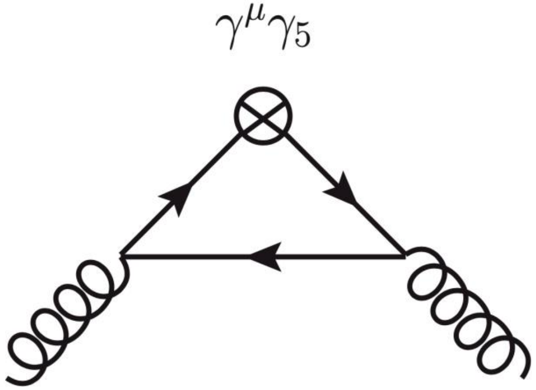
Unraveling the Axial Anomaly: Its Role in the ‘Spin Crisis’ and the ‘$U_A(1)$ Problem’



Theory Seminar JLab

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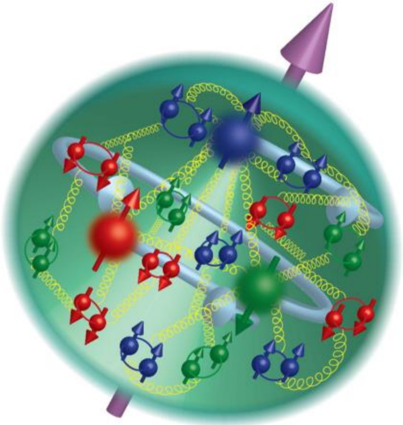
January 2025



Jefferson Lab

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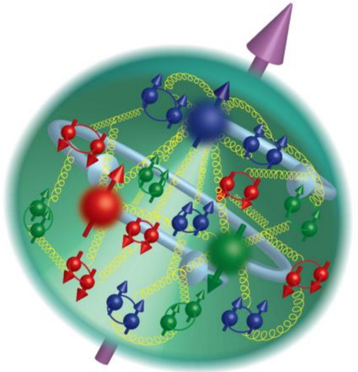
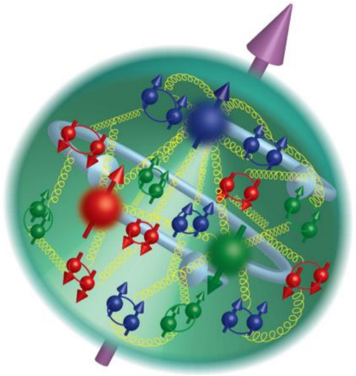
Outline

- Spin Crisis and $U_A(1)$ problem and its relationship with DIS.
- Anomaly poles in the triangle diagram?
- Non-local version of the problem: DVCS and GPDs.
- Conclusions.



Spin crisis or ‘g1 problem’ (JM)

$$(\Delta u + \Delta d + \Delta s)s^{\mu} \equiv \Sigma(\mu^2)s^{\mu}$$



Naive picture: contribution to the spin of the proton comes ***only*** from the spin of the *quarks* and *anti-quarks* ***and*** it is ***given*** by *Σ.*

Doesn’t take into account other possible contributions: angular momentum of partons and gluon helicity.

Gives *Σ* the interpretation of contribution of quarks and anti-quark spin only.

Jaffe and Manohar 1990

Spin crisis or ‘g1 problem’ (JM)

$$\int_0^1 \mathrm{d}x\, g_1^{\mathrm{p}}(x,Q^2) = \frac{1}{18}\left(3F + D + 2\Sigma(Q^2)\right)\left(1 - \frac{\alpha_s}{\pi} + \mathrm{O}(\alpha_s^2)\right) + \mathrm{O}\left(\frac{\Lambda^2}{Q^2}\right)$$

Measurable quantities

F and *D* already known from experiments, EMC collaboration measured g_1 in 1987, then ‘*g1 problem*’ (last to be measured).

But even before the ‘spin crisis’, it was ***already not expected*** that *Σ=1.*

Assuming: $\Delta s(Q^2_{\mathrm{EMC}}) \cong 0$ ⇒ $\Sigma(Q^2_{\mathrm{EMC}})_{\mathrm{EJ}} = 3F - D = 0.60 \pm 0.12$

Ellis-Jaffe ***1974***

In this picture 60% of the spin of the proton is carried by the spin of quarks and anti-quarks.

Spin crisis or ‘g1 problem’ (JM)

Results from EMC:

$$\int_0^1 \mathrm{d}x g_1^{\mathrm{p}}(x) = 0.126 \pm 0.010 \pm 0.015$$

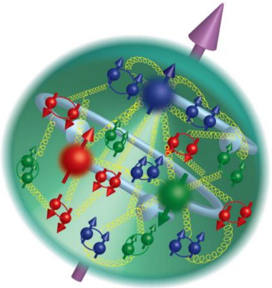
$$\Delta u(Q^2_{\mathrm{EMC}}) = 0.78 \pm 0.08\,,$$

$$\Delta s(Q^2_{\mathrm{EMC}}) = -0.16 \pm 0.08$$

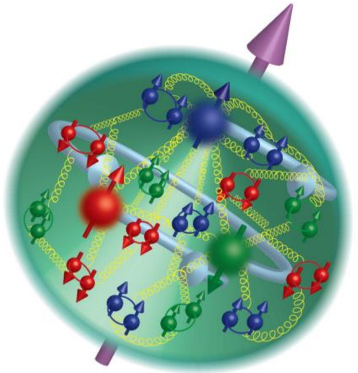
Q^2_{EMC}~ 10 GeV^2: strange quarks?

$$\Delta d(Q^2_{\mathrm{EMC}}) = -0.50 \pm 0.08\,,$$

$$\Sigma(Q^2_{\mathrm{EMC}}) = 0.13 \pm 0.19$$



Spin crisis or ‘g1 problem’ (AR-CCM)



Naive picture: contribution to the spin of the proton comes ***only*** from the spin of the *quarks* and *anti-quarks* ***and*** it is ***given*** by *Σ.*

AR picture: there is an important gluonic contribution of order α_s to $\Sigma(Q^2)$: Δg contribution could be large and compensates $\alpha_s \sim 0.25$.

$$\Delta g \equiv \int_0^1 dx\, [g_+(x,Q^2) - g_-(x,Q^2)]$$

This is a measurable quantity and CCM proposed measurements of *Δg* in di-jet production.

