Dispersion-theoretical methods for three-body decays and Primakoff reactions

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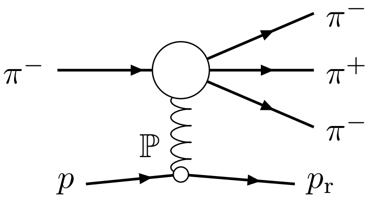
Contents

- Analysis of rescattering effects in 3π final states
  [DS, T. Isken, B. Kubis, M. Mikhasenko, M. Niehus
  Eur. Phys. J. C 83, 510 (2023); arXiv:2212.11767]
- Dispersive analysis of the Primakoff reaction $\gamma K \to K\pi$
  [Maximilian Dax, DS, Bastian Kubis
  Eur. Phys. J. C 81, 221 (2021); arXiv:2012.04655]
- Polarizabilities from Kaon Compton scattering
  [DS, Jan Luca Dammann, Bastian Kubis
  in preparation]

Analysis of rescattering effects in 3π final states

COMPASS experiment



- large data set on diffractive $\pi^-\pi^-\pi^+$ production
- allows to study $I^G = 1^-$ processes
- $m_{3\pi}$ mass range from 0.5 to 2.5GeV
- set of 88 partial waves
- highly relevant question for COMPASS PWA framework:

[COMPASS; 2015]

to what extent and how are the $\pi^+\pi^-$ partial waves modified by the presence of the third spectator pion?

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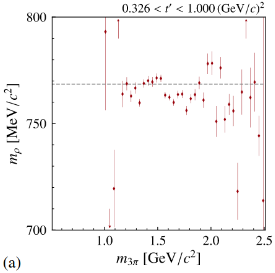

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- highly relevant question for COMPASS PWA framework:

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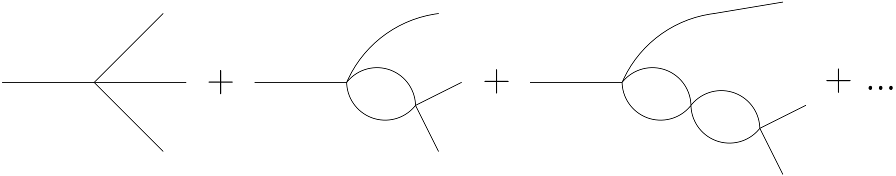
  to what extent and how are the $\pi^+\pi^-$ partial waves modified by the presence of the third spectator pion?
- ρ-mass dependence on $m_{3\pi}$

Motivation

- different decay processes that decay into $\rho\pi$ final states
- effects of the ρ described by $\pi\pi$ P-wave phase shift



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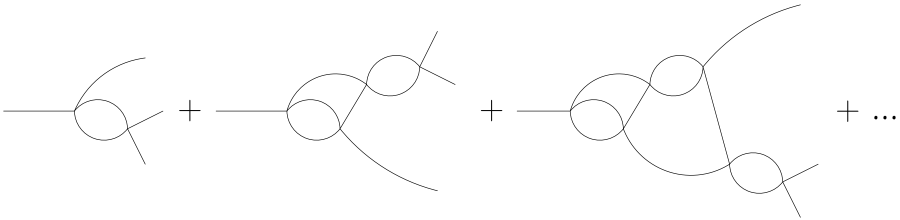
- include full rescattering effects
- Khuri–Treiman equations [Khuri, Treiman; 1960]



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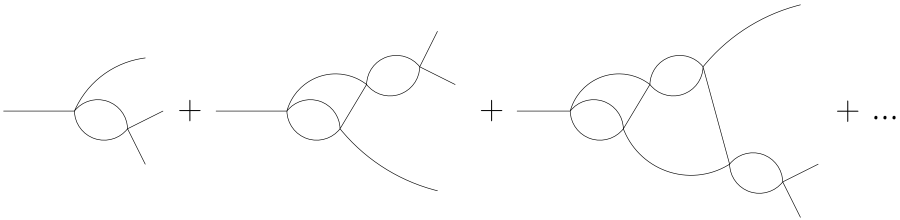
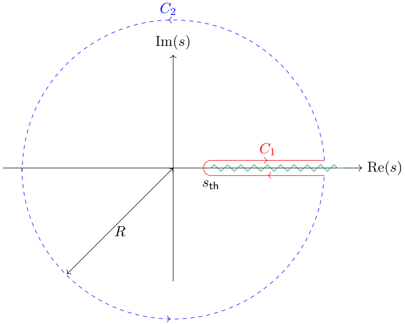


- which statistics to observe rescattering effects, depending on process and energy?

Dispersive formalism

- analyticity ($\simeq$ causality) & Cauchy's integral formula

$$f(s) = \frac{1}{2\pi i} \oint_{\partial U} \frac{f(s')}{s' - s} \mathrm{d}s'$$



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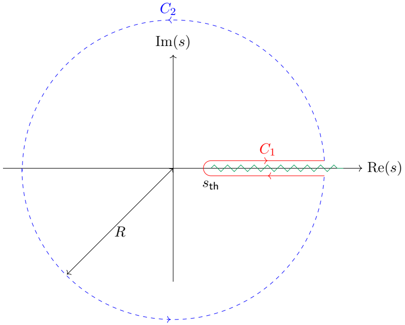

