Near threshold heavy quarkonium photoproduction

Feng Yuan
Lawrence Berkeley National Laboratory
arXiv: 2111.07034
Heavy quarkonium production near threshold: a very hot topic

Near threshold heavy vector meson photoproduction at LHC and EicC,
Ya-Ping Xie, V.P. Gonçalves, e-Print: 2103.12568 [hep-ph]

QCD Analysis of Near-Threshold Photon-Proton Production of Heavy Quarkonium.
Yuxun Guo, Xiangdong Ji, Yizhuang Liu, e-Print: 2103.11506 [hep-ph]

Trace Anomaly of Proton Mass with Vector Meson Near-Thresholds Photoproduction
Wei Kou, Rong Wang, Xurong Chen e-Print: 2103.10017 [hep-ph]

Nucleon mass radii and distribution: Holographic QCD, Lattice QCD and GlueX data,
Kiminad A. Mamo, Ismail Zahed, e-Print: 2103.03186 [hep-ph]

$\phi$-meson lepto-production near threshold and the strangeness $D$-term,
Yoshitaka Hatta, Mark Strikman e-Print: 2102.12631 [hep-ph]

Proton mass decomposition: naturalness and interpretations,
Xiangdong Ji, e-Print: 2102.07830 [hep-ph]

Extraction of the proton mass radius from the vector meson photoproductions near thresholds,
Rong Wang, Wei Kou, Ya-Ping Xie, Xurong Chen e-Print: 2102.01610 [hep-ph]

The mass radius of the proton,
Dmitri E. Kharzeev, e-Print: 2102.00110 [hep-ph]

Quantum Anomalous Energy Effects on the Nucleon Mass,
Xiangdong Ji, Yizhuang Liu e-Print: 2101.04483 [hep-ph]
The proton mass distribution/mass radius is one of focuses.

All these are models:
- Vector meson dominance model
- Holographic model

How rigorous (in QCD) that we can measure the proton mass distribution?
Different methods have been applied

- Which gravitational form factors contribute
  - VDM: scalar gravitational form factor, Kharzeev and others
  - Holographic model and QCD analysis: all form factors, maybe dominated by C-form factor, Hatta et al, Ji et al, Zahed et al

- Two-gluon or three-gluon?
This talk focuses on the special kinematics: large momentum transfer

We can compute both the cross section and the form factors separately in perturbative QCD, then we can check that if there is/not a direct connection between the near threshold production and the gluonic gravitational form factors (and how)
Outline

- Intro: What are the gravitational form factors
- Light-cone wave functions/distribution amplitudes
- Form factor calculations
- Threshold heavy quarkonium production
- Discussions
Rutherford scattering

The Scattering of $\alpha$ and $\beta$ Particles by Matter and the Structure of the Atom

E. Rutherford, F.R.S.*
Philosophical Magazine
Series 6, vol. 21
May 1911, p. 669-688

Discovery of nuclei, as point-like particle in atomic matter
Finite size of nucleon (charge radius)

- Rutherford scattering with electron

- Deviation from the point-like particle is described by the form factors

RevModPhys.28.214
EM vs Gravitational Form Factors

EM current

Photon

Graviton

Energy-Momentum current

Kobzarev-Okun 1992; Pagels 1966; Ji 1997
Where to study: through GPDs

- **Wigner distributions** (Belitsky, Ji, Yuan)

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<td>$F_i(t)$, form factors, elastic scattering</td>
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<td>$A_{n,0}(t) + 4\xi^2 A_{n,2}(t) + \ldots$, generalized form factors, lattice calculations</td>
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Where $\xi = 0$.