Altarelli and Ross 1988
Carlitz, Collins and Muller 1988

Spin crisis or ‘g1 problem’ (AR)

$$(\Delta u + \Delta d + \Delta s)s^\mu \equiv \Sigma(\mu^2)s^\mu = \langle N(ps)| \sum_{b=1}^{3} \bar{\psi}_b \gamma_\mu \gamma_5 \psi_b |N(ps)\rangle$$

The ‘singlet’ axial current is not conserved even in *massless* QCD. Here we assume N_f ***massless*** quarks.

$$j_5^\mu = \sum_i^{N_f} = \tilde{q}_i \gamma^\mu \gamma_5 q_i, \quad \partial_\mu j_5^\mu = \frac{\alpha_s N_f}{8\pi} \sum_i^{8} \epsilon_{\mu\nu\rho\sigma} F_i^{\mu\nu} F_i^{\rho\sigma}$$

*The divergence of the axial current is the divergence of a **gluonic operator**.*

$$k_\mu = \frac{\alpha_s}{2\pi} \epsilon_{\mu\nu\lambda\sigma} \mathrm{Tr}\left[A^\nu \left(F^{\lambda\sigma} - \frac{2}{3} A^\lambda A^\sigma\right)\right]$$

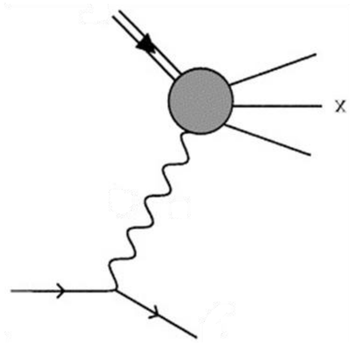
$$j_5^\mu = \tilde{j}_5{}^\mu + k^\mu$$

$$\boxed{\Delta q = \widetilde{\Delta q} - \frac{\alpha_s}{2\pi} N_f \, \Delta g}$$

Main idea of AR: ‘**pure *gluonic’*** *term* ***cancels*** *the* ***‘pure quark’*** *term, restoring the idea that the ‘pure quark’ contribution to the spin of the proton is large.*

DIS, the box diagram and its relation with the axial anomaly

REMINDER:



DIS: polarized lh→lX collisions gives $g_1(x,Q^2)$.

For the 'hadronic part'..

… use the optical theorem and factorization theorem to get cross sections.

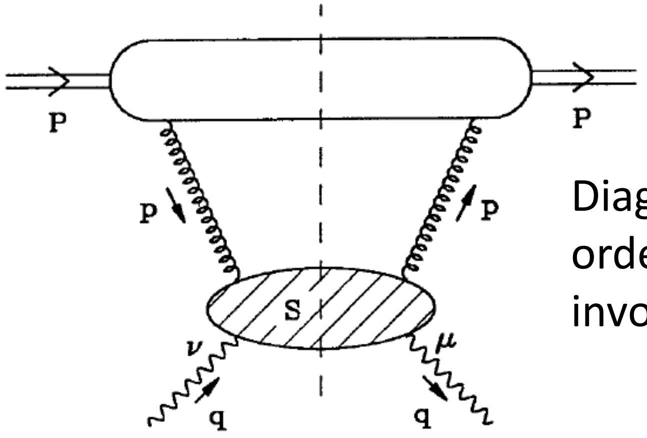
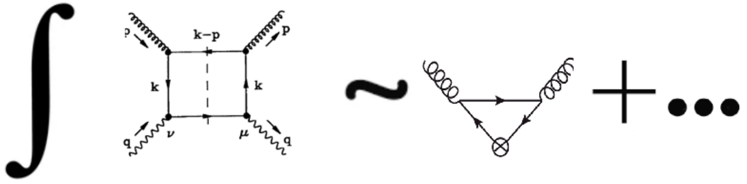


Diagram appearing to order α_s . Its hard part involves the box diagram.



x integration.

$$\left\langle p, s \left| \sum_{b=1}^{3} \bar{\psi}_b \gamma_\mu \gamma_5 \psi_b \right|_{\mu^2} \right| p, s \right\rangle$$

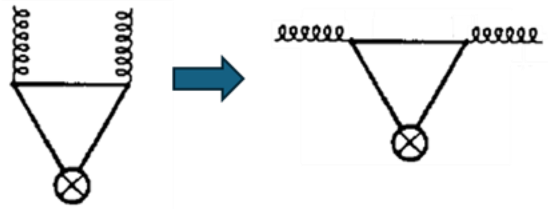
$\int g_1(x,Q^2)\, dx$ includes a term with the anomalous axial current.

Main criticisms (JM-BQ)

$$\Delta q = \widetilde{\Delta q} - \frac{\alpha_s}{2\pi} N_f \, \Delta g$$

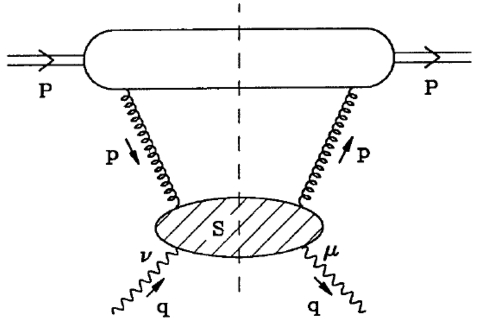
The operator k^μ (Chern-Simons current) is:

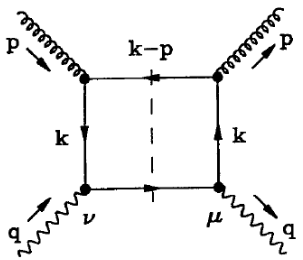
1. gauge dependent (already noticed by AR)
2. not well defined (***non-local in the usual sense***): 'anomaly poles'.



$$\Delta q = \widetilde{\Delta q} - \frac{\alpha_s}{2\pi} N_f \, \Delta g$$

AR and CCM agree in the 'hard' order α_s prefactor of Δg *using different IR regulators.*





BQ: ***If one regulates 'properly'*** (gauge invariant way) the ***IR divergences*** of the box diagram, one obtains ***a vanishing prefactor.***

Bodwin and Qiu 1989

Our take on the ‘anomaly poles’ (JM)

regularization-independent. In QCD, the pole at $l^2 = 0$ is unphysical and is cancelled by non-triangle contributions to the matrix element of A^0_μ. With the aid of this we can decompose the matrix element of A^0_μ into the contribution from the triangle and the rest, using eq. (5.10),

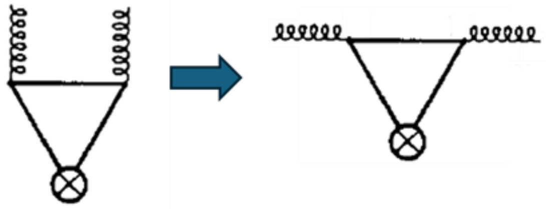
$$\Sigma(t)s^\mu + l\cdot sl^\mu h(t) = \frac{l\cdot sl^\mu}{l^2}\kappa(t) + \left(s^\mu - \frac{l\cdot sl^\mu}{l^2}\right)\lambda(t)\,, \qquad (5.19)$$

where the first term on the right comes from the triangle and the second is the rest. The cancellation of the anomaly pole, as well as the Ward identity, eq. (5.14), requires $\lambda(0) = \kappa(0)$. Note that the triangle contribution is singular as $l^\lambda \to 0$: the answer depends on the direction in which the limit is taken. [Of course, $\Sigma(0)$ is finite and independent of the direction of the limit.] Apparently, the (forward) calculation of refs. [12, 13] is equivalent to taking the limit so that $l\cdot sl^\mu/l^2 \to s^\mu$.

To summarize: a study of the $U(1)_A$ Ward identity in QCD for $m = 0$ reveals that it is impossible in principle to separate a gluonic and an “intrinsic quark” contribution to Σ. It also suggests how, ignoring certain non-perturbative effects, one could erroneously arrive at the separation proposed in refs. [12, 13].

How true is this statement?

Our take on the ‘anomaly poles’ (CFLMPR)



Is this diagram divergent in the collinear limit?
‘Anomaly pole’ in DIS (box diagram)?

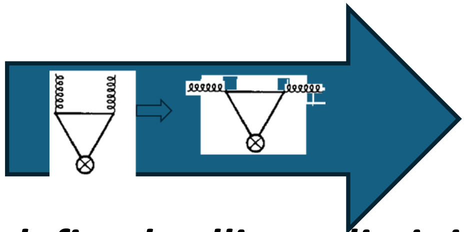
Non-perturbative effect?

