Gluon helicity distributions

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Outline

Proton spin puzzle

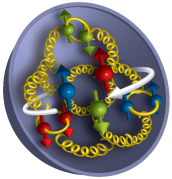
What is the decomposition of the proton spin [Nucl. Phys. B **337**, 509-546 (1990)]?

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + \Delta G + L_g$$

- current extraction of $\Delta\Sigma$ is around 0.3
- spin can be extracted from parton distribution functions (PDFs)
- orbital angular momentum can be extracted from GPDs

Parton distribution functions

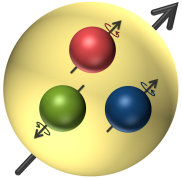
- Probability to find a quark i or a gluon g in a hadron h carrying a fraction x of the hadron's momentum.



$$f_{i/h}(x)=\frac{1}{4\pi}\int \mathrm{d}\xi^- e^{-\imath x\xi^- P^+}\left\langle h(P,S)\middle|\overline{\psi}_i\left(0,\xi^-,\vec{0}\right)\Gamma\mathcal{G}\psi_i(0)\middle|h(P,S)\right\rangle$$

$$f_{g/h}(x)=\frac{-1/\imath}{4\pi x P^+}\int \mathrm{d}\xi^- e^{-\imath x\xi^- P^+}\left\langle h(P,S)\middle|F^{+\alpha}\left(0,\xi^-,\vec{0}\right)\mathcal{G}F^+_\alpha(0)/\widetilde{F}^+_\alpha(0)...\middle|h(P,S)\right\rangle$$

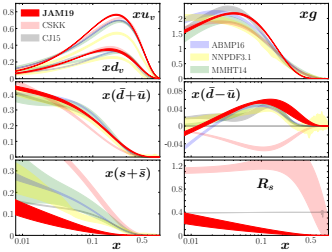
$$\mathcal{G}=P\exp\left(\imath g\int_0^{\xi^-}\mathrm{d}y^- A^+\left(0,y^-,\vec{0}\right)\right),\quad \Gamma=\gamma^+ \text{ or } \gamma^+\gamma^5,\quad \widetilde{F}^{\mu\nu}=\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F^{\rho\sigma}$$



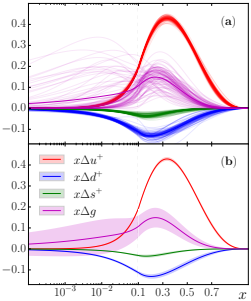
- Spin-averaged (unpolarized): $f = f^\uparrow + f^\downarrow$

- Spin-dependent (polarized): $\Delta f = f^\uparrow - f^\downarrow$

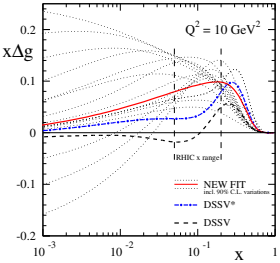
Motivations



PRD **101**, 074020 (2020)



PRD **93**, 074005 (2016)



PRL **113**, 012001 (2014)

Global QCD analysis

Global QCD analysis of high-energy scattering reactions

- Factorization theorems
- Bayesian inference
- Monte Carlo sampling
- Multi-step strategy
- Mellin transformation

Global QCD analysis - Bayesian inference

$$\mathrm{d}\sigma^{\mathrm{DIS}} = \sum_i H_i^{\mathrm{DIS}} \otimes f_i$$

$$\mathrm{d}\sigma^{\mathrm{DY}} = \sum_{i,j} H_i^{\mathrm{DY}} \otimes f_i \otimes f_j$$

$$\mathrm{d}\sigma^{\mathrm{jet}} = \sum_{i,j} H_i^{\mathrm{jet}} \otimes f_i \otimes f_j$$

hadron structure

posterior belief

prior belief

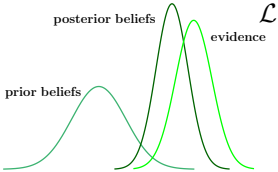
$$\rho(\vec{p}|\mathrm{data}) \sim \mathcal{L}(\vec{p}|\mathrm{data})\pi(\vec{p})$$

likelihood

polynomial

$$f_i(x) = n_i x^{\alpha_i}(1-x)^{\beta_i} P(x)$$

$$\vec{p} = (n_i, \alpha_i, \beta_i, \ldots)$$



$$\mathcal{L}(\vec{p}|\mathrm{data}) = \exp\left(-\frac{1}{2}\chi^2(\vec{p}|\mathrm{data})\right)$$