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\begin{aligned}
f(s) &= \frac{1}{2\pi i}\oint_{\partial U}\frac{f(s')}{s'-s}\mathrm{d}s' \\
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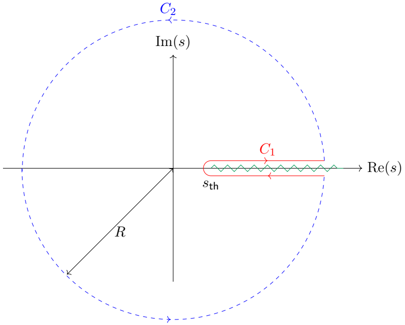
&= \frac{1}{\pi}\int_{s_{\text{th}}}^{\infty}\frac{\mathrm{Im}\, f(s')}{s'-s-i\varepsilon}\mathrm{d}s'
\end{aligned}
$$



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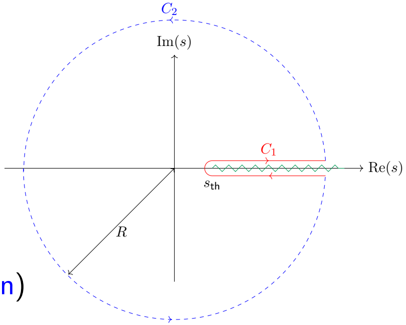
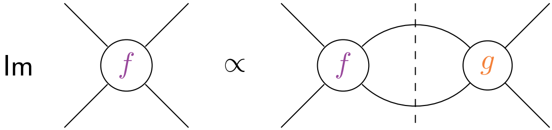


- unitarity relation ($\simeq$ prob. conservation)

$$\mathrm{Im}\,f(s) \propto f(s)\cdot g^*(s)$$



- obeys Watson's final state theorem [Watson, 1954]

Khuri–Treiman equations

- amplitude for $\omega, \phi \to 3\pi$

$$\mathcal{F}(s,t,u) = \mathcal{F}(s) + \mathcal{F}(t) + \mathcal{F}(u)$$

- unitarity condition

$$f_1(s) = \mathcal{F}(s) + \widehat{\mathcal{F}}(s)$$
$$\text{disc}\mathcal{F}(s) = 2i\left(\mathcal{F}(s) + \cancel{\widehat{\mathcal{F}}(s)}\right)\sin\delta(s)e^{-i\delta(s)}\theta(s - 4M_\pi^2)$$

- partial wave f_1 with angular momentum $\ell = 1$
- $\pi\pi$ P-wave phase shift δ
- homogeneous solution with $\widehat{\mathcal{F}}(s) = 0$ gives Omnès function

[Omnès; 1958]

$$\Omega(s) = \exp\left(\frac{s}{\pi}\int_{4M_\pi^2}^{\infty} \mathrm{d}s' \frac{\delta(s')}{s'(s'-s)}\right)$$

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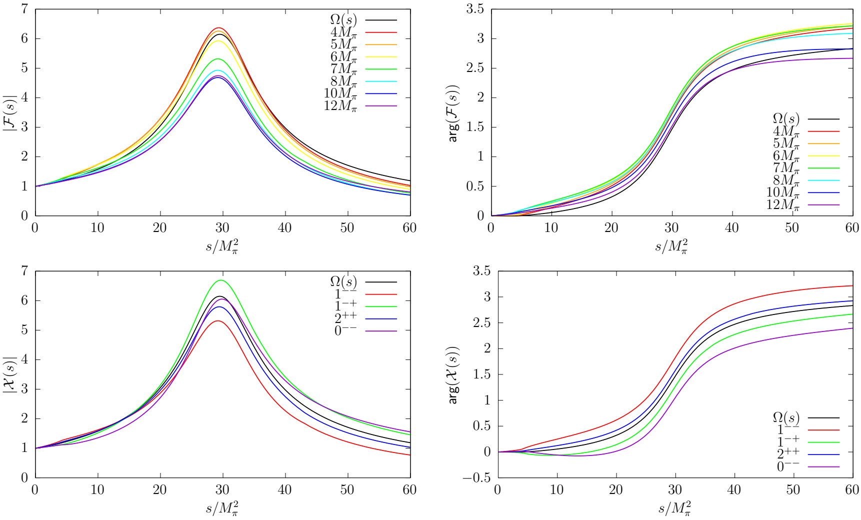
- partial wave f_1 with angular momentum $\ell = 1$
- $\pi\pi$ P-wave phase shift δ
- inhomogeneous solution leads to KT equations

$$\mathcal{F}(s) = \Omega(s)\left(P_{n-1}(s) + \frac{s^n}{\pi}\int_{4M_\pi^2}^{\infty}\frac{\mathrm{d}s'}{s'^n}\frac{\sin\delta(s')\widehat{\mathcal{F}}(s')}{|\Omega(s')|(s'-s)}\right)$$