- massive degenerate quarks (*m*)
- off-forward (*Δ=p’-p*)
- off-shell gluons ($p^2=p'^2$)
- contracted with polarization vectors ($\varepsilon.p= \varepsilon'.p'=0$)

$$A_1^\mu = -2i\,\varepsilon^{\mu\,\epsilon\,\epsilon'^*P}\,, \qquad A_2^\mu = \frac{2i}{\Delta^2}\,\Delta^\mu\,\varepsilon^{\epsilon\,\epsilon'^*P\,\Delta}\,, \qquad A_3^\mu = \frac{4i}{\Delta^2}\,(\epsilon\cdot P\,\varepsilon^{\mu\,\epsilon'^*P\,\Delta} + \epsilon'^*\cdot P\,\varepsilon^{\mu\,\epsilon\,P\,\Delta})\,.$$

$$\Gamma_5^\mu = G_1(\Delta^2)\left(A_1^\mu + \frac{\Delta^2}{\Delta^2-4p^2}\,A_3^\mu\right) + G_2(\Delta^2)\,A_2^\mu\,.$$

General form depends on *just two form factors*:
(‘Ward’ and ‘Schouten’ identities).



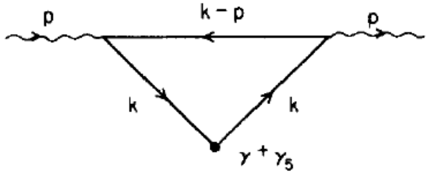
Well defined collinear limit in our perturbative calculation.

$$\lim_{\Delta^2\to 0}\Gamma_5^\mu = G_1(0;m,p^2)\,A_1^\mu$$

Castelli, Freese, Lorcé, Metz, Pasquini and Rodini 2024

Our take on the 'anomaly poles' (CFLMPR)

AR and CCM's calculation for the forward limit.



Question: Is the anomaly present in the forward limit?

$$i\underline{\Delta_\mu}\,\Gamma_5^\mu = \langle g(p',\lambda')\,|\,\partial_\mu J_5^\mu(0)\,|\,g(p,\lambda)\rangle\,.$$

The *intuition says no*: since in the forward limit Δ^μ is zero, so the divergence of the current doesn't give us information of the current in the forward limit…?

Let's see how things work in reality.
First, define a form factor *D* for the divergence of the current:

$$\langle g(p',\lambda')\,|\,\partial_\mu J_5^\mu(0)\,|\,g(p,\lambda)\rangle = -2\,\varepsilon^{\epsilon\,\epsilon'^{*}P\,\Delta}D(\Delta^2)$$

Our take on the 'anomaly poles' (CFLMPR)

Is the anomaly present in the forward limit?

$$\langle g(p',\lambda')\,|\,\partial_\mu J_5^\mu(0)\,|\,g(p,\lambda)\rangle = -2\,\varepsilon^{\epsilon\,\epsilon'^*\,P\,\Delta} D(\Delta^2)$$

$G(\Delta^2; m, 0) = D(\Delta^2; m, 0)$ On-shell gluons.

$G_1(0; m, p^2) = D(0; m, p^2)$ Collinear gluons.

Now, we define from the form factors of the axial current:

$$G = G_1 + G_2$$

$G_2(0; m, p^2) = 0$

Explicit calculation.

$$\lim_{\Delta^2\to 0} \Gamma_5^\mu = G_1(0; m, p^2)\, A_1^\mu$$

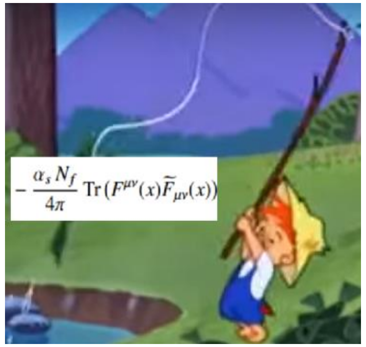
We find that in the collinear limit, the axial current is ***totally*** defined in terms of its divergence. And the ***collinear limit is well defined***.

$$\partial_\mu J_5^\mu(x) = \sum_q 2im_q\,\bar{q}(x)\,\gamma_5\,q(x) - \frac{\alpha_s N_f}{4\pi}\,\mathrm{Tr}\left(F^{\mu\nu}(x)\widetilde{F}_{\mu\nu}(x)\right)$$

Can we identify the anomaly in CCM paper?

REMINDER: The anomaly is related to the UV divergences and regularization of γ_5 . If you naively calculate the 'anomaly' in ***4 dimensions*** in a ***massless quark theory*** you ***get zero.***

The ***anomaly arises*** from an ***ε/ε cancellation.***

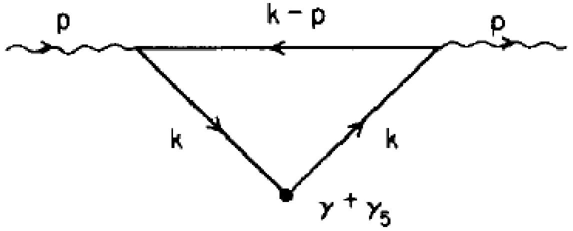


$$\Gamma_5^{+} = \mp \frac{\alpha_s N_f p^{+}}{\pi^2} \int_0^1 dx \int \frac{d^{n-2}k_T}{[k_T^2+m^2+P^2x(1-x)]^2}\left[(k_T^2+m^2)(1-2x)-2m^2(1-x)-2\left(\frac{n-4}{n-2}\right)k_T^2(1-x)\right]$$

x integration vanishes (doesn't contribute)

massive theory

ε/ε cancellation!



$$\partial_\mu J_5^\mu(x) = \sum_q 2im_q\,\bar{q}(x)\,\gamma_5\,q(x) - \frac{\alpha_s N_f}{4\pi}\,\mathrm{Tr}\,(F^{\mu\nu}(x)\widetilde{F}_{\mu\nu}(x))$$

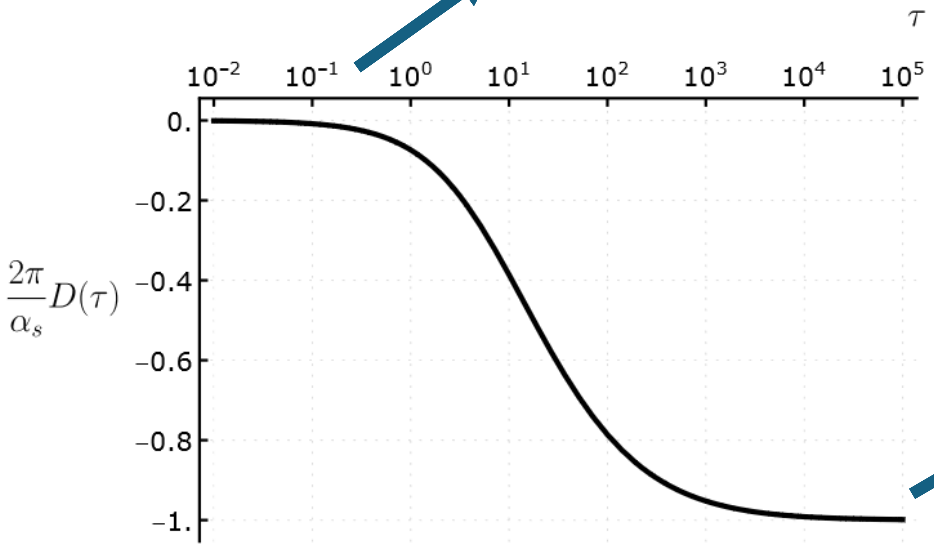
Our take on the ‘anomaly poles’ (CFLMPR)

On-shell gluons, massive quarks and ‘anomaly pole’ ***cancellation in the form factor G (or D).***

Anomaly term cancels with the mass term (infinite quark mass) and chiral symmetry restorage.