$$\chi^2 = \frac{1}{N}\sum_{i=1}^{N}\frac{(E_i - T_i)^2}{\alpha_i^2}$$

JAM numerical framework

To evolve convoluted DGLAP equations in x space

$$\frac{\partial \boldsymbol{q}(x,t)}{\partial t} = \frac{\alpha_s(t)}{2\pi} \int_x^1 \boldsymbol{P}\left(\frac{x}{z}, \alpha_s(t)\right) \boldsymbol{q}(z,t) \frac{1}{z}\, \mathrm{d}z$$

we apply Mellin transformation

$$\widetilde{f}(N) = \int_0^\infty x^{N-1} f(x)\, \mathrm{d}x$$

to remove the convolution

$$\frac{\partial \widetilde{\boldsymbol{q}}(N,t)}{\partial t} = \frac{\alpha_s(t)}{2\pi} \widetilde{\boldsymbol{P}}(N, \alpha_s(t)) \widetilde{\boldsymbol{q}}(N,t)$$

JAM numerical framework

- Differential cross section in x space is

$$\mathrm{d}\sigma = \int \mathrm{d}x\,\mathrm{d}y\, f_i(x) f_j(y) \mathcal{H}$$

where $\mathcal{H}$ is the perturbative part

JAM numerical framework

- Differential cross section in x space is

$$\mathrm{d}\sigma = \int \mathrm{d}M\,\mathrm{d}N\,\widetilde{f}_i(M)\widetilde{f}_j(N) \times \int \mathrm{d}x\,\mathrm{d}y\,x^{-M}y^{-N}\mathcal{H}$$

- Write PDFs in Mellin space

JAM numerical framework

- Differential cross section in x space is

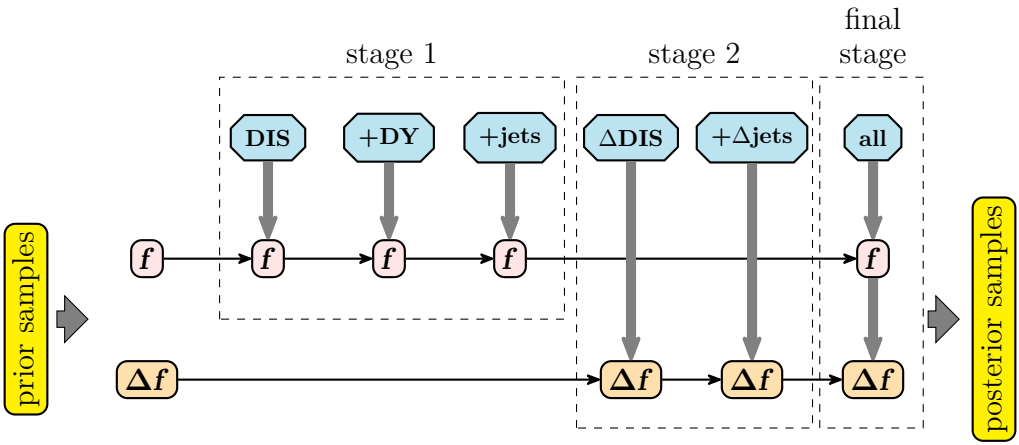
$$f(x) = nx^{\alpha}(1-x)^{\beta}\left(1 + c\sqrt{x} + dx\right)$$

- Write PDFs in Mellin space

$$\begin{aligned}\widetilde{f}(N) =& nB(N+\alpha, \beta+1) + \\ & ncB(N+\alpha+0.5, \beta+1) + \\ & ndB(N+\alpha+1, \beta+1)\end{aligned}$$

- $\widetilde{f}_i(N)$ takes simple form for typical parameterization

Multistep strategy



Data resampling

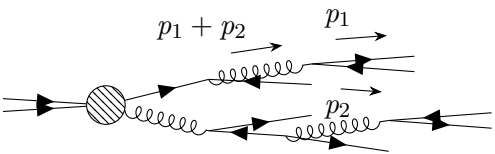
- We generate many resampled data sets $\widetilde{\sigma}_i$ with random numbers R_i and quadrature sum α_i of uncorrelated uncertainties

$$\widetilde{\sigma}_i = \sigma_i + R_i \alpha_i$$

- Monte Carlo sampling is used to avoid fits being trapped in local minima.
- Data are divided by experiment and included with multiple steps for fits to better converge.
- We currently include unpolarized data: DIS, Drell-Yan, jet production, and polarized data: inclusive DIS, jet production (total of **3576** data points).

