Solving the medium-induced radiation spectrum for an arbitrary number of scatterings

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@ Brookhaven National Lab, Feb 14 2020

CA, Liliana Apolinario, Fabio Dominguez: arXiv: 2002.01517

Jefferson Lab
EXPLORING THE NATURE OF MATTER

Motivation

Hard probes

- Production of high-energy parton unlikely to interfere with the medium formation
  Good probe!

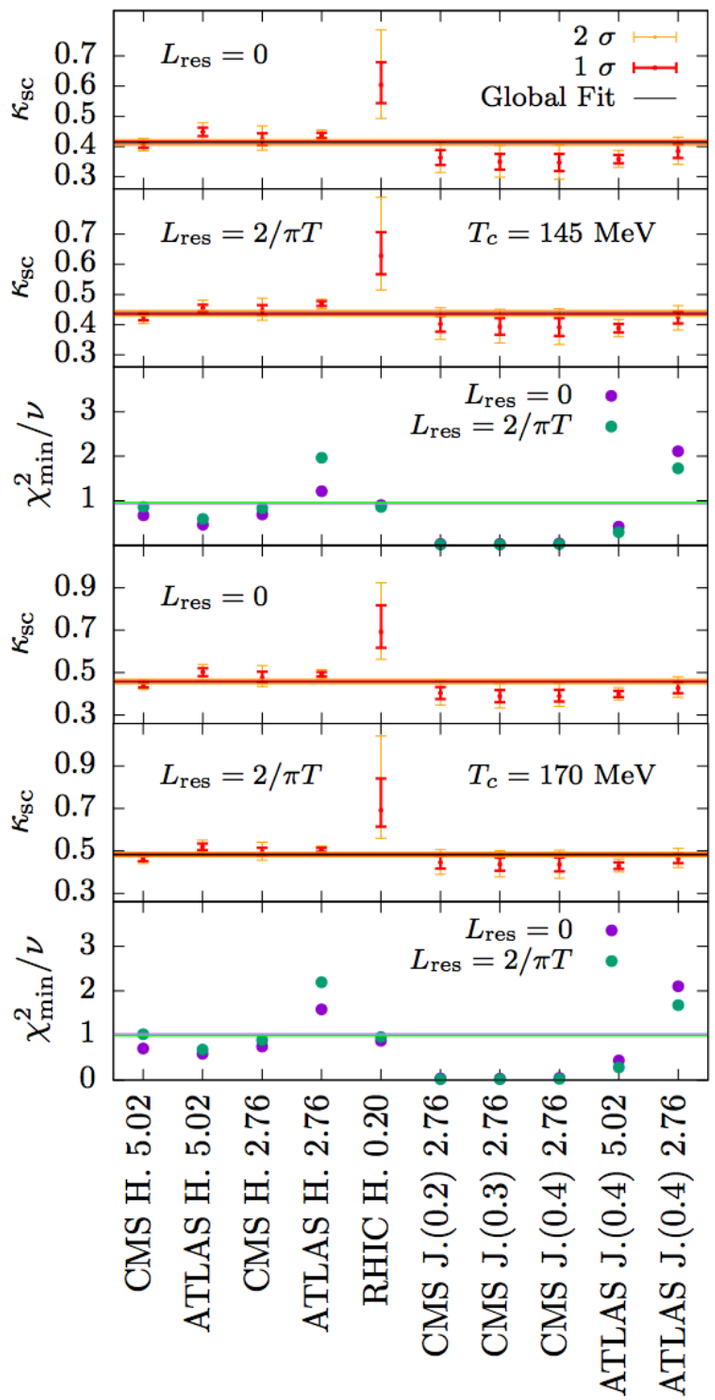
- Versatile: access to different time and energy scales

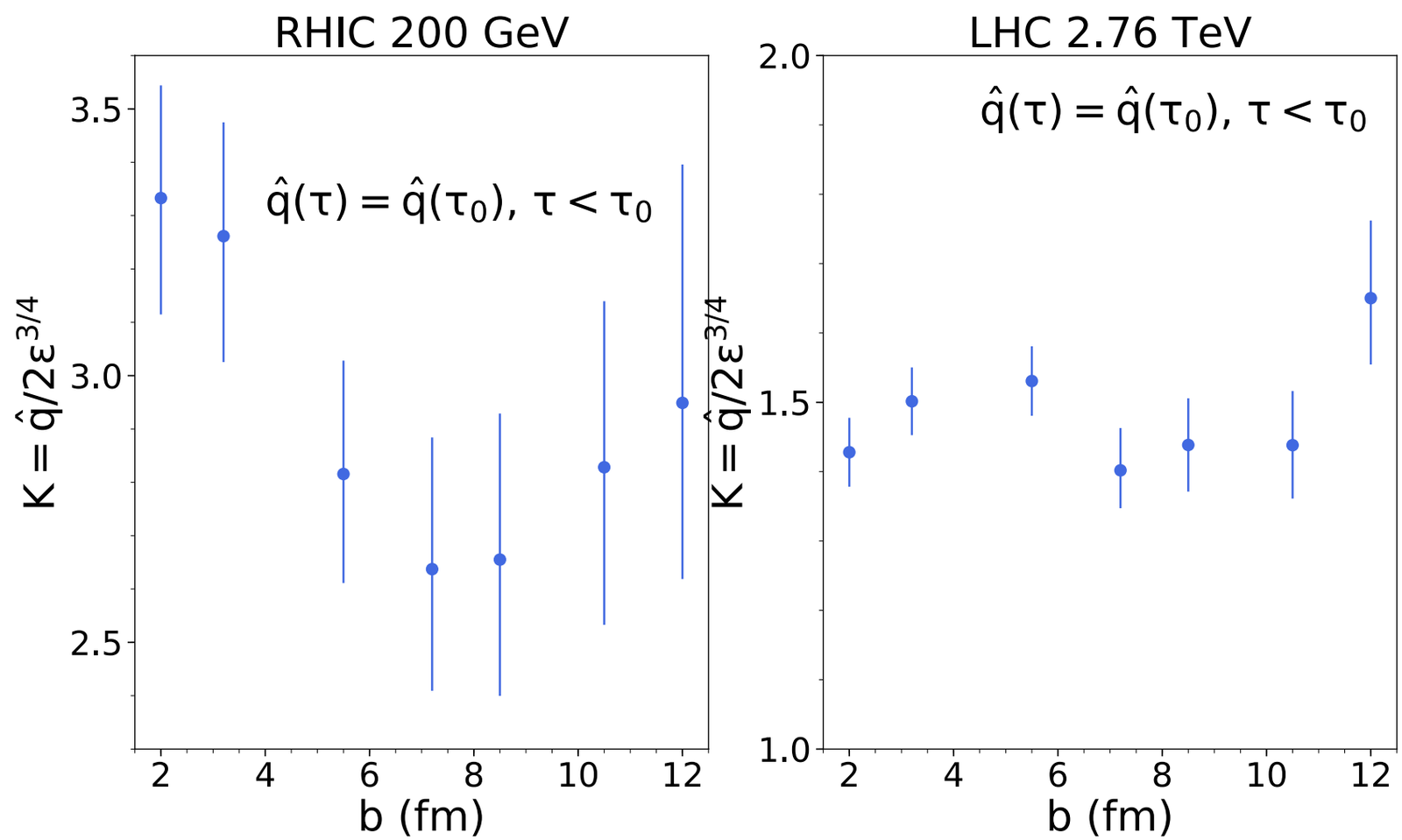
- Jet quenching → Extracting the properties of the QGP

- **Medium-induced gluon radiation** is the main mechanism of energy loss

The jet quenching parameter

Casalderrey et al. 1808.07386





CA, Armesto, Luzum, Salgado, Zurita arXiv:1606.04837

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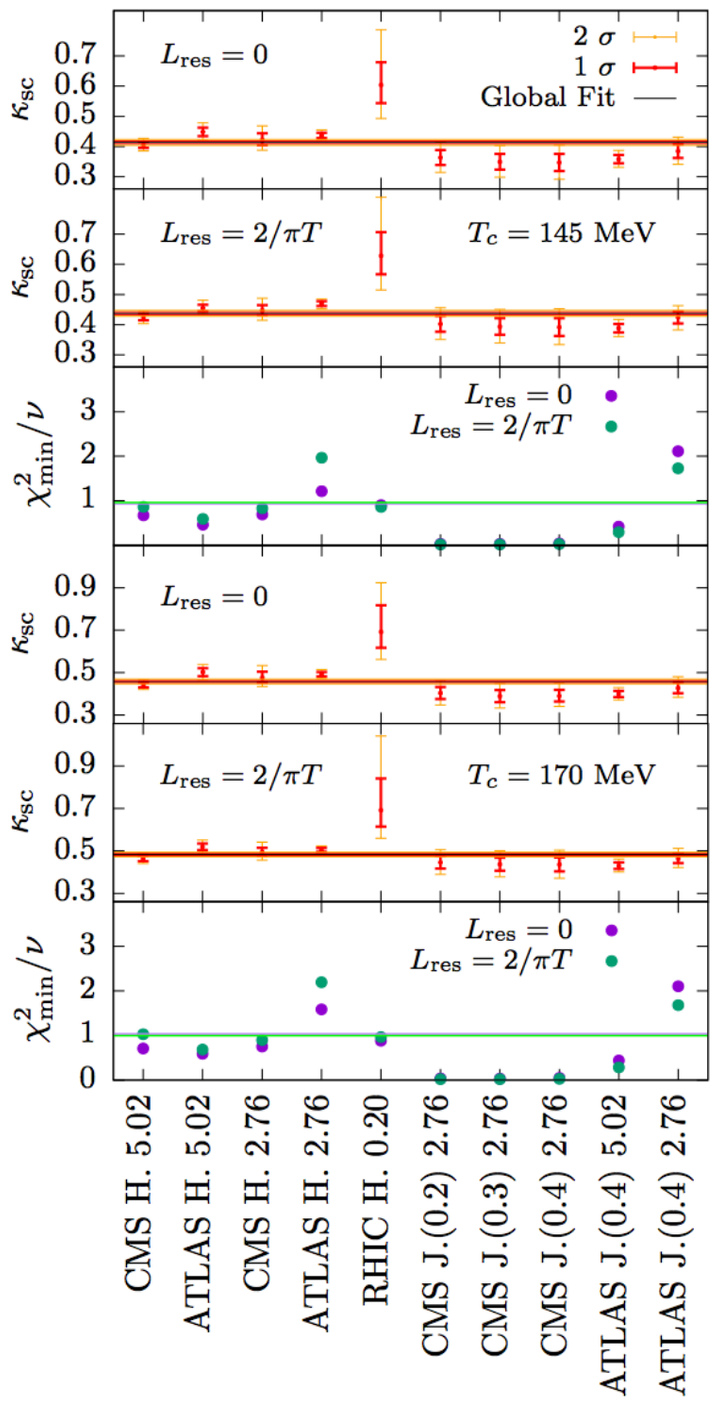
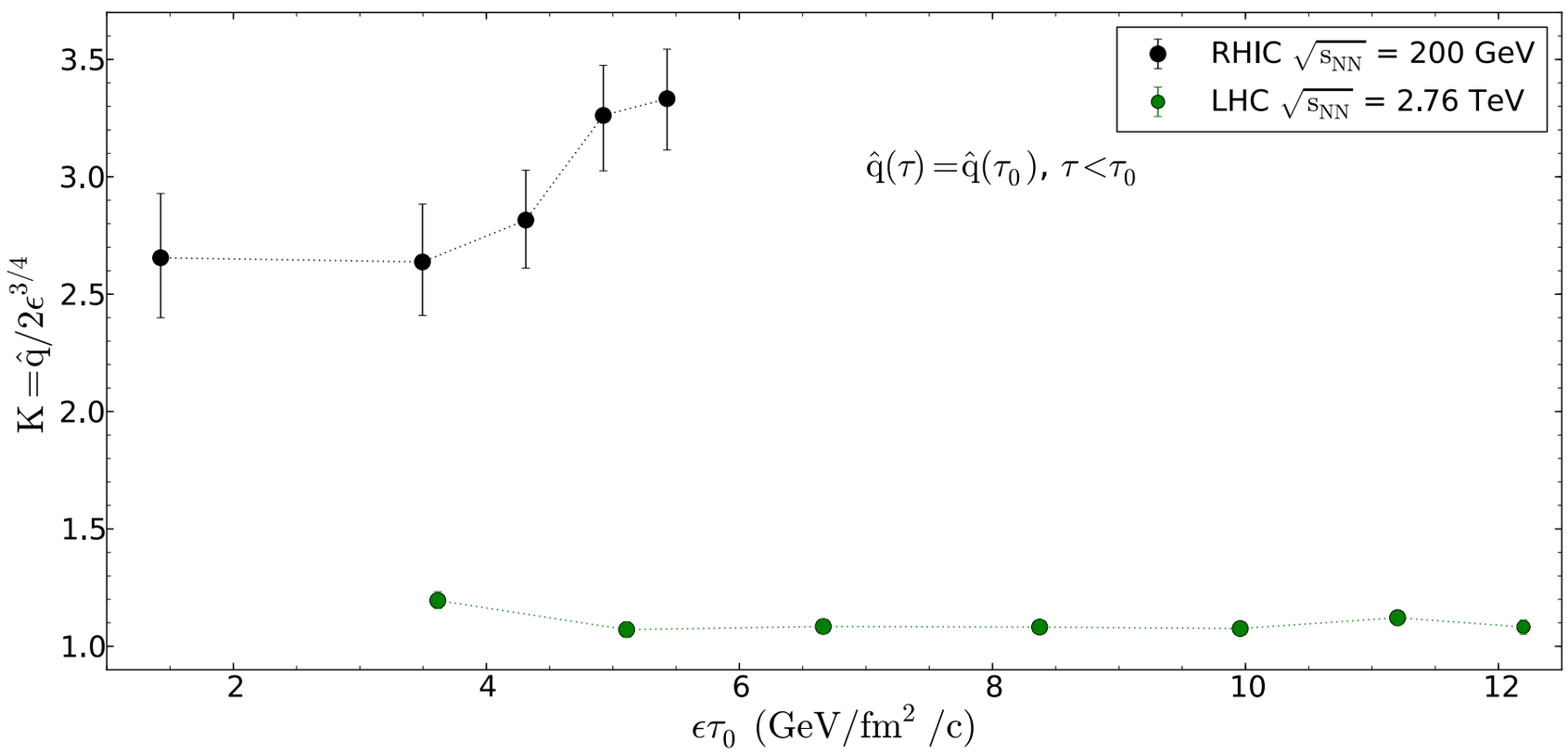