4D process:

- $\int d^2 b_T$
What do we learn

- My view: one aspect of the parton tomography in hadrons, because they are part of GPDs
  - Proton spin sum rule is derived from these form factors

- C-form factors
  - Pressure, shear force: Polyakov-Schweitzer 2018
  - Momentum-current gravitation multipoles: Ji-Liu, 2021

- Reconstruct the proton mass
  - Ji 1996; Ji 2021; Ji-Liu 2021
  - Hatta-Rajan-Tanaka 2018;
  - Metz-Pasquini-Rodini 2020

\[
\langle P', s' | T_{a}^{\mu\nu}(0) | P, s \rangle = \bar{u}_{s}(P') \left[ A_{a}(t) \gamma^{(\mu} \bar{P}^{\nu)} + B_{a}(t) \frac{i \bar{P}^{(\mu} \sigma^{\nu)} \rho}{2 \Lambda} + C_{a}(t) \frac{\Delta^{\mu} \Delta^{\nu} - g^{\mu\nu} \Delta^{2}}{\Lambda} + \bar{C}_{a}(t) \Lambda g^{\mu\nu} \right] u_{s}(P),
\]

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Form factors at large momentum transfer
Perturbatively, one can compute the form factors at large momentum transfer.

Nucleon wave function

\[ \phi(x, \tilde{Q}) \]

\[ T_H(x, y, Q) \]

\[ \phi^*(y, \tilde{Q}) \]

Lepage-Brodsky 1980
Efremov-Radyushkin 1980
Light-cone Wave Functions

- They are building blocks for the hadron structure

\[ |P\rangle = \sum_{n, \lambda_i} \int \prod_i \frac{dx_i d^2 k_{\perp i}}{\sqrt{x_i} 16\pi^3} \phi_n(x_i, k_{\perp i}, \lambda_i) |n : x_i, k_{\perp i}, \lambda_i\rangle \]

- Fock state of n-partons: momentum fractions, transverse momenta, helicities

- Can be used to calculate the form factors, GPDs, and hard exclusive scattering amplitudes, including near threshold heavy quarkonium production

Lepage-Brodsky 1980
Nucleon’s 3-quarks WF

According to the general structure, six independent light-cone wave functions for three quarks component:

\[ |P \uparrow \rangle_{1/2} = \int d[1]d[2]d[3] \left( \bar{\psi}^{(1)}(1,2,3) \right) \]

\[ \times \frac{\epsilon^{abc}}{\sqrt{6}} u_{a \uparrow}^\dagger(1) \left( u_{b \downarrow}^\dagger(2) d_{c \uparrow}^\dagger(3) - d_{b \downarrow}^\dagger(2) u_{c \uparrow}^\dagger(3) \right) |0\rangle \]

\[ |P \downarrow \rangle_{1/2} = \int d[1]d[2]d[3] \left( (k_1^x - i k_1^y) \bar{\psi}^{(3)}(1,2,3) + (k_2^x - i k_2^y) \bar{\psi}^{(4)}(1,2,3) \right) \]

\[ \times \frac{\epsilon^{abc}}{\sqrt{6}} \left( u_{a \downarrow}^\dagger(1) u_{b \uparrow}^\dagger(2) d_{c \uparrow}^\dagger(3) - d_{a \downarrow}^\dagger(1) u_{b \uparrow}(2) u_{c \uparrow}^\dagger(3) \right) |0\rangle \]

\[ L_z = 0 \]

\[ |L_z| = 1 \]
Distribution amplitudes

- Integrate out the transverse momentum
  - Twist-three (leading-twist)
    \[ \Phi_3(y_i) = -2\sqrt{6} \int \frac{d^2 \vec{k}'_1 \cdot d^2 \vec{k}'_2 \cdot d^2 \vec{k}'_3}{(2\pi)^6} \delta^{(2)}(\vec{k}'_1 + \vec{k}'_2 + \vec{k}'_3) \bar{\psi}^{(1)}(1, 2, 3) \]
  - Twist-four (Braun-Fries-Mahnke-Stein 2000)

