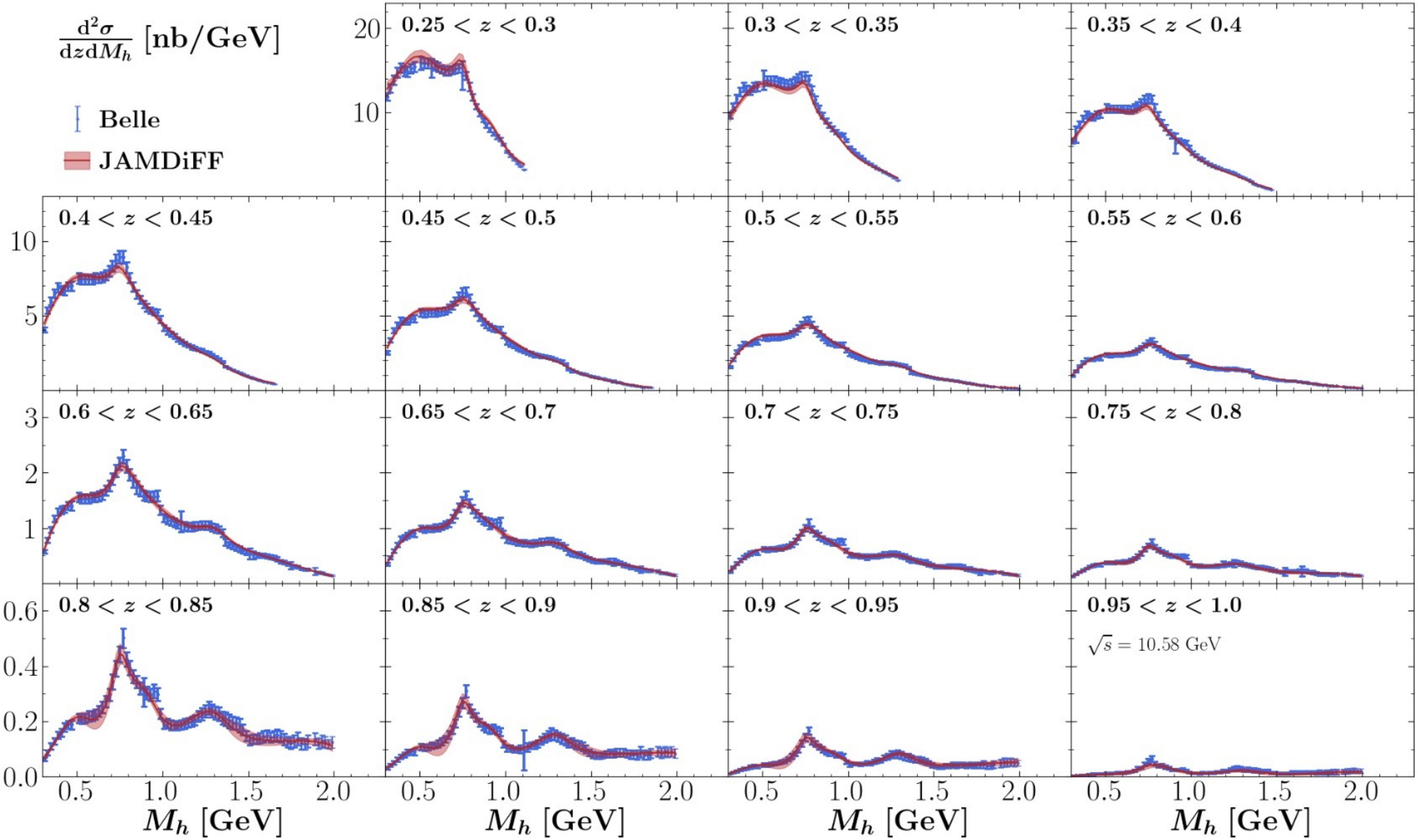
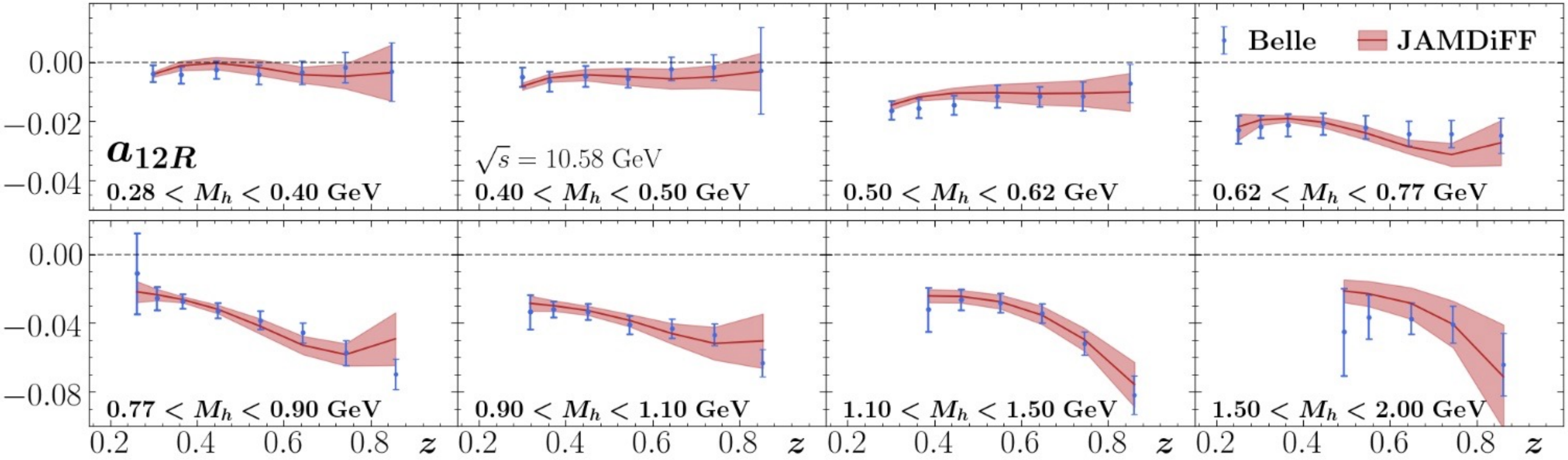
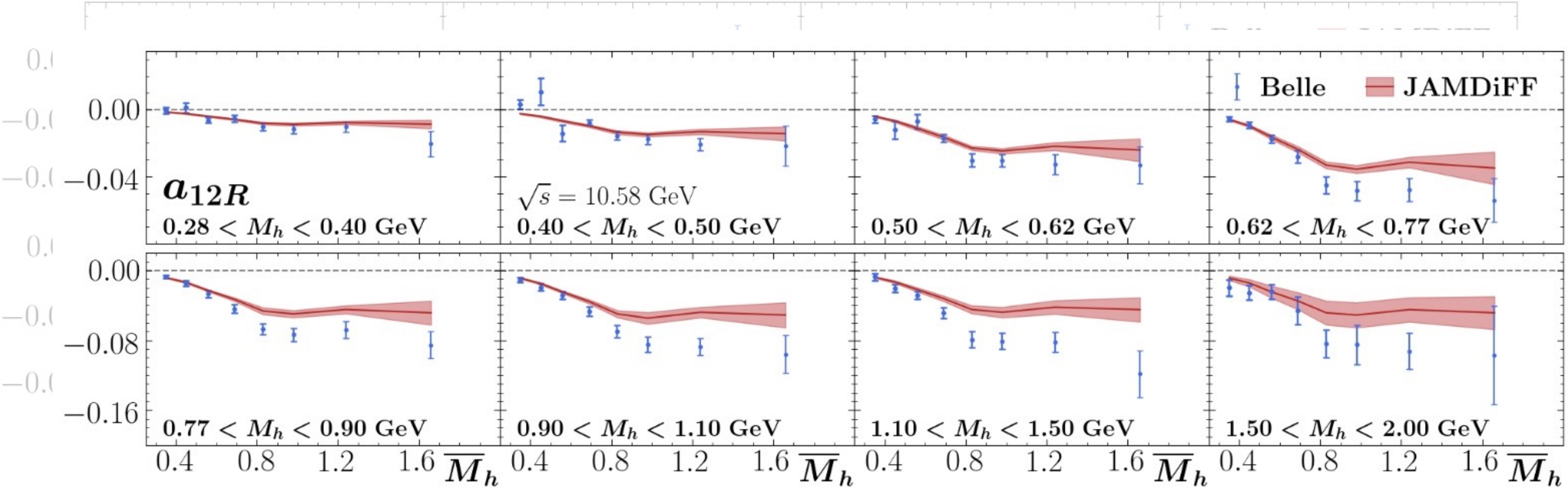
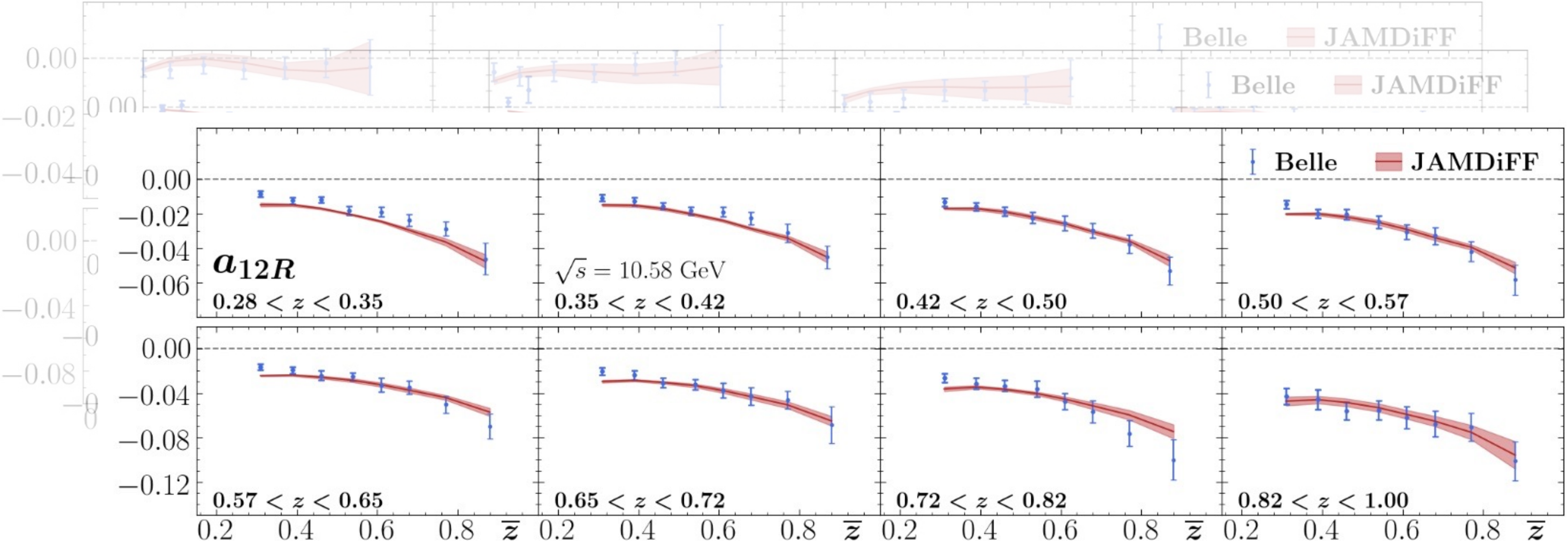
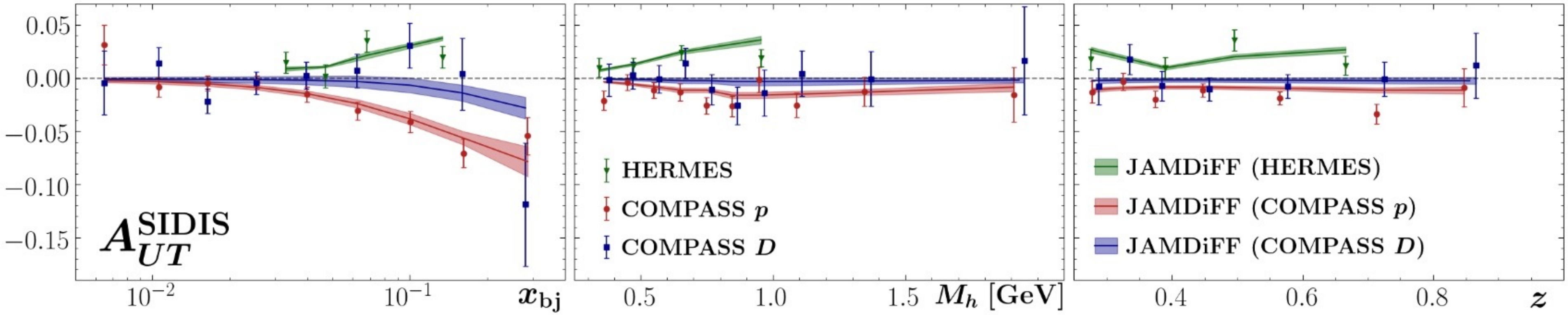
ETMC δu [30]	1	—	0.23
ETMC δd [30]	1	—	0.70
PNDME δu [25]	1	—	7.02
PNDME δd [25]	1	—	0.10
Total	1631	**1.34**	**1.34**