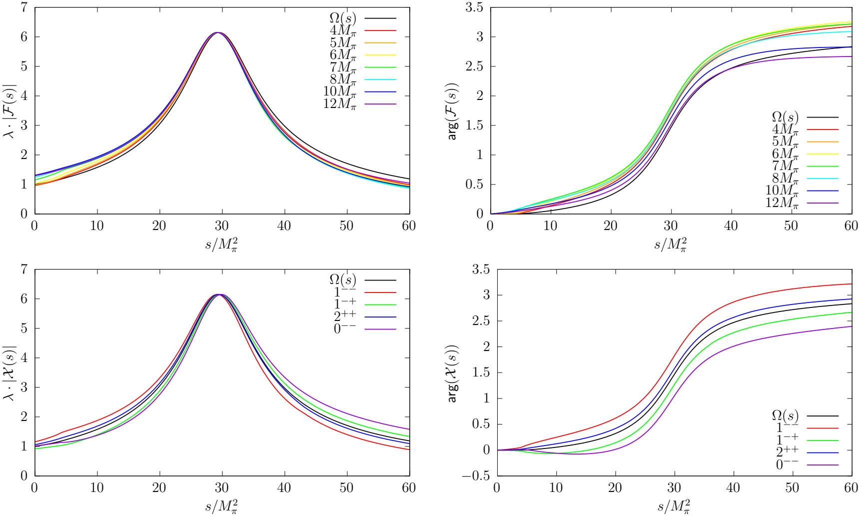
Decays

- four different processes
- angular-momentum quantum numbers of 3π system
- $(I^G)J^{PC} = (0^-)0^{--}$ (exotic), $(0^-)1^{--}$ (ω, ϕ),
  $(1^-)1^{-+}$ (π_1), $(1^-)2^{++}$ (a_2)
- restrictive analysis with only one free parameter: $\rightarrow$ normalization
- only consider P-waves
- not include processes that involve S-waves (1^{++}, a_1)
- only elastic unitarity

Basis functions (normalized $s = 0$)



Basis functions (normalized $s = M_\rho^2$)



Log-likelihood differences

- KT equations set as truth
- to which extent do Omnès functions reproduce these?
- probability density function

$$f(s,t) = \frac{|\mathcal{M}(s,t)|^2}{\int_D |\mathcal{M}(s,t)|^2 \mathrm{d}s\mathrm{d}t}$$

- likelihood function

$$L(\mathbb{D}, f) = \prod_{i=1}^{N} f(s_i, t_i)$$

- log-likelihood difference

$$\Delta\mathcal{L}(\mathbb{D}) = \ln(L(\mathbb{D}, f^{\text{Omnès}})) - \ln(L(\mathbb{D}, f^{\text{KT}}))$$

Log-likelihood differences

- use Kullback-Leibler divergence d_{KL} and variance ν_{KL}

  [Kullback, Leibler; 1951]

- probability of $\Delta\mathcal{L} > 0$ via cumulative distribution function

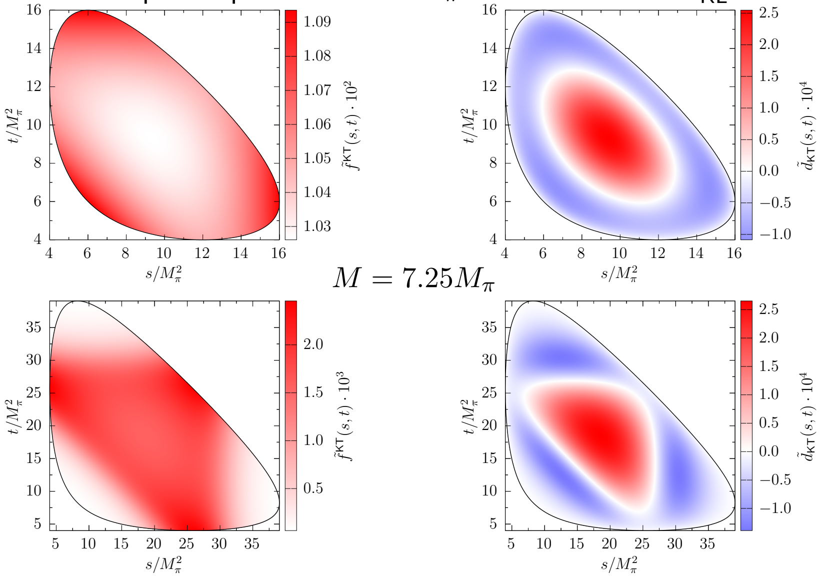
$$q(N) = 1 - \mathcal{N}(0, \mu(N), \sigma(N)) \quad \mu(N) = -N d_{\mathsf{KL}} \quad \sigma(N) = \sqrt{N \nu_{\mathsf{KL}}}$$

- number of events

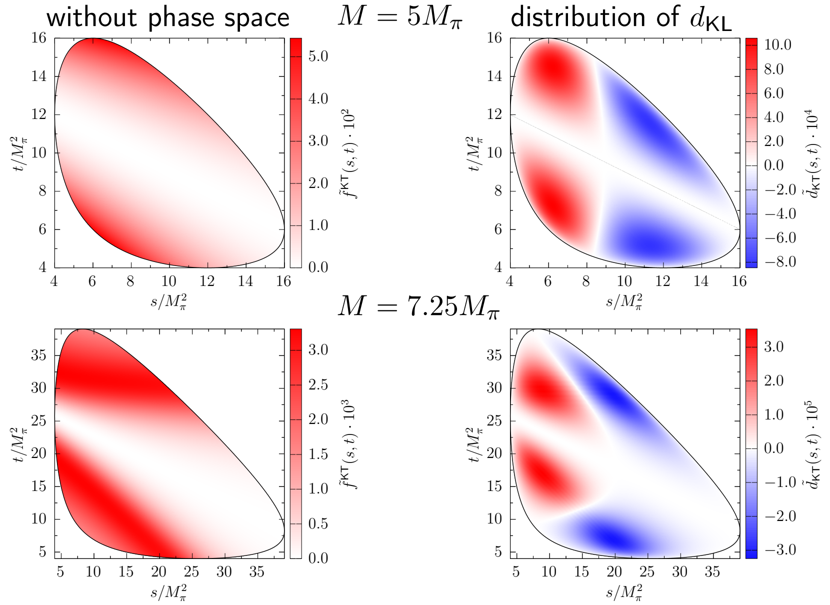
$$N(q) = 2\nu_{\mathsf{KL}} \left(\frac{\mathrm{erf}^{-1}(1 - 2q)}{d_{\mathsf{KL}}} \right)^2$$

