



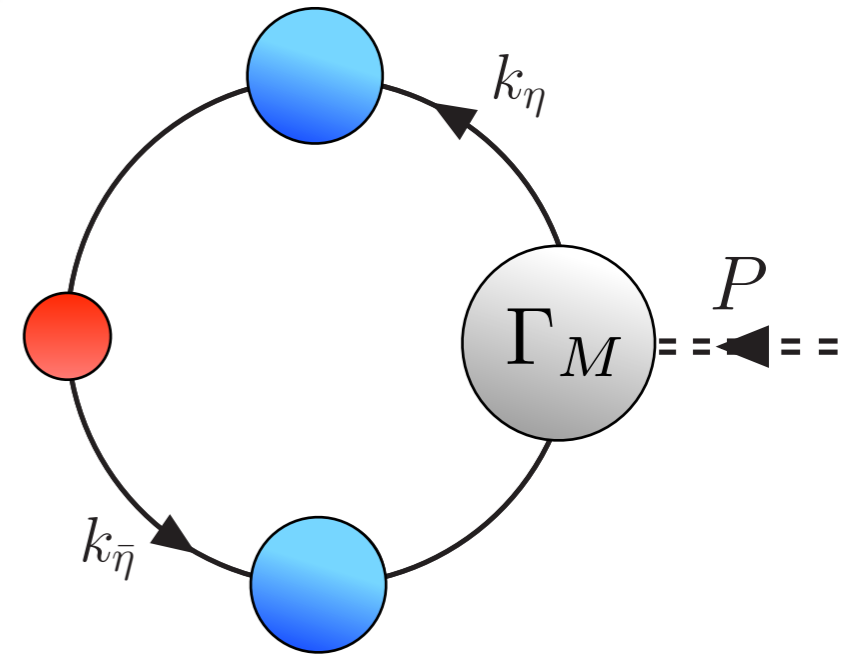
Universidad de Sucre

INCLUYENTE, INTEGRADA Y PARTICIPATIVA



$$\phi_M(x, \mu)$$

$$\delta(n \cdot k_\eta - xn \cdot P) \gamma_5 \gamma \cdot n$$



Meson Distribution Amplitudes from Bethe-Salpeter Wave Functions

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JLab Theory Seminar

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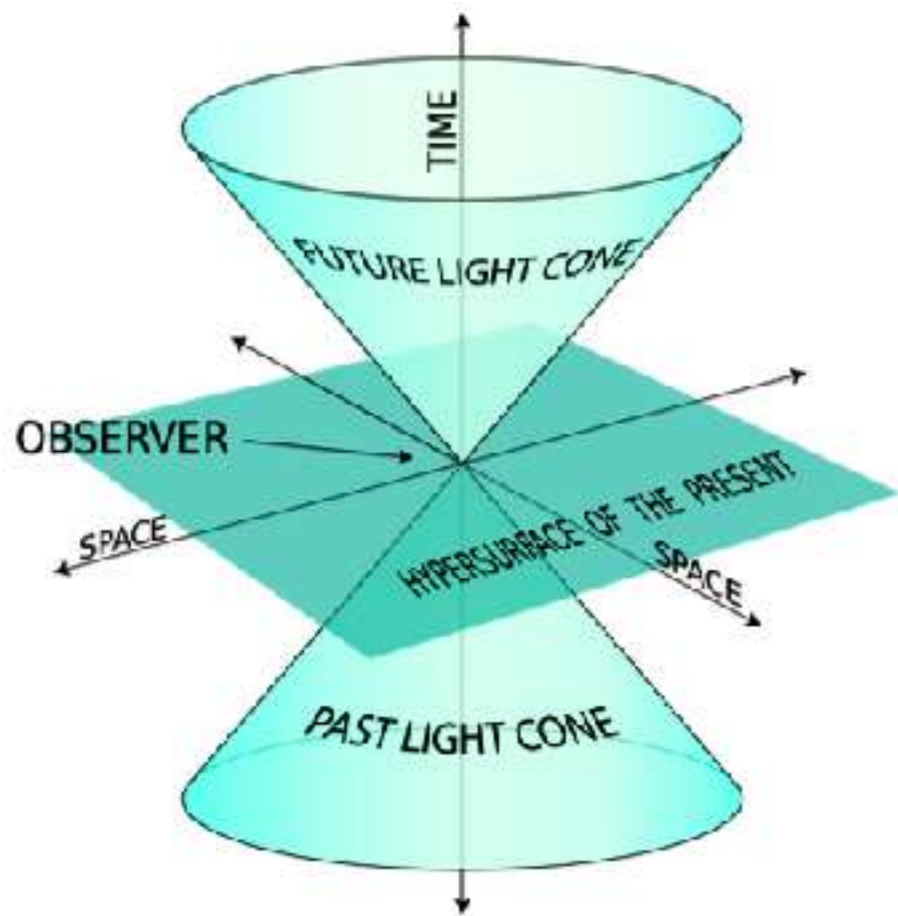
"WE COLLABORATE. I'M AN EXPERT, BUT NOT AN AUTHORITY, AND DR. GELPIS IS AN AUTHORITY, BUT NOT AN EXPERT."

Work in collaboration with

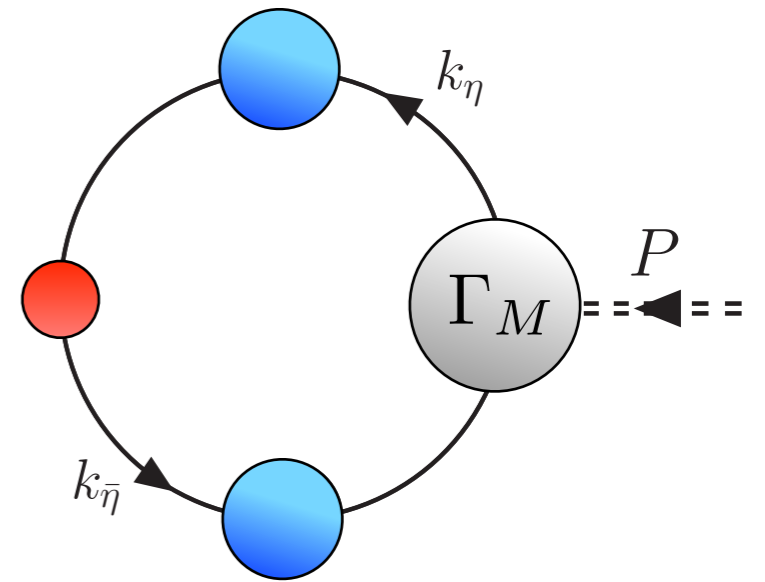
- Bruno El-Bennich, Universidade Cidade de São Paulo, Brazil.
- Roberto Correa Da Silveira, Universidade Cidade de São Paulo, Brazil.
- Eduardo Rojas, Universidad de Nariño, Colombia
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Contents

- **Light - cone distribution amplitudes**
- **Nonperturbative continuum tools for QCD**
- **Extracting Distribution Amplitudes**
- **Conclusions and work in progress**



$$\delta(n \cdot k_\eta - x n \cdot P) \gamma_5 \gamma \cdot n$$



Light-Cone Distribution Amplitudes

$$\phi_M(x, \mu)$$

Light-Cone Distribution Amplitudes

$$\phi_M(x, \mu)$$

● Hadronic light-cone distribution amplitudes (LCDAs) were introduced four decades ago in the context of QCD description of hard exclusive reactions.

► **QCD** Factorization involves matrix elements which are convolution integrals

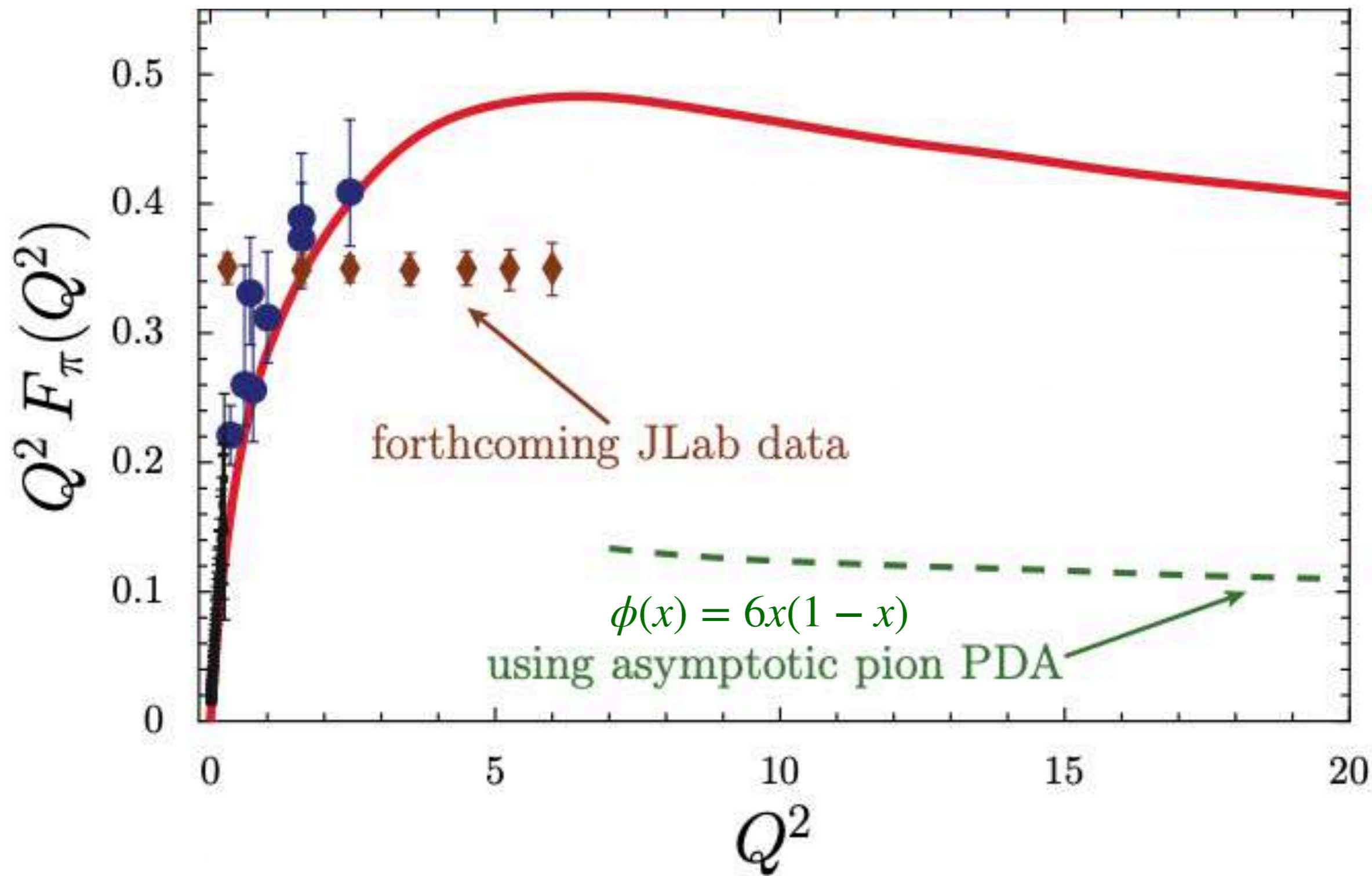
$$\langle \pi^+ \pi^- | (\bar{u}b)_{V-A} (\bar{u}d)_{V-A} | \bar{B}_d \rangle \rightarrow \int_0^1 d\xi du dv \phi_B(\xi) \phi_\pi(u) \phi_\pi(v) T(\xi, u, v; m_b)$$

$T(\xi, u, v; m_b)$ → **Hard - scattering kernels (perturbative)**

$\phi_B(u), \phi_\pi(v)$ → **Meson's Light-front distribution Amplitudes (LCDAS)**
(nonperturbative)

$$\phi_M(x) \neq \phi^{\text{asy}}(x) = 6x(1-x)$$

Light-Cone Distribution Amplitudes



Light-Cone Distribution Amplitudes

● The LCDAs are scale-dependent functions can be understood as the closest relative of quantum mechanical wave functions in quantum field theory

● $\phi_M(x, \mu)$ expresses the light-front fraction of the hadron's momentum carried by a valence quark. Allows for a probability interpretation of partons.

● x is the light-front momentum fraction: $x = k^+/P^+$ and μ the renormalization scale.

● **Leading-twist LCDA for pseudoscalar meson**

$$\begin{aligned} &\langle 0 | \bar{q}_f(y_2 n) \mathcal{W} [y_2 n, y_1 n] \gamma \cdot n \gamma_5 q_g(y_1 n) | M(P) \rangle \\ &= i f_M n \cdot P \int_0^1 dx e^{-i n \cdot P (y_1 x + y_2 \bar{x})} \phi_M(x, \mu) \end{aligned}$$

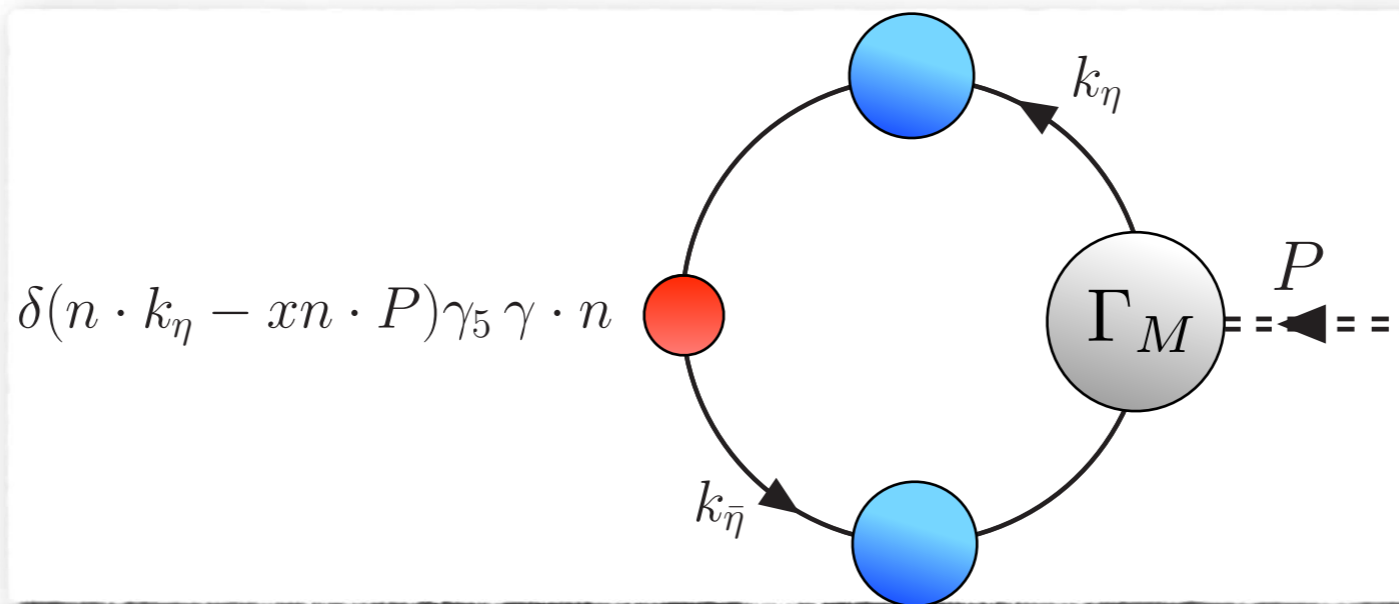
$$f_M = \text{weak decay constant}$$

$$n^2 = 0$$

$$n \cdot P = -m_M$$

Light-Cone Distribution Amplitudes

Light-cone projection of the Bethe-Salpeter wave function



$$f_M \phi_M(x, \mu) = \frac{\mathcal{Z}_2 N_c}{\sqrt{2}} \text{Tr}_D \int^\Lambda \frac{d^4 k}{(2\pi)^4} \delta(n \cdot k_\eta - x n \cdot P) \gamma_5 \gamma \cdot n \chi_M(k_\eta, k_{\bar{\eta}})$$

$\chi_M(k_\eta, k_{\bar{\eta}}) := S(k_\eta) \Gamma_M(k, P) S(k_{\bar{\eta}}) \rightarrow$ Meson's Bethe-Salpeter wave function (BSWF)

$\Gamma_M(k, P) \rightarrow$ Meson's Bethe-Salpeter Amplitude

$S(k_\eta) \rightarrow$ Quark propagator

L. Chang, I.C. Cloët, J.J. Cobos-Martínez, C.D. Roberts, S.M.Schmidt, P. C. Tandy, Phys. Rev. Lett. 110 (2013)
J. Segovia, L. Chang, I. C. Cloët, C. D. Roberts, S. M. Schmidt, Phys. Lett. B 731 (2014)

Light-Cone Distribution Amplitudes

- $\phi_M(x, \mu)$ is directly accessible from the **Light-Cone Wave Function (LCWF)** by integrating over the meson's transverse momentum \mathbf{k}_\perp^2 .

$$f_M \phi_M(x, \mu) = \frac{1}{(2\pi)^3} \int^{\mu^2} dk_\perp^2 \psi_M^{\uparrow\downarrow\pm\downarrow\uparrow}(x, \mathbf{k}_\perp^2)$$

where $\psi(x, k_\perp)$ is the Fourier transform of the positive-energy projection of the Bethe-Salpeter wave function evaluated at equal time in coordinate space.

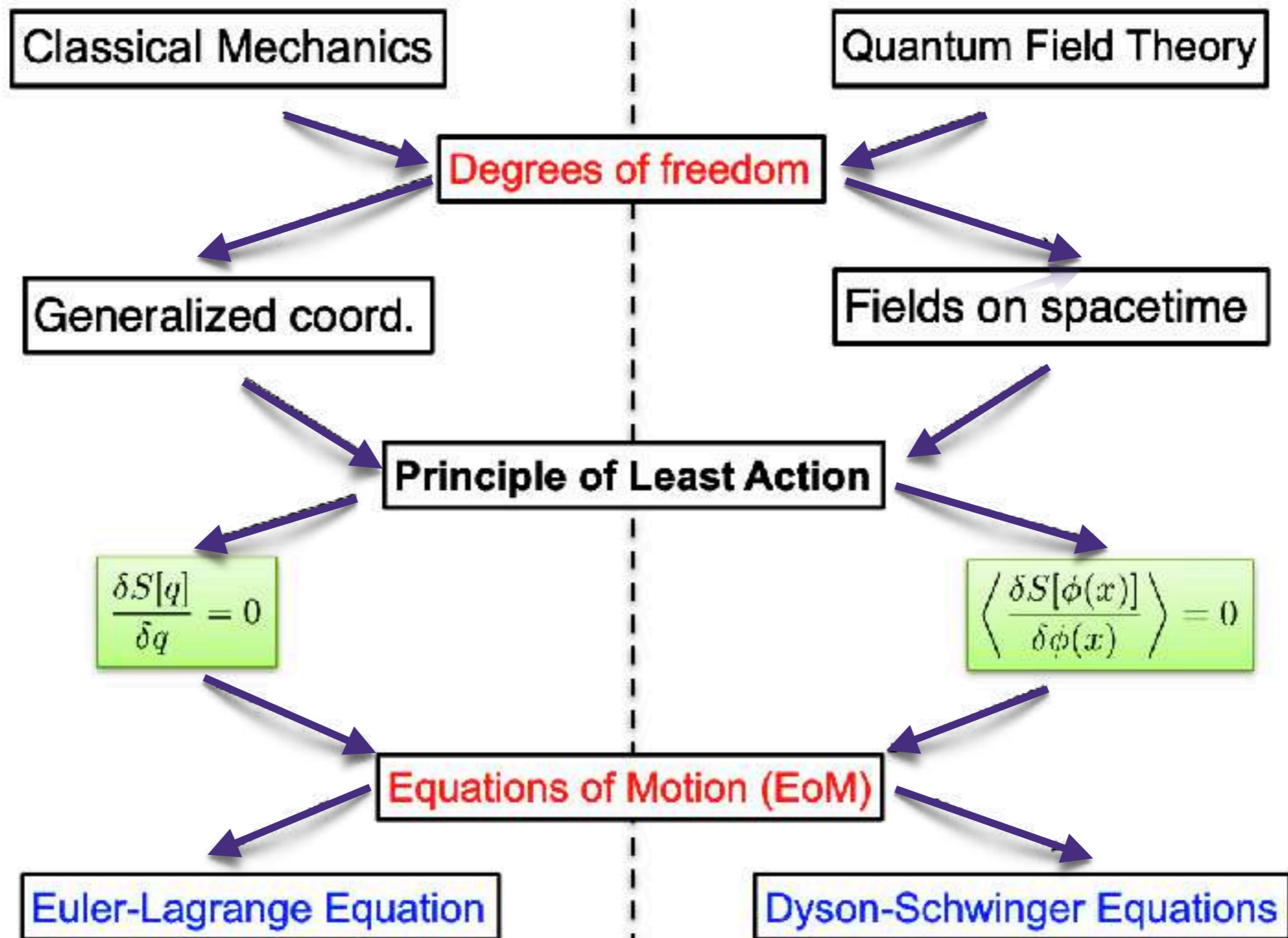
- Also, from the **LCWFs** we can access to structure functions:

GPDs

PDFs

FFs

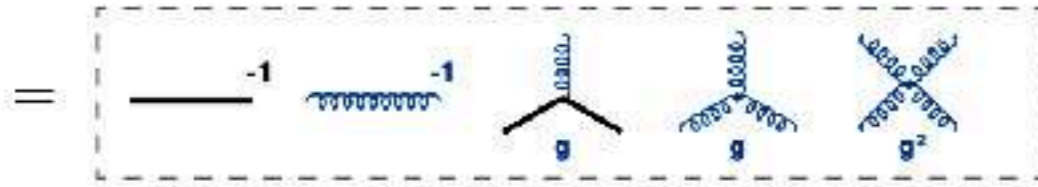
Non-perturbative continuum tools for QCD



QCD's Dyson-Schwinger Equations (DSEs)

QCD's classical action:

$$S = \int d^4x [\bar{\psi} (\not{\partial} + ig\not{A} + m) \psi + \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu}]$$



Quantum "effective action":

$$\int \mathcal{D}[\psi, \bar{\psi}, A] e^{-S} = e^{-\Gamma}$$



DSE = Quantum equation of motion: obtained from path integral, relate n-point functions

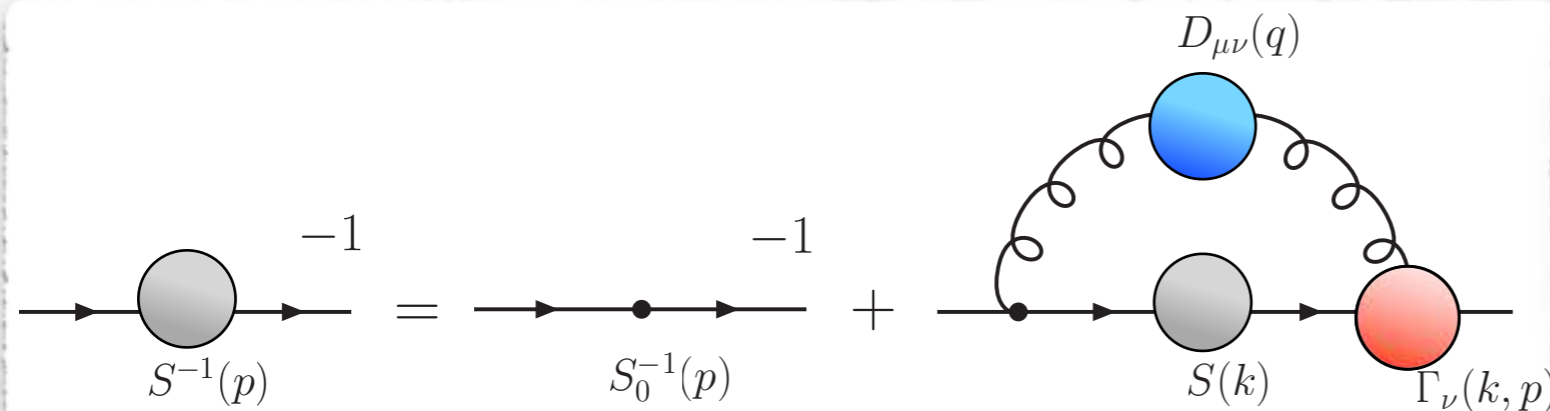
- Infinitely many coupled equations
- Continuum methods: Reproduce perturbation theory, but non-perturbative
- Systematic truncations: neglect higher n-point functions to obtain **closed system**

Capture two emergent phenomena:

- **Dynamical chiral symmetry breaking**
- **Confinement**

QCD's Dyson-Schwinger Equations (DSEs)

Quark Dyson-Schwinger Equation



Quark mass function

$$M_f(p^2) = \frac{B_f(p^2, \mu^2)}{A_f(p^2, \mu^2)}$$

$$S_f^{-1}(p) = Z_2^f (i\gamma \cdot p + m_f^{\text{bm}}) + Z_1^f g^2 \int^\Lambda \frac{d^4 k}{(2\pi)^4} D_{\mu\nu}^{ab}(q) \frac{\lambda^a}{2} \gamma_\mu S_f(k) \Gamma_{\nu,f}^b(k, p)$$

Quark propagator

$$\begin{aligned} S_f^{-1}(p) &= i\gamma \cdot p A_f(p^2, \mu) + B_f(p^2, \mu) \\ &= A_f(p^2, \mu) [i\gamma \cdot p + M_f(p^2)] \end{aligned}$$

Renormalization condition:

$$A_f(p^2) = A_f(p^2) \Big|_{p^2=\mu^2} = 1$$

● $D_{\mu\nu}^{ab}(q)$ = Dressed gluon propagator

● $\Gamma_\nu^b(k, p)$ = Dressed quark-gluon vertex

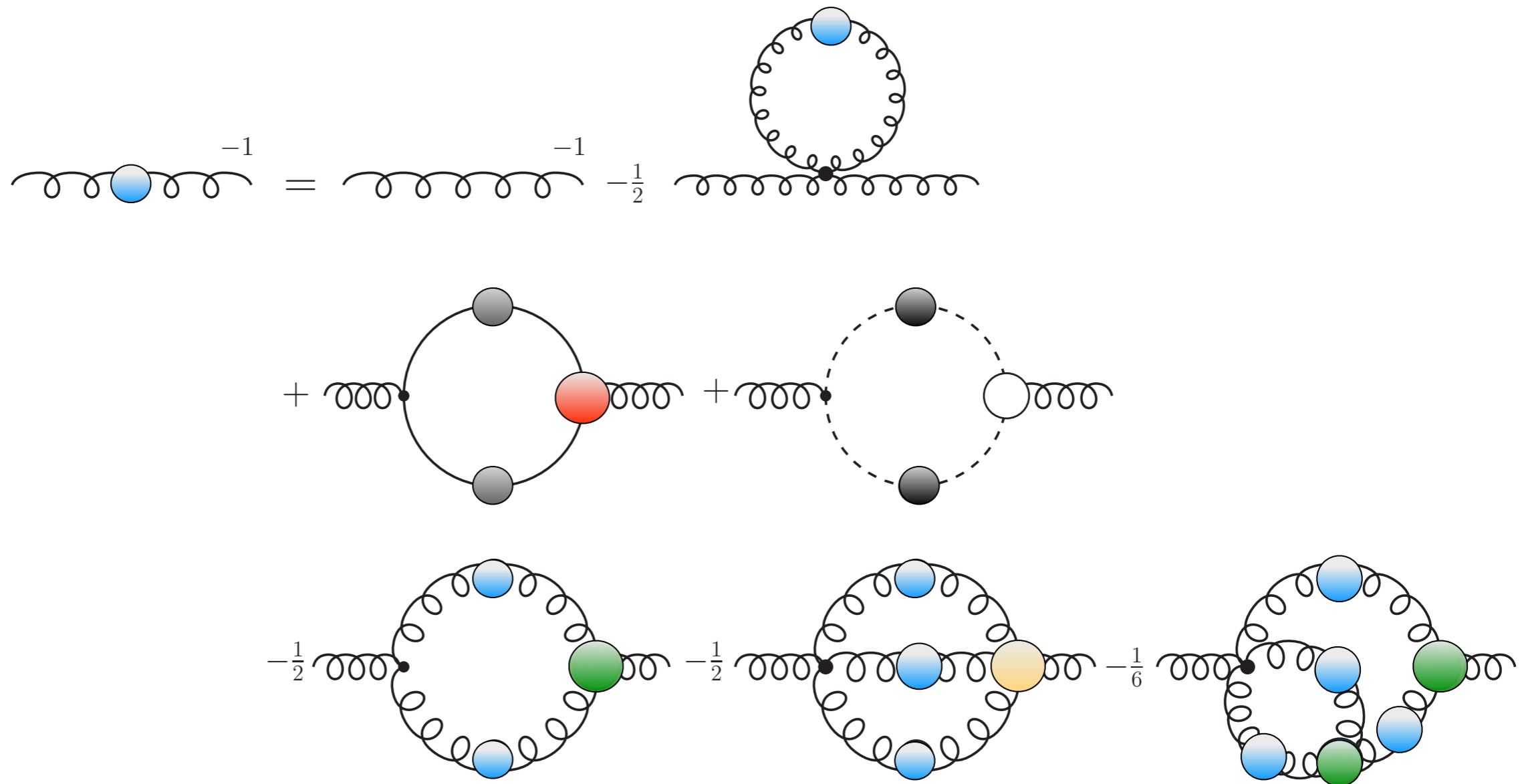
● Z_2 = Quark wave function renormalization constant

● Z_4 = Quark-gluon vertex renormalization constant

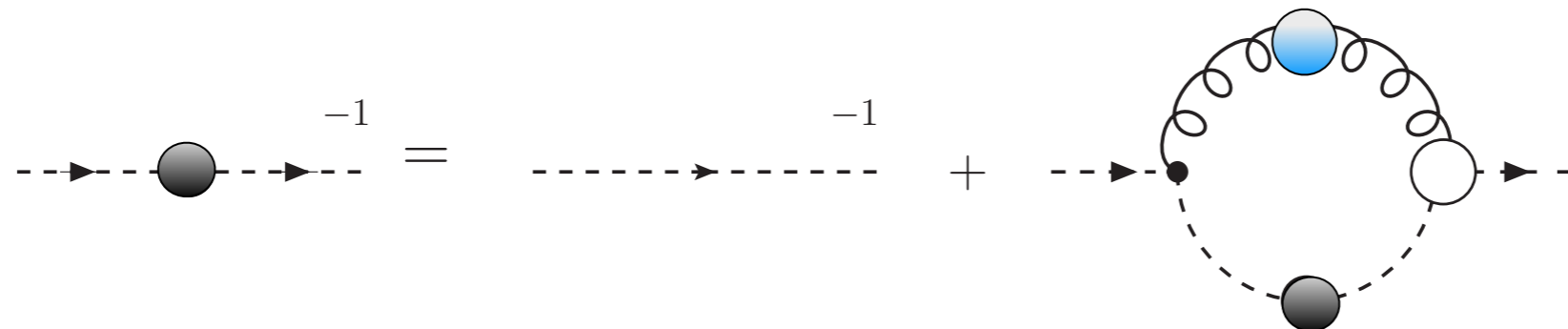
$$S_f^{-1}(p) \Big|_{p^2=\mu^2} = i\gamma \cdot p + m_f(\mu)$$

QCD's Dyson-Schwinger Equations (DSEs)

Gluon

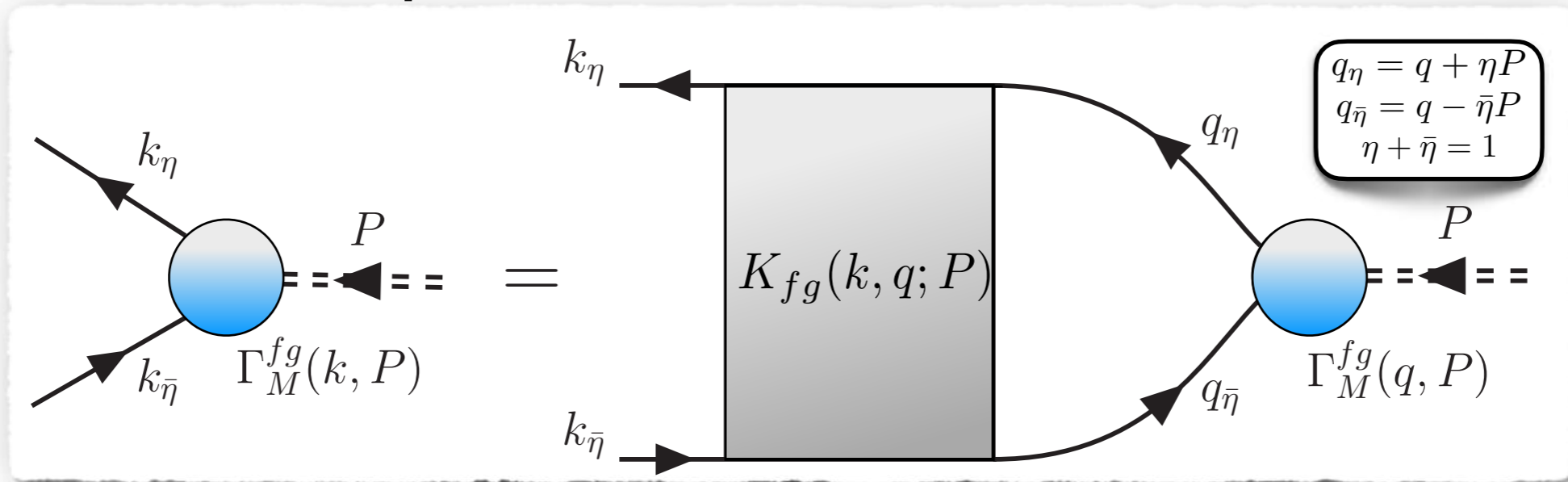


Ghost



Bethe-Salpeter Equations for QCD bound-states

✦ BSE = Bound-state equation for meson



$$\Gamma_M^{fg}(k, P) = \int \frac{d^4 q}{(2\pi)^4} K_{fg}(k, q; P) S_f(q_\eta) \Gamma_M^{fg}(q, P) S_g(q_{\bar{\eta}})$$

● $K_{fg}(q, k; P)$ = Quark-antiquark scattering kernel

● $S_f(q_\eta)$ = Dressed quark propagator

● $\Gamma_M^{fg}(k, P)$ = Meson's Bethe-Salpeter Amplitude (BSA)

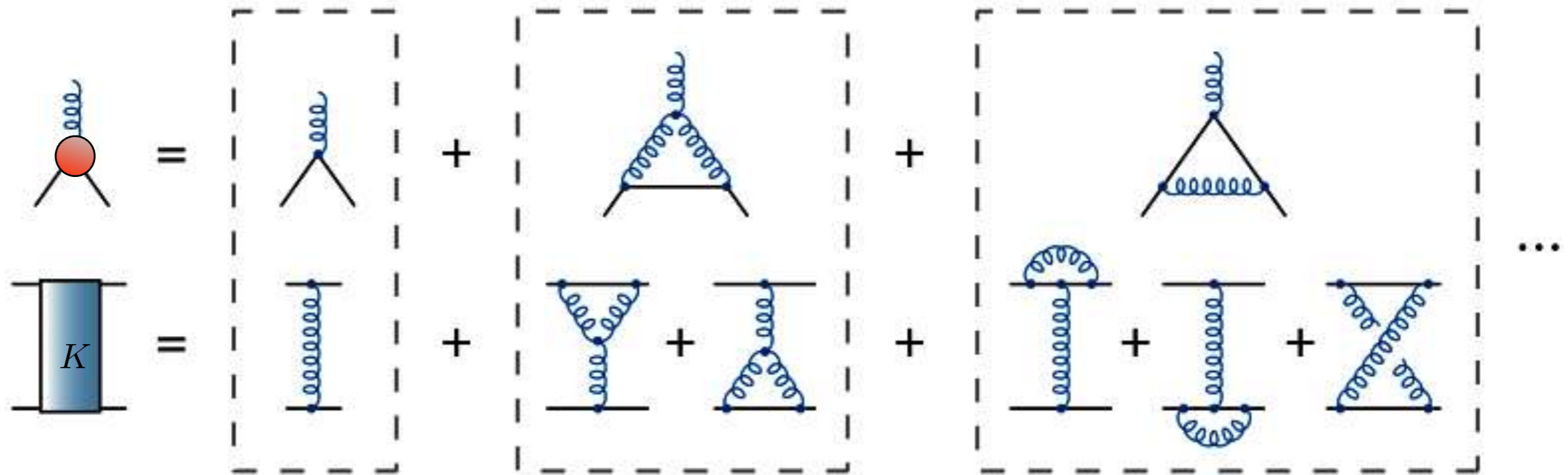
$$\Gamma_M(k, P) = \sum_{i=1}^N T_M^i(k, P) F_i(k, P)$$

General solution for Poincaré
invariant pseudoscalar BSA

$$\Gamma_M^{fg}(k, P) = \gamma_5 \left[i E_M^{fg}(k, P) + \gamma \cdot P F_M^{fg}(k, P) + \gamma \cdot k k \cdot P G_M^{fg}(k, P) + \sigma_{\mu\nu} k_\mu P_\nu H_M^{fg}(k, P) \right]$$

Truncation schemes and symmetries

- **DSE/BSE:** Kernel can be derived in accordance with chiral symmetry



Truncation must preserve **AV-WTI**, which ensures that we will have massless pions in the chiral limit.

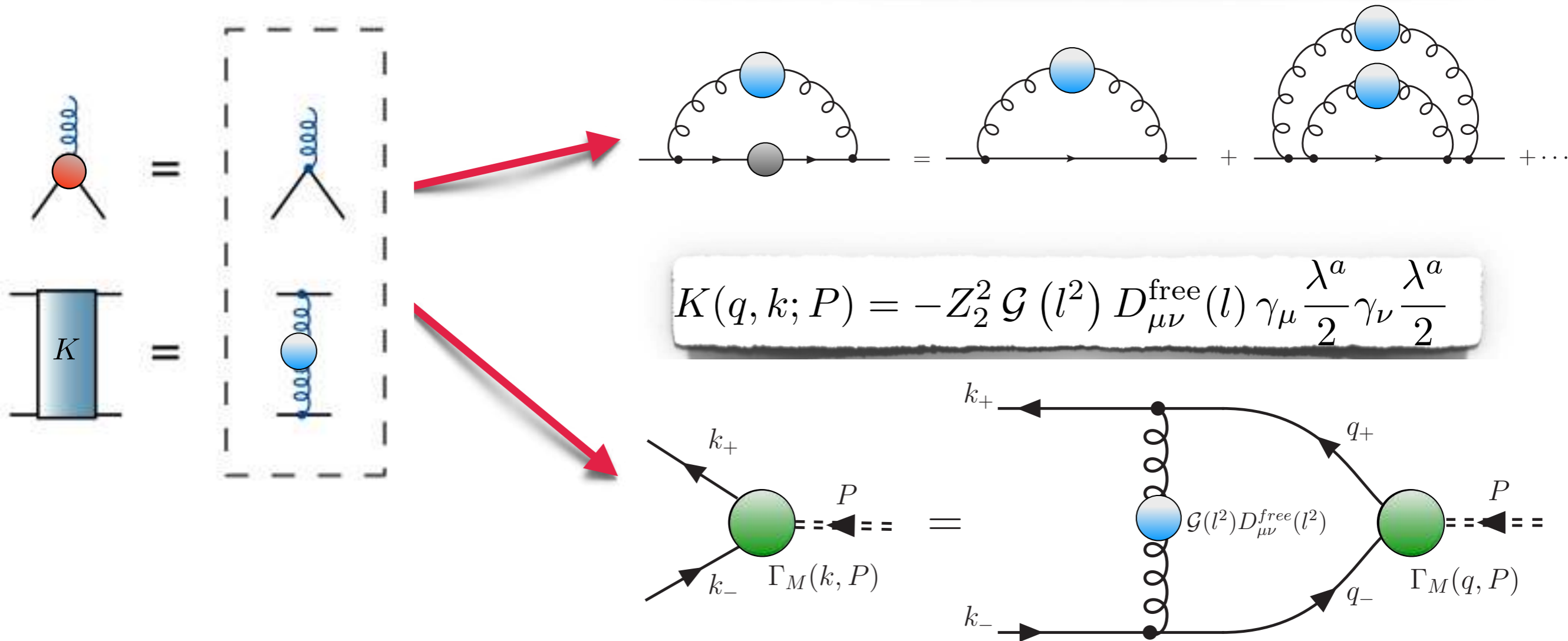
AV-WTI:
$$P_\mu \Gamma_{5\mu}^{fg}(k; P) = S_f^{-1}(k_\eta) i\gamma_5 + i\gamma_5 S_g^{-1}(k_{\bar{\eta}})$$

Axial vector-vertex:

$$\Gamma_{5\mu}^{fg}(k; P) = Z_2^f \gamma_5 \gamma_\mu + \int^\Lambda \frac{d^4 q}{(2\pi)^4} K_{fg}(q, k; P) S_f(q_\eta) \Gamma_{5\mu}^{fg}(q; P) S_g(q_{\bar{\eta}})$$

Rainbow-Ladder truncation

Leading truncation



$\mathcal{G}(q)$: Effective gluon interaction

Gluon interaction model

An interaction ansatz for $\mathcal{G}(q^2)$ that has proven its merits in meson and baryon phenomenology can be decomposed as

$$\frac{\mathcal{G}_f(q^2)}{q^2} = \mathcal{G}_f^{\text{IR}}(q^2) + 4\pi\tilde{\alpha}_{\text{PT}}(q^2)$$

Infrared part: Two-Models

MT-Model: $\mathcal{G}_f^{\text{IR}}(q^2) = \frac{4\pi^2}{\omega_f^6} q^2 D_f e^{-q^2/\omega_f^2}$

QC-Model: $\mathcal{G}_f^{\text{IR}}(q^2) = \frac{8\pi^2}{\omega_f^4} D_f e^{-q^2/\omega_f^2}$ ✓