Energy-centrality dependence difficult to reconcile



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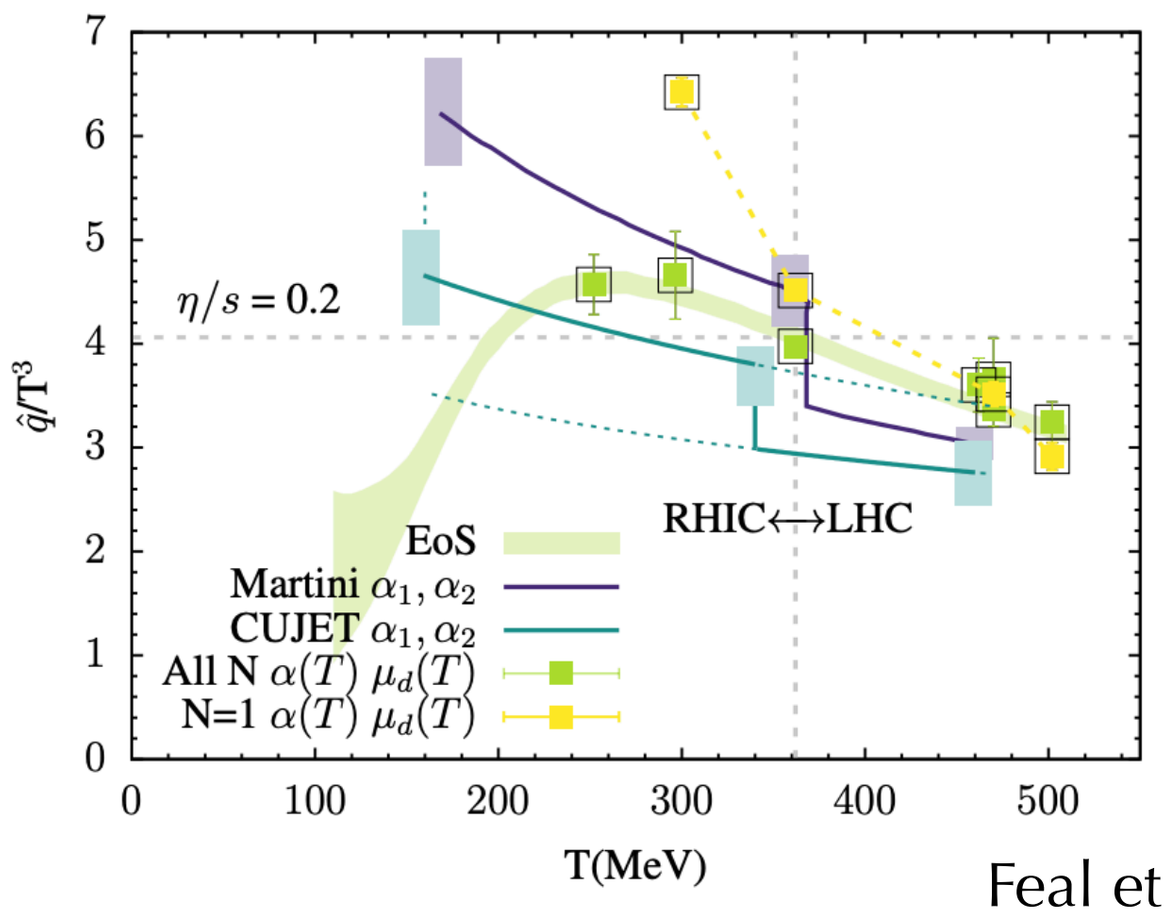
The jet quenching parameter

- Resummation of all scatterings beyond the Gaussian approximation within a MC approach

- One dimensional expansion only



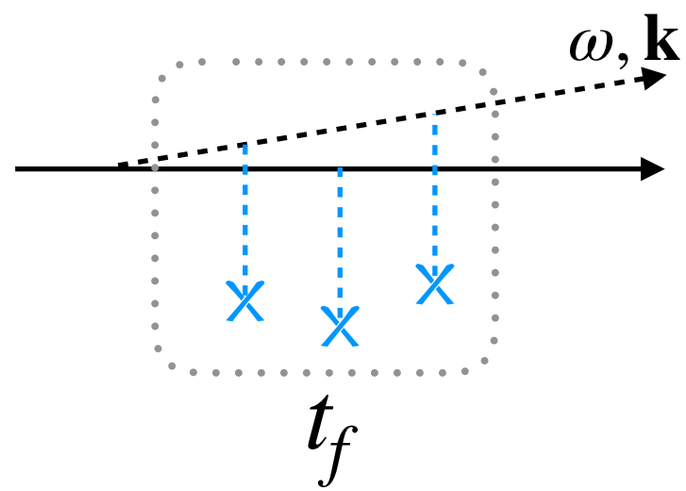
Feal et al. 1911.01309

- Centrality/energy puzzle seems to disappear

Introduction

LPM effect

- During the formation time of the gluon multiple scatterings act **coherently**



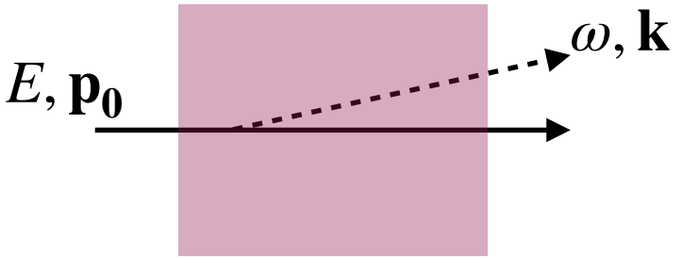
Suppression of the spectrum for **large formation times** $t_f \sim \dfrac{\omega}{\mathbf{k}^2}$

$$\omega \frac{dI}{d\omega} = \alpha_s \sqrt{\frac{\omega_c}{\omega}}$$

Landau, Pomeranchuk, Migdal
for QED

Assumptions

- The gluon is emitted at small angles and $\mu \ll \mathbf{k}, \mathbf{p_0} \ll \omega, \mathbf{E}$



- The radiation is due to elastic scatterings mediated by gluons

- The interactions are instantaneous. The medium is seen as recoilless background field

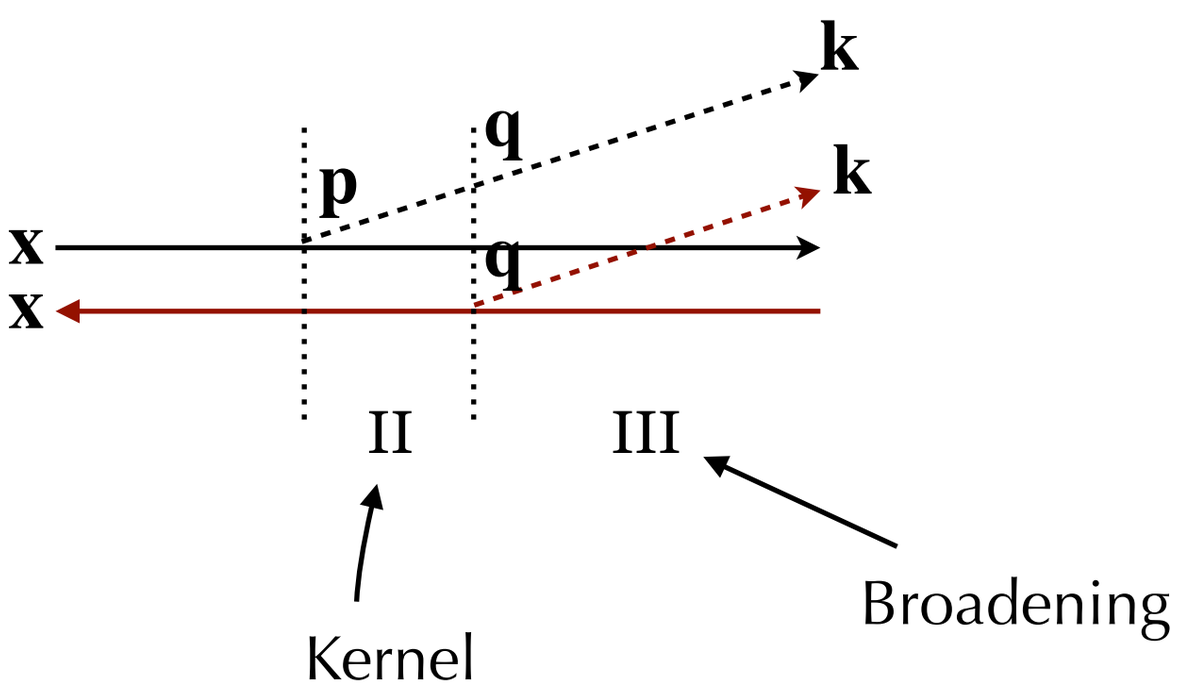
- The details of the parton-medium interaction are given by the dipole cross section

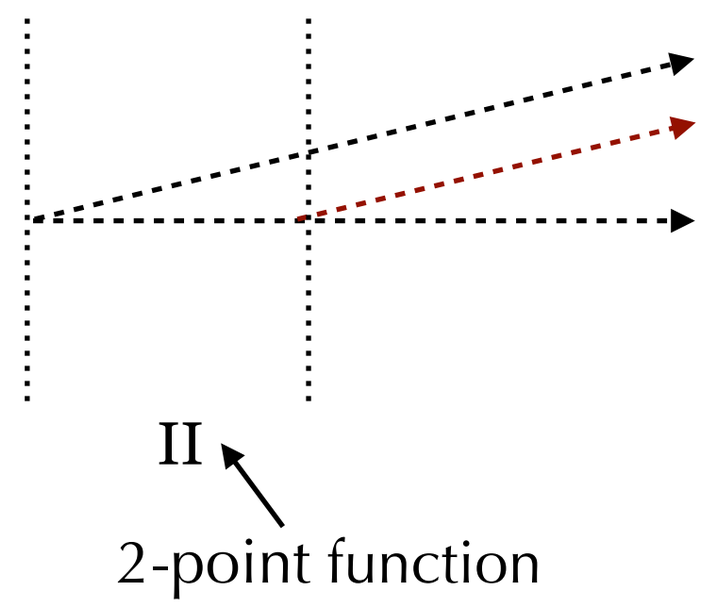
Medium-induced gluon spectrum

- For a soft emitted gluon ($\omega \ll E$)

$$\omega \frac{dI}{d\omega d^2\boldsymbol{k}} = \frac{2\alpha_s C_R}{(2\pi)^2\omega^2} \mathrm{Re} \int_0^\infty dt' \int_0^{t'} dt \int_{\boldsymbol{pq}} \boldsymbol{p}\cdot\boldsymbol{q}\, \widetilde{\mathcal{K}}(t',\boldsymbol{q};t,\boldsymbol{p})\mathcal{P}(\infty,\boldsymbol{k};t',\boldsymbol{q})$$

BDMPS-Z





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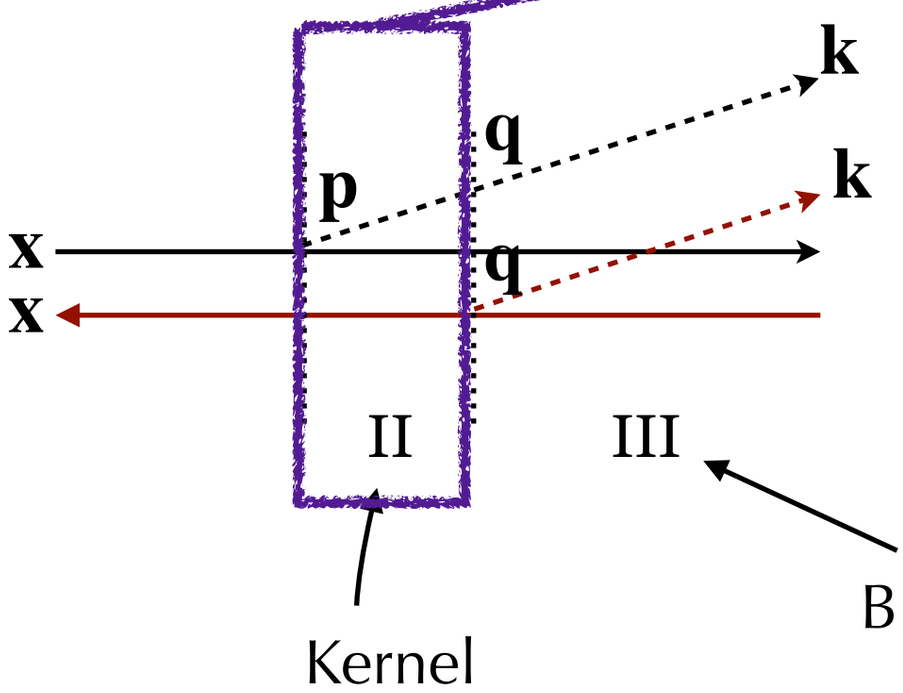

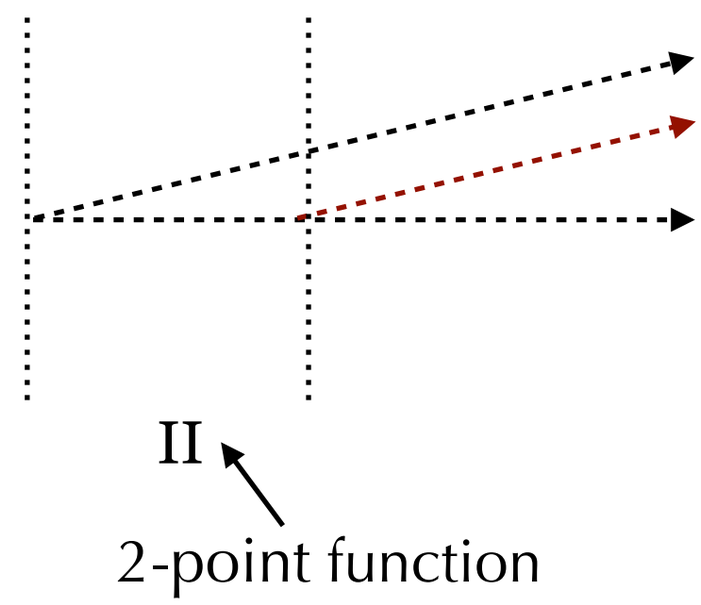

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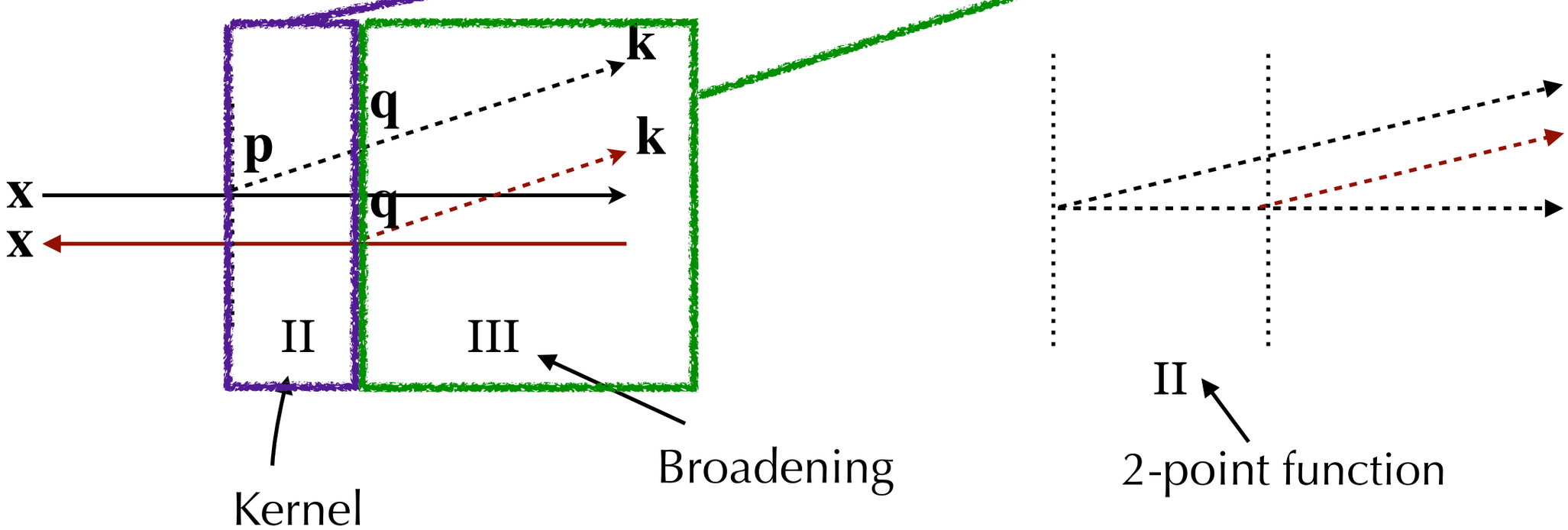

