Proton mass decompositions: a fresh look

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Based on Rodini et al., 2004.03704 **(QED case)**

Metz et. al, 2006.11171 **(QCD case)**

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Outline

Energy-Momentum Tensor (EMT): definitions and renormalization

Scheme dependence and scheme choice (QED example)

EMT matrix elements: form factors in QED and QCD

Review of different decompositions

Numerical results and scheme-dependence exploration

Purpose

Understanding the origin of the proton mass

Is it possible to measure the individual (i.e. quark and gluon) contributions?

Is it possible to measure the trace anomaly?

Is there a physically meaningful interpretation of the different decompositions?

Which are the quantities that can give us more information about the proton mass decompositions?

Definition of the Energy-Momentum Tensor

$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = T_C^{\mu\nu} + \partial_\rho \Phi^{\rho\mu\nu}$$

$$T_e^{\mu\nu} = Z_2 \; \bar{\psi} \frac{i}{4} \gamma^{\{\mu} \overleftrightarrow{\partial}^{\nu\}} \psi - Z_2 \mu^{2\varepsilon} \; e \bar{\psi} \gamma^{\{\mu} A^{\nu\}} \psi$$

$$T_\gamma^{\mu\nu} = -Z_3 \; F^{\mu\alpha} F^{\nu}{}_{\alpha} + Z_3 \; \frac{g^{\mu\nu}}{4} F^{\alpha\beta} F_{\alpha\beta}$$

$$T^{\mu\nu} = T_e^{\mu\nu} + T_\gamma^{\mu\nu}$$

Blaschke et al., Nucl.Phys.B 912 (2016) 192

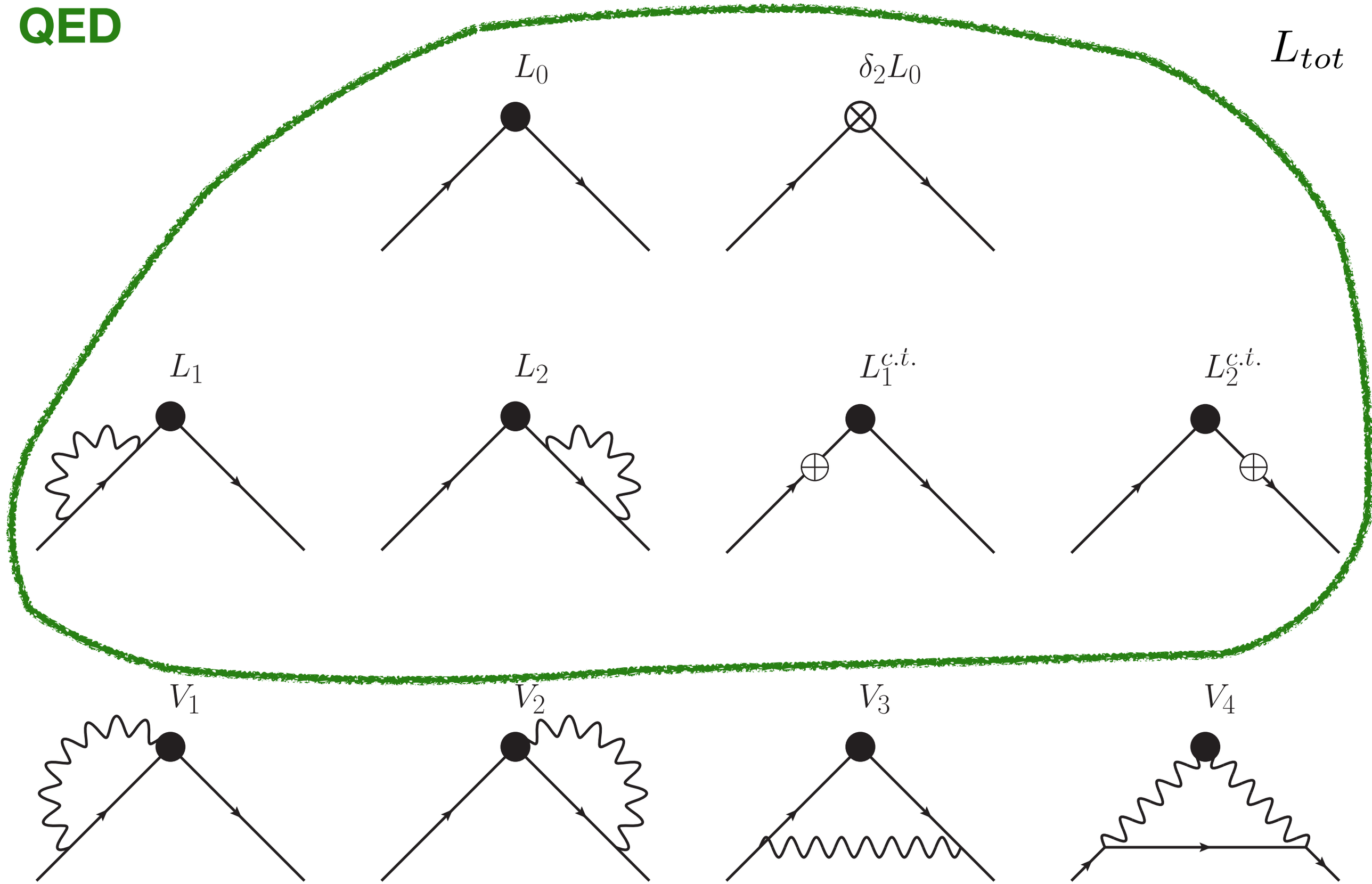
Lagrangian renormalization is understood

$$T^{\mu\nu} = \mathcal{O}_1 + \frac{\mathcal{O}_2}{4} + \mathcal{O}_3$$

$$\mathcal{O}_1 = -F^{\mu\alpha} F^{\nu}{}_{\alpha} \qquad \mathcal{O}_2 = g^{\mu\nu} F^{\alpha\beta} F_{\alpha\beta}$$

$$\mathcal{O}_3 = \frac{i}{4} \, \bar{\psi} \, \gamma^{\{\mu} \overleftrightarrow{D}^{\nu\}} \, \psi \qquad \mathcal{O}_4 = g^{\mu\nu} m \bar{\psi} \psi$$

QED



Renormalize fields, coupling and masses is not enough.

We must also renormalize the operators:

$$\mathcal{O}_{1,R} = Z_T \mathcal{O}_1 + Z_M \mathcal{O}_2 + Z_L \mathcal{O}_3 + Z_S \mathcal{O}_4$$
$$\mathcal{O}_{2,R} = Z_F \mathcal{O}_2 + Z_C \mathcal{O}_4$$
$$\mathcal{O}_{3,R} = Z_\psi \mathcal{O}_3 + Z_K \mathcal{O}_4 + Z_Q \mathcal{O}_1 + Z_B \mathcal{O}_2$$
$$\mathcal{O}_{4,R} = \mathcal{O}_4$$

$$Z_T + Z_Q = 1$$
$$Z_L + Z_\psi = 1$$
$$Z_M + Z_B + \frac{Z_F}{4} = \frac{1}{4}$$
$$Z_S + Z_K + \frac{Z_C}{4} = 0$$

Hatta et al., JHEP 12 (2018) 008

Tanaka, JHEP 01 (2019) 120

The total EMT is not affected by the additional renormalization

$$T_R^{\mu\nu} = T^{\mu\nu}$$

$$T^\mu{}_\mu = (T_R)^\mu{}_\mu = (T^\mu{}_\mu)_R = (1+\gamma_m)(m\bar{\psi}\psi)_R + \frac{\beta}{2g}(F^{\alpha\beta}F_{\alpha\beta})_R$$

However, in general, trace and renormalization do not commute

$$\mathrm{Tr}[O_R^{\mu\nu}] \neq (\mathrm{Tr}[O^{\mu\nu}])_R$$

In particular $T_{q,R}^{\mu\nu} = \mathcal{O}_{3,R}$ $\qquad T_{g,R}^{\mu\nu} = \mathcal{O}_{1,R} + \frac{\mathcal{O}_{2,R}}{4}$

$$(T_{q,R})^\mu{}_\mu = (1+y)(m\bar{\psi}\psi)_R + x\,(F^{\alpha\beta}F_{\alpha\beta})_R$$

$$(T_{g,R})^\mu{}_\mu = (\gamma_m - y)(m\bar{\psi}\psi)_R + \left(\frac{\beta}{2g} - x\right)(F^{\alpha\beta}F_{\alpha\beta})_R$$

