Towards a unification of gluon distributions at small and moderate x

Yacine Mehtar-Tani (BNL/RBRC)

@ Jefferson Lab
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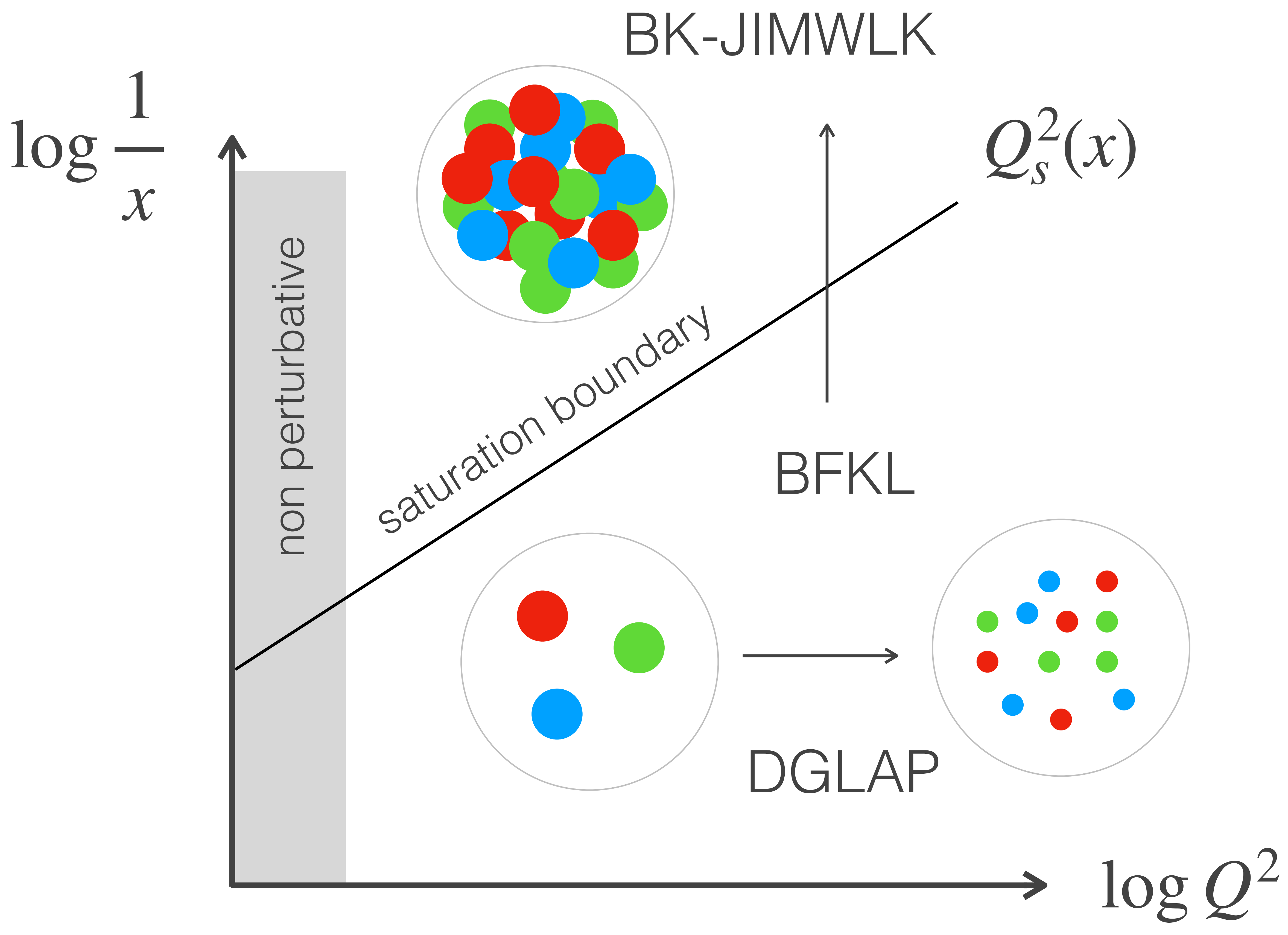
In collaboration with Renaud Boussarie

Outline

- Introduction
- BK equation and the NLO crisis
- Where is x hiding?
- Digression: transverse gauge links at small x
- Revisiting the shock-wave approximation
- Road test: inclusive DIS
- A novel unintegrated gluon distribution
- Summary and outlook

Introduction

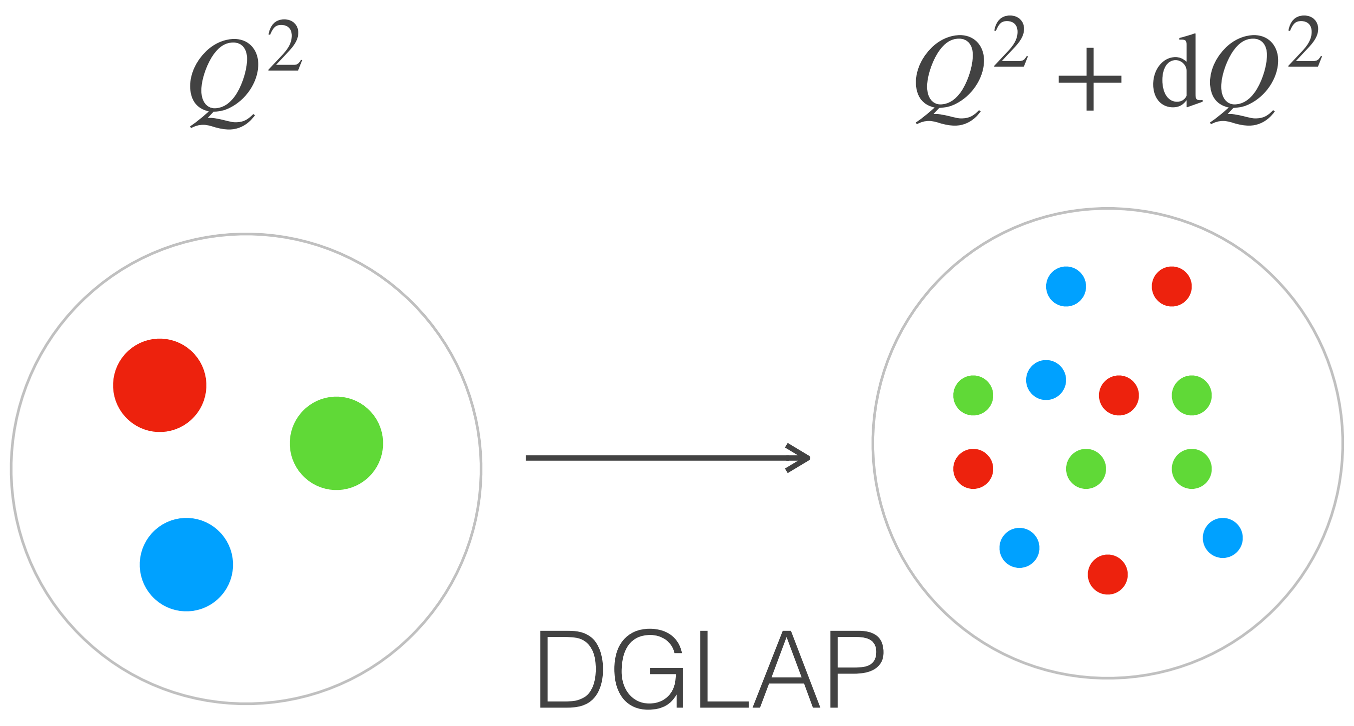
Proton structure and QCD evolution

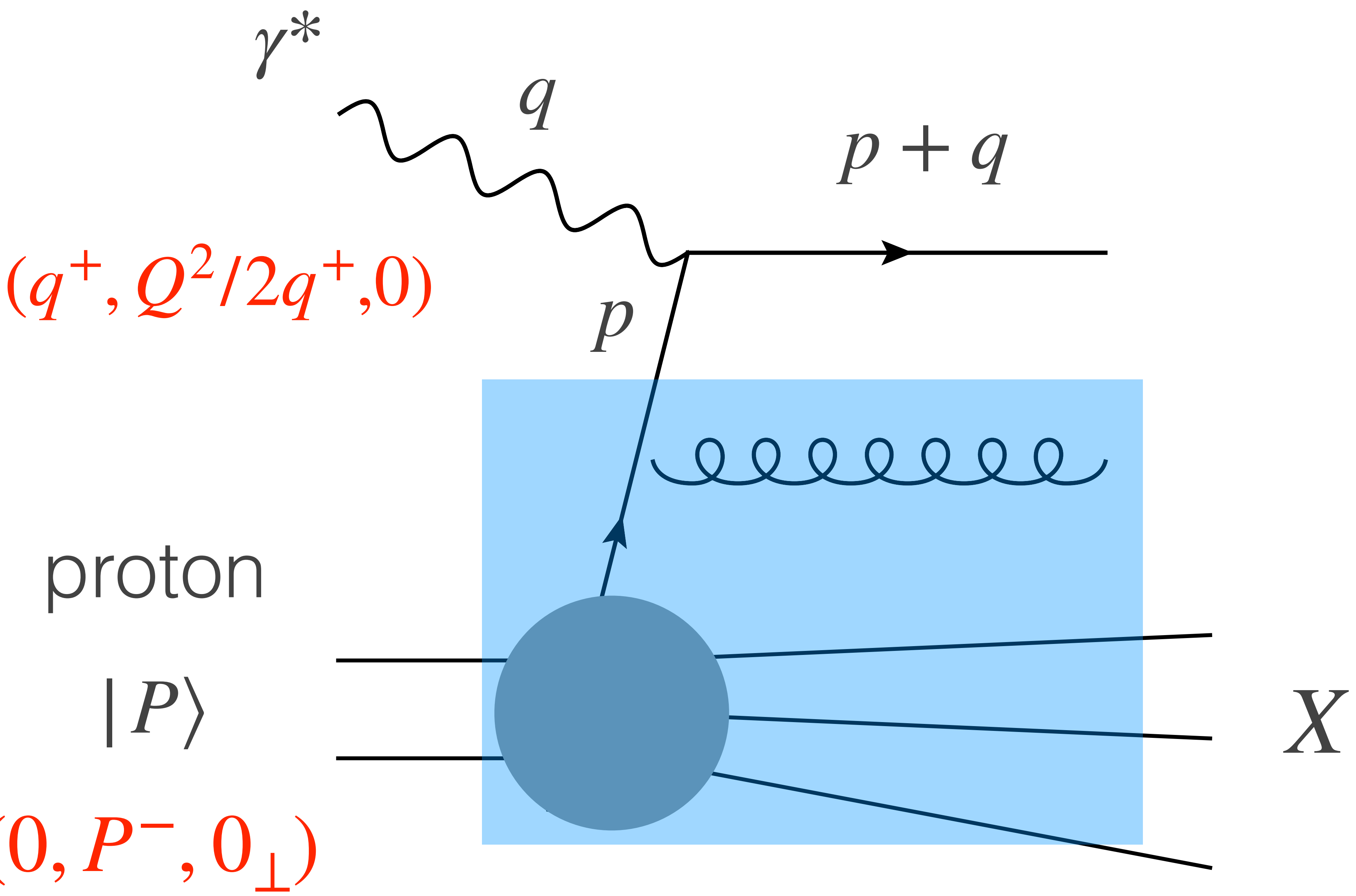


- at large Q^2 and moderate $x \sim Q^2/s$ the proton can be described as a collection of weakly interacting partons
- partonic interpretation breaks down beyond leading twist
- at small x gluon occupation number increases and eventually saturates due to non-linear effects (gluon recombination)

[Venugopalan, McLerran (MV), Balitsky, Kovchegov (BK)
Jallilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner (JIMWLK) (1993-2001)]

DIS kinematics





$q \equiv (q^+, Q^2/2q^+, 0)$

$P \equiv (0, P^-, 0_\perp)$

- DIS kinematics: hadronic part

$$Q^2 = -q^2 > 0 \qquad x_{Bj} \equiv \frac{Q^2}{2P\cdot q} = \frac{Q^2}{s}$$

- Mass condition

$$p = xP + k_\perp \qquad k_\perp \ll Q$$

$$(p+q)^2 = 0$$

$$\Rightarrow \quad -Q^2 + xP^- q^+ \simeq 0 \quad \Rightarrow \quad \boxed{x = x_{Bj}}$$

- At leading order we recover the Parton model:

$$F_2(x) \sim xq(x)$$

- Light-cone variables:

$$q^+ = \frac{q^0 + q^3}{\sqrt{2}} \qquad q^- = \frac{q^0 - q^3}{\sqrt{2}}$$

DGLAP: Bjorken limit $Q^2 \sim s$

[Dokshitzer, Gribov, Lipatov, Altarelli, Parisi (1972-77)]

- DGLAP evolution resums large collinear logarithms $\bar{\alpha} \log Q/\mu = \bar{\alpha} \int_\mu^Q \frac{\mathrm{d}k_\perp}{k_\perp}$
- successive emissions are strongly ordered in transverse momentum

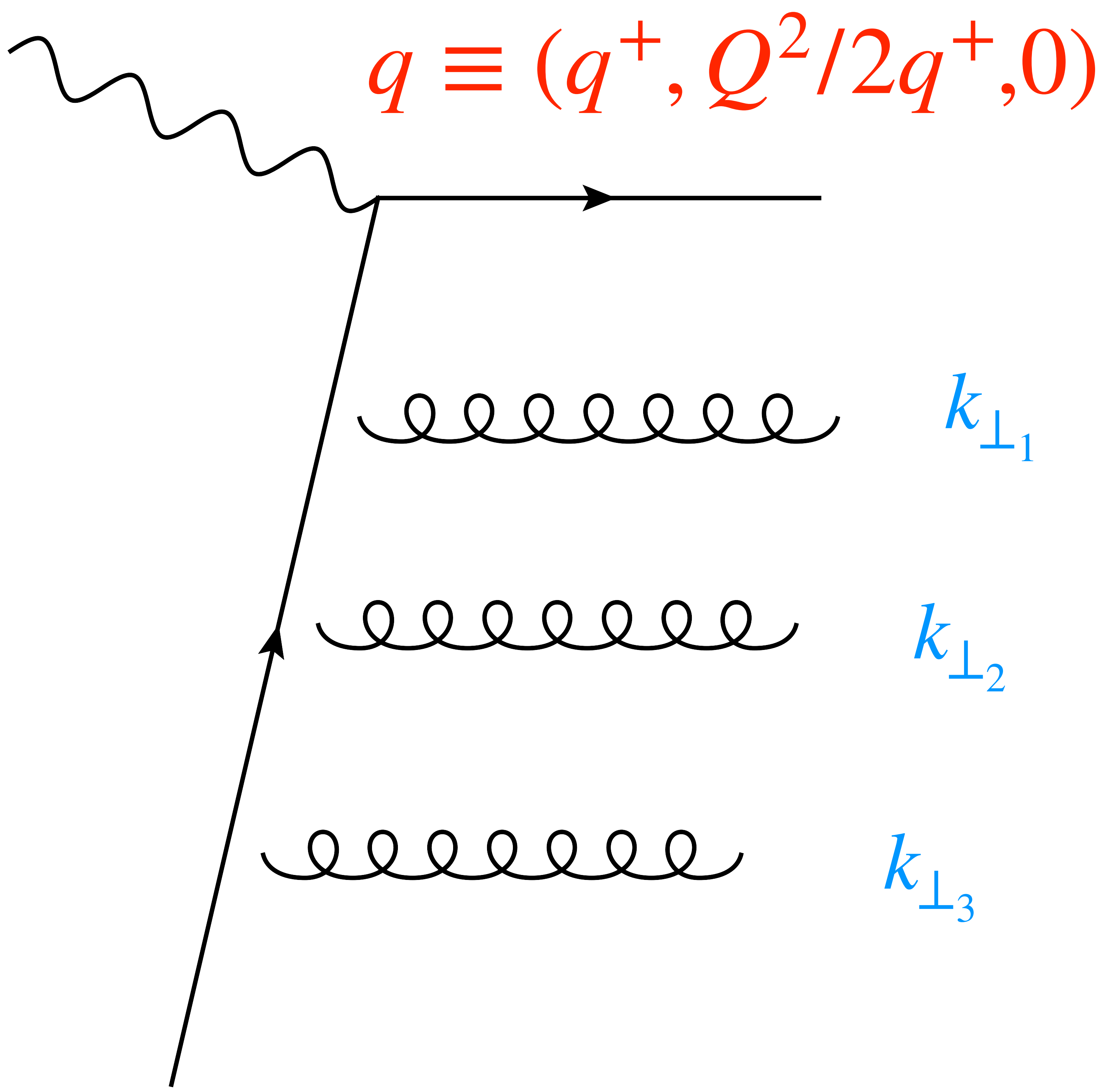
$$Q \gg k_{\perp_1} \gg k_{\perp_2} \gg \ldots \gg \mu$$