Experimental definitions for jets

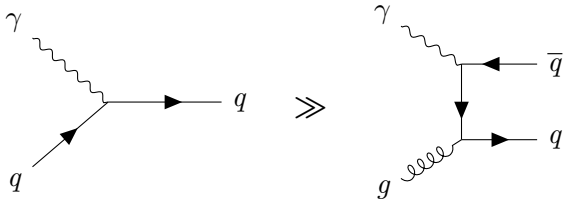
- Jets are collimated spray of hadrons in high-energy particle reactions.
- Jets are manifestations of collinear enchancement.
- Probes hadron structure (PDFs) without details of hadronization (fragmentation functions), in contrast to SIDIS.
- Jets are defined up to specific clustering algorithms (cone, k_T, anti-k_T...).



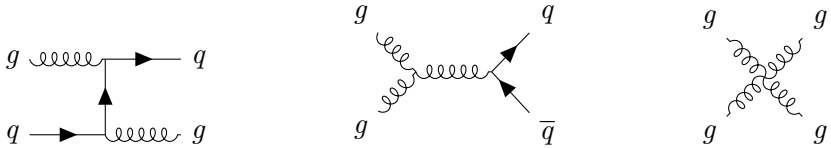
$$\frac{1}{(p_1+p_2)^2} = \frac{1}{2E_1E_2(1-\cos(\theta))}$$

Sensitivity to gluon PDFs

In **inclusive DIS**, sensitivity to gluon PDF only appears at NLO:



On the other hand, in **jet production**, gluon diagrams appear at LO:



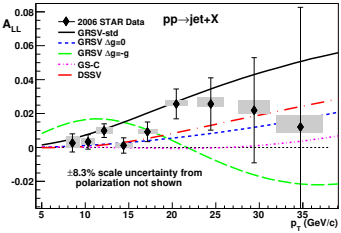
Jet in polarized collisions

- RHIC measures double longitudinal spin asymmetry

$$A_{LL}^{\text{jets}} = \frac{\sigma^{++} - \sigma^{+-}}{\sigma^{++} + \sigma^{+-}} = \frac{\Delta\sigma(\Delta g, \ldots)}{\sigma(g, \ldots)}$$

  - $\sigma^{+\pm}$ are differential cross sections when proton beams have equal & opposite helicity
  - denominator is spin-averaged cross section

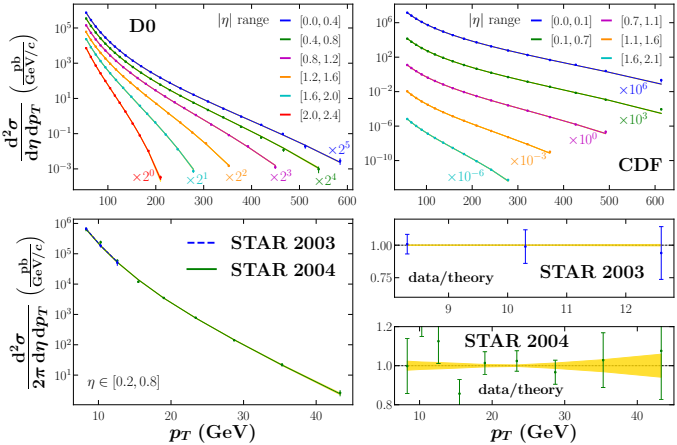
- A_{LL}^{jets} is sensitive to unpolarizd PDFs $\rightarrow$ perform **simultaneous** analysis to check



PRD **86**, 032006 (2012)

Fits to jet in unpolarized collisions

Good agreement between theory and Tevatron data

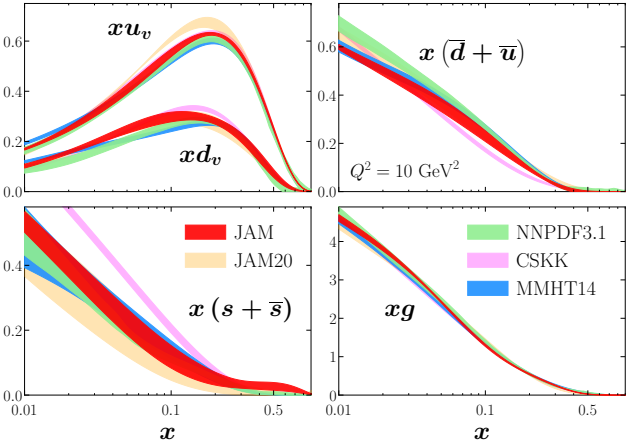


First inclusion of unpolarized RHIC jets!

arXiv:2201.02075 [hep-ph] (Y. Zhou *et al.*)

Fits to jet in unpolarized collisions

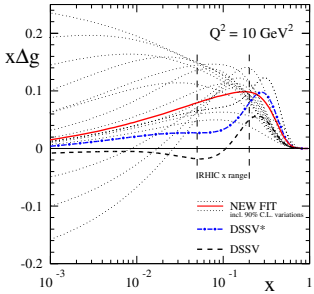
Differences are caused by choices of datasets.