Hatta et al., JHEP 12 (2018) 008

Tanaka, JHEP 01 (2019) 120

How to fix the counterterms

$Z_{F,C}$ Are known from R. Tarrach, Nucl. Phys. B 196 (1982) 45

$Z_{L,T,Q,\psi}$ Are given by the evolution equations for the second moment of the flavor-singlet unpolarized parton distributions. Tanaka, JHEP 01 (2019) 120

$$Z_M = \frac{Z_T}{d} - \frac{Z_F}{d}\left(1 - \frac{\beta}{2g} + x\right)$$

$$Z_S = -\frac{Z_L}{d} - \frac{Z_C}{d}\left(1 - \frac{\beta}{2g} + x\right) - \frac{y - \gamma_m}{d}$$

$$Z_B = \frac{Z_Q}{d} + \frac{x}{d} Z_F$$

$$Z_K = -\frac{Z_\psi}{d} + \frac{x}{d} Z_C + \frac{1+y}{d}$$

Hatta et al., JHEP 12 (2018) 008

Gauge-field operator

$$\widetilde{\mathcal{O}}_{1,R} = \mathcal{O}_{1,R} + \frac{1}{4}\left(1 - \frac{\beta}{2g} + x\right)\mathcal{O}_{2,R} + \frac{y - \gamma_m}{4}\mathcal{O}_{4,R},$$

$$\widetilde{\mathcal{O}}_{3,R} = \mathcal{O}_{3,R} - \frac{x}{4}\mathcal{O}_{2,R} - \frac{1+y}{4}\mathcal{O}_{4,R}$$

Tanaka, JHEP 01 (2019) 120

Hatta et al., JHEP 12 (2018) 008

Fixing x and y it corresponds to choose a scheme

Diagonal schemes:

D1: $x = 0 \quad y = \gamma_m$ Rodini et al., 2004.03704

D2: $x = 0 \quad y = 0$ Metz et. al, 2006.11171

$$(T_{q,R})^{\mu}{}_{\mu} = (1+y)(m\bar{\psi}\psi)_R + x\,(F^{\alpha\beta}F_{\alpha\beta})_R$$

$$(T_{g,R})^{\mu}{}_{\mu} = (\gamma_m - y)(m\bar{\psi}\psi)_R + \left(\frac{\beta}{2g} - x\right)(F^{\alpha\beta}F_{\alpha\beta})_R$$

MS scheme: Tanaka, JHEP 01 (2019) 120 Hatta et al., JHEP 12 (2018) 008

MSbar scheme Metz et al., 2006.11171

MS scheme

Tanaka, JHEP 01 (2019) 120

Impose vanishing finite contributions to the derived counterterms

$$Z_X = \delta_{X,T} + \delta_{X,\psi} + \delta_{X,F} + \frac{a_X}{\epsilon} + \frac{b_X}{\epsilon^2} + \frac{c_X}{\epsilon^3} + \ldots$$

From the definition of $Z_{M,S}$

$$\frac{1}{32}\left[(8 + 4a_T + 2b_T + c_T + \ldots) - \left(1 + x - \frac{\beta}{2g}\right)(8 + 4a_F + 2b_F + c_F + \ldots)\right] = 0$$

$$\frac{1}{32}\left[-(4a_L + 2b_L + c_L + \ldots) - \left(1 + x - \frac{\beta}{2g}\right)(4a_C + 2b_C + c_C + \ldots) + 8(\gamma_m - y)\right] = 0$$

MSbar

Metz et al., 2006.11171

The finite contribution in MSbar counterterms is uniquely determined by the divergent part

$$Z\big|_{\overline{\mathrm{MS}}} = (1,0) + \alpha_s \frac{\bar{a}_1}{\epsilon} S_\epsilon + \alpha_s^2 \left(\frac{\bar{b}_2}{\epsilon^2} + \frac{\bar{b}_1}{\epsilon} \right) S_\epsilon^2 + \alpha_s^3 \left(\frac{\bar{c}_3}{\epsilon^3} + \frac{\bar{c}_2}{\epsilon^2} + \frac{\bar{c}_1}{\epsilon} \right) S_\epsilon^3$$

$$S_\epsilon\big|_{\overline{\mathrm{MS}}_1} = \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}$$

$$S_\epsilon\big|_{\overline{\mathrm{MS}}_2} = (4\pi e^{-\gamma_E})^\epsilon$$

$$Z_B = \frac{Z_Q}{d} + \frac{x}{d} Z_F$$

$$x = x_1 \alpha + x_2 \alpha^2 + ...$$

What we find (in MSbar 1)

$$O(\alpha_s) : \frac{1}{8} \Big(\bar{a}_{1,Q} + 2\bar{a}_{1,Q} \left(\log(4\pi) - \gamma_E \right) + 2x_1 \Big)$$

What we want

$$O(\alpha_s) : \frac{1}{4} \bar{a}_{1,Q} (\log(4\pi) - \gamma_E)$$

QED

$$Z_T + Z_Q = 1$$
$$Z_L + Z_\psi = 1$$
$$Z_M + Z_B + \frac{Z_F}{4} = \frac{1}{4}$$
$$Z_S + Z_K + \frac{Z_C}{4} = 0$$

$$x = 0, \quad y = \begin{cases} \frac{\alpha}{3\pi} & \overline{\mathrm{MS}} \\ \gamma_m = \frac{3\alpha}{2\pi} & \mathrm{D}_1 \end{cases}$$

$$Z_F = 1 + \frac{\beta(e)}{e}\Delta_{\mathrm{UV}} \qquad Z_C = 2\gamma_m \Delta_{\mathrm{UV}}$$

$$Z_T = 1 \qquad Z_Q = 0 \qquad Z_\psi = 1 + \frac{2\alpha}{3\pi}\Delta_{\mathrm{UV}}$$

$$Z_L = -\frac{2\alpha}{3\pi}\Delta_{\mathrm{UV}} \qquad Z_M = -\frac{\alpha}{12\pi}\Delta_{\mathrm{UV}} \qquad Z_B = 0$$

$$Z_S = \begin{cases} -\frac{7\alpha}{12\pi}\Delta_{\mathrm{UV}} & \overline{\mathrm{MS}} \\ -\frac{7\alpha}{24\pi} - \frac{7\alpha}{12\pi}\Delta_{\mathrm{UV}} & \mathrm{D}_1 \end{cases} \qquad Z_K = \begin{cases} -\frac{\alpha}{6\pi}\Delta_{\mathrm{UV}} & \overline{\mathrm{MS}} \\ \frac{7\alpha}{24\pi} - \frac{\alpha}{6\pi}\Delta_{\mathrm{UV}} & \mathrm{D}_1 \end{cases}$$

$$\Delta_{\mathrm{UV}} = \frac{1}{\varepsilon} + \log(4\pi) - \gamma_{\mathrm{E}}$$

Rodini et al., 2004.03704

EMT matrix elements

$$\langle \mathcal{O} \rangle = \langle P | \mathcal{O} | P \rangle$$

$$\langle \mathcal{O} \rangle = \langle e(P) | \mathcal{O} | e(P) \rangle$$

Ji, Phys. Rev. Lett. 78 (1997) 610

Two form factors for the forward limit

$$\langle T^{\mu\nu}_{i,R} \rangle = 2P^\mu P^\nu A_i(0) + 2M^2 g^{\mu\nu} \bar{C}_i(0)$$

$$i = q, g$$

$$\langle T^{\mu\nu}_{i,R} \rangle = \left(2P^\mu P^\nu - \frac{M^2}{2} g^{\mu\nu} \right) A^R_i(0) + \frac{M^2}{2} g^{\mu\nu} \left(A^R_i(0) + 4\bar{C}^R_i(0) \right)$$

In Light-Front Quantization can be linked to the average parton longitudinal momentum fraction

$$A_q(0) + A_g(0) = 1 \qquad \bar{C}_q(0) + \bar{C}_g(0) = 0$$

How to obtain the form factors: QED (1/2)

1-loop calculation!

$$L^R_{tot}(\Delta=0) = 2P^\mu P^\nu \left(1 + \frac{\alpha}{\pi\varepsilon_{\rm IR}} - \frac{\alpha}{\pi} - \frac{\alpha\mathcal{L}}{4\pi}\right) \qquad \mathcal{L} = \log\left(\frac{\mu^2}{m^2}\right)$$

$$(V_1+V_2)^R(\Delta=0) = \begin{cases} 2P^\mu P^\nu \left(-\frac{\alpha\mathcal{L}}{2\pi} - \frac{3\alpha}{2\pi}\right) - 2m^2 g^{\mu\nu}\left(\frac{\alpha\mathcal{L}}{4\pi} + \frac{\alpha}{4\pi}\right) & \overline{\rm MS} \\ 2P^\mu P^\nu \left(-\frac{\alpha\mathcal{L}}{2\pi} - \frac{3\alpha}{2\pi}\right) - 2m^2 g^{\mu\nu}\left(\frac{\alpha\mathcal{L}}{4\pi} - \frac{\alpha}{24\pi}\right) & {\rm D}_1 \end{cases}$$

$$V_3^R(\Delta=0) = 2P^\mu P^\nu \left(-\frac{\alpha}{\pi\varepsilon_{\rm IR}} + \frac{14\alpha}{9\pi} + \frac{\alpha\mathcal{L}}{12\pi}\right) + 2m^2 g^{\mu\nu}\left(\frac{5\alpha\mathcal{L}}{12\pi} + \frac{7\alpha}{36\pi}\right)$$

$$V_4^R(\Delta=0) = \begin{cases} 2P^\mu P^\nu \left(\frac{2\alpha\mathcal{L}}{3\pi} + \frac{17\alpha}{18\pi}\right) + 2m^2 g^{\mu\nu}\left(-\frac{\alpha\mathcal{L}}{6\pi} + \frac{\alpha}{18\pi}\right) & \overline{\rm MS} \\ 2P^\mu P^\nu \left(\frac{2\alpha\mathcal{L}}{3\pi} + \frac{17\alpha}{18\pi}\right) + 2m^2 g^{\mu\nu}\left(-\frac{\alpha\mathcal{L}}{6\pi} - \frac{17\alpha}{72\pi}\right) & {\rm D}_1 \end{cases}$$