\[
\Psi_4(x_1, x_2, x_3) = -\frac{2\sqrt{6}}{x_2 M} \int \frac{d^2 \vec{k}_1 \cdot d^2 \vec{k}_2 \cdot d^2 \vec{k}_3}{(2\pi)^6} \delta^{(2)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \times \vec{k}_2 \cdot \left[ \bar{k}_1 \psi^{(3)}(1, 2, 3) + \bar{k}_2 \bar{\psi}^{(4)}(1, 2, 3) \right].
\]

\[
\Phi_4(x_2, x_1, x_3) = -\frac{2\sqrt{6}}{x_3 M} \int \frac{d^2 \vec{k}_1 \cdot d^2 \vec{k}_2 \cdot d^2 \vec{k}_3}{(2\pi)^6} \delta^{(2)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \times \vec{k}_3 \cdot \left[ \bar{k}_1 \psi^{(3)}(1, 2, 3) + \bar{k}_2 \bar{\psi}^{(4)}(1, 2, 3) \right].
\]
Form factor calculations

Compute the partonic scattering amplitudes, convert to hadron’s
Leading-twist: direct integration of $k_t$, higher-twist: need $k_t$-expansion

- Two gluon exchanges are needed to generate large momentum transfer
- Helicity-non-flip has power behavior, $F_1 \sim 1/t^2$
- Helicity-flip amplitude has power behavior, $F_2 \sim 1/t^3$

Brodsky-Lepage 1981 for $F_1$
Belitsky-Ji-Yuan 2002 for $F_2$
Gravitational form factors:
No much difference, only some surprises

- **Pion case**

\[
\langle P' | T^\mu_\nu_g | P \rangle = 2 \bar{P}^\mu P^\nu A_\pi^g(t) \\
+ \frac{1}{2} (\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2) C_\pi^g(t) + 2m^2 g^{\mu\nu} \bar{C}_\pi^g(t)
\]

\[
A_\pi^g(t) = C_\pi^g(t) = \frac{4m^2}{t} \bar{C}_\pi^g(t)
\]

\[
= \frac{4\pi \alpha_s C_F}{-t} \int dx_1 dy_1 \phi^*(y_1) \phi(x_1) \left( \frac{1}{x_1 \bar{x}_1} + \frac{1}{y_1 \bar{y}_1} \right)
\]

- \(A_g = C_g!!\)
- \(C\)bar cancels between quarks and gluons
  - Quark part from GPD quark at large-\(t\)
    (Hoodbhoy-Ji-Yuan 2003)

Tong-Ma-Yuan, 2103.12047;
Different from Tanaka, PRD 2018

- May introduce difficulty in the interpretation, since integral over \(t\) is not convergent

Polyakov-Schweitzer 2018
Freese-Miller 2021
\[ \langle P', s' \mid T^{\mu\nu}_a(0) \mid P, s \rangle = \bar{u}_s(P') \left[ A_a(t) \gamma^{(\mu} \bar{P}^{\nu)} \right. \\
+ B_a(t) \frac{i \bar{P}^{(\mu} \sigma^{\nu)} \rho \Delta \rho}{2\Lambda} + C_a(t) \frac{\Delta^{\mu} \Delta^{\nu} - g^{\mu\nu} \Delta^2}{\Lambda} \\
\left. + \bar{C}_a(t) \Lambda g^{\mu\nu} \right] u_s(P), \]

- No contribution from three-gluon vertex diagram
- \( A_g \sim 1/t^2 \)
- \( B_g, C_g \) scale as \( 1/t^3 \), \( \bar{C}_g \) scales as \( 1/t^2 \)

\[ \mathcal{A} = \frac{4\pi^2 \alpha_s^2 C_B^2}{3t^2} \left( I_{13} + I_{12} + I_{31} + I_{32} \right), \quad I_{ij} = \frac{x_i + y_i}{\bar{x}_i \bar{y}_i x_i x_j y_i y_j} \]

Tong-Ma-Yuan, 2103.12047
Threshold photoproduction of heavy quarkonium
Near threshold production: kinematics

