Valance structure of pion from lattice QCD

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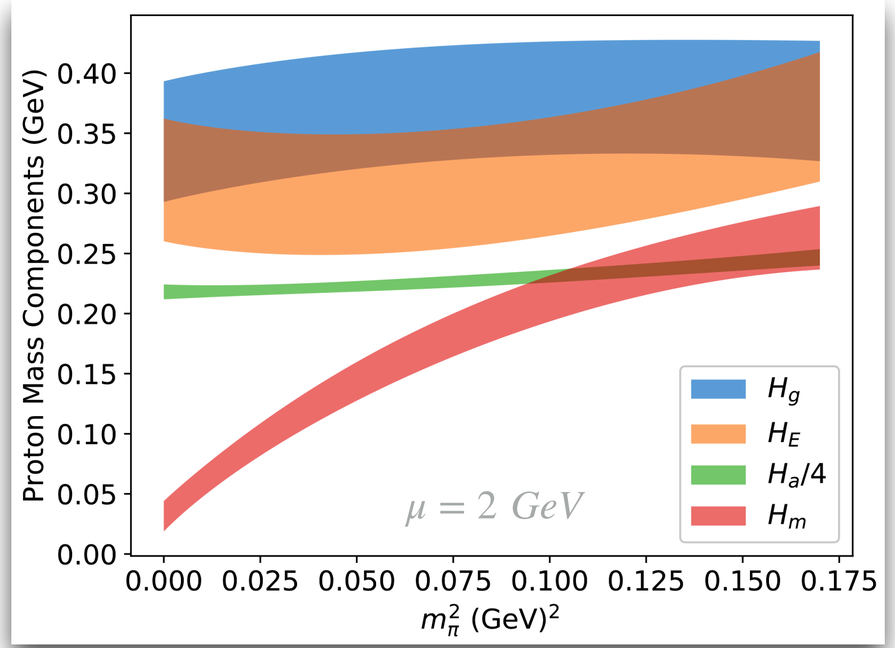
based on: 1905.06349 2007.06590

- significant contributions of gluons (trace-anomaly) to proton mass, even in the chiral limit

$$M_p = -\langle T_{\mu\mu}\rangle = \langle H_m\rangle + \langle H_a\rangle$$

H_a : QCD trace anomaly
H_m : $m_q \times$ quark condensate

X.-D. Ji: Phys. Rev. Lett. 74, 1071 (1995)



$$M_p = -\langle T_{44}\rangle = \langle H_m\rangle + \langle H_E\rangle + \langle H_g\rangle + \langle H_a\rangle/4$$

χQCD: Phys. Rev. Lett. 121, 212001 (2018)

H_g : gluon field energy; H_E : quark energy

- significant contributions of gluons (trace-anomaly) to proton mass, even in the chiral limit

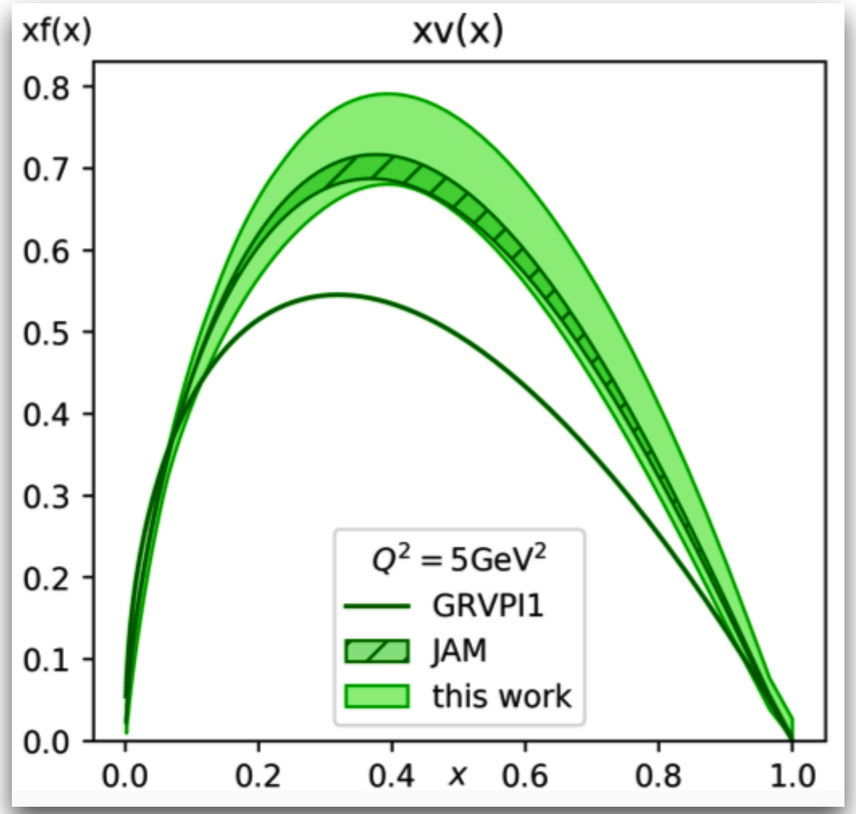
- contribution of gluons (trace-anomaly) to pion mass ?

$$m_q \to 0 \ : -\langle T_{\mu\mu} \rangle = \langle H_m \rangle + \langle H_a \rangle = M_\pi \to 0$$

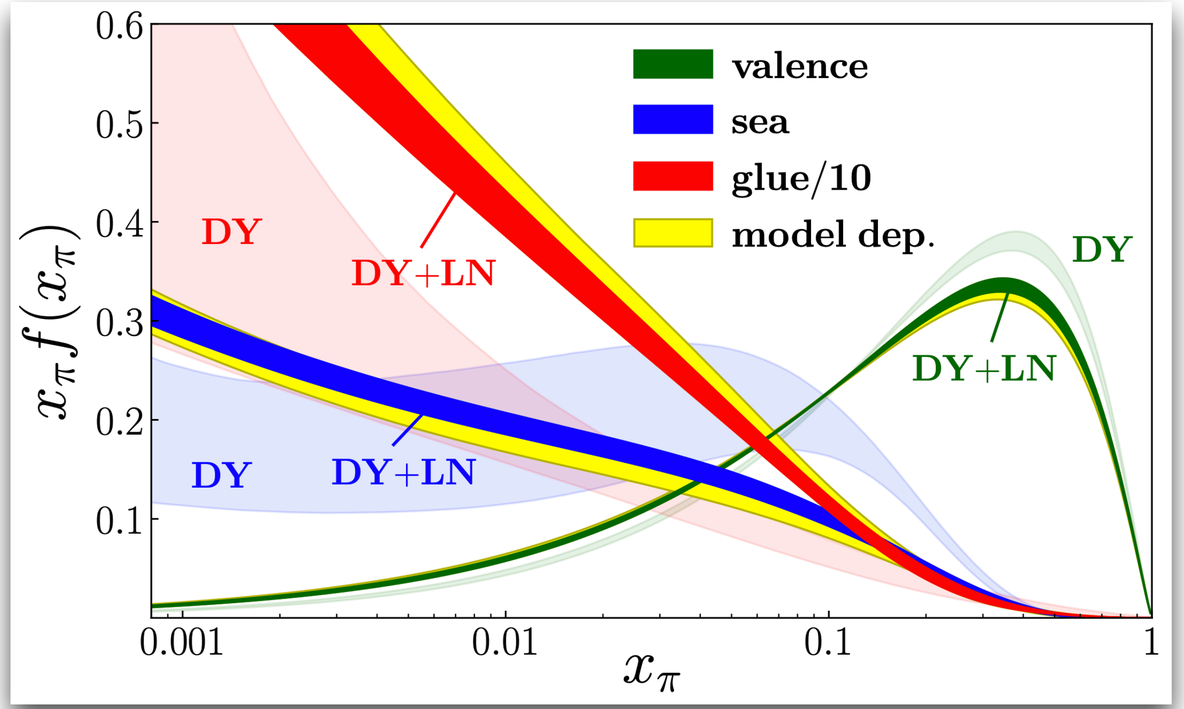
a puzzling dichotomy !!

understanding the role of gluons in hadron mass generation will remain incomplete without explaining the *absence* of pions mass

where are the gluons? … valance quark PDF of pion



xFitter: Phys. Rev. D102, 014040 (2020)



JAM: Phys. Rev. Lett. 121, 152001 (2018)

where are the gluons? … valance quark PDF of pion

different for different models/fits

$$q_v^\pi(x) = Ax^a(1-x)^b\left[1 + \text{sub leading}\right]$$

data: Fermilab E615 Phys. Rev. D39, 92 (1989)