Rodini et al., 2004.03704

How to obtain the form factors: QED (2/2)

$$A_e^R(0) = 1 - \frac{2\alpha\mathcal{L}}{3\pi} - \frac{17}{18}\frac{\alpha}{\pi} \qquad \mathcal{L} = \log\left(\frac{\mu^2}{m^2}\right)$$

$$A_\gamma^R(0) = \frac{2\alpha\mathcal{L}}{3\pi} + \frac{17}{18}\frac{\alpha}{\pi}$$

$$\bar{C}_e^R(0) = \begin{cases} \frac{\alpha\mathcal{L}}{6\pi} - \frac{\alpha}{18\pi}, & \overline{\mathrm{MS}} \\ \\ \frac{\alpha\mathcal{L}}{6\pi} + \frac{17}{72}\frac{\alpha}{\pi}, & \mathrm{D}_1 \end{cases}$$

$$\bar{C}_\gamma^R(0) = \begin{cases} -\frac{\alpha\mathcal{L}}{6\pi} + \frac{\alpha}{18\pi}, & \overline{\mathrm{MS}} \\ \\ -\frac{\alpha\mathcal{L}}{6\pi} - \frac{17}{72}\frac{\alpha}{\pi}, & \mathrm{D}_1 \end{cases}$$

Rodini et al., 2004.03704

How to obtain the form factors: QCD

Two phenomenological inputs are required

$$\frac{3}{2}m^2 a = \langle \widetilde{\mathcal{O}}^{00}_{3,R} \rangle_{\mathbf{P}=0} \qquad \frac{3}{2}m^2(1-a) = \langle \widetilde{\mathcal{O}}^{00}_{1,R} \rangle_{\mathbf{P}=0}$$

$$2m^2 b = \langle (1+\gamma_m) m\bar{\psi}\psi \rangle_{\mathbf{P}=0} \qquad 2m^2(1-b) = \frac{\beta(g)}{2g} \langle (F^{\mu\nu}F_{\mu\nu})_R \rangle_{\mathbf{P}=0}$$

$$\langle (T_{i,R})^{\mu}_{\mu} \rangle \propto A_i^R(0) + 4\bar{C}_i^R(0)$$

1

From CT18NNLO PDF extraction at $\mu = 2\ \mathrm{GeV}$

$$a = a_q \quad (1-a) = a_g$$

$$A_q(0) \equiv a_q = 0.586 \pm 0.013$$

$$A_g(0) \equiv a_g = 1 - a_q = 0.414 \pm 0.013$$

Hou et al.,1908.11394

How to obtain the form factors: QCD

2

$$\sigma_u + \sigma_d = \sigma_{\pi N} = \frac{\langle P|\, \hat{m}\, \bar{u}u \,|P\rangle}{2M} \qquad \sigma_s = \frac{\langle P|\, m_s\, \bar{s}s \,|P\rangle}{2M} \qquad \sigma_c = \frac{\langle P|\, m_c\, \bar{c}c \,|P\rangle}{2M}$$

Alarcon et al., Phys.Rev.D 85 (2012) 051503

Alarcon et al., Phys.Lett.B 730 (2014) 342

Hoferichter et al., Phys.Rev.Lett. 115 (2015) 092301

πN scattering data analysed with ChPT

$$\sigma_{\pi N}\big|_{\mathrm{ChPT}} = (59 \pm 7)\,\mathrm{MeV} \qquad \sigma_s\big|_{\mathrm{ChPT}} = (16 \pm 80)\,\mathrm{MeV}$$

Lattice QCD

Alexandrou et al., 1909.00485

$$\sigma_{\pi N}\big|_{\mathrm{LQCD}} = (41.6 \pm 3.8)\,\mathrm{MeV} \qquad \sigma_s\big|_{\mathrm{LQCD}} = (39.8 \pm 5.5)\,\mathrm{MeV}$$

$$\sigma_c\big|_{\mathrm{LQCD}} = (107 \pm 22)\,\mathrm{MeV}$$

Scenario A

CT18NNLO for a_q, a_g **ChPT for the sigma terms** $\sigma_{\pi N}, \sigma_s$

Scenario B

CT18NNLO for a_q, a_g **LQCD for the sigma terms** $\sigma_{\pi N}, \sigma_s, \sigma_c$

Strong coupling constant from RunDec: Chetyrkin et al., Comput.Phys.Commun. 133 (2000) 43

$O(\alpha_s^3)$ **results and analysis of convergence** Metz et. al, 2006.11171

Hatta: QED

Rodini et al., 2004.03704

$$\bar{m} = \bar{m}_e + \bar{m}_\gamma = \frac{1}{\langle e(P)|e(P)\rangle} \int d^3x \left(\langle (T_{e,R}(x))^\mu_\mu \rangle + \langle (T_{\gamma,R}(x))^\mu_\mu \rangle \right)$$

In the rest frame

$$\frac{\bar{m}_e}{m} = A_e^R(0) + 4\bar{C}_e^R(0) = 1 + y - \gamma_m = \begin{cases} 1 - \frac{7\alpha}{6\pi} & \overline{\mathrm{MS}} \\ 1 & \mathrm{D}_1 \end{cases}$$

$$\frac{\bar{m}_\gamma}{m} = A_\gamma^R(0) + 4\bar{C}_\gamma^R(0) = \gamma_m - y = \begin{cases} \frac{7\alpha}{6\pi} & \overline{\mathrm{MS}} \\ 0 & \mathrm{D}_1 \end{cases}$$

In a moving frame

$$\frac{\bar{m}_i}{m} = \frac{m}{E} \left(\frac{\bar{m}_i}{m} \right)_{\mathrm{rest}}$$

Hatta: QCD

Hatta et. al., JHEP 12 (2018) 008

$$M = \bar{M}_q + \bar{M}_g = \frac{1}{\langle P|P\rangle} \int d^3x \Big(\langle \, (T_{q,R}(x))^{\mu}{}_{\mu} \, \rangle + \langle \, (T_{g,R}(x))^{\mu}{}_{\mu} \, \rangle \Big)$$