Dalitz plots for 1^{--}: ω, ϕ

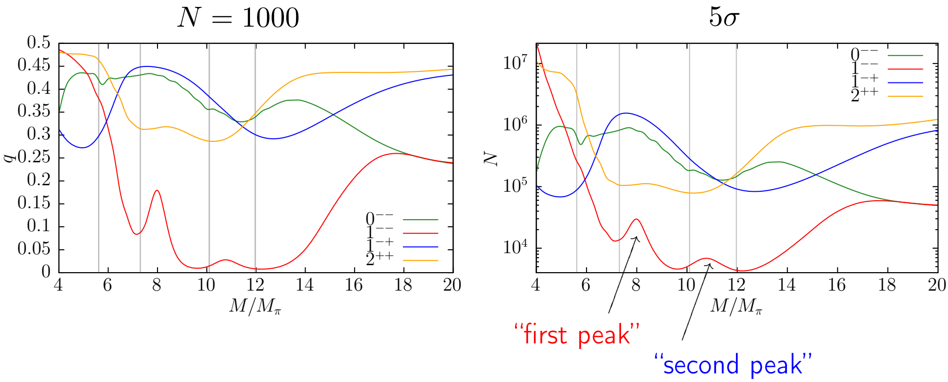
without phase space $M = 5M_\pi$ distribution of d_{KL}



Dalitz plots for 1^{-+}: π_1



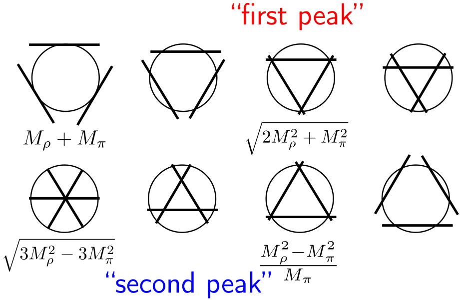
Results



- form dominated by angular momentum quantum number
- rise for limit $M \to \infty$ (but inelastic effects)
- two peaks in 1^{--}

Results

- schematic Dalitz plots for different decay masses



- difference decreases when ρ bands cross in Dalitz plot
- approx. $\sqrt{2M_\rho^2 + M_\pi^2}$
- second peak also due to kinematic effect

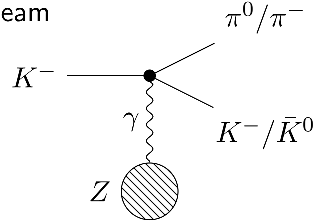
Conclusion

- compare Omnès to KT solutions:
  when are rescattering effects visible?
- in 0^{--}, 1^{-+} and 2^{++} many events ($\mathcal{O}(10^5) - \mathcal{O}(10^6)$) are needed for all decay masses
- for 1^{--} strong dependence on decay mass
- at ω mass: small difference [BESIII; 2018]
- at ϕ mass: rescattering effects easy to observe [KLOE; 2003]

Dispersive analysis of the Primakoff reaction $\gamma K \to K\pi$

Motivation

- pion production in the Coulomb field of a heavy nucleus
- $\gamma^{(*)}\pi \to \pi\pi$ investigated [Hoferichter et al., 2012, 2017], [Niehus et al., 2021]
- AMBER experiment planned (+ OKA experiment)
- upgrade from pion beam to charged-kaon beam



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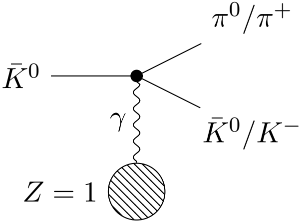

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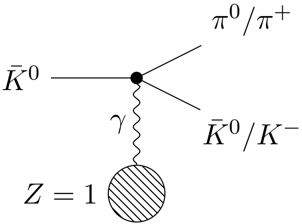
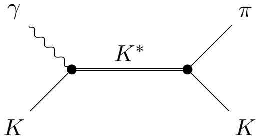
- K_L-facility with neutral-kaon beam
- proton target $\to$ weaker Coulomb field
- combine knowledge about
  - resonances ($K^*(892)$) at higher energies (radiative couplings)
  - chiral anomaly at $s = t = u = 0$

$$F_{KK\pi\gamma} = \frac{e}{4\pi^2 F_\pi^3}$$





Chiral anomaly

- quantum effect, model-independent prediction for QCD at low energies
- Wess, Zumino, Witten [1971,1983] for processes with odd intrinsic parity
- prime example: $\pi^0 \to \gamma\gamma$
- additionally three-pseudoscalar-photon processes
  $\gamma\pi \to \pi\pi$ and $\gamma K \to K\pi$