- The variable related to $x = k^-/P^- \sim 1$

$$k_1^- \sim k_2^- \sim \ldots \sim P^- \sim \sqrt{s}$$

- $k^- \sim t_f^{-1}$ is the inverse formation time scale
- It follows that $k^+ = k_\perp^2/k^-$'s are strongly ordered

$$q^+ \gg k_1^+ \gg k_2^+ \gg \ldots \gg \frac{\mu^2}{P^-}$$



$P \equiv (0, P^-, 0_\perp)$ $\quad s = q^+ P^-$

BFKL: Regge limit $Q^2 \ll s$

[Balitsky, Fadin, Kuraev, Lipatov, (1976-78)]

- BFKL evolution resums large soft logarithms $\bar{\alpha} \log s^{-1} = \bar{\alpha} \int_{q_0^+}^{q^+} \frac{\mathrm{d}k^+}{k^+}$
- successive emissions are strongly ordered in longitudinal momenta

$$\sqrt{s} \sim q^+ \gg k_1^+ \gg k_2^+ \gg \ldots \gg q_0^+$$

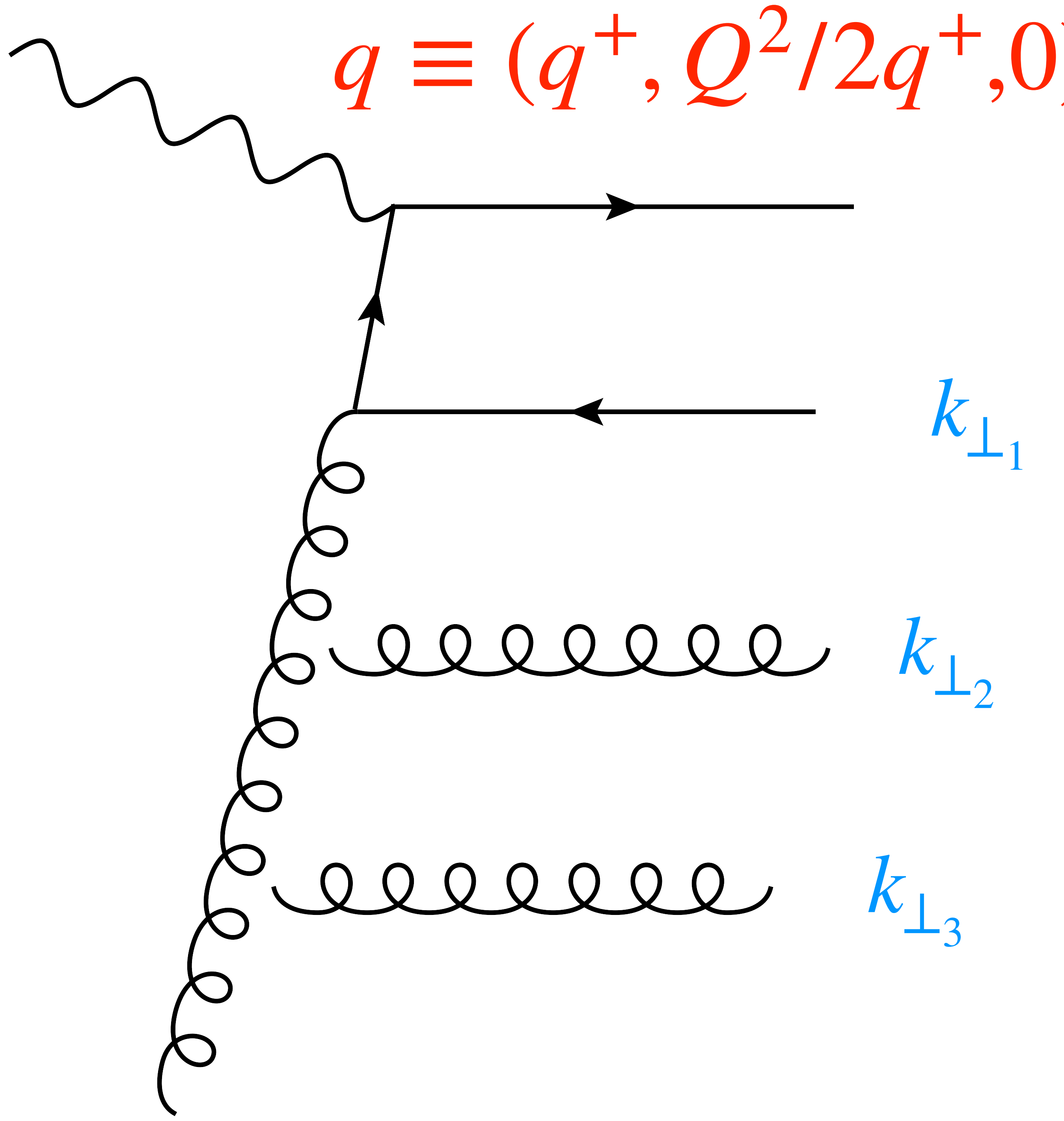
- And the variable $x = k^-/P^- \ll 1$

$$k_1^- \ll k_2^- \ll \ldots \ll P^- \sim \sqrt{s}$$

- $k^- \sim t_f^{-1}$ is the inverse formation time scale
- Transverse momenta are of same order

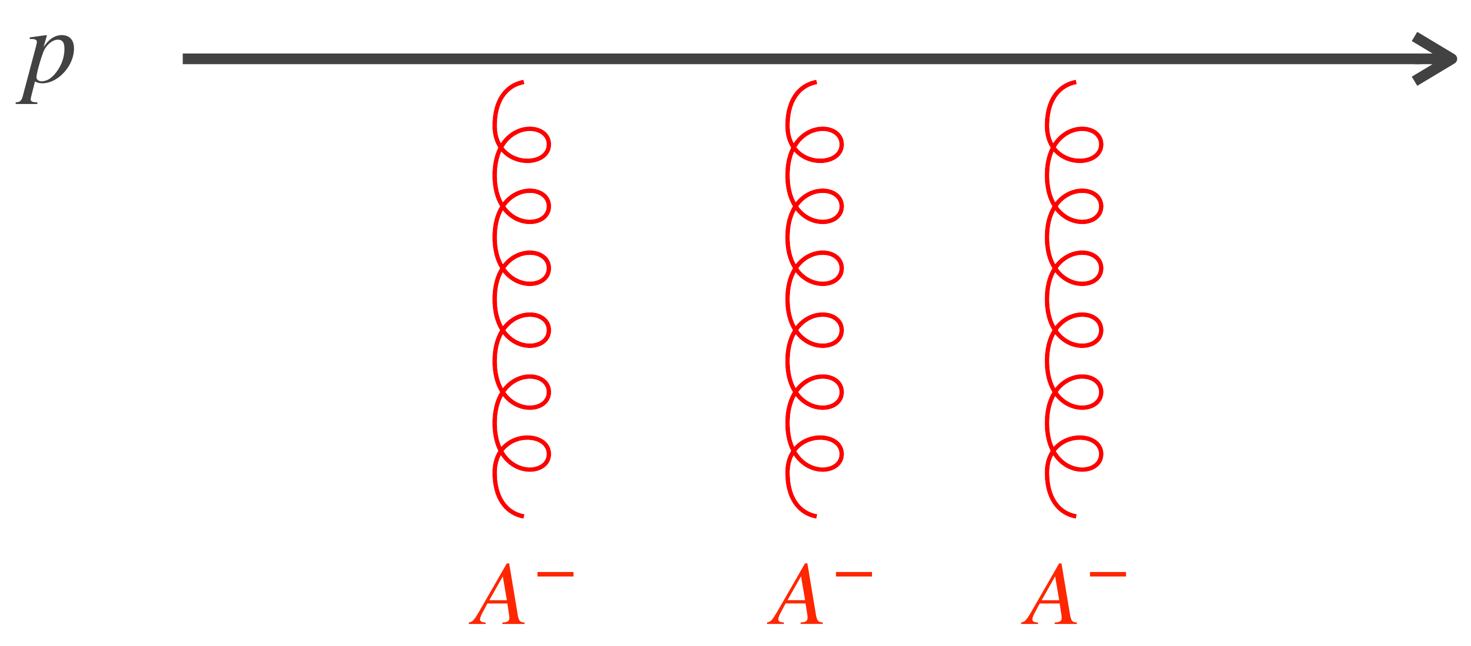
$$Q \sim k_{\perp_1} \sim k_{\perp_2} \sim \ldots \sim \mu$$

$q \equiv (q^+, Q^2/2q^+, 0)$



$P \equiv (0, P^-, 0_\perp)$ $\quad s = q^+ P^-$

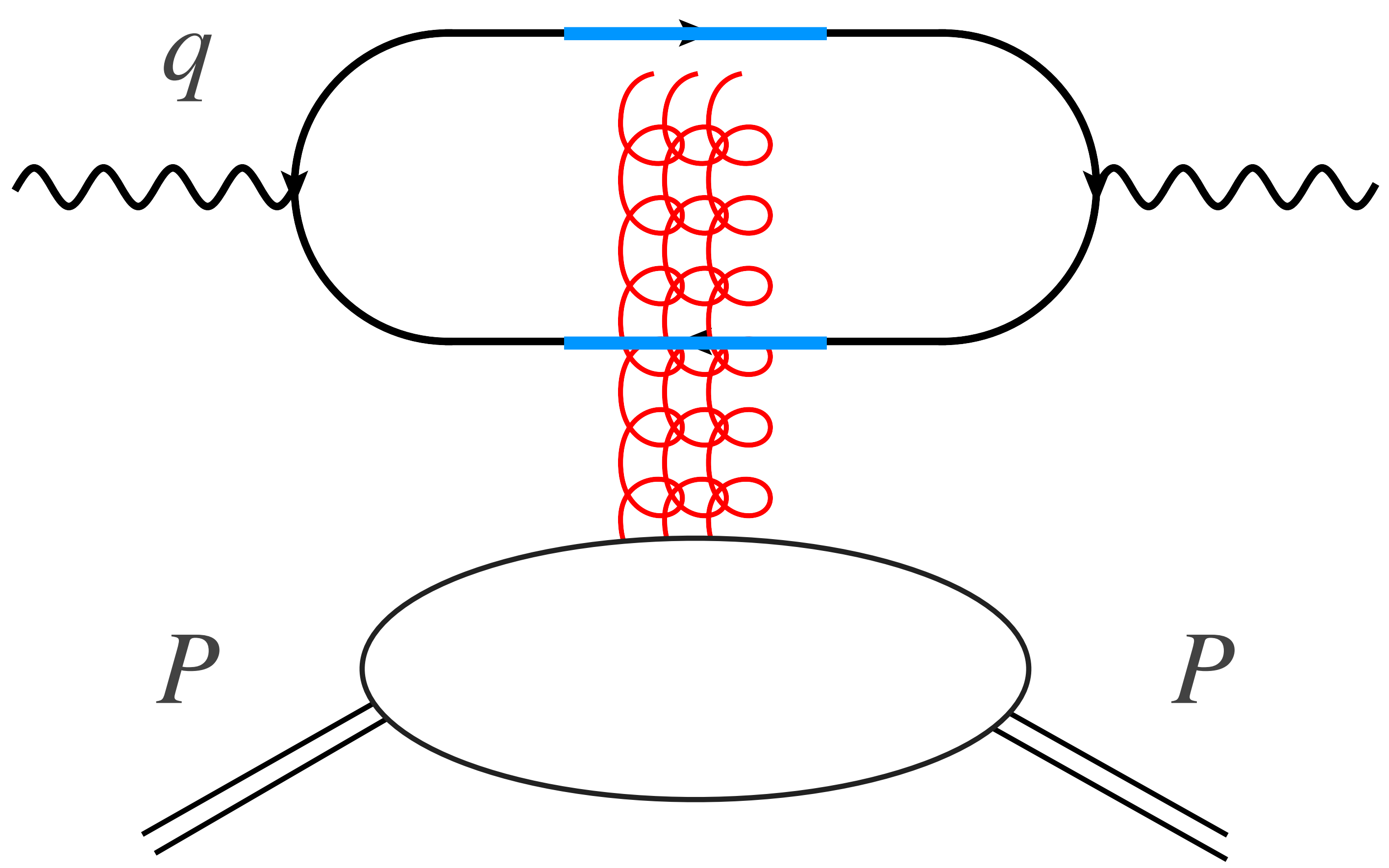
The dipole model in DIS



- the relevant d.o.f. in the saturation regime are strong classical fields $g A^- \sim 1$
- eikonal lines: $p^+ \gg k^+$ and $k^- \ll P^-$

- path ordered Wilson line

$$U_{\boldsymbol{x}} \equiv [+\infty, -\infty]_{\boldsymbol{x}} = P \exp \left[ig \int_{-\infty}^{+\infty} \mathrm{d}x^+ A^-(x^+, \boldsymbol{x}) \right]$$



- dominant contribution in DIS: dipole scattering

$$\langle P \,|\, \mathrm{Tr}\; U_{\boldsymbol{x}_1} \; U^\dagger_{\boldsymbol{x}_2} \,|\, P \rangle$$

shock wave limit: $t_f \equiv Q^2/q^+ \gg 1/P^- \;\; \rightarrow x = Q^2/s \ll 1$

BK and the NLO crisis

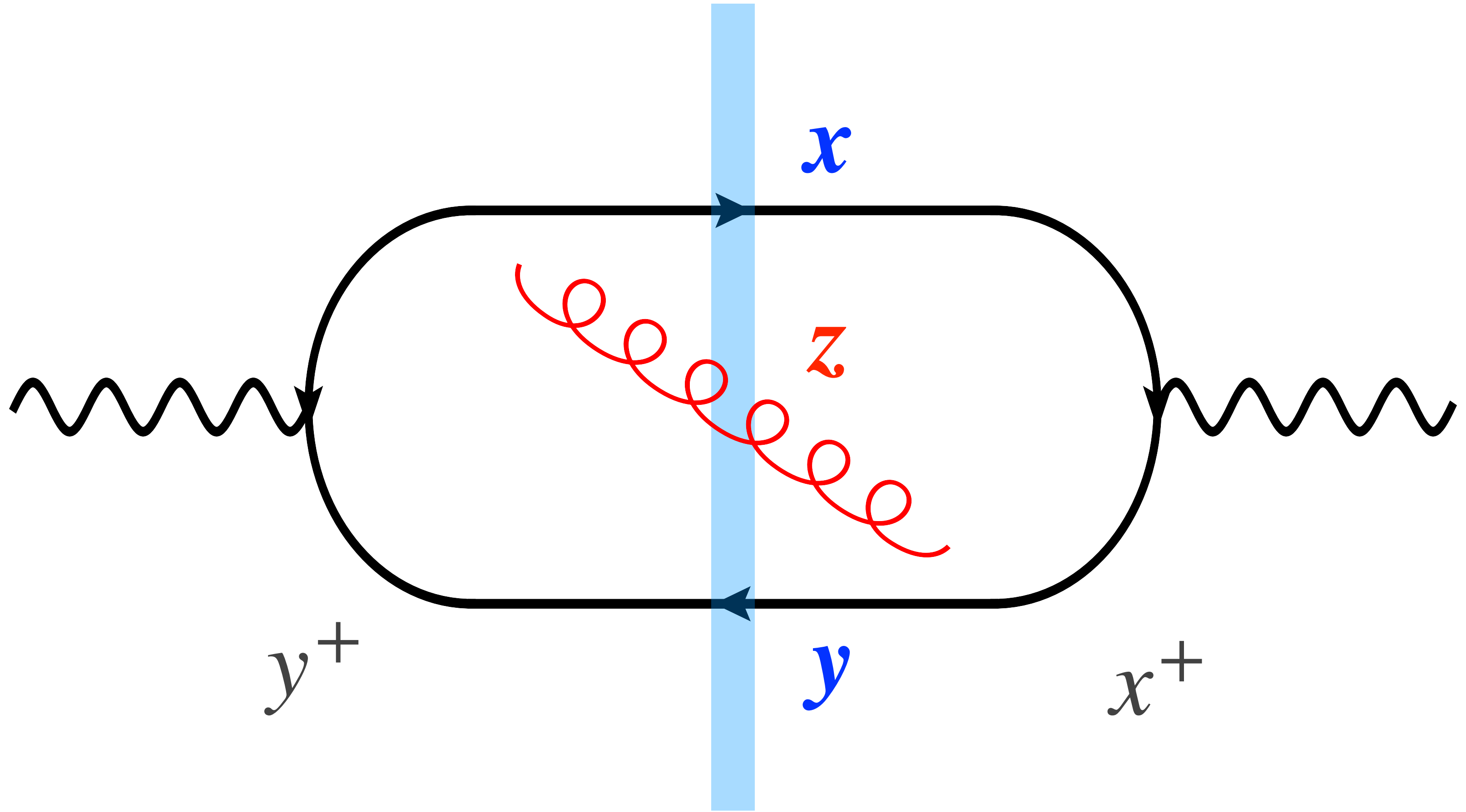
BK equation at LO

- Balitsky-Kovchegov (1996-1999) equation describes the non-linear evolution of the dipole scattering amplitude as function of the rapidity