In the rest frame

$$\bar{M}_i = M \left(A_i(0) + 4\bar{C}_i(0) \right)$$

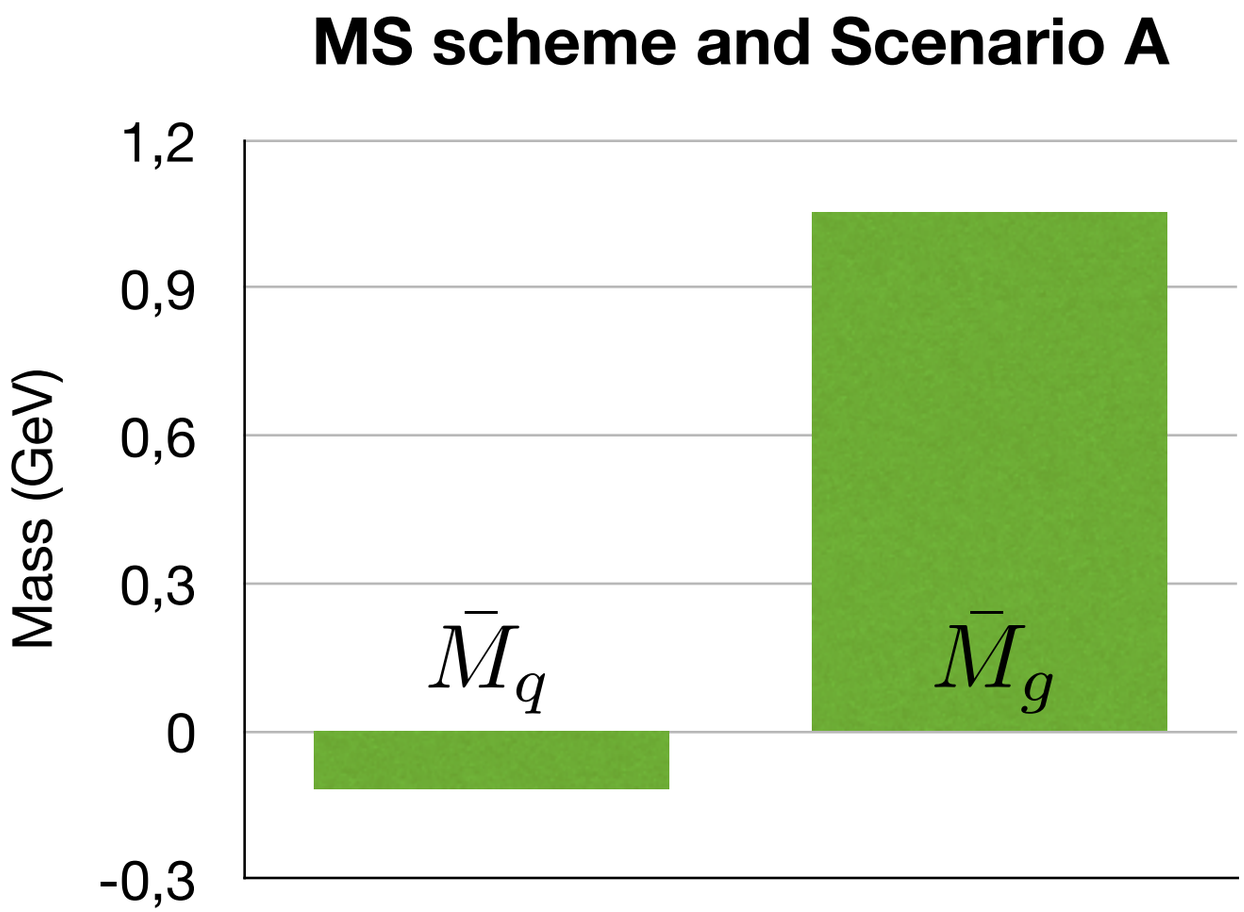
In a moving frame

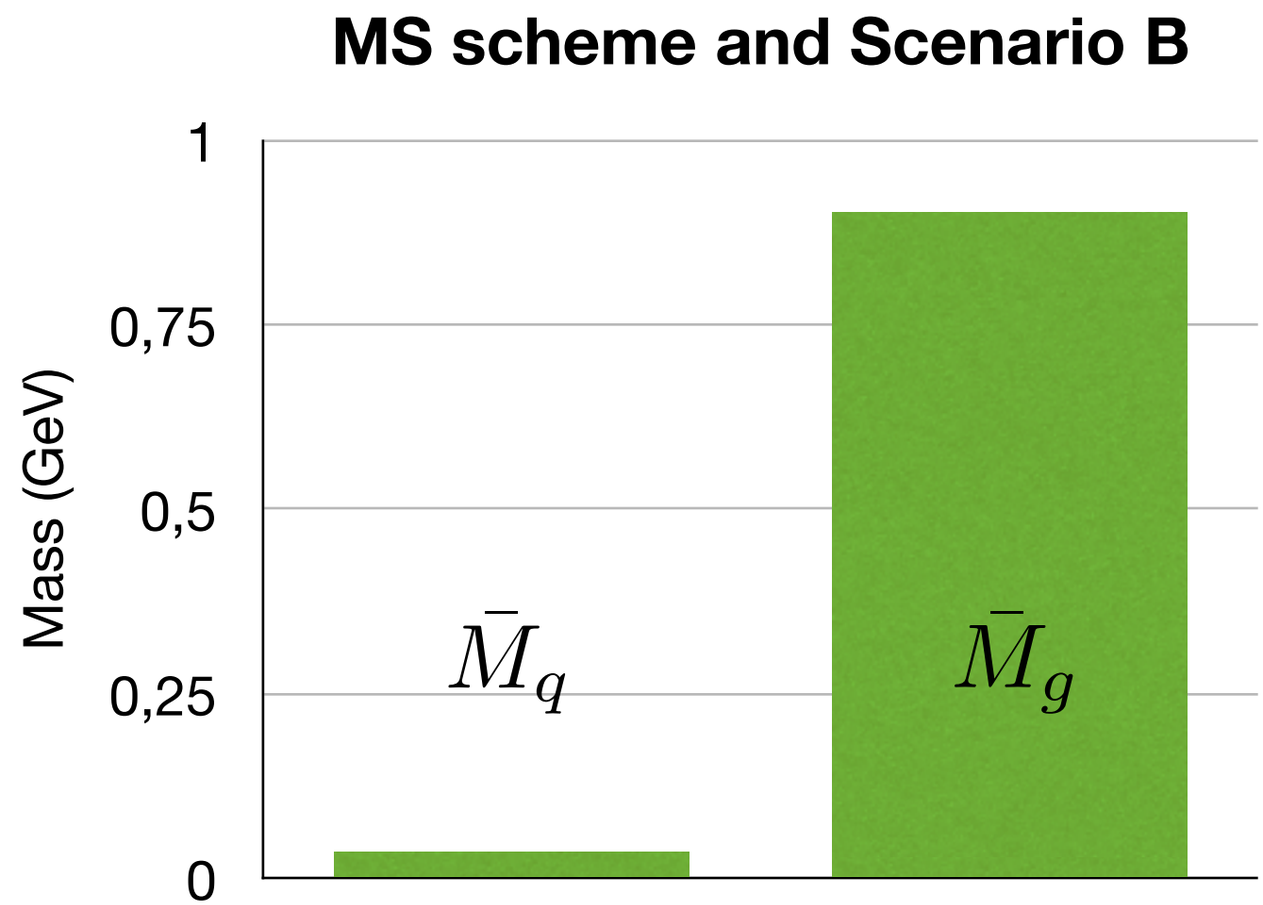
$$\bar{M}_i = \frac{M^2}{E} \left(A_i(0) + 4\bar{C}_i(0) \right)$$

Hatta: QCD

Hatta et. al., JHEP 12 (2018) 008

Metz et. al, 2006.11171





Lorcé (two terms, QED)

$$m = U_e + U_\gamma$$

In the rest frame

$$U_i = \left.\frac{\langle \int d^3x \; T^{00}_{i,R} \rangle}{\langle e(P)|e(P)\rangle}\right|_{\mathbf{P}=0} = m\left(A^R_i(0) + \bar{C}^R_i(0)\right)$$

$$U_e = \begin{cases} m\left(1 - \frac{\alpha\mathcal{L}}{2\pi} - \frac{\alpha}{\pi}\right) & \overline{\mathrm{MS}} \\ m\left(1 - \frac{\alpha\mathcal{L}}{2\pi} - \frac{17\alpha}{24\pi}\right) & \mathrm{D}_1 \end{cases} \qquad U_\gamma = \begin{cases} m\left(\frac{\alpha\mathcal{L}}{2\pi} + \frac{\alpha}{\pi}\right) & \overline{\mathrm{MS}} \\ m\left(\frac{\alpha\mathcal{L}}{2\pi} + \frac{17\alpha}{24\pi}\right) & \mathrm{D}_1 \end{cases}$$

In a moving frame

$$U_i = EA^R_i(0) + \frac{m^2}{E}\bar{C}^R_i(0)$$

Lorcé (two terms)

Lorcé, Eur.Phys.J.C 78 (2018) 2, 120 | Metz et. al, 2006.11171

$$U_i = M\big(A_i(0) + \bar{C}_i(0)\big) \qquad H_i = \int d^3x\, T^{00}_{i,R} = \int d^3x\, \mathcal{H}_i(x)$$

$$M = U_q + U_g = \frac{1}{\langle P|P\rangle}\Big(\langle\, H_{q,R}\,\rangle + \langle\, H_{g,R}\,\rangle\Big)$$

$$T^{00}_{q,R} = (m\bar{\psi}\psi)_R + (\psi^\dagger\, i\boldsymbol{D}\cdot\boldsymbol{\alpha}\,\psi)_R$$

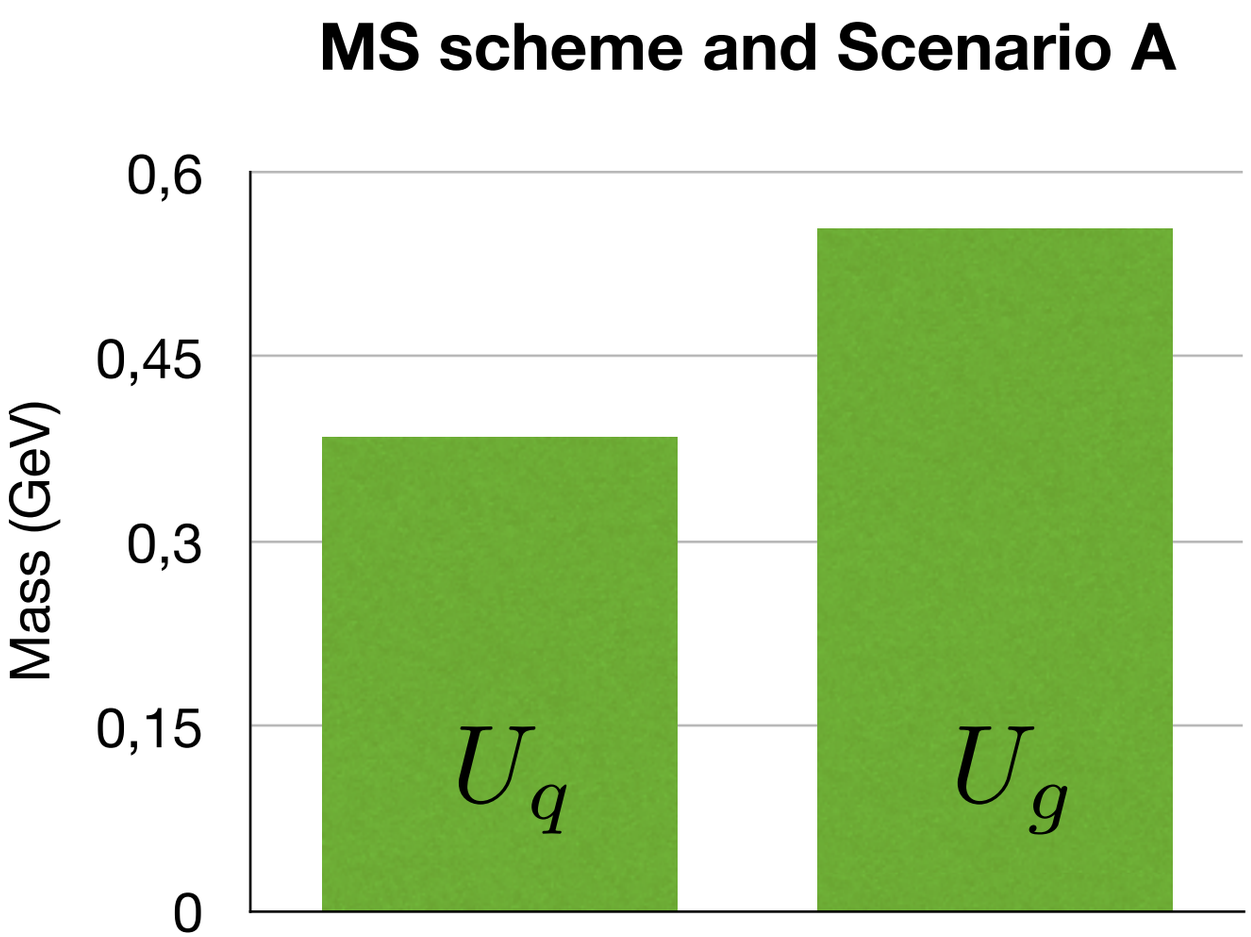
$$T^{00}_{g,R} = \frac{1}{2}(E^2 + B^2)_R$$