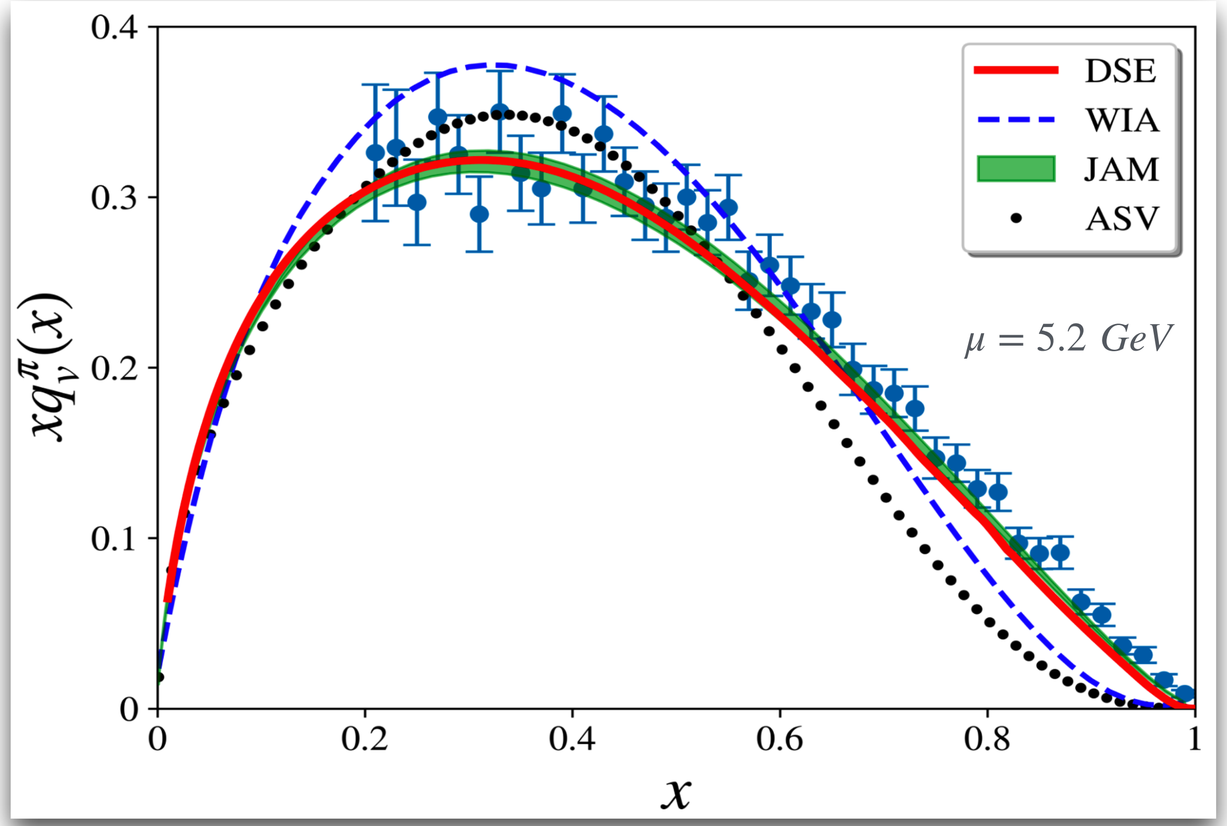
pQCD: $b \simeq 2$ dominated by one-gluon exchange
Berger, Brodsky: Phys. Rev. Lett. 42, 940

JAM: $b \simeq 1$ Phys. Rev. Lett. 121, 152001 (2018)

ASV: $b \simeq 1.3$ NLL threshold resummation
Phys. Rev. Lett. 105, 252003 (2010)

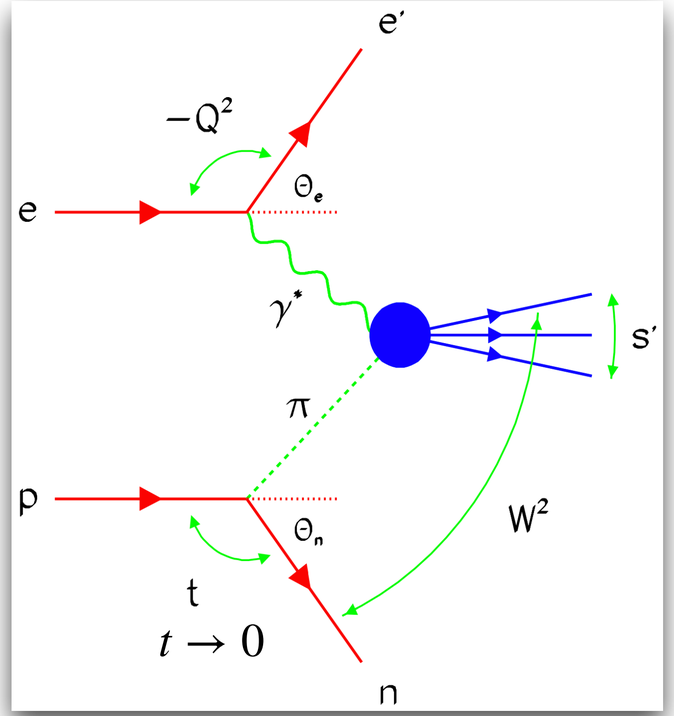
WIA: $b \simeq 2$ DSE, rainbow-ladder truncation
Nguyen et al: Phys. Rev. C83, 062201 (2011)

DSE: $b \simeq 2$ another DSE w/ different quark dressing, different contributions of gluons
Phys. Rev. Lett. 124, 042002 (2020)



Bednar, Cloet, Tandy: Phys. Rev. Lett. 124, 042002 (2020)

Sullivan process w/ off-shell pion



whitepaper: Eur. Phys. J. A55,190 (2019)

Pion and Kaon Structure at the Electron-Ion Collider

abstract

1 GeV mass-scale that characterizes atomic nuclei appear; why does it have the observed value; and, enigmatically, why are the composite Nambu-Goldstone (NG) bosons in quantum chromodynamics (QCD) abnormally light in comparison? In this perspective, we provide an analysis of the mass budget of the pion and proton in QCD; discuss the special role of the kaon, which lies near the boundary between dominance of strong and Higgs mass-generation mechanisms; and explain the need for a coherent effort in QCD phenomenology and continuum calculations, in exa-scale computing as provided by lattice QCD, and in experiments to make progress in understanding the origins of hadron masses and the distribution of that mass within them. We compare the unique capabilities foreseen at the electron-ion collider (EIC) with those at the hadron-electron ring accelerator (HERA), the

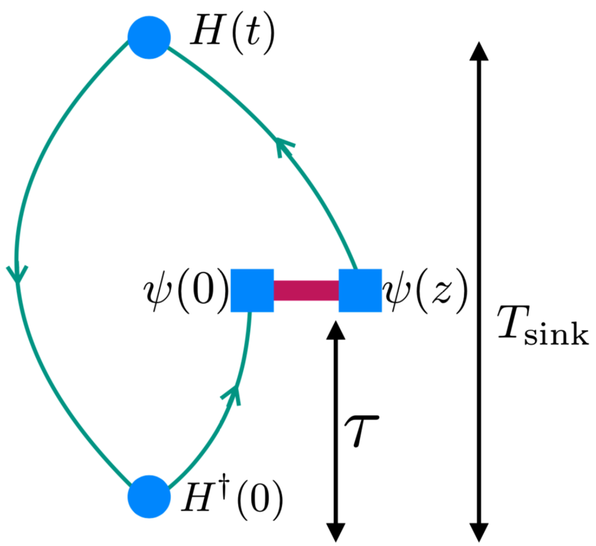
journey from Euclidean time to light-cone …

- bare matrix element, extract ground-state aptitude

$$Q_b(z, p_z) = \langle H(p_z,0) \,|\, \overline{\psi}(0,\tau)\gamma_0 W(0,z)\psi(z,\tau) \,|\, H(p_z, T_{\text{sink}})\rangle$$

equal-time, non-local operator within boosted hadron

$W(0,z)$: Wilson line from *0* to *z*



- renormalized matrix element $Q(z, p_z, p^R) = Z(z, p^R) Q_b(z, p_z)$ non-perturbative RI-MOM

by computing the same matrix element for Landau-gauge fixed off-shell quarks with $p^2 > 0$

renormalization condition: $Z(z, p^R, a) Q_b(z, p = p^R, a) \equiv e^{izp_z^R}$ Izubuchi et. al.: Phys. Rev. D 98, 056004 (2018)

- renormalization group invariant ratio $\mathcal{M}(z, p_z, p_z^0) = \dfrac{Q_b(z, p_z)}{Q_b(z, p_z^0)}$

$p_z^0 = 0$ Radyushkin: Phys. Rev. D96, 034025 (2017)

$p_z^0 > 0$ BNL et. al.: Phys. Rev. D102, 074504 (2020)