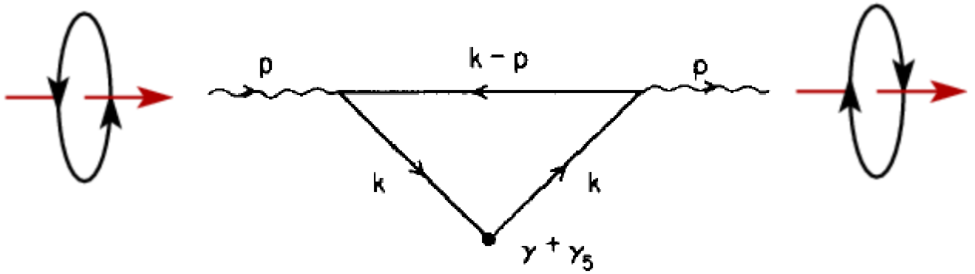
Perturbative results…

$$\tau = -\Delta^2/m^2$$



Anomaly dominated (chiral limit).

… and a general argument (no ‘anomaly pole’).



The axial current operator flips the helicity of the gluon. But for collinear kinematics (*τ=0*) the transition is forbidden due to conservation of angular momentum.

The $U_A(1)$ problem in a nutshell (Weinberg)

The axial anomaly is thought to be related with the ***U_A (1)* problem.**

There are *9 Goldstone bosons* related to the *8 non-singlet currents* and *1 singlet current*.

$$(j_5^a)^\mu = \bar{\psi}\gamma_\mu\gamma_5 T^a \psi, \quad j_5^\mu = \bar{\psi}\gamma_\mu\gamma_5\psi$$

Meson	Mass (MeV/c²)
π^+/π^- (Pions)	139.5
π^0 (Pion)	134.9
K^+/K^- (Kaons)	493.6
K^0 (Kaons)	497.6
η (Eta)	547.8
η' (Eta prime)	957.7

Why is the mass of η' (9th *lightest singlet SU(3) pseudoscalar meson*) so heavy in comparison with the other mesons?

The one great puzzle presented by all such theories has to do with the status of the η particle. Quark-gluon models always entail a U(1) axial-vector current A_5^μ, whose conservation is broken only by the $\mathcal{P}$ and $\mathfrak{N}$ quark masses. In consequence, the usual arguments of current algebra require an isoscalar pseudoscalar Goldstone boson with a mass of the same order of magnitude as the pion mass. Indeed, we shall show in Sec. II that under plausible assumptions the mass of this light particle L must be less than $\sqrt{3}\, m_\pi$,[6] so that it cannot be identified with the observed η particle.

Weinberg 1975

The $U_A(1)$ problem solution: t'Hooft-Witten-Veneziano

The proposed solution (t'Hooft) involves field configurations that have non-zero expectation value of the ϑ term of QCD: the second term of the action can be written as a divergence of a current.

$$Z = \int dA_\mu \exp i \int \mathrm{Tr}\left[-\frac{1}{4}F_{\mu\nu}^2 + \frac{g^2\theta}{16\pi^2 N_c} F_{\mu\nu}\tilde{F}_{\mu\nu}\right]$$

These field configurations must be ***non-perturbative (for instance: instantons).***

Witten and Veneziano propose an explanation considering a large *N* limit. The idea is to separate the nonet mass degeneracy in the chiral limit: quark loops appear in the second order in the *1/N* expansion.

t'Hooft 1976. Witten-Veneziano 1979

Connection of both problems? (JM)

Discussion for external nucleon matrix elements with full massless QCD.

Massless QCD: non-singlet currents matrix elements: $\langle N(p's)|\,\bar{\psi}\gamma_\mu\gamma_5 T^a\psi\,|N(ps)\rangle = \Delta q^a(t)s_\mu + (l\cdot s)l_\mu h^a(t)$

Ward identity $\Delta q^a(t) + th^a(t) = 0$

$$h^a(t) = \frac{f^a(0)}{t} + \text{non-singular terms}$$

$\mapsto$ $\Delta q^a(0) = -f^a(0)$

Δq^a is related to the singular term of *h^a(t)*.

Massless singlet current: without *η'* mass into consideration (wrong picture):

$$\langle N(p's)|\,\bar{\psi}\gamma_\mu\gamma_5\psi\,|N(ps)\rangle = \Sigma(t)s_\mu + (l\cdot s)l_\mu h(t)$$

AnomalousWard identity $\Sigma(t) + th(t) = \kappa(t)$

$$h(t) \stackrel{?}{=} \frac{h(0)}{t} + \text{non-singular terms}$$

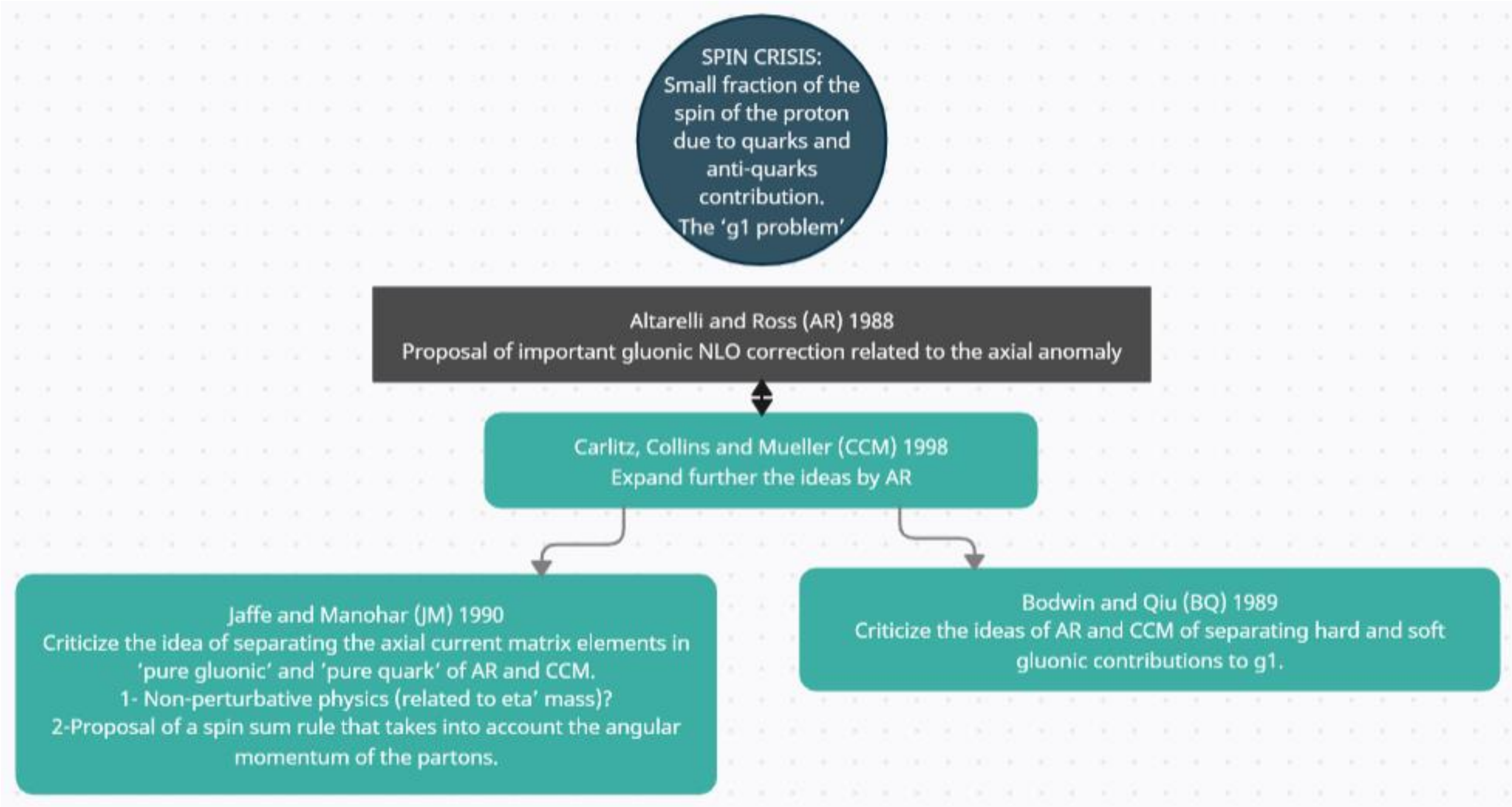
$\mapsto$ $\Sigma(0) = -f(0) + \kappa(0)$