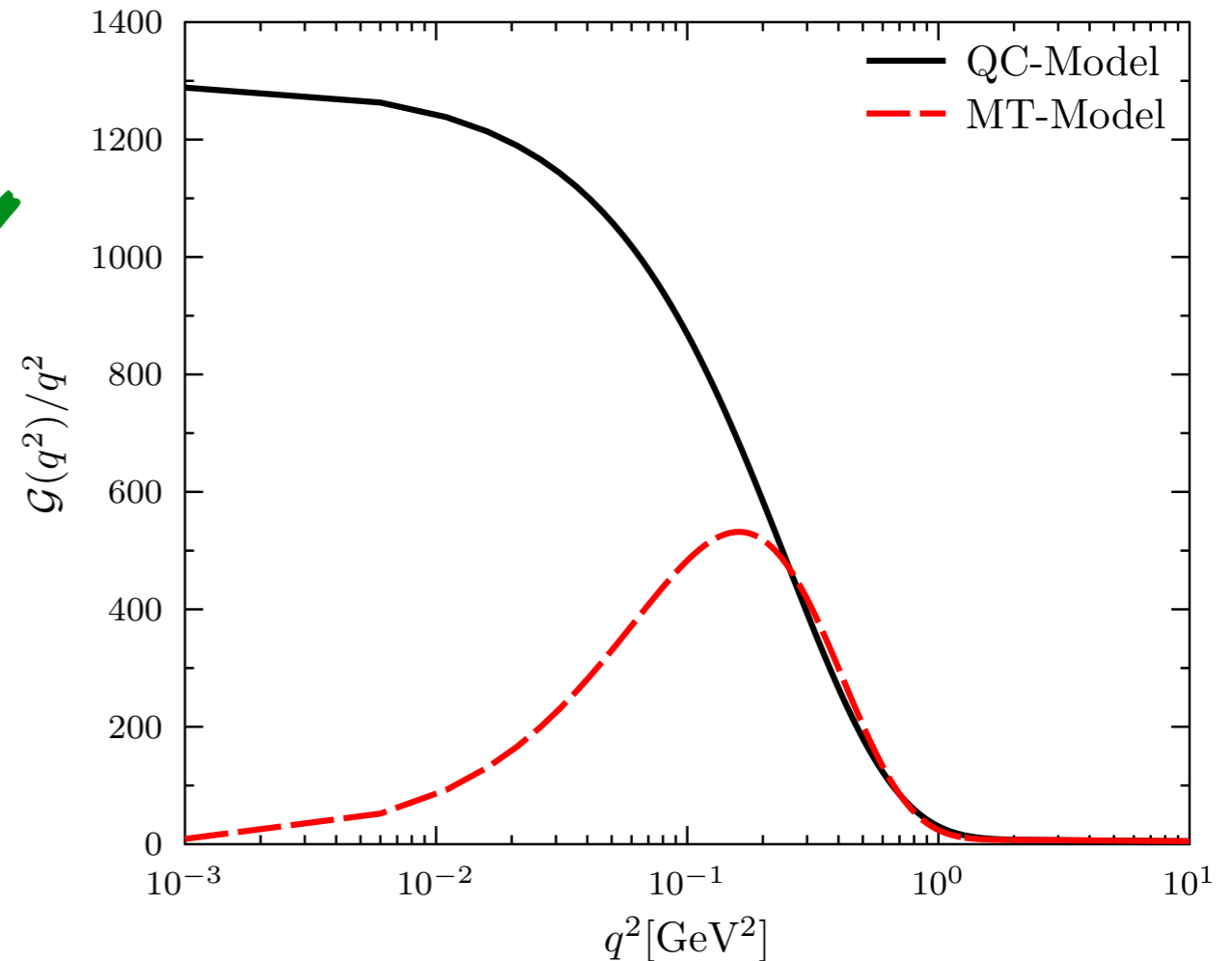
Ultraviolet part: Model

$$4\pi\tilde{\alpha}_{\text{PT}}(q^2) = \frac{8\pi^2\gamma_m\mathcal{F}(q^2)}{\ln\left[\tau + \left(1 + q^2/\Lambda_{\text{QCD}}^2\right)^2\right]}$$

$$\gamma_m = 12/(33 - 2N_f), \quad N_f = 4, \quad \Lambda_{\text{QCD}} = 0.234 \text{ GeV}$$

$$\mathcal{F}(q^2) = [1 - \exp(-q^2/4m_t^2)]/q^2, \quad m_t = 0.5 \text{ GeV}$$

$$\tau = e^2 - 1$$



P. Maris and P. C. Tandy, Phys. Rev. C 60, 055214 (1999)

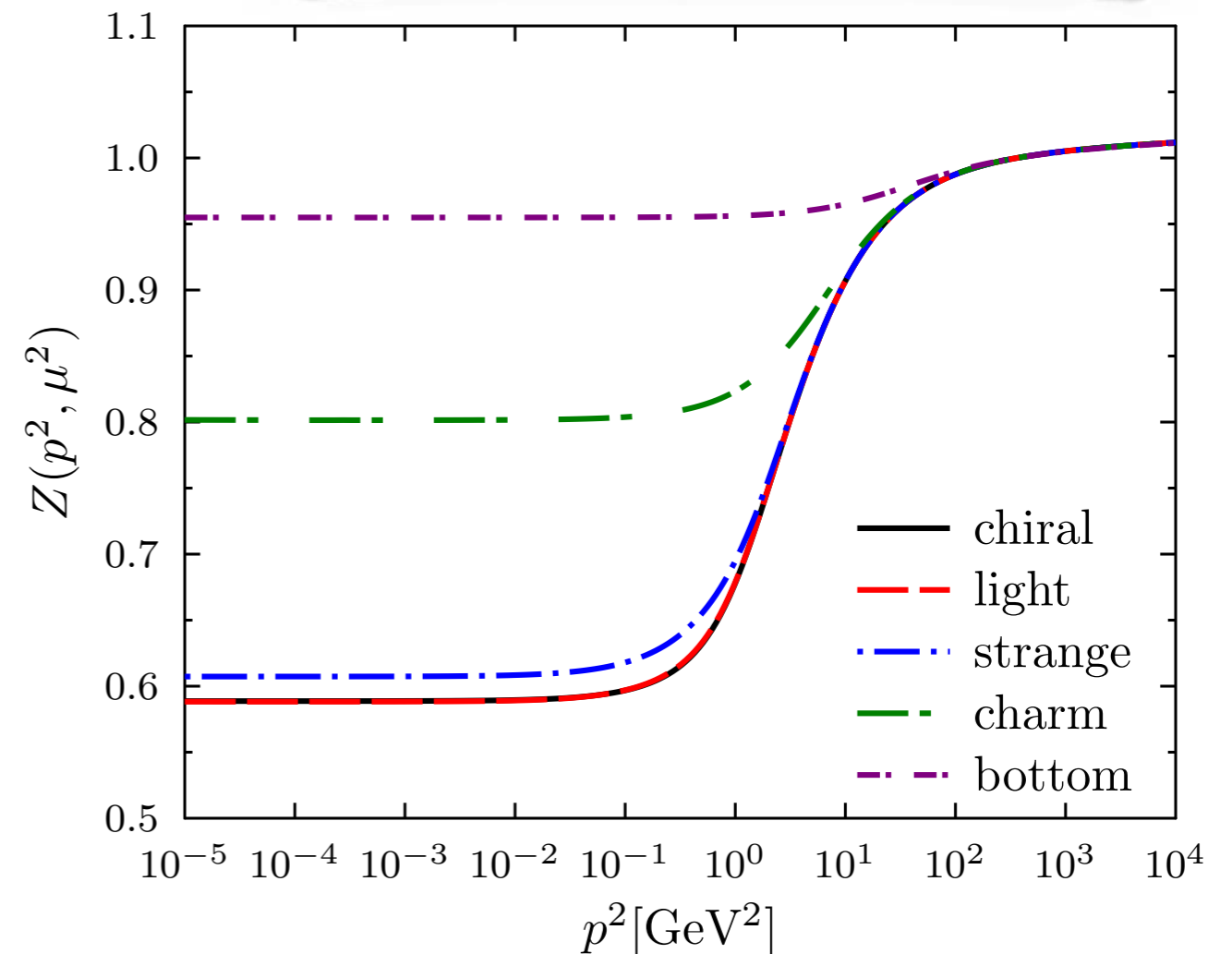
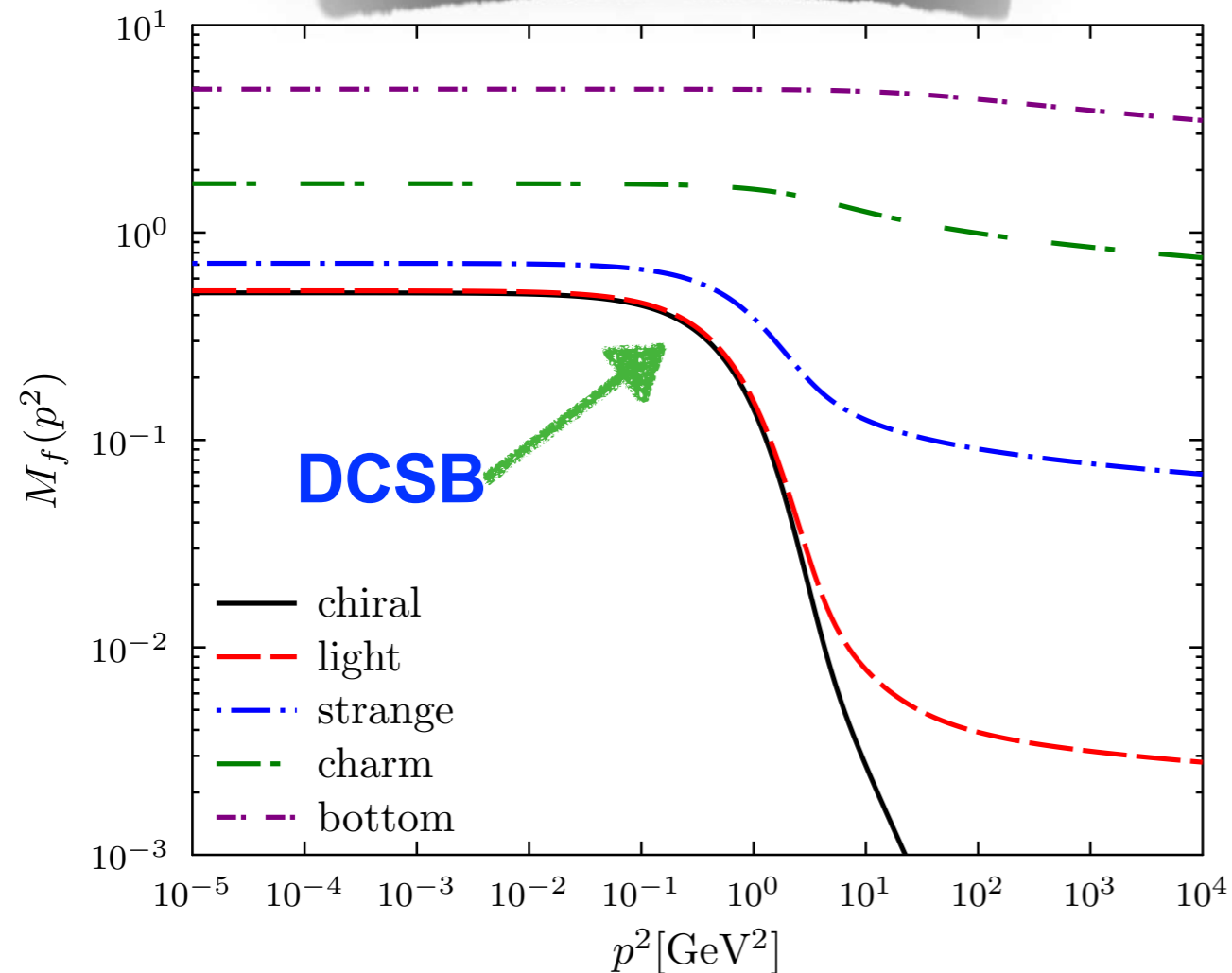
S. X. Qin, L. Chang, Y. X. Liu, C. D. Roberts and D. J. Wilson, Phys. Rev. C 84, 042202 (2011)

Numerical solutions: Quark DSE

REAL AXIS SOLUTION

$$M_f(p^2) = \frac{B_f(p^2, \mu^2)}{A_f(p^2, \mu^2)}$$

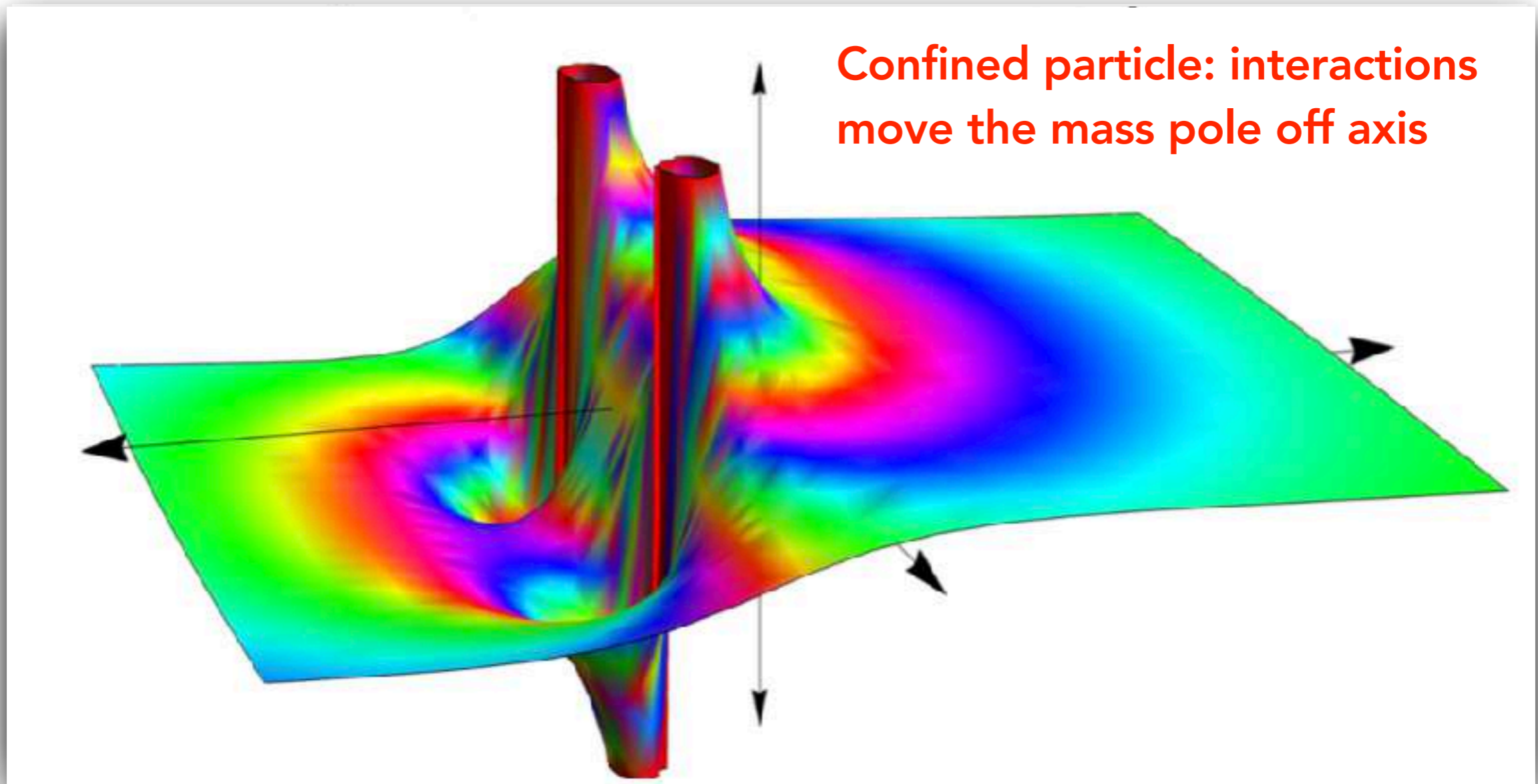
$$Z_f(p^2, \mu^2) = 1/A_f(p^2, \mu^2)$$



Quark Propagator: Dynamical chiral symmetry breaking generates 'constituent-quark masses'.

Numerical solutions: Quark DSE

- Dramatic change in the analytic structure of the quark propagator:



Complex conjugate singularities characterized by a dynamically generated mass scale.

Numerical solutions: Quark DSE

Quark Propagator: In order to obtain bound-states masses, we need to know the quark propagator in a parabolic region in the complex plane.

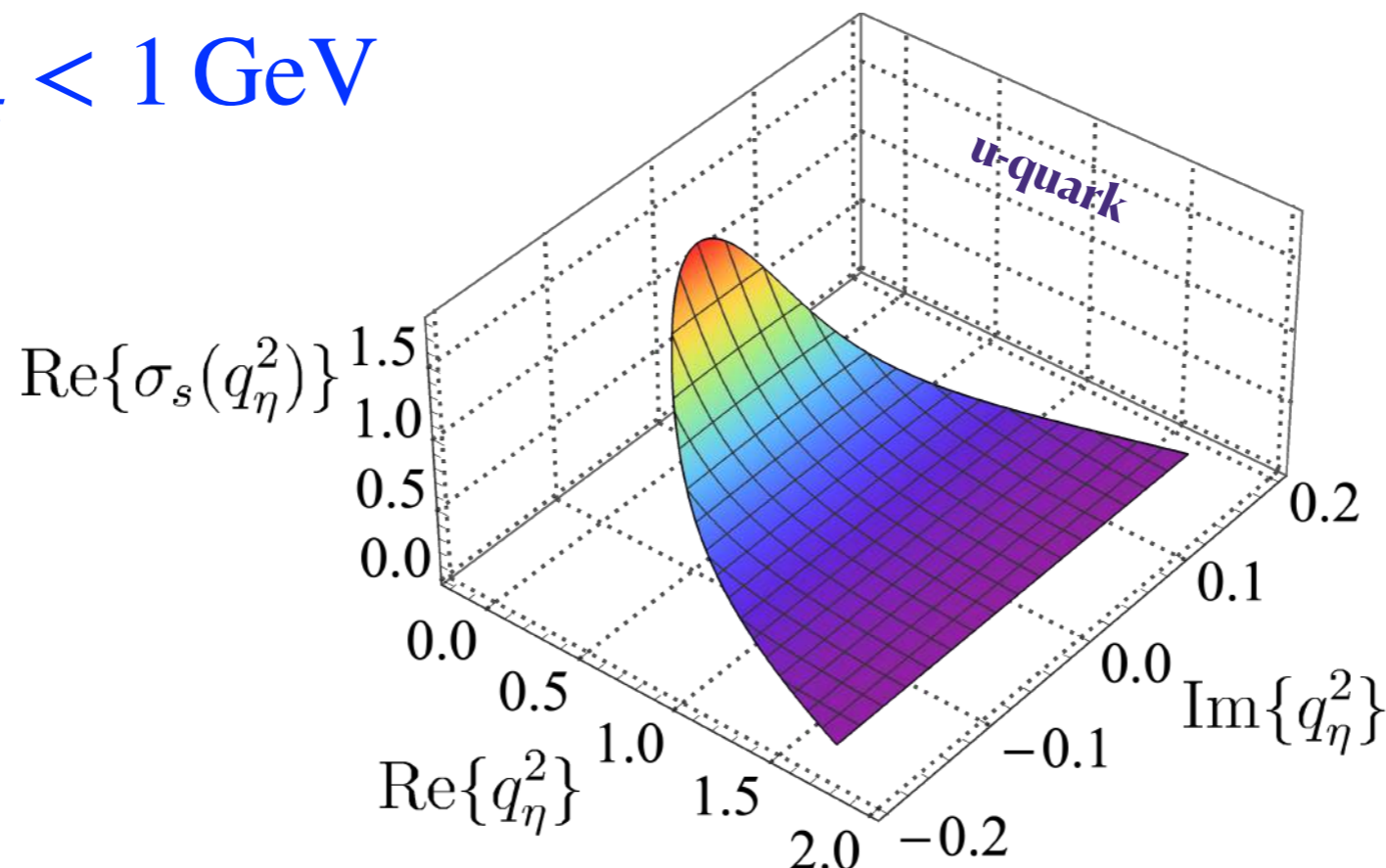
COMPLEX PLANE SOLUTION

$$S(q_\eta) = -i\gamma \cdot q_\eta \sigma_v(q_\eta^2) + \sigma_s(q_\eta^2)$$
$$q_\eta^2 = q^2 - \eta^2 m_M^2 + 2i\eta m_M |q| z_q$$

$$\sigma_v^f(q_\eta^2) = \frac{A_f(q_\eta^2)}{q_\eta^2 A_f^2(q_\eta^2) + B_f^2(q_\eta^2)}$$
$$\sigma_s^f(q_\eta^2) = \frac{B_f(q_\eta^2)}{q_\eta^2 A_f^2(q_\eta^2) + B_f^2(q_\eta^2)}$$

● **Light meson:** For instant, the pion with $\eta = 1/2$.

$$m_M = m_\pi < 1 \text{ GeV}$$



Numerical solutions: Quark DSE

Quark Propagator: In order to obtain bound-states masses, we need to know the quark propagator in a parabolic region in the complex plane.