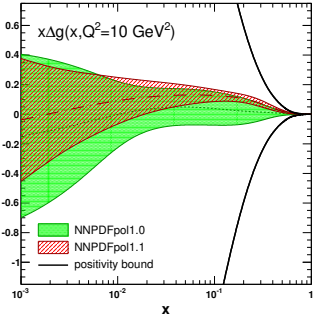
An overall good agreement is found.

arXiv:2201.02075 [hep-ph]

Polarized PDFs - previous extractions



PRL **113**, 012001 (2014)



Nucl.Phys.B **887** (2014) 276

Extracted with SU(3) flavor symmetry and positivity constraints

Polarized PDFs - theory assumptions

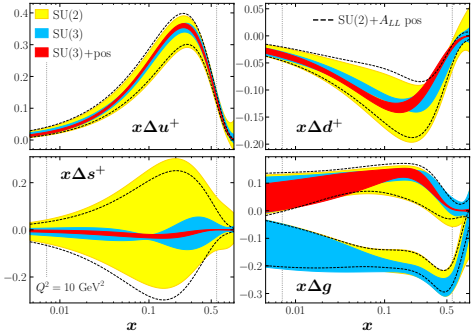
1. **SU(2)** flavor symmetry only
2. **SU(2)** and **SU(3)**
3. **SU(2)** and **SU(3)**, and **PDF positivity**

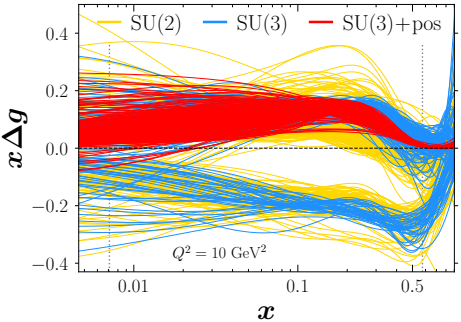
more constraints

more biases

less data driven

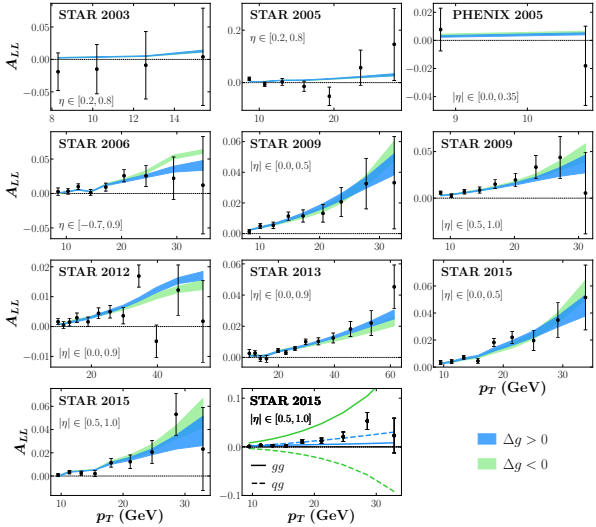
Polarized PDFs - results





$$\Delta q^+ = \Delta q + \Delta\overline{q},\ q = u, d, s$$

Gluon helicity - jet A_{LL} and Δg

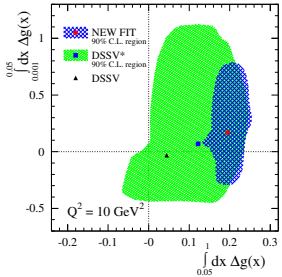


Good agreement with data for $\Delta g > 0$ and $\Delta g < 0$

Large cancellation between gg and qg channels for $\Delta g < 0$

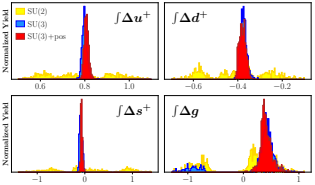
Polarized PDFs - $\Delta G = \int_{0.05}^{1} \Delta g \, \mathrm{d}x$

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + \Delta G + L_g$$



PRL **113**, 012001 (2014)

- **SU(2)**:
  - $\Delta g > 0$: 0.20 ± 0.13
  - $\Delta g < 0$: -0.56 ± 0.12
- **SU(3)**:
  - $\Delta g > 0$: 0.27 ± 0.03
  - $\Delta g < 0$: -0.61 ± 0.04
- **SU(3) + pos**: 0.25 ± 0.03
- **DSSV14**: 0.2 ± 0.05