1st term (analogy): quark contribution.
2nd term: gluon contribution.

↓ Zero even in chiral QCD! (*η'*)

This discussion involves non-perturbative effects (bound states: mesons)
In perturbation theory there is no *η'*, we should find ***a pole in h(t)*** in ***chiral*** QCD (dimensional analysis) OR ***h(t)=0 to finite orders in perturbation theory.***

The early stages of the 'spin crisis'



Connection of both problems? 30 years later… (TV papers)

We find that in both asymptotics, the matrix element for the $g_1(x_B, Q^2)$ structure function is identically controlled by the triangle anomaly, which has an infrared pole in the forward scattering limit. As we discussed at length in the introduction, the cancellation of this pole involves a subtle interplay of perturbative and nonperturbative physics that is deeply related to the $U_A(1)$ problem in QCD. As a further consequence, our results bring up important questions regarding the applicability of QCD factorization to observables that are sensitive to the anomaly.

From what I showed you already…

We made a calculation with an explicit form physical polarization vectors and ***didn't find*** the '***anomalous pole'*** in the ***triangle diagram*** when considering the forward limit.

When considering massive quarks as IR regulators, the form factor that defines the current already vanishes: ***consistent with conservation of angular momentum!***

Our calculations don't show inconsistencies with factorization in DIS.

Connection of both problems? 30 years later… (JM-TV)

$$S^\mu \Sigma(Q^2) = S^\mu \tilde{\Sigma}(Q^2) + 2\, n_f \frac{1}{M_N} \langle P, S | K^\mu | P, S \rangle \,, \tag{11}$$

where $S^\mu \tilde{\Sigma}(Q^2) = \frac{1}{M_N} \langle P, S | \tilde{J}^5_\mu | P, S \rangle$. Here $\tilde{J}^5_\mu = J^5_\mu - 2 n_f K_\mu$ is a conserved current since the anomaly satisfies the equation

$$\partial_\mu J^\mu_5 = 2\, n_f\, \partial_\mu K^\mu \,, \tag{12}$$

with the Chern-Simons current K^μ defined to be

$$K_\mu = \frac{\alpha_S}{4\pi} \epsilon_{\mu\nu\rho\sigma} \left[A^\nu_a \left(\partial^\rho A^\sigma_a - \frac{1}{3} g f_{abc} A^\rho_b A^\sigma_c \right) \right] . \tag{13}$$

One possible explanation for the small value of $\Delta\Sigma$, advanced early [2, 3, 53] after the EMC discovery, is that if first moment of g_1 were providing information on $\tilde{\Sigma}$ rather than Σ, that would provide a potential resolution of the spin crisis with the framework of the parton model itself. More specifically, it was argued on the basis of the gauge structure of K^μ that one could write

$$\tilde{\Sigma}(Q^2) = \Sigma(Q^2) - \frac{n_f \alpha_S}{2\pi} \Delta G \,, \tag{14}$$

where ΔG is the gluon helicity pdf. If ΔG is large, this would provide a natural explanation of the spin crisis. However as pointed out by Jaffe and Manohar, this identification is intrinsically problematic because while the gluon helicity pdf ΔG is manifestly gauge invariant, the same cannot be said of the Chern-Simons current. The latter is not gauge invariant under large gauge transformations U, which give,

$$K_\mu \to K_\mu + i \frac{g}{8\pi^2} \epsilon_{\mu\nu\alpha\beta} \partial^\nu \left((U^\dagger \partial^\alpha U) A^\beta \right) + \frac{1}{24\pi^2} \epsilon_{\mu\nu\alpha\beta} \left[(U^\dagger \partial^\nu U)(U^\dagger \partial^\alpha U)(U^\dagger \partial^\beta U) \right] . \tag{15}$$

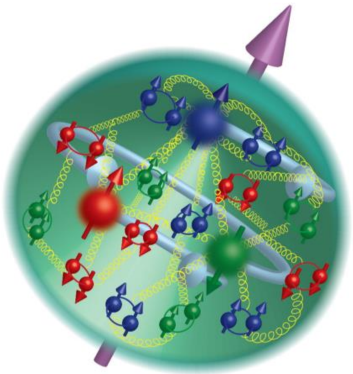
The resolution of the problem, as discussed by Jaffe and Manohar [4] (see also [54]), lies in how one takes limits when $U_A(1)$ is broken. This is because the breaking of this symmetry lifts an apparent pole in the forward scattering amplitude. Indeed this is the fundamental reason why the η'-meson gets a mass (distinct from the pseudoscalar octet) in QCD [20, 29, 55, 56]. We will now spell out the argument as sketched in [4]. For our convenience, and that of the reader familiar with their paper, we will use their notations.

Trying to separate pure gluon (pure quark) contributions through the Chern-Simons and a conserved current is problematic: *observables must be related to gauge invariant objects. We agree with this point.*

This is true, but the poles appear with non-perturbative effects (not the box diagram).