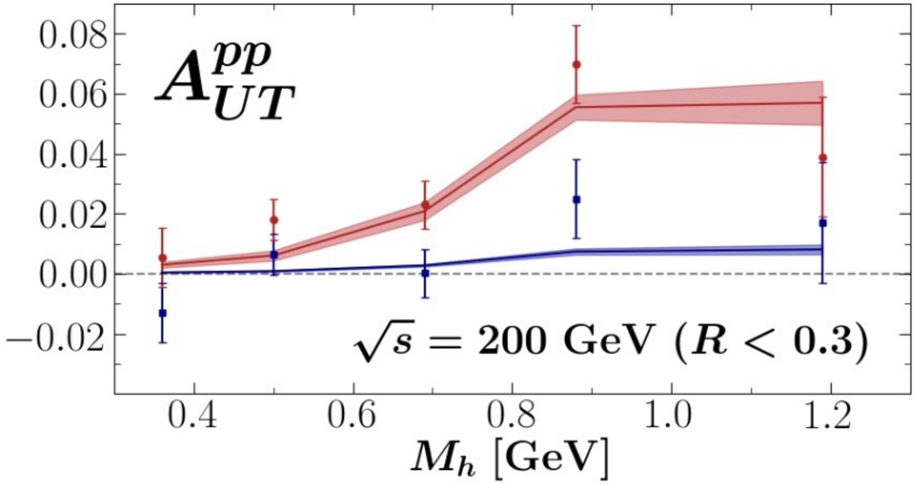

$\frac{d^2\sigma}{dz dM_h}$ [nb/GeV]
Belle
JAMDiFF
0.25 < z < 0.3
0.3 < z < 0.35
0.35 < z < 0.4
0.4 < z < 0.45
0.45 < z < 0.5
0.5 < z < 0.55
0.55 < z < 0.6
0.6 < z < 0.65
0.65 < z < 0.7
0.7 < z < 0.75
0.75 < z < 0.8
0.8 < z < 0.85
0.85 < z < 0.9
0.9 < z < 0.95
0.95 < z < 1.0
$\sqrt{s}$ = 10.58 GeV
20
10
5
3
2
1
0.6
0.4
0.2
0.0
0.5
1.0
1.5
2.0
M_h [GeV]

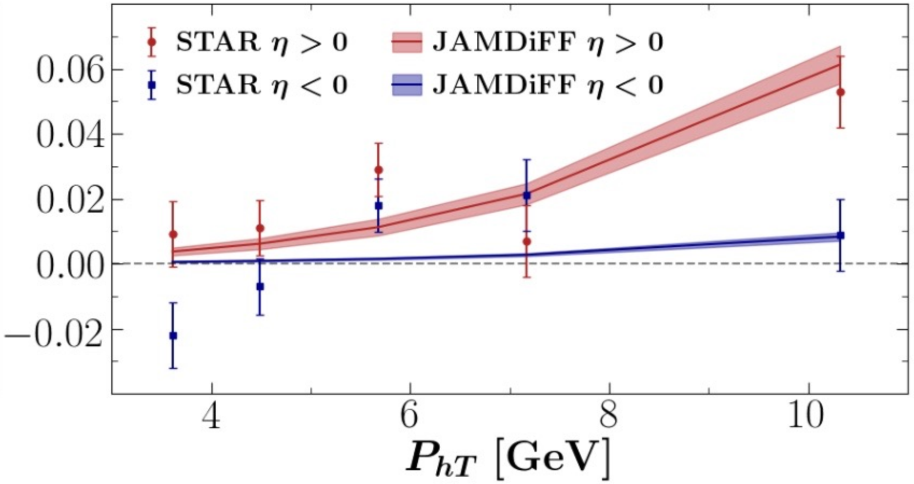

a_{12R}
$\sqrt{s} = 10.58$ GeV
Belle
JAMDiFF
$0.28 < M_h < 0.40$ GeV
$0.40 < M_h < 0.50$ GeV
$0.50 < M_h < 0.62$ GeV
$0.62 < M_h < 0.77$ GeV
$0.77 < M_h < 0.90$ GeV
$0.90 < M_h < 1.10$ GeV
$1.10 < M_h < 1.50$ GeV
$1.50 < M_h < 2.00$ GeV
0.00
−0.02
−0.04
−0.08
0.2
0.4
0.6
0.8
z