- leading twist factorization in momentum space

X. Ji: Phys. Rev. Lett. 110, 262002 (2013)

$$q(x, p_z, p^R) = \frac{1}{4\pi}\int_0^\infty dz e^{-izp_z x} Q(z, p_z, p^R)$$

quasi-PDF: F. T at fixed p_z

$$q(x, p_z, p^R) = \int \frac{dy}{|y|} f(x,\mu)\, C\left(\frac{x}{y}, \frac{\mu}{yp_z}, \frac{p_\perp^R}{p_z^R}, \frac{yp_z}{p_z^R}\right) + \mathcal{O}\left(\frac{m_h^2}{xp_z^2}, \frac{\Lambda_{QCD}^2}{xp_z^2}\right)$$

MS-bar PDF

coefficient function

target mass, higher twist

NLO: Stewart, Zhao: Phys. Rev. D 97, 054512 (2018)
NNLO: Chen, Wang, Zhu: Phys. Rev. D102, 011503 (2020)

- leading twist factorization in coordinate space

  pseudo-PDF, short distance expansion

Radyushkin: Phys. Rev. D96, 034025 (2017)
Phys. Lett. B781, 433 (2018)

Karpie, Orginos, Zafeiropoulos: JHEP 11, 178 (2018)

BNL et. al.: Phys. Rev. D102, 074504 (2020)

$$\mathcal{M}(z, p_z, p_z^0) = \frac{\sum_{n=0}^{\infty} \frac{(-izp_z)^n}{n!} c_n(z^2\mu^2) \langle x^n \rangle(\mu)}{\sum_{n=0}^{\infty} \frac{(-izp_z^0)^n}{n!} c_n(z^2\mu^2) \langle x^n \rangle(\mu)} + \mathcal{O}\left(z^2 m_h^2, z^2 \Lambda_{\mathrm{QCD}}\right)$$

coefficient function

PDF moments

target mass, higher twist

$$\langle x^n \rangle = \int_{-1}^{1} dx x^n f(x, \mu)$$

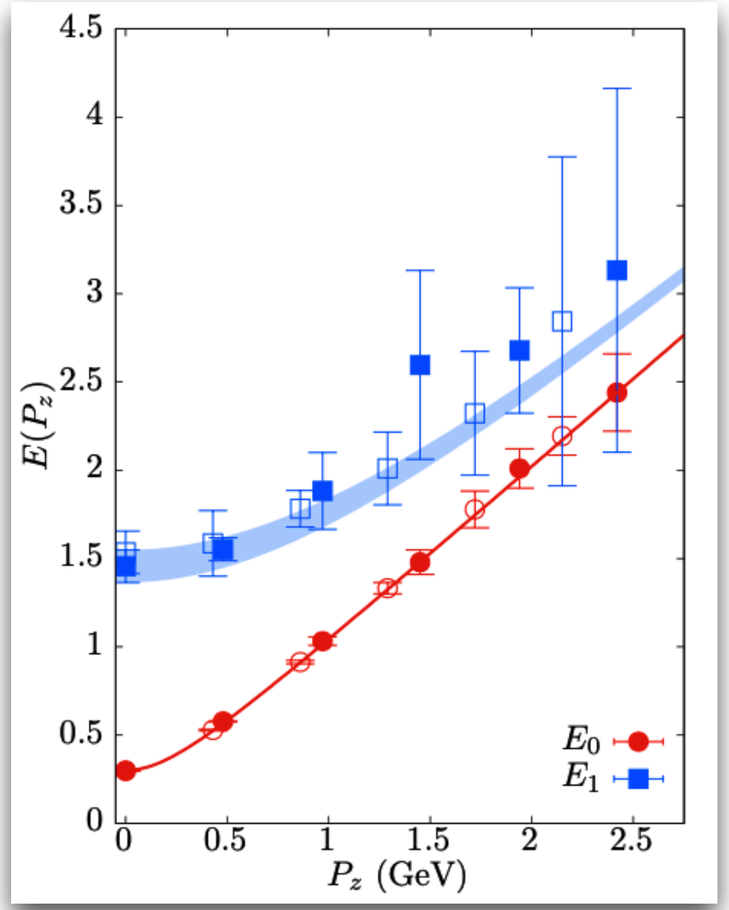
NLO: Izubuchi et. al.: Phys. Rev. D 98, 056004 (2018)
NNLO: Li, Ma, Qiu: 2006.12370

chasing after pions …

- Wilson-Clover valance quarks: $m_\pi^{val} \approx 300$ MeV
- boosts: $p_z \leq 2.4$ GeV
- lattice spacings: $a = 0.04,\ 0.06$ fm
- lattice sizes: $64^4,\ 48 \times 64^3;\ \ m_\pi^{val} L \approx 4$
- 2+1 flavor HISQ HotQCD gauge configurations:
  $m_\pi^{sea} \simeq 160$ MeV, $m_K^{sea} \simeq 500$ MeV

BNL et. al.: Phys.Rev. D100, 034516 (2019)

