Generalized Parton Distributions and partonic contributions to the QCD energy momentum tensor

March 11, 2019
Jefferson Laboratory

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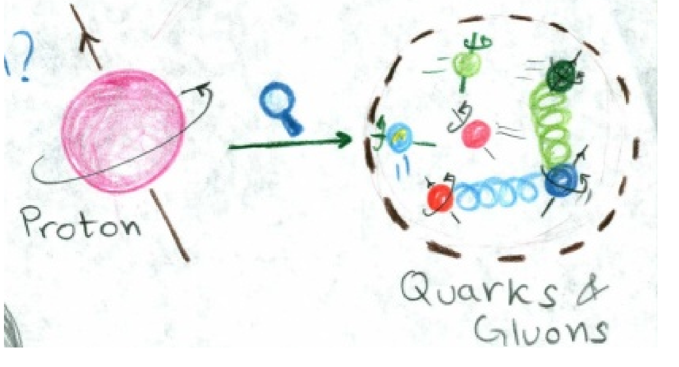


Collaborators

- Simonetta Liuti (University of Virginia)
- Michael Engelhardt (New Mexico State University)
- Aurore Courtoy (Mexico University)

Orbital Angular Momentum

- Simonetta Liuti (University of Virginia)
- Kent Yagi (University of Virginia)

Equation of State Neutron stars

- Yoshitaka Hatta (BNL, Kyoto University)
- Kazuhiro Tanaka (Juntendo University, KEK)
- Di-Lun Yang (Nishina Center, RIKEN)

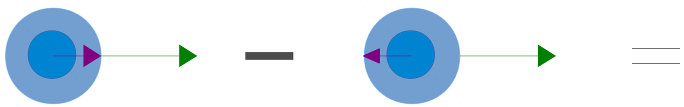
Trace Anomaly and Proton Mass

Thanks!!

Outline

- Partonic Orbital Angular Momentum
- Wandzura Wilczek and genuine twist three contributions to twist three GPDs
- QCD trace anomaly and the mass of the proton
- Measuring these in experiment
- The equation of state of neutron stars at short distances

Proton Spin Crisis



$g_1^P(x)$ Quark Spin Contribution