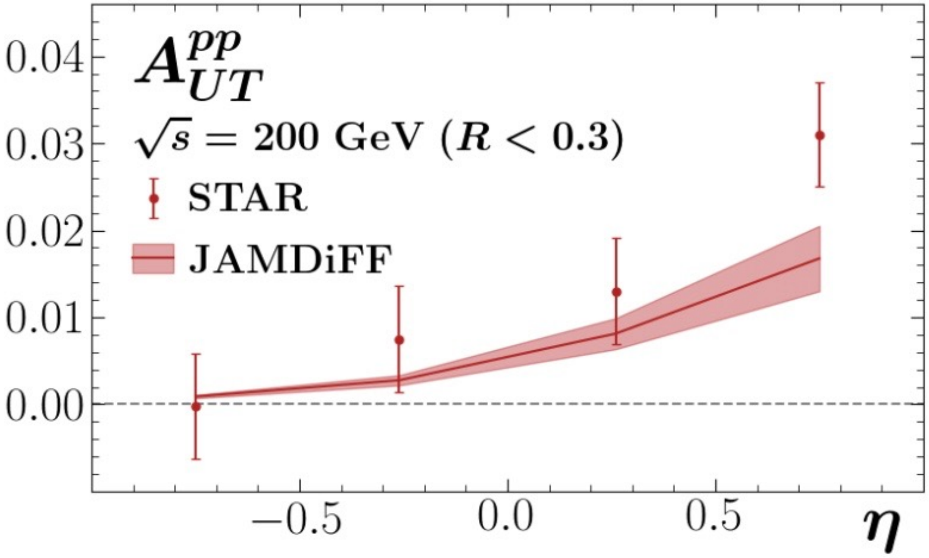

a_{12R}
$\sqrt{s} = 10.58$ GeV
Belle
JAMDiFF
$0.28 < M_h < 0.40$ GeV
$0.40 < M_h < 0.50$ GeV
$0.50 < M_h < 0.62$ GeV
$0.62 < M_h < 0.77$ GeV
$0.77 < M_h < 0.90$ GeV
$0.90 < M_h < 1.10$ GeV
$1.10 < M_h < 1.50$ GeV
$1.50 < M_h < 2.00$ GeV
0.00
−0.04
0.00
−0.08
−0.16
0.4
0.8
1.2
1.6
$\overline{M}_h$

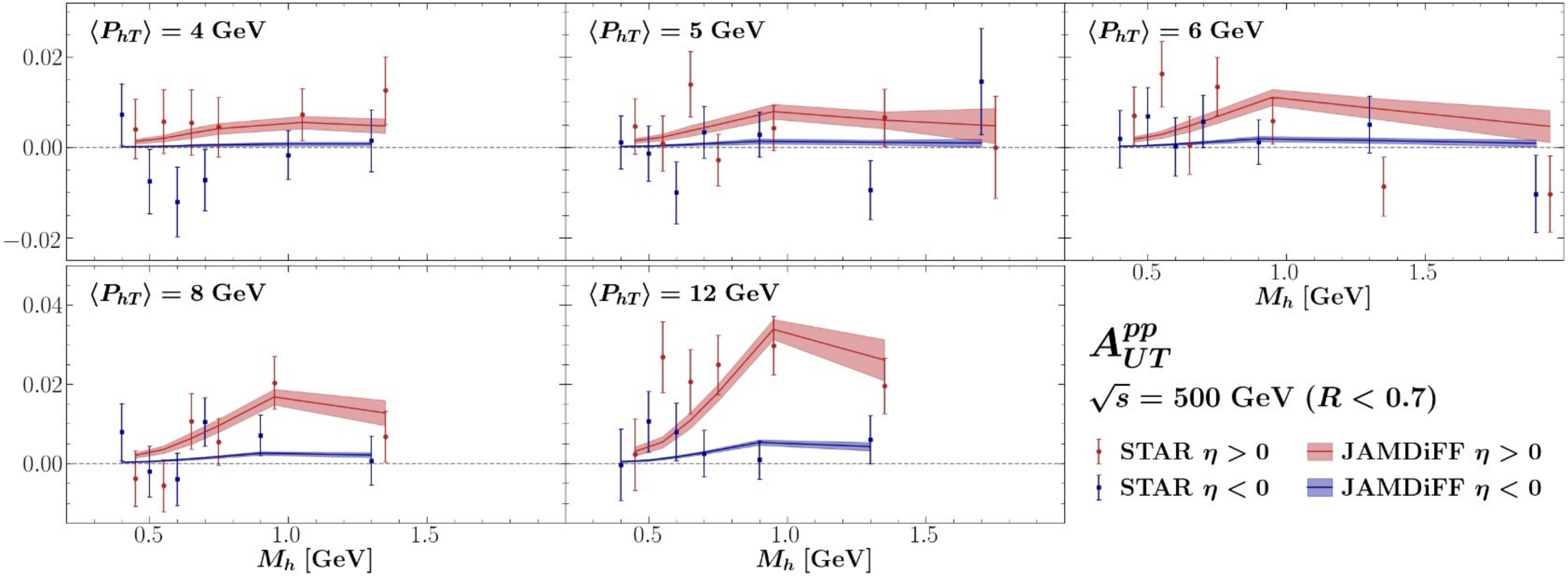

Belle
JAMDiFF
a_{12R}
$\sqrt{s} = 10.58$ GeV
$0.28 < z < 0.35$
$0.35 < z < 0.42$
$0.42 < z < 0.50$
$0.50 < z < 0.57$
$0.57 < z < 0.65$
$0.65 < z < 0.72$
$0.72 < z < 0.82$
$0.82 < z < 1.00$
0.00
−0.02
−0.04
−0.06
−0.08
−0.12
0.2
0.4
0.6
0.8
$\overline{z}$

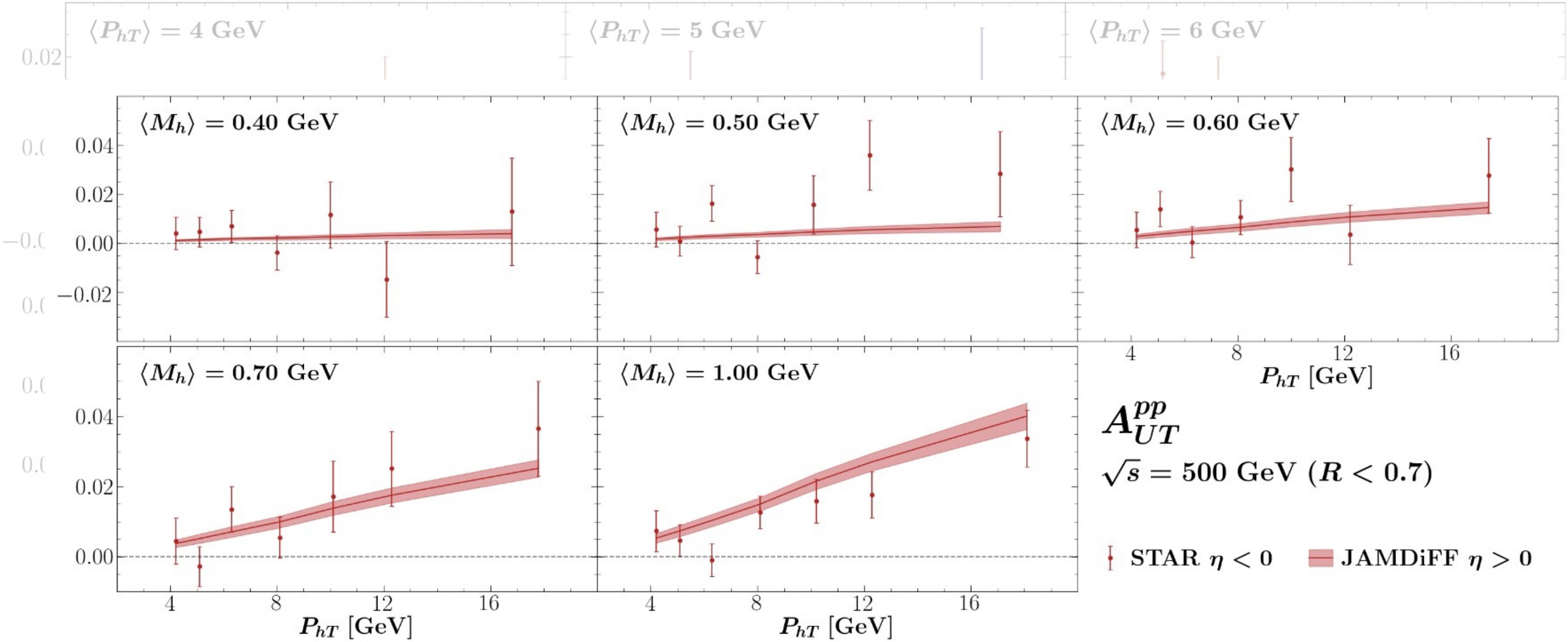

0.05
0.00
−0.05
−0.10
−0.15
A_{UT}^{SIDIS}
10^{-2}
10^{-1}
x_{bj}
0.5
1.0
1.5
M_h [GeV]
0.4
0.6
0.8
z
HERMES
COMPASS p
COMPASS D
JAMDiFF (HERMES)
JAMDiFF (COMPASS p)
JAMDiFF (COMPASS D)