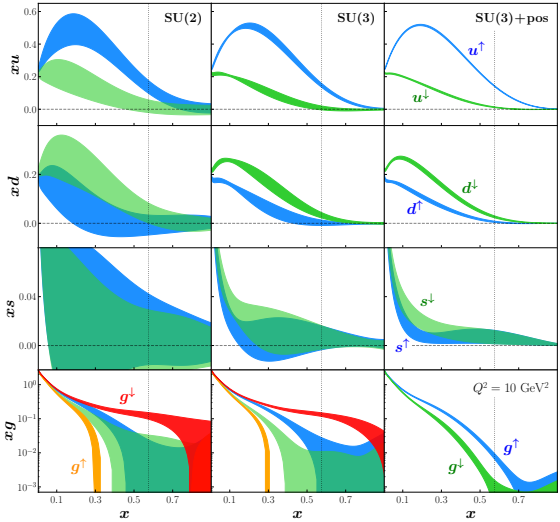


arXiv:2201.02075 [hep-ph]

Helicity basis PDFs

$u^{\uparrow/\downarrow}$ and $d^{\uparrow/\downarrow}$ are well separated

$s^{\uparrow/\downarrow}$ can be hardly distinguished

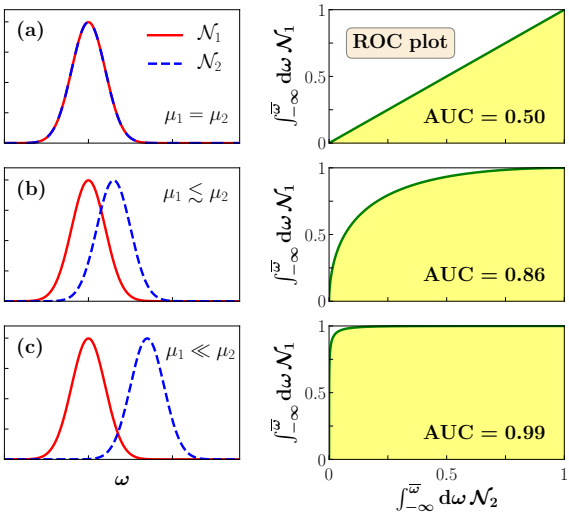


First simultaneous extraction of $f^\uparrow$ and $f^\downarrow$!

$g^{\uparrow/\downarrow}$ for $\Delta g > 0$ (blue and green) and $\Delta g < 0$ (orange and red)

ROC and AUC

- indistinguishable
- somewhat distinguishable
- clearly distinguishable

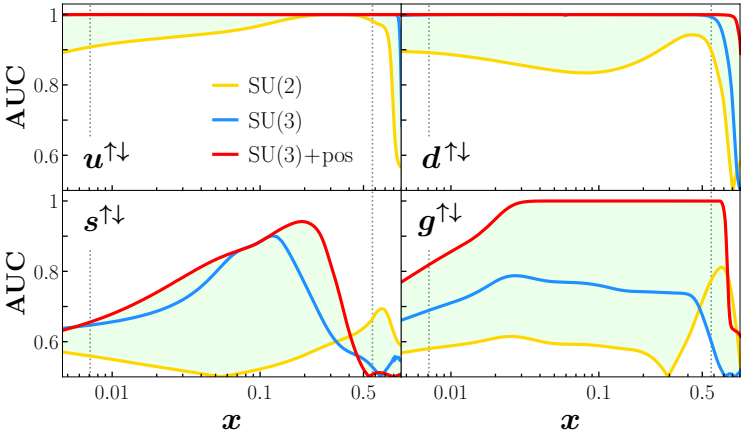


ROC: *receiver operating characteristic curve*

AUC: *area under curve of ROC*

AUC for helicity PDFs

- 0.5: can barely discriminate $f^{\uparrow}$ and $f^{\downarrow}$
- 1.0: can clearly discriminate $f^{\uparrow}$ and $f^{\downarrow}$



Currently working on...

- polarized SIDIS with large q_T: linear dependence on Δg at LO
- di-jets: also sensivite Δg at LO
- $\pi^{\pm}$ and $K^{\pm}$ production in pp collisions

Collaboration

In collaboration with Nobuo Sato and Wally Melnitchouk in Jefferson Lab.

And thanks to Carlota Andrés, Patrick Barry, Christopher Cocuzza, Eric Moffat for helpful discussions!