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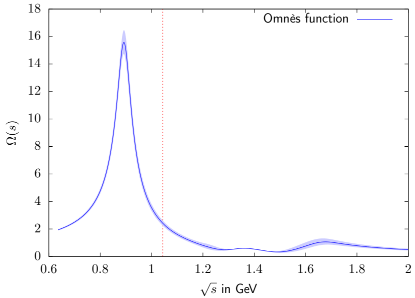
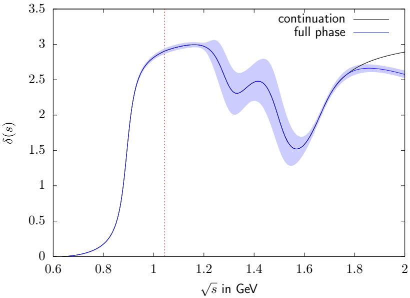
- prime example: $\pi^0 \to \gamma\gamma$
- additionally three-pseudoscalar-photon processes
  $\gamma\pi \to \pi\pi$ and $\gamma K \to K\pi$
- prediction at zero energies $s = t = u = 0$ and zero masses
- only depend on e, F_π (and F_K)
- leading ChPT prediction, low-energy theorems

Homogeneous Omnès problem

- $K\pi$ P-wave phase shift from [Peláez and Rodas, 2016] $\rightarrow$ no S wave
- $I = 1/2$ phase shift contains $K^*(892)$, $K^*(1410)$ and $K^*(1680)$
- very well constrained up to the $K\eta$ threshold
- Omnès function: $\Omega(s)$ [Omnès, 1958]





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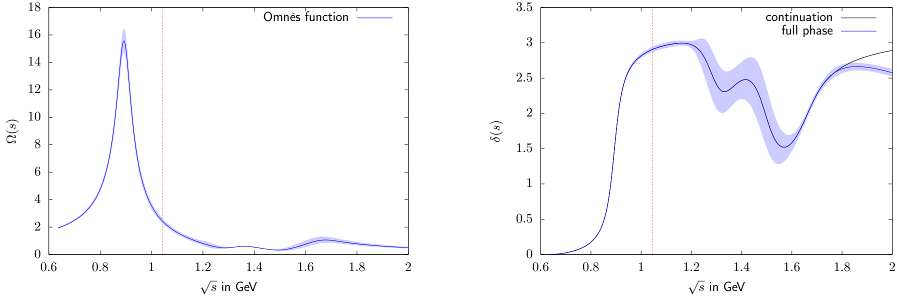

- $I = 3/2$ phase shift is $|\delta(s)| < 3^\circ$ for $s < (1.74\,\text{GeV})^2$
- approximate it with $\delta(s) = 0 \Rightarrow \Omega(s) = 1$

Reconstruction theorem

- separate kinematic prefactor $\mathcal{M} = i\varepsilon_{\mu\nu\alpha\beta}\epsilon^{\mu}p_1^{\nu}p_2^{\alpha}p_0^{\beta}\mathcal{F}(s,t,u)$
- decompose scalar amplitude $\mathcal{F}(s,t,u)$ using isospin and $s \leftrightarrow u$ symmetry into single variable amplitudes
- one example for $\gamma\bar{K}^0 \to \bar{K}^0\pi^0$

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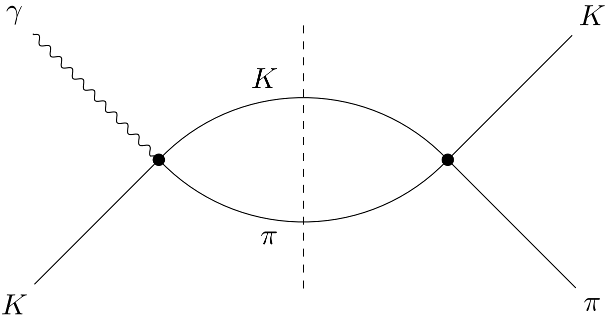

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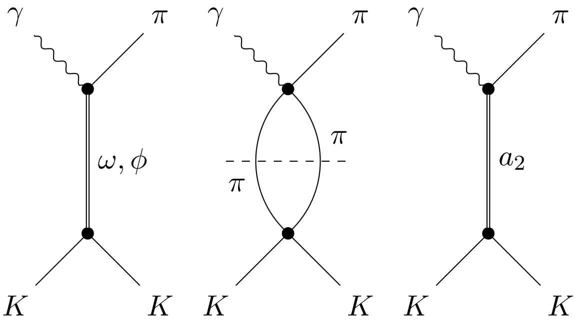

Fixed t-channel resonances

- narrow resonances ω, ϕ fixed by VMD exchange
- coupling via partial widths $\omega, \phi \to \bar{K}K$ and $\omega, \phi \to \pi^0\gamma$

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- broad ρ resonance via dispersive $\pi\pi$ reconstruction
- dispersion integral containing $\gamma\pi \to \pi\pi$ and $\pi\pi \to K\bar{K}$ amplitudes
- $\gamma\pi \to \pi\pi$ amplitude from [Hoferichter et al., 2012]
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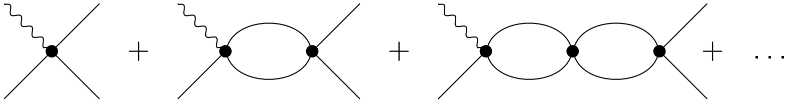
- D-wave a_2 resonance via tensor meson dominance

Inhomogeneous Omnès problem

- representation of homogeneous solution $\Omega(s)$



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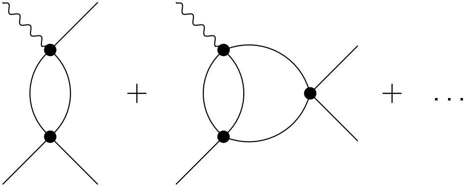