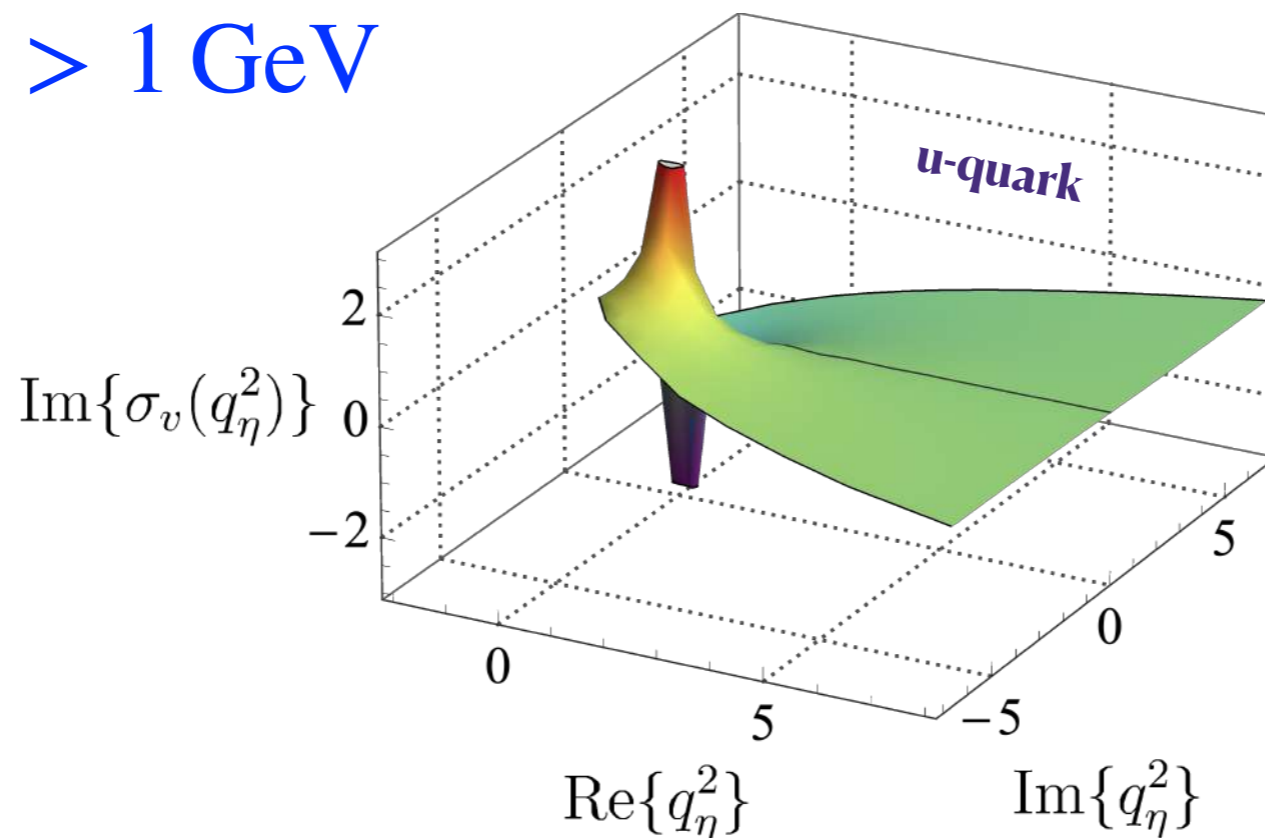
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● **Heavy meson:** For instant, the D meson with ($\eta = 0.5$)

$$m_M = m_D > 1 \text{ GeV}$$



Numerical solutions: Quark DSE

Quark Propagator: In order to obtain bound-states masses, we need to know the quark propagator in a parabolic region in the complex plane.

COMPLEX PLANE SOLUTION

$$S(q_\eta) = -i\gamma \cdot q_\eta \sigma_v(q_\eta^2) + \sigma_s(q_\eta^2)$$

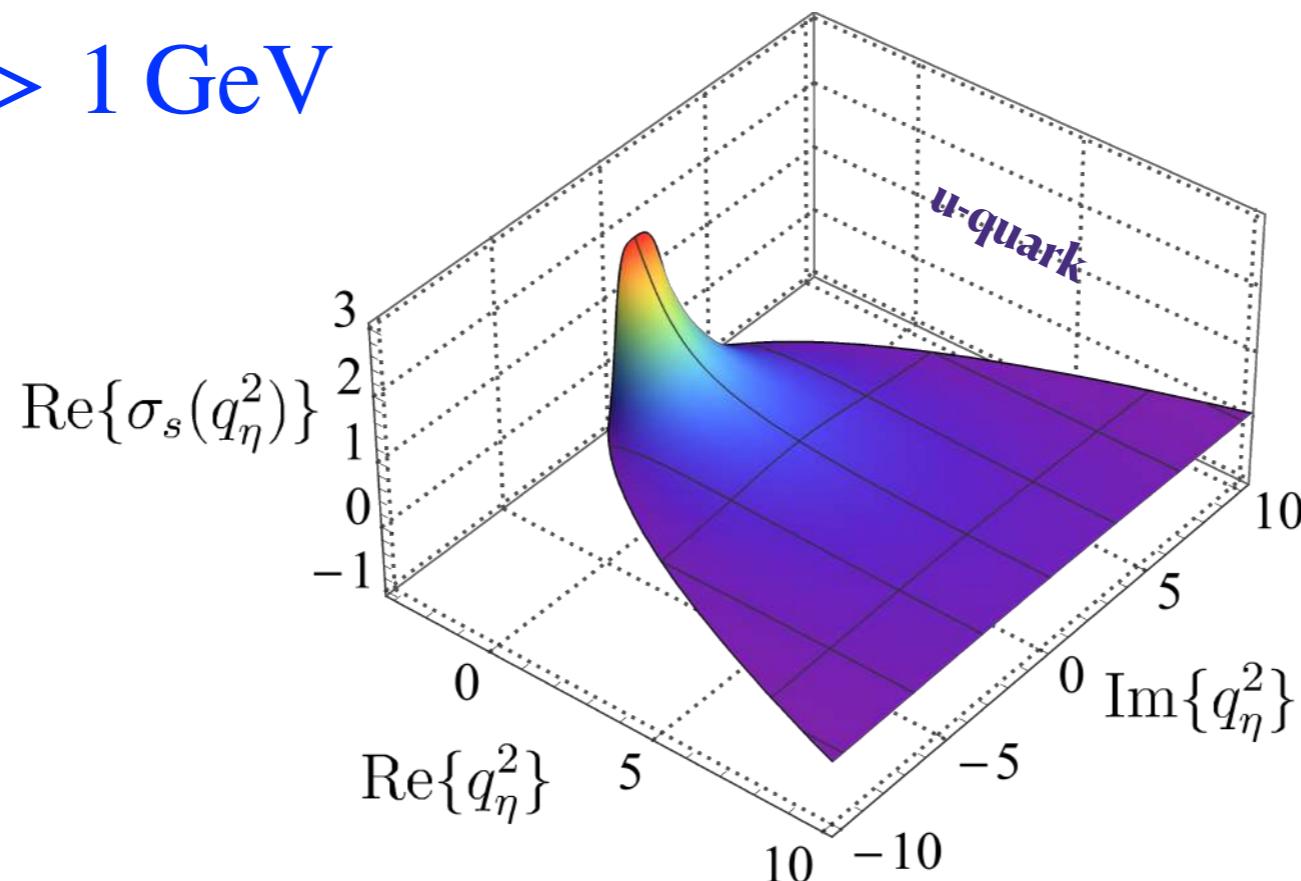
$$q_\eta^2 = q^2 - \eta^2 m_M^2 + 2i\eta m_M |q| z_q$$

$$\sigma_v^f(q_\eta^2) = \frac{A_f(q_\eta^2)}{q_\eta^2 A_f^2(q_\eta^2) + B_f^2(q_\eta^2)}$$

$$\sigma_s^f(q_\eta^2) = \frac{B_f(q_\eta^2)}{q_\eta^2 A_f^2(q_\eta^2) + B_f^2(q_\eta^2)}$$

● **Heavy meson:** For instant, the D meson with $\eta = 0.2$

$$m_M = m_D > 1 \text{ GeV}$$



Numerical solutions: BSE

For the bound state masses we solve an artificial eigenvalue problem:

$$\lambda_n(P^2)\Gamma_M^{fg}(k, P) = \int \frac{d^4q}{(2\pi)^4} K_{fg}(k, q; P) S_f(q_\eta) \Gamma_M^{fg}(q, P) S_g(q_{\bar{\eta}})$$

$$\Gamma_M(k, P) = \sum_{i=1}^N T_M^i(k, P) F_i(k^2, z_k, P^2), \quad z_k = k \cdot P / |k| |P|$$

$$\mathcal{F}_i(k, P) = \sum_{m=0}^{\infty} {}^m \mathcal{F}_i(k^2, P^2) U_m(z_k)$$

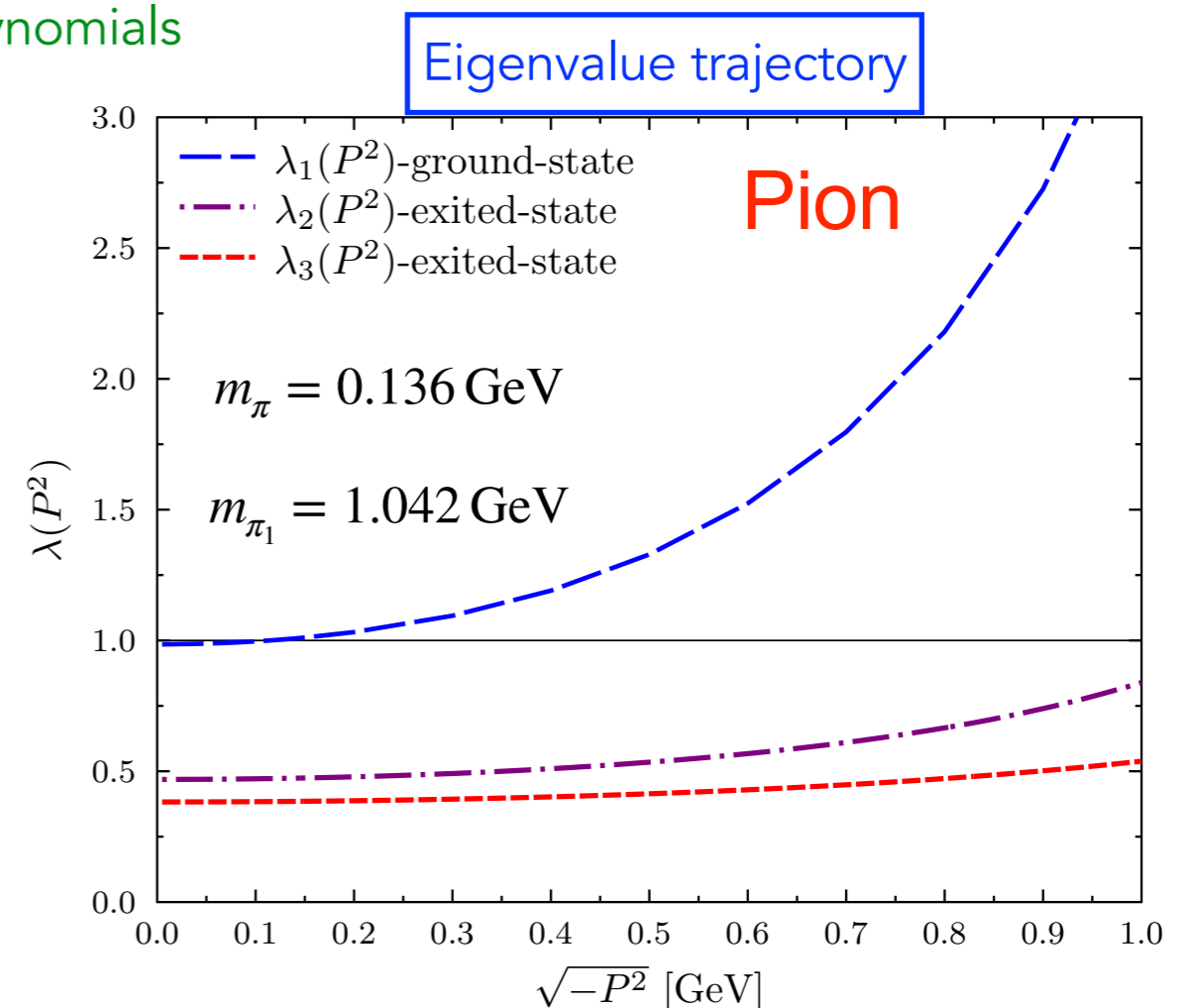
Chebyshev polynomials

Chebyshev's Moments

$$\frac{2}{\pi} \int_0^1 \sqrt{1-x^2} U_m(x) U_n(x) dx = \delta_{mn}$$

$$\lambda(P^2) {}^m \mathcal{F}^{\alpha i}(P) = (\mathbf{K}_\beta^\alpha)_{n j}^{m i}(P) {}^n \mathcal{F}^{\beta j}(P)$$

Basically to obtain the meson mass we have to find the root of $\lambda(P^2) - 1 = 0$.



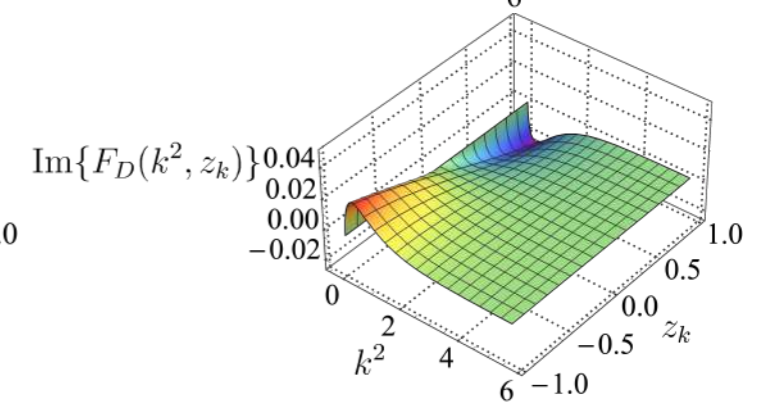
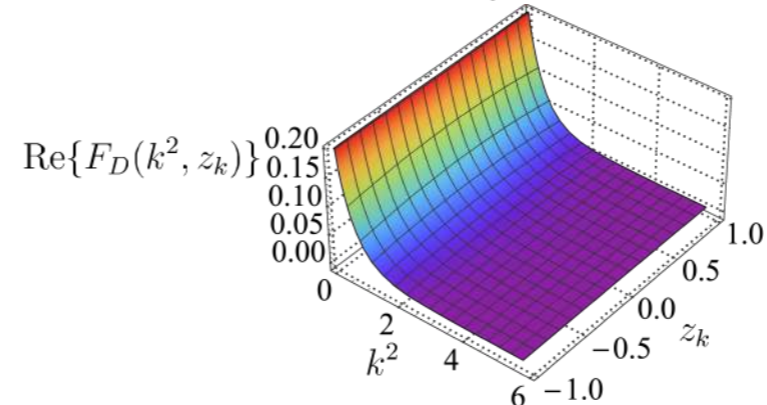
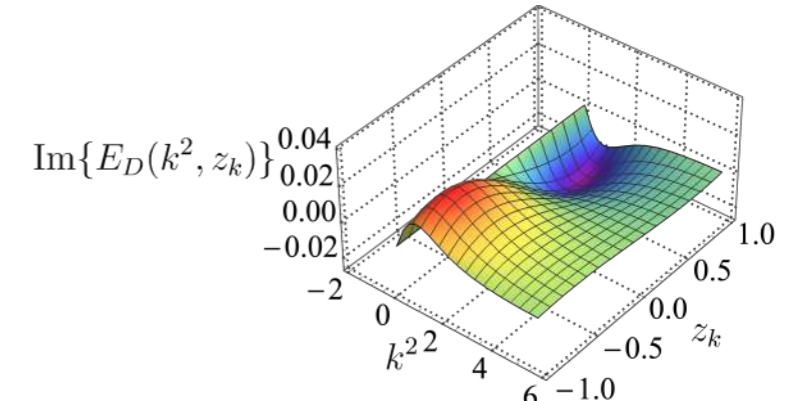
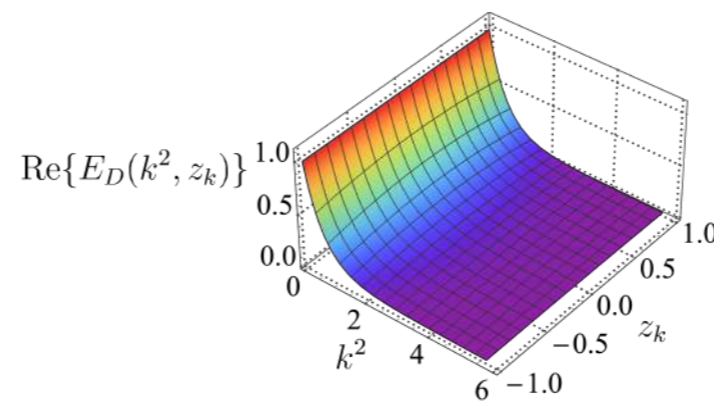
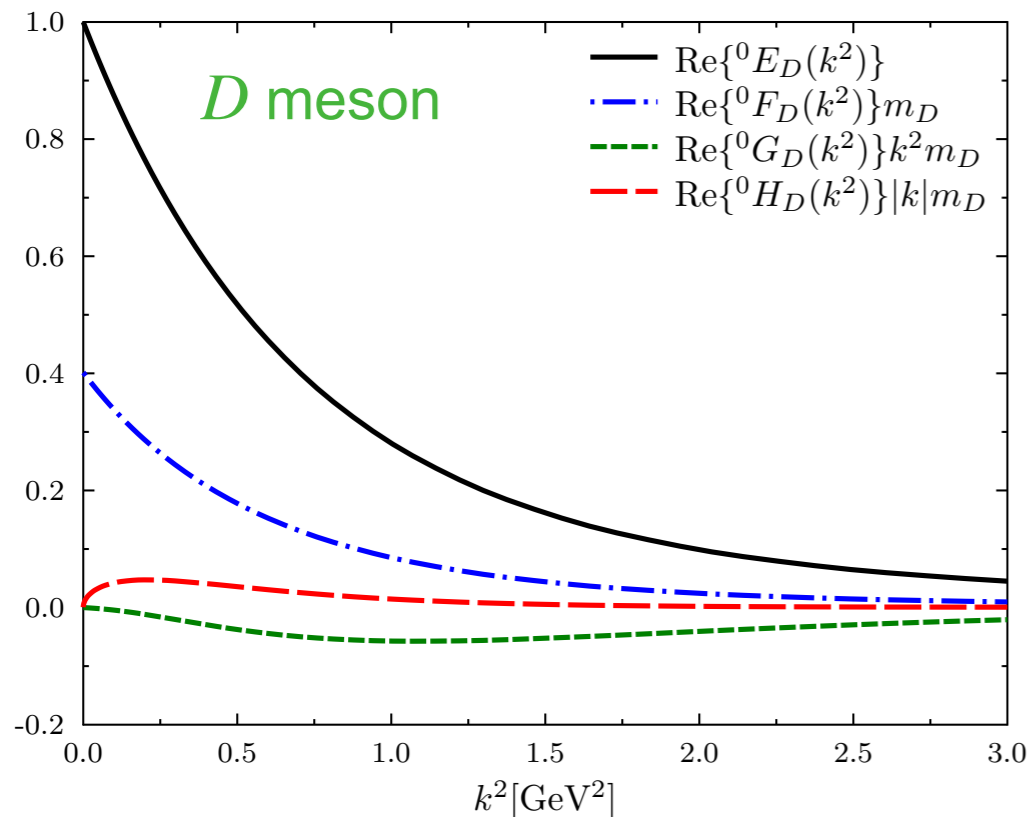
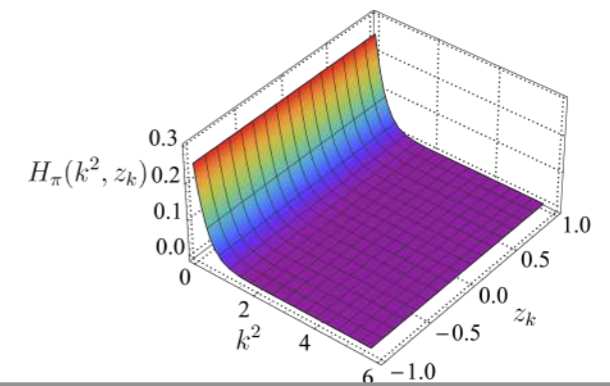
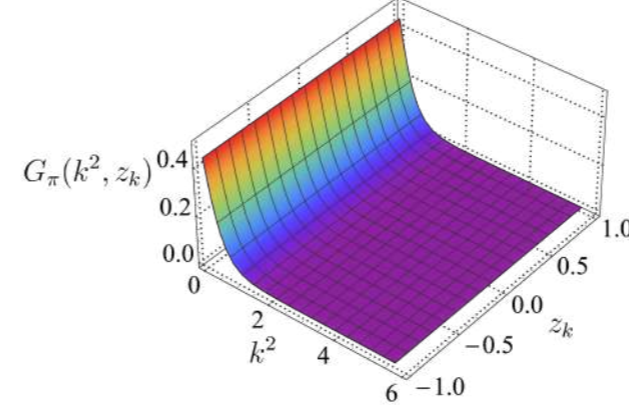
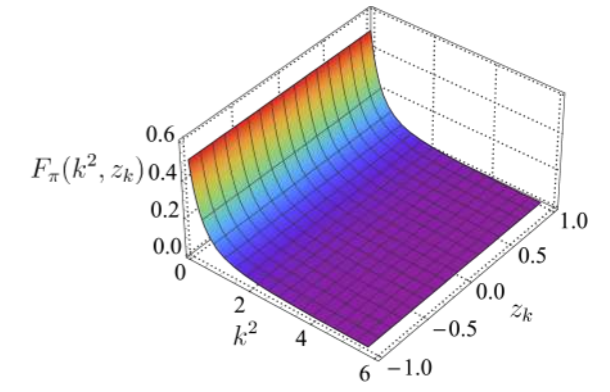
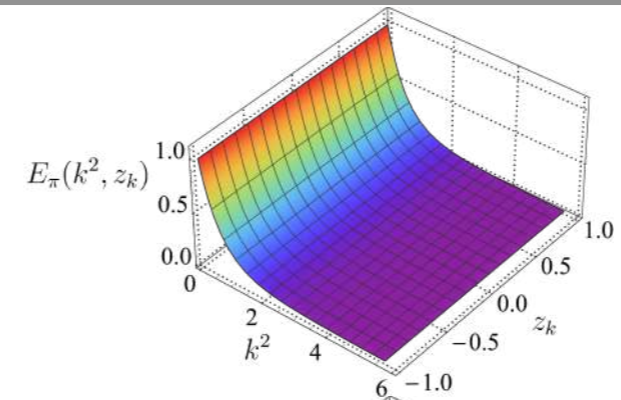
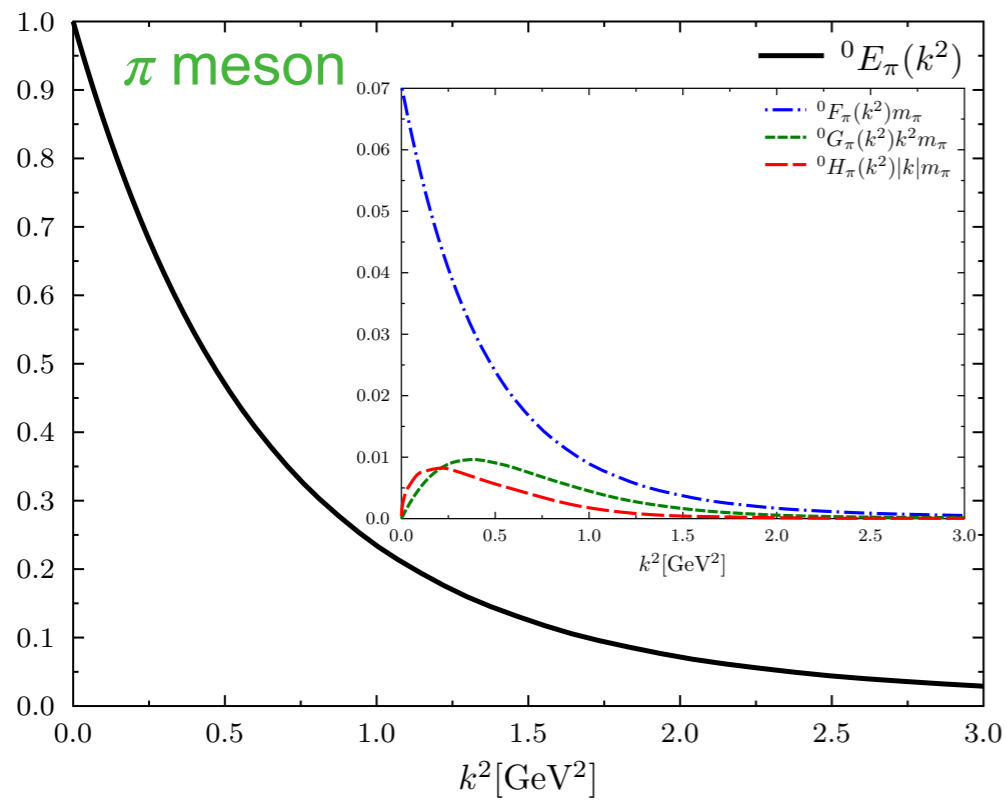
Pseudoscalar mesons