- Two limits
  - Threshold: \( \chi = \frac{M_V^2 + 2M_p M_V}{W_{\gamma p}^2 - M_p^2} \) \( \Rightarrow 1 \), (1-\( \chi \)) a small parameter, Brodsky et al 2001
  - Heavy quark limits

\[ W_{\gamma p}^2 \sim M_V^2 \gg (-t) \gg \Lambda_{QCD}^2, \]

\[ p_1 \cdot k_\gamma \sim p_1 \cdot k_\psi \sim M_V^2 \]
\[ p_2 \cdot k_\gamma \sim p_2 \cdot k_\psi \ll M_V^2 \]
NRQCD for heavy quarkonium production  Bodwin-Braaten-Lepage 1995

- Propagators are of order heavy quark mass, $\sim 1/M_\gamma$
- Take transverse polarization for the incoming photon

$$\mathcal{M}_{\psi,ab}^{\mu\nu} = \frac{\delta^{ab} N_\psi \left[ \epsilon_{\psi}^* \cdot \epsilon_\gamma \mathcal{W}_T^{\mu\nu} + \epsilon_{\psi}^* \cdot k \mathcal{W}_L^{\mu\nu} + \mathcal{W}_S^{\mu\nu} \right]}{k_1 \cdot k_\gamma k_2 \cdot k_\gamma}$$

$$\mathcal{W}_T^{\mu\nu} = -k_1 \cdot k_\gamma k_2 \cdot k_\gamma g^{\mu\nu} - k_1 \cdot k_2 k_\gamma^{\mu} k_\gamma^{\nu} + k_1 \cdot k_\gamma k_2^{\mu} k_\gamma^{\nu} + k_2 \cdot k_\gamma k_1^{\mu} k_\gamma^{\nu}$$

$$\mathcal{W}_L^{\mu\nu} = k_1 \cdot k_\gamma \epsilon_\gamma^{\nu} k_2^{\mu} + k_2 \cdot k_\gamma \epsilon_\gamma^{\mu} k_1^{\nu}$$

$$\mathcal{W}_S^{\mu\nu} = -k_1 \cdot k_2 \left( k_1 \cdot k_\gamma \epsilon_{\psi}^{*\mu} \epsilon_\gamma^{\nu} + k_2 \cdot k_\gamma \epsilon_{\psi}^{*\nu} \epsilon_\gamma^{\mu} + k_1 \cdot \epsilon_{\psi}^{*\nu} k_\gamma^{\mu} \epsilon_\gamma^{\nu} + k_2 \cdot \epsilon_{\psi}^{*\mu} k_\gamma^{\nu} \epsilon_\gamma^{\mu} \right).$$
Vanishing of three-gluon exchange

- Suggested by Brodsky et al., 2001, and widely accepted by exp. and claimed that
  - Two-gluon exchange suppressed by \((1-x)^2\), where three-gluon dominates at threshold
- Due to C-parity conservation, there is no contribution from the three-gluon exchange
Couple to the Nucleon

- Additional gluon exchange to generate large-$t$
- Nucleon spin configurations
  - Helicity conserved
  - Helicity-flip
Partonic scattering: 1

\[ \bar{U}(p_2)\gamma^\mu U(p_1)\text{Tr}\left[\frac{1+\gamma^5}{2}\phi_2 \cdots \gamma^\nu \cdots \frac{1+\gamma^5}{2}\phi_1 \cdots\right] \]

- \( k_1 \) attaches the helicity-up quark line
Partonic scattering: II

$k_1$ attaches the helicity-down quark line

\[ \bar{U}(p_2)\gamma^\rho U(p_1)\text{Tr} \left[ \frac{1 + \gamma_5}{2} p_2 \cdots \gamma^\nu \cdots \frac{1 + \gamma_5}{2} p_1 \cdots \gamma^\rho \cdots \right] \]

\[ \bar{U}(p_2)\gamma^\mu U(p_1) \bar{P}^\nu \]
Final amplitude

\[ A_3 = \langle J/\psi(\epsilon_\psi), N'_\uparrow | \gamma(\epsilon_\gamma), N_\uparrow \rangle \]