$Y \equiv \log \frac{q^+}{\Lambda^+}$

$$S_Y(\boldsymbol{x}-\boldsymbol{y}) \equiv \frac{1}{N_c} \left\langle \mathrm{Tr}\, U(\boldsymbol{x})\, U^\dagger(\boldsymbol{y}) \right\rangle_Y$$

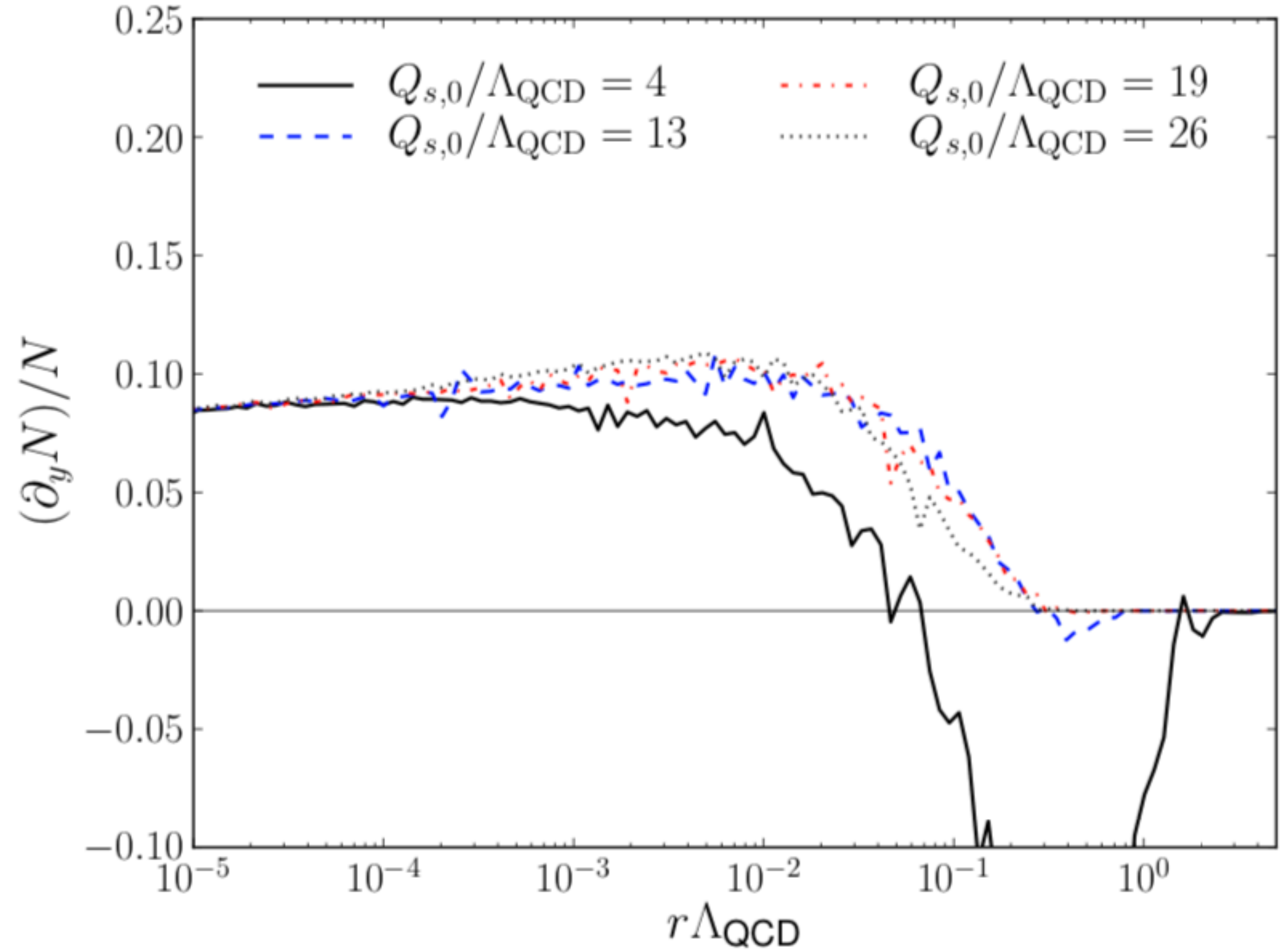
$$\frac{\partial}{\partial Y} S_Y(\boldsymbol{x}-\boldsymbol{y}) = \bar{\alpha} \int \mathrm{d}\boldsymbol{z} \frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2} \left[S_Y(\boldsymbol{x}-\boldsymbol{z}) S_Y(\boldsymbol{z}-\boldsymbol{y}) - S_Y(\boldsymbol{x}-\boldsymbol{y}) \right]$$



BK and the NLO crisis

[Balitsky and Chirilli (2008)]

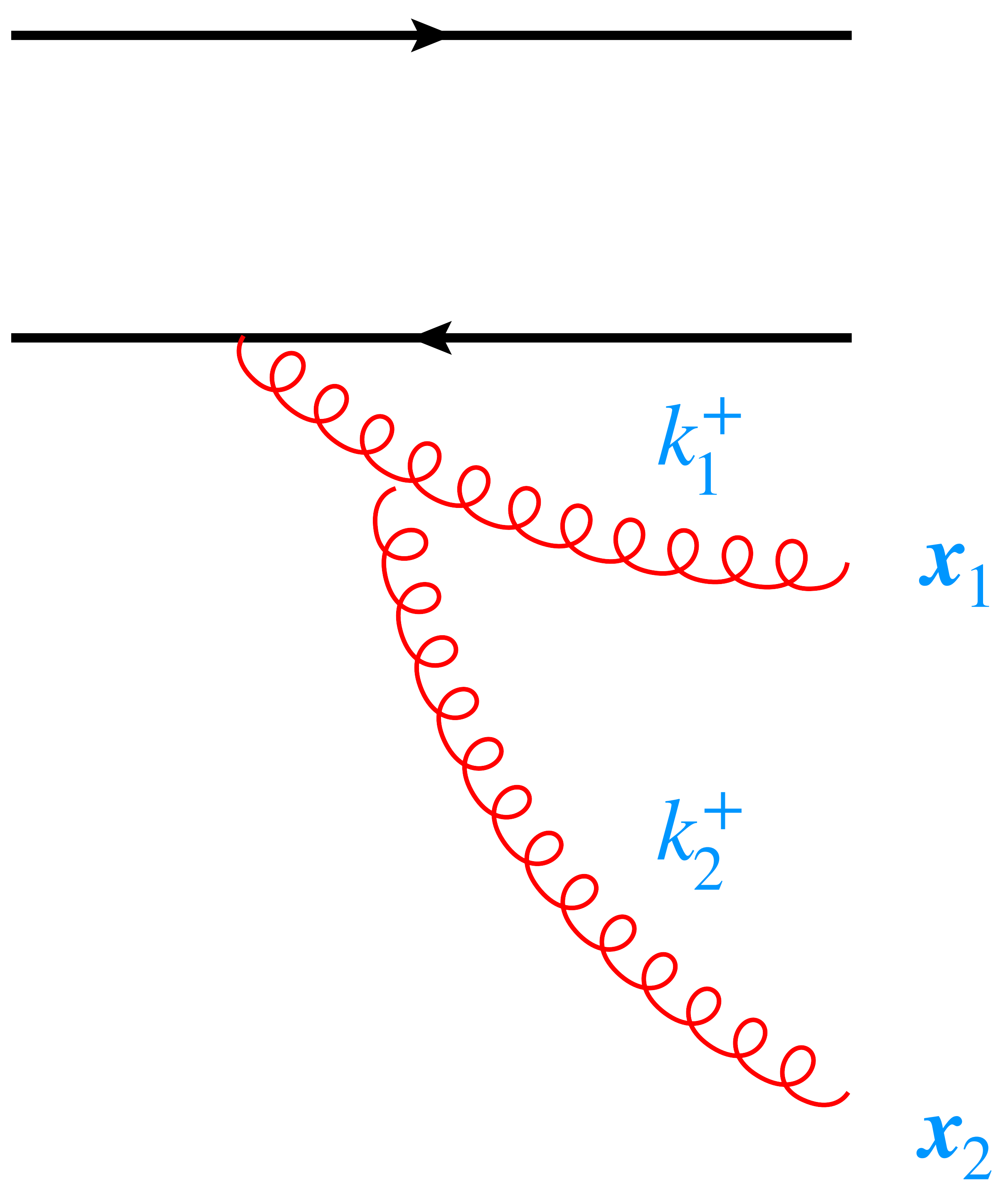
- At NLO BK equation was found to be numerically unstable



[Lappi and Mäntysaari (2015)]

BK and the NLO crisis

- This is due to the fact that the rapidity variable a $Y \equiv \log \frac{q^+}{\Lambda^+}$ evolves independently from $\boldsymbol{x}$
- As a result small dipoles that radiate larger dipoles generate large collinear logarithms when $k_1^- \sim k_2^- \Rightarrow$ NLO > LO



formation ime ordering

$$k_1^- = \boldsymbol{x}_1^2 k_1^+ < k_2^- = \boldsymbol{x}_2^2 k_2^+ \quad \Rightarrow \quad \frac{\boldsymbol{x}_1^2}{\boldsymbol{x}_2^2} < \frac{k_2^+}{k_1^+} \ll 1$$

$$k_1^+ \gg k_2^+$$

[Beuf (2014) Ducloué, Iancu, Mueller, Soyez, Triantafyllopoulos (2015-2019)]

BK and the NLO crisis

[Beuf (2014) Ducloué, Iancu, Mueller, Soyez, Triantafyllopoulos (2015-2019)]

- Several solutions have been proposed: implementing kinematic bound, resummation of double logarithms. However,

    - they spoil the renormalization picture established at LO
    - exhibit scheme dependence
    - no operator definition for systematic order by order calculations
    - not discussed at the level of observables: inclusive DIS

Where is x hiding?

Where is x hiding?

- In the shock wave [Balitsky (1996)] approach there is no explicit dependence on the longitudinal momentum fraction of the gluons in the target
- The standard trick is to identify rapidity variables $Y = \eta$

$$\eta \equiv \log \frac{1}{x_{Bj}}$$

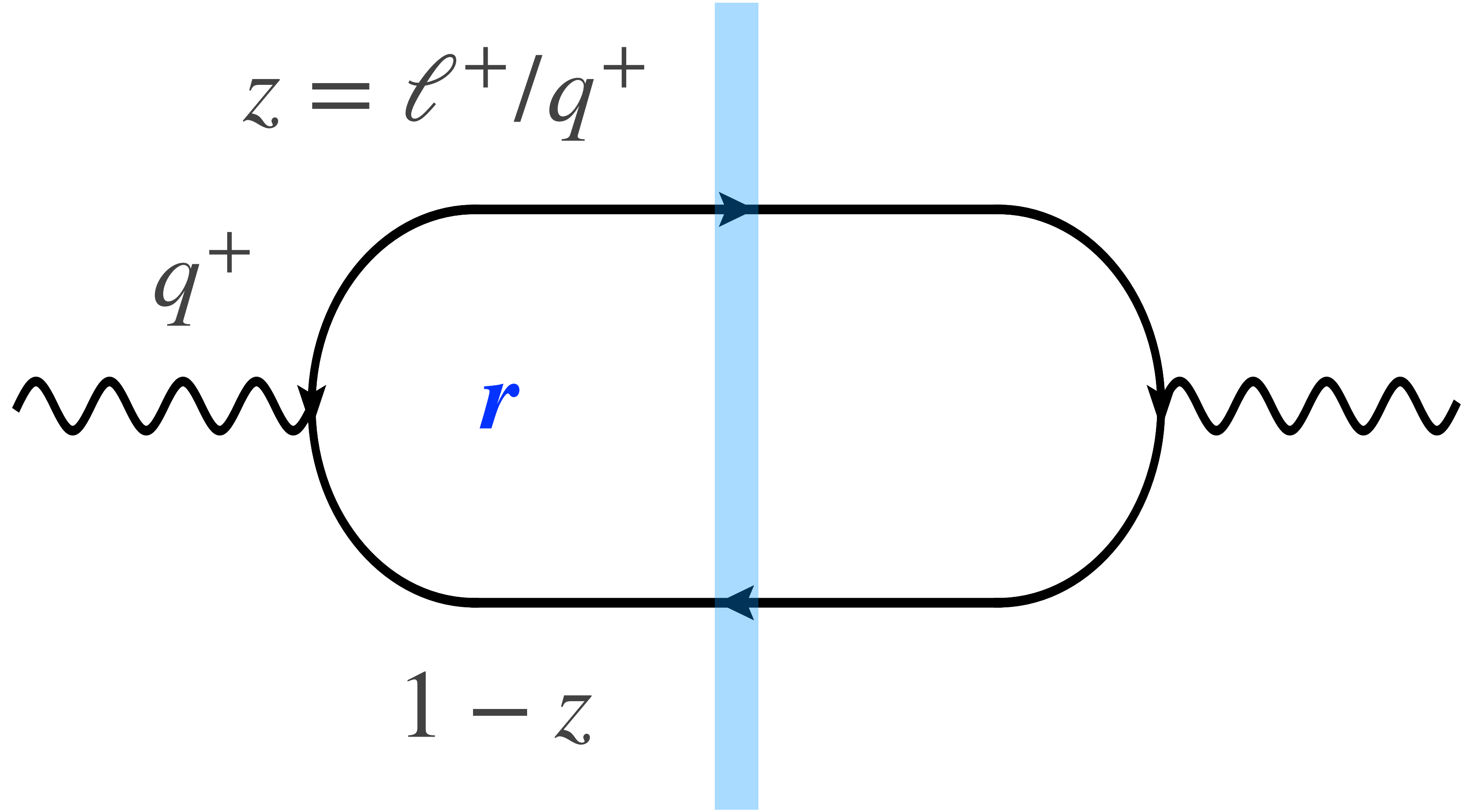
$$Y \equiv \log \frac{q^+}{q_0^+}$$

transverse photon wave function

$$\varphi(\boldsymbol{r}) = \frac{\boldsymbol{r}\cdot\boldsymbol{\epsilon}_\lambda}{|\boldsymbol{r}|} K_1\left(\sqrt{z(1-z)|\boldsymbol{r}|^2 Q^2}\right)$$

cross-section

$$\sigma(x_{Bj}, Q^2) \sim e^2 \int_0^1 \mathrm{d}z\, P(z)\ |\varphi(\boldsymbol{r})|^2\ \langle \mathrm{Tr} U(\boldsymbol{r}) U^\dagger(0)\rangle_Y$$



- It is instructive to check the collinear limit

Where is x hiding?

