Gluon helicity distributions

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# Outline

1. Introduction
2. Global QCD analysis
3. Jets
4. Polarized PDFs
5. Helicity

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Gluon helicity distributions

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Proton spin puzzle

What is the decomposition of the proton spin [Nucl. Phys. B 337, 509-546 (1990)]?

\[ \frac{1}{2} = \frac{1}{2} \Delta \Sigma + L_q + \Delta G + L_g \]

- current extraction of \( \Delta \Sigma \) is around 0.3
- spin can be extracted from parton distribution functions (PDFs)
- orbital angular momentum can be extracted from GPDs
Parton distribution functions

- Probability to find a quark $i$ or a gluon $g$ in a hadron $h$ carrying a fraction $x$ of the hadron’s momentum.

$$f_{i/h}(x) = \frac{1}{4\pi} \int d\xi^+ e^{-ix\xi^-} P^+ \langle h(P,S) \vert \bar{\psi}_i(0,\xi^-,0) \Gamma G \psi_i(0) \vert h(P,S) \rangle$$

$$f_{g/h}(x) = \frac{-1/ix}{4\pi P^+} \int d\xi^+ e^{-ix\xi^-} P^+ \langle h(P,S) \vert F^\alpha(0,\xi^-,0) \Gamma G F^\alpha_\alpha(0)/\tilde{F}^\alpha_\alpha(0)... \vert h(P,S) \rangle$$

$$g = P \exp \left( ig \int_0^{\xi^-} dy^- A^+(0,y^-,0) \right), \quad \Gamma = \gamma^+ \text{ or } \gamma^+ \gamma^5, \quad \tilde{F}^\mu\nu = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F^\rho\sigma$$

- Spin-averaged (unpolarized): $f = f^\uparrow + f^\downarrow$

- Spin-dependent (polarized): $\Delta f = f^\uparrow - f^\downarrow$
Motivations

Introduction

Global analysis

Jets

Δf

Helicity

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PRD 101, 074020 (2020)

PRD 93, 074005 (2016)

PRL 113, 012001 (2014)

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Global QCD analysis

Global QCD analysis of high-energy scattering reactions

- Factorization theorems
- Bayesian inference
- Monte Carlo sampling
- Multi-step strategy
- Mellin transformation
Global QCD analysis - Bayesian inference

\[
\begin{align*}
    \text{d}\sigma^{\text{DIS}} &= \sum_i H_i^{\text{DIS}} \otimes f_i \\
    \text{d}\sigma^{\text{DY}} &= \sum_{i,j} H_{ij}^{\text{DY}} \otimes f_i \otimes f_j \\
    \text{d}\sigma^{\text{jet}} &= \sum_{i,j} H_{ij}^{\text{jet}} \otimes f_i \otimes f_j \\
    \rho(p | \text{data}) &\sim \mathcal{L}(p | \text{data}) \pi(p) \\
\end{align*}
\]

Posterior belief

Prior belief

Likelihood

Evidence

\[
\begin{align*}
    f_i(x) &= n_i x^{\alpha_i}(1 - x)^{\beta_i} P(x) \\
    p &= (n_i, \alpha_i, \beta_i, \ldots)
\end{align*}
\]

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To evolve convoluted DGLAP equations in $x$ space

$$\frac{\partial q(x, t)}{\partial t} = \frac{\alpha_s(t)}{2\pi} \int_x^1 P\left(\frac{x}{z}, \alpha_s(t)\right) q(z, t) \frac{1}{z} \, dz$$

we apply Mellin transformation

$$\tilde{f}(N) = \int_0^\infty x^{N-1} f(x) \, dx$$

to remove the convolution

$$\frac{\partial \tilde{q}(N, t)}{\partial t} = \frac{\alpha_s(t)}{2\pi} P(N, \alpha_s(t)) \tilde{q}(N, t)$$
Differential cross section in $x$ space is

$$d\sigma = \int dx \, dy \, f_i(x) f_j(y) \mathcal{H}$$

where $\mathcal{H}$ is the perturbative part.
Differential cross section in $x$ space is

$$d\sigma = \int dM \, dN \, \tilde{f}_i(M) \tilde{f}_j(N) \times \int dx \, dy \, x^{-M} y^{-N} \mathcal{H}$$

Write PDFs in Mellin space
**JAM numerical framework**

- Differential cross section in $x$ space is
  \[ f(x) = nx^\alpha(1 - x)^\beta(1 + c\sqrt{x} + dx) \]

- Write PDFs in Mellin space
  \[ \tilde{f}(N) = nB(N + \alpha, \beta + 1) + ncB(N + \alpha + 0.5, \beta + 1) + ndB(N + \alpha + 1, \beta + 1) \]

- $\tilde{f}_i(N)$ takes simple form for typical parameterization
Multistep strategy

Prior samples

stage 1

DIS
+f
+jets

Δf
+f
+f

stage 2

ΔDIS
+Δjets

Δf
Δf
Δf

final stage

all

posterior samples

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Data resampling

- We generate many resampled data sets $\tilde{\sigma}_i$ with random numbers $R_i$ and quadrature sum $\alpha_i$ of uncorrelated uncertainties

$$\tilde{\sigma}_i = \sigma_i + R_i \alpha_i$$

- Monte Carlo sampling is used to avoid fits being trapped in local minima.