	M_P	M_P^{exp}	ϵ_{M_P} [%]	f_P	$f_P^{\text{exp/QCD}}$	ϵ_{f_P} [%]
$\pi(u\bar{d})$	0.140	0.138	1.45	$0.094^{+0.001}_{-0.001}$	0.092(1)	2.17
$K(u\bar{s})$	0.494	0.494	0	$0.110^{+0.001}_{-0.001}$	0.110(2)	0
$D(c\bar{d})$	$1.867^{+0.008}_{-0.004}$	1.864	0.11	$0.144^{+0.001}_{-0.001}$	0.150 (0.5)	4.00
$D_s(c\bar{s})$	$2.015^{+0.021}_{-0.018}$	1.968	2.39	$0.179^{+0.004}_{-0.003}$	0.177(0.4)	1.13
$\eta_c(c\bar{c})$	$3.012^{+0.003}_{-0.039}$	2.984	0.94	$0.270^{+0.002}_{-0.005}$	0.279(17)	3.23
$\eta_b(b\bar{b})$	$9.392^{+0.005}_{-0.004}$	9.398	0.06	$0.491^{+0.009}_{-0.009}$	0.472(4)	4.03
$B(u\bar{b})$	$5.277^{+0.008}_{-0.005}$	5.279	0.04	$0.132^{+0.004}_{-0.002}$	0.134(1)	4.35
$B_s(s\bar{b})$	$5.383^{+0.037}_{-0.039}$	5.367	0.30	$0.128^{+0.002}_{-0.003}$	0.162(1)	20.5
$B_c(c\bar{b})$	$6.282^{+0.020}_{-0.024}$	6.274	0.13	$0.280^{+0.005}_{-0.002}$	0.302(2)	10.17

$$f_P P_\mu = \frac{Z_2 N_c}{\sqrt{2}} \int^\Lambda \frac{d^4 k}{(2\pi)^4} \text{Tr}_D \left[i\gamma_5 \gamma_\mu S_f(k_\eta) \Gamma_P^{fg}(k, P) S_g(k_{\bar{\eta}}) \right]$$

F. E. S, R. C. da Silveira, J. Cobos-Martínez, B. El-Bennich and E. Rojas, Eur. Phys. J. C 80, 955 (2020).

Pseudoscalar mesons



Extracting LCDAs

$$\phi_M(x, \mu)$$

- Projection of BS wave-function onto the light front : $\chi_M(k_\eta, k_{\bar{\eta}}) := S(k_\eta)\Gamma_M(k, P)S(k_{\bar{\eta}})$

$$f_M \phi_M(x, \mu) = \frac{\mathcal{Z}_2 N_c}{\sqrt{2}} \text{Tr}_D \int^{\Lambda} \frac{d^4 k}{(2\pi)^4} \delta(n \cdot k_\eta - x n \cdot P) \gamma_5 \gamma \cdot n \chi_M(k_\eta, k_{\bar{\eta}})$$

Not accessible in Euclidean space

- Computing Mellin moments

$$\langle x^m \rangle = \int_0^1 dx x^m \phi_M(x, \mu) \quad \langle x^0 \rangle = \int_0^1 dx \phi_M(x, \mu) = 1$$

→
$$\langle x^m \rangle = \frac{\mathcal{Z}_2 N_c}{\sqrt{2} f_M} \text{Tr}_D \int^{\Lambda} \frac{d^4 k}{(2\pi)^4} \frac{(n \cdot k_\eta)^m}{(n \cdot P)^{m+1}} \gamma_5 \gamma \cdot n \chi_M(k_\eta, k_{\bar{\eta}})$$

- Interpolation of quark propagator and Bethe-Salpeter amplitudes

$$S_f(q) = \sum_{k=1}^N \left[\frac{z_k^f}{i\gamma \cdot q + m_k^f} + \frac{(z_k^f)^*}{i\gamma \cdot q + (m_k^f)^*} \right] \quad \mathcal{F}_i(k, P) = \sum_{j=0}^2 \int_{-1}^1 dz \rho_{\nu_j}(z) \frac{\mathcal{U}_j \Lambda_{\mathcal{F}_i}^{2n_j}}{(k^2 + z k \cdot P + \Lambda_{\mathcal{F}_i}^2)^{n_j}}$$

Extracting LCDAs

$$\phi_M(x, \mu)$$

Reconstructing LCDAs

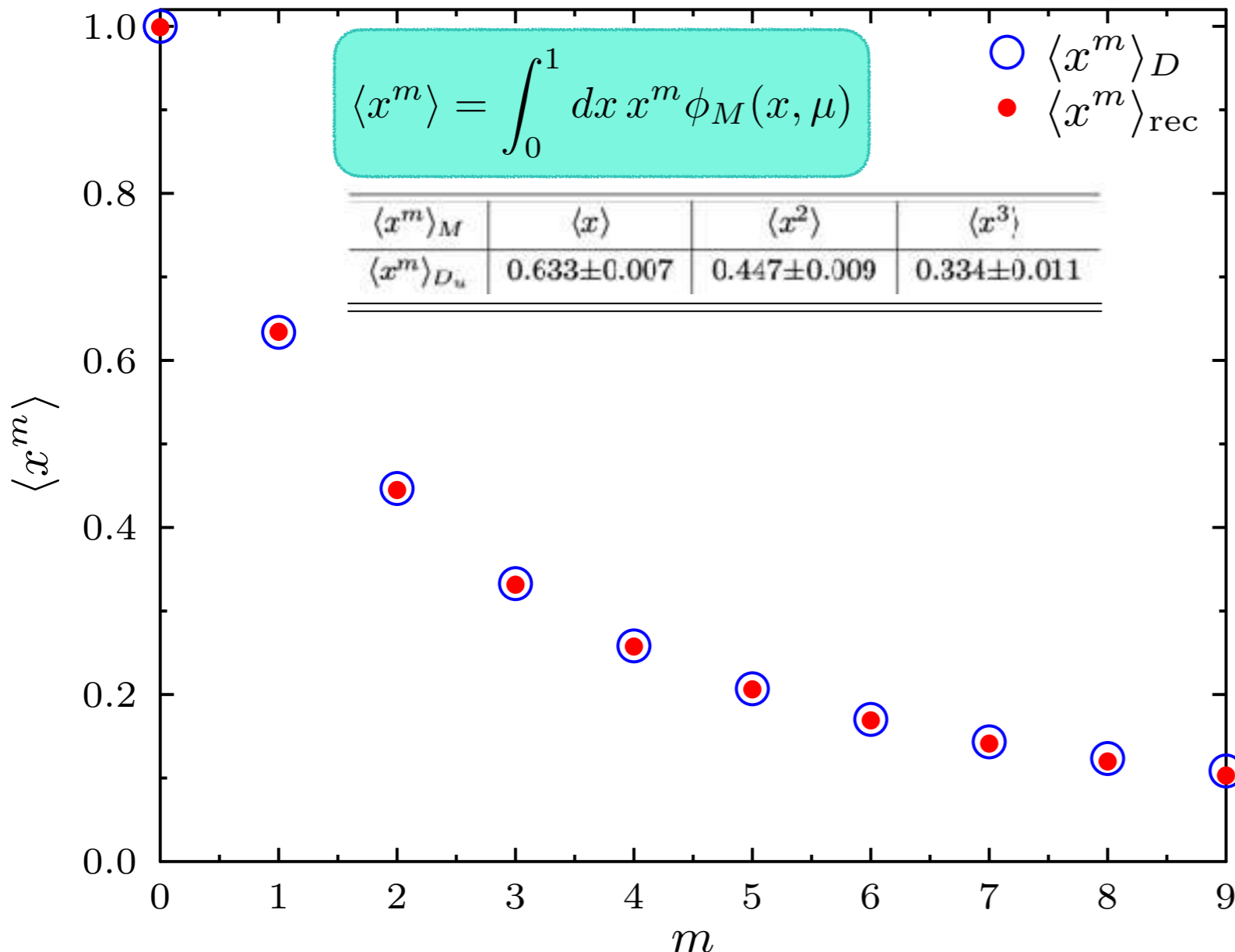
$$\phi_{\pi}^{\text{rec.}}(x, \mu) = \mathcal{N}(\alpha) [x\bar{x}]^{\alpha-1/2} [1 + a_2 C_2^{\alpha} (2x - 1)]$$

$$\phi_H^{\text{rec.}}(x, \mu) = \mathcal{N}(\alpha, \beta) 4x\bar{x} e^{4\alpha x\bar{x} + \beta(x - \bar{x})}$$

$$\langle x^m \rangle = \int_0^1 dx x^m \phi_M(x, \mu)$$

○ $\langle x^m \rangle_D$
● $\langle x^m \rangle_{\text{rec}}$

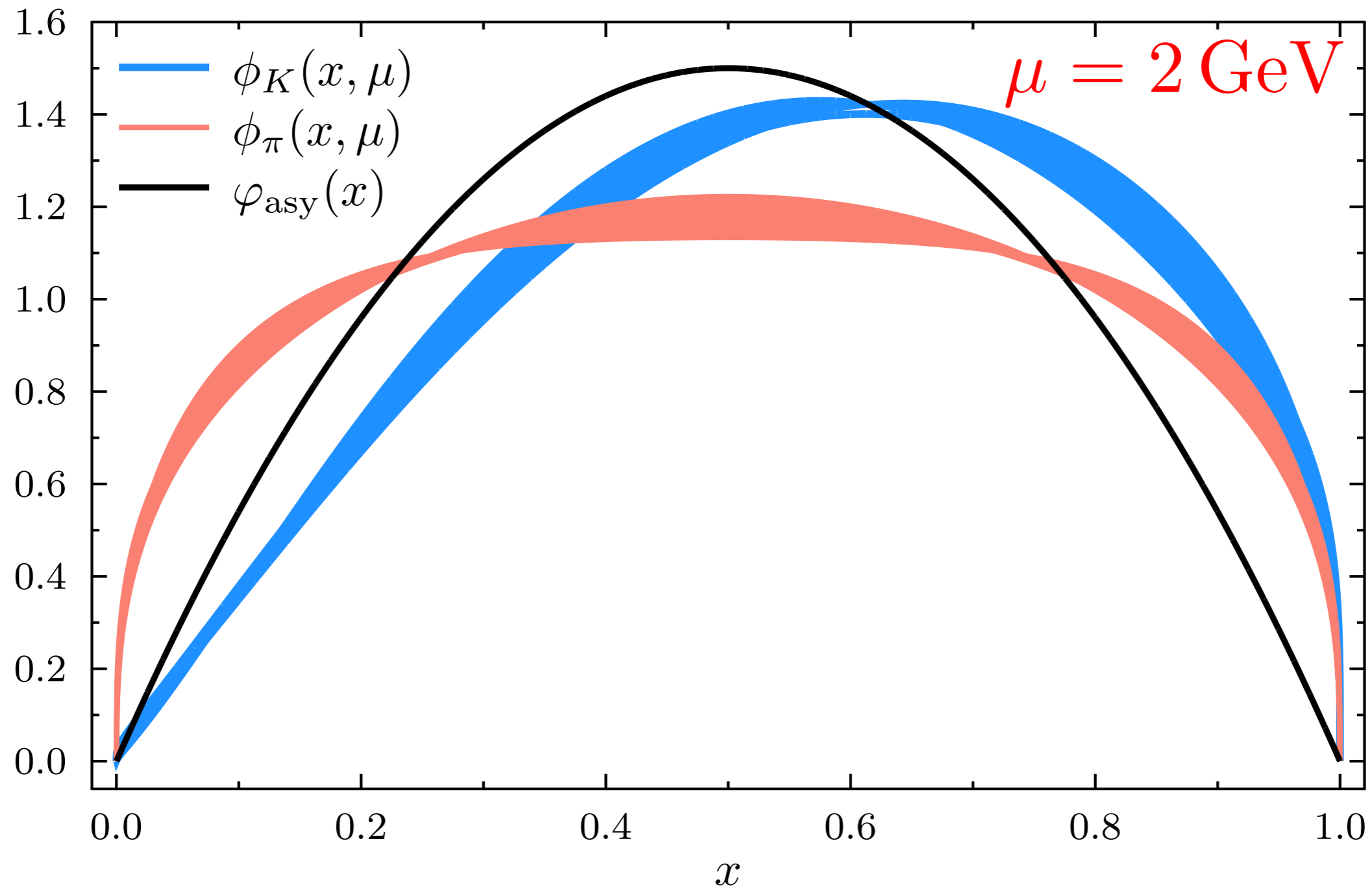
$\langle x^m \rangle_M$	$\langle x \rangle$	$\langle x^2 \rangle$	$\langle x^3 \rangle$
$\langle x^m \rangle_{D_u}$	0.633 ± 0.007	0.447 ± 0.009	0.334 ± 0.011



$$\epsilon(\alpha, \beta) = \sum_{m=1}^{m_{\text{max}}} \left| \frac{\langle x^m \rangle_{\text{rec.}}}{\langle x^m \rangle_H} - 1 \right|$$

Extracting LCDAs

$$\phi_M(x, \mu)$$

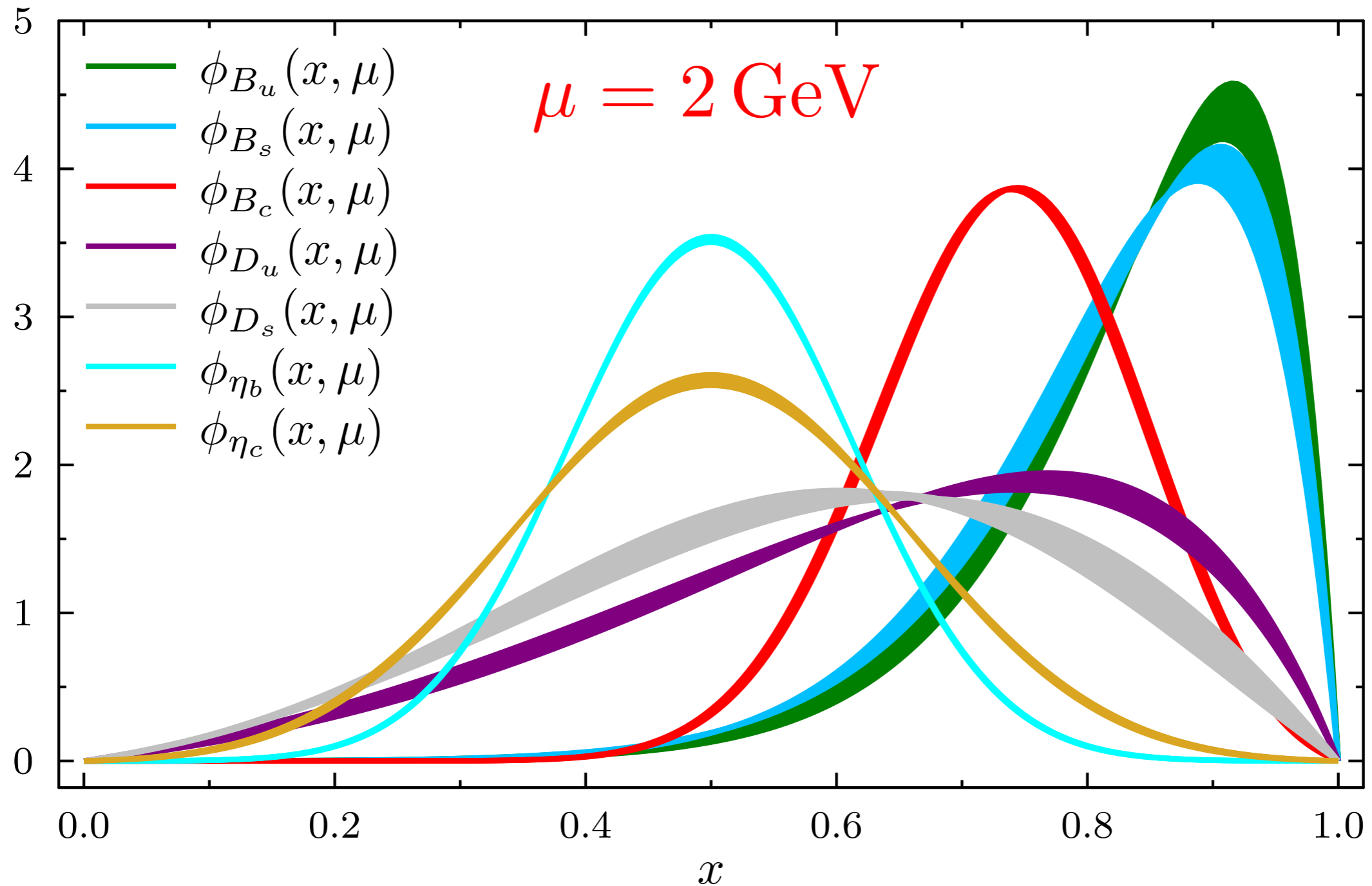


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F. E. S, R. C. da Silveira, J. Cobos-Martínez, B. El-Bennich and E. Rojas, Eur. Phys. J. C 80, 955 (2020)

Extracting LCDAs

$$\phi_M(x, \mu)$$



F. E. S, R. C. da Silveira, J. Cobos-Martínez, B. El-Bennich and E. Rojas, Eur. Phys. J. C 80, 955 (2020)

Extracting LCDAs

● LCDAs for vector mesons:

PHYSICAL REVIEW D **106**, L091504 (2022)

Letter

D^* and D_s^* distribution amplitudes from Bethe-Salpeter wave functions

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We report on the first calculation of the longitudinal and transverse light front distribution amplitudes of the D^* and D_s^* mesons and their first four moments. As a by-product, we also obtain these distribution amplitudes for the ρ , ϕ , K^* , and J/Ψ mesons and confirm a prediction of lattice QCD for the vector kaon: while the longitudinal distribution amplitude is almost symmetric, the transverse one is oblique implying that the strange quark carries more momentum.