- In the leading twist: expanding for small dipoles $r_\perp \sim 1/Q \ll b_\perp$

$$U(\boldsymbol{b}+\frac{\boldsymbol{r}}{2}) = U(\boldsymbol{b}) + r^i \partial^i U(\boldsymbol{b})$$

$$\partial^i U(\boldsymbol{b}) = \int_{-\infty}^{+\infty} \mathrm{d}x^+ \left([+\infty, x^+]\, \partial^i A^-(x^+)\, [+\infty, x^+] \right)_{\boldsymbol{b}} \qquad \text{where} \qquad \partial^i A^- \equiv F^{i-}$$

- we recover the gluon PDF …

$$\int \mathrm{d}\boldsymbol{b} \left\langle \mathrm{Tr}\, U(\boldsymbol{b}+\frac{\boldsymbol{r}}{2})\, U^\dagger(\boldsymbol{b}-\frac{\boldsymbol{r}}{2}) \right\rangle_Y \approx \boldsymbol{r}^2\, xg(1/\boldsymbol{r}^2, x) + O(\boldsymbol{r}^4)$$

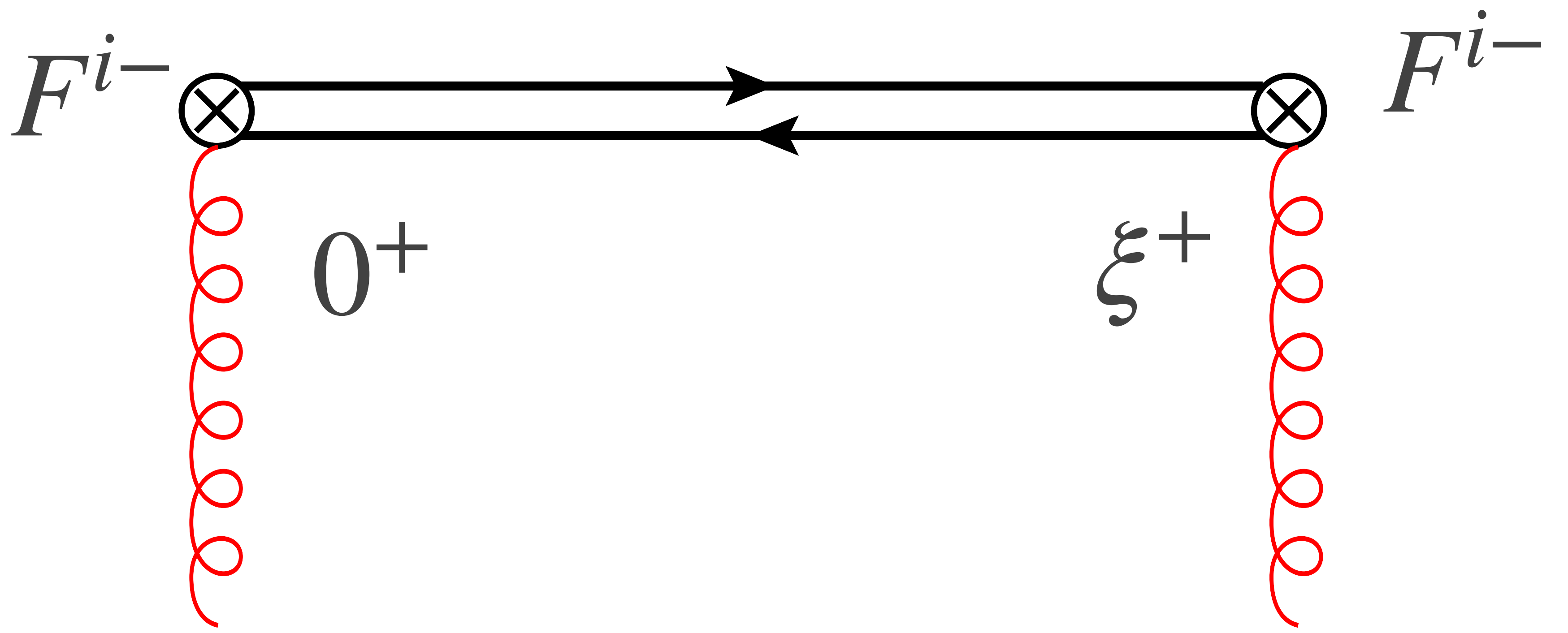
Where is x hiding?

- …except for the phase that encodes to $x = k^-/P^-$ dependence

$$xg(x,\mu^2) = \frac{2}{P^-}\int_{\boldsymbol{b}}\int_{x^+,y^+} \left\langle \mathrm{Tr}\,[y^+,x^+]\,F^{i-}(x^+)\,[y^+,x^+]\,F^{i-}(y^+)\right\rangle_Y$$

- the complete gluon PDF writes (the integral over x^+ and $\boldsymbol{b}$ cancel against $\langle P|P\rangle$)

$$xg(x,\mu^2) = 2\int \frac{\mathrm{d}\xi^+}{(2\pi)P^-}\,\mathrm{e}^{ixP^-\xi^+}\left\langle P\right|\mathrm{Tr}\,[0,\xi^+]\,F^{i-}(\xi^+)\,[\xi^+,0]\,F^{i-}(0)\left|P\right\rangle$$



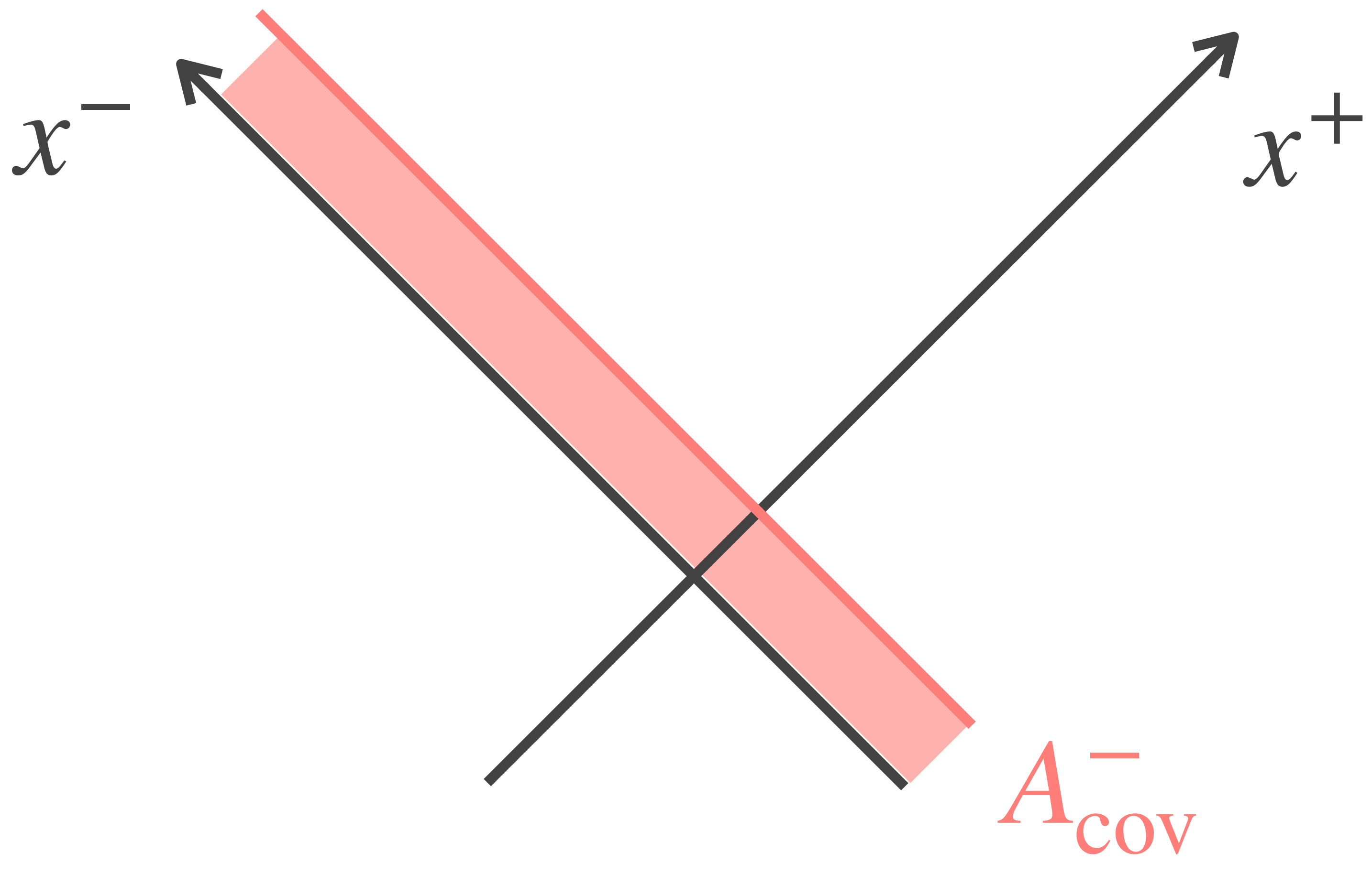
Where is x hiding?

- Parametrically we have. $\xi^+ = x^+ - y^+ \sim 1/P^-$
- Then the phase scales as. $i\,x\,P^- \xi^+ \sim x$
- In shock wave one assumes: $x_{Bj} \ll x \ll 1 \;\Rightarrow$ one can neglect the phase.

  This is the BFKL-BK phase space
- On the other hand a potentially large collinear logarithm lives in the region $x_{Bj} \ll x \sim 1$
- This might not be so relevant at leading order but at NLO dilute spots in the proton wave function may generate large collinear logarithms

Digression:
background field and transverse gauge links

Background field and transverse gauge links



target sources

$$J^\nu(x) \rightarrow J^-(x^+, x_\perp)$$

and $\quad J^+ = J_\perp = 0$

- consider a target boosted along the $-z$ direction close to the light cone. Due to time dilation the target color sources are "frozen" in the - direction

- Yang-Mills equations $[D_\mu, F^{\mu\nu}] = J^\nu$ can be solved exactly (together with the continuity equation $[D_\mu, J^\mu] = 0$) in covariant gauge $\partial \cdot A = 0$ (or light-cone gauge $A^+ = 0$)

$$A^-_{\rm cov} = -\frac{1}{\partial^2_\perp} J^- \quad \text{and} \quad A^+ = A_\perp = 0$$

Background field and transverse gauge links

- under an arbitrary gauge rotation $\Omega(x^+, x_\perp)$ the target field transforms as

$$A^- \;\to\; \Omega_{\boldsymbol{x}}(x^+)\; A^-_{\rm cov}\;(x^+, \boldsymbol{x})\; \Omega^{-1}_{\boldsymbol{x}}(x^+) \;-\; \frac{1}{ig}\Omega_{\boldsymbol{x}}(x^+)\;\partial^-\;\Omega^{-1}_{\boldsymbol{x}}(x^+)$$

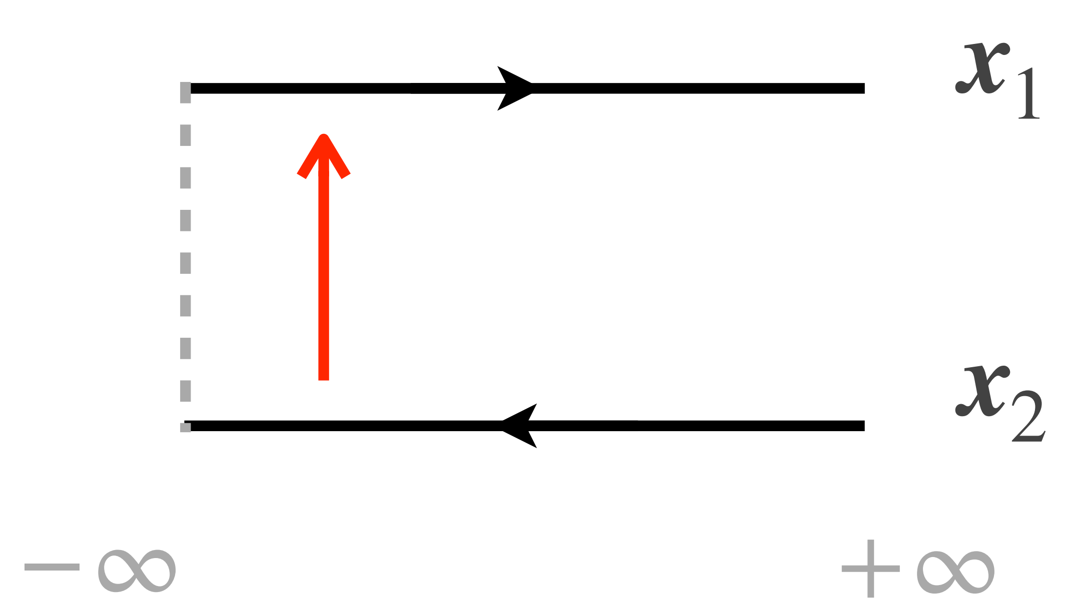
$$\boldsymbol{A}^{\,i} \;\to\; -\frac{1}{ig}\;\Omega_{\boldsymbol{x}}(x^+)\;\partial^i\;\Omega^{-1}_{\boldsymbol{x}}(x^+)$$

- exploiting the residual gauge freedom we can generate a transverse pure gauge
- N.B.: the partonic picture is manifest in the LC-gauge $A^- = 0$ (with $A_\perp \neq 0$)
- small x observables are (in the dilute/dense limit) more naturally expressed in the wrong LC-gauge $A^- \neq 0$ (with $A_\perp = 0$).
- in order to connect to the partonic interpretation one needs to deal with transverse fields

Background field and transverse gauge links

- geometric interpretation of the all twist resummation

$$U_{\boldsymbol{x}_1} = U_{\boldsymbol{x}_2} - \boldsymbol{r}^i \int_0^1 \mathrm{d}s\ (\partial^i U_{\boldsymbol{x}_2 + s\boldsymbol{r}})$$



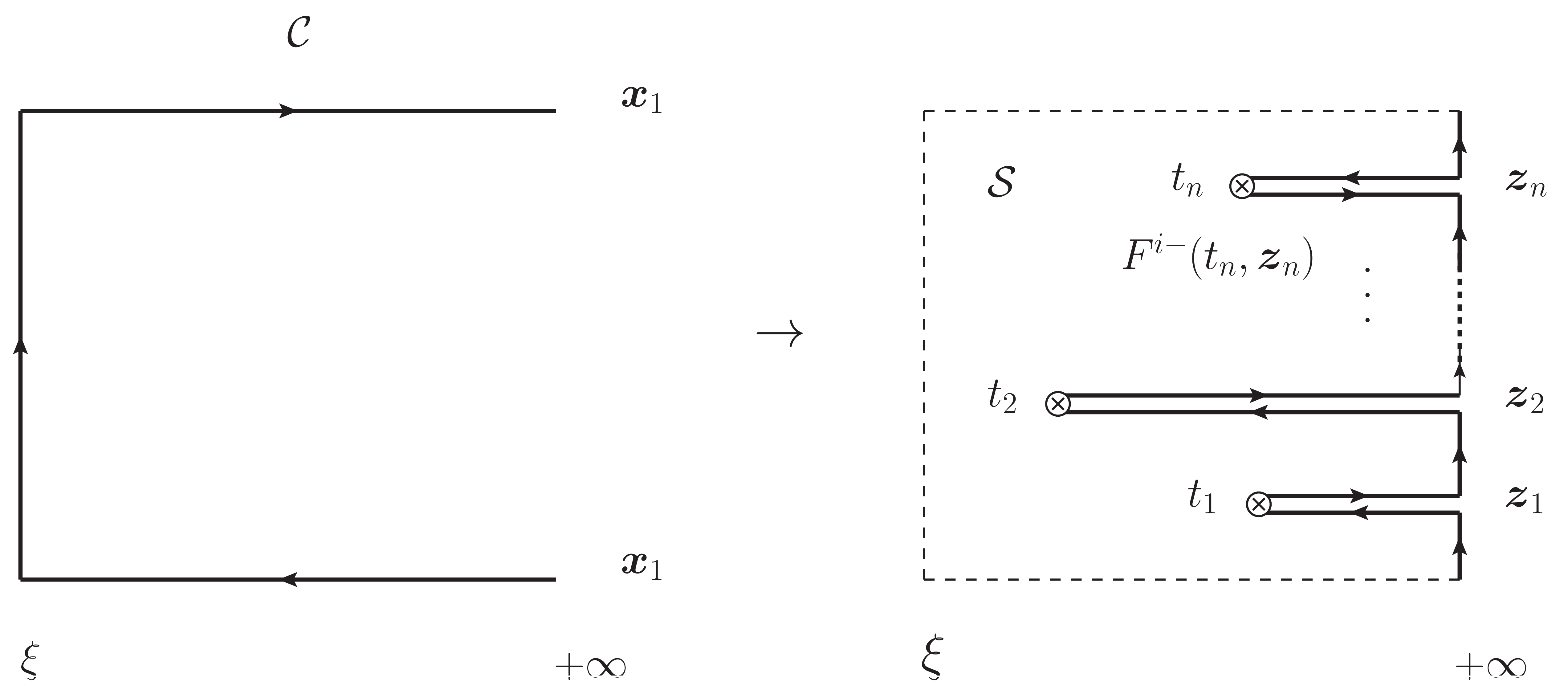
- and noticing that $\frac{1}{ig}(\partial^i U_{\boldsymbol{x}}) U_{\boldsymbol{x}} \equiv A^i(\boldsymbol{x})$, one can express the dipole operator (in the background field A^-) as a transverse gauge link:

$$U_{\boldsymbol{x}_1} U^\dagger_{\boldsymbol{x}_2} = [\boldsymbol{x}_1, \boldsymbol{x}_2] = 1 - ig \int_{\boldsymbol{x}_2}^{\boldsymbol{x}_1} \mathrm{d}\boldsymbol{z}\, \boldsymbol{A}^i(\boldsymbol{z})\ [\boldsymbol{z}, \boldsymbol{x}_2]$$