\[ = \int [dx][dy] \Phi(x_1, x_2, x_3) \Phi^*(y_1, y_2, y_3) \frac{1}{(-t)^2} \]

\[ \times \bar{U}_\uparrow(p_2) k_\gamma U_\uparrow(p_1) \mathcal{M}_\psi^{(3)}(\epsilon_\gamma, \epsilon_\psi, \{x_i\}, \{y_i\}) \]

\[ \mathcal{M}^{(3)} = \epsilon_\psi^* \cdot \epsilon_\gamma \frac{8e_c e g_s^6}{27 \sqrt{3 M_\psi^7}} \psi_J(0) (2\mathcal{H}_3 + \mathcal{H'}_3) \]

\[ \mathcal{H}_3 = I_{13} + I_{31} + I_{12} + I_{32}, \quad I_{ij} = \frac{1}{x_i x_j y_i y_j \bar{x}_i \bar{y}_i} \]

Similar structure as \( A_g \) form factor or GPD \( H_g \) contribution
Amplitude squared

\[ |A_3|^2 = (1 - \chi)G_\psi G_{p3}(t)G_{p3}^*(t) \]

\[ G_\psi = |N_\psi|^2 = \frac{384\pi^2e_c^2\alpha(4\pi\alpha_s)^2}{N_c^2M_\psi^3} \langle 0|\phi_\psi^{(3S_1)}|0 \rangle \]

\[ G_{p3}(t) = \frac{8\pi^2\alpha_s^2C_B^2}{3t^2} \int [dx][dy]\Phi_3(\{x\})\Phi_3^*(\{y\})[2H_3 + \mathcal{H}_3'] \]

- Suppressed at the threshold, \( \chi \to 1 \)
- This behavior is similar to \( H_g \) contribution to \( J/\psi \) production in the GPD formalism with \( 1-\xi \) suppression factor
  - Hoodbhoy 1996, see also, Koempel-Kroll-Metz-Zhou 2012, Guo-Ji-Liu 2021
- Power behavior of \( 1/t^4 \)
Twist-four contribution

\[ A_4 = \langle J/\psi(\epsilon_\psi), N^\uparrow \mid \gamma(\epsilon_\gamma), N_\downarrow \rangle \]
\[ = \int [dx][dy] \Psi_4(\{x\}) \Phi_3^*(\{y\}) M_\psi^{(4)}(\{x\}, \{y\}) \]
\[ \times \bar{U}_\uparrow(p_2) U_\downarrow(p_1) \frac{M_p}{(-t)^3} , \]

\[ |A_4|^2 = \tilde{m}_t^2 G_\psi G_{p4}(t) G_{p4}^*(t) \quad \tilde{m}_t^2 = M_p^2/(-t) \]

\[ G_{p4}(t) = \frac{C_B^2(4\pi\alpha_s)^2}{12t^2} \int [dx][dy] \Phi_3(y_1, y_2, y_3) \]
\[ \times \{ x_3 \Phi_4(x_1, x_2, x_3) T_4 \phi(\{x\}, \{y\}) \]
\[ + x_1 \Psi_4(x_2, x_1, x_3) T_4 \psi(\{x\}, \{y\}) \} , \]

- Helicity-flip amplitude
- kt-expansion, similar to F_2 form factor
- There is no interference between twist-3 and twist-4
- Power behavior \( \sim 1/t^5 \)

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Connection to the gluonic gravitational form factors
There is no direct connection to the gluonic gravitational form factors

- **Scattering amplitude**

\[ G_{p3}(t) = \int [dx][dy] \Phi_3(x_1, x_2, x_3) \Phi_3^*(y_1, y_2, y_3) \times [2\mathcal{H}_3 + \mathcal{H}_3'] , \]

\[ \mathcal{H}_3 = \frac{8\pi^2 \alpha_s^2 C_B^2}{3t^2} (I_{13} + I_{31} + I_{12} + I_{32}) \]

\[ I_{ij} = \frac{1}{x_i x_j y_i y_j x_i^2 y_i} \]