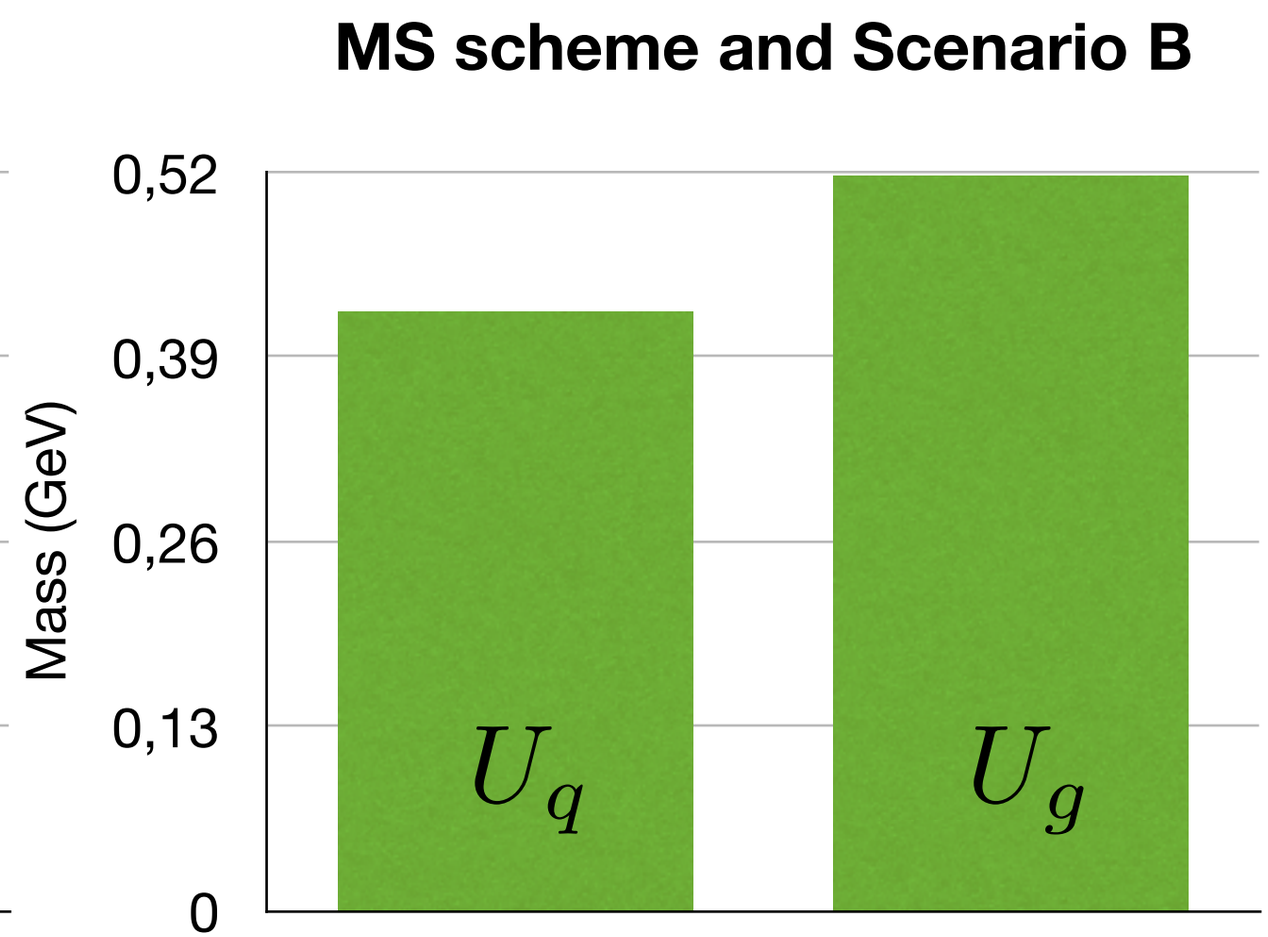
$$\Big\langle (\psi^\dagger\, i\boldsymbol{D}\cdot\boldsymbol{\alpha}\,\psi)_R + \frac{1}{2}(E^2 + B^2)_R \Big\rangle$$

$$= \Big\langle \gamma_m (m\bar{\psi}\psi)_R + \frac{\beta}{2g}(F^{\alpha\beta}F_{\alpha\beta})_R \Big\rangle$$

Lorcé (two terms)

Lorcé, Eur.Phys.J.C 78 (2018) 2, 120 | Metz et. al, 2006.11171





Ji

Ji, Phys.Rev. D52 (1995) 271

Ji, Phys.Rev.Lett. 74 (1995) 1071

$$T^{\mu\nu} = \bar{T}^{\mu\nu} + \hat{T}^{\mu\nu} \qquad \hat{T}^{\mu\nu} = \frac{1}{4} g^{\mu\nu} T^{\alpha}{}_{\alpha}$$

$$(\mathcal{H}'_q)_{\rm Ji} = \left[(\bar{T}^{00}_q)_R\right]_{\rm Ji} = \left(\psi^\dagger \left(i\boldsymbol{D}\cdot\boldsymbol{\alpha}\right)\psi\right)_R + \frac{3}{4} m\bar{\psi}\psi$$

$$(\mathcal{H}'_m)_{\rm Ji} = \left[(\hat{T}^{00}_q)_R\right]_{\rm Ji} = \frac{1+\gamma_m}{4} m\bar{\psi}\psi$$

$$(\mathcal{H}'_g)_{\rm Ji} = \left[(\bar{T}^{00}_g)_R\right]_{\rm Ji} = \frac{1}{2}\left(E^2 + B^2\right)_R$$

$$(\mathcal{H}'_a)_{\rm Ji} = \left[(\hat{T}^{00}_g)_R\right]_{\rm Ji} = -\frac{\beta(g)}{4g}\left(E^2 - B^2\right)_R$$

$$\widetilde{\mathcal{O}}_{1,R} = \mathcal{O}_{1,R} + \frac{1}{4}\left(1 - \frac{\beta}{2g} + x\right)\mathcal{O}_{2,R} + \frac{y-\gamma_m}{4}\mathcal{O}_{4,R},$$

$$\widetilde{\mathcal{O}}_{3,R} = \mathcal{O}_{3,R} - \frac{x}{4}\mathcal{O}_{2,R} - \frac{1+y}{4}\mathcal{O}_{4,R}$$

Tanaka, JHEP 01 (2019) 120

Hatta et al., JHEP 12 (2018) 008

Ji (the revisited version)