Formalism

Medium-induced gluon spectrum

- **Emission** off a parton with energy E of a **soft gluon** $(\omega \ll E)$

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BDMPS-Z

- Broadening

$$\mathcal{P}(t'',\boldsymbol{k};t',\boldsymbol{q}) \equiv \int d^2\boldsymbol{z}\, e^{-i(\boldsymbol{k}-\boldsymbol{q})\cdot\boldsymbol{z}} \exp\left\{-\frac{1}{2}\int_{t'}^{t''} ds\, n(s)\,\sigma(\boldsymbol{z})\right\}$$

- Emission Kernel

$$\mathcal{K}(t',\boldsymbol{z};t,\boldsymbol{y}) \equiv \int_{\boldsymbol{pq}} e^{i(\boldsymbol{q}\cdot\boldsymbol{z}-\boldsymbol{p}\cdot\boldsymbol{y})}\, \widetilde{\mathcal{K}}(t',\boldsymbol{q};t,\boldsymbol{p})$$

$$= \int_{\boldsymbol{r}(t)=\boldsymbol{y}}^{\boldsymbol{r}(t')=\boldsymbol{z}} \mathcal{D}\boldsymbol{r} \exp\left[\int_t^{t'} ds \left(\frac{i\omega}{2}\dot{\boldsymbol{r}}^2 - \frac{1}{2}n(s)\sigma(\boldsymbol{r})\right)\right]$$

Difficult to solve numerically for realistic $\sigma(\mathbf{r})$

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Medium information

Difficult to solve numerically for realistic $\sigma(\mathbf{r})$

Analytic limits

- **<u>Harmonic oscillator</u>** approximation

Perturbative tails neglected

$$n(s)\sigma(\mathbf{r}) \approx \frac{1}{2}\hat{q}(s)\mathbf{r}^2 + \mathcal{O}(\mathbf{r^2}\ln\mathbf{r^2})$$

The Kernel can be analytically computed for a static medium

Parameters: $\hat{q}\,,\,L$

- **<u>Opacity expansion</u>**

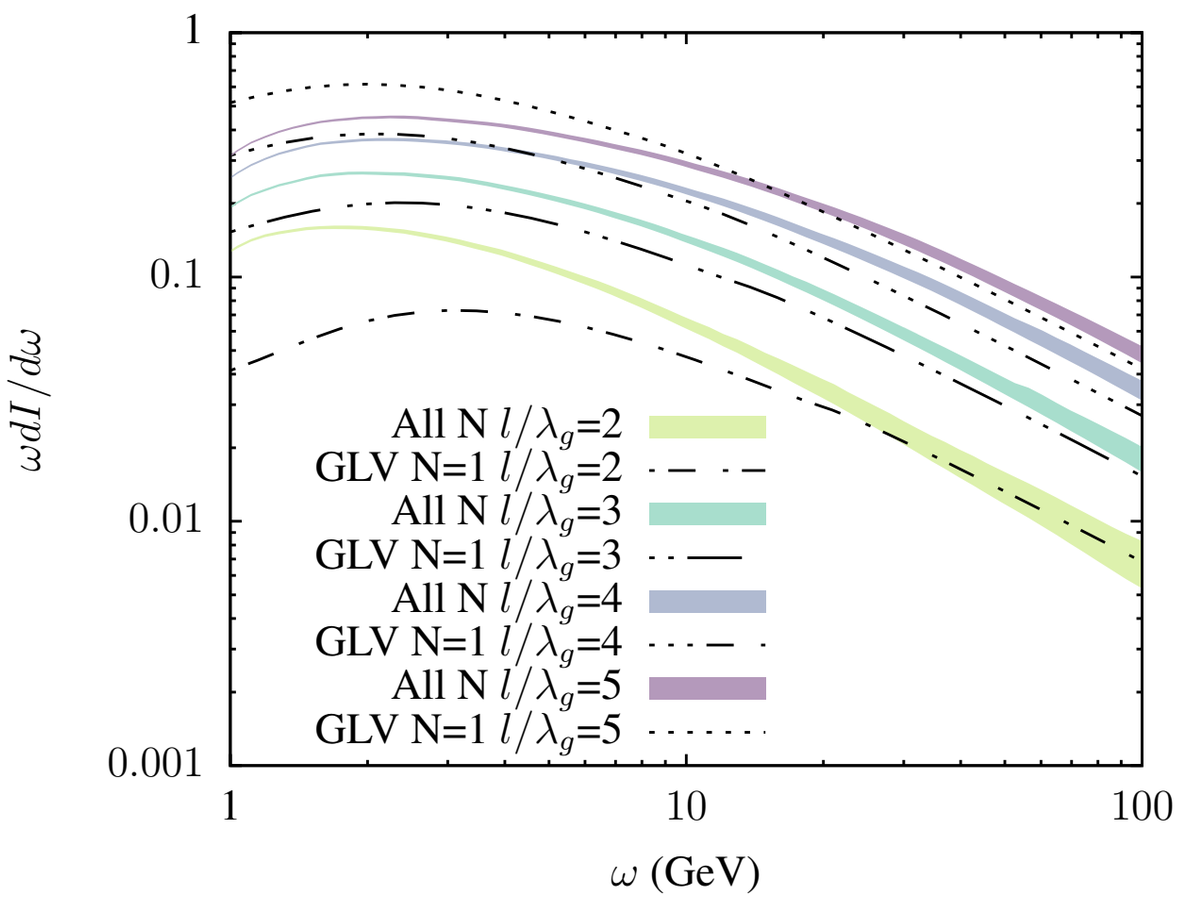
$\sigma(\mathbf{r})$ is taken as the full Yukawa cross-section

The integrand in the Kernel is expanded in powers of $\left(n(s)\sigma(\mathbf{r})\right)^N$

$N = 1$: **First opacity** or **GLV** approximation

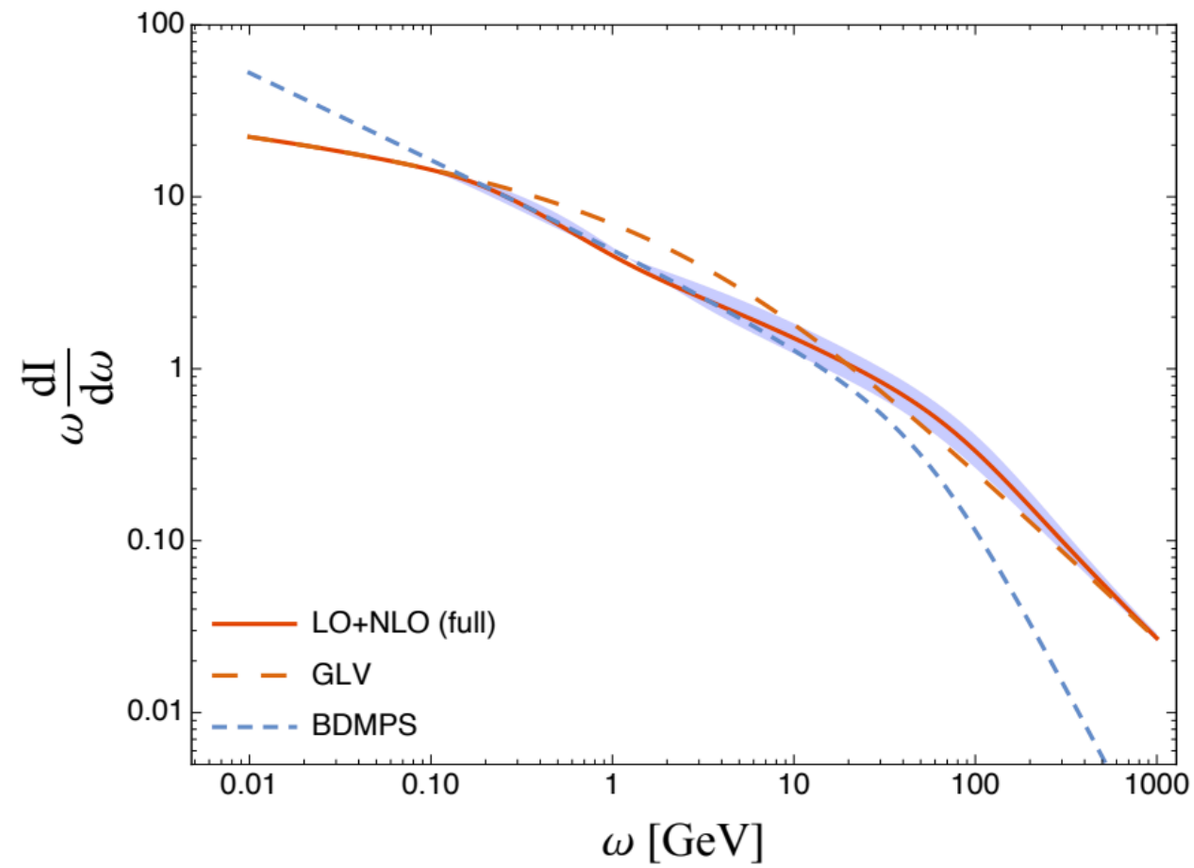
Latest developments

- Full resummation of all scatterings within a MC approach



Feal and Vazquez, arXiv:1810.02645

- Analytical expansion around the HO



Mehtar-Tani and Tywoniuk, arXiv:1910.02032

Mehtar-Tani, arXiv:1903.00506

Reorganizing the spectrum

$$\omega \frac{dI}{d\omega d^2\boldsymbol{k}} = \frac{2\alpha_s C_R}{(2\pi)^2\omega^2} \mathrm{Re} \int_0^\infty dt' \int_0^{t'} dt \int_{\boldsymbol{pq}} \boldsymbol{p}\cdot\boldsymbol{q}\, \widetilde{\mathcal{K}}(t',\boldsymbol{q};t,\boldsymbol{p})\mathcal{P}(\infty,\boldsymbol{k};t',\boldsymbol{q})$$

- The gluon emission can take place inside or outside the medium
- Medium-vacuum interferences need to be properly taken into account
- But we can perform analytically the integral in t' by using:

$$\mathcal{P}(t'',\boldsymbol{k};t',\boldsymbol{q}) = (2\pi)^2\,\delta^{(2)}(\boldsymbol{k}-\boldsymbol{q}) - \frac{1}{2}\int_{t'}^{t''} ds\, n(s) \int_{\boldsymbol{k}'} \sigma(\boldsymbol{k}'-\boldsymbol{q})\mathcal{P}(t'',\boldsymbol{k};s,\boldsymbol{k}')\,,$$

$$\widetilde{\mathcal{K}}(t',\boldsymbol{q};t,\boldsymbol{p}) = (2\pi)^2\,\delta^{(2)}(\boldsymbol{q}-\boldsymbol{p})\, e^{-i\frac{p^2}{2\omega}(t'-t)} - \frac{1}{2}\int_t^{t'} ds\, n(s) \int_{\boldsymbol{k}'} \sigma(\boldsymbol{q}-\boldsymbol{k}') e^{-i\frac{q^2}{2\omega}(t'-s)} \widetilde{\mathcal{K}}(s,\boldsymbol{k}';t,\boldsymbol{p})\,.$$

Reorganizing the spectrum II

$$\omega \frac{dI}{d\omega d^2\boldsymbol{k}} = \frac{2\alpha_s C_R}{(2\pi)^2\omega^2} \mathrm{Re} \int_0^\infty dt' \int_0^{t'} dt \int_{\boldsymbol{pq}} \boldsymbol{p}\cdot\boldsymbol{q}\, \widetilde{\mathcal{K}}(t',\boldsymbol{q};t,\boldsymbol{p})\mathcal{P}(\infty,\boldsymbol{k};t',\boldsymbol{q})$$

- Performing the t′-integral (and subtracting the vacuum term) brings a factor of $\frac{2\,i\,\omega}{\mathbf{q}^2}$
- The result is proportional to: $i\omega\left(\frac{\boldsymbol{l}}{\boldsymbol{l}^2} - \frac{\boldsymbol{q}}{\boldsymbol{q}^2}\right)\sigma(\boldsymbol{l}-\boldsymbol{q})$
- The spectrum, once the vacuum contribution has been subtracted, is:

$$\omega \frac{dI}{d\omega d^2\boldsymbol{k}} = \frac{2\alpha_s C_R}{(2\pi)^2\omega} \mathrm{Re} \int_0^\infty ds\, n(s) \int_0^s dt \int_{\boldsymbol{pql}} i\boldsymbol{p}\cdot\left(\frac{\boldsymbol{l}}{\boldsymbol{l}^2} - \frac{\boldsymbol{q}}{\boldsymbol{q}^2}\right)\sigma(\boldsymbol{l}-\boldsymbol{q})\widetilde{\mathcal{K}}(s,\boldsymbol{q};t,\boldsymbol{p})\mathcal{P}(\infty,\boldsymbol{k};s,\boldsymbol{l})$$

Solving the spectrum

- Now the s-integration is an integration over the position of the scatterings:

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Guarantees convergence of $\boldsymbol{q}$ and $\boldsymbol{l}$ integrals as $\sigma(\mathbf{q}) \sim 1/\mathbf{q}^4$ for large $\boldsymbol{q}$

- The propagators satisfy Schwinger-Dyson equations:

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$$\partial_t \widetilde{\mathcal{K}}(s,\boldsymbol{q};t,\boldsymbol{p}) = \frac{i\boldsymbol{p}^2}{2\omega} \widetilde{\mathcal{K}}(s,\boldsymbol{q};t,\boldsymbol{p}) + \frac{1}{2} n(t) \int_{\boldsymbol{k}'} \sigma(\boldsymbol{k}'-\boldsymbol{p}) \widetilde{\mathcal{K}}(s,\boldsymbol{q};t,\boldsymbol{k}')$$

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- We write the spectrum as a convolution

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$$\boldsymbol{\psi}(s,\boldsymbol{k};t,\boldsymbol{p}) = \int_{\boldsymbol{q}} \boldsymbol{\phi}(L,\boldsymbol{k};s,\boldsymbol{q})\, \widetilde{\mathcal{K}}(s,\boldsymbol{q};t,\boldsymbol{p})$$

$$\boldsymbol{\phi}(\tau,\boldsymbol{k};s,\boldsymbol{q}) = n(s) \int_{\boldsymbol{l}} \left(\frac{\boldsymbol{l}}{\boldsymbol{l}^2} - \frac{\boldsymbol{q}}{\boldsymbol{q}^2} \right) \sigma(\boldsymbol{l}-\boldsymbol{q}) \mathcal{P}(\tau,\boldsymbol{k};s,\boldsymbol{l})$$

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- Satisfying:

Oscillations

$$\partial_t \boldsymbol{\psi}(s,\boldsymbol{k};t,\boldsymbol{p}) = \boxed{\frac{i\boldsymbol{p}^2}{2\omega} \boldsymbol{\psi}(s,\boldsymbol{k};t,\boldsymbol{p})} + \frac{1}{2} n(t) \int_{\boldsymbol{k}'} \sigma(\boldsymbol{k}'-\boldsymbol{p}) \boldsymbol{\psi}(s,\boldsymbol{k};t,\boldsymbol{k}')$$