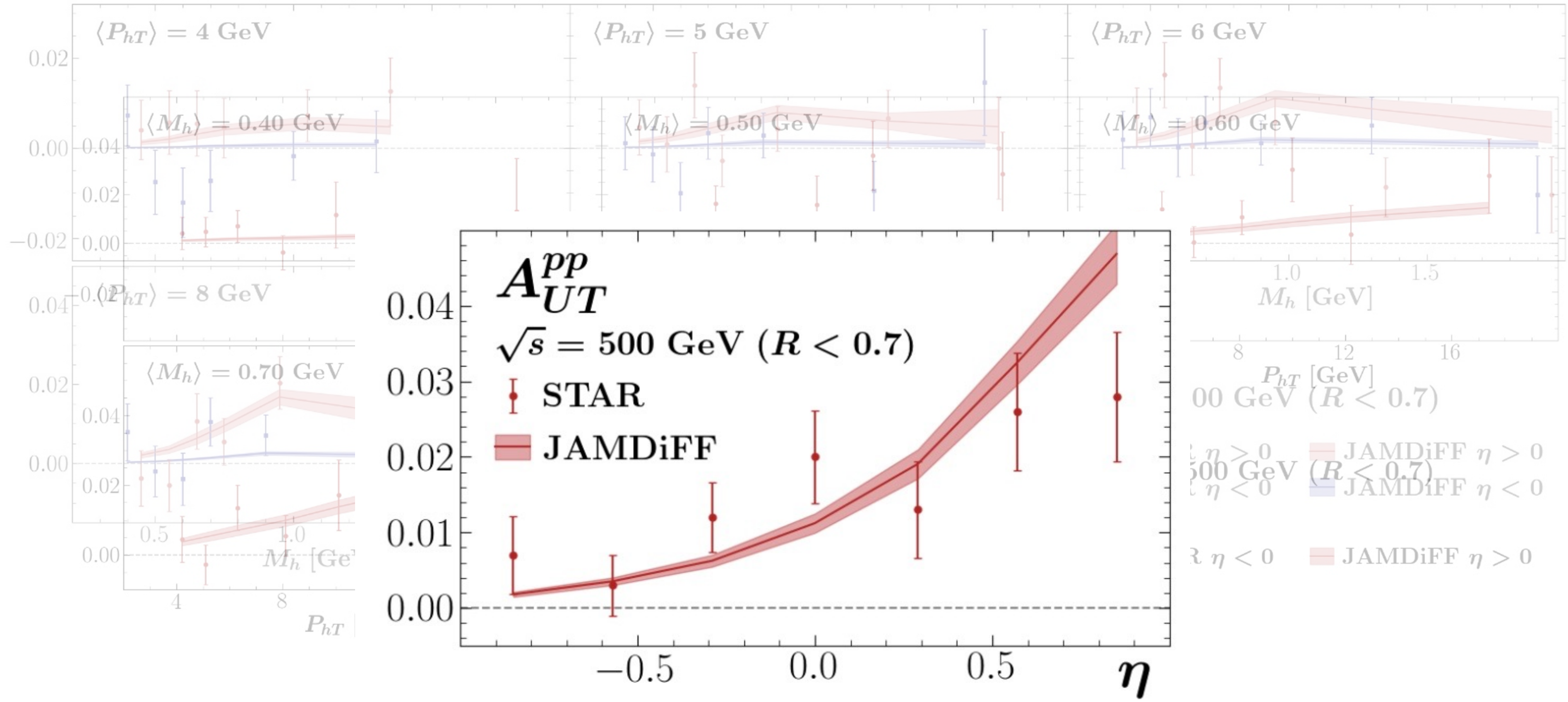

A_{UT}^{pp}
$\sqrt{s} = 200$ GeV ($R < 0.3$)
0.08
0.06
0.04
0.02
0.00
−0.02
0.4
0.6
0.8
1.0
1.2
M_h [GeV]

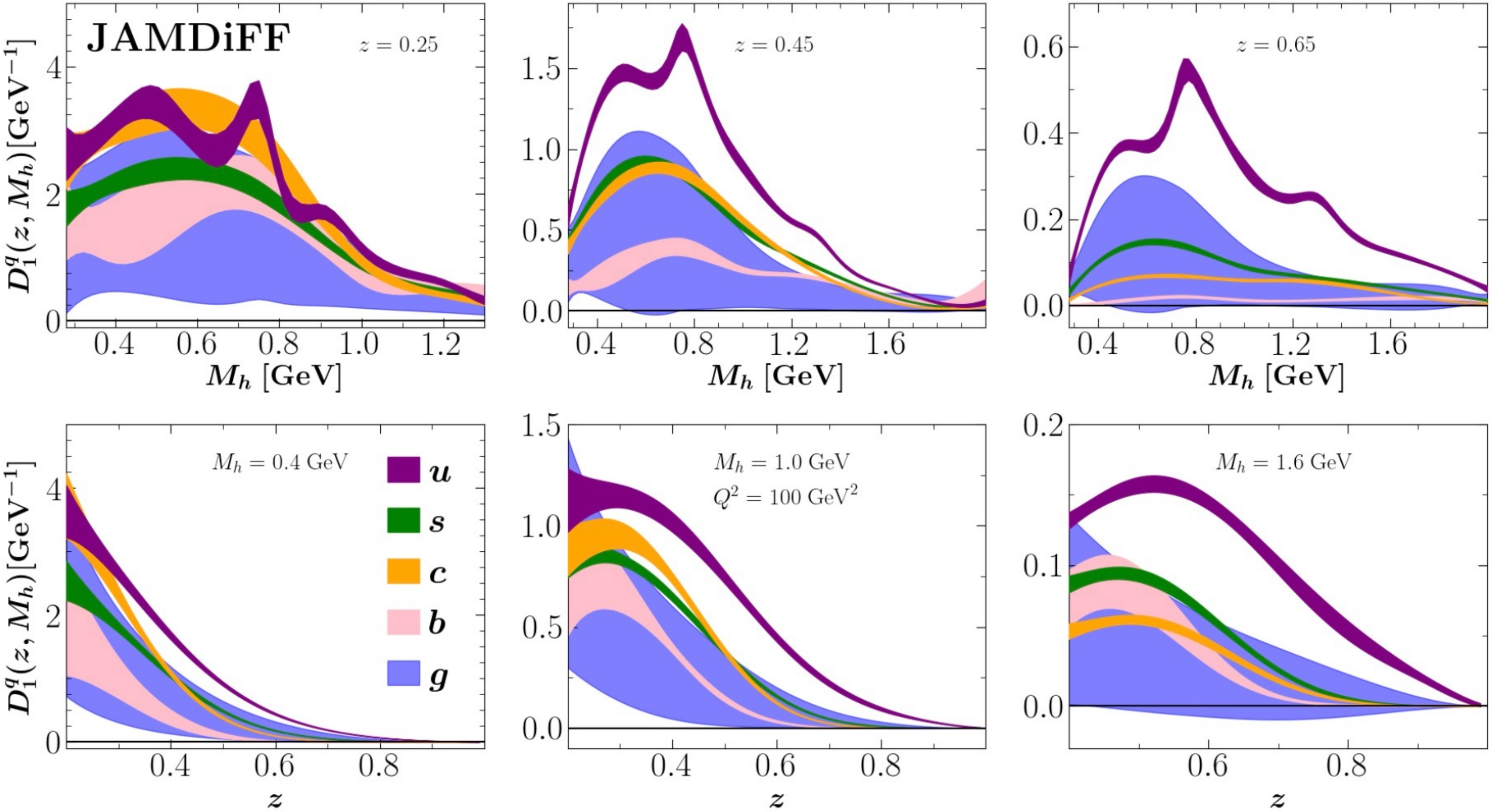

STAR $\eta > 0$
STAR $\eta < 0$
JAMDiFF $\eta > 0$
JAMDiFF $\eta < 0$
0.06
0.04
0.02
0.00
−0.02
4
6
8
10
P_{hT} [GeV]