- Data are divided by experiment and included with multiple steps for fits to better converge.

- We currently include unpolarized data: DIS, Drell-Yan, jet production, and polarized data: inclusive DIS, jet production (total of 3576 data points).
Jets are collimated spray of hadrons in high-energy particle reactions.

Jets are manifestations of collinear enhancement.

Probes hadron structure (PDFs) without details of hadronization (fragmentation functions), in contrast to SIDIS.

Jets are defined up to specific clustering algorithms (cone, $k_T$, anti-$k_T$...).

\[
\frac{1}{(p_1 + p_2)^2} = \frac{1}{2E_1E_2(1 - \cos(\theta))}
\]
Sensitivity to gluon PDFs

In inclusive DIS, sensitivity to gluon PDF only appears at NLO:

On the other hand, in jet production, gluon diagrams appear at LO:

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Jet in polarized collisions

- RHIC measures double longitudinal spin asymmetry

\[
A_{LL}^{jets} = \frac{\sigma^{++} - \sigma^{+-}}{\sigma^{++} + \sigma^{+-}} = \frac{\Delta \sigma(\Delta g, \ldots)}{\sigma(g, \ldots)}
\]

- \(\sigma^{\pm}\) are differential cross sections when proton beams have equal & opposite helicity
- denominator is spin-averaged cross section

- \(A_{LL}^{jets}\) is sensitive to unpolarized PDFs \(\rightarrow\) perform simultaneous analysis to check

PRD 86, 032006 (2012)
Fits to jet in unpolarized collisions

[Graphs showing data and theory comparisons for D0 and CDF experiments, with notes about good agreement between theory and Tevatron data and first inclusion of unpolarized RHIC jets.]

Fits to jet in unpolarized collisions

An overall good agreement is found.

Differences are caused by choices of datasets.

\[ x u_v \]
\[ x d_v \]
\[ x (d + u) \]
\[ x (s + \bar{s}) \]

\[ Q^2 = 10 \text{ GeV}^2 \]


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Polarized PDFs - previous extractions

PRL 113, 012001 (2014)


Extracted with SU(3) flavor symmetry and positivity constraints
Polarized PDFs - theory assumptions

1. SU(2) flavor symmetry only

2. SU(2) and SU(3)

3. SU(2) and SU(3), and PDF positivity

more constraints

more biases

less data driven
Polarized PDFs - results

\[ \Delta q^+ = \Delta q + \Delta \bar{q}, \quad q = u, d, s \]
Gluon helicity - jet $A_{LL}$ and $\Delta g$

Good agreement with data for $\Delta g > 0$ and $\Delta g < 0$

Large cancellation between $gg$ and $qg$ channels for $\Delta g < 0$
Global analysis

Introduction

Jets

Helicity

Polarized PDFs - $\Delta G = \int_{0.05}^{1} \Delta g \, dx$

$$\frac{1}{2} = \frac{1}{2} \Delta \Sigma + L_q + \Delta G + L_g$$

- **SU(2):**
  - $\Delta g > 0$: $0.20 \pm 0.13$
  - $\Delta g < 0$: $-0.56 \pm 0.12$

- **SU(3):**
  - $\Delta g > 0$: $0.27 \pm 0.03$
  - $\Delta g < 0$: $-0.61 \pm 0.04$

- **SU(3) + pos:** $0.25 \pm 0.03$

- **DSSV14:** $0.2 \pm 0.05$

PRL 113, 012001 (2014)

Helicity basis PDFs

- $u^\uparrow / \downarrow$ and $d^\uparrow / \downarrow$ are well separated.
- $s^\uparrow / \downarrow$ can be hardly distinguished.
- First simultaneous extraction of $f^\uparrow$ and $f^\downarrow$!
- $g^\uparrow / \downarrow$ for $\Delta g > 0$ (blue and green) and $\Delta g < 0$ (orange and red).

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ROC and AUC

- indistinguishable
- somewhat distinguishable
- clearly distinguishable

ROC: *receiver operating characteristic curve*
AUC: *area under curve of ROC*

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AUC for helicity PDFs

- 0.5: can barely discriminate $f^{↑}$ and $f^{↓}$
- 1.0: can clearly discriminate $f^{↑}$ and $f^{↓}$
Currently working on...

- polarized SIDIS with large $q_T$: linear dependence on $\Delta g$ at LO
- di-jets: also sensitive $\Delta g$ at LO
- $\pi^\pm$ and $K^\pm$ production in $pp$ collisions
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