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$$\omega \frac{dI}{d\omega d^2\boldsymbol{k}} = \frac{2\alpha_s C_R}{(2\pi)^2\omega} \mathrm{Re} \int_0^L ds\, n(s) \int_0^s dt \int_{\boldsymbol{pql}} i\boldsymbol{p} \cdot \left(\frac{\boldsymbol{l}}{\boldsymbol{l}^2} - \frac{\boldsymbol{q}}{\boldsymbol{q}^2} \right) \sigma(\boldsymbol{l}-\boldsymbol{q}) \widetilde{\mathcal{K}}(s,\boldsymbol{q};t,\boldsymbol{p}) \mathcal{P}(L,\boldsymbol{k};s,\boldsymbol{l})$$

- We write the spectrum as a convolution

$$\omega \frac{dI}{d\omega d^2\mathbf{k}} = \frac{2\alpha_s C_R}{(2\pi)^2\omega} \mathrm{Re} \int_0^L ds\, n(s) \int_0^s dt \int_{\mathbf{p}} i\,\mathbf{p} \cdot \psi(s,\mathbf{k};t,\mathbf{p})$$

$$\boldsymbol{\psi}(s,\boldsymbol{k};t,\boldsymbol{p}) = \int_{\boldsymbol{q}} \boldsymbol{\phi}(L,\boldsymbol{k};s,\boldsymbol{q})\, \widetilde{\mathcal{K}}(s,\boldsymbol{q};t,\boldsymbol{p}) \qquad \boldsymbol{\phi}(\tau,\boldsymbol{k};s,\boldsymbol{q}) = n(s) \int_{\boldsymbol{l}} \left(\frac{\boldsymbol{l}}{\boldsymbol{l}^2} - \frac{\boldsymbol{q}}{\boldsymbol{q}^2} \right) \sigma(\boldsymbol{l}-\boldsymbol{q}) \mathcal{P}(\tau,\boldsymbol{k};s,\boldsymbol{l})$$

- Switching to the interaction picture

$$\boldsymbol{\psi}_I(s,\boldsymbol{k};t,\boldsymbol{p}) = e^{\frac{i\boldsymbol{p}^2}{2\omega}(s-t)} \boldsymbol{\psi}(s,\boldsymbol{k};t,\boldsymbol{p})$$

Final equations

1. Solve:

$$\partial_\tau \boldsymbol{\phi}(\tau, \boldsymbol{k}; s, \boldsymbol{q}) = -\frac{1}{2} n(\tau) \int_{\boldsymbol{k}'} \sigma(\boldsymbol{k} - \boldsymbol{k}') \boldsymbol{\phi}(\tau, \boldsymbol{k}'; s, \boldsymbol{q})$$

With initial condition

$$\boldsymbol{\phi}(s, \boldsymbol{k}; s, \boldsymbol{q}) = n(s) \left(\frac{\boldsymbol{k}}{\boldsymbol{k}^2} - \frac{\boldsymbol{q}}{\boldsymbol{q}^2} \right) \sigma(\boldsymbol{k} - \boldsymbol{q})$$

2. Solve:

$$\partial_t \boldsymbol{\psi}_I(s, \boldsymbol{k}; t, \boldsymbol{p}) = \frac{1}{2} n(t) \int_{\boldsymbol{k}'} e^{\frac{i\boldsymbol{p}^2}{2\omega}(s-t)} \sigma(\boldsymbol{k}' - \boldsymbol{p}) e^{-\frac{i\boldsymbol{k}'^2}{2\omega}(s-t)} \boldsymbol{\psi}_I(s, \boldsymbol{k}; t, \boldsymbol{k}')$$

with initial condition

$$\boldsymbol{\psi}_I(s, \boldsymbol{k}; s, \boldsymbol{p}) = \boldsymbol{\phi}(L, \boldsymbol{k}; s, \boldsymbol{p})$$

3. Compute:

$$\omega \frac{dI}{d\omega d^2\mathbf{k}} = \frac{2\alpha_s C_R}{(2\pi)^2 \omega} \mathrm{Re} \int_0^L ds \int_0^s dt \int_{\mathbf{p}} i\, e^{-i\frac{\mathbf{p}^2}{2\omega}(s-t)} \mathbf{p} \cdot \psi_I(s, \mathbf{k}; t, \mathbf{p})$$

Energy distribution

- The energy distribution is obtained by integrating the k_T-differential spectrum with the constraint: $k < \omega$

$$\omega \frac{dI}{d\omega} = \int_0^{\omega^2} \omega \frac{dI}{d\omega d^2\mathbf{k}^2} d\mathbf{k}^2$$

This is the $\bar{R}$ finite case

- We can also remove this kinematic constraint:

$$\omega \frac{dI}{d\omega} = \int_0^{\infty} \omega \frac{dI}{d\omega d^2\mathbf{k}^2} d\mathbf{k}^2$$

This is also known as $\bar{R} \to \infty$ limit

Results

The set-up

- For simplicity : $n(s) = n_0$

- Two types of Parton-medium interactions considered: $\sigma(\mathbf{r}) = \int_{\mathbf{q}} V(\mathbf{q})\left(1 - e^{i\mathbf{q}\mathbf{r}}\right)$

  - Yukawa

$$V(\mathbf{q}) = \frac{8\pi\mu^2}{\left(\mathbf{q}^2 + \mu^2\right)^2}$$

Parameters: n_0 , L , μ

  - Hard thermal loop

$$\frac{1}{2} n\, V(\mathbf{q}) = \frac{g_s^2 N_c\, \mu^2 T}{\mathbf{q}^2\left(\mathbf{q}^2 + \mu^2\right)}$$