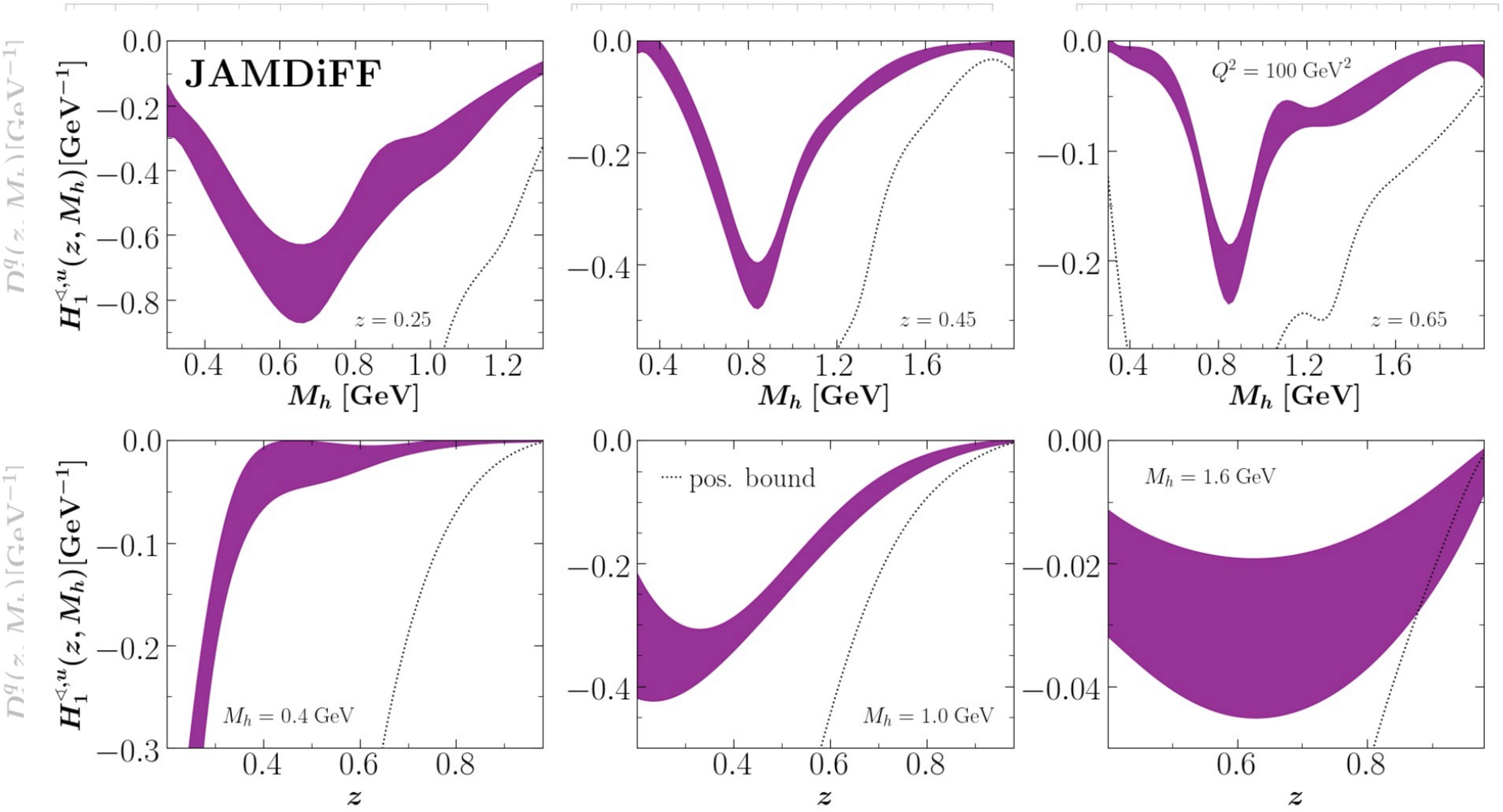

A_{UT}^{pp}
$\sqrt{s} = 200$ GeV ($R < 0.3$)
STAR
JAMDiFF
0.04
0.03
0.02
0.01
0.00
−0.5
0.0
0.5
η

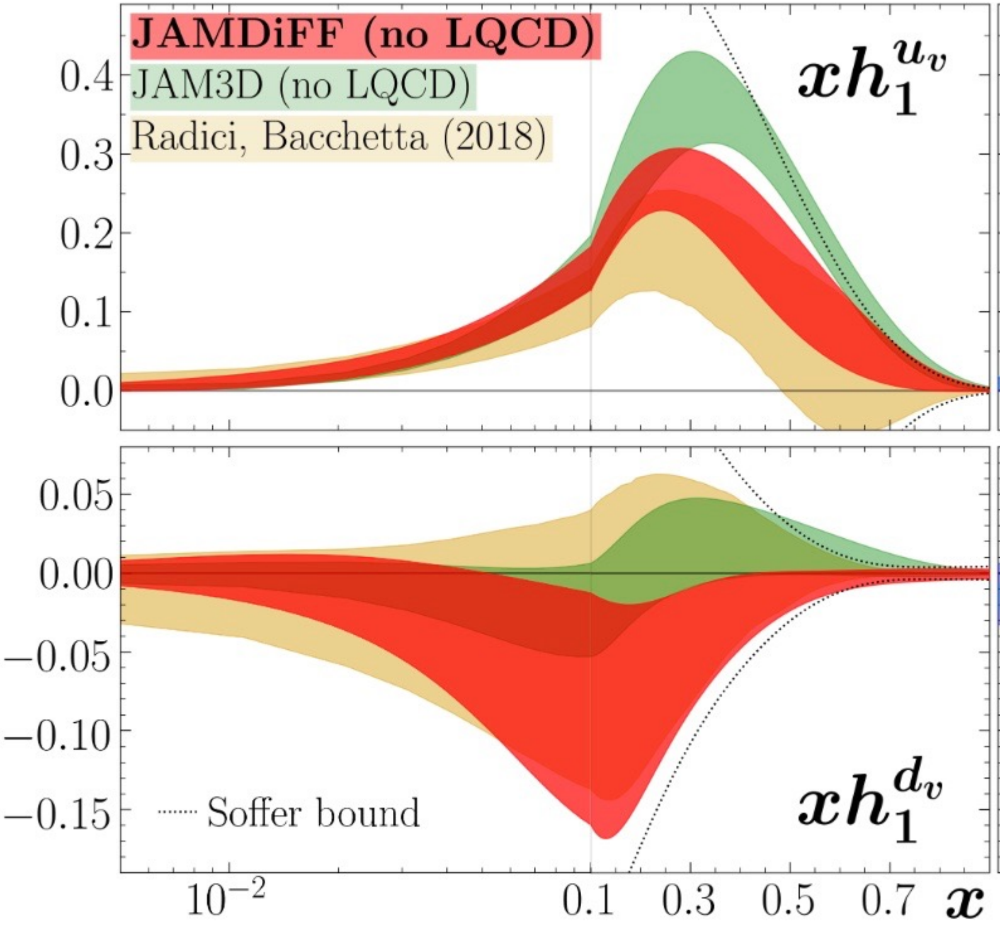

$\langle P_{hT} \rangle = 4$ GeV
$\langle P_{hT} \rangle = 5$ GeV
$\langle P_{hT} \rangle = 6$ GeV
$\langle P_{hT} \rangle = 8$ GeV
$\langle P_{hT} \rangle = 12$ GeV
0.02
0.00
−0.02
0.04
0.02
0.00
0.5
1.0
1.5
M_h [GeV]
A_{UT}^{pp}
$\sqrt{s} = 500$ GeV $(R < 0.7)$
STAR $\eta > 0$
STAR $\eta < 0$
JAMDiFF $\eta > 0$
JAMDiFF $\eta < 0$


$\langle P_{hT}\rangle = 4$ GeV
$\langle P_{hT}\rangle = 5$ GeV
$\langle P_{hT}\rangle = 6$ GeV
$\langle M_h\rangle = 0.40$ GeV
$\langle M_h\rangle = 0.50$ GeV
$\langle M_h\rangle = 0.60$ GeV
$\langle M_h\rangle = 0.70$ GeV
$\langle M_h\rangle = 1.00$ GeV
0.04
0.02
0.00
−0.02
4
8
12
16
P_{hT} [GeV]
A_{UT}^{pp}
$\sqrt{s} = 500$ GeV ($R < 0.7$)
STAR $\eta < 0$
JAMDiFF $\eta > 0$