Rodini et al., 2004.03704 | Metz et. al, 2006.11171

$$\mathcal{H}'_q = \bar{T}^{00}_{q,R} = (\psi^\dagger\, i\boldsymbol{D}\cdot\boldsymbol{\alpha}\,\psi)_R + (m\bar{\psi}\psi)_R - \frac{1+y}{4}(m\bar{\psi}\psi)_R - \frac{x}{4}(F^{\alpha\beta}F_{\alpha\beta})_R$$

$$\mathcal{H}'_m = \hat{T}^{00}_{q,R} = \frac{1+y}{4}(m\bar{\psi}\psi)_R + \frac{x}{4}(F^{\alpha\beta}F_{\alpha\beta})_R$$

$$\mathcal{H}'_g = \bar{T}^{00}_{g,R} = \frac{1}{2}(E^2+B^2)_R + \frac{y-\gamma_m}{4}(m\bar{\psi}\psi)_R - \frac{1}{4}\left(\frac{\beta}{2g} - x\right)(F^{\alpha\beta}F_{\alpha\beta})_R$$

$$\mathcal{H}'_a = \hat{T}^{00}_{g,R} = \frac{\gamma_m - y}{4}(m\bar{\psi}\psi)_R + \frac{1}{4}\left(\frac{\beta}{2g} - x\right)(F^{\alpha\beta}F_{\alpha\beta})_R$$

$$M = M_q + M_m + M_g + M_a = \frac{\langle\, H_q \,\rangle}{\langle P|P\rangle} + \frac{\langle\, H_m \,\rangle}{\langle P|P\rangle} + \frac{\langle\, H_g \,\rangle}{\langle P|P\rangle} + \frac{\langle\, H_a \,\rangle}{\langle P|P\rangle}$$

$$\mathcal{H}_q \equiv \left(\tilde{T}_q^{00}\right)_R = \mathcal{H}'_q + c_{qm}\,\mathcal{H}'_m + c_{qa}\,\mathcal{H}'_a$$

$$\mathcal{H}_m \equiv \left(\check{T}_q^{00}\right)_R = (1 - c_{qm})\,\mathcal{H}'_m + c_{ma}\,\mathcal{H}'_a$$

$$\mathcal{H}_g \equiv \left(\tilde{T}_g^{00}\right)_R = \mathcal{H}'_g + c_{ga}\,\mathcal{H}'_a$$

$$\mathcal{H}_a \equiv \left(\check{T}_g^{00}\right)_R = (1 - c_{qa} - c_{ma} - c_{ga})\,\mathcal{H}'_a$$

$$c_{qm} = \frac{(-3+y)\frac{\beta}{2g} + x(3-\gamma_m)}{(1+y)\frac{\beta}{2g} - x(1+\gamma_m)}$$

$$c_{qa} = -c_{ma} = \frac{4x}{(1+y)\frac{\beta}{2g} - x(1+\gamma_m)}$$

$$c_{ga} = 1 \quad c_{gm} = 0$$

$$\mathcal{H}_q = (\psi^\dagger\, i\boldsymbol{D}\cdot\boldsymbol{\alpha}\,\psi)_R$$

$$\mathcal{H}_m = (m\bar{\psi}\psi)_R$$

$$\mathcal{H}_g = \frac{1}{2}(E^2 + B^2)_R$$

$$\mathcal{H}_a = 0$$

Ji (QED)

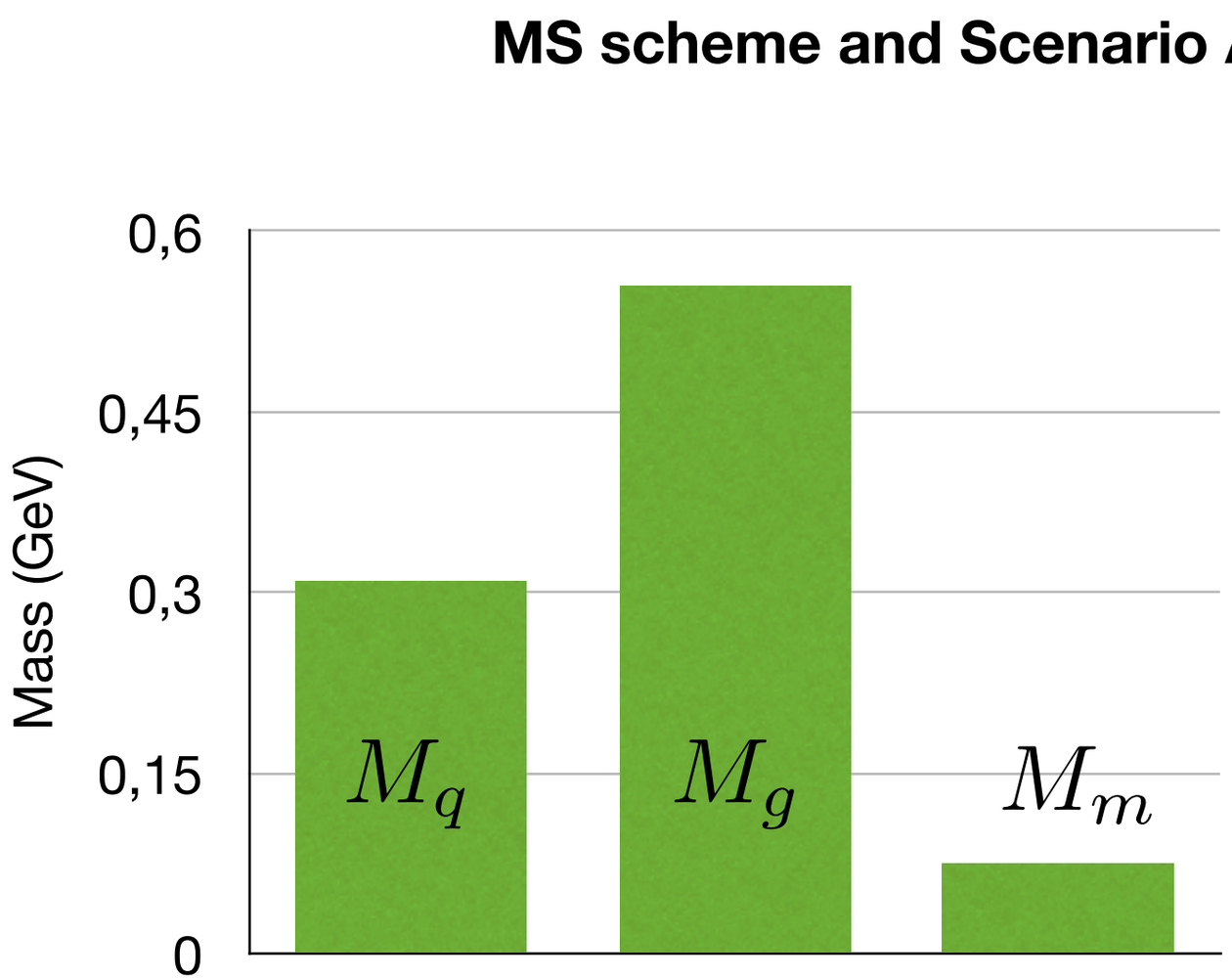
$$\frac{3}{2}m^2 a = \langle \widetilde{\mathcal{O}}^{00}_{3,R} \rangle_{\mathbf{P}=0}$$
$$2m^2 b = \langle (1+\gamma_m) m\bar{\psi}\psi \rangle_{\mathbf{P}=0}$$