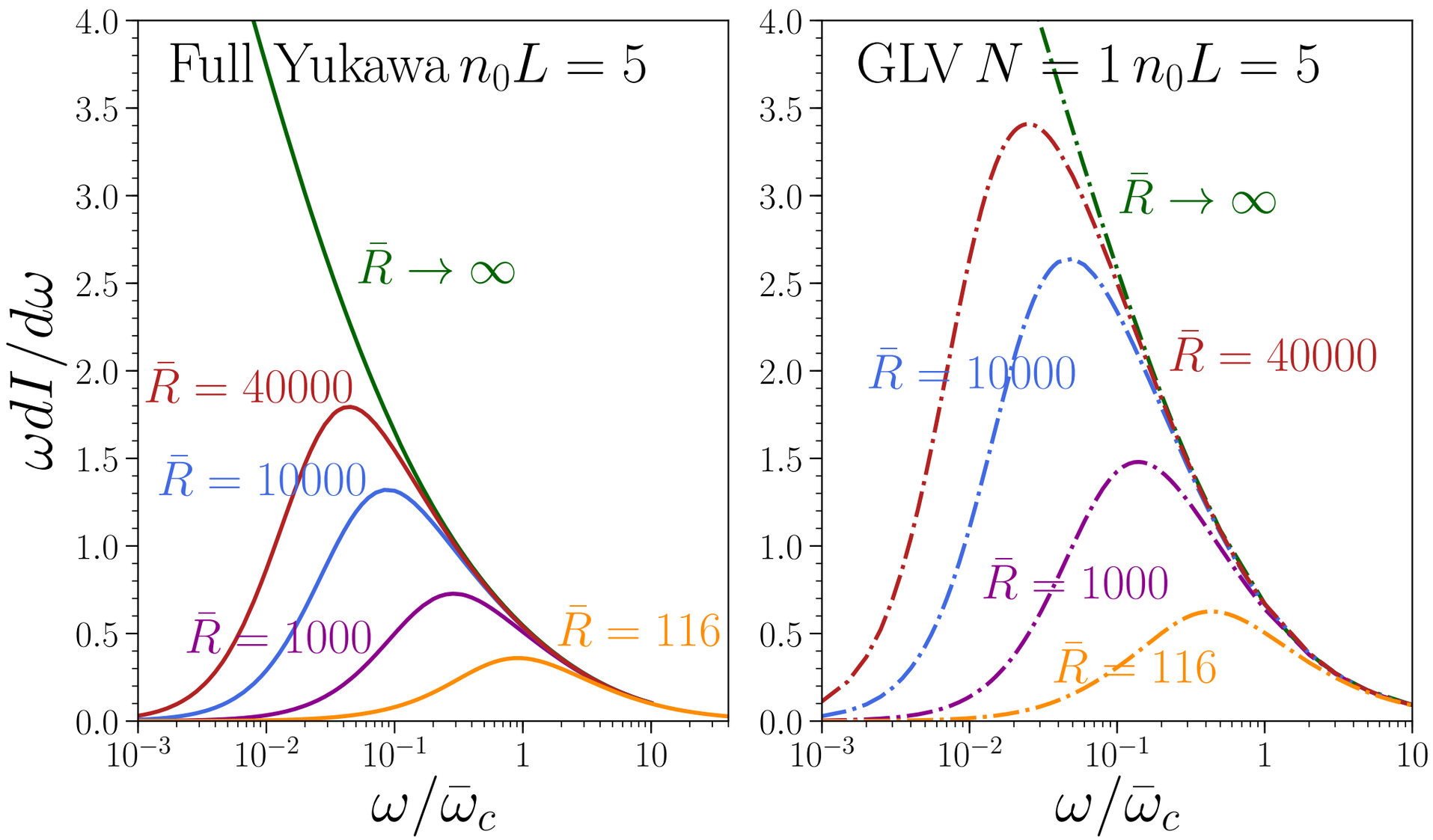
Parameters: T, L , μ

Yukawa-type interaction

Energy distribution

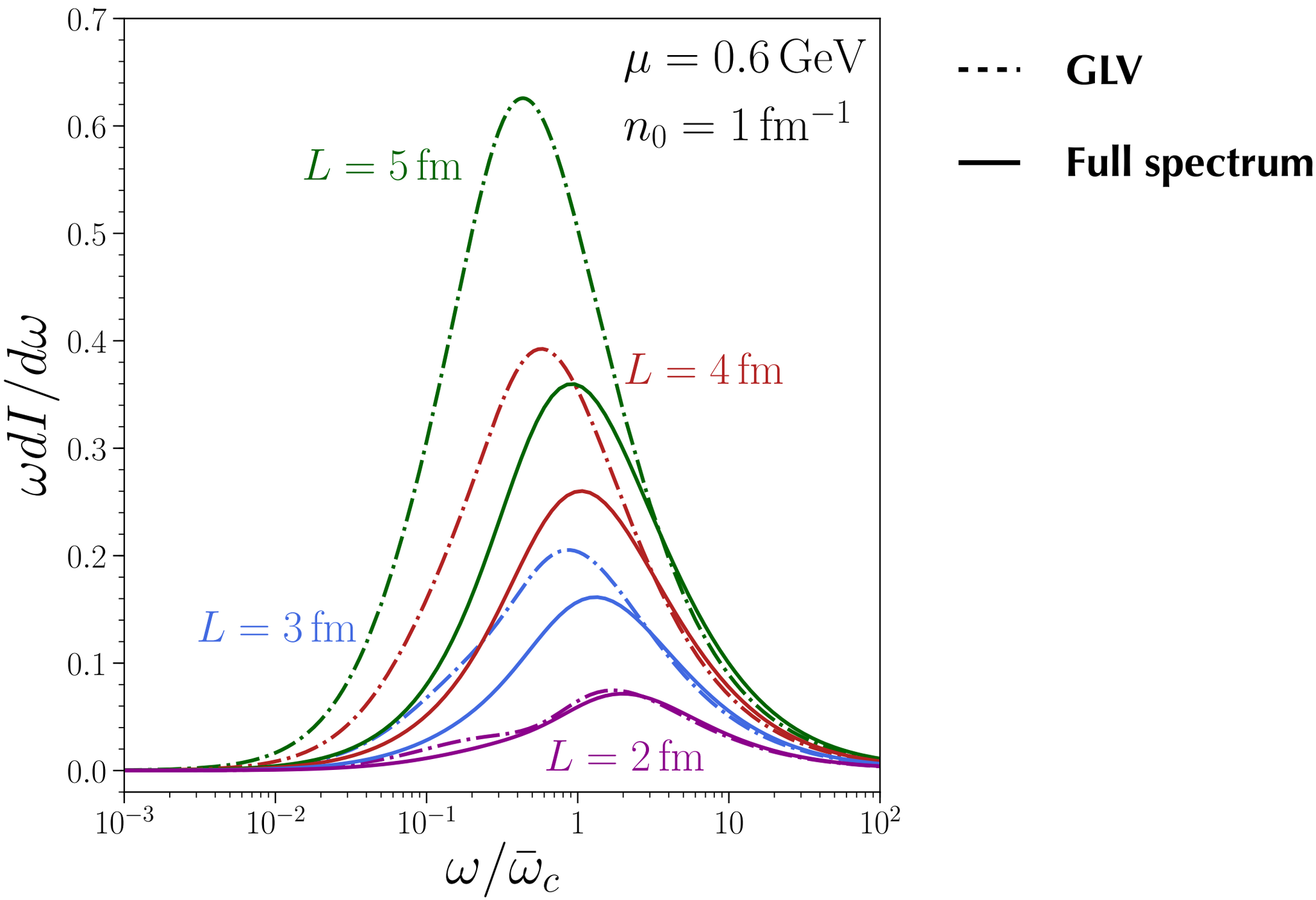
$$\bar{R} = \bar{\omega}_c L$$

$$\bar{\omega}_c = \frac{1}{2}\mu^2 L$$

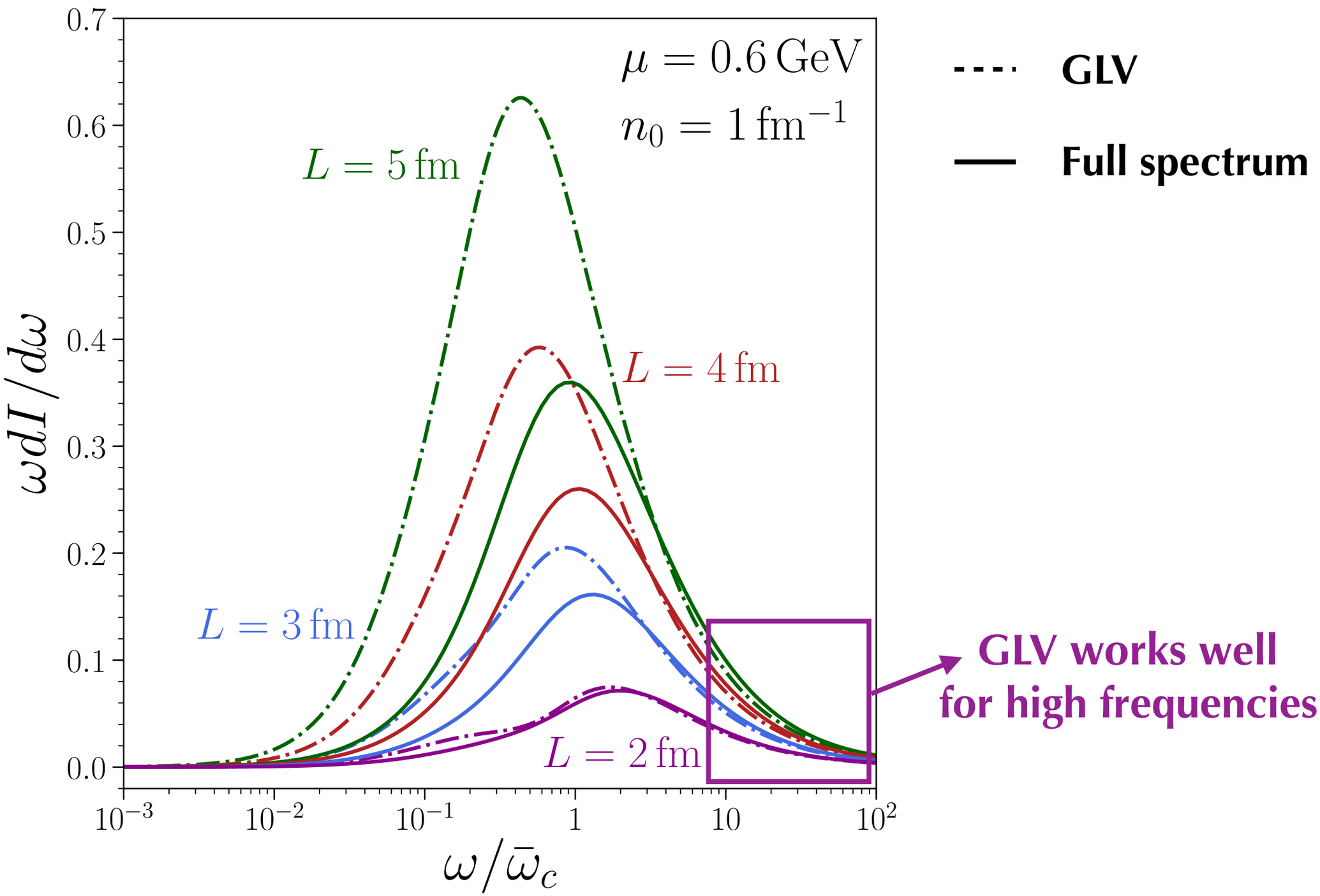


The GLV approximation underpredicts the spectrum

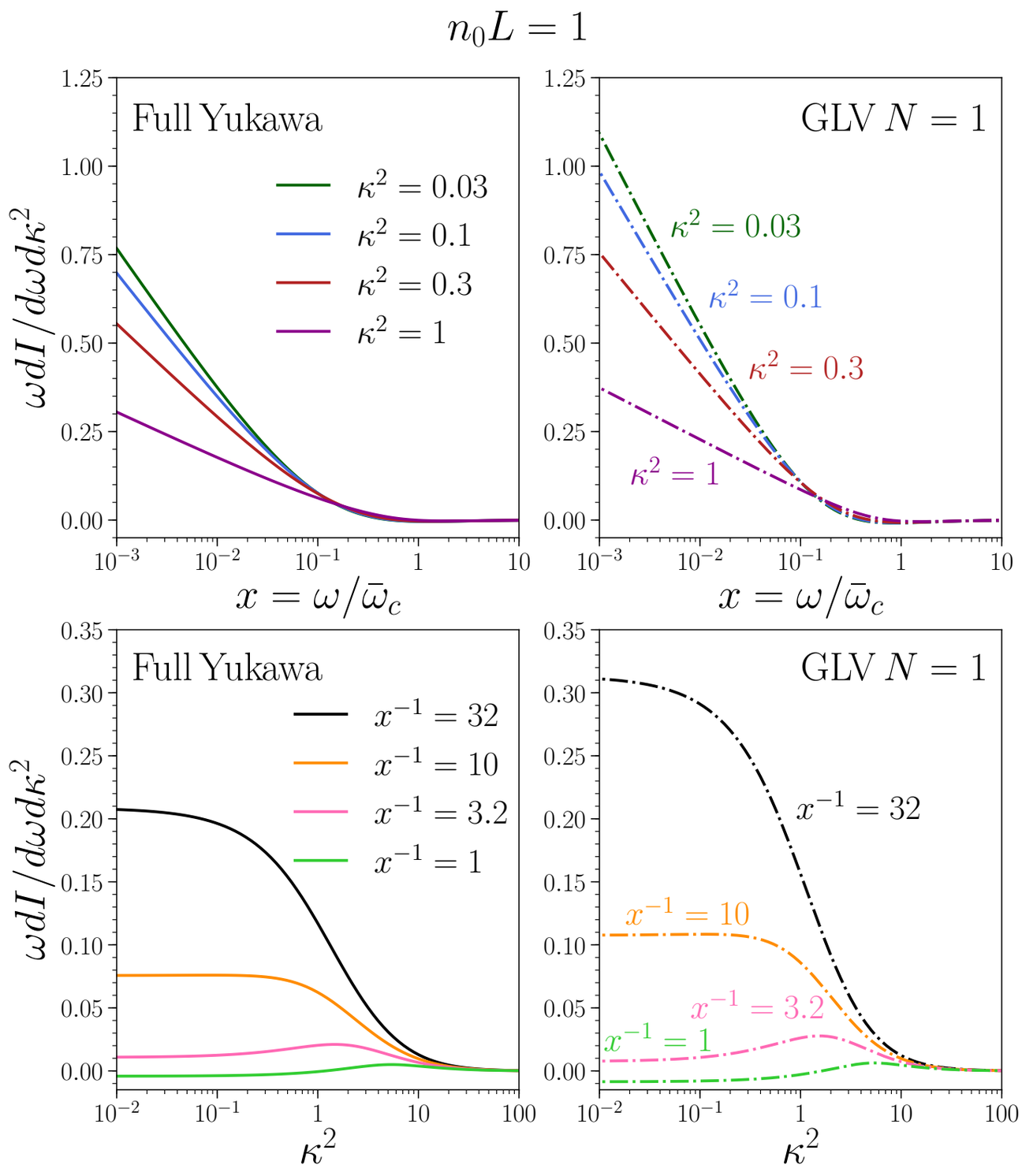
Energy distribution



Energy distribution



k_T-differential spectrum



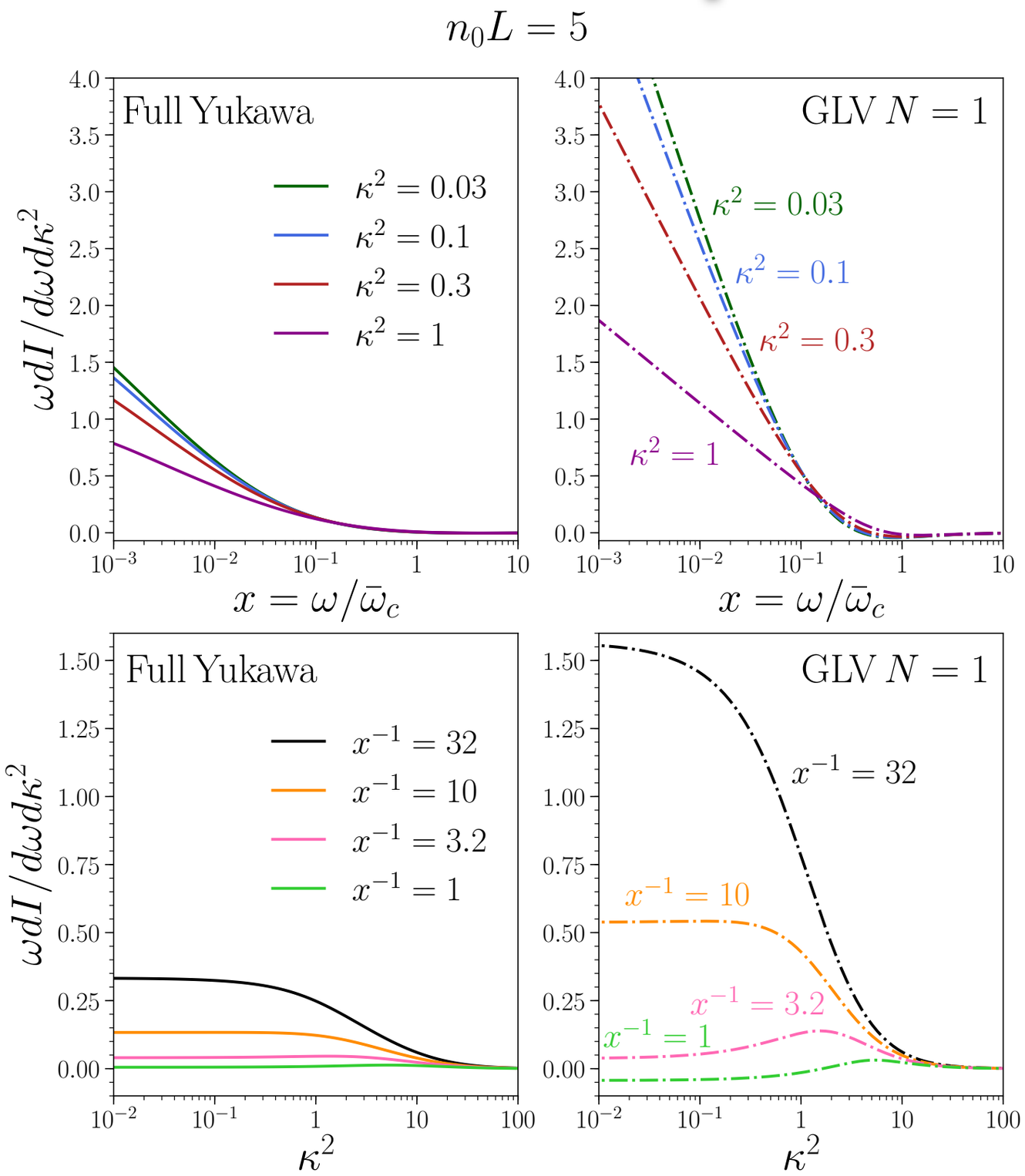
Discrepancies increase when decreasing ω

Discrepancies increase when decreasing κ^2

$$\bar{\omega}_c = \frac{1}{2}\mu^2 L$$

$$\kappa^2 = \frac{k^2}{\mu^2}$$

k_T-differential spectrum



Discrepancies larger

for $n_0 L = 5$

than for $n_0 L = 1$

$$\bar{\omega}_c = \frac{1}{2}\mu^2 L$$

$$\kappa^2 = \frac{k^2}{\mu^2}$$

Comparing with the HO

- The natural parameters of the HO are: $\hat{q}\,,\ L$

$$\left.\begin{aligned} n_0\,\sigma(\mathbf{r}) &= \frac{1}{2}\hat{\mathbf{q}}\,r^2 \\ \sigma(\mathbf{r}) &= \int_{\mathbf{q}} V(\mathbf{q})\,\left(1-e^{i\mathbf{q}\mathbf{r}}\right) \end{aligned}\right\} \quad \hat{q}L \approx \int_{\mathbf{q}} \frac{1}{2}\mathbf{q}^2\cos^2\phi V(\mathbf{q}) \sim (n_0 L)\,\mu^2 \ln\sqrt{\frac{q_{max}}{\mu}}$$

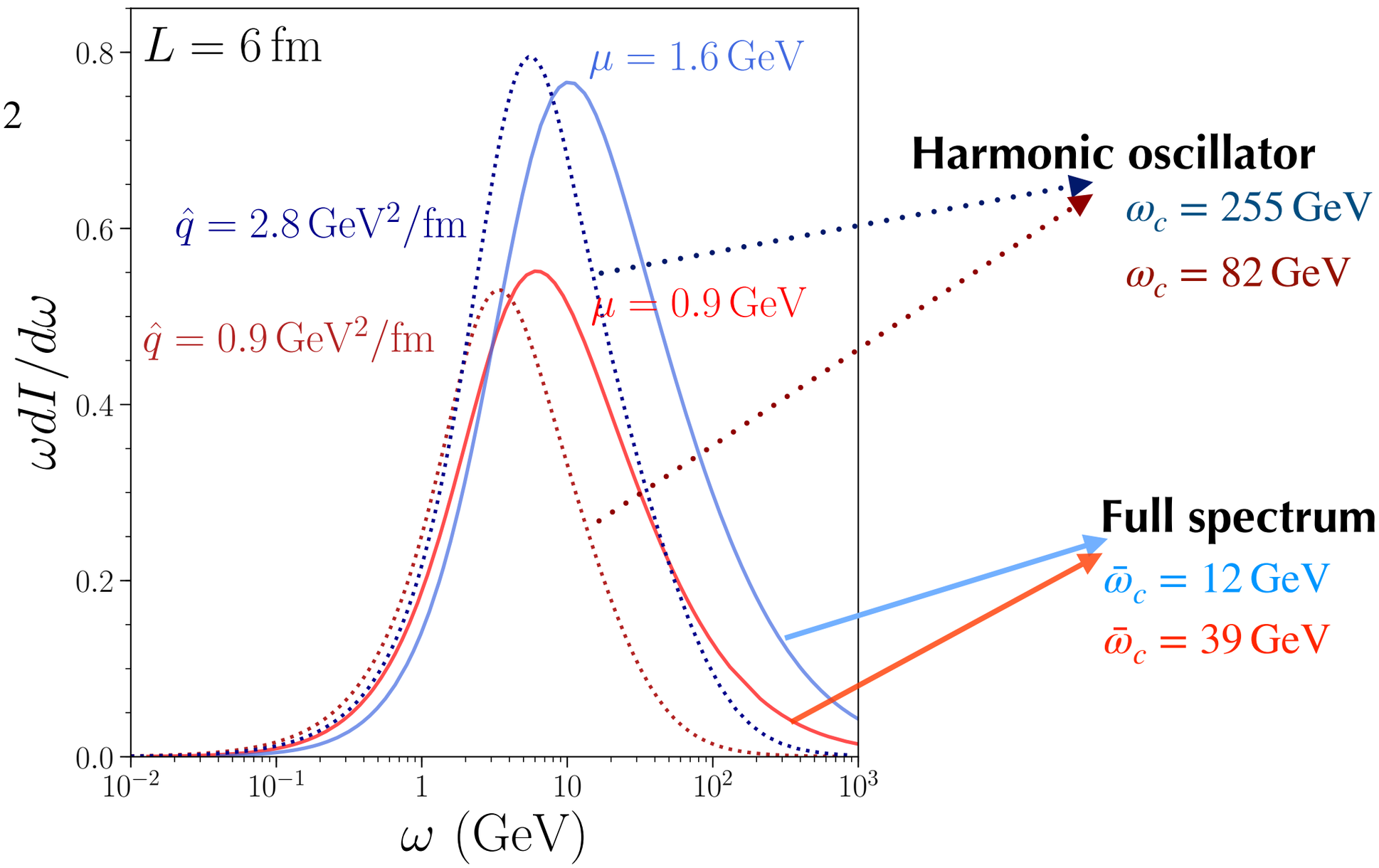
- We set $\ln\sqrt{\frac{q_{max}}{\mu}} = 1.3$

- This allows us to make **only qualitative comparisons** between the full-resumed spectrum and the HO approximation

Energy distribution

WARNING: Not a direct comparison!