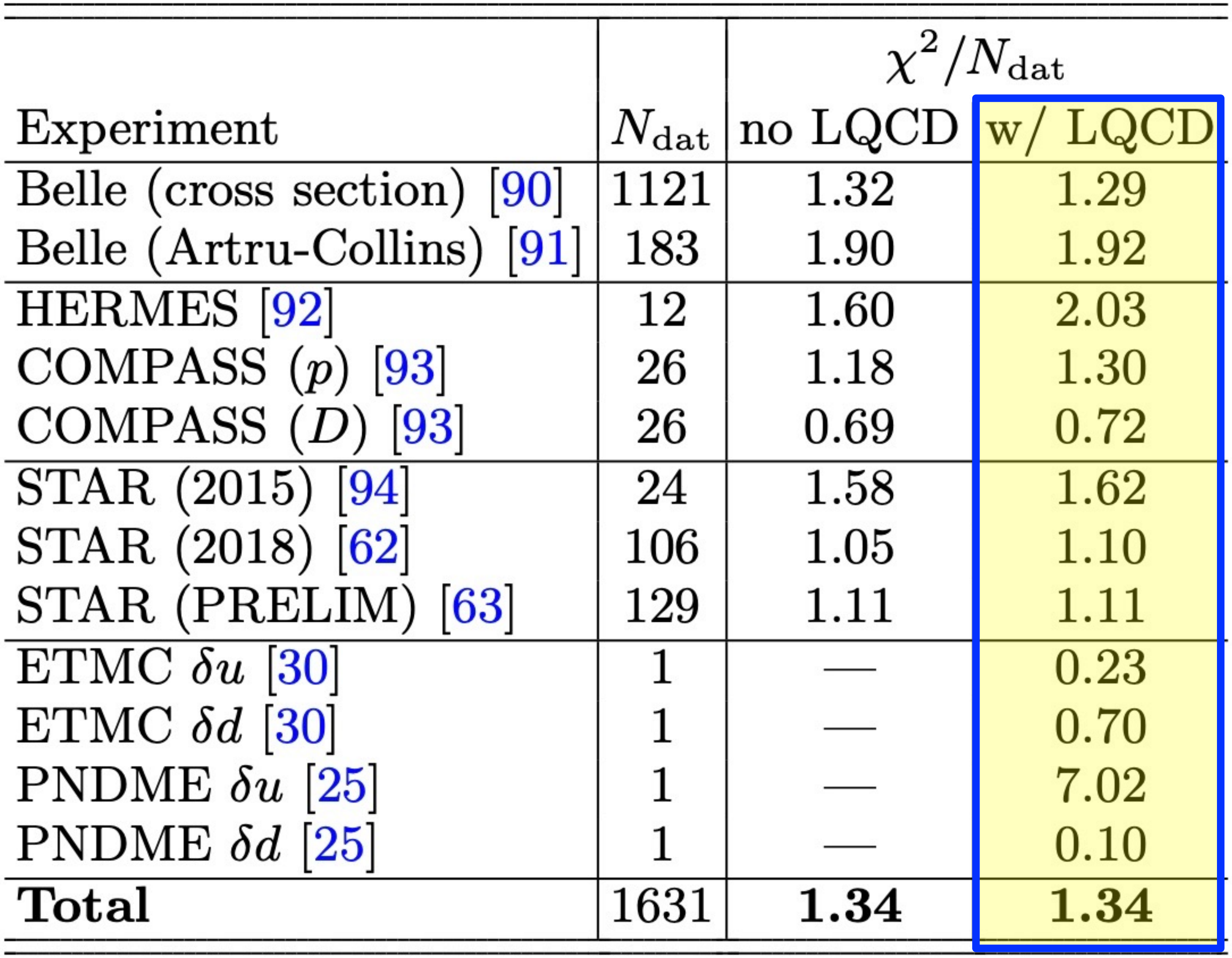

A_{UT}^{pp}
$\sqrt{s} = 500$ GeV ($R < 0.7$)
STAR
JAMDiFF
0.04
0.03
0.02
0.01
0.00
−0.5
0.0
0.5
η
$\langle P_{hT} \rangle = 4$ GeV
$\langle P_{hT} \rangle = 5$ GeV
$\langle P_{hT} \rangle = 6$ GeV
$\langle P_{hT} \rangle = 8$ GeV
$\langle M_h \rangle = 0.40$ GeV
$\langle M_h \rangle = 0.50$ GeV
$\langle M_h \rangle = 0.60$ GeV
$\langle M_h \rangle = 0.70$ GeV
M_h [GeV]
P_{hT} [GeV]
JAMDiFF $\eta > 0$
JAMDiFF $\eta < 0$


JAMDiFF
$z = 0.25$
$z = 0.45$
$z = 0.65$
$D_1^q(z, M_h)\,[\mathrm{GeV}^{-1}]$
M_h [GeV]
0
2
4
0.4
0.6
0.8
1.0
1.2
0.0
0.5
1.0
1.5
1.6
0.2
0.4
0.6
$M_h = 0.4$ GeV
$M_h = 1.0$ GeV
$Q^2 = 100$ GeV^2
$M_h = 1.6$ GeV
0.1
z
u
s
c
b
g


JAMDiFF
$H_1^{\sphericalangle,u}(z,M_h)$ [GeV^{-1}]
$D_1^q(z,M_h)$ [GeV^{-1}]
$z = 0.25$
$z = 0.45$
$z = 0.65$
$Q^2 = 100$ GeV^2
M_h [GeV]
pos. bound
$M_h = 0.4$ GeV
$M_h = 1.0$ GeV
$M_h = 1.6$ GeV
z
0.0
−0.2
−0.4
−0.6
−0.8
−0.1
−0.3
0.00
−0.02
−0.04
0.4
0.6
0.8
1.0
1.2
1.6



- JAMDiFF (no LQCD) finds agreement with Radici, Bacchetta (2018) with a slightly larger u_v function at larger x
- The u_v function for JAMDiFF (no LQCD) is smaller than JAM3D (no LQCD) but the d_v function is larger in magnitude