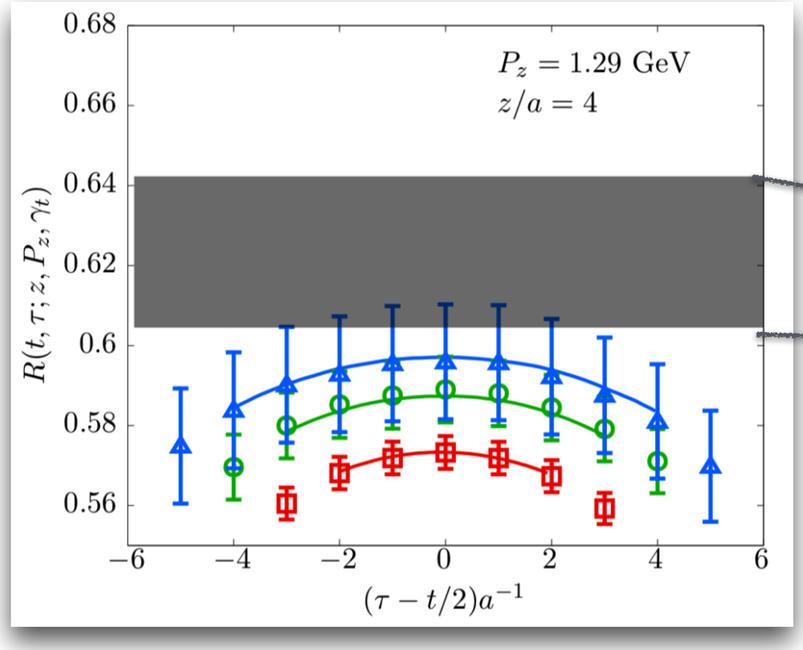
energy levels of boosted pion

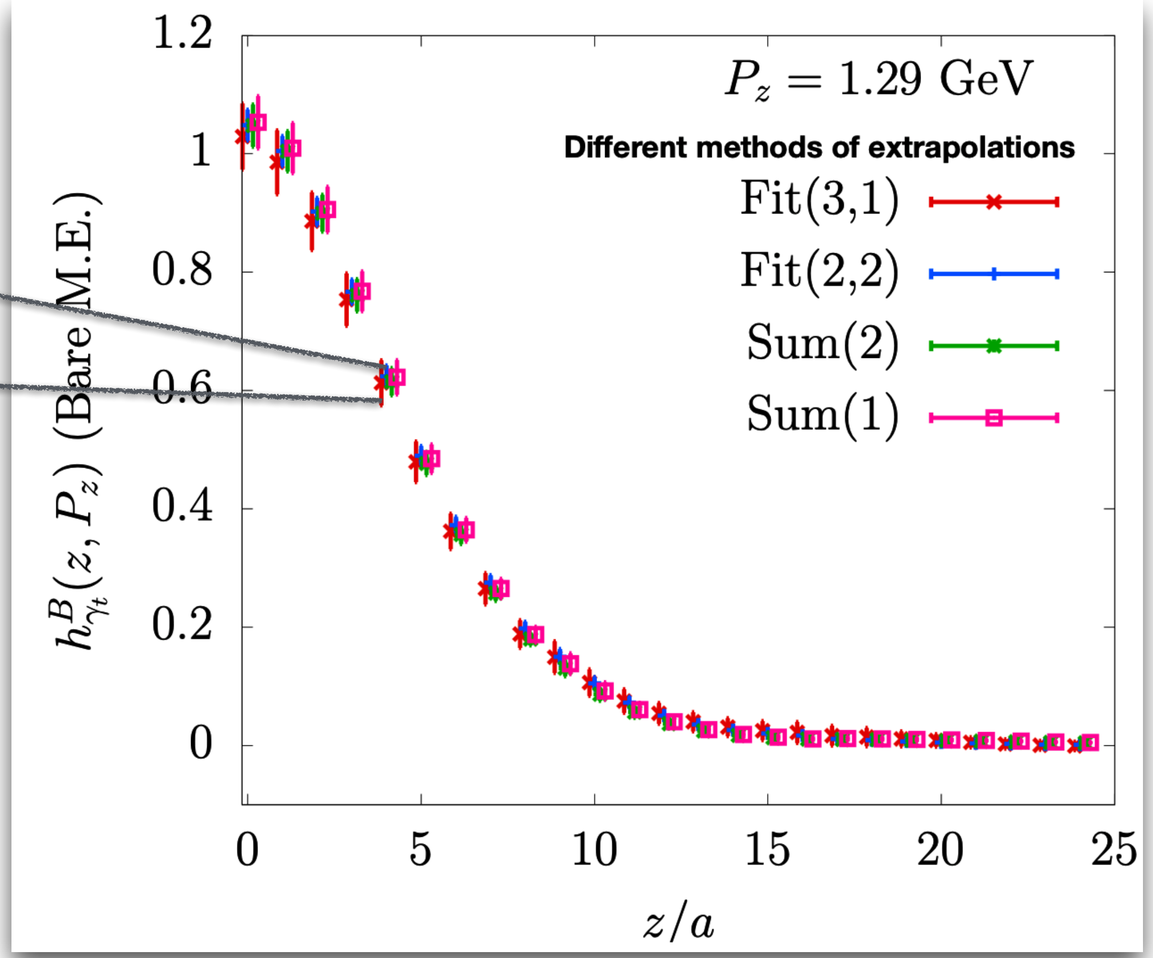


BNL et. al.: 2007.06590

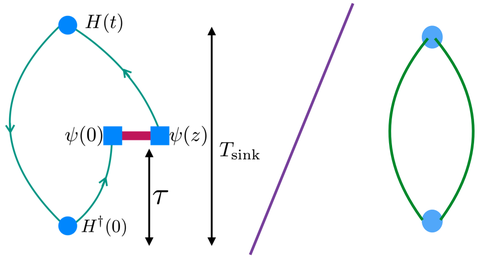
probing boosted pions

bare 3-pt operators in z-space

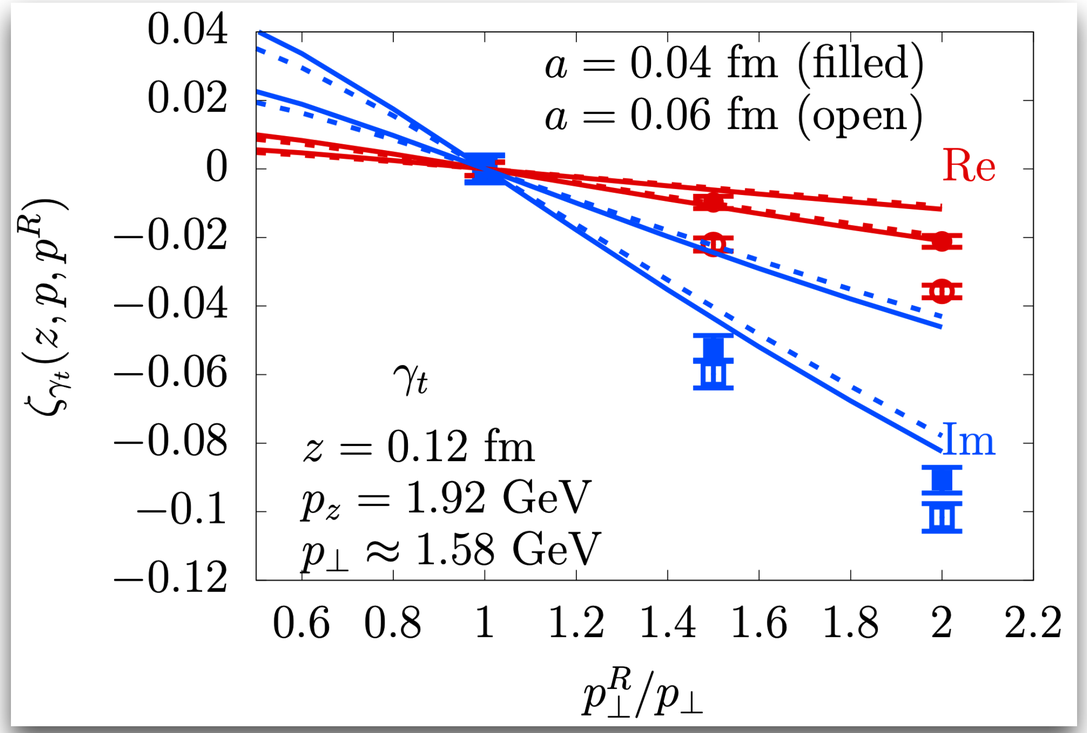




ratio of 3-pt to 2-pt corr. function



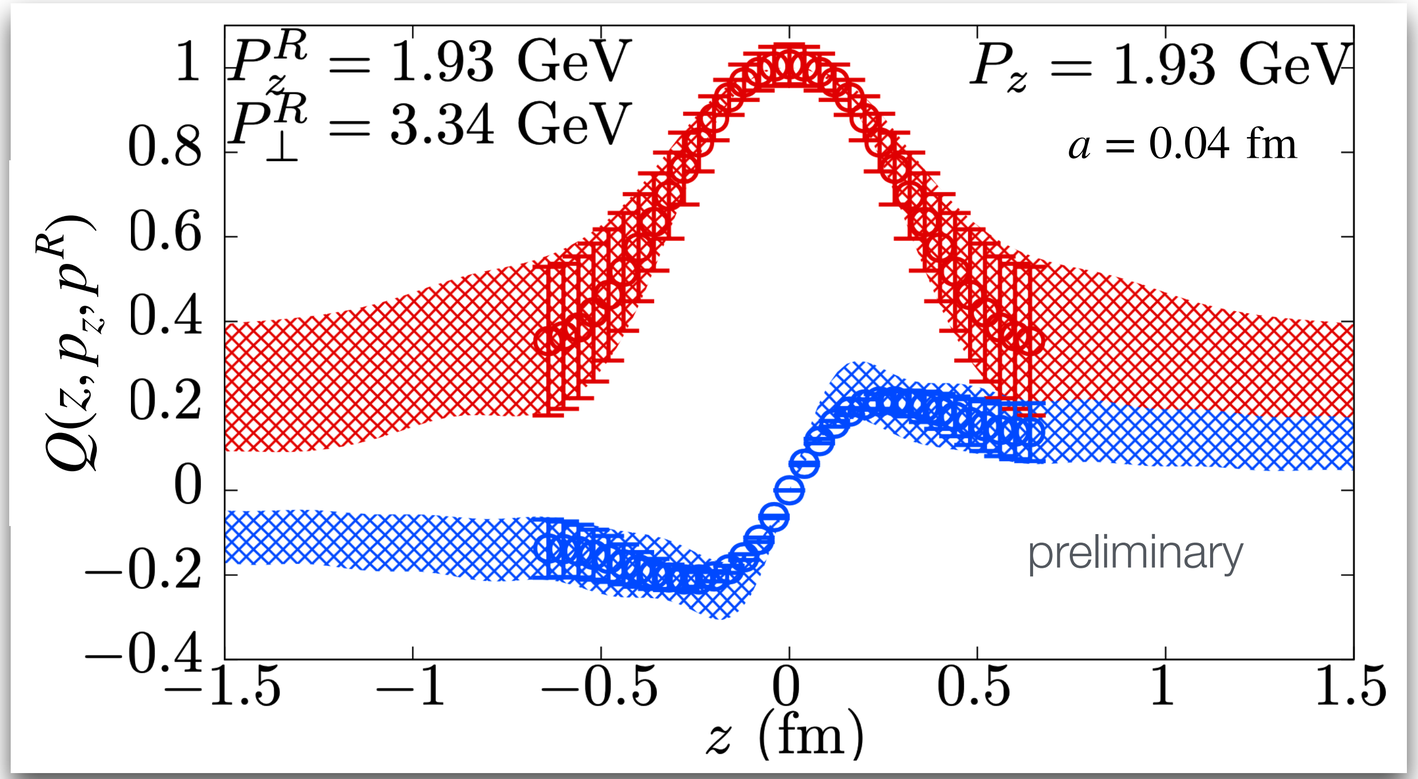
scale dependence of non-perturbative renormalization factor



real part, imaginary part; lines: 1-loop pQCD

$$\xi_{\gamma_t}(z, p, p^R) = Z_{\gamma_t}(z, p^R)/Z_{\gamma_t}(z, p) - 1 \simeq \partial \ln[Z_{\gamma_t}(z, p)]/\partial p\,|_{p^R}$$