$\hat{q}L = 1.3\,(n_0 L)\mu^2$

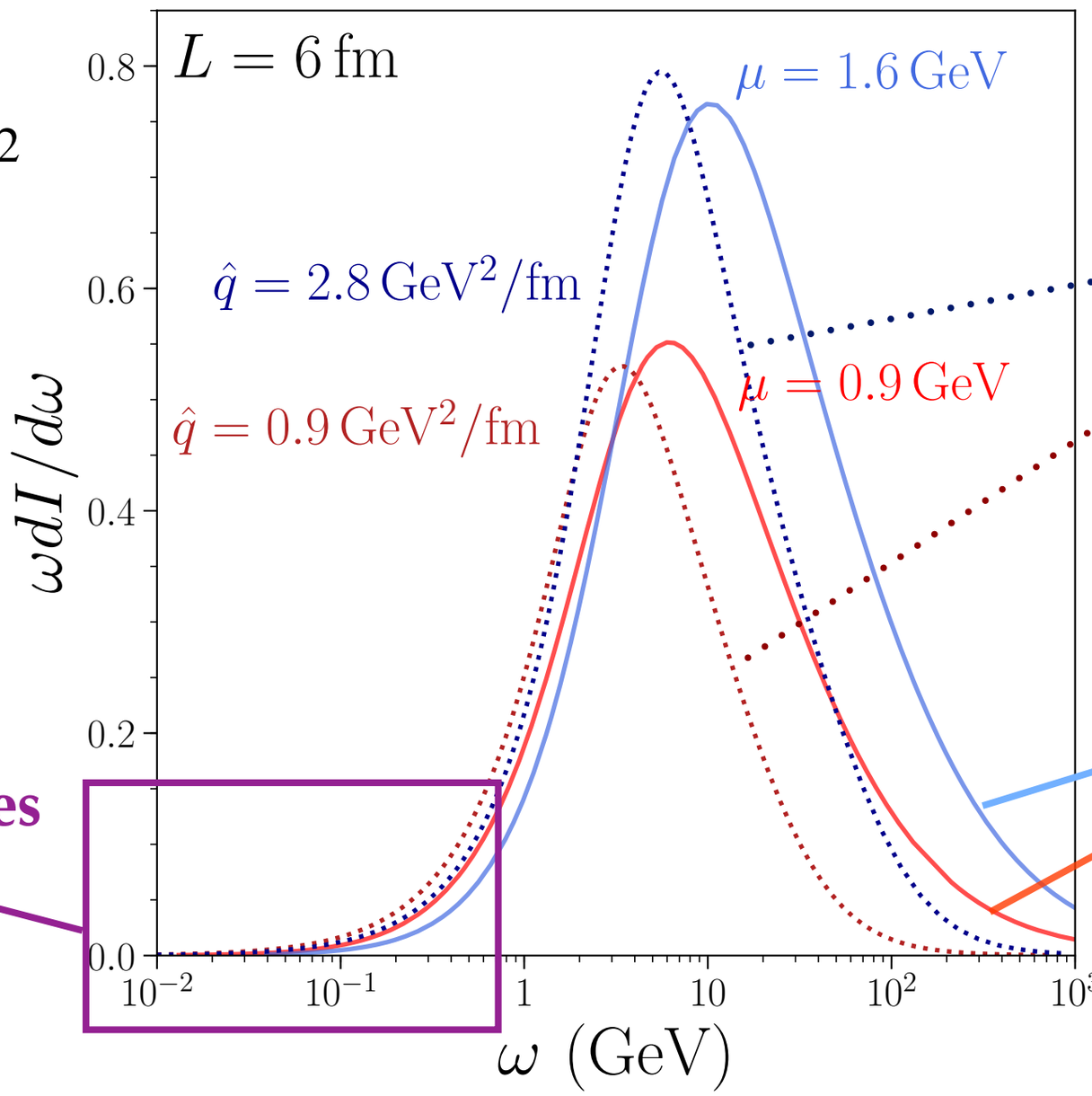


The full-resumed distribution is harder than the HO

Energy distribution

WARNING: Not a direct comparison!

$\hat{q}L = 1.3\,(n_0 L)\mu^2$



Harmonic oscillator

$\omega_c = 255\,\mathrm{GeV}$

$\omega_c = 82\,\mathrm{GeV}$

Full spectrum

$\bar{\omega}_c = 12\,\mathrm{GeV}$

$\bar{\omega}_c = 39\,\mathrm{GeV}$

HO works well for low frequencies

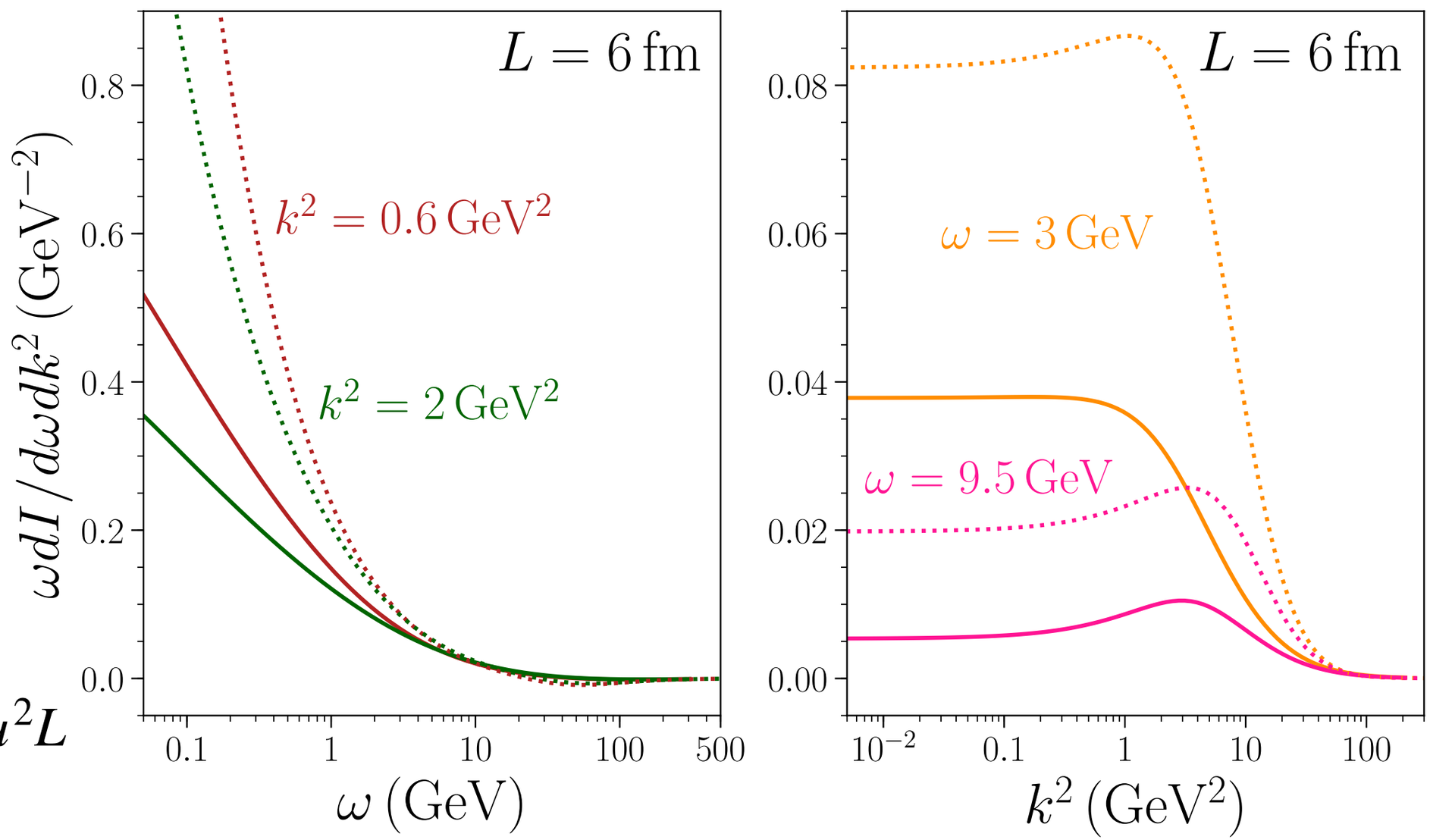
The full-resumed distribution is harder than the HO

k_T-differential spectrum

WARNING: Not a direct comparison!