Tarasov and Venugopalan 2020-2021

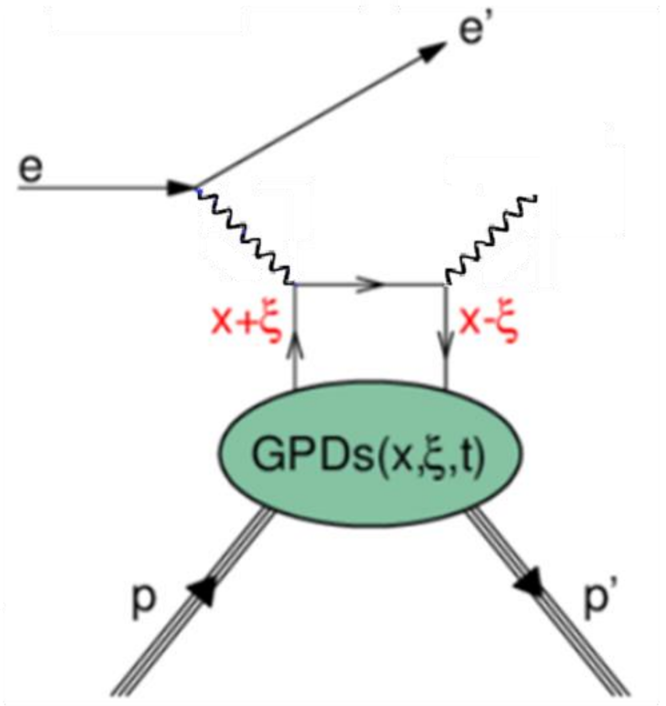
A different species:
DVCS and the axial anomaly



DVCS, the box diagram and its relationship with the axial anomaly

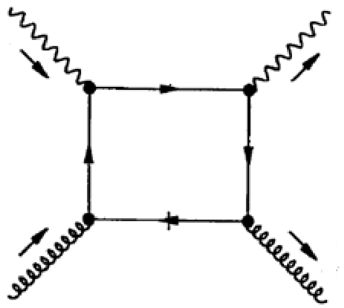
REMINDER: the 'exclusive' process lh→ lhɣ

For certain kinematics (DVCS), the 'leading' contribution for this process is given by this simple diagram



Deeply Virtual Compton Scattering (DVCS)

The box diagram contributes to the next order in α_s

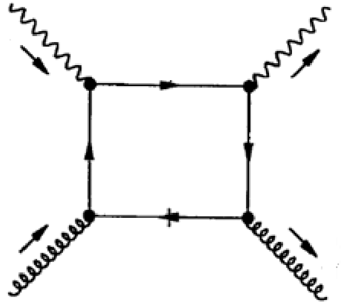


DVCS, the box diagram and its relationship with the axial anomaly

DVCS deals with a more general object than DIS, that is defined off-forward: dealing with amplitudes instead of cross sections through optical theorem as in DIS.

We assume factorization, and for the hard part, to order α_s we find the box-diagram contribution.

DIS: Forward limit and take the imaginary part of the box.



DVCS: off-forward matrix elements and in the light-cone limit and on-shell final photon.

As there is a connection with the anomaly in DIS, there should be a connection in DVCS.

DVCS: **definition**, constraints and results

Non-local light-cone separated version of the axial current between gluon states

$$F_{\lambda\lambda'}^{[\gamma^+\gamma_5]}(x,\Delta) = \int \frac{dz^-}{4\pi} e^{ik\cdot z} \langle g(p',\lambda') | \bar{q}(-\tfrac{z}{2})\, \gamma^+\gamma_5\, \mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\, q(\tfrac{z}{2}) | g(p,\lambda)\rangle\Big|_{z^+=0,\vec{z}_\perp=\vec{0}_\perp}$$

Wilson line to preserve
gauge invariance.

We work with:

- On-shell gluons (simplicity: less variables and no longitudinal polarizations).
- Massive degenerate quarks regularizing the IR limit.

Using Ward identity, Bose symmetry and Schouten identity we find the decomposition:

$$F_{\lambda\lambda'}^{[\gamma^+\gamma_5]}(x,\Delta) = (B_1 - B_2 + \xi B_3 + B_4)\, H_1(x,\xi,\Delta^2) + B_2\, H_2(x,\xi,\Delta^2)$$

DVCS: definition, **constraints** and results

$$H_1(x,\xi,\Delta^2) = \frac{1}{2(1-\xi^2)}\left(F_{++}^{[\gamma^+\gamma_5]}(x,\Delta) - F_{--}^{[\gamma^+\gamma_5]}(x,\Delta)\right), \qquad H_2(x,\xi,\Delta^2) = -\frac{1}{2\xi}\left(F_{+-}^{[\gamma^+\gamma_5]}(x,\Delta) - F_{-+}^{[\gamma^+\gamma_5]}(x,\Delta)\right)$$

General analysis that ***does not*** rely on perturbative analysis.

H_2 is a helicity flip GPD → Conservation of angular momentum implies that H_2 must vanish when $\Delta^2 = 0$.

H_1 and H_2 are dimensionless, so they depend on ratios of m^2, Δ^2 and μ^2 (dim. reg. scale) → Taking the collinear limit makes sense only if we have another mass scale (m).

First moment of the GPDs give the form factors, and recall that for the local axial current with external on-shell gluons we had one form factor G only.

$$\int_{-1}^{1} dx\, H_1(x,\xi,\Delta^2) = 0\,, \qquad \int_{-1}^{1} dx\, H_2(x,\xi,\Delta^2) = G(\Delta^2)\,.$$

So H_2 is related with the anomaly.

DVCS: definition, constraints and **results**

$$
H_1(x,\xi,\Delta^2;m) = \frac{\alpha_s}{4\pi}
\begin{cases}
\frac{2x-1-\xi^2}{1-\xi^2}\left[\frac{1}{\varepsilon} - \ln\frac{m^2}{\bar{\mu}^2}\right] - 1 \\
+\frac{4+(1+\xi^2)\kappa - 2x(\kappa+2)}{1-\xi^2}\frac{1}{\sqrt{\kappa(\kappa+4)}}\ln\frac{\sqrt{\kappa+4}+\sqrt{\kappa}}{\sqrt{\kappa+4}-\sqrt{\kappa}} & \xi \le x \le 1\,, \\
-\frac{(1-\xi)(\xi+x)}{2\xi(1+\xi)}\left[\frac{1}{\varepsilon} - \ln\frac{m^2}{\bar{\mu}^2}\right] - \frac{\xi+x}{2\xi} \\
-\frac{\xi^2(2-x)-x}{2\xi(1-\xi^2)}\ln\left[1+\frac{(1-\xi)(\xi+x)(\xi^2+\xi(1-x)-x)\kappa}{4\xi^2(1-x)^2}\right] \\
+\frac{4+(1+\xi^2)\kappa-2x(\kappa+2)}{2(1-\xi^2)}\frac{1}{\sqrt{\kappa(\kappa+4)}}\ln\frac{h_+}{h_-} \; + (x\to -x) & -\xi \le x \le \xi\,,
\end{cases}
$$

$$
H_2(x,\xi,\Delta^2;m) = \frac{\alpha_s}{4\pi}
\begin{cases}
\frac{2(1-x)}{1-\xi^2}\left[-1+\frac{2}{\sqrt{\kappa(\kappa+4)}}\ln\frac{\sqrt{\kappa+4}+\sqrt{\kappa}}{\sqrt{\kappa+4}-\sqrt{\kappa}}\right] & \xi \le x \le 1\,, \\
\frac{2}{1+\xi}\left[-\frac{\xi+x}{2\xi}+\frac{1-x}{1-\xi}\frac{1}{\sqrt{\kappa(\kappa+4)}}\ln\frac{h_+}{h_-}\right] + (x\to -x) & -\xi \le x \le \xi\,.
\end{cases}
$$

First order in perturbation theory. Degenerate massive quarks.