- dipole operator can be expessed in terms of tranverse link operators

Background field and transverse gauge links

- non-Abelian Stokes theorem: more generally, the dipole operator can be written as a path ordered tower of "twisted" field strength tensor (i.e. dressed with future pointing Wilson lines)



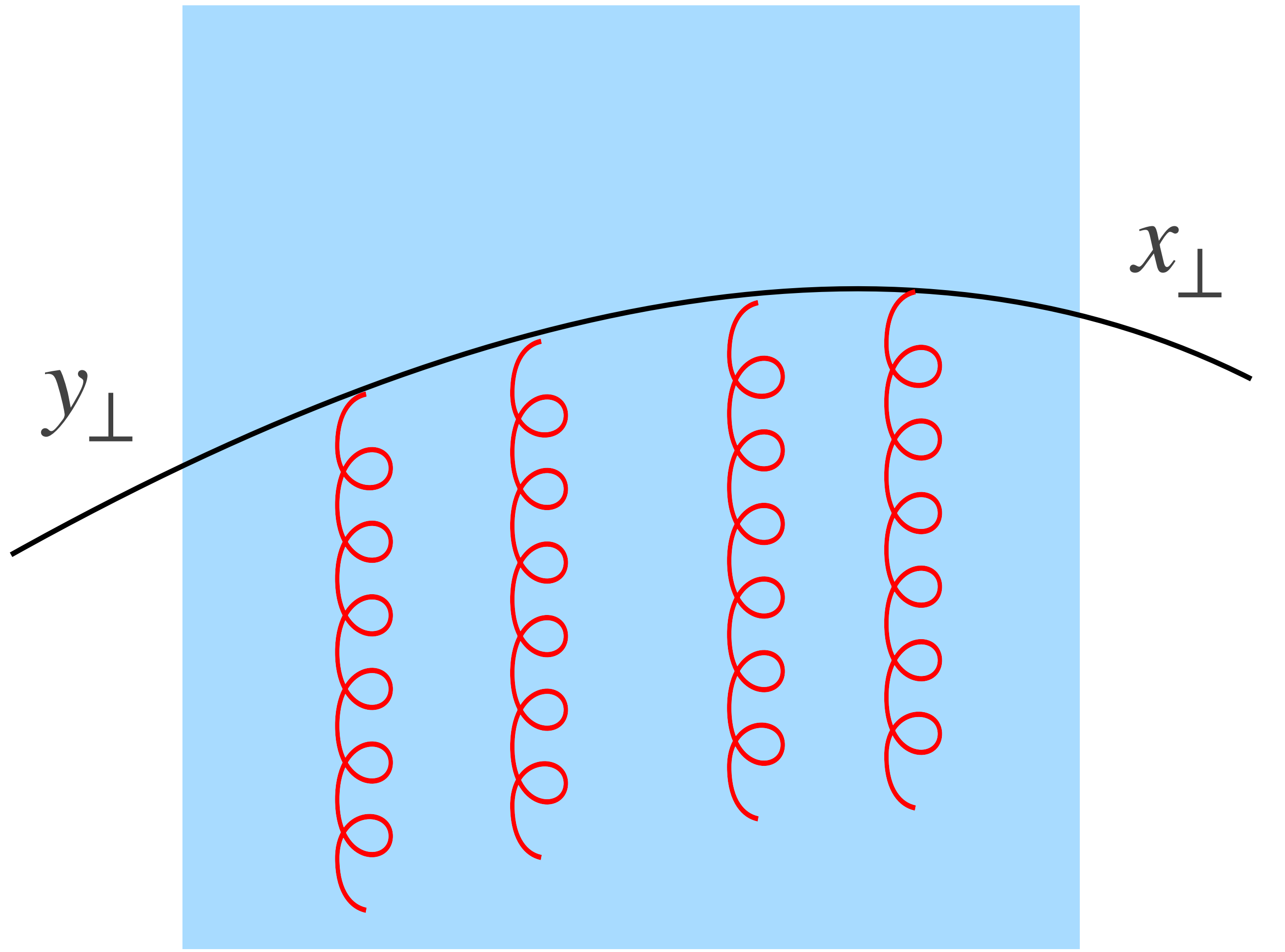
[Fishbane, Gasiorowicz, Kaus (1981) Wiedemann (2000) YMT, Boussarie (2020)]

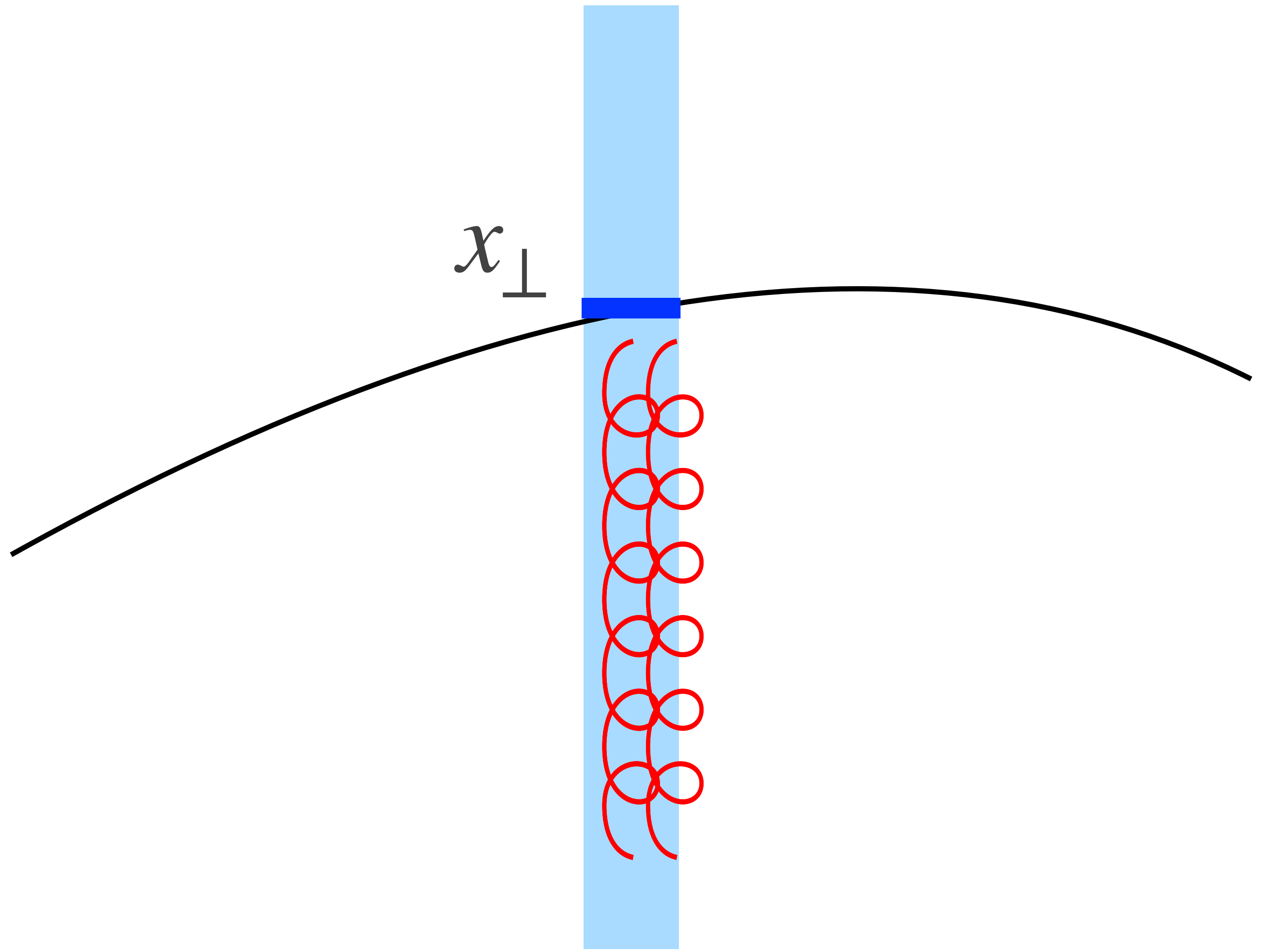
$$U_{x_2} U^\dagger_{x_1} \equiv P \exp\left[-ig \int_S \mathrm{d}t \mathrm{d}z \, [+\infty, x^+]_x \, F^{i-}(x^+, x) \, [x^+, +\infty]_x \right]$$

Revisiting the Shock Wave Approximation

Revisiting the Shock Wave Approximation

- Under the assumption that all transverse momenta are of same order along the ladder, the leading power $1/s$ is obtained letting $p^+ \to \infty$ for any particle propagating inside the shock wave





propagator

$$\mathscr{G}_{p^+}(x^+, y^+) = \left[i\frac{\partial}{\partial x^+} - \frac{\hat{p}_\perp^2}{2p^+} - gA^- \right]^{-1}$$

sock wave limit

$$\lim_{p^+ \to +\infty} \mathscr{G}_{p^+}(x^+, \boldsymbol{x};\, y^+, \boldsymbol{y}) = \delta(\boldsymbol{x} - \boldsymbol{y})\, U_{\boldsymbol{x}}(x^+, y^+)$$

Revisiting the Shock Wave Approximation

- This limit neglects quantum diffusion

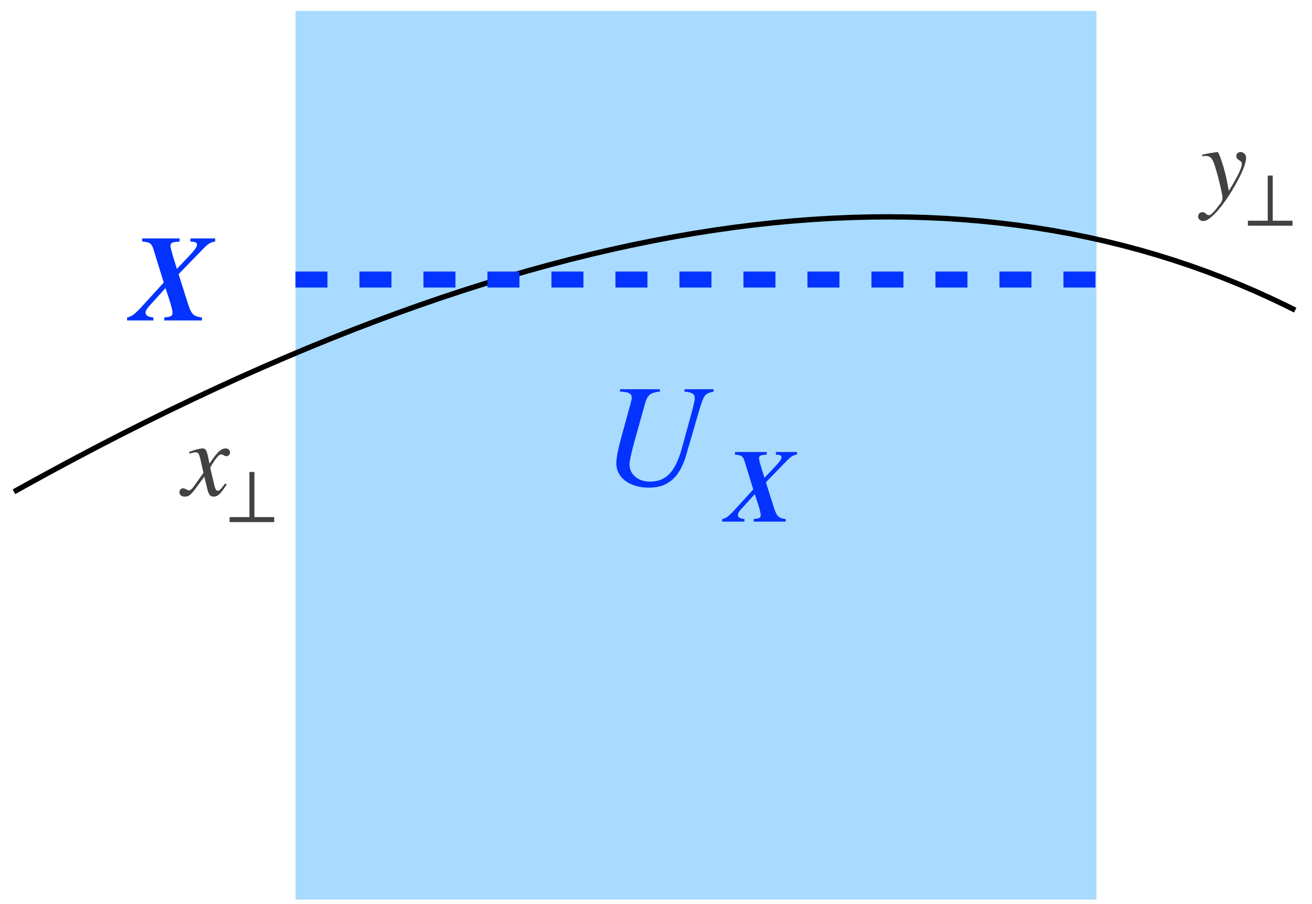
$$\mathcal{G}^0_{p^+}(x^+, \boldsymbol{x}; y^+, \boldsymbol{y}) = \frac{p^+}{2i\pi\,\Delta x^+}\, \mathrm{e}^{\,i\frac{(\boldsymbol{x}-\boldsymbol{y})^2\, p^+}{\Delta x^+}}$$

- it is important when $(\Delta \boldsymbol{x})^2 \sim \Delta x^+/p^+ \sim s^{-1}$
- In effect, the phase relates the transverse dynamics to longitudinal dynamics, this is the phase that appears in the definition of PDF's
- It encodes the information about k^-'s in the target. It is expected to be non-negligible away from the strongly ordered region in k^-.