DOI: [10.1103/PhysRevD.106.L091504](https://doi.org/10.1103/PhysRevD.106.L091504)

Extracting LCDAs

Bound-state equation for vector mesons

$$\Gamma_\mu(k; P) = \int^\Lambda \frac{d^4 q}{(2\pi)^4} K(k, q; P) S_f(q_+) \Gamma_\mu(q; P) S_g(q_-)$$

$$\Gamma_\mu(k; P) = \sum_{i=1}^8 T_\mu^i(k, P) F_i(k^2, z_k, P^2)$$

$$T_\mu^1(k, P) = i\gamma_\mu^T,$$

$$T_\mu^2(k, P) = i[3k_\mu^T(\gamma \cdot k^T) - \gamma_\mu^T(k^T)^2],$$

$$T_\mu^3(k, P) = i(k \cdot P)k_\mu^T \gamma \cdot P,$$

$$T_\mu^4(k, P) = i[\gamma_\mu^T \gamma \cdot P(\gamma \cdot k^T) + k_\mu^T \gamma \cdot P],$$

$$T_\mu^5(k, P) = k_\mu^T,$$

$$T_\mu^6(k, P) = (k \cdot P)[\gamma_\mu^T(\gamma \cdot k^T) - (\gamma \cdot k^T)\gamma_\mu^T],$$

$$T_\mu^7(k, P) = \gamma_\mu^T \gamma \cdot P - \gamma \cdot P \gamma_\mu^T - 2T_\mu^8(k, P),$$

$$T_\mu^8(k, P) = \hat{k}_\mu^T(\gamma \cdot \hat{k}^T)\gamma \cdot P,$$

where $a_\mu^T = T_{\mu\nu} a_\nu$, with $T_{\mu\nu} = \delta_{\mu\nu} - P_\mu P_\nu / P^2$, $P \cdot a^T = 0$ for any four-vector a_μ and $\hat{k}^T \cdot \hat{k}^T = 1$.

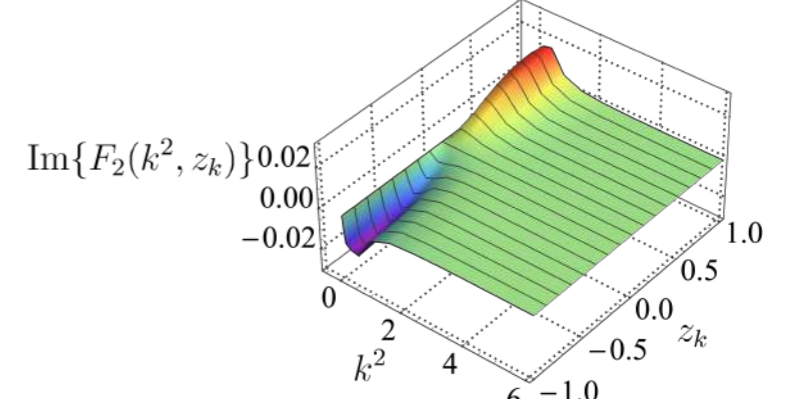
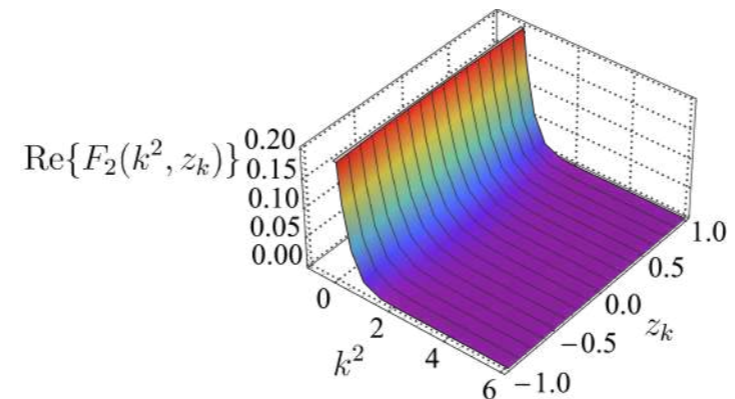
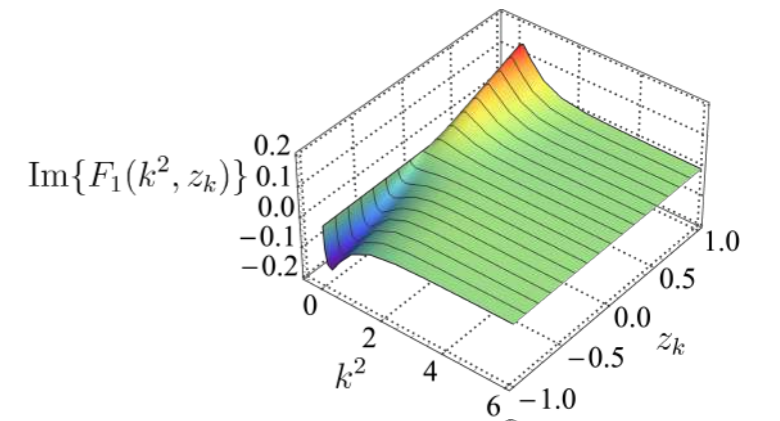
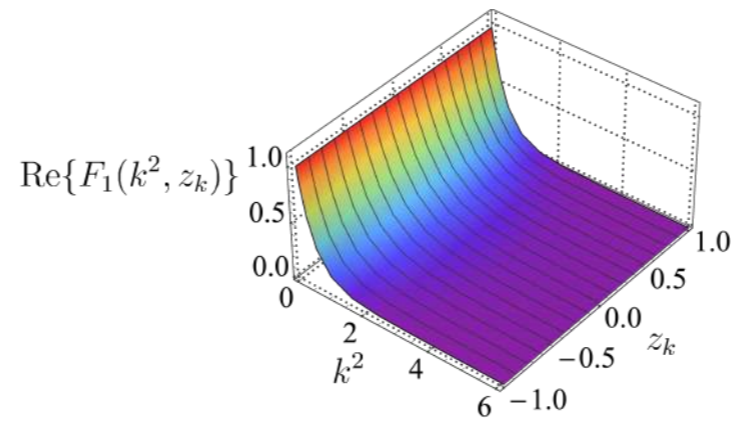
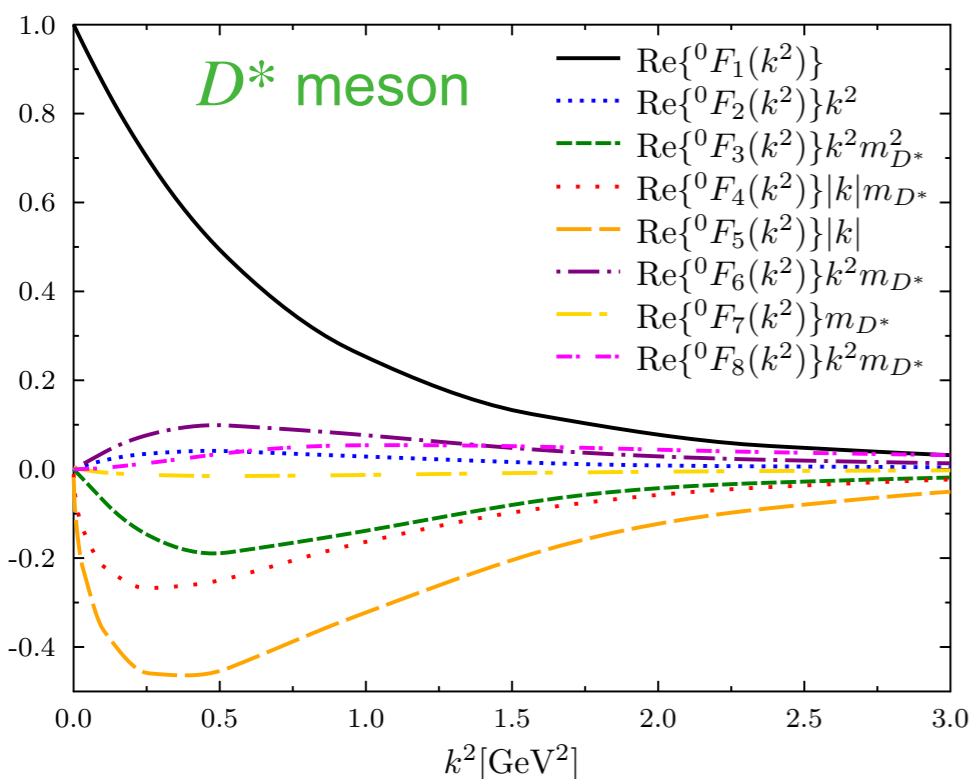
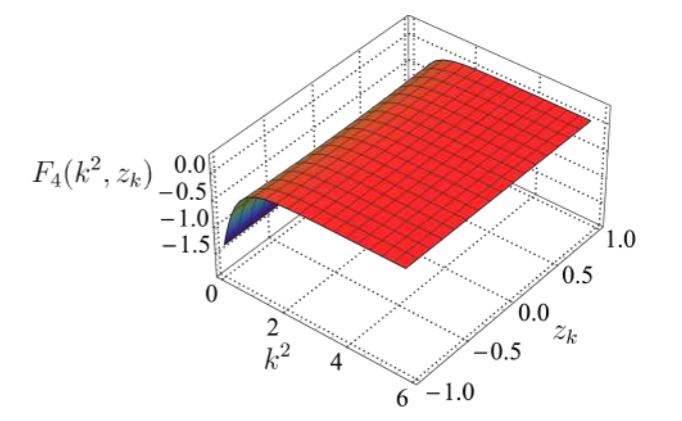
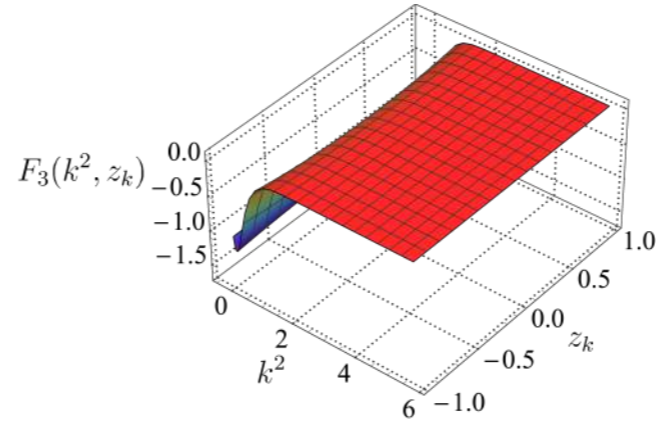
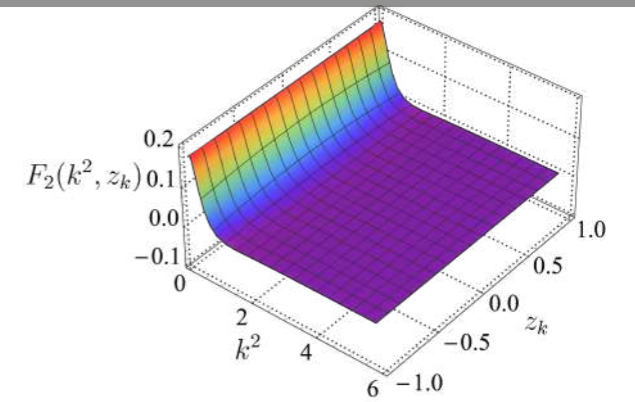
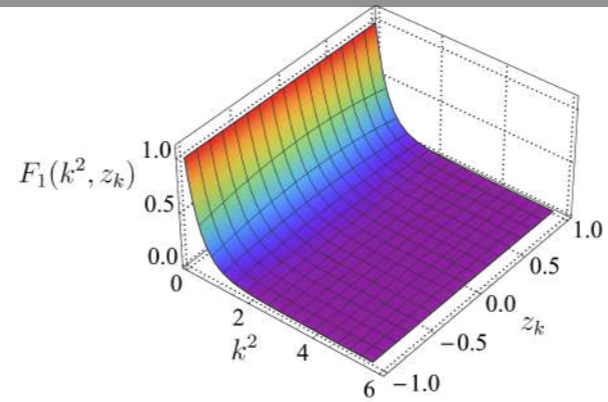
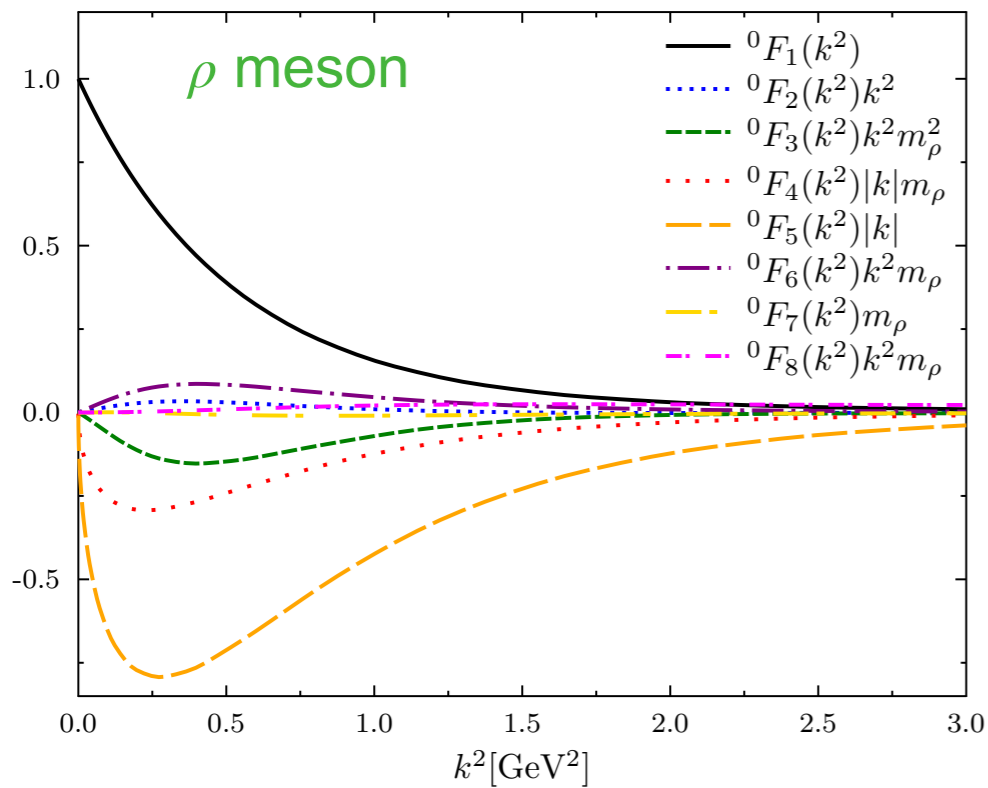
Extracting LCDAs

● Bound-state equation for vector mesons

	M_V	M_V^{exp}	ϵ_{M_V} [%]	f_V	$f_V^{\text{exp/lQCD}}$	ϵ_{f_V} [%]
$\rho(u\bar{u})$	0.730	0.775	5.81	0.145	0.153(1)	5.23
$\phi(s\bar{s})$	1.070	1.019	5.20	0.187	0.168(1)	11.31
$K^*(u\bar{s})$	0.942	0.896	5.13	0.177	0.159(1)	11.32
$D^*(c\bar{d})$	2.021	2.009	0.60	0.165	0.158(6)	4.43
$D_s^*(c\bar{s})$	2.169	2.112	2.70	0.205	0.190(5)	7.90
$J/\psi(c\bar{c})$	3.124	3.097	0.87	0.277	0.294(5)	5.78
$\Upsilon(b\bar{b})$	9.411	9.460	0.52	0.594	0.505(4)	17.62

$$f_V M_V = \frac{Z_2 N_c}{3\sqrt{2}} \int^\Lambda \frac{d^4 k}{(2\pi)^4} \text{Tr}_D \left[\gamma_\mu S_f(k_\eta) \Gamma_{V\mu}^{fg}(k, P) S_g(k_{\bar{\eta}}) \right]$$

Extracting LCDAs



Extracting LCDAs

● LCDAs for vector mesons:

► For a longitudinally polarized vector meson

$$f_v(n \cdot P) \phi_{\parallel}(z, \mu) = m_v \text{Tr}_{\text{CD}} Z_2 \int_q \delta(n \cdot q_+ - x n \cdot P) \gamma \cdot n n_{\nu} \chi_{\nu}(q; P)$$

► For a transversely polarized vector meson

$$f_v^{\perp} m_v^2 \phi_{\perp}(z, \mu) = (n \cdot P) \text{Tr}_{\text{CD}} Z_T \int_q \delta(n \cdot q_+ - x n \cdot P) \sigma_{\mu\nu} P_{\mu} \chi_{\nu}(q; P)$$

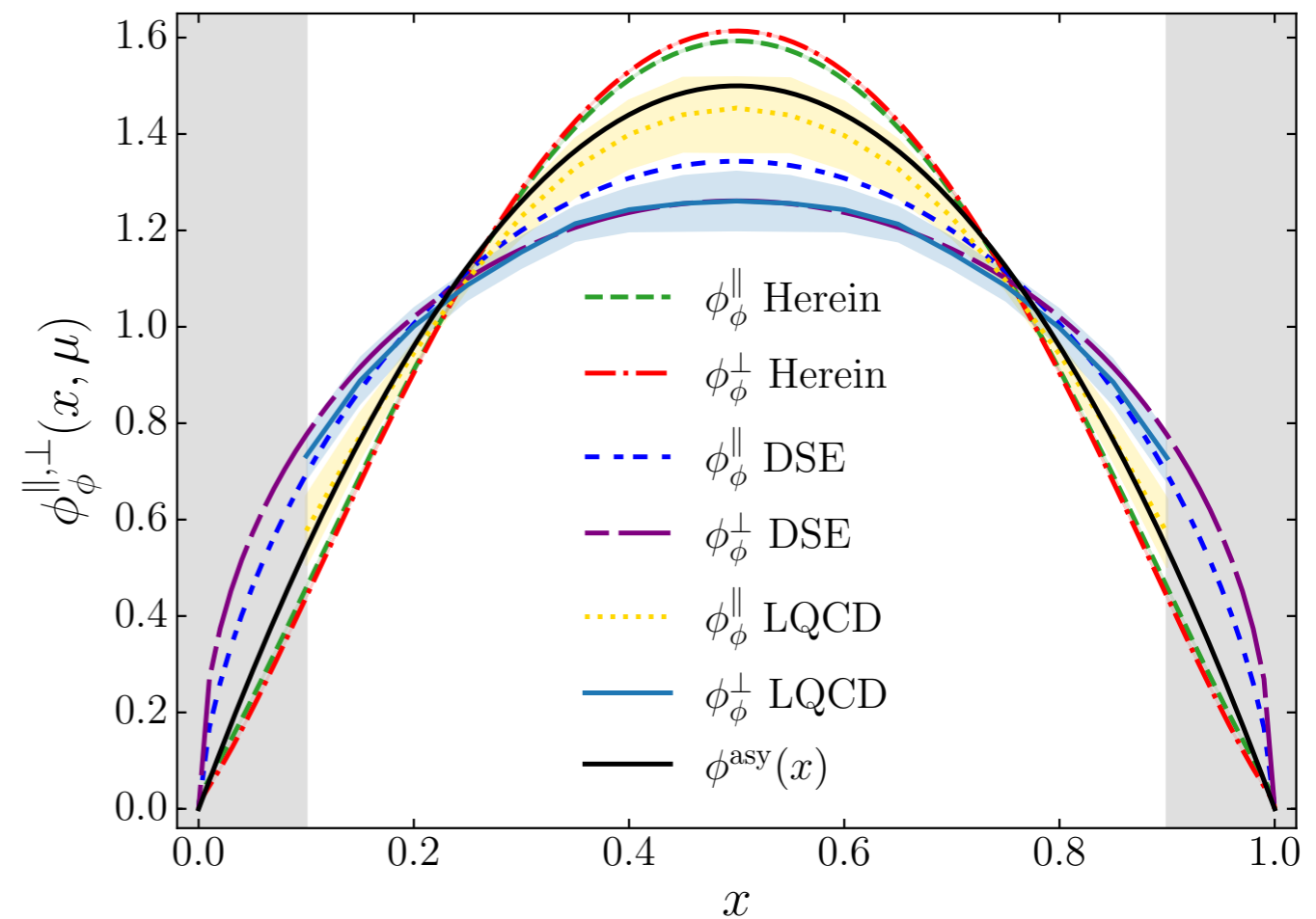
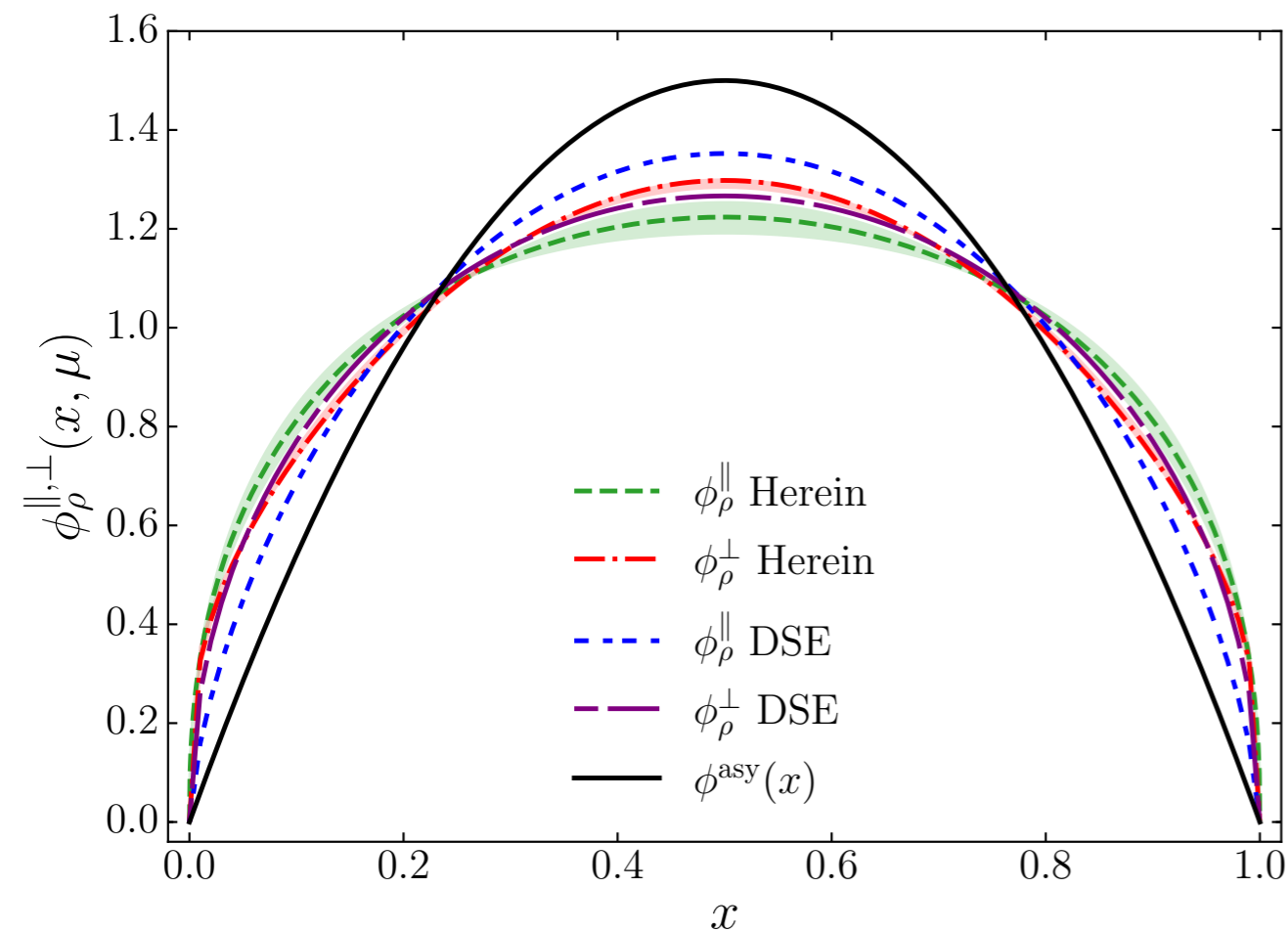
$$\chi_{\nu}(q; P) = S_f(q_+) \Gamma_{\nu}(q; P) S_g(q_-),$$