DVCS: definition, constraints and **results**

Collinear limit $\tau = \frac{-\Delta^2}{m^2} \to 0$

$$H_1(x,\xi,\Delta^2;m) = \frac{\alpha_s}{4\pi}\begin{cases} \frac{2x-1-\xi^2}{1-\xi^2}\left[\frac{1}{\varepsilon} - \ln\frac{m^2}{\bar{\mu}^2} - 1\right] + O(\tau) & \xi \le x \le 1\,, \\ -\frac{1-\xi}{1+\xi}\left[\frac{1}{\varepsilon} - \ln\frac{m^2}{\bar{\mu}^2} - 1\right] + O(\tau) & -\xi \le x \le \xi\,, \end{cases}$$

H_1 GPD does not vanish, but its first moment does.

$$H_2(x,\xi,\Delta^2;m) = \frac{\alpha_s}{4\pi}\begin{cases} -\frac{(1-x)^3}{3(1-\xi^2)^2}\tau + O(\tau^2) \overset{\tau\to 0}{\to} 0 & \xi \le x \le 1\,, \\ -\frac{(\xi+x)^2(\xi^2+2\xi(1-x)-x)}{12\xi^3(1+\xi)^2}\tau + (x\to -x) + O(\tau^2) \overset{\tau\to 0}{\to} 0 & -\xi \le x \le \xi\,. \end{cases}$$

As expected, the GPD H_2 vanishes in the collinear limit.

DVCS: definition, constraints and **results**

Massless limit:

$$H_2(x,\xi,\Delta^2;m) = \frac{\alpha_s}{4\pi}\begin{cases} -\frac{2(1-x)}{1-\xi^2} + O\left(\frac{\ln\tau}{\tau}\right) \overset{\tau\to\infty}{\to} -\frac{2(1-x)}{1-\xi^2} & \xi \le x \le 1\,, \\ -\frac{2}{1+\xi} + O\left(\frac{\ln\tau}{\tau}\right) \overset{\tau\to\infty}{\to} -\frac{2}{1+\xi} & -\xi \le x \le \xi\,. \end{cases}$$

We recover the results from BHV (calculation with massless quarks).

The GPD H_2 is given purely by the anomaly contribution:

$$\int_{-1}^{1} dx\, H_1(x,\xi,\Delta^2) = 0\,, \qquad \int_{-1}^{1} dx\, H_2(x,\xi,\Delta^2) = G(\Delta^2)\,.$$

$$F^{[\gamma^+\gamma_5]}_{\lambda\lambda'}(x,\Delta) = \int \frac{dz^-}{4\pi}\, e^{ik\cdot z}\, \langle g(p',\lambda')\,|\,\bar{q}(-\tfrac{z}{2})\,\gamma^+\gamma_5\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\,q(\tfrac{z}{2})\,|\,g(p,\lambda)\rangle\Big|_{z^+=0,\vec{z}_\perp=\vec{0}_\perp}$$

$$= (B_1 - B_2 + \xi B_3 + B_4)\,H_1(x,\xi,\Delta^2) + \underline{B_2\,H_2(x,\xi,\Delta^2)}\,,$$

In BHV, the structure B_2 does not appear explicitly contracted with physical polarization vectors, and it seems to have a pole in Δ^2.

$$B_2 = \frac{-2\xi}{\Delta^2}\,\varepsilon^{P\,\Delta\,\epsilon\,\epsilon'^*}$$

Bhattacharya, Hatta and Vogelsang 2023

Recent work (MT-BHS)

Relation between light-cone separated operators that reduces to the 'usual' anomalous Ward identity for massless quarks.

$$\Delta_\mu P^+ \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p'|\bar{\psi}(-z^-/2) W \gamma^\mu \gamma_5 \psi(z^-/2)|p\rangle = \frac{n_f \alpha_s M}{2\pi} \tilde{C}^{anom} \otimes \tilde{\mathcal{F}}(x,\xi,t) + O_F(x,\xi,t).$$

Divergence of the non-local current.

Non-local anomaly term.

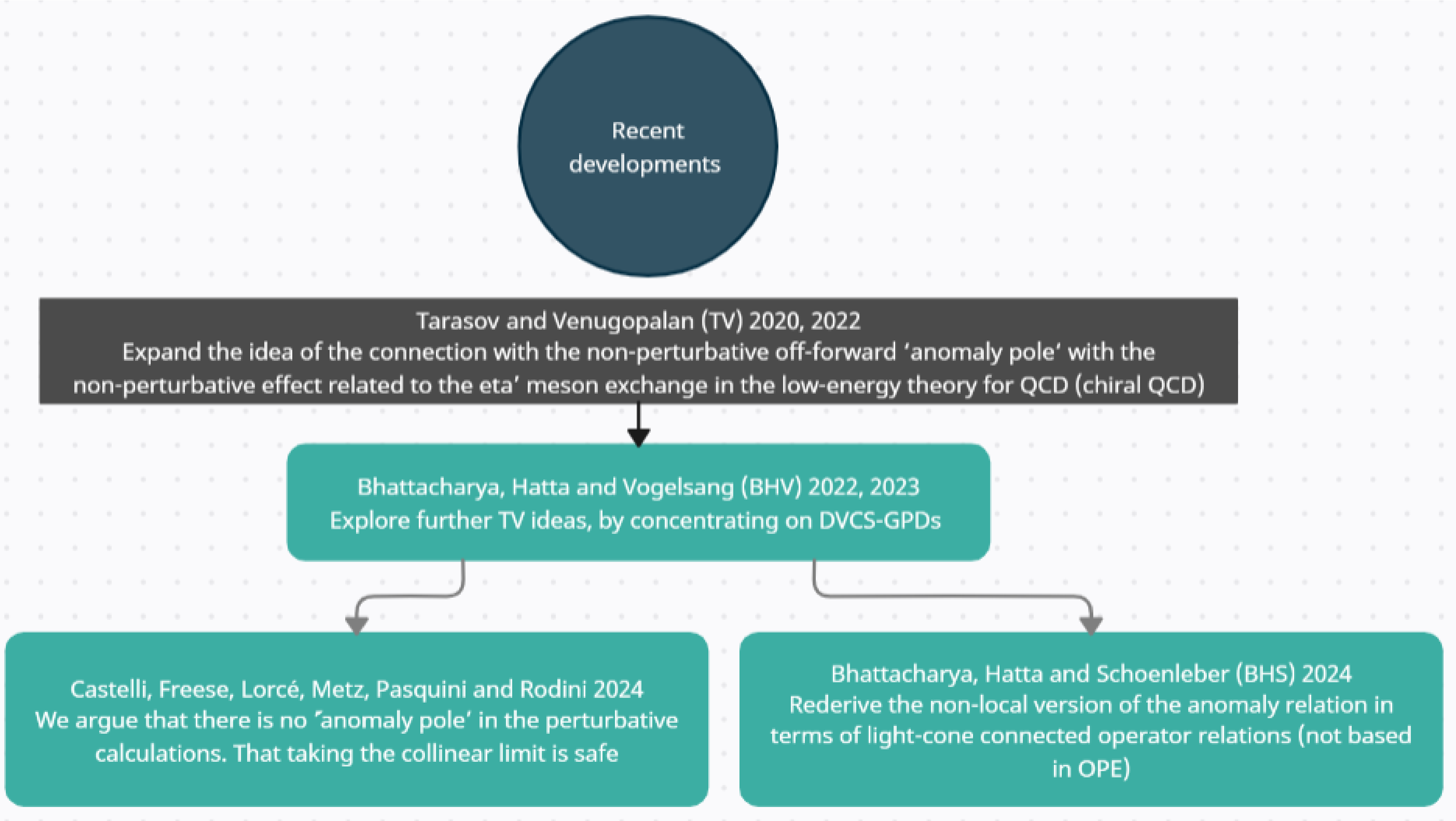
Extra term not related to the anomaly.

$$\tilde{\mathcal{F}}(x,\xi,t) \equiv \frac{iP^+}{M} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p'|F^{\mu\nu}(-z^-/2) \tilde{W} \tilde{F}_{\mu\nu}(z^-/2)|p\rangle$$

$$\int dx O_F(x,\xi,t) = 0.$$

Mueller and Teryaev 1997- Bhattacharya, Hatta and Schoenleber 2024

Recent developments (~30 years later)



Most recent development… (TV)

Claims:

The ‘anomaly pole’ is regulated in QED and pQCD by the quark mass, but not in QCD.