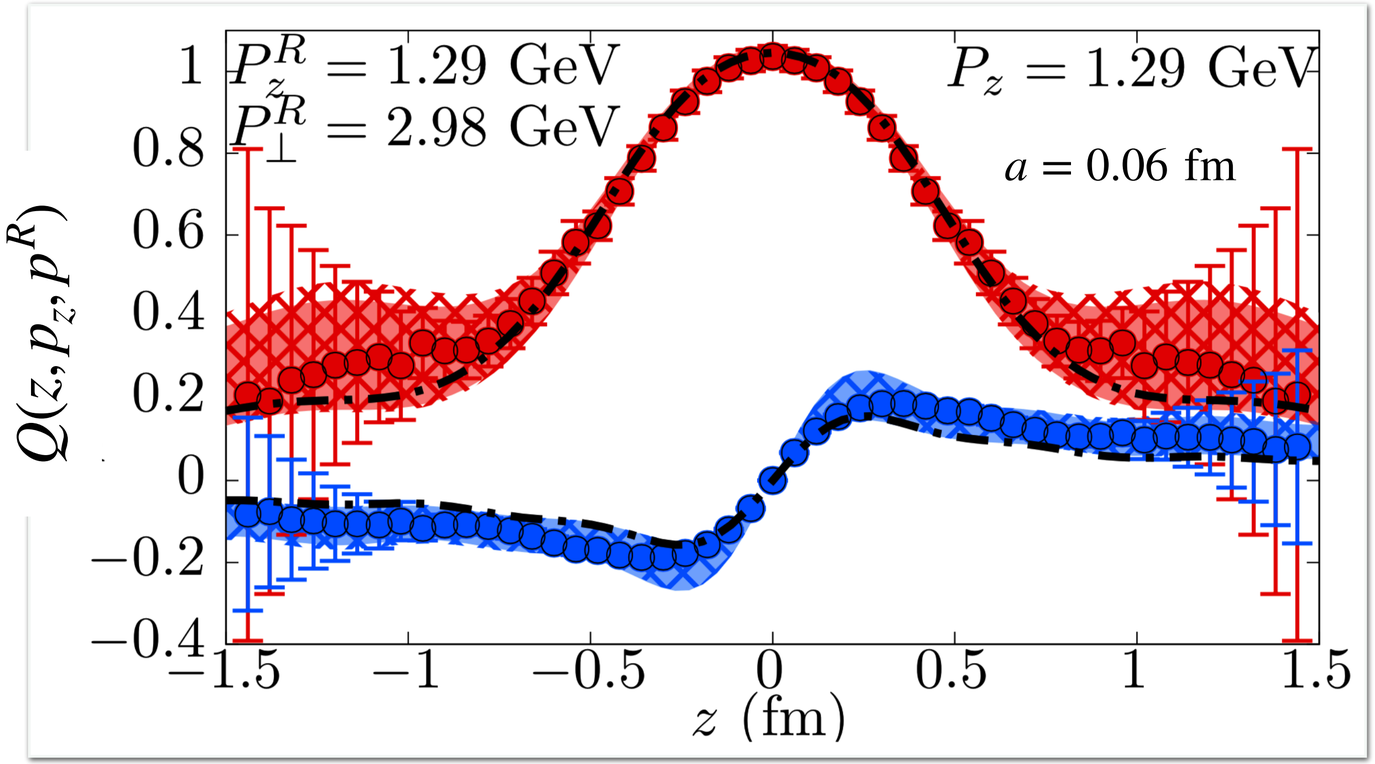
renormalized 3-pt operator in z-space



real part, imaginary part

Fourier transform w.r.t. z → RI-MOM qPDF $\xrightarrow{\text{pQCD matching}}$ $\overline{\text{MS}}$ PDF

comparison with JAM in z-space

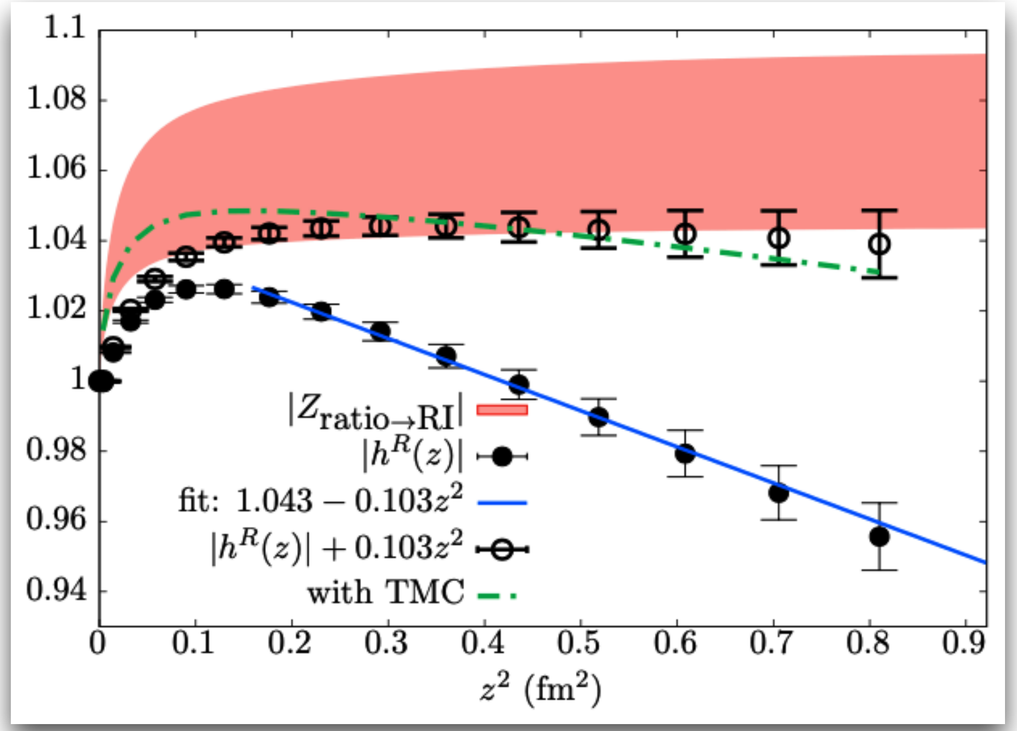


real part, imaginary part

dashed black lines: reconstructions from JAM global fits by reversing the procedure

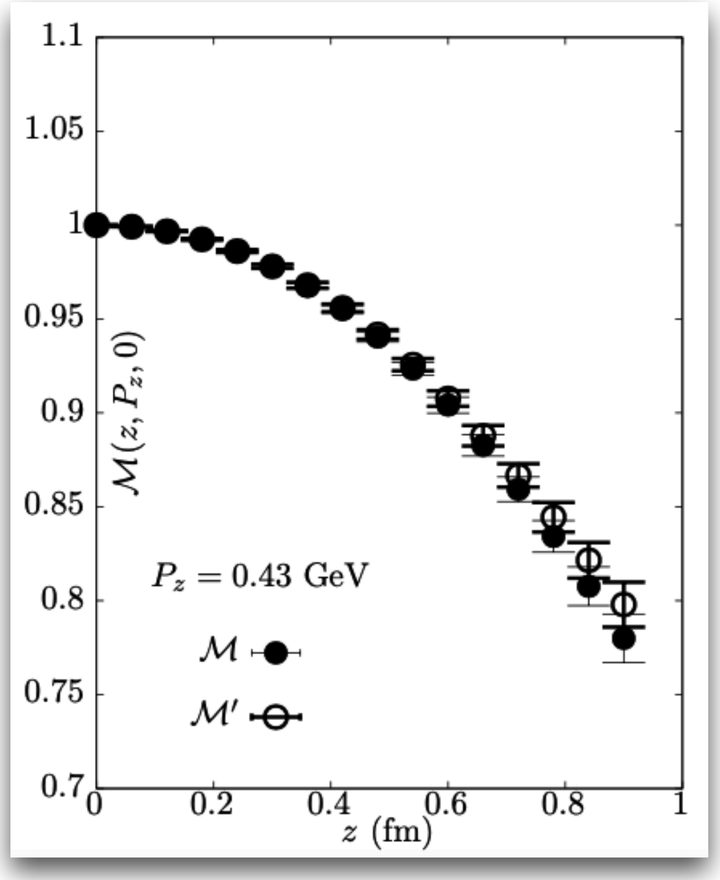
assessing higher twist corrections

RI-MOM renormalized



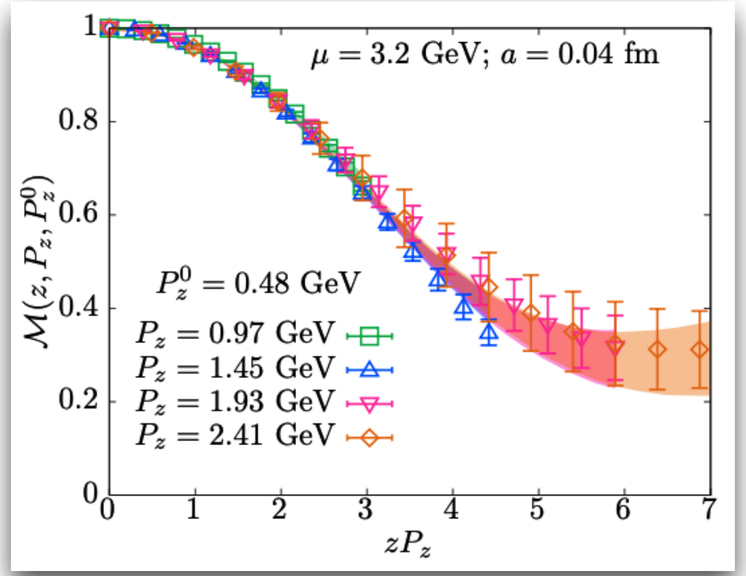
$$Q(z, p_z = 0; p_z^R = 1.29\ GeV, p_\perp^R = 2.98\ GeV)$$