Extracting LCDAs

$$\phi_M(x, \mu)$$

Longitudinal and transverse LCDAs (ρ and ϕ mesons)

$$\phi_{V\text{rec}}^{\parallel,\perp}(x, \mu) = \mathcal{N}(\alpha)[x\bar{x}]^{\alpha-\frac{1}{2}} \left[1 + \sum_{n=1}^N a_n C_n^\alpha(2x-1) \right]$$



Fei Gao, Lei Chang, et al., Phys. Rev. D 90, 014011(2014).

Lattice QCD: Lattice Parton Collaboration, PRL 127 (2021)

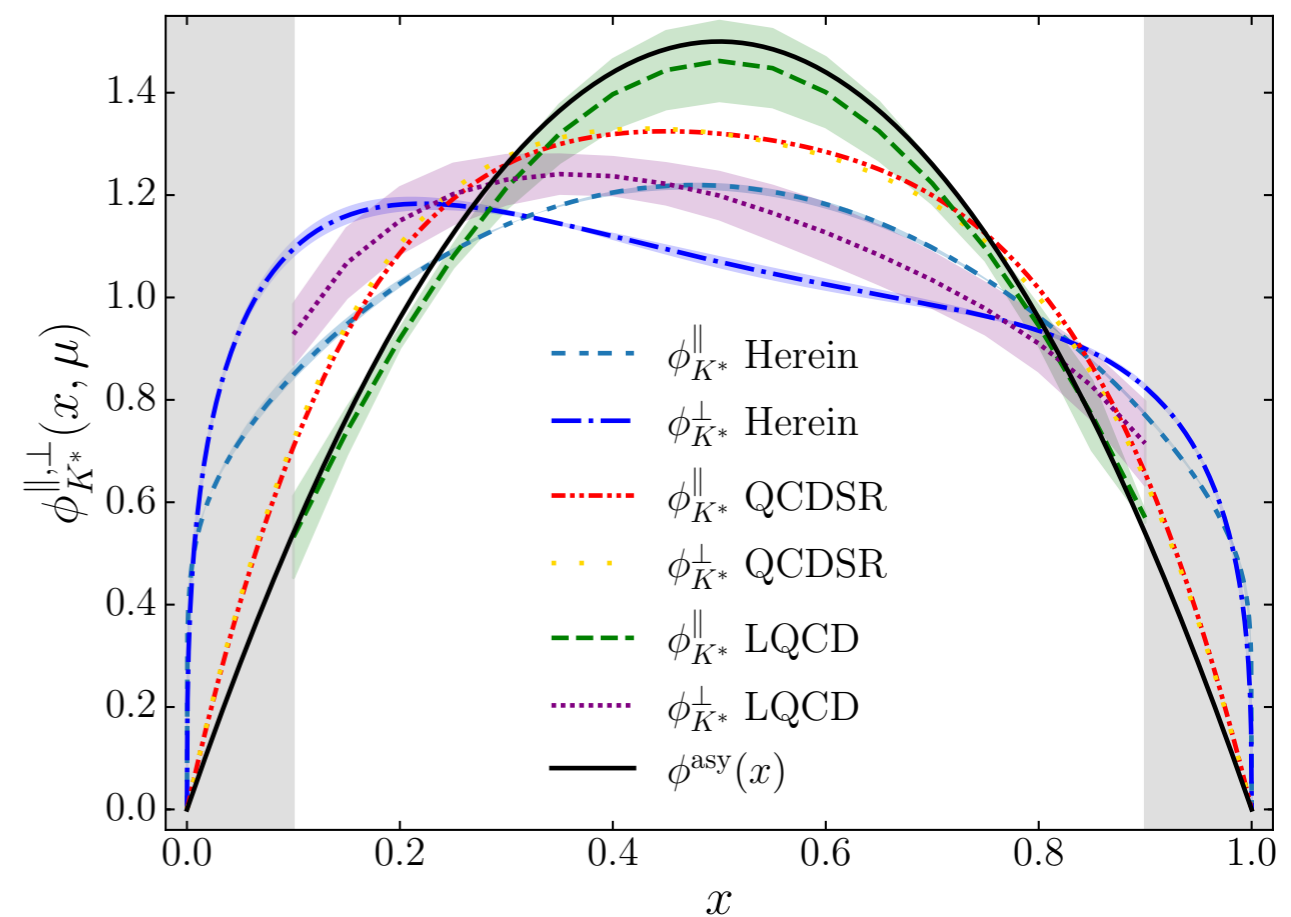
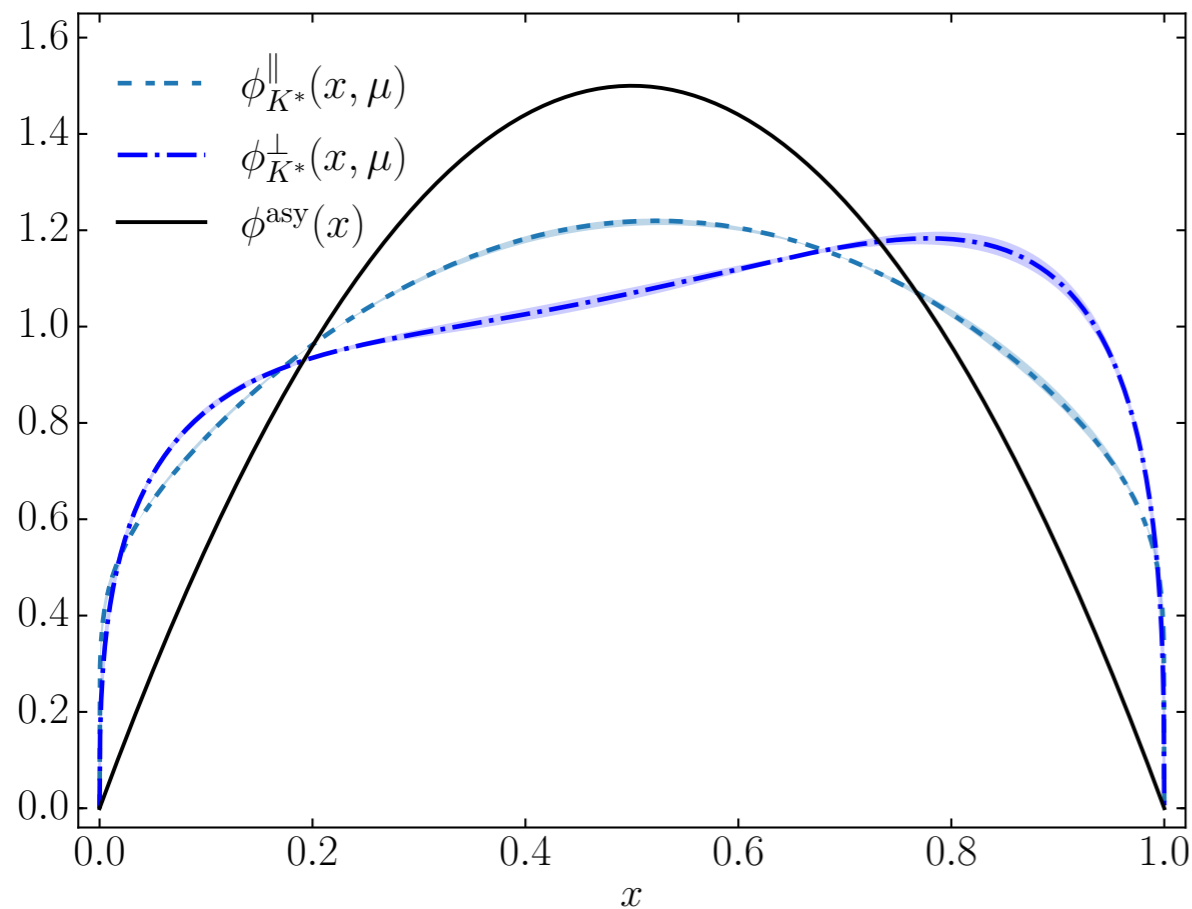
F. E. S, R. C. da Silveira, B. El-Bennich, Phys. Rev. D 106, L091504 (2022).

Extracting LCDAs

$$\phi_M(x, \mu)$$

Longitudinal and transverse LCDAs (K^* meson)

$$\phi_{V\text{rec}}^{\parallel,\perp}(x, \mu) = \mathcal{N}(\alpha)[x\bar{x}]^{\alpha-\frac{1}{2}} \left[1 + \sum_{n=1}^N a_n C_n^\alpha(2x-1) \right]$$



F. E. S, R. C. da Silva, B. El-Bennich, Phys. Rev. D 106, L091504 (2022).

Lattice QCD: Lattice Parton Collaboration, PRL 127 (2021)

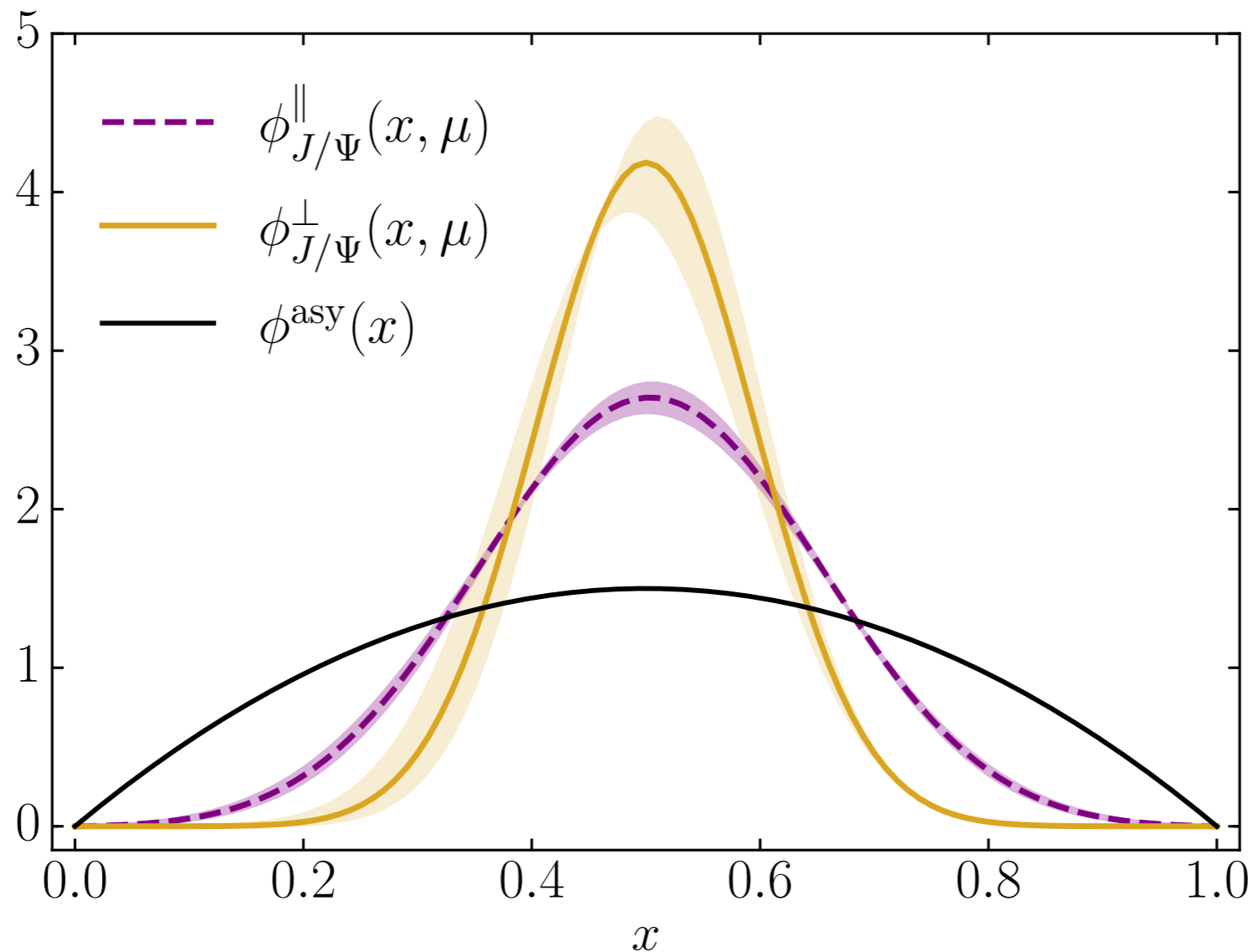
QCD Sum Rule: P. Ball, V. M. Braun and A. Lenz, JHEP 08 (2007)

Extracting LCDAs

$$\phi_M(x, \mu)$$

Longitudinal and transverse LCDAs

$$\phi_{V_{\text{rec}}}^{\parallel, \perp}(x, \mu) = \mathcal{N}(\alpha, \beta) 4x\bar{x} e^{4\alpha x\bar{x} + \beta(x-\bar{x})}$$



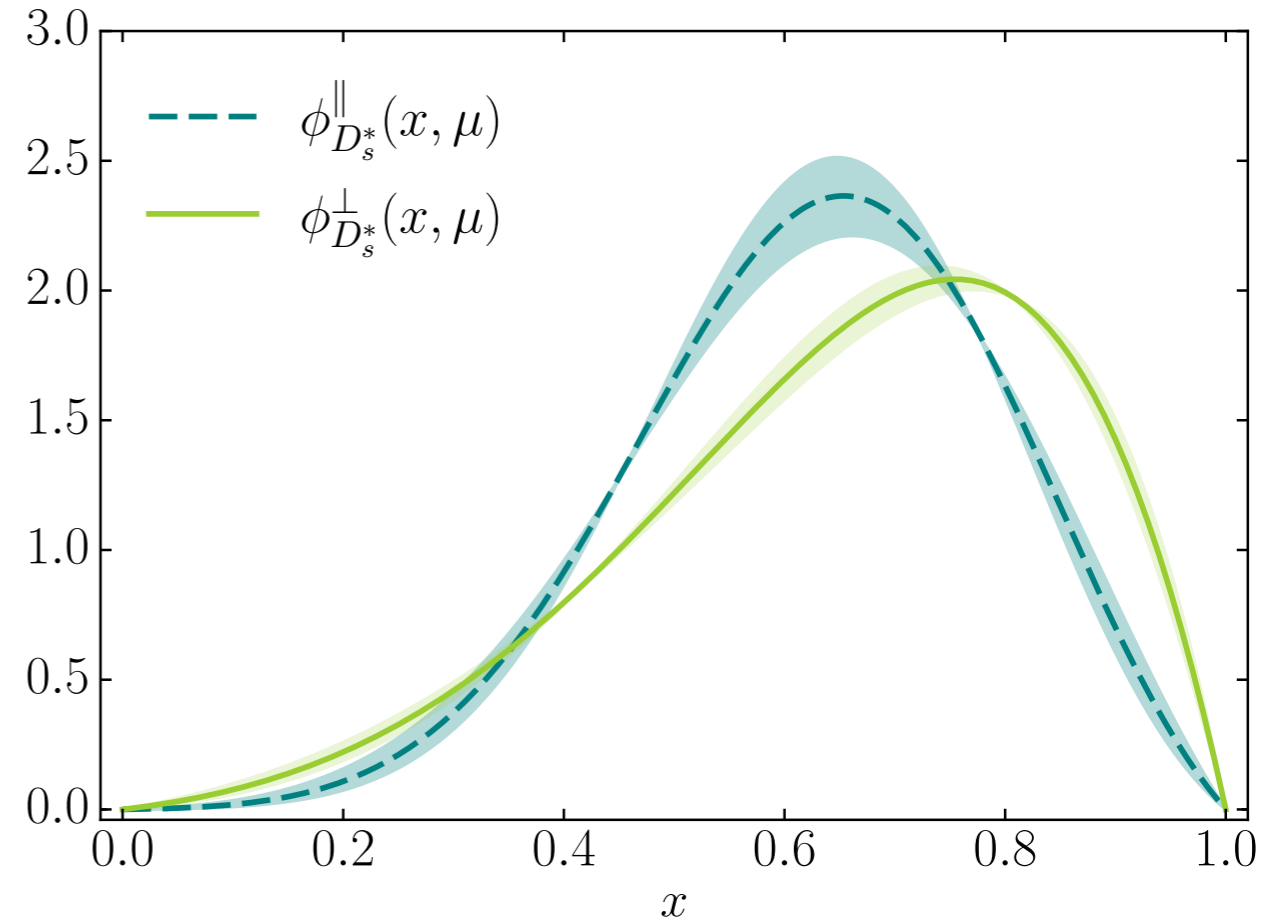
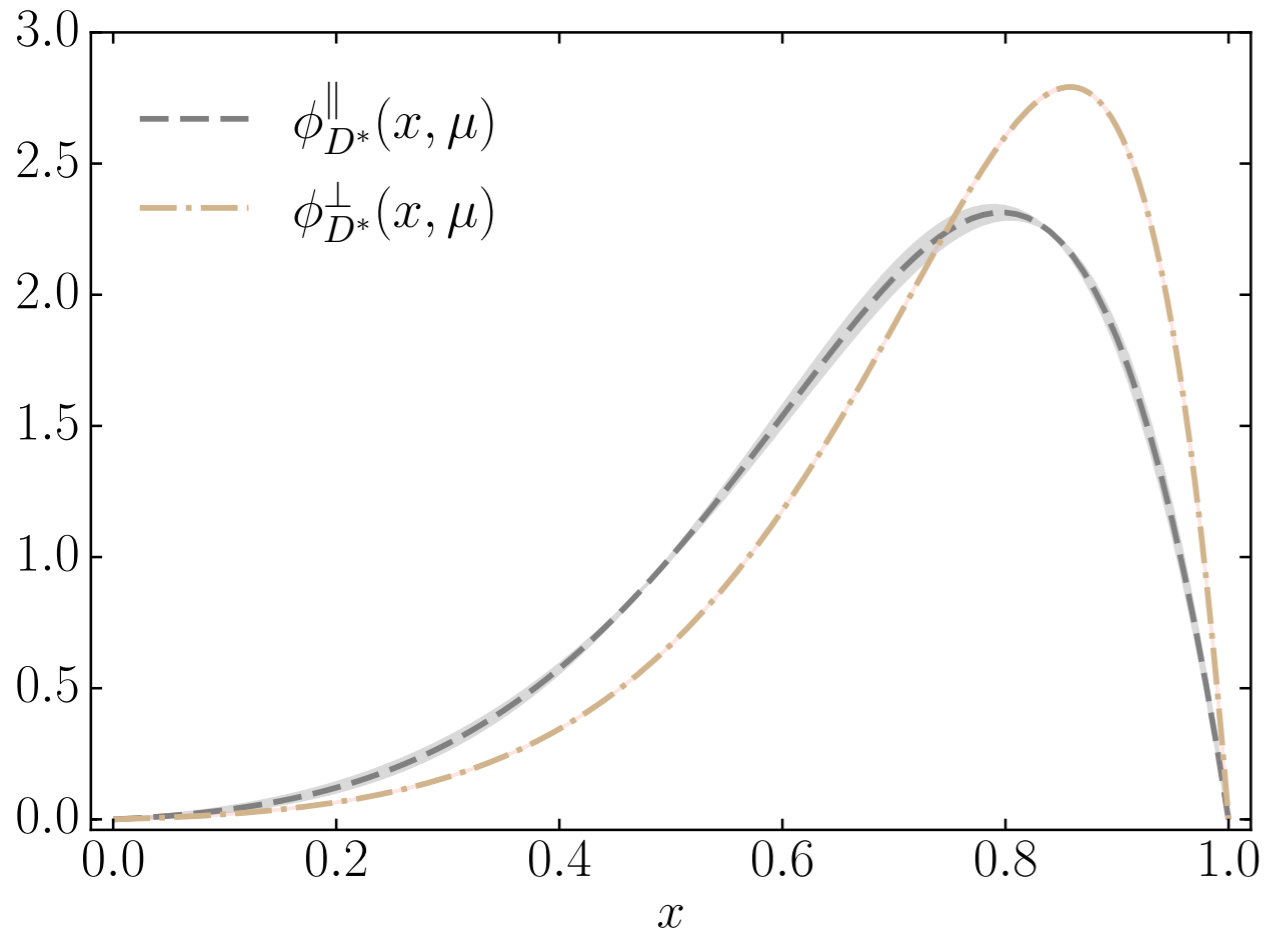
F. E. S, R. C. da Silveira, B. El-Bennich, Phys. Rev. D 106, L091504 (2022).