Basis functions

- solution depends on subtraction polynomials linearly
- construct basis functions that correspond to one subtraction constant

$$\mathcal{F}(s) = \sum_{i=0}^{N} \alpha_i \mathcal{F}_i(s)$$

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- reduces computational effort dramatically
- fit/matching can be done using the basis functions
- can be provided for experimental analysis

Subtraction schemes

- how many free parameters are there?
- we find

$$\mathcal{F}^{(i)}(s) = \Omega(s)\left(P_{n-1}^{(i)}(s) + \frac{s^n}{\pi}\int_{s_{\text{th}}}^{\infty}\frac{\mathrm{d}s'}{s'^n}\frac{\widehat{\mathcal{F}}^{(i)}(s')\sin(\delta^{(i)}(s'))}{|\Omega(s')|(s'-s)}\right)$$

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- two/four free parameters
- can be fixed via cross-section data, which is not available
- predict cross-section via available input of anomaly and coupling

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- how many free parameters are there?
- we find

$$\mathcal{F}^{(i)}(s) = \Omega(s)\left(P_{n-1}^{(i)}(s) + \frac{s^n}{\pi}\int_{s_{\text{th}}}^{\infty}\frac{\mathrm{d}s'}{s'^n}\frac{\widehat{\mathcal{F}}^{(i)}(s')\sin(\delta^{(i)}(s'))}{|\Omega(s')|(s'-s)}\right)$$

- we consider two subtraction schemes with $n = 1$ and $n = 2$
- two/four free parameters
- can be fixed via cross-section data, which is not available
- predict cross-section via available input of anomaly and coupling
- future goal: extract these quantities from data!

Matching: anomaly

- calculate chiral Wess–Zumino–Witten [1971,1983] anomaly

$$\mathcal{F}^{-0/00}(s=0,t=0,u=0) = F_{KK\pi\gamma} = \frac{e}{4\pi^2 F_\pi^3}$$

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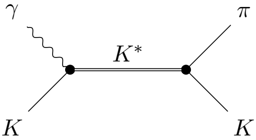

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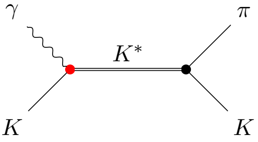

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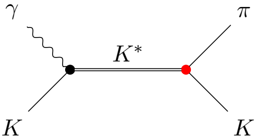
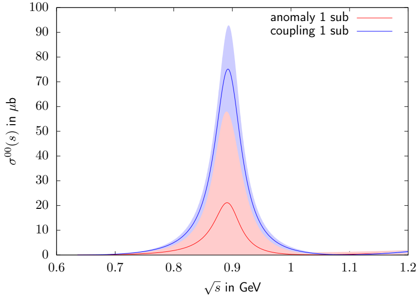
- $K\pi$ coupling and K^* pole position from Roy–Steiner analysis [Peláez, Rodas, Ruiz de Elvira, 2019]

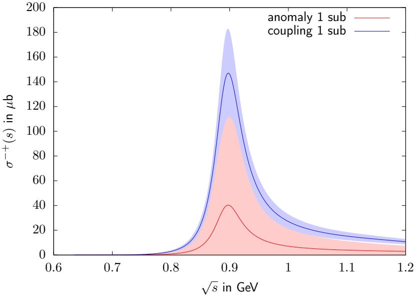


Matching: anomaly or coupling

- minimal subtraction scheme
- fully determined by the anomaly or coupling



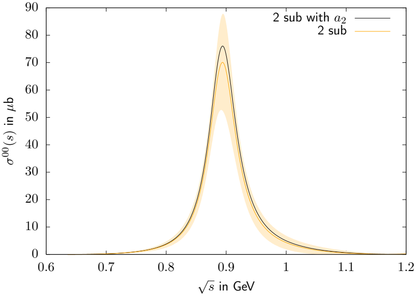
$\gamma\bar{K}^0 \to \bar{K}^0\pi^0$



$\gamma\bar{K}^0 \to K^-\pi^+$

Matching: anomaly and coupling

- twice subtracted scheme with and without a_2 resonance



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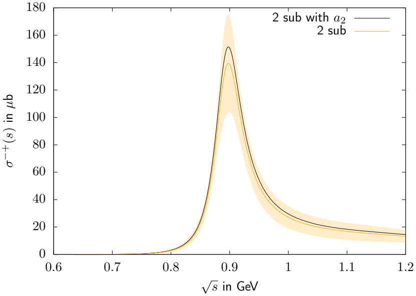
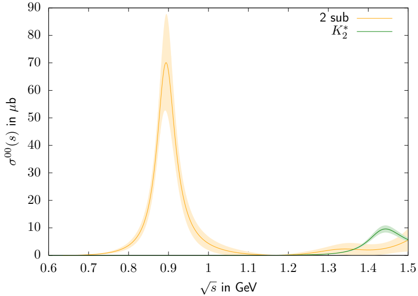


$$\gamma \bar{K}^0 \to K^- \pi^+$$

D-wave $K_2^*(1430)$

- use Lagrangians [Ecker and Zauner, 2007], [Plenter and Kubis, 2015]
- for neutral channel the radiative coupling is only an upper limit $\Gamma_{K_2^* \to K^0\gamma} < 5.4\,$keV [Alavi-Harati et al. (KTeV), 2001]