Revisiting the Shock Wave Approximation

- Instead of taking the limit $p^+ \to +\infty$ we can simply expand around the constant transverse coordinate (classical trajectory)

$$\boldsymbol{X} \equiv \frac{\boldsymbol{x} + \boldsymbol{y}}{2}$$



- At leading power we we recover a Wilson line multiplied by a phase

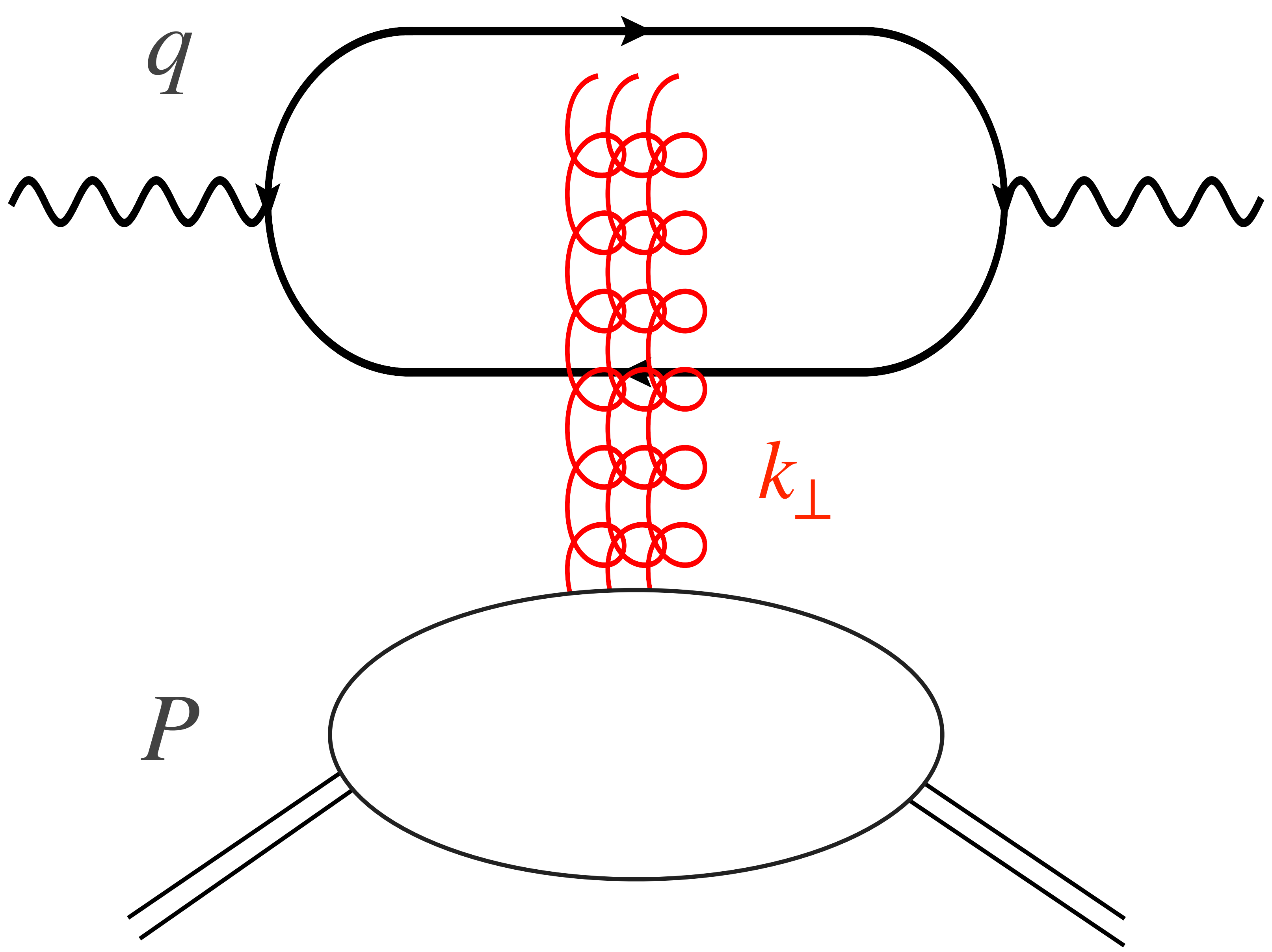
$$\mathcal{G}_{p^+}(x^+, \boldsymbol{x}; y^+, \boldsymbol{y}) = \mathcal{G}_0(\boldsymbol{x} - \boldsymbol{y}, x^+ - y^+)\, U_{\boldsymbol{X}}(x^+, y^+) + \ldots$$

- Taking again the limit $p^+ \to +\infty$ we recover the shock wave
- The ellipses are subleading terms suppressed as $(\boldsymbol{x} - \boldsymbol{y})^i(\partial^i U)_{\boldsymbol{X}}$

Road Test:
Inclusive DIS

Inclusive DIS beyond shock wave

$q \equiv (q^+, q^-, 0_\perp)$



$P \equiv (0^+, P^-, 0_\perp)$

1. ordering in k^+
2. gluonic target
3. leading power in $k_\perp^2/s$

Inclusive DIS beyond shock wave

- Working assumptions:

  1. We shall assume a large separation between the target modes and the quantum fluctuations (here the quark dipole)

[Balitsky and Braun (1988)]

$k^+ > \Lambda^+$	$k^+ < \Lambda^+$
quantum	classical
$\psi_q(x)$	$A_c^-(x^+, \boldsymbol{x}) \sim \delta(k^+)$

NB: strong ordering in k^+ is common to small x and leading twist DGLAP

$$k_1^+ = \frac{k_1^-}{k_{\perp_1}^2} \ll k_2^+ = \frac{k_2^-}{k_{\perp_2}^2}$$

[Balitsky and Trasov (2015)]

Inclusive DIS beyond shock wave

- Working assumptions:

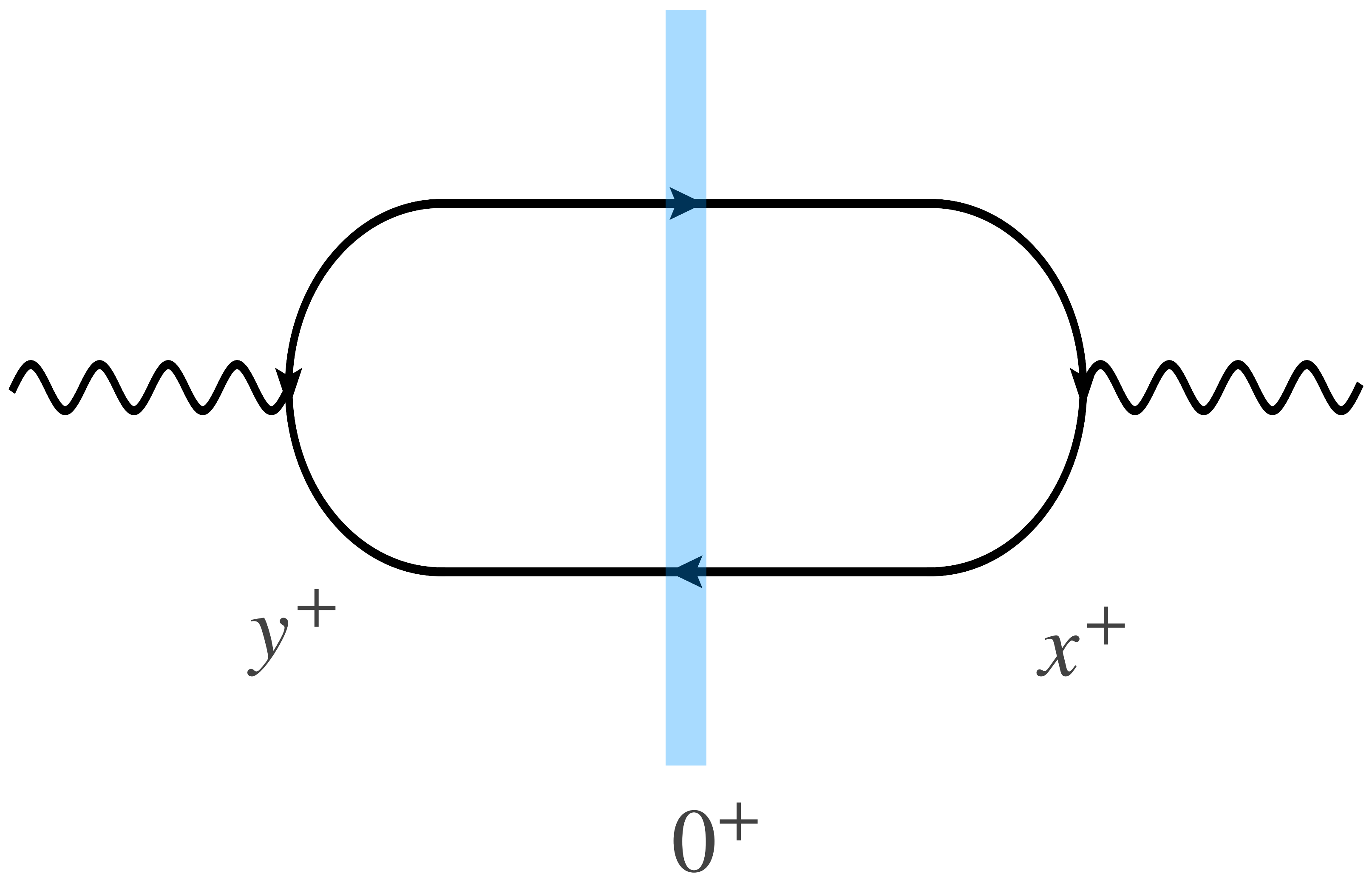
2. We shall neglect power suppressed terms of the form

$$o\left(\frac{k_\perp^2}{s}\right)$$

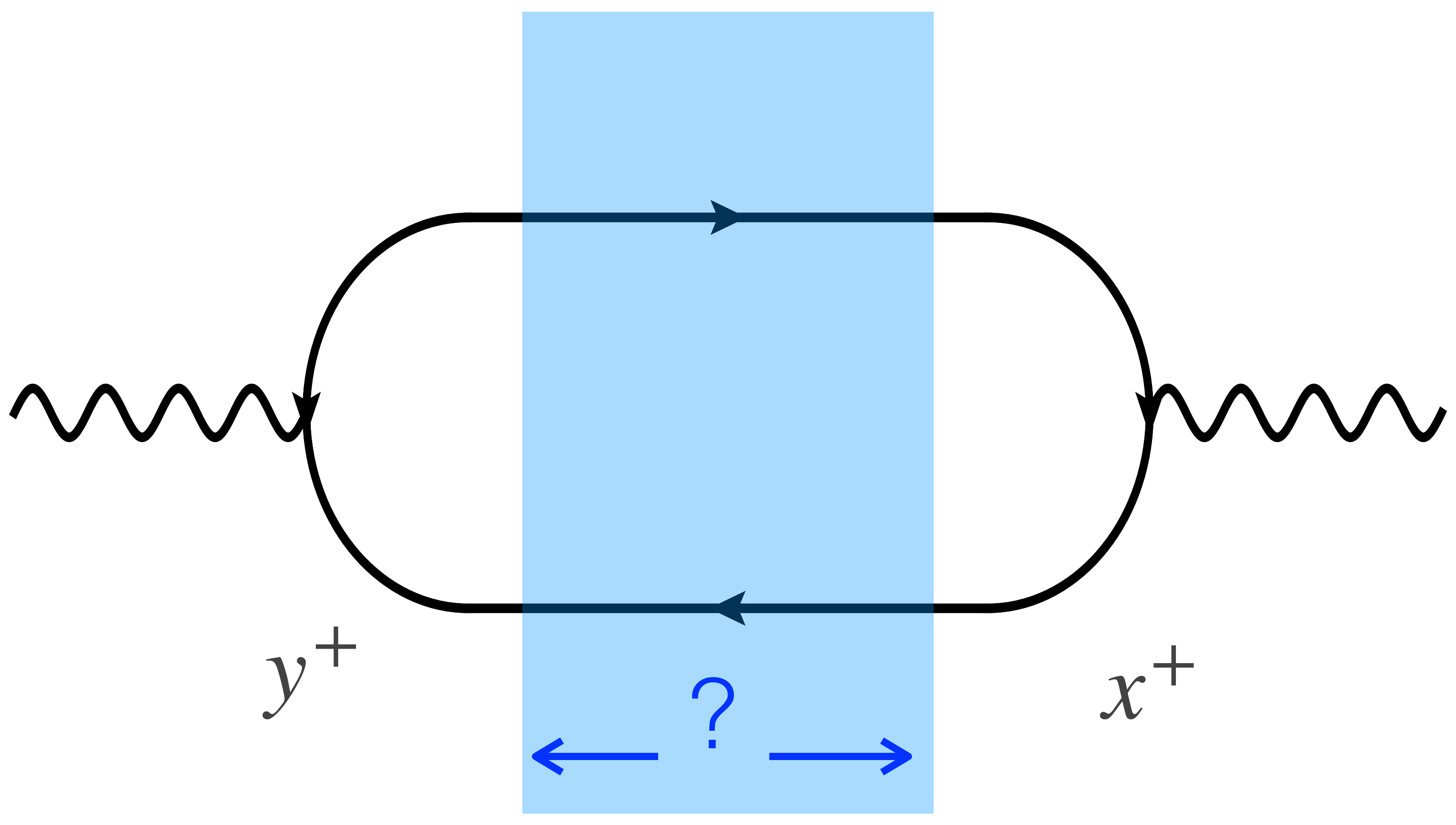
- Bjorken limit: If $s \sim Q^2$ it is higher twist $k_\perp/Q$
- Regge limit: If $s \gg Q^2$ it is suppressed by a powers of the energy

Inclusive DIS beyond shock wave

- In the shock wave approximation the times of the photon splitting into quark antiquark pair are integrated form $0 < x^+ < +\infty$ and $-\infty < y^+ < 0$ which yields the photon wave functions

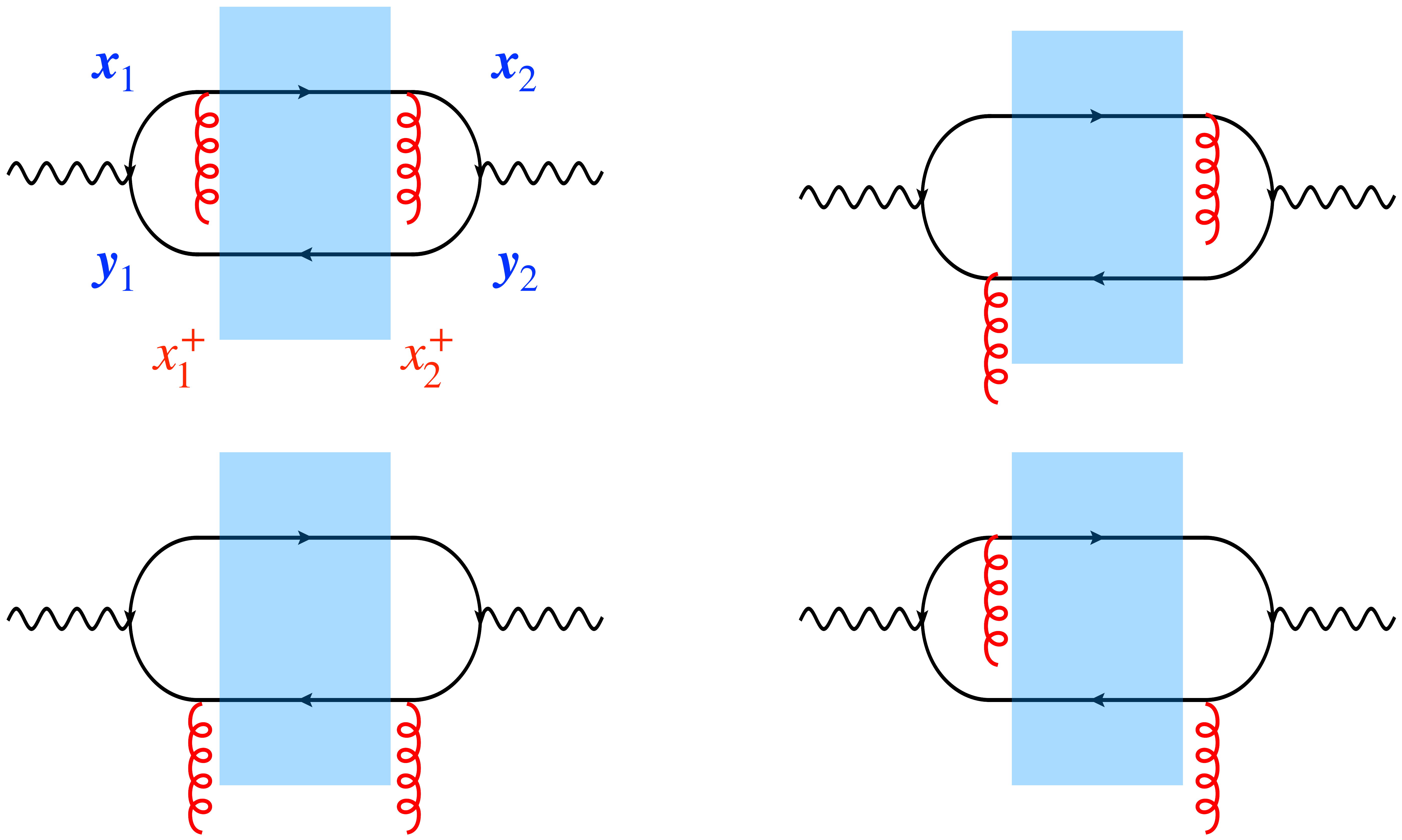


- What are the integration limits of the vertices if one relaxes the shock wave approximation?
- What is the longitudinal extent of the shock wave ?