Extracting LCDAs

$$\phi_M(x, \mu)$$

Longitudinal and transverse LCDAs

$$\phi_{V\text{rec}}^{\parallel, \perp}(x, \mu) = \mathcal{N}(\alpha, \beta) 4x\bar{x} e^{4\alpha x\bar{x} + \beta(x-\bar{x})}$$



F. E. S, R. C. da Silveira, B. El-Bennich, Phys. Rev. D 106, L091504 (2022).

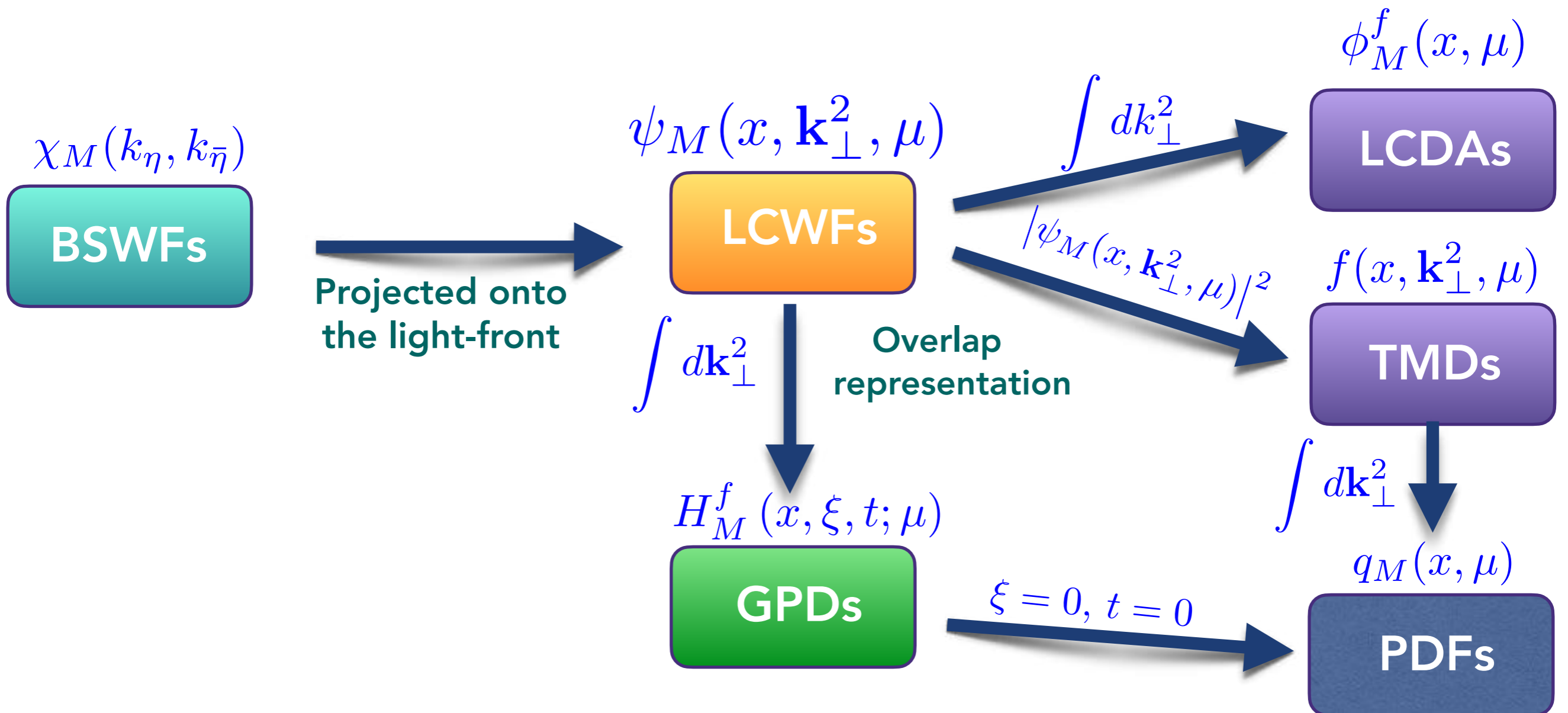


Light-cone wave functions and quantum information

- Bruno El-Bennich, Universidade Cidade de São Paulo, Brazil.
- Gastão Krein, IFT-UNESP, Brazil.
- Ian Cloët, Argonne National Lab, US.

Light-Cone Wave Functions (LCWFs)

- From the **LCWFs** we can access to structure functions:



- **The inputs:** Solutions from **DSE** and **Bound-State** equations.

Light-Cone Wave Functions (LCWFs)

- The **LCWFs** are obtained from the **BSWF** via the light front projections:

$$\psi_M^{\uparrow\downarrow}(x, \mathbf{k}_\perp^2) = \sqrt{3}i \int \frac{dk^+ dk^-}{\pi} \delta(xP^+ - k^+) \text{Tr}_D[\gamma^+ \gamma_5 \chi(k, P)],$$

$$\psi_M^{\uparrow\uparrow}(x, \mathbf{k}_\perp^2) = -\frac{\sqrt{3}i}{\mathbf{k}_\perp^2} \int \frac{dk^+ dk^-}{\pi} \delta(xP^+ - k^+) \text{Tr}_D[i\sigma_{+i} k_T^i \gamma_5 \chi(k, P)]$$

- With the **LCWF** one can readily derive two distributions:

- The leading-twist **TMD**

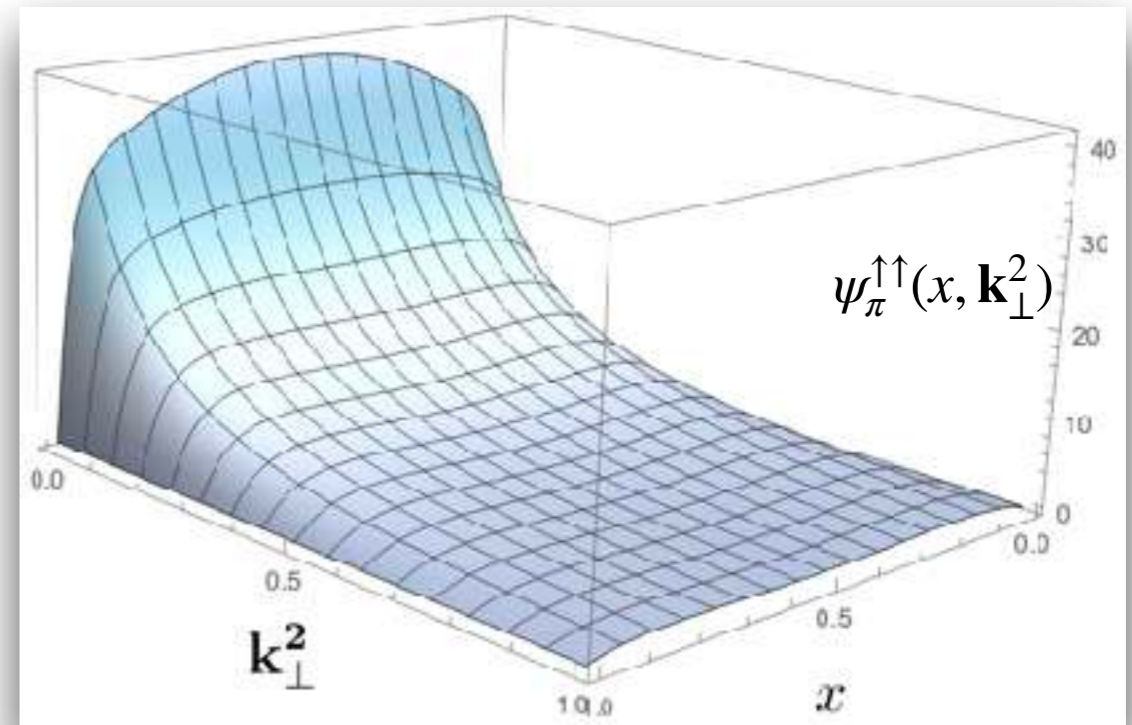
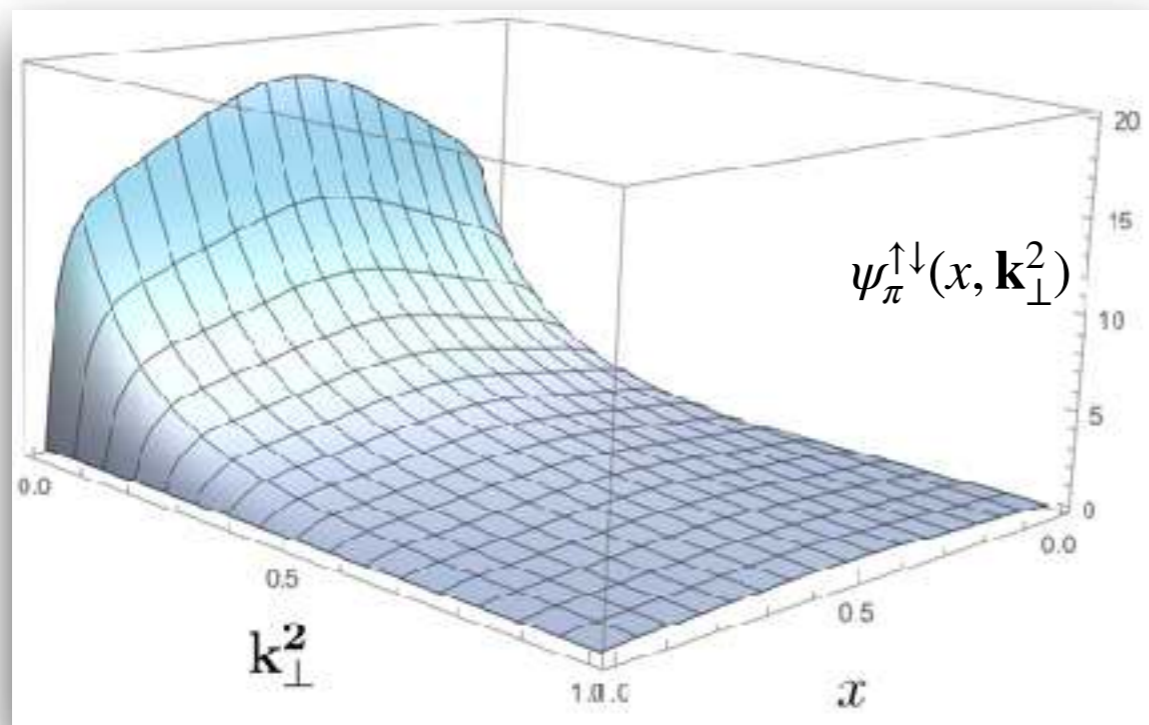
$$f_M(x, \mathbf{k}_\perp^2, \mu) = \frac{1}{(2\pi)^3} \left| \psi_M^{\uparrow\downarrow}(x, \mathbf{k}_\perp^2, \mu) + \mathbf{k}_\perp^2 \psi_M^{\uparrow\uparrow}(x, \mathbf{k}_\perp^2, \mu) \right|^2$$

- The **PDF**

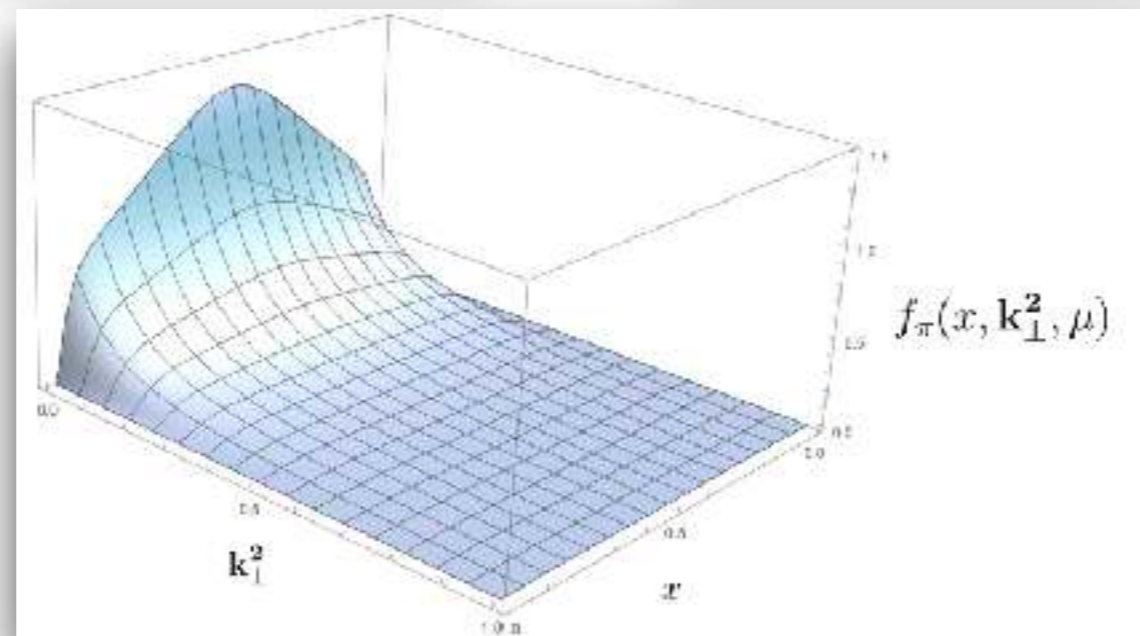
$$q_M(x, \mu) = \int d\mathbf{k}_\perp^2 f_M(x, \mathbf{k}_\perp^2, \mu)$$

Light-Cone Wave Functions (LCWFs)

- Pion LCWF



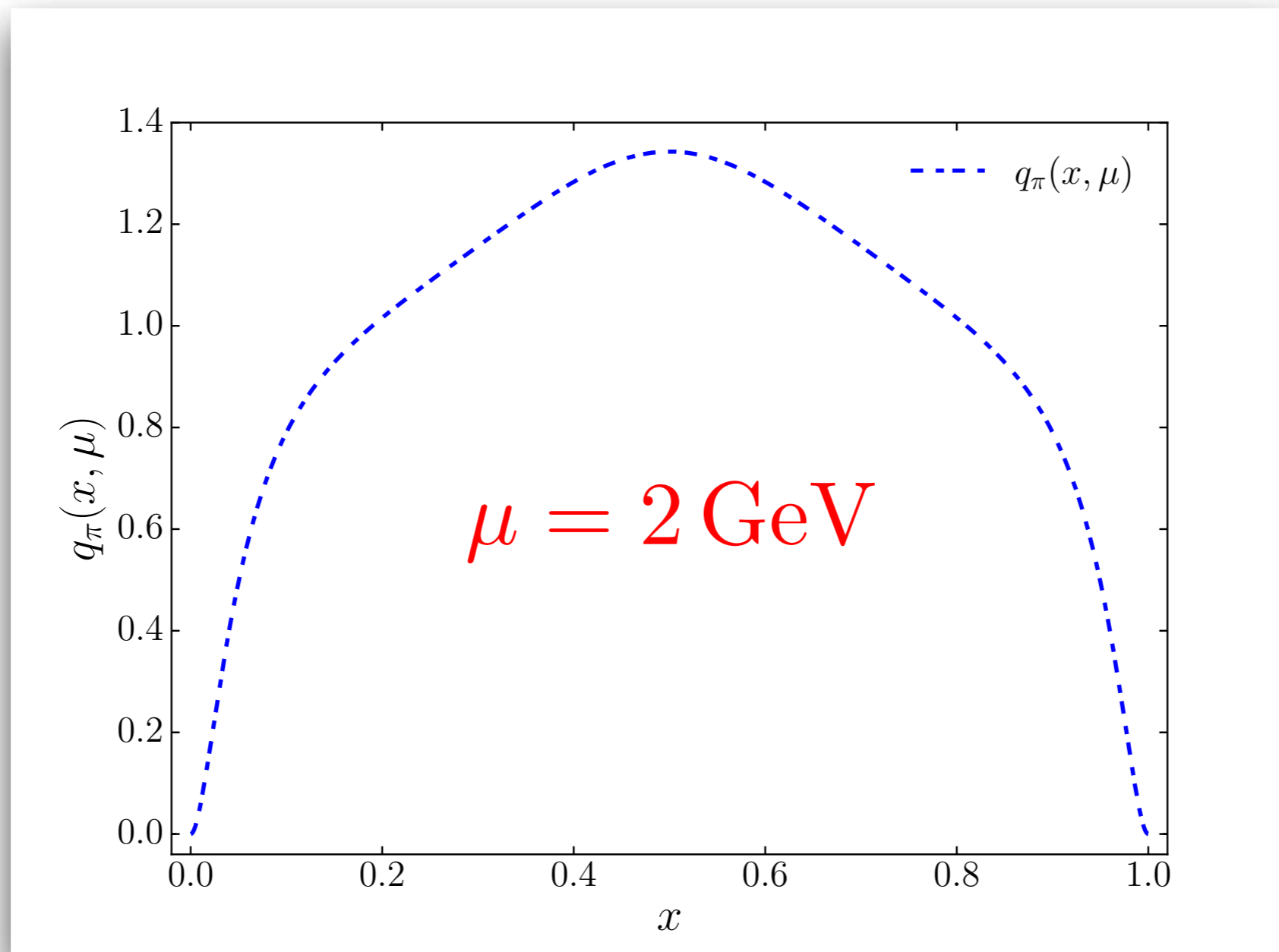
- Pion TMD



Light-Cone Wave Functions (LCWFs)

- Pion PDF

$$q_M(x, \mu) = \int d\mathbf{k}_\perp^2 f_M(x, \mathbf{k}_\perp^2, \mu)$$



Entanglement entropy of constituent quarks in the pion LCWF

- Entropy of the valence distribution of the valence quark distribution in the pion.

$$s(\mu^2) = - \int_0^1 \{ q_\pi^u(x, \mu^2) \log[q_\pi^u(x, \mu^2)] + q_\pi^d(x, \mu^2) \log[q_\pi^d(x, \mu^2)] \} dx$$

- This definition is related to the **Von Neumann entropy** for a quantum object in a non-pure state.

- Explore how this entropy evolves as a function of μ^2 .

- Explore how the entropy behaves when the coupling constant in the gap and bound-state equations decreases or increases.

This study represents an initial step into the world of quantum information from my standpoint of view as a hadron physicist.

Conclusions

- Good reproduction of the meson mass spectrum and their weak decay constants for pseudoscalar and vector channels .
- First predictions for **LCDAs** of vector D and D_s mesons.
- **LCDAs** of the J/Ψ and D_s^* mesons can readily be used in diffractive vector-meson production which are of interest to the experimental program of the **EIC**.
- We also are interested in study the entropy of the valence distribution of the valence quark distribution in the pion.



Universidad de Sucre

INCLUYENTE, INTEGRADA Y PARTICIPATIVA



Thank you