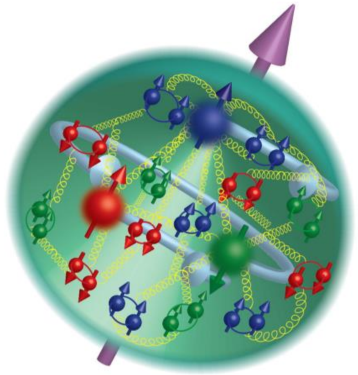
We first considered the simple QED (or equivalently, pQCD) case. In this case one replaces the scalar source term Φ is simply set equal to the quark mass at the end of the computation. As we will discuss shortly, for QCD in general, Φ cannot be take as a constant; functional derivatives of the effective action with respect to this quantity are what generate the chiral condensate. In the simpler QED and perturbative QCD case, performing the worldline computation (with the details provided in Appendix A) we showed that indeed the PVV triangle exactly cancels the AVV triangle in the forward limit demonstrating that there is no anomaly pole. The result for QED was first shown by Adler and worked out in further detail by Adler and Bardeen. It also provided the essence of the recent claim for pQCD in [32].

Tarasov and Venugopalan 2025

Summary

- Introduction of the ‘spin crisis’ and recent developments.
- We didn’t focus the discussion the *role of the angular momentum* of the partons in the *spin sum rules.*
- There is no anomaly pole in the matrix elements of the axial current to first order in perturbation theory in several kinematics for external gluons.
- The anomaly is present in the collinear limit.

Thank you



Backup slides

Results for Σ: experiments

The COMPASS Collaboration performed new measurements of the longitudinal double spin asymmetry $A_1^{\mathrm{p}}(x,Q^2)$ and the longitudinal spin structure function $g_1^{\mathrm{p}}(x,Q^2)$ of the proton in the range $0.0025 < x < 0.7$ and in the DIS region, $1 < Q^2 < 190\,(\mathrm{GeV}/c)^2$, thus extending the previously covered kinematic range [3] towards large values of Q^2 and small values of x. The new data improve the statistical precision of $g_1^{\mathrm{p}}(x)$ by about a factor of two for $x \lesssim 0.02$.

The world data for g_1^{p}, g_1^{d} and g_1^{n} were used to perform a NLO QCD analysis, including a detailed investigation of systematic effects. This analysis thus updates and supersedes the previous COMPASS QCD analysis [17]. It was found that the contribution of quarks to the nucleon spin, $\Delta\Sigma$, lies in the interval 0.26 and 0.36 at $Q^2 = 3\,(\mathrm{GeV}/c)^2$, where the interval limits reflect mainly the large uncertainty in the determination of the gluon contribution.

The COMPASS collaboration 2015

Results for Σ: lattice QCD

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + J_g$$

$$\frac{1}{2}\Delta\Sigma = 0.200(17) \quad \text{with} \quad \frac{1}{2}\Delta u = 0.414(16)\,, \quad \frac{1}{2}\Delta d = -0.194(10)\,, \quad \frac{1}{2}\Delta s = -0.021(5)\,.$$

$$J_q = 0.398(60) \quad \text{with} \quad J_u = 0.296(40)\,, \quad J_d = 0.058(40)\,, \quad J_s = 0.046(20)$$

$$L_q = 0.198(62) \quad \text{with} \quad L_u = -0.118(43)\,, \quad L_d = 0.252(41)\,, \quad L_s = 0.067(21)$$

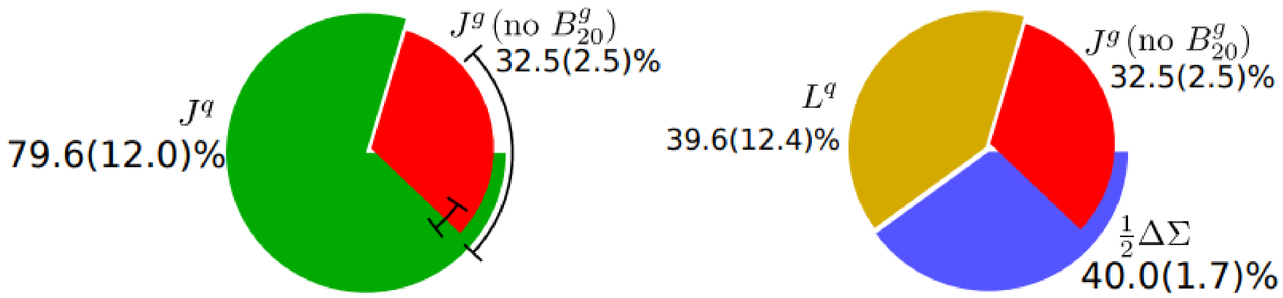


Figure 2: Graphical representation of the proton spin contribution of quarks and gluons from lattice QCD results.

Wiese et al 2016

Reminder on the Goldstone theorem

$$\langle \frac{\partial^2 V}{\partial \Phi_k \partial \Phi_i} T^a_{ij} \Phi_j + \frac{\partial V}{\partial \Phi_i} T^a_{ik} \rangle = 0 \; . \qquad (200)$$

Remembering that $\mathcal{M}^2_{ki} = \langle \frac{\partial^2 V}{\partial \Phi_k \partial \Phi_i} \rangle$ is the scalar mass matrix, we obtain

$$\mathcal{M}^2_{ki} \, (T^a v)_i \; = \; 0 \; . \qquad (201)$$

We therefore found the general form of the **Goldstone theorem**: *If the vacuum is not invariant under a symmetry generator $T^a v \neq 0$, then $T^a v$ is an eigenvector of the mass matrix $\mathcal{M}^2$ corresponding to a zero eigenvalue.*

- Pions $\pi \sim q \, \bar{q}$ are *pseudo-Goldstones* for the breaking of the *chiral* $\rightarrow$ vector symmetries $U(3)_L \times U(3)_R \rightarrow SU(3)_V \times U(1)_B$ (see figure 8). They are not exactly massless (therefore the name "pseudo") due to a small explicit breaking coming from quark masses. Pions are (pseudo)scalar particles, but not elementary, they are quark-antiquark bound states. In this case we talk about *dynamical* symmetry breaking.

Observation : The $U(1)_A$ symmetry is broken by quantum anomalies, there is no corresponding goldstone boson.