		$\chi^2/N_{\rm dat}$	
Experiment	$N_{\rm dat}$	no LQCD	w/ LQCD
Belle (cross section) [90]	1121	1.32	1.29
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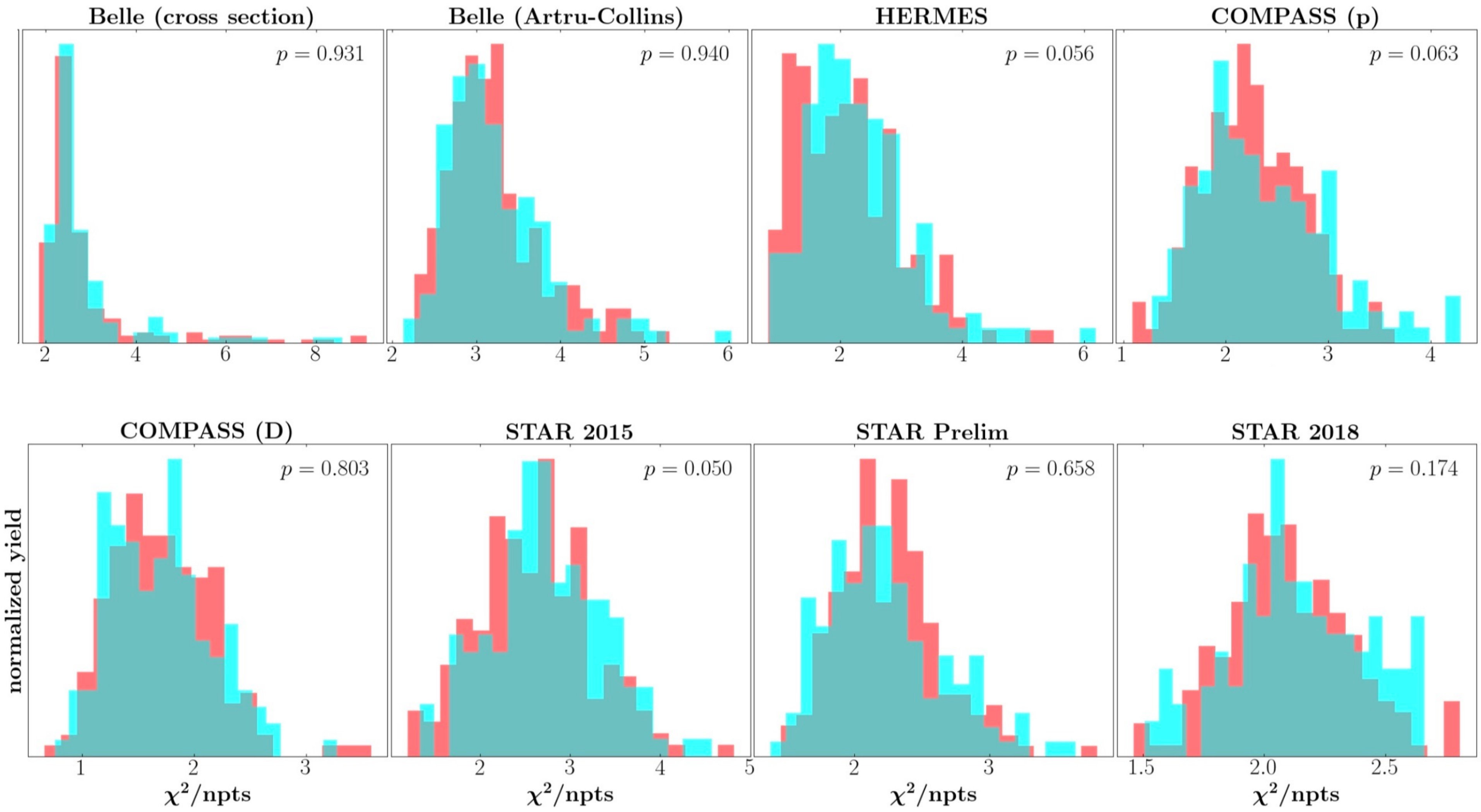
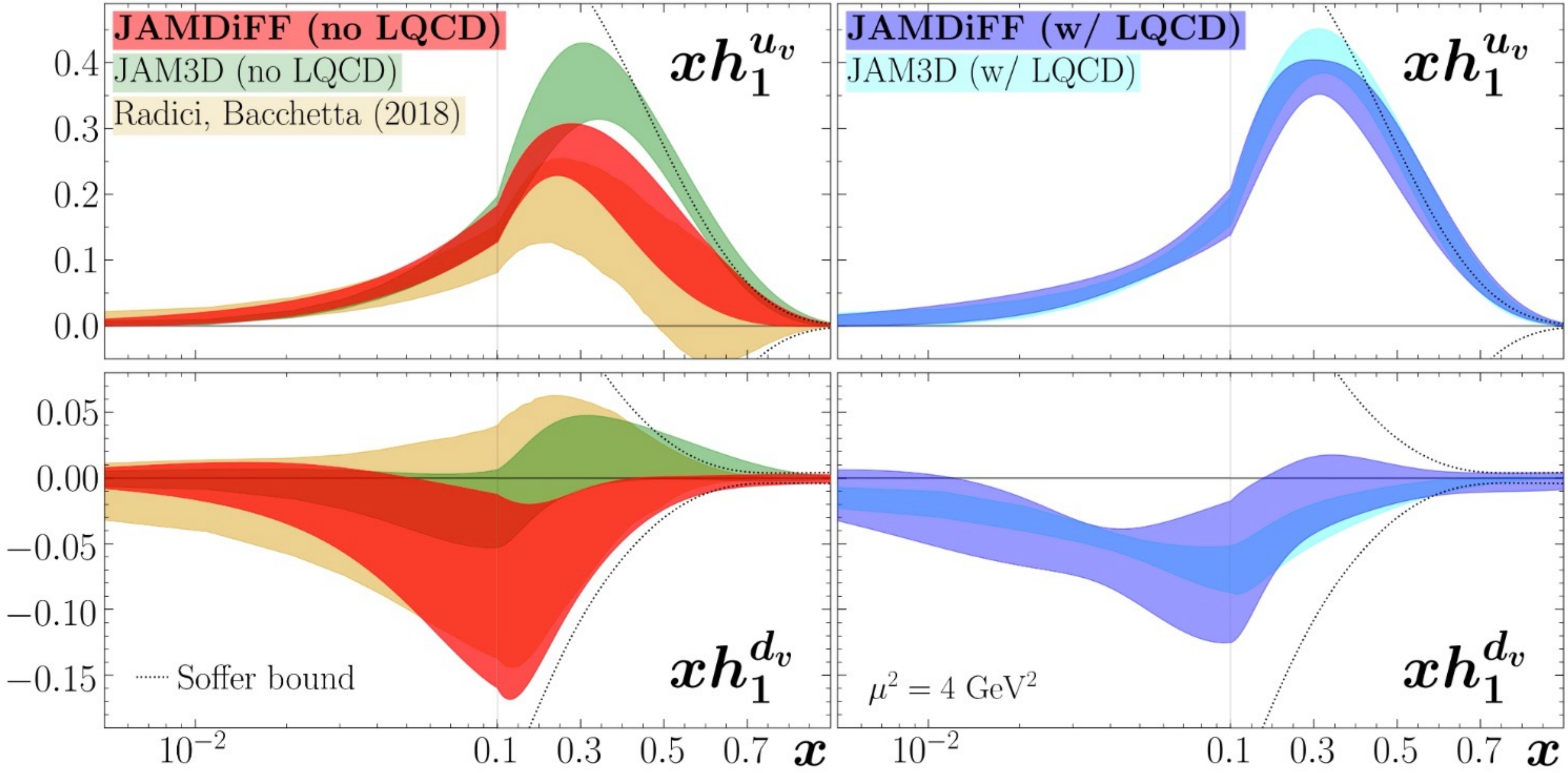


🟥 JAMDiFF (no LQCD)

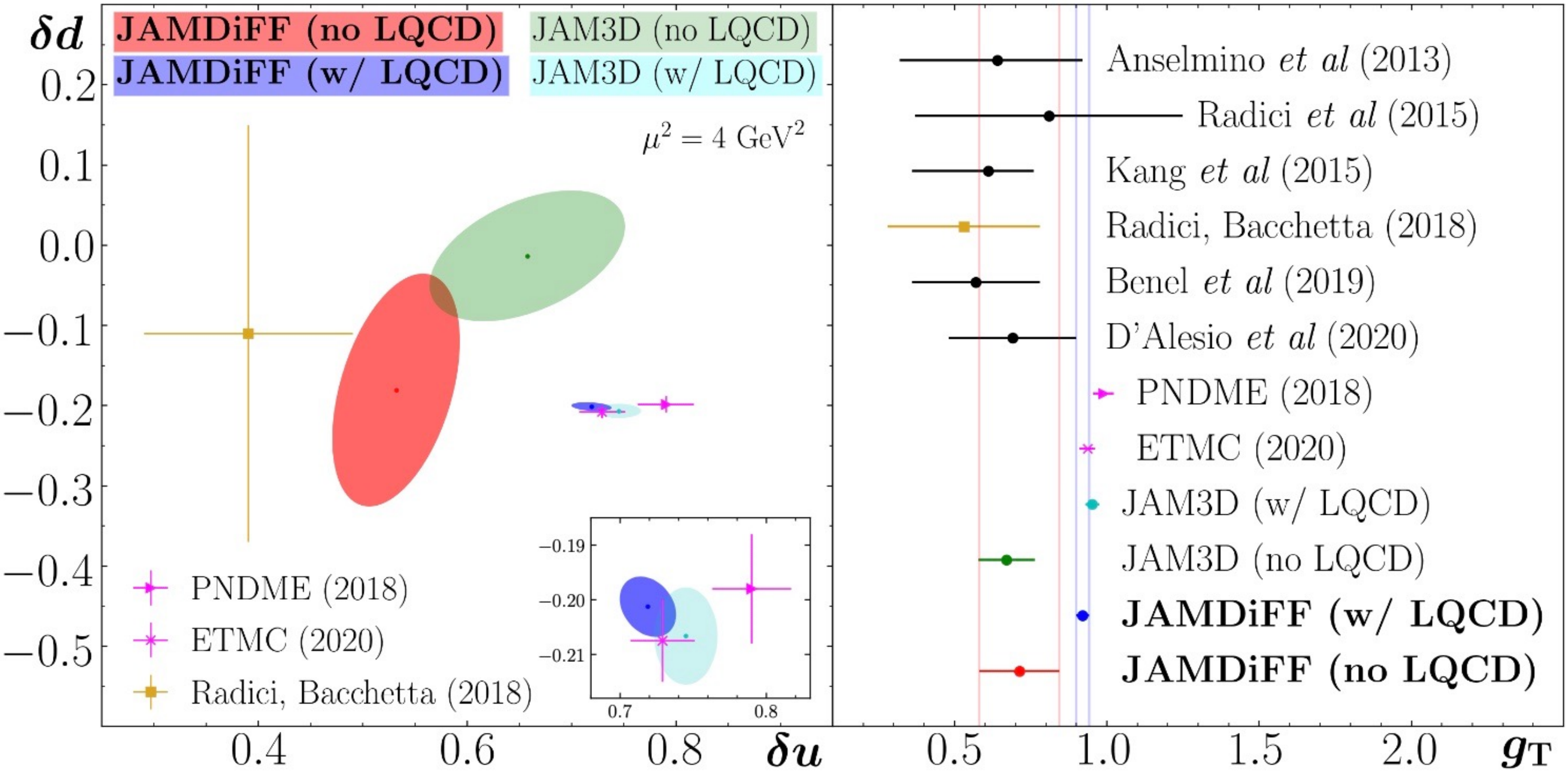
🟦 JAMDiFF (w/ LQCD)