···· **HO**

—— **Full spectrum**

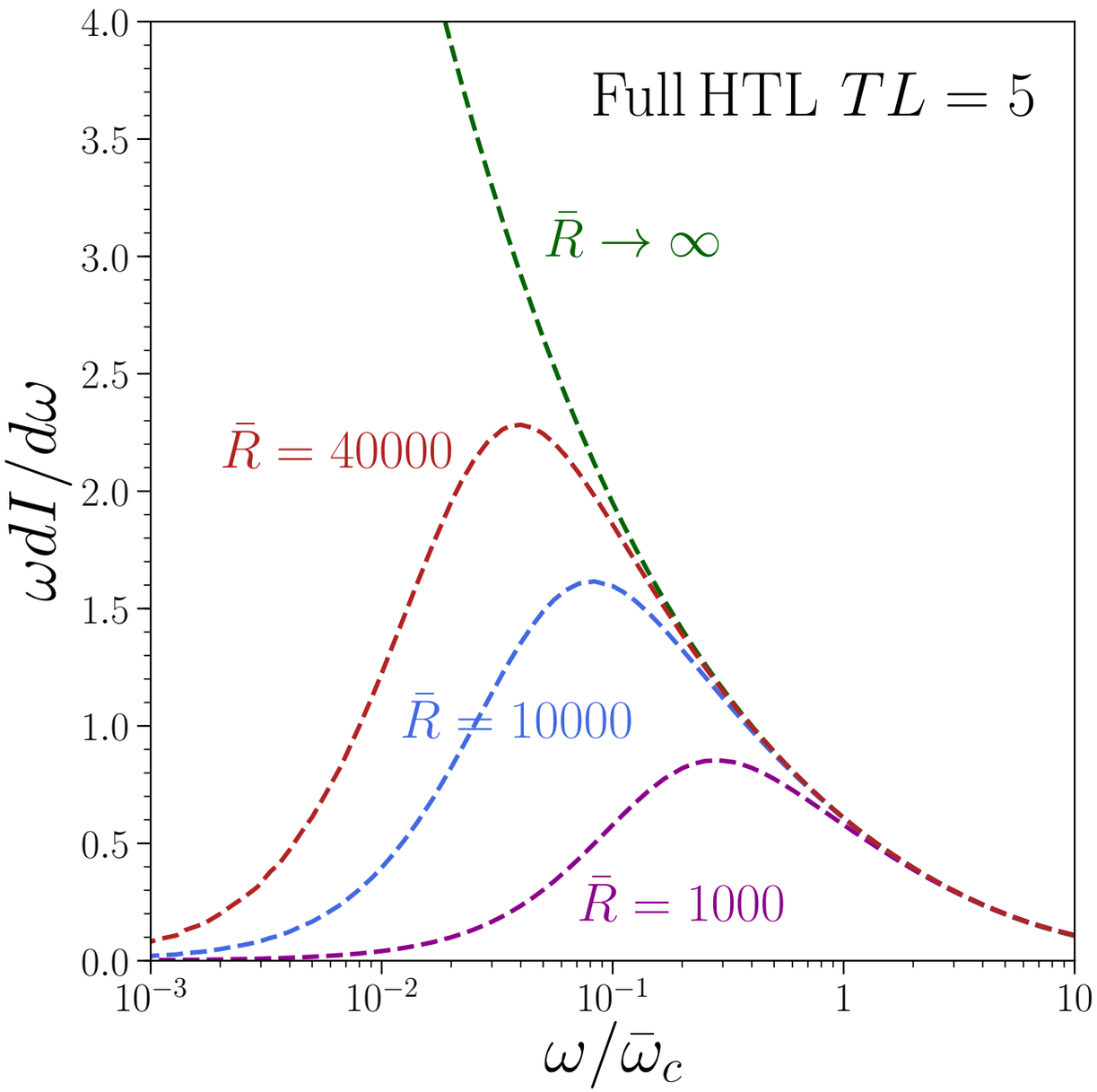


$$\bar{\omega}_c = \frac{1}{2}\mu^2 L$$

$$\kappa^2 = \frac{k^2}{\mu^2}$$

HTL-type interaction

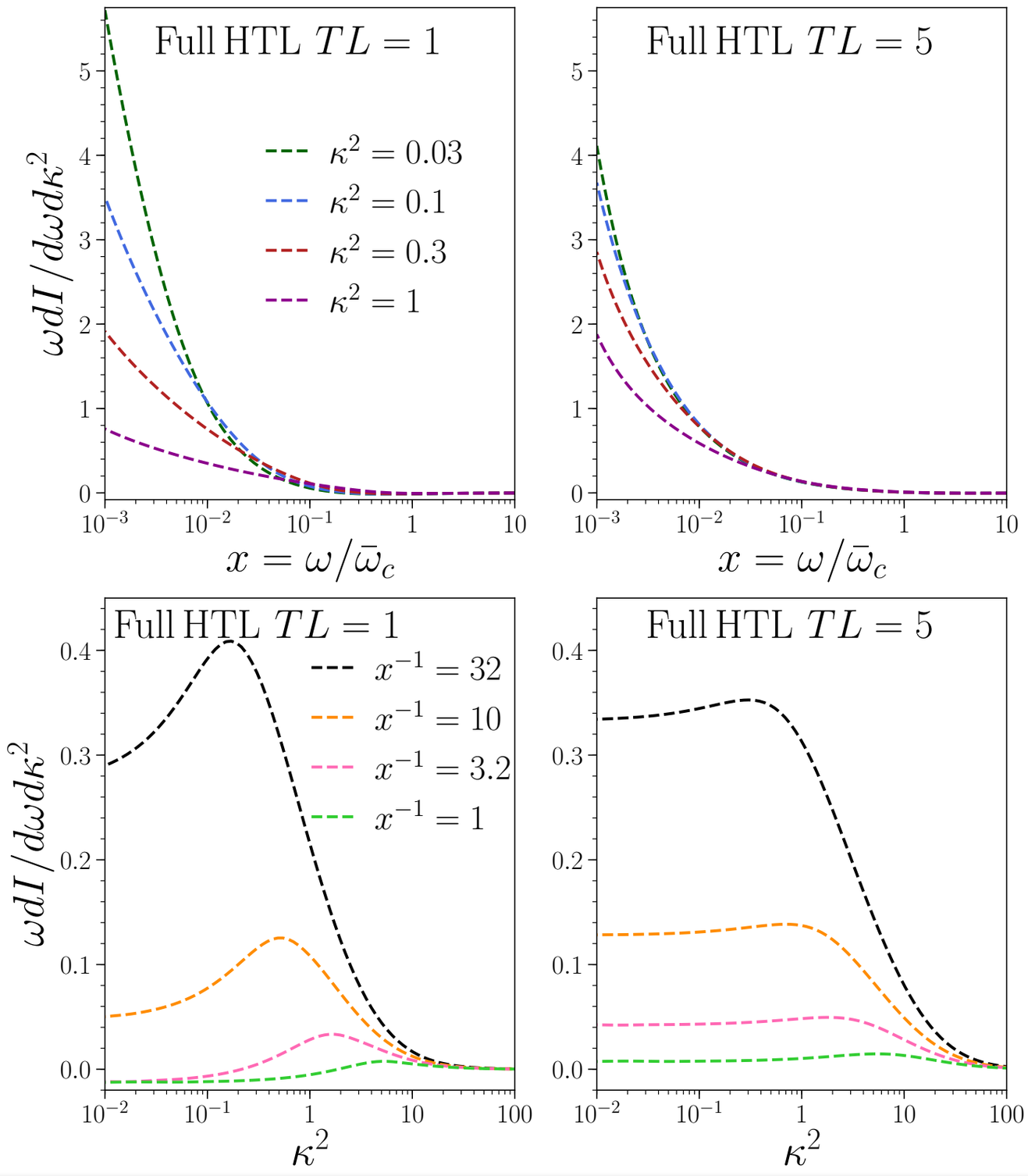
Energy distribution



$$\bar{R} = \bar{\omega}_c L$$

$$\bar{\omega}_c = \frac{1}{2}\mu^2 L$$

k_T-differential spectrum

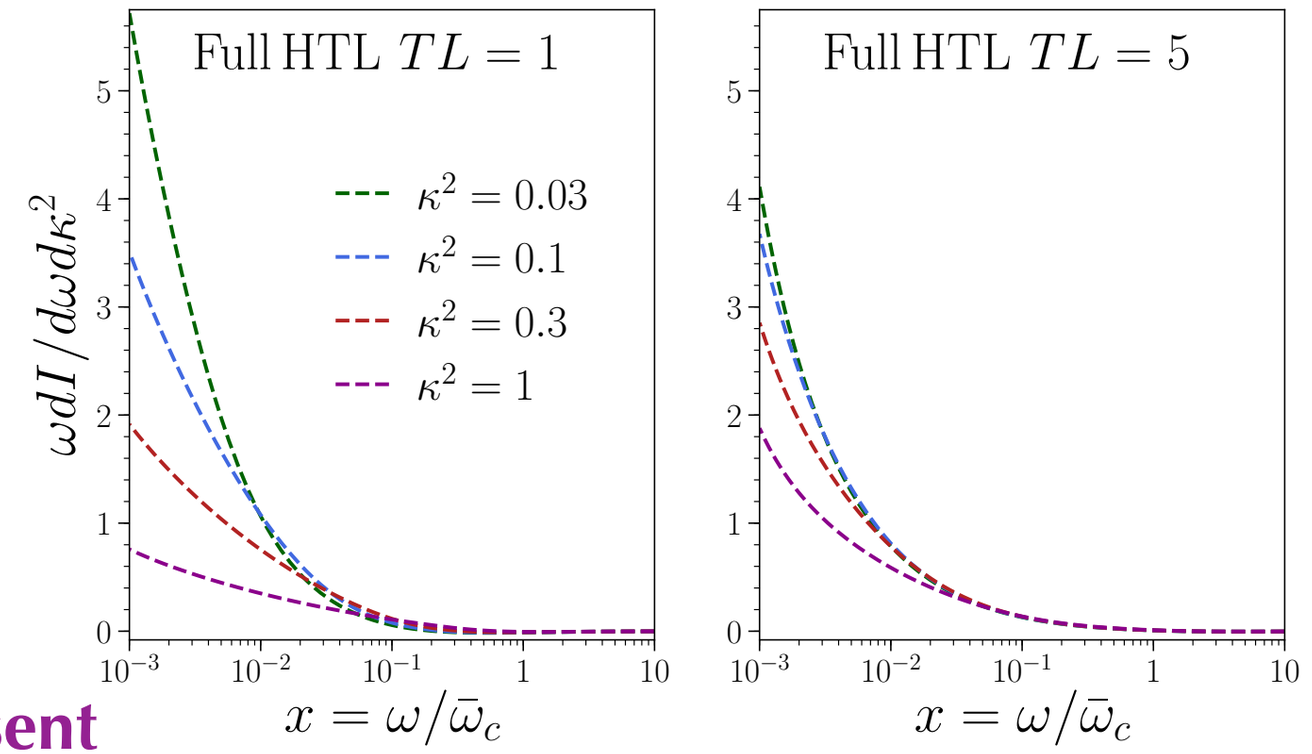


The discrepancies with respect to Yukawa decrease when increasing *TL*

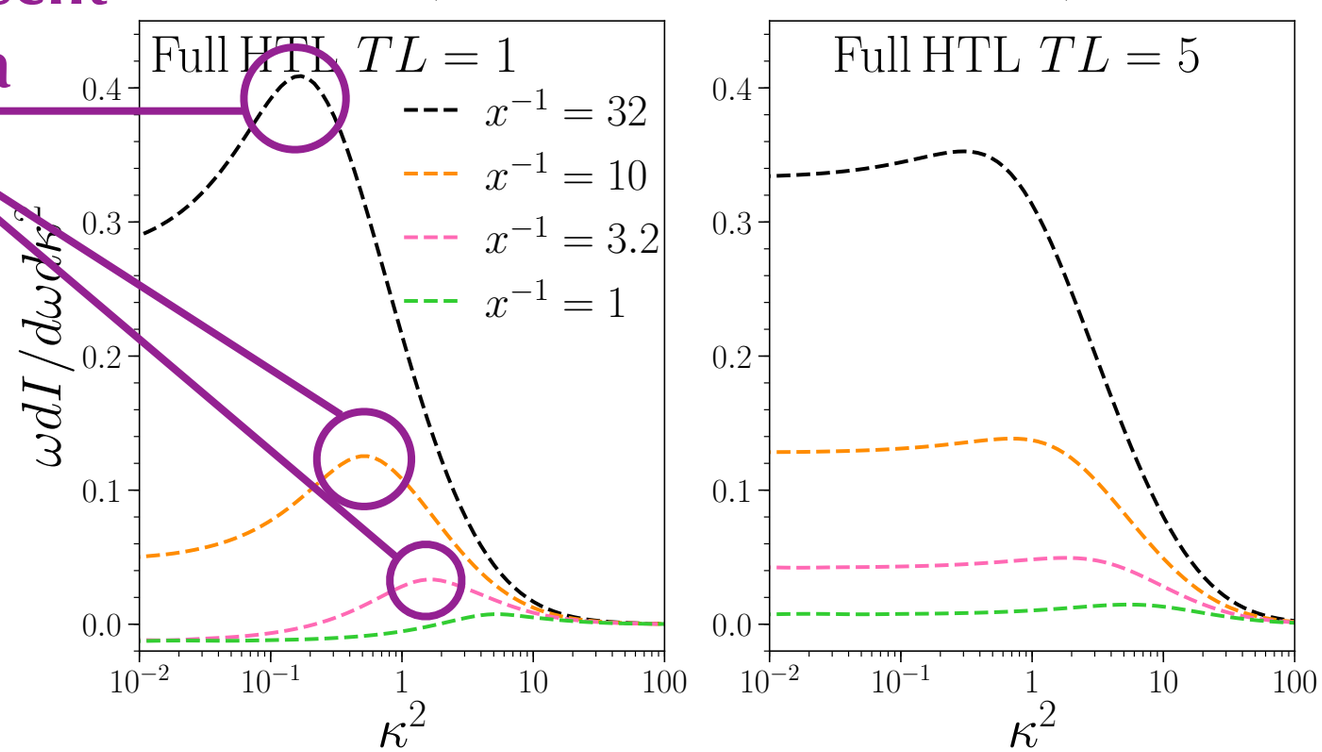
$$\bar{\omega}_c = \frac{1}{2}\mu^2 L$$

$$\kappa^2 = \frac{k^2}{\mu^2}$$

k_T-differential spectrum



Peaks not present for Yukawa



$$\bar{\omega}_c = \frac{1}{2}\mu^2 L$$

$$\kappa^2 = \frac{k^2}{\mu^2}$$

The discrepancies with respect to Yukawa decrease when increasing *TL*

Conclusions

- We derive an **analytical expression** for the medium-induced gluon radiation spectrum in the soft limit with **resummation of all multiple scatterings**

- By using Schwinger-Dyson equations, we can evaluate this spectrum for **realistic parton-medium interactions**

- Two interaction models analyzed: Yukawa and HTL

- The full-resummed spectrum is smaller than the GLV result

  GLV works well for large gluon energies and transverse momenta

- The HO goes faster to zero for large gluon energies than the full-resummed one

  HO is close to the full result for low gluon energies

Outlook

- Relaxing the soft gluon approximation

- Exploring non-static media

- Phenomenology: nuclear modification factor
  high-p_T asymmetries

- Radiation pattern of an antenna

Beyond the soft approximation

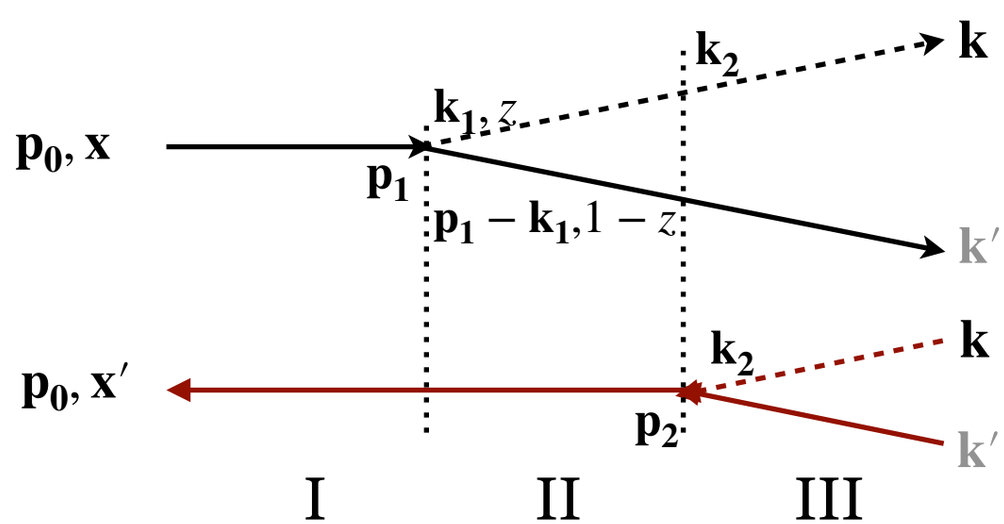
Beyond the soft approximation

- The emitted gluon carries a fraction of energy z of the energy of the parent quark

$$z\frac{dI}{dzd^2\boldsymbol{k}} = \frac{\alpha_s P_{g\leftarrow q}(z)}{(2\pi)^2 z(1-z)^2 E^2}\,\mathrm{Re}\int_0^\infty dt' \int_0^{t'} dt \int_{\boldsymbol{p}_1\boldsymbol{p}_2\boldsymbol{k}_1\boldsymbol{k}_2} \mathcal{P}_q(t,\boldsymbol{p}_1;0,\boldsymbol{p}_0)$$
$$\times\,(\boldsymbol{k}_1 - z\boldsymbol{p}_1)\cdot(\boldsymbol{k}_2 - z\boldsymbol{p}_2)\;\widetilde{\mathcal{K}}^{(3)}(t',\boldsymbol{k}_2 - z\boldsymbol{p}_2;t,\boldsymbol{k}_1 - z\boldsymbol{p}_1;\boldsymbol{p}_2 - \boldsymbol{p}_1)$$
$$\times\,\mathcal{P}_g(\infty,\boldsymbol{k};t',\boldsymbol{k}_2)$$

Blaizot, Dominguez, Iancu, Mehtar-Tani arXiv:1209.4585

Apolinario, Armesto, Milhano, Salgado arXiv:1407.0599



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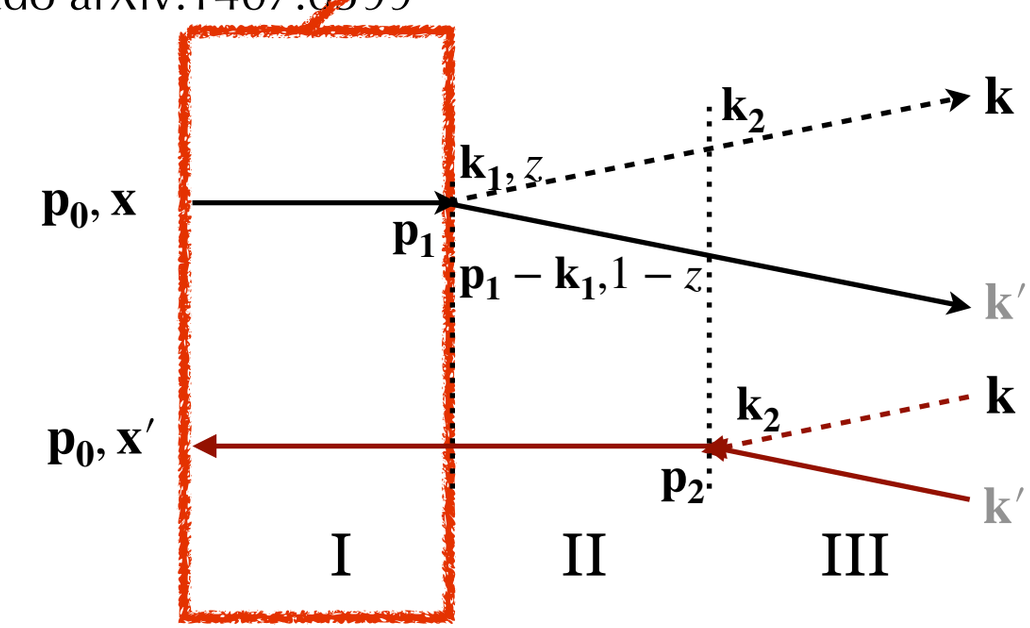

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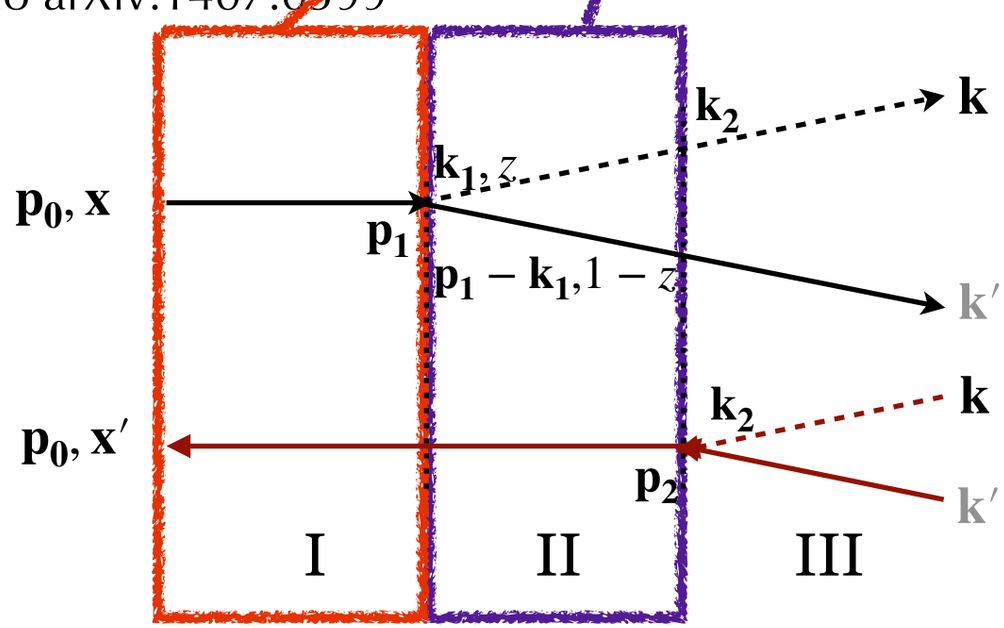