- **Gluonic Form Factors**

\[ A_g(t) = \int [dx][dy] \Phi_3(x_1, x_2, x_3) \Phi_3^*(y_1, y_2, y_3) \times [2\mathcal{A}_3 + \mathcal{A}_3'] , \]

\[ \mathcal{A} = \frac{4\pi^2 \alpha_s^2 C_B^2}{3t^2} (I_{13} + I_{12} + I_{31} + I_{32}) , \]

\[ I_{ij} = \frac{x_i + y_i}{\bar{x}_i \bar{y}_i x_i x_j y_i y_j} \]
Discussion: construct the gluon operators

- Take the leading contribution of heavy quark mass limit

\[ \mathcal{M}^{\mu\nu}_\psi = N_\psi \epsilon^*_\psi \cdot \epsilon_\gamma \frac{k_{\gamma,\alpha} k_{\gamma,\beta}}{k_1 \cdot k_\gamma k_2 \cdot k_\gamma} \mathcal{W}^{\alpha\beta\mu\nu}_T \]

\[ \mathcal{W}^{\alpha\beta\mu\nu}_T = -k_1^\alpha k_2^\beta g^{\mu\nu} - k_1 \cdot k_2 g^{\alpha\mu} g^{\beta\nu} + k_1^\nu k_2^\beta g^{\alpha\mu} + k_2^\mu k_1^\alpha g^{\beta\nu}, \]
Connect to gravitational form factors?

\[ A \propto \int d^4 \eta_1 d^4 \eta_2 d^4 k_1 d^4 k_2 e^{ik_1 \cdot \eta_1 + ik_2 \cdot \eta_2} \frac{k_\gamma^\alpha k_\gamma^\beta}{k_1 \cdot k_\gamma k_2 \cdot k_\gamma} \langle N' | F^\alpha \rho(\eta_1) F^{\beta \rho}(\eta_2) | N \rangle. \]

- We have to make approximations: the two gluons in the t-channel carry the same momentum.
It is a long stretch to make this connection

- The QCD dynamics involved in the on-shell photon (massless) transition to a massive heavy quarkonium does not allow a simple interpretation.
Compare to the GPD formalism
Discussion: compare to the GPD formalism

\[ M(\varepsilon_V, \varepsilon) = \frac{8\sqrt{2\pi\alpha_s(M_V)}}{M_V^2} \phi^*(0) G(t, \xi)(\varepsilon_V^* \cdot \varepsilon) \]

- **Guo-Ji-Liu 2021** argue that this can also apply near the threshold
  - Very strong argument
- **Large-t gluon GPD can be calculated in perturbative QCD**
  - **Ji-Hoodbhoy-Yuan 2004**, for quark GPDs
  - **Consistency is checked**

Earlier references in GPD formalism:
- **Hoodbhoy 1996; Koempel-Kroll-Metz-Zhou 2012 and references therein**
Taylor expansion of the hard coefficient

\[ G(t, \xi) = \sum_{n=0}^{\infty} \frac{1}{\xi^{2n+2}} \int_{-1}^{1} dx x^{2n} F_g(x, \xi, t) \]

- Only the leading term corresponds to the gluonic gravitational form factors

\[ G(t, \xi) = \frac{1}{\xi^2 (\bar{P}^+)^2} \left< P' \right| \frac{1}{2} \sum_{\alpha, i} F_{\alpha, i}^+ (0) F_{\alpha, i}^+ (0) \left| P \right> \]

\[ = \frac{1}{2\xi^2 (\bar{P}^+)^2} \left< P' \right| T_{g,}^{++} \left| P \right> , \]

Boussarie-Hatta 2020;
Hatta-Strikman 2021;
Guo-Ji-Liu 2021;
What does this approximation mean?