Inclusive DIS beyond shock wave

- Solution: extracting the first and last interactions provides a physical boundary to the shock wave
- we have 4 contributions
- This seems involved at first glance



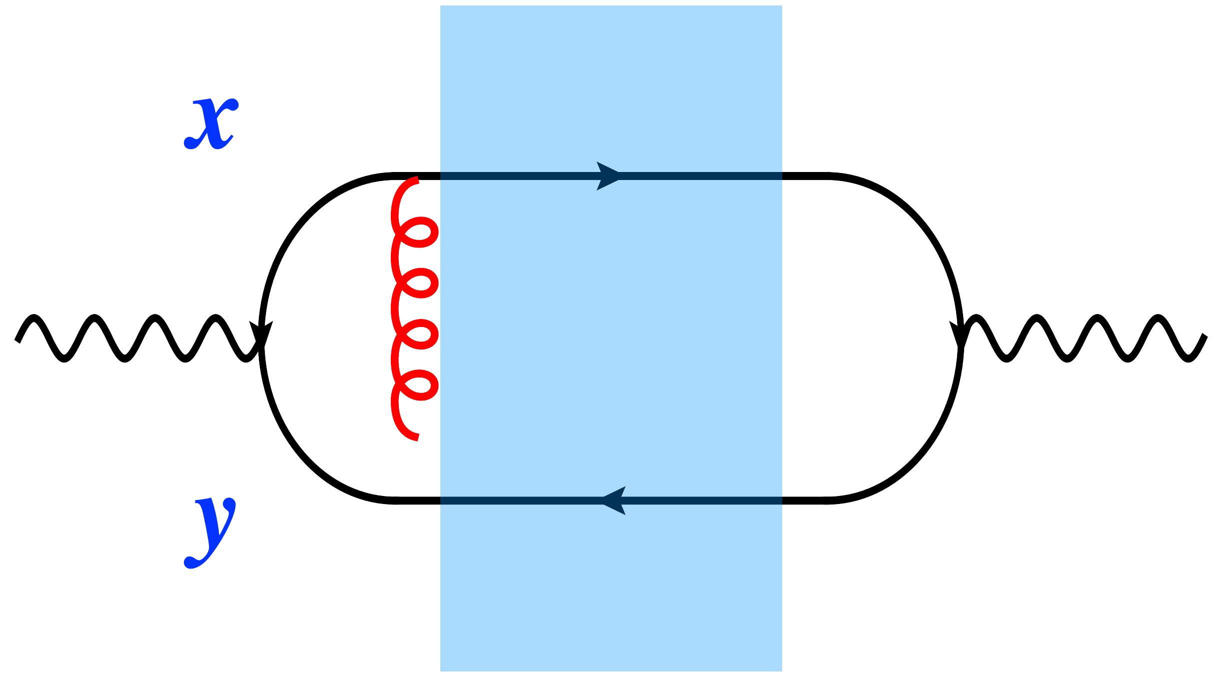
Inclusive DIS beyond shock wave

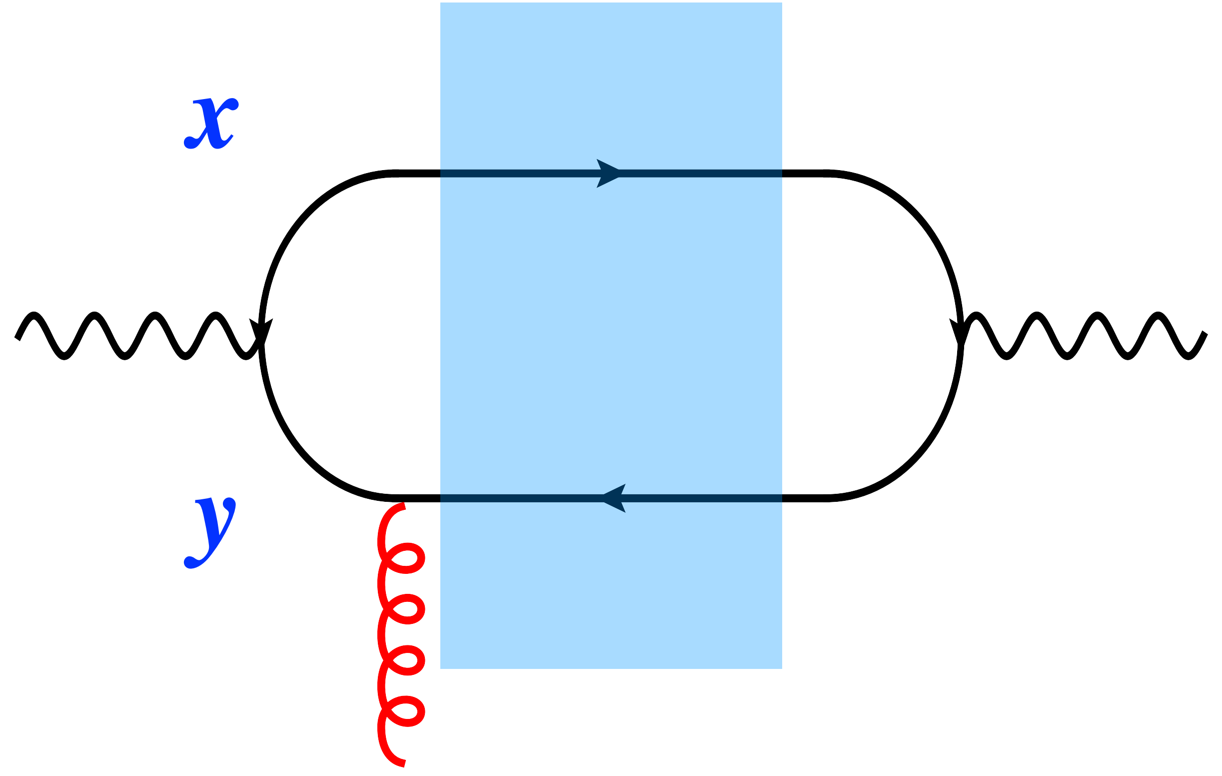
- Consider the left side of the diagram first
- The two gluon fields combine to generate field strength tensor

$$A^-(\boldsymbol{x}) - A^-(\boldsymbol{y}) = \int_0^1 \mathrm{d}s\, r^i \partial^i A^-(\boldsymbol{y} + s\boldsymbol{r})$$

$$= \int_0^1 \mathrm{d}z^i\, F^{i-}(\boldsymbol{z})$$

- Where $\boldsymbol{z}(s) \equiv s\,\boldsymbol{x} + (1-s)\,\boldsymbol{y}$ is a straight line trajectory in the transverse plane





$\boldsymbol{r} = \boldsymbol{x} - \boldsymbol{y}$

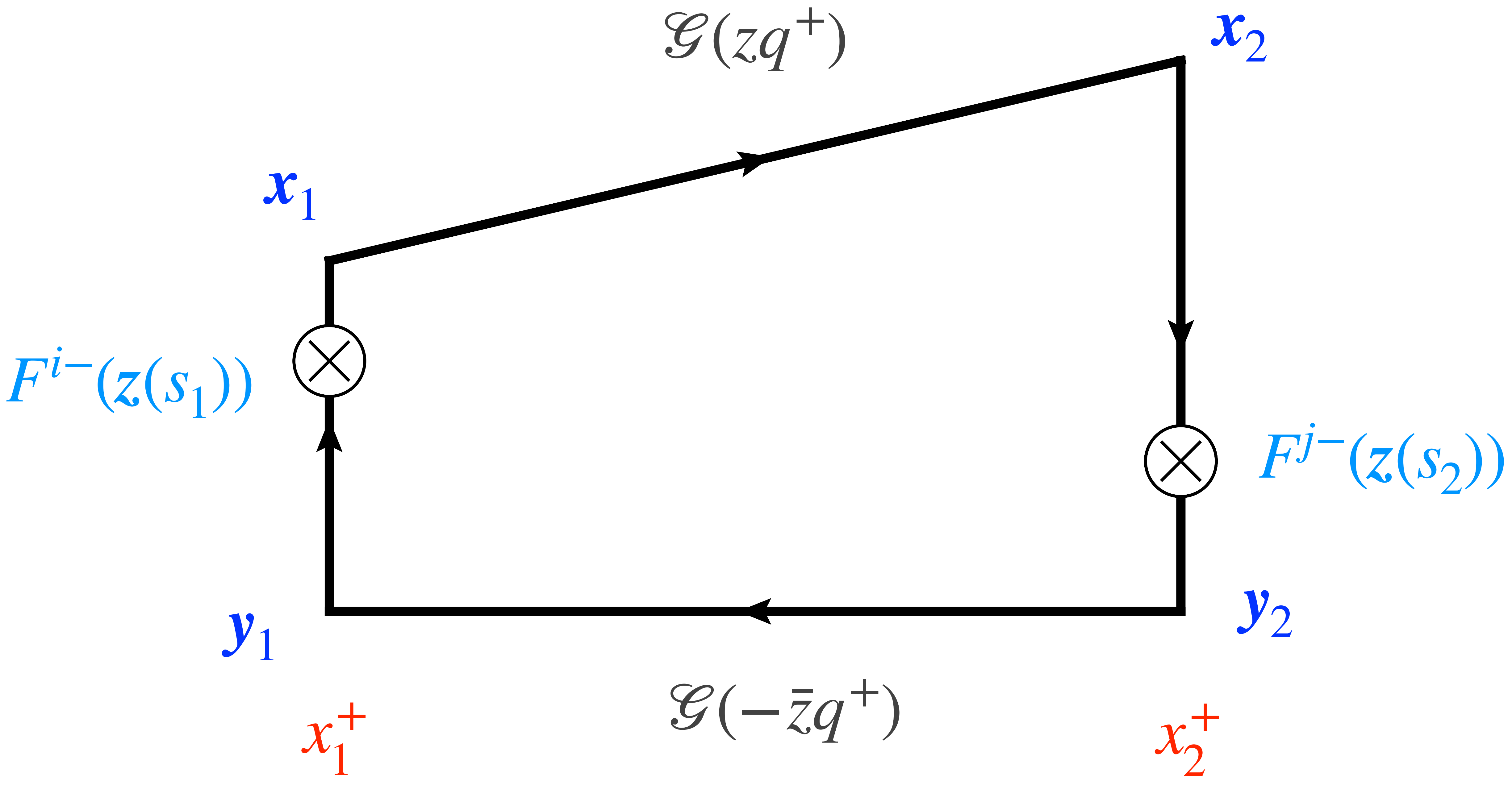
Inclusive DIS beyond shock wave

- applying the same trick to the r.h.s. we obtain the following hadronic operator

$$O_{\text{dipole}}(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{y}_1, \boldsymbol{y}_2, x_1^+, x_2^+) \equiv \int_{\boldsymbol{y}_1}^{\boldsymbol{x}_1} \mathrm{d}z_1^i \int_{\boldsymbol{y}_2}^{\boldsymbol{x}_2} \mathrm{d}z_2^j$$

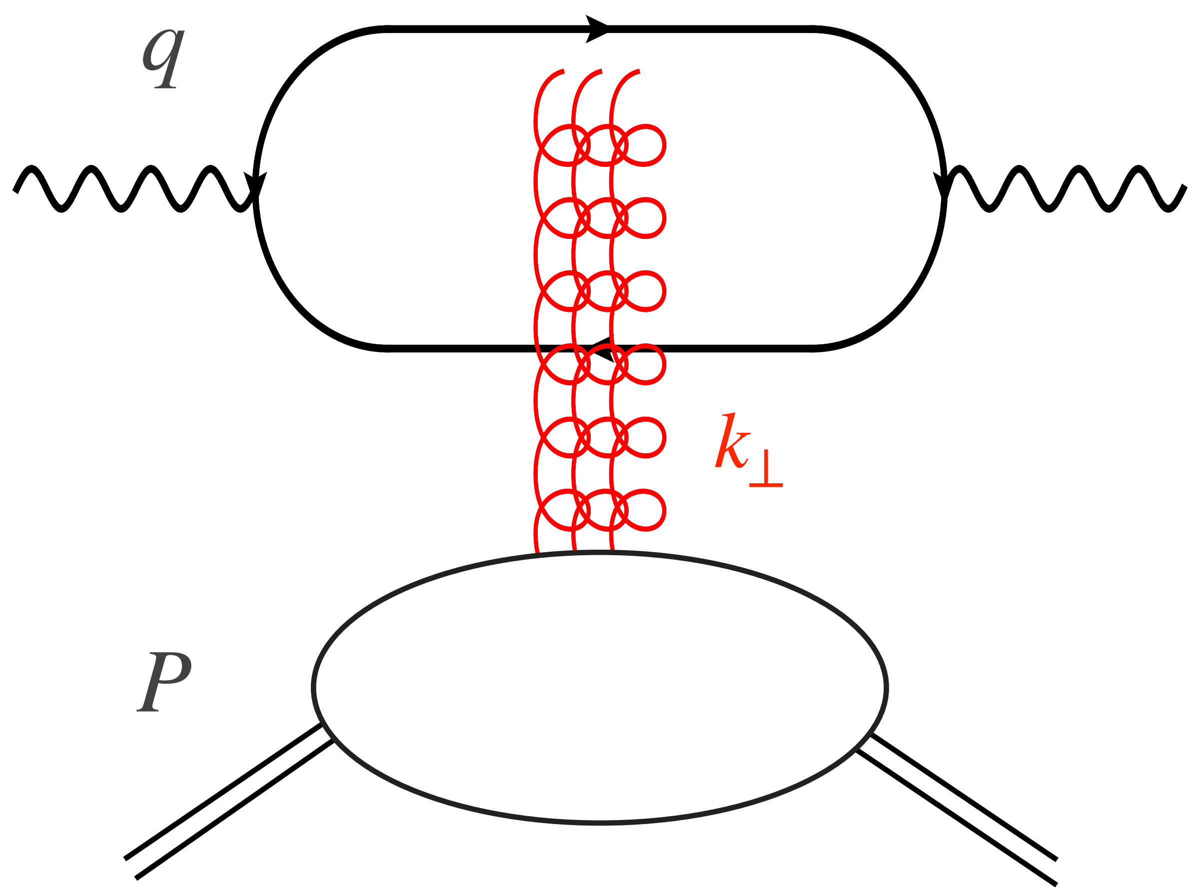
$$\left\langle P \,|\, \mathrm{Tr}\, \mathcal{G}(\boldsymbol{y}_2, x_2^+ ; \boldsymbol{y}_1\, x_1^+ \,|\, (1-z)q^+)\, F^{j-}(x_1^+, \boldsymbol{z}_1)\, \mathcal{G}(\boldsymbol{x}_1, x_1^+ ; \boldsymbol{x}_2, x_2^+ \,|\, zq^+)\, F^{i-}(x_2^+, \boldsymbol{z}_2) \,|\, P \right\rangle$$

- performing a gauge rotation leads to transverse gauge links: explicit gauge invariance
- explicit dependence on + momenta of the dipole zq^+ and $(1-z)q^+$



Inclusive DIS beyond shock wave

- DIS cross-section takes now a form similar to that of the shock wave. The wave functions are identical



$$\sigma(x_{Bj}, Q^2) \sim e^2 \int_0^1 \mathrm{d}\, z\, P(z) \int_{\boldsymbol{r},\boldsymbol{b}} \mathrm{d}\,\boldsymbol{r}\ |\varphi(z(1-z)|\boldsymbol{r}|^2 Q^2)|^2\ \langle \mathrm{Tr} U(\boldsymbol{r}) U^\dagger(0) \rangle_Y$$

$$\sigma(x_{Bj}, Q^2) \sim e^2 \int_0^1 \mathrm{d}\, z\, P(z) \int_{\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{b}_1,\boldsymbol{b}_2} \varphi(z(1-z)|\boldsymbol{r}_1|^2 Q^2)\ \varphi^*(z(1-z)|\boldsymbol{r}_2|^2 Q^2)$$

$$\times\, \mathrm{e}^{i\, x_{Bj} P^-(x_2^+ - x_2^+)}\ \langle P | O_{\mathrm{dipole}}(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{b}_1, \boldsymbol{b}_2, x_1^+, x_2^+ | z) | P \rangle$$