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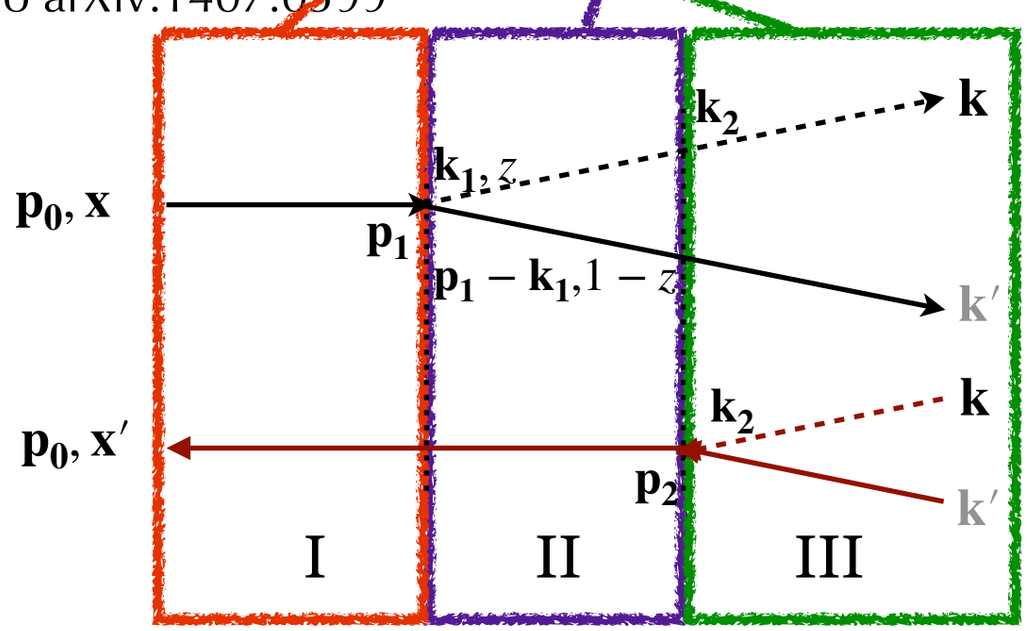

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$$\times\,\mathcal{P}_g(\infty,\boldsymbol{k};t',\boldsymbol{k}_2)$$

- The emission kernel is

$$\widetilde{\mathcal{K}}^{(3)}(t_2,\boldsymbol{q}_2;t_1,\boldsymbol{q}_1;\boldsymbol{l}) = \int_{\boldsymbol{u}_1\boldsymbol{u}_2\boldsymbol{v}} e^{-i(\boldsymbol{u}_2\cdot\boldsymbol{q}_2 - \boldsymbol{u}_1\cdot\boldsymbol{q}_1 + \boldsymbol{v}\cdot\boldsymbol{l})}$$
$$\times\int_{\boldsymbol{u}(t_1)=\boldsymbol{u}_1}^{\boldsymbol{u}(t_2)=\boldsymbol{u}_2}\mathcal{D}\boldsymbol{u}\exp\left\{\frac{iz(1-z)E}{2}\int_{t_1}^{t_2} d\xi\,\dot{\boldsymbol{u}}^2 - \frac{1}{2}\int_{t_1}^{t_2} d\xi\, n(\xi)\sigma_3(\boldsymbol{u}(\xi),\boldsymbol{v};z)\right\}$$

$$\sigma_3(\boldsymbol{u},\boldsymbol{v};z) = \frac{1}{2}\sigma(\boldsymbol{u}) + \frac{1}{2}\sigma((1-z)\boldsymbol{u}+\boldsymbol{v}) - \frac{1}{2N_c^2}\sigma(-z\boldsymbol{u}+\boldsymbol{v}).$$

Beyond the soft approximation

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$$\widetilde{\mathcal{K}}^{(3)}(t_2,\boldsymbol{q}_2;t_1,\boldsymbol{q}_1;\boldsymbol{l}) = (2\pi)^4\delta^{(2)}(\boldsymbol{l})\delta^{(2)}(\boldsymbol{q}_2-\boldsymbol{q}_1)e^{-i\frac{\boldsymbol{q}_2^2}{2z(1-z)E}(t_2-t_1)}$$
$$-\frac{1}{2}\int_{t_1}^{t_2} ds\, n(s)\, e^{-i\frac{\boldsymbol{q}_2^2}{2z(1-z)E}(t_2-s)}\int_{\boldsymbol{q}'\boldsymbol{l}'}\tilde{\sigma}_3(\boldsymbol{q}_2-\boldsymbol{q}',\boldsymbol{l}-\boldsymbol{l}';z)\widetilde{\mathcal{K}}^{(3)}(s,\boldsymbol{q}';t_1,\boldsymbol{q}_1;\boldsymbol{l}')$$

$$\tilde{\sigma}_3(\boldsymbol{q},\boldsymbol{l};z) = \int_{\boldsymbol{u}\boldsymbol{v}} e^{-i(\boldsymbol{u}\cdot\boldsymbol{q}+\boldsymbol{v}\cdot\boldsymbol{l})}\sigma_3(\boldsymbol{u},\boldsymbol{v};z)$$
$$= (2\pi)^2\frac{1}{2}\left[\delta^{(2)}(\boldsymbol{l})\sigma(\boldsymbol{q}) + \delta^{(2)}((1-z)\boldsymbol{l}-\boldsymbol{q})\sigma(\boldsymbol{l}) - \frac{1}{N_c^2}\delta^{(2)}(\boldsymbol{q}+z\boldsymbol{l})\sigma(\boldsymbol{l})\right]$$

Energy spectrum without kinematic constraint

- Integrating in $\boldsymbol{k}$ over the full transverse plane ($\bar{R} \to \infty$)

$$z\frac{dI}{dz} = \frac{\alpha_s P_{g\leftarrow q}(z)}{(1-z)E}\,\mathrm{Re}\int_0^L ds\, n(s)\int_0^s dt\int_{\boldsymbol{q}_1\boldsymbol{q}_2\boldsymbol{l}} i\boldsymbol{q}_1\cdot\left(\frac{\boldsymbol{l}}{\boldsymbol{l}^2}-\frac{\boldsymbol{q}_2}{\boldsymbol{q}_2^2}\right)$$
$$\times\ \tilde{\sigma}'(\boldsymbol{l}-\boldsymbol{q}_2;z)\,\widetilde{\mathcal{K}}'(s,\boldsymbol{q}_2;t,\boldsymbol{q}_1)$$

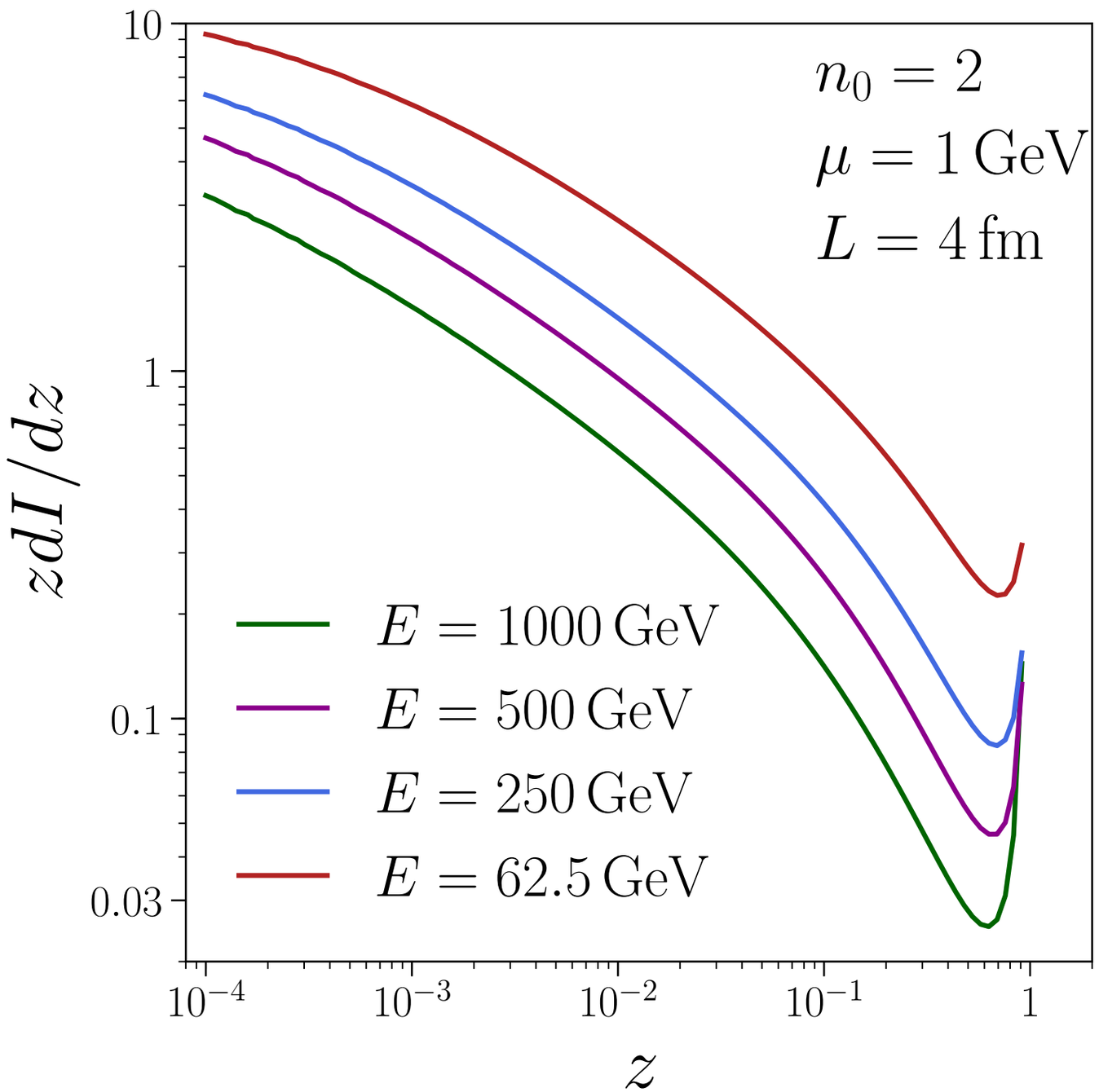
- The emission kernel is

$$\widetilde{\mathcal{K}}'(s,\boldsymbol{q}_2;t,\boldsymbol{q}_1) = \int_{\boldsymbol{l}}\widetilde{\mathcal{K}}^{(3)}(s,\boldsymbol{q}_2;t,\boldsymbol{q}_1;\boldsymbol{l}) = \int_{\boldsymbol{u}_1\boldsymbol{u}_2} e^{-i(\boldsymbol{u}_2\cdot\boldsymbol{q}_2-\boldsymbol{u}_1\cdot\boldsymbol{q}_1)}$$
$$\times\int_{\boldsymbol{u}(t)=\boldsymbol{u}_1}^{\boldsymbol{u}(s)=\boldsymbol{u}_2}\mathcal{D}\boldsymbol{u}\exp\left\{\frac{iz(1-z)E}{2}\int_t^s d\xi\,\dot{\boldsymbol{u}}^2-\frac{1}{2}\int_t^s d\xi\, n(\xi)\sigma_3(\boldsymbol{u}(\xi),0;z)\right\}$$

$$\tilde{\sigma}'(\boldsymbol{q};z) = \frac{1}{2}\sigma(\boldsymbol{q})+\frac{1}{2(1-z)^2}\sigma\left(\frac{\boldsymbol{q}}{1-z}\right)-\frac{1}{2N_c^2z^2}\sigma\left(\frac{\boldsymbol{q}}{z}\right)$$

Results

$\bar{R} \to \infty$



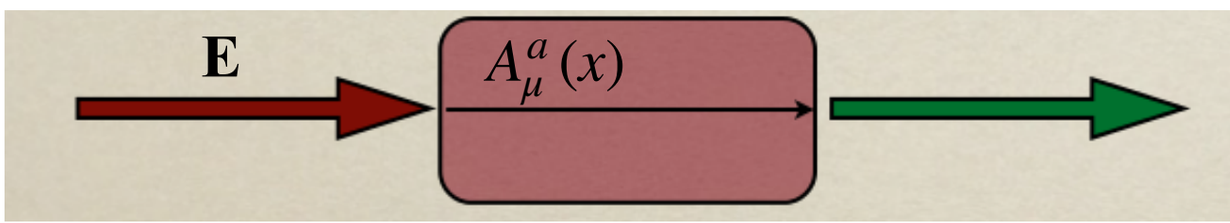
Thanks!

Eikonal and beyond

QCD vertices unmodified. Propagators modified

Color rotation

$E \to \infty$



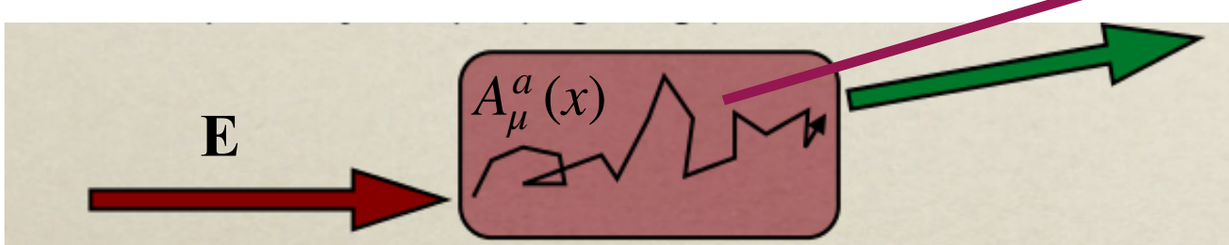
$E \gg p_T \gg \mu$

Wilson line

$$W(\mathbf{x}_\perp) = \mathscr{P} \exp\left\{ ig \int dx_+ \, A_-(x_+, \mathbf{x}_\perp) \right\}$$

Color rotation + p_T-broadening

Brownian motion



$E > p_T \gg \mu$

Green function

$$G(t_2, \mathbf{x}_{2\perp}; t_1, \mathbf{x}_{1\perp} | E) = \mathscr{P} \int \mathscr{D}\mathbf{r}_\perp(x_+) \exp\left\{ i\frac{E}{2} \int dx_+ \left[\frac{d\mathbf{r}_\perp}{dx_+}\right]^2 + ig \int dx_+ \, A_-(x_+, \mathbf{x}_\perp) \right\}$$

GLV spectrum

- With kinematic constraint

$$\omega \frac{dI^{N=1}}{d\omega} = 2\frac{\alpha_s C_R}{\pi}(n_0 L)\,\gamma \int_0^\infty dr\, \frac{r - sin(r)}{r^2} \times \left(\frac{1}{r+\gamma} - \frac{1}{\sqrt{((\bar{R}/2\gamma) + r + \gamma)^2 - 4r\bar{R}/2\gamma}} \right)$$

- No kinematic constraint

$$\lim_{\bar{R}\to\infty} \omega \frac{dI^{N=1}}{d\omega} = 2\,\frac{\alpha_s C_R}{\pi}\,(n_0\,L)\,\gamma \int_0^\infty dr\, \frac{1}{r+\gamma}\,\frac{r - sin(r)}{r^2}$$

$$\simeq 2\,\frac{\alpha_s C_R}{\pi}\,(n_0\,L) \begin{cases} \log\left[\frac{\bar{\omega}_c}{\omega}\right] & \text{for} \quad \bar{\omega}_c > \omega \\ \frac{\pi}{4}\,\frac{\bar{\omega}_c}{\omega} & \text{for} \quad \bar{\omega}_c < \omega \end{cases}$$

HO spectrum

- No kinematic constraint

$$\lim_{R\to\infty} \omega \frac{dI}{d\omega} = \frac{2\alpha_s C_R}{\pi} \ln \left| \cos\left[(1+i)\sqrt{\frac{\omega_c}{2\omega}} \right] \right| .$$

$$\omega_c = \frac{1}{2}\hat{q}L^2$$

$$R = \omega_c L$$

$$\lim_{R\to\infty} \omega \frac{dI}{d\omega} \simeq \frac{2\alpha_s C_R}{\pi} \begin{cases} \sqrt{\frac{\omega_c}{2\,\omega}} & \text{for} \quad \omega < \omega_c \\ \frac{1}{12}\left(\frac{\omega_c}{\omega}\right)^2 & \text{for} \quad \omega > \omega_c \end{cases}$$