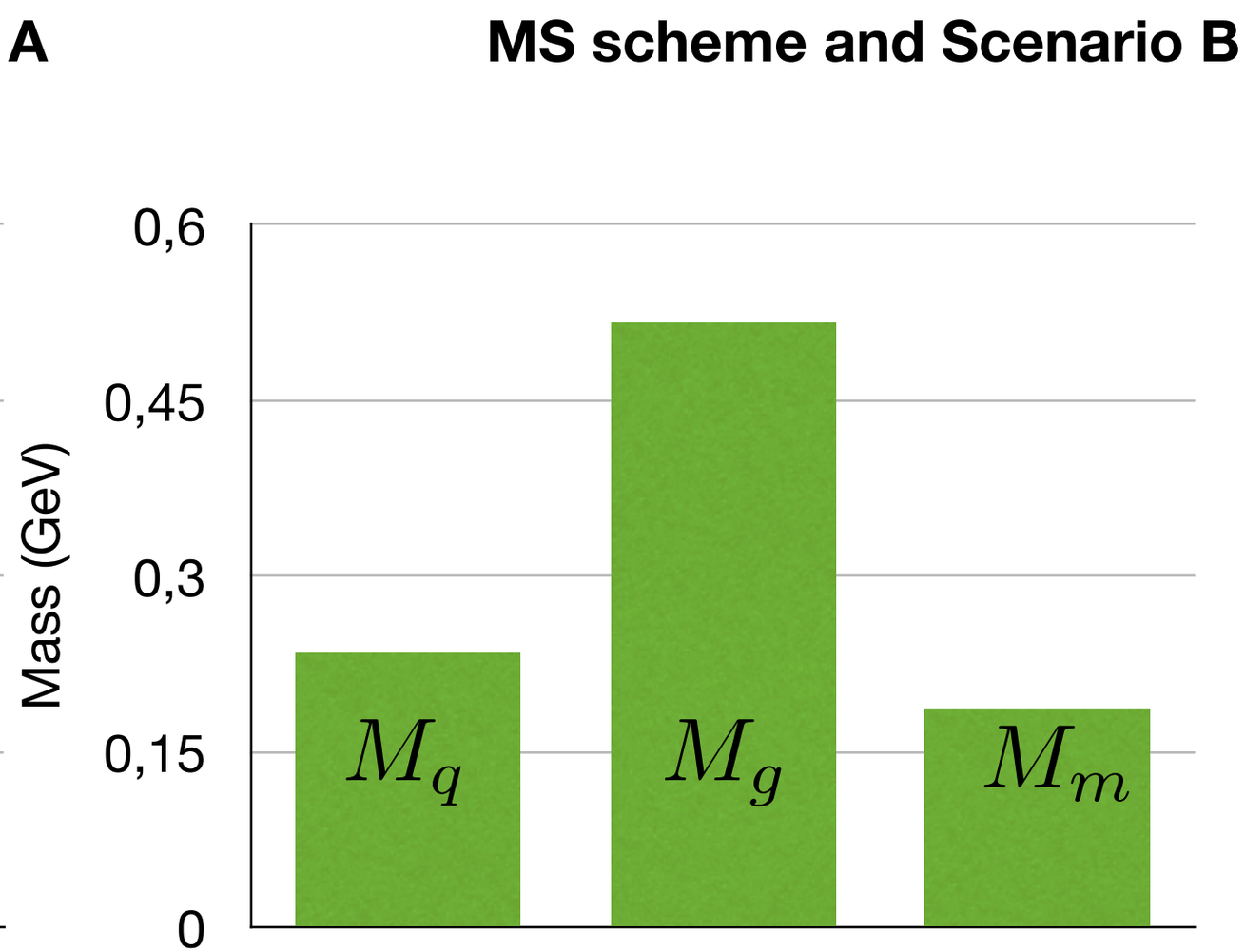
In the rest frame

$$m_e = \frac{3}{4}ma + \frac{m}{4}\left(x(1-b)\frac{2e}{\beta(e)} + b\frac{y-3}{1+\gamma_m} \right)$$
$$m_m = \frac{mb}{1+\gamma_m}$$
$$m_\gamma = \frac{3}{4}m(1-a) + \frac{m(1-b)}{4}\left(1 - x\frac{2e}{\beta(e)}\right) + mb\frac{\gamma_m - y}{4(1+\gamma_m)}$$
$$m_a = 0$$

$$\frac{m_e}{m} = \begin{cases} \frac{\alpha}{2\pi} - \frac{\alpha\mathcal{L}}{2\pi} & \overline{\mathrm{MS}} \\ \frac{19\alpha}{24\pi} - \frac{\alpha\mathcal{L}}{2\pi} & \mathrm{D}_1 \end{cases}, \qquad \frac{m_m}{m} = \begin{cases} 1 - \frac{3\alpha}{2\pi} & \overline{\mathrm{MS}} \\ 1 - \frac{3\alpha}{2\pi} & \mathrm{D}_1 \end{cases}$$
$$\frac{m_\gamma}{m} = \begin{cases} \frac{\alpha}{\pi} + \frac{\alpha\mathcal{L}}{2\pi} & \overline{\mathrm{MS}} \\ \frac{17\alpha}{24\pi} + \frac{\alpha\mathcal{L}}{2\pi} & \mathrm{D}_1 \end{cases}$$

Ji (QCD)





Rodini et al., 2004.03704 | Metz et. al, 2006.11171 | Ji, Phys.Rev.Lett. 74 (1995) 1071 | Ji, Phys.Rev. D52 (1995) 271

Lorcé (four terms, the revisited version)

Lorcé, Eur.Phys.J.C 78 (2018) 2, 120 | Rodini et al., 2004.03704 | Metz et. al, 2006.11171

$$M = \tilde{U}_q + \check{U}_q + \tilde{U}_g + \check{U}_g$$

$$\frac{\langle \bar{T}^{00}_{i,R} \rangle}{\langle P|P \rangle} = \frac{3}{4}(U_i + W_i) \qquad \frac{\langle \hat{T}^{00}_{i,R} \rangle}{\langle P|P \rangle} = \frac{1}{4}(U_i - 3W_i)$$

$$\tilde{U}_q = (3 + c_{qm})\,\frac{U_q}{4} + c_{qa}\,\frac{U_g}{4}$$

$$\check{U}_q = (1 - c_{qm})\,\frac{U_q}{4} + c_{ma}\,\frac{U_g}{4}$$

$$\tilde{U}_g = (3 + c_{ga})\,\frac{U_g}{4} = U_g$$

$$\check{U}_g = (1 - c_{qa} - c_{ma} - c_{ga})\,\frac{U_g}{4} = 0$$

Lorcé (QED)

Lorcé, Eur.Phys.J.C 78 (2018) 2, 120 | Rodini et al., 2004.03704

$$\frac{\tilde{U}_e}{m} = \begin{cases} \frac{\alpha}{3\pi} & \overline{\mathrm{MS}} \\ \frac{3\alpha}{2\pi} & \mathrm{D}_1 \end{cases}$$

$$\frac{\check{U}_e}{m} = \begin{cases} 1 - \frac{4\alpha}{3\pi} - \frac{\alpha\mathcal{L}}{2\pi} & \overline{\mathrm{MS}} \\ 1 - \frac{53\alpha}{24\pi} - \frac{\alpha\mathcal{L}}{2\pi} & \mathrm{D}_1 \end{cases}$$

$$\frac{\tilde{U}_\gamma}{m} = \begin{cases} \frac{\alpha}{\pi} + \frac{\alpha\mathcal{L}}{2\pi} & \overline{\mathrm{MS}} \\ \frac{17\alpha}{24\pi} + \frac{\alpha\mathcal{L}}{2\pi} & \mathrm{D}_1 \end{cases}$$

$$c_{em} = \frac{(3-y)\frac{\beta(e)}{2e} - x(3-\gamma_m)}{-(1+y)\frac{\beta(e)}{2e} + x(1+\gamma_m)}$$