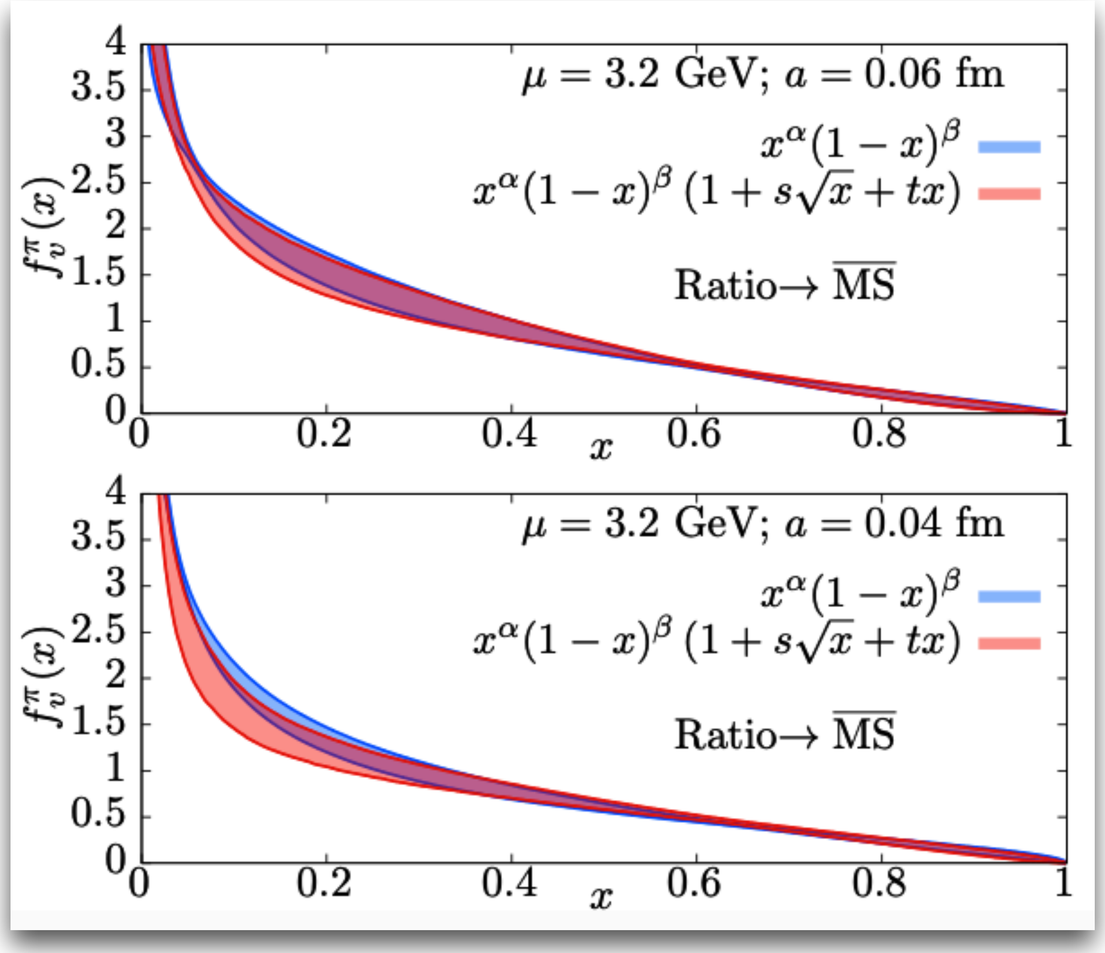
ratio scheme



$$\mathscr{M}(z, p_z = 0.43\ GeV, p_z^0 = 0)$$

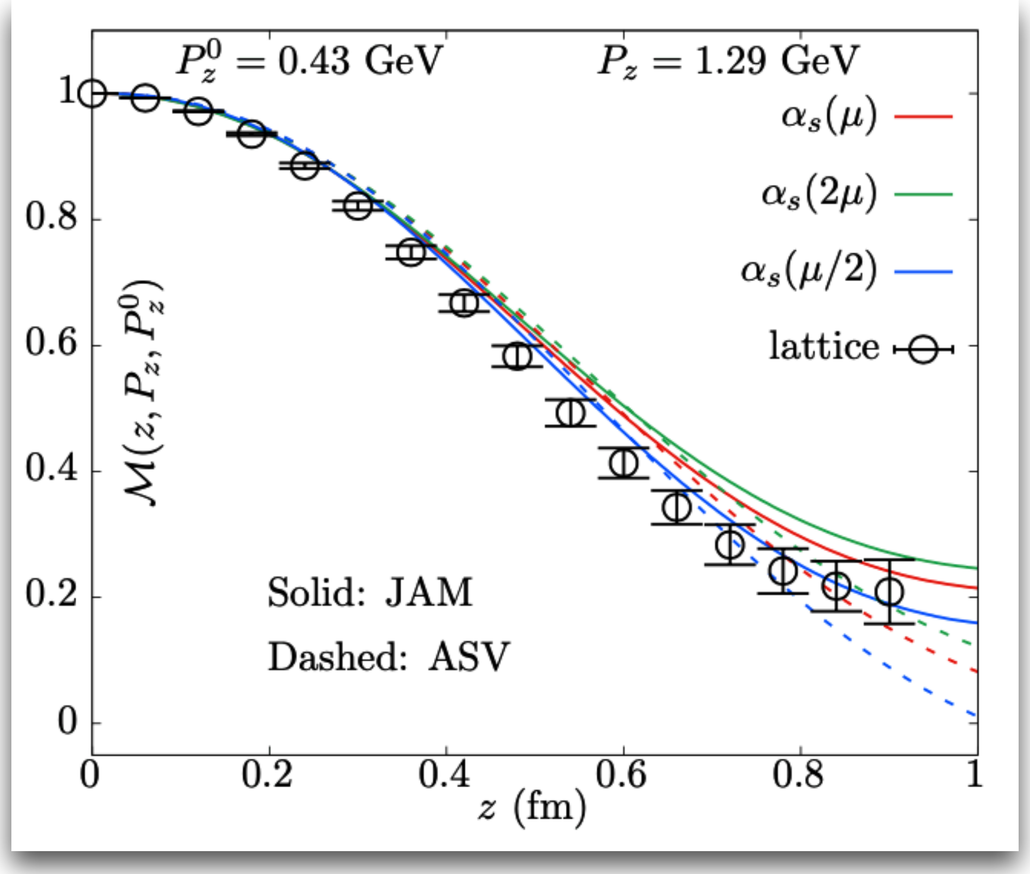
model fits …


$\mu = 3.2$ GeV; $a = 0.04$ fm
$P_z^0 = 0.48$ GeV
$P_z = 0.97$ GeV
$P_z = 1.45$ GeV
$P_z = 1.93$ GeV
$P_z = 2.41$ GeV
$\mathcal{M}(z, P_z, P_z^0)$
zP_z

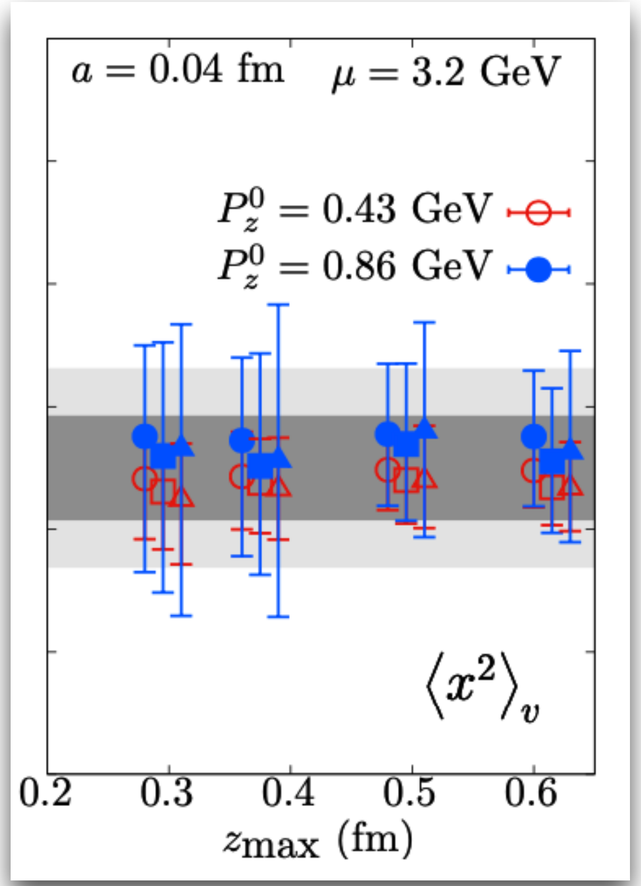

$\mu = 3.2$ GeV; $a = 0.06$ fm
$x^\alpha(1-x)^\beta$
$x^\alpha(1-x)^\beta\,(1+s\sqrt{x}+tx)$
Ratio→ $\overline{\text{MS}}$
$f_v^\pi(x)$
x
$\mu = 3.2$ GeV; $a = 0.04$ fm
$x^\alpha(1-x)^\beta$
$x^\alpha(1-x)^\beta\,(1+s\sqrt{x}+tx)$
Ratio→ $\overline{\text{MS}}$
$f_v^\pi(x)$
x