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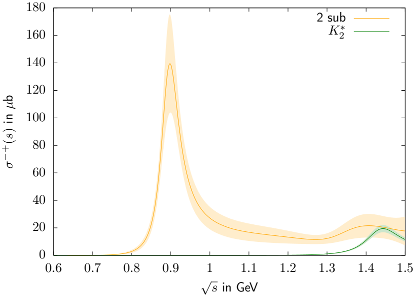


$\gamma\bar{K}^0 \to K^-\pi^+$

- D-wave relevant in charged channels above $1.35\,$GeV [Bacho, 2021]

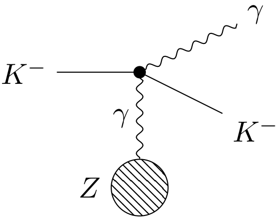
Summary and outlook

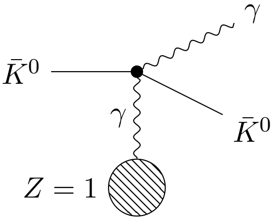
- constructed a dispersive solution for the Primakoff reaction $\gamma K \to K\pi$ for all charge configurations
- input: fixed t-channels and $K\pi$ phase shift
- using the basis functions a fit to COMPASS++, KLF (or OKA) data is possible to determine the free parameters
- matching: use radiative couplings and chiral anomaly to predict the free parameters
- future goal: extract these quantities using a fit to data

Polarizabilities from kaon Compton scattering

Motivation

- same setup as before $\rightarrow$ different final state
- again pion case already investigated
- most recent measurement: COMPASS 2015
- $\sqrt{s} < 3.5 M_\pi$ $\rightarrow$ more than 350MeV
- difficulty for kaon: K^* resonance





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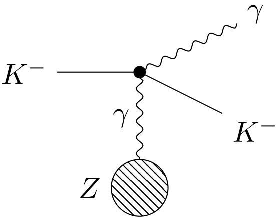
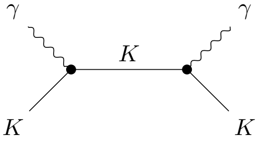
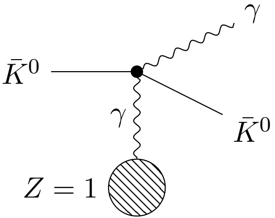
- difficulty for kaon: K^* resonance
- main contribution: Born term







Definitions

- two helcitity amplitudes

$$\mathcal{F}_{++}(s,t) = \mathcal{F}_{--}(s,t) = 4(M_K^4 - su)\mathcal{B}(s,t)$$
$$\mathcal{F}_{+-}(s,t) = \mathcal{F}_{-+}(s,t) = -\frac{t}{2}\mathcal{A}(s,t) + t(t-4M_K^2)\mathcal{B}(s,t)$$

- differential cross section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\alpha_{\text{em}}^2}{4s}\left(|\mathcal{F}_{++}(s,t)|^2 + |\mathcal{F}_{+-}(s,t)|^2\right)$$

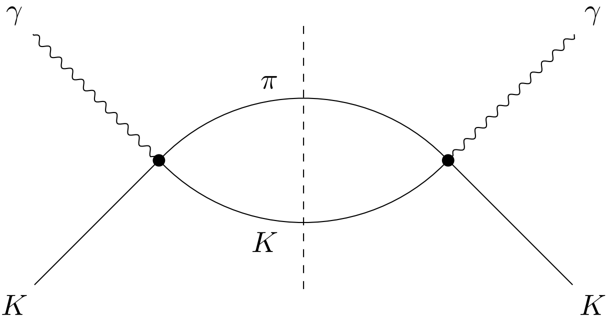
- polarizabilities

$$\pm\frac{2\alpha_{\text{em}}}{M_K t}\widehat{\mathcal{F}}_{+\pm}(M_K^2,t) = \alpha_1 \pm \beta_1 + \frac{t}{12}(\alpha_2 \pm \beta_2) + \mathcal{O}(t^2)$$

where
$$\widehat{\mathcal{F}}_{+\pm}(s,t) = \mathcal{F}_{+\pm}(s,t) - \mathcal{F}_{+\pm}^{\text{Born}}(s,t)$$

K^* resonance

- use known $\gamma K \to K\pi$ amplitudes
- calculate imaginary part for $\gamma K \to \gamma K$
- use dispersion relation



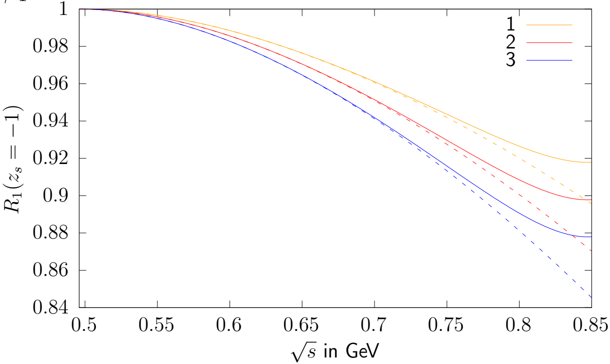
Ratios

- define ratios similar to COMPASS analysis

$$R_1(s,t) = \frac{\mathrm{d}\sigma/\mathrm{d}\Omega(s,t)}{\mathrm{d}\sigma/\mathrm{d}\Omega(s,t)\Big|_{\alpha_1=\beta_1=0}}$$

$$\bar{R}_1(s) = \int_{-1}^{0} \mathrm{d}z R_1(s,t)$$

colors: different $\alpha_1 - \beta_1$
bands: different $\alpha_1 + \beta_1$



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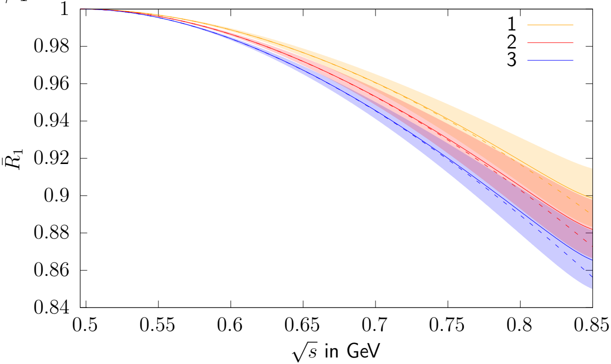