$$c_{ea} = \frac{4x}{(1+y)\frac{\beta(e)}{2e} - x(1+\gamma_m)}$$

$$c_{\gamma m} = 0$$

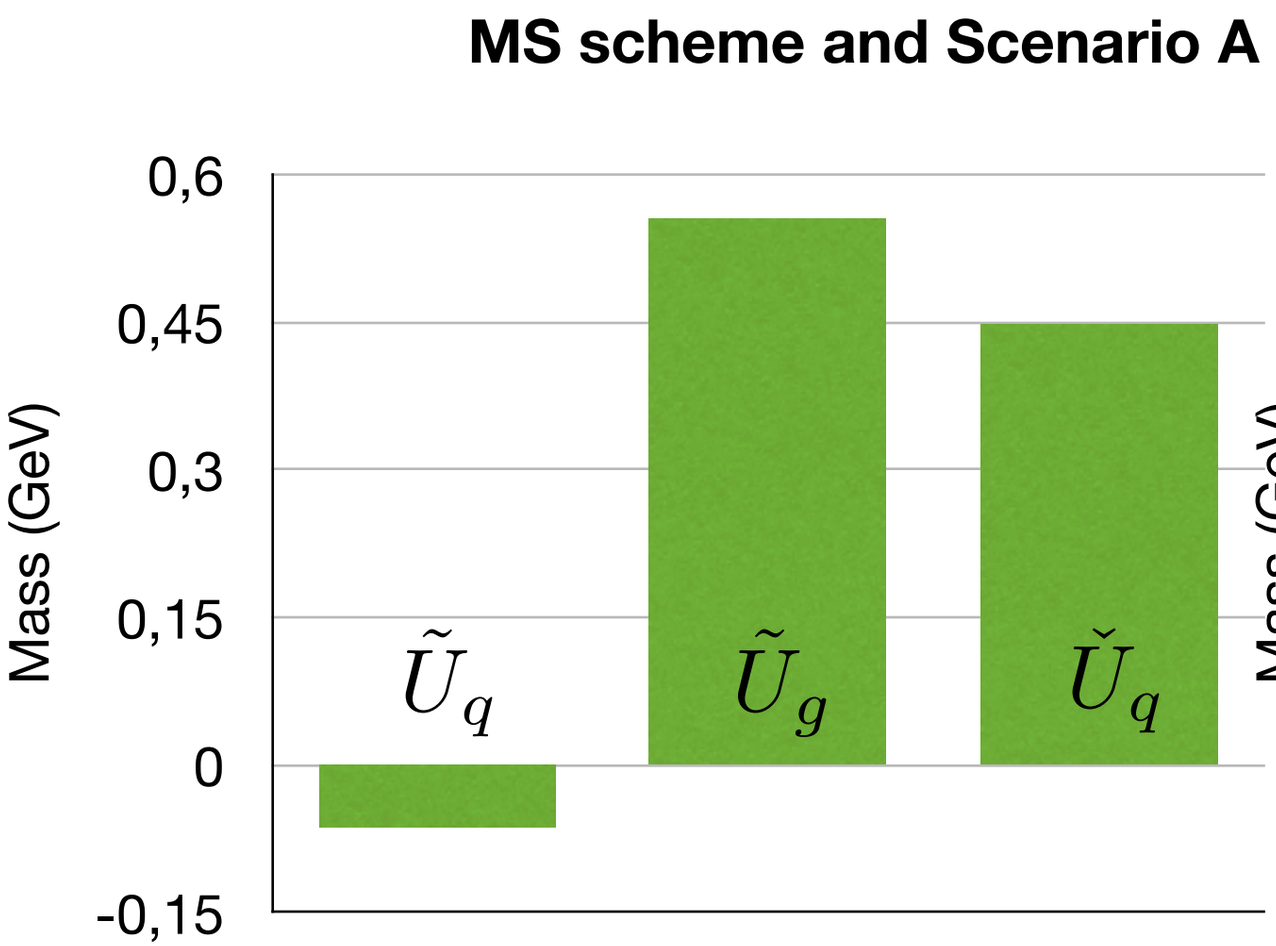
$$c_{\gamma a} = 1$$

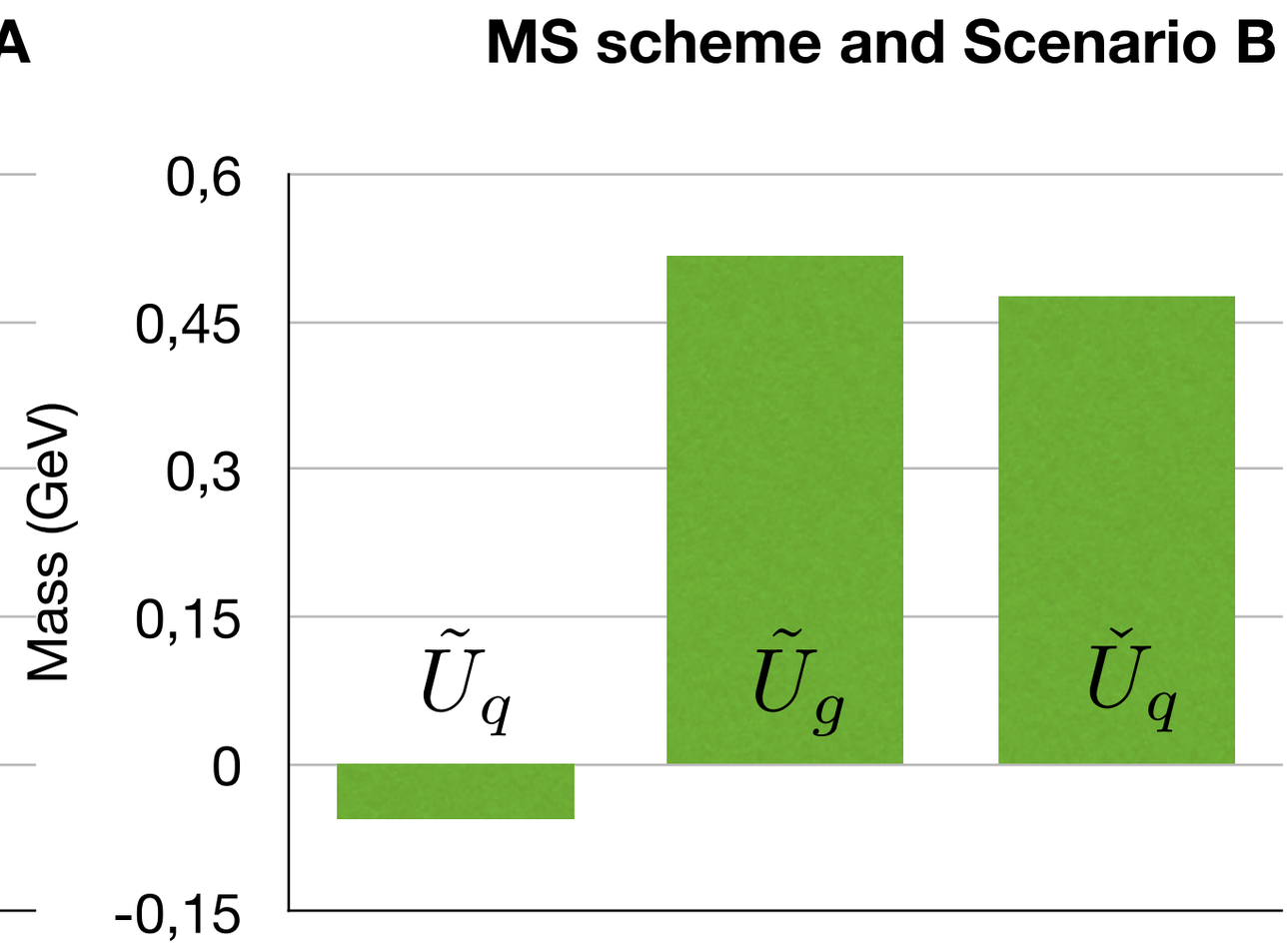
$$c_{ma} = -c_{ea}$$

Lorcé (four terms)

Lorcé, Eur.Phys.J.C 78 (2018) 2, 120

Metz et. al, 2006.11171





Going to a moving frame

$$\frac{\int d^3x \ \langle \widetilde{\mathcal{O}}^{00}_{3,R} \rangle}{\langle e(P)|e(P)\rangle}\Big|_{\mathbf{P}=0} = \frac{3}{4}am \to a\left(E - \frac{m^2}{4E}\right)$$

$$\frac{\int d^3x \ \langle (1+\gamma_m) m\bar{\psi}\psi \rangle}{\langle e(P)|e(P)\rangle}\Big|_{\mathbf{P}=0} = mb \to b\frac{m^2}{E}$$

Ji

$$\frac{m_e}{E} = \begin{cases} \frac{E^2-m^2}{E^2} + \frac{\alpha}{\pi}\left(-\frac{17}{18} + \frac{13m^2}{9E^2} - \frac{2\mathcal{L}}{3} + \frac{\mathcal{L}m^2}{6E^2}\right) & \overline{\mathrm{MS}} \\ \frac{E^2-m^2}{E^2} + \frac{\alpha}{\pi}\left(-\frac{17}{18} + \frac{125m^2}{72E^2} - \frac{2\mathcal{L}}{3} + \frac{\mathcal{L}m^2}{6E^2}\right) & \mathrm{D}_1 \end{cases}$$

$$\frac{m_m}{E} = \begin{cases} \frac{m^2}{E^2}\left(1 - \frac{3\alpha}{2\pi}\right) & \overline{\mathrm{MS}} \\ \frac{m^2}{E^2}\left(1 - \frac{3\alpha}{2\pi}\right) & \mathrm{D}_1 \end{cases}$$

$$\frac{m_\gamma}{E} = \begin{cases} \frac{\alpha}{\pi}\left(\frac{17}{18} + \frac{m^2}{18E^2} + \frac{2\mathcal{L}}{3} - \frac{\mathcal{L}m^2}{6E^2}\right) & \overline{\mathrm{MS}} \\ \frac{\alpha}{\pi}\left(\frac{17}{18} - \frac{17m^2}{72E^2} + \frac{2\mathcal{L}}{3} - \frac{\mathcal{L}m^2}{6E^2}\right) & \mathrm{D}_1 \end{cases}$$

Lorcé

$$\frac{\tilde{U}_e}{E} = \begin{cases} \frac{\alpha}{3\pi} & \overline{\mathrm{MS}} \\ \frac{3\alpha}{2\pi} & \mathrm{D}_1 \end{cases}$$

$$\frac{\check{U}_e}{E} = \begin{cases} 1 + \frac{\alpha}{\pi}\left(-\frac{23}{18} - \frac{m^2}{18E^2} - \frac{2\mathcal{L}}{3} + \frac{\mathcal{L}m^2}{6E^2}\right) & \overline{\mathrm{MS}} \\ 1 + \frac{\alpha}{\pi}\left(-\frac{22}{9} + \frac{17m^2}{72E^2} - \frac{2\mathcal{L}}{3} + \frac{\mathcal{L}m^2}{6E^2}\right) & \mathrm{D}_1 \end{cases}$$

$$\frac{\tilde{U}_\gamma}{E} = \begin{cases} \frac{\alpha}{\pi}\left(\frac{17}{18} + \frac{m^2}{18E^2} + \frac{2\mathcal{L}}{3} - \frac{\mathcal{L}m^2}{6E^2}\right) & \overline{\mathrm{MS}} \\ \frac{\alpha}{\pi}\left(\frac{17}{18} - \frac{17m^2}{72E^2} + \frac{2\mathcal{L}}{3} - \frac{\mathcal{L}m^2}{6E^2}\right) & \mathrm{D}_1 \end{cases}$$

How strong the scheme dependence is?

Metz et. al, 2006.11171

		MS	$\overline{\text{MS}}_1$	$\overline{\text{MS}}_2$	D1	D2
Scenario A	M_q	0.309 ± 0.044	0.194 ± 0.033	0.178 ± 0.032	0.362 ± 0.045	0.357 ± 0.051
	M_m	0.075 ± 0.080	0.075 ± 0.080	0.075 ± 0.080	0.075 ± 0.080	0.075 ± 0.080
	M_g	0.555 ± 0.036	0.669 ± 0.047	0.686 ± 0.048	0.502 ± 0.035	0.507 ± 0.029
Scenario B	M_q	0.234 ± 0.006	0.135 ± 0.003	0.120 ± 0.003	0.286 ± 0.006	0.272 ± 0.008
	M_m	0.187 ± 0.023	0.187 ± 0.023	0.187 ± 0.023	0.187 ± 0.023	0.187 ± 0.023
	M_g	0.517 ± 0.017	0.617 ± 0.020	0.631 ± 0.020	0.465 ± 0.017	0.479 ± 0.015

Conclusions

Renormalization of multiple local operators

Different decompositions → **Two-term decompositions**

Different decompositions → **Four-term decompositions**

Summary

Form factors as fundamental building blocks for all the decompositions.

Scheme dependence prevents a unique interpretation of the terms of the decompositions

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