The analyses with and without LQCD are statistically equivalent



- ➢ JAMDiFF (w/LQCD) agrees with JAM3D (w/LQCD*) - nontrivial since the lattice data only constrains the full moment of the transversity PDFs

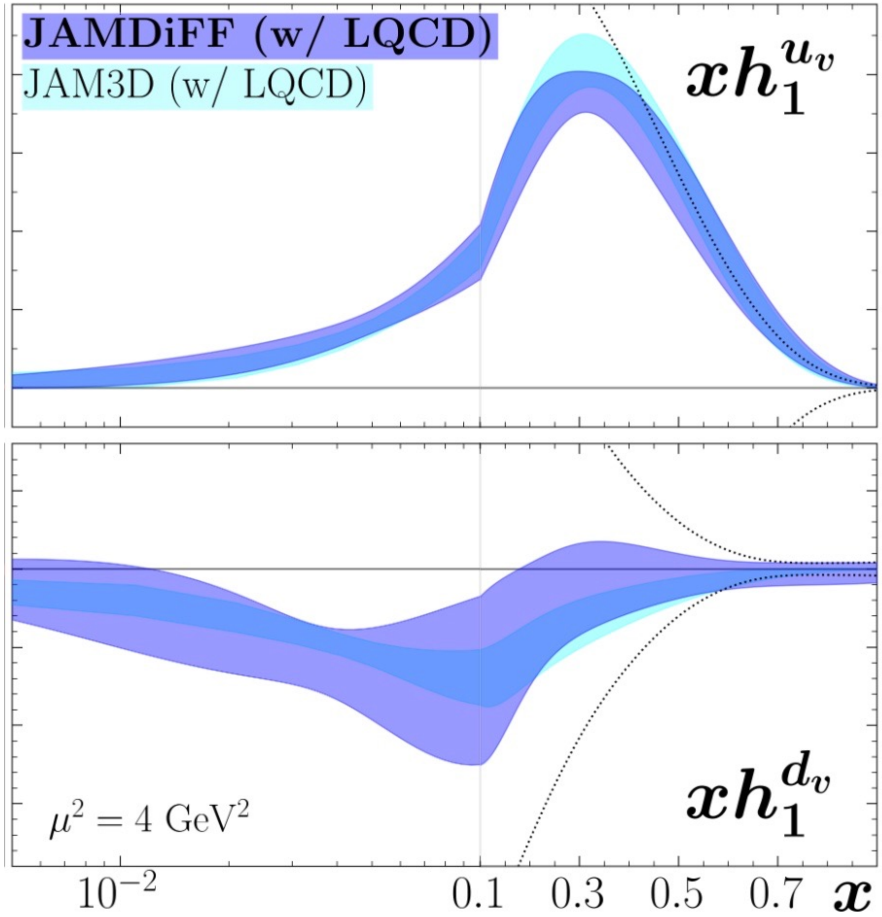
*The JAM3D (w/LQCD) analysis is slightly modified from the published version: antiquarks are now included (with $\bar{u} = -\bar{d}$) and $\delta u, \delta d$ from ETMC and PNDME are both included in the fit (rather than just g_T from ETMC)

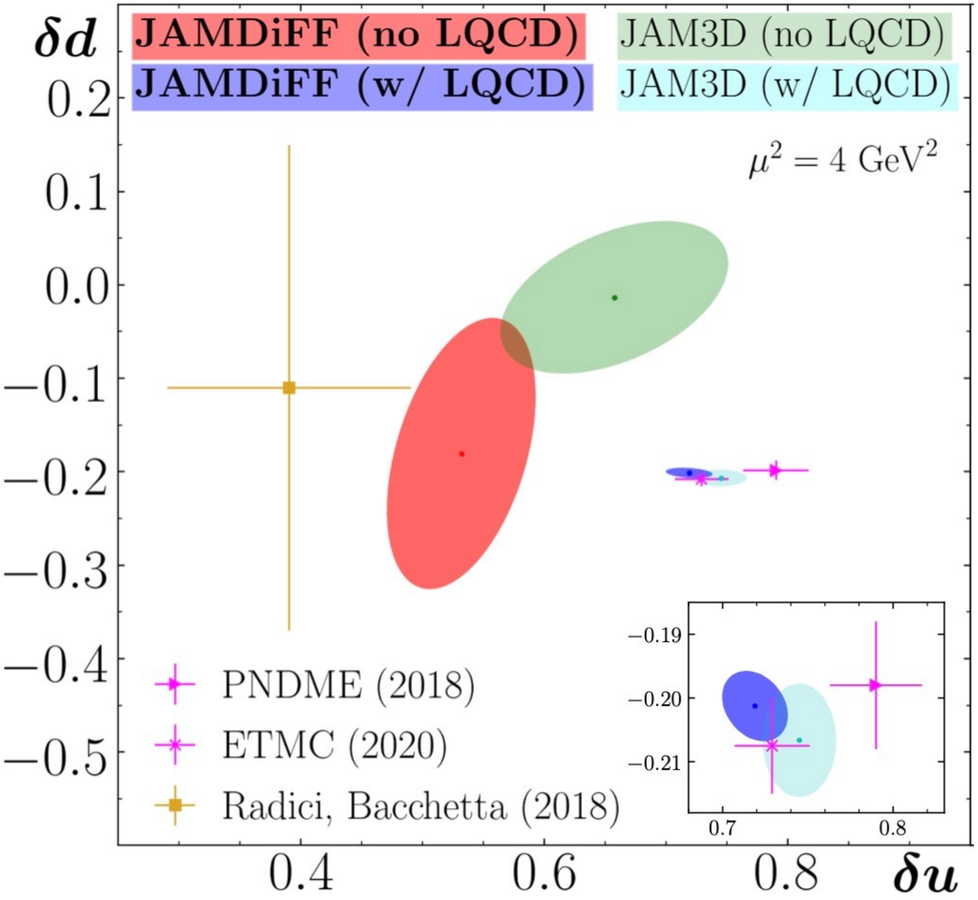


- ➢ JAMDiFF (no LQCD) agrees within errors with JAM3D (no LQCD) and Radici, Bacchetta (2018) for the tensor charges

- ➢ Similar to the JAM3D analysis, **JAMDiFF also finds compatibility with lattice once that data is included in the fit, *and can still describe the experimental data well***

- Possible explanation for the surprising shift (~3-4σ difference with lattice to a ~0.3-2σ difference) in the phenomenological values of the tensor charges once LQCD data is included in the analysis:
  - The experimental measurements are sensitive to the *x*-dependence of the transversity PDFs, not the full moment like the lattice data.
  - When integrating over *x*, small but consistent changes in the PDF as a function of *x* can accumulate into a large change in the tensor charge while not significantly affecting the description of the experimental data.
  - Before drawing a conclusion about the compatibility between LQCD tensor charges and experimental data, one needs first to include both in the analysis.
  - One should only be concerned if the description of the lattice data remains poor even after its inclusion and/or if the description of the experimental data suffers significantly.





JAM3D and JAMDiFF agree on the x-dependence of transversity and also can successfully include lattice QCD data on the tensor charges in the analyses, thus showing for the first time the universal nature of all available information on transversity and the tensor charges of the nucleon

Summary and Outlook

Summary

- We have introduced a ***new definition of dihadron fragmentation functions that is consistent with a number density interpretation***, giving these functions a clear physical meaning

- We have performed separate QCD global analyses of TSSAs in TMD/collinear twist-3 single-hadron observables and in dihadron fragmentation measurements, also studying the role of lattice QCD in our fits

- The tensor charges of the nucleon are quantities of particular interest - they are fundamental properties of the nucleon that have connections to QCD phenomenology, *ab initio* lattice QCD computations, model calculations, and low-energy beyond the Standard Model studies (e.g., beta decay, EDM)

Recent analyses by the JAM Collaboration show agreement between single-hadron and dihadron approaches for extracting transversity as well as compatibility with lattice QCD tensor charges, thus showing for the first time the universal nature of all this information

Outlook

- Further refinements/improvements:
  - TMD/collinear twist-3: include lattice tensor charge data and hadron-in-jet Collins effect measurements with CSS evolution in the analysis, …
  - Dihadron: other groups including lattice tensor charge data; unpolarized *pp* cross section data to better constrain $D_1^g(z, M_h)$; NLO calculations, …
  - "Universal" analysis where TMD/collinear twist-3 ***and*** dihadron measurements are fit simultaneously
  - Incorporate proper small-*x* evolution for transversity (Kovchegov, Sievert (2019))
  - Using pseudo-PDF or quasi-PDF approaches, lattice can now compute $h_1(x)$ (Egerer, et al. (2021); Alexandrou, et al. (2022)) - eventually can include data into phenomenology (more constraining than the tensor charge data)