- The leading term is equivalent to: **no x-dependence** in the hard part

$$A(x, \xi) \equiv \frac{1}{x + \xi - i0} - \frac{1}{x - \xi + i0}$$

- Which means that **same momentum** for the two gluons

$$A \propto \frac{k_\gamma^\alpha k_\gamma^\beta}{\langle k_1 \cdot k_\gamma k_2 \cdot k_\gamma \rangle} \langle T^{\alpha \beta}_g \rangle$$
- If the GPD gluon distributions have power behaviors $(\xi^2-x^2)^2$, this approximation is not that bad
  - Guo-Ji-Liu 2021, Hatta-Strikman 2021, ~20% corrections
  - Power behavior is supported by evolution at asymptotic scale (see, review by Markus Diehl and references therein)

- However
  - There may not be a simple Taylor expansion at high order perturbative QCD
  - At low/moderate scale, a power behavior may not be manifest for the GPD gluon distributions
    - Cross section contribution has divergence around $\xi=x$, the Taylor expansion will be much more subtle
An example

\[ t^2 H_u p(x, \xi) \]

\( \xi = 0.0 \)
\( \xi = 0.4 \)
\( \xi = 0.7 \)

Ji-Hoodbhoy-Yuan PRL2004
Discussion: higher order corrections

- $\alpha_s$ corrections will be important to check if the power counting results would be strongly modified
  - Four-gluon exchange contribution (*brought by Stan*), either power suppressed or $\alpha_s$ corrections to two-gluon exchange

- If/how the final state interactions between the proton and heavy quarkonium will improve/modify the factorization arguments in our/others’ calculations
  - Relativistic corrections in NRQCD are worthwhile to explore
Phenomenology
Phenomenology application: Differential cross section

\[ \frac{d\sigma}{dt}|_{-t \gg \Lambda_{QCD}^2} = \frac{1}{16\pi (W_{\gamma p}^2 - M_p^2)^2} \left( |A_3|^2 + |A_4|^2 \right) \]

\approx \frac{1}{(-t)^4} \left[ (1 - \chi)N_3 + \tilde{m}_t^2 N_4 \right] ,

- Total cross section contributions
- Smooth to low-\(t\) by replace \(-t \rightarrow \Lambda^2 \cdot t\)
- Parametric comparison, only power counting
- Clearly, twist-4 dominates near threshold
Twist-four fit to GlueX data

$E_\gamma \approx 10.7\text{GeV}$

GlueX Coll, PRL2019
Comments

- Power behavior for near threshold heavy quarkonium production at large-$t$ is derived, and the leading contribution comes from non-zero OAM three-quark state, scales as $1/t^5$
  - Agree with the GlueX data
  - Precision data in the future will be able to test different power
Predictions for $\psi'$

\[ \frac{d\sigma}{dt} (\text{nb/GeV}^2) \]

\[ W (\text{GeV}) \]

- $\gamma p \rightarrow J/\psi p$
- $\gamma p \rightarrow \psi' p$
Predictions for $Y$ (1S,2S)

$\gamma p \rightarrow Y(1S) \ p$

$\gamma p \rightarrow Y(2S) \ p$

- $\sigma (nb)$
  - $\gamma p \rightarrow Y(1S) \ p$
  - $\gamma p \rightarrow Y(2S) \ p$

$t (GeV^2)$

$W (GeV)$
Summary

- It is hard to build a direct connection between the near threshold photoproduction of heavy quarkonium and the gluonic gravitational form factors
  - All previous results/claims should be re-evaluated
- Looking forward: phenomenological study in terms of the gluon GPDs is greatly needed
  - Indirect connection to the gluonic gravitational form factors
Gluon landscape from future EIC, e.g., through diffractive quarkonium production

- Cover energy range from threshold to high energy
- Potential to have detailed study of gluon GPDs

Gryniuk, Vanderhaeghen 1806.08205
Joosten, Meziani, 1802.02616