Ratios

- define ratios similar to COMPASS analysis

$$R_2(s,t) = \frac{\mathrm{d}\sigma/\mathrm{d}\Omega(s,t)}{\mathrm{d}\sigma/\mathrm{d}\Omega(s,t)\Big|_{\alpha_1=\beta_1=K^*=0}}$$

$$\bar{R}_2(s) = \int_{-1}^{0} \mathrm{d}z R_2(s,t)$$

colors: different $\alpha_1 - \beta_1$
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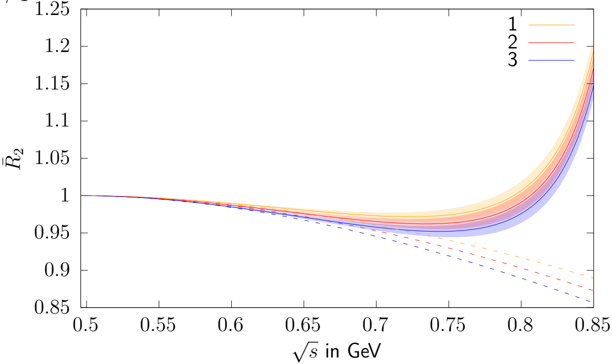
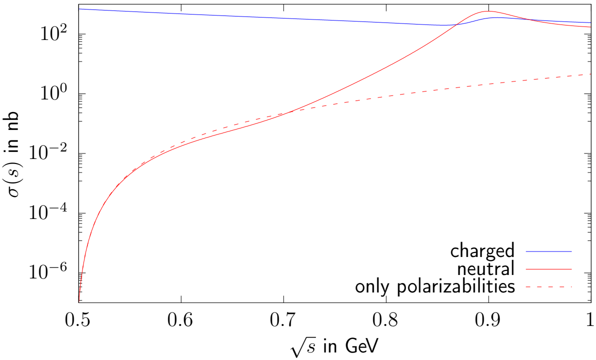


Neutral kaon

- in principle possible
- however no Born terms $\rightarrow$ heavily suppressed

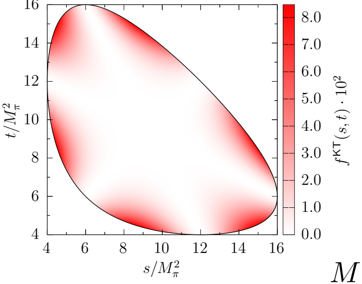


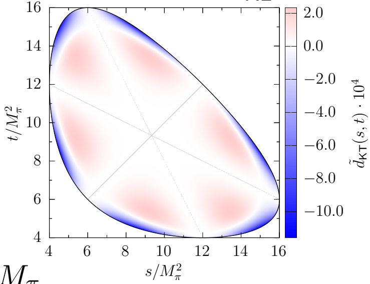
Summary

- to do: replace ChPT loops by dispersive amplitude
- suggest to use $\bar{R}_1$ for extraction of kaon polarizabilities
- global analysis together with $\gamma K \to K\pi$ amplitudes

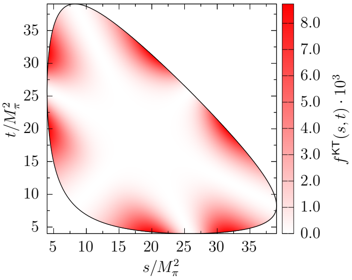
Dalitz plots for 0^{--}

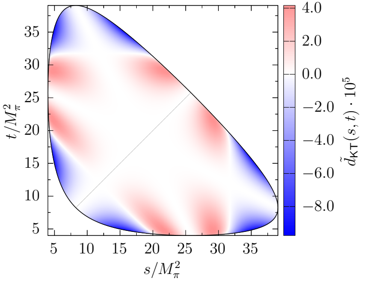
without phase space $M = 5M_\pi$ distribution of d_{KL}





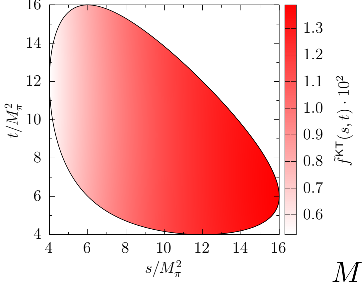
$M = 7.25M_\pi$

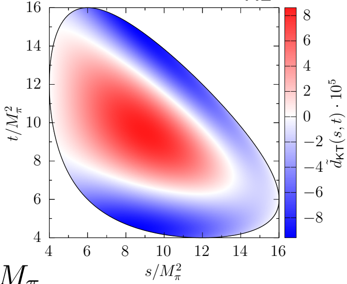




Dalitz plots for 2^{++}

without phase space $M = 5M_\pi$ distribution of d_{KL}





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