Inclusive DIS beyond shock wave

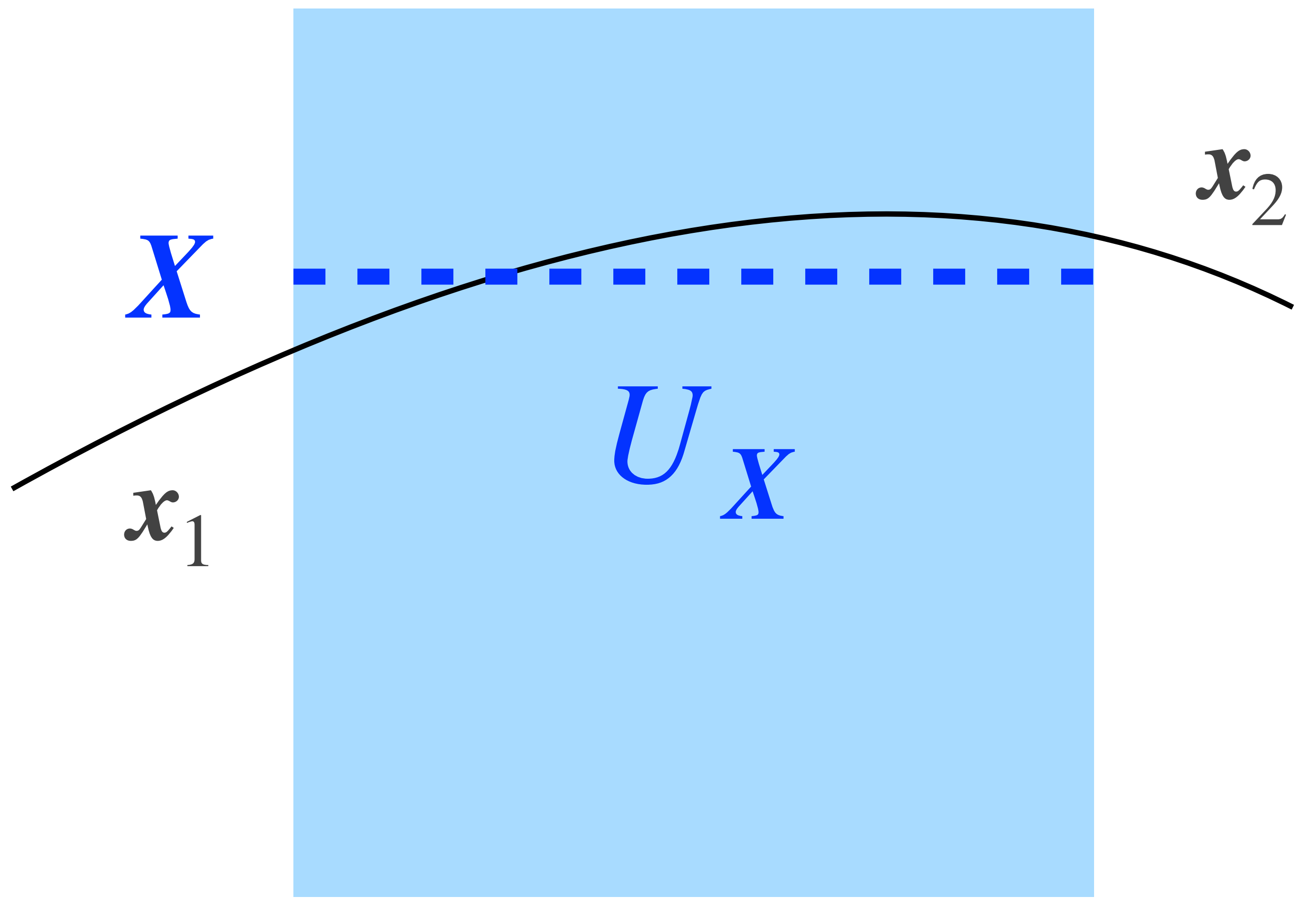
- We have constructed a gauge invariant hadronic operator that has explicit dependence on longitudinal variables
- The hard matrix element is composed of the same wave functions as in shock wave albeit non-local in transverse position space: dipole transverse size a x_1^+ and x_2^+ are not equal $\boldsymbol{r}_1 \neq \boldsymbol{r}_2$
- Bialas, Navlet and Peschanski (2000) noticed that the dipole model (with locality in transverse space) is inconsistent with x dependence

A Novel Unintegrated Gluon Distribution

Unintegrated Gluon Distribution

- Now we apply the third working assumption, that is the (modified) eikonal propagation

$$\mathcal{G}_{p^+}(x^+, \boldsymbol{x}_2; y^+, \boldsymbol{x}_1) = \mathcal{G}_0(\boldsymbol{x}_2 - \boldsymbol{x}_1, x_2^+ - y_1^+)\, U_{\boldsymbol{X}}(x_2^+, x_1^+) + \dots$$



- On may Fourier transform w.r.t. $\boldsymbol{u} = \boldsymbol{x}_2 - \boldsymbol{x}_1$

$$\mathcal{G}_{p^+}(x_2, x_1^+, \boldsymbol{X}; \boldsymbol{\ell}) = \mathrm{e}^{i\frac{\boldsymbol{\ell}^2}{2zq^+}\Delta x^+}\, U_{\boldsymbol{X}}(x_2^+, x_1^+) + \dots$$

- Note the factorization of the phase that encodes the dependence on the longitudinal momentum
- $\boldsymbol{\ell}$ is the average transverse momentum of the quark

Factorization formula for DIS at arbitrary x

- Combining all three phases we obtain

$$ik^-\Delta x^+ \;\equiv\; i\,\frac{\ell^2 + z\bar{z}Q^2}{2z\bar{z}\,q^+}\,\Delta x^+$$

- This is nothing but Feynman x that we encountered when deriving the DGLAP limit

$$x_F \equiv \frac{\ell^2 + z\bar{z}Q^2}{2z\bar{z}\,s}$$

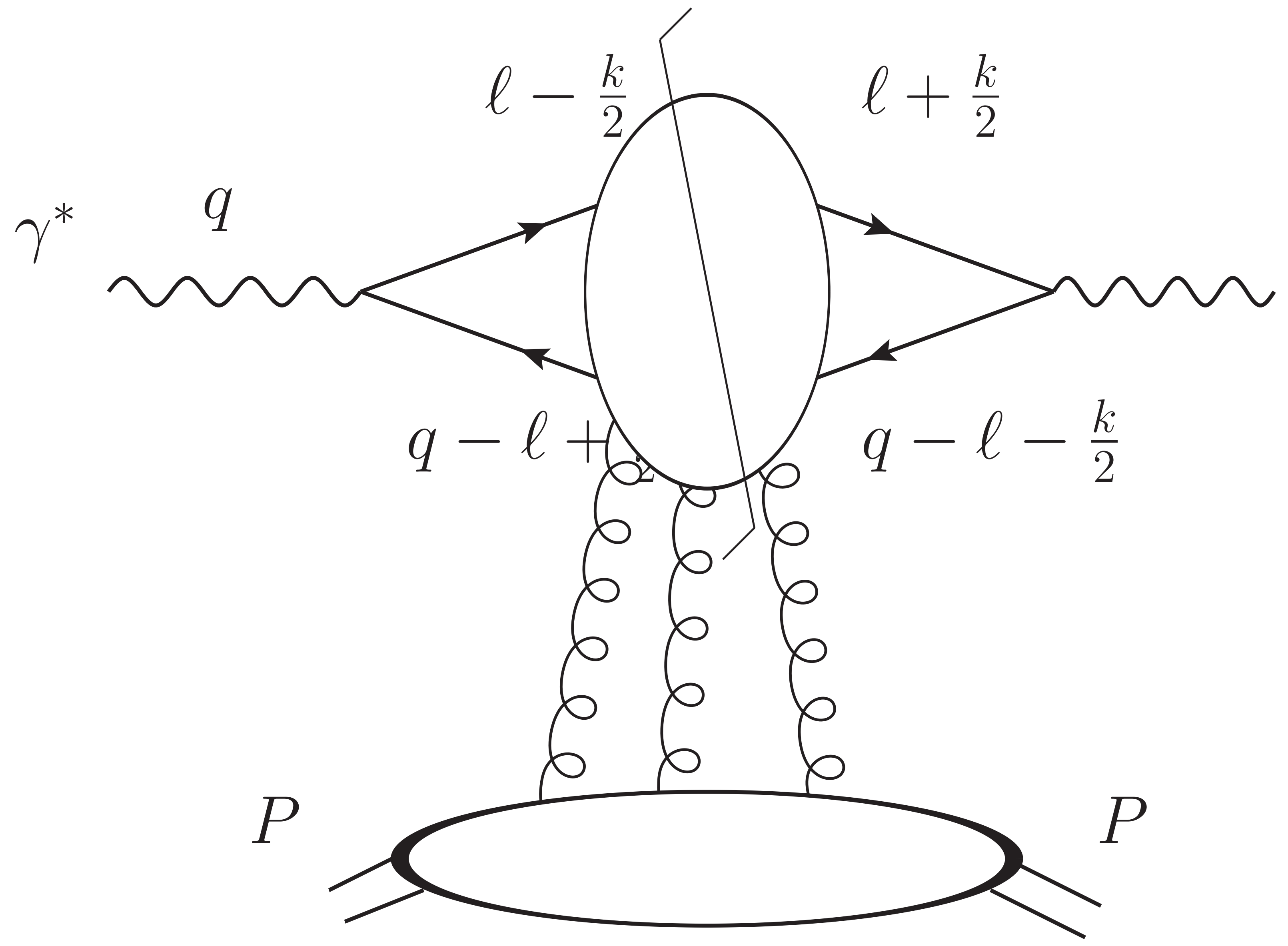
Factorization formula for DIS at arbitrary x

- After integration over the factorized free propagators that lead to the Feynman x phase, we obtain the factorization formula (for the transverse photon cross-section), in momentum space,

$$\sigma(x_{Bj}, Q^2) \sim e^2 \int_0^1 \mathrm{d}\, z\, P(z) \int_0^1 \mathrm{d}x \int_{\boldsymbol{\ell},\boldsymbol{k}} \partial^i \varphi\left(\boldsymbol{\ell} - \frac{\boldsymbol{k}}{2}\right) \partial^j \varphi^* \left(\boldsymbol{\ell} + \frac{\boldsymbol{k}}{2}\right) \delta\left(x - x_{Bj} - \frac{\boldsymbol{\ell}^2}{2z\bar{z}q^+}\right)$$

$$\times\, xG^{ij}(x, \boldsymbol{k}) \;+\; O\left(k_\perp^2/s\right)$$

- $\varphi(\boldsymbol{\ell})$ is the Fourier transform of the photon wave function (shock wave)
- The delta function relates x in the gluon distribution to x_{Bj}



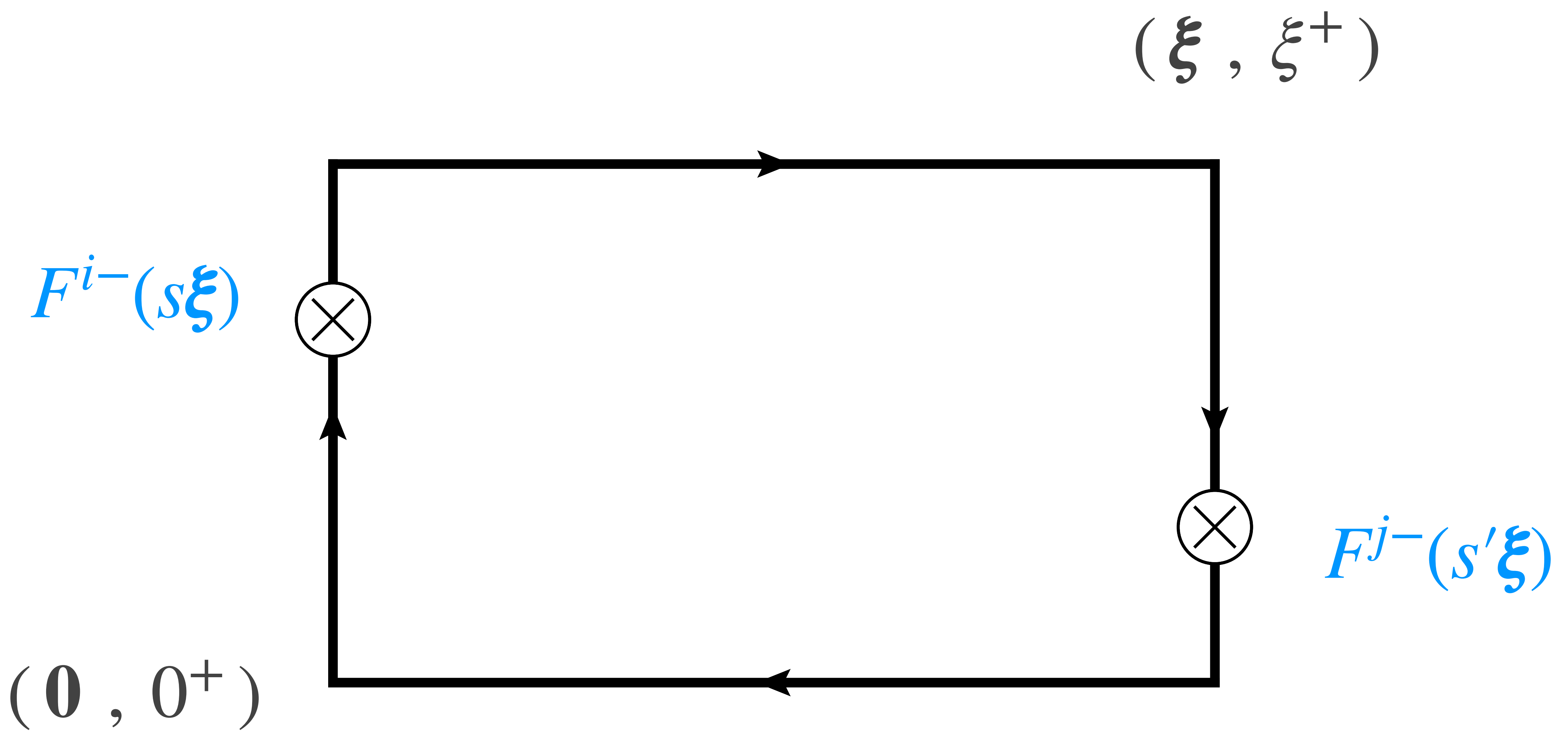
Factorization formula for DIS at arbitrary x

- The unintegrated gluon distribution $xG^{ij}(x, \boldsymbol{k})$ writes

$$xG^{ij}(x, k_\perp) \equiv 2 \int \frac{d\xi^+ d\boldsymbol{\xi}}{(2\pi)^3 P^-} e^{\,i x P^- \xi^+ - i\boldsymbol{k}\cdot\boldsymbol{\xi}} \left\langle P \,|\, \mathrm{Tr}\,[\,0\,,\xi^+]_{\boldsymbol{\xi}}\, F^{j-}\,(\xi^+, s'\boldsymbol{\xi})\,[\,\xi^+\,,0]_{\boldsymbol{0}}\, F^{i-}\,(\,0\,, s\boldsymbol{\xi})\,|\, P \right\rangle$$

- integrating over $k_\perp$ yields $\xi_\perp = 0$ and we recover the gluon PDF
- at small x we recover shock wave

$$\xi^i \xi^j G^{ij}(x=0, \boldsymbol{\xi}) \to \langle P \,|\, \mathrm{Tr}\; U_{\boldsymbol{\xi}}\; U^\dagger_{\boldsymbol{0}} \,|\, P \rangle$$



Summar and Outlook

Summary and outlook

- We revisited the shock wave approximation in high energy scattering by performing a gradient expansion around the classical trajectory of partons
- The leading power accounts for both small and moderate x limit
- We have calculated in this framework gluon induced DIS. We have obtained in particular a new factorization formula involving a novel unintegrated gluon distribution
- Outlook: quantum evolution, application to other observables such as DVCS
- Potential probe of gluon saturation on the lattice