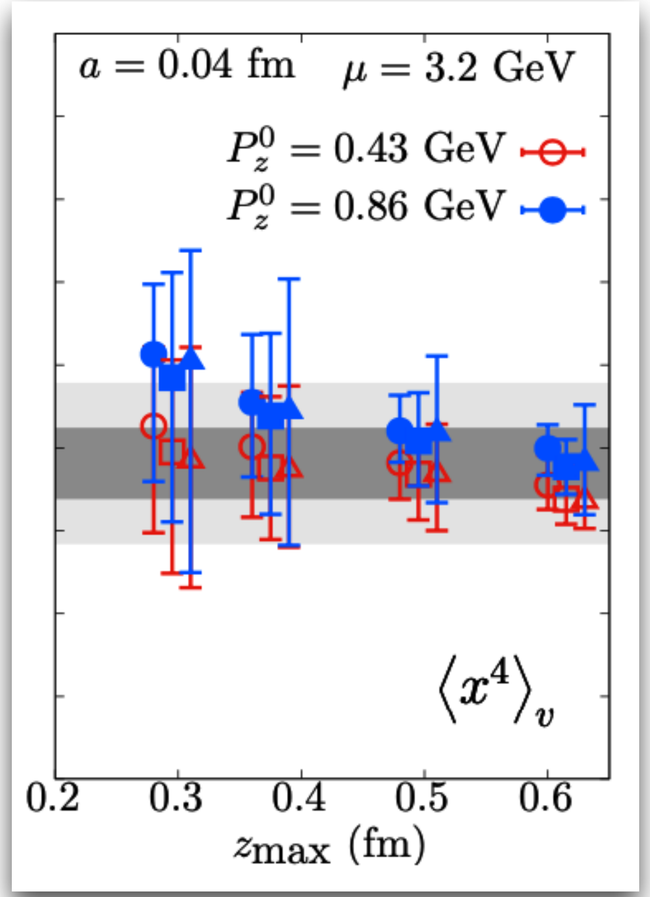
no fits …

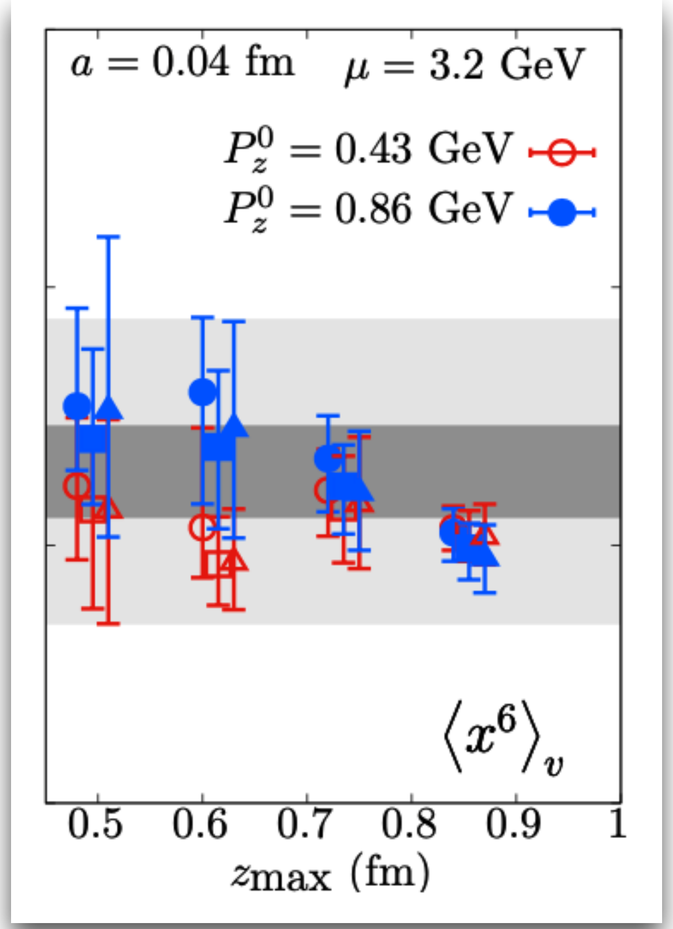
comparisons in z-space



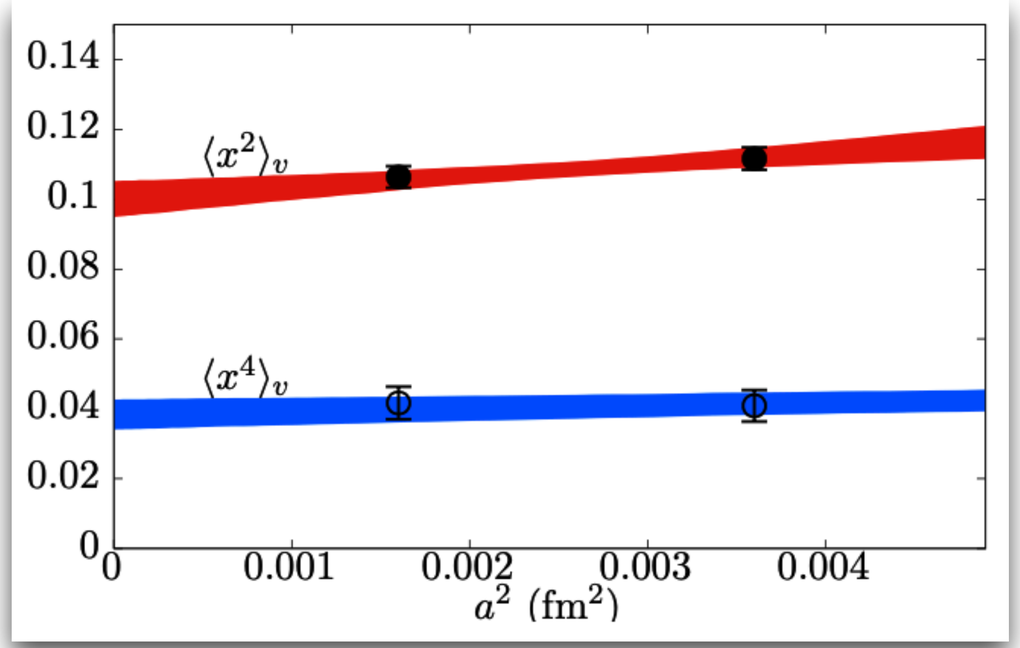
moments of PDF

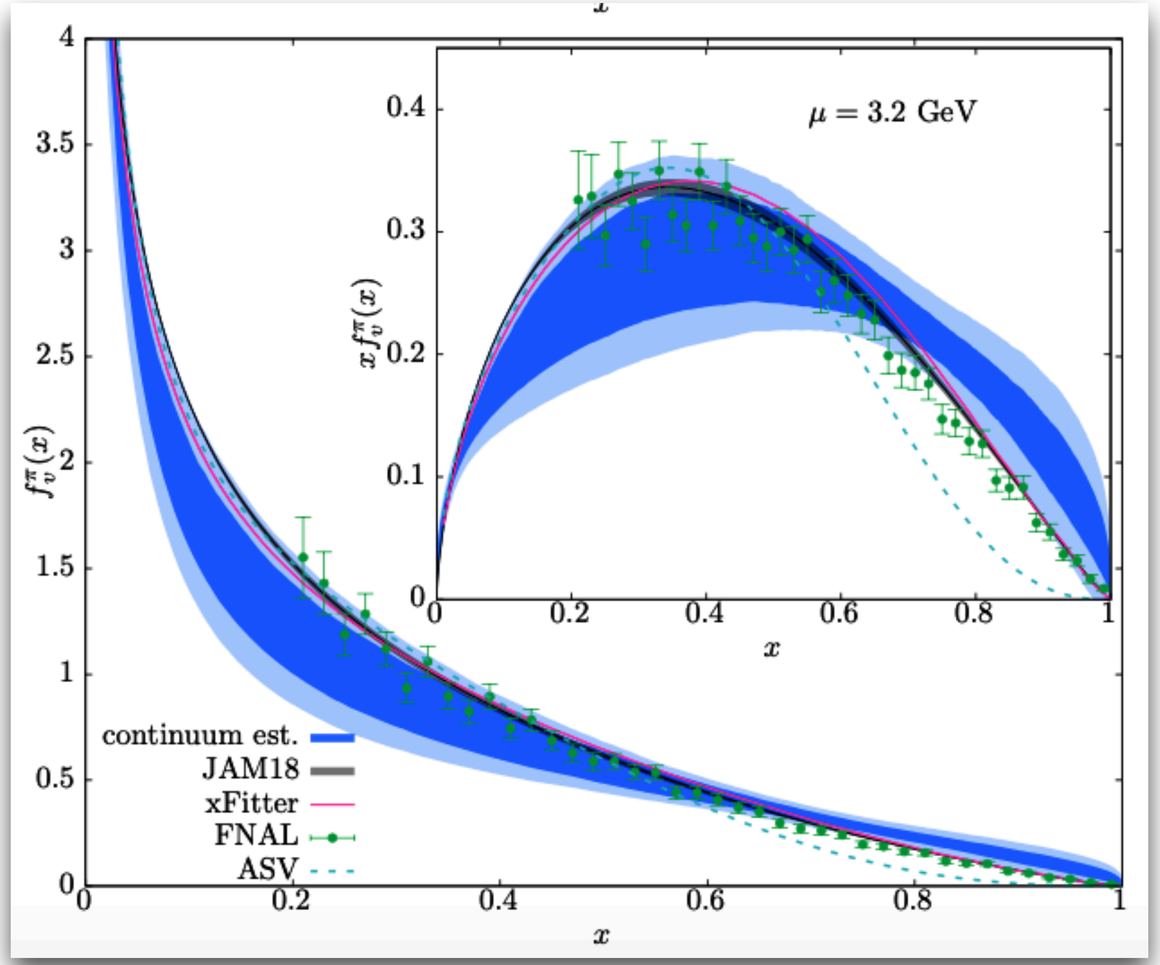






summary …

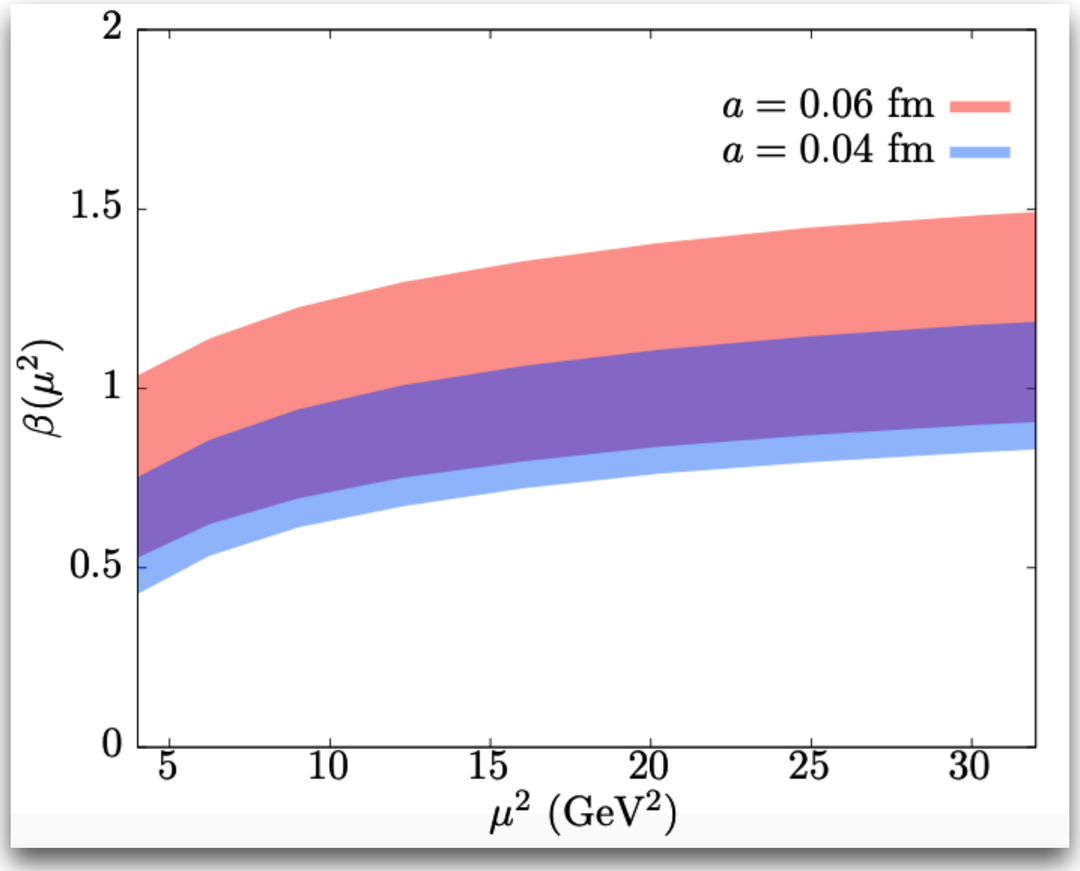

0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
$\langle x^2 \rangle_v$
$\langle x^4 \rangle_v$
0
0.001
0.002
0.003
0.004
a^2 (fm^2)


$\mu = 3.2$ GeV
$f_v^\pi(x)$
$x f_v^\pi(x)$
x
continuum est.
JAM18
xFitter
FNAL
ASV

back up …

Method	a (fm)	$\langle x \rangle_v$	$\langle x^2 \rangle_v$	$\langle x^3 \rangle_v$	$\langle x^4 \rangle_v$	α	β	s	t
(a) Model independent analysis	0.06		0.1088(48)(58)		0.0346(57)(73)				
	0.04		0.1050(43)(39)		0.0382(44)(54)				
	$a \to 0$		0.0993(71)(54)		0.0356(39)(60)				
(b) 2-parameter	0.06	0.2470(92)(52)	0.1122(54)(51)	0.0649(53)(62)	0.0423(52)(60)	-0.33(15)(11)	1.02(37)(32)		
	0.04	0.2289(96)(44)	0.1083(47)(34)	0.0652(49)(36)	0.0444(48)(34)	-0.51(10)(05)	0.66(24)(20)		
	$a \to 0$	0.216(19)(08)	0.1008(69)(43)	0.0604(39)(46)	0.0408(37)(44)	-0.55(15)(08)	0.66(34)(22)		
(c) 4-parameter	0.06	0.2457(92)(61)	0.1121(54)(50)	0.0649(53)(62)	0.0420(51)(59)	-0.40(16)(14)	1.11(41)(34)	-0.14(16)(20)	1.0(1.0)(1.2)
	0.04	0.2253(98)(45)	0.1080(46)(34)	0.0647(47)(38)	0.0436(43)(38)	-0.61(13)(06)	0.86(22)(25)	-0.20(24)(19)	2.5(1.9)(2.5)
	$a \to 0$	0.213(19)(08)	0.1009(68)(42)	0.0607(40)(47)	0.0410(40)(47)	-0.61(16)(08)	0.77(26)(30)	-0.19(27)(17)	1.5(2.0)(1.7)
(d) large-n asymptotics	0.06		0.1093(48)(53)		0.0365(44)(58)		1.40(25)(30)		
	0.04		0.1050(49)(37)		0.0392(38)(43)		1.12(24)(20)		
	$a \to 0$		0.0996(71)(61)		0.0386(56)(58)		1.15(23)(22)		
(e) Effective β	0.06						1.73(39)(37)		
	0.04						1.53(21)(25)		
	$a \to 0$						1.55(34)(27)		

TABLE III. Summary of results from analyses presented this paper at $\mu = 3.2$ GeV. Each row is a type of analysis, namely — (a) the model-independent estimates of the moments (b,c) fits to 2- and 4-parameter PDF ansatz, (d) a semi-model independent analysis based on modeling $\langle x^n \rangle_v$ by the asymptotic formula for $n \geq 4$, and (e) the estimate of exponent β from $\beta_{\text{eff}}(n = 4)$. The columns are the outcomes; namely, the value of the first four valence moments $\langle x^n \rangle_v$, the parameters α, β, s, t in the PDF ansatz $f_v^\pi(x) \sim x^\alpha (1-x)^\beta (1 + s\sqrt{x} + tx)$. For each analysis, values from two different lattice spacings a are given, and also our continuum expectations, denoted by $a \to 0$, based on a^2 extrapolation are also given. For these estimates, we used the ratio scheme with $n_z^0 = 1, 2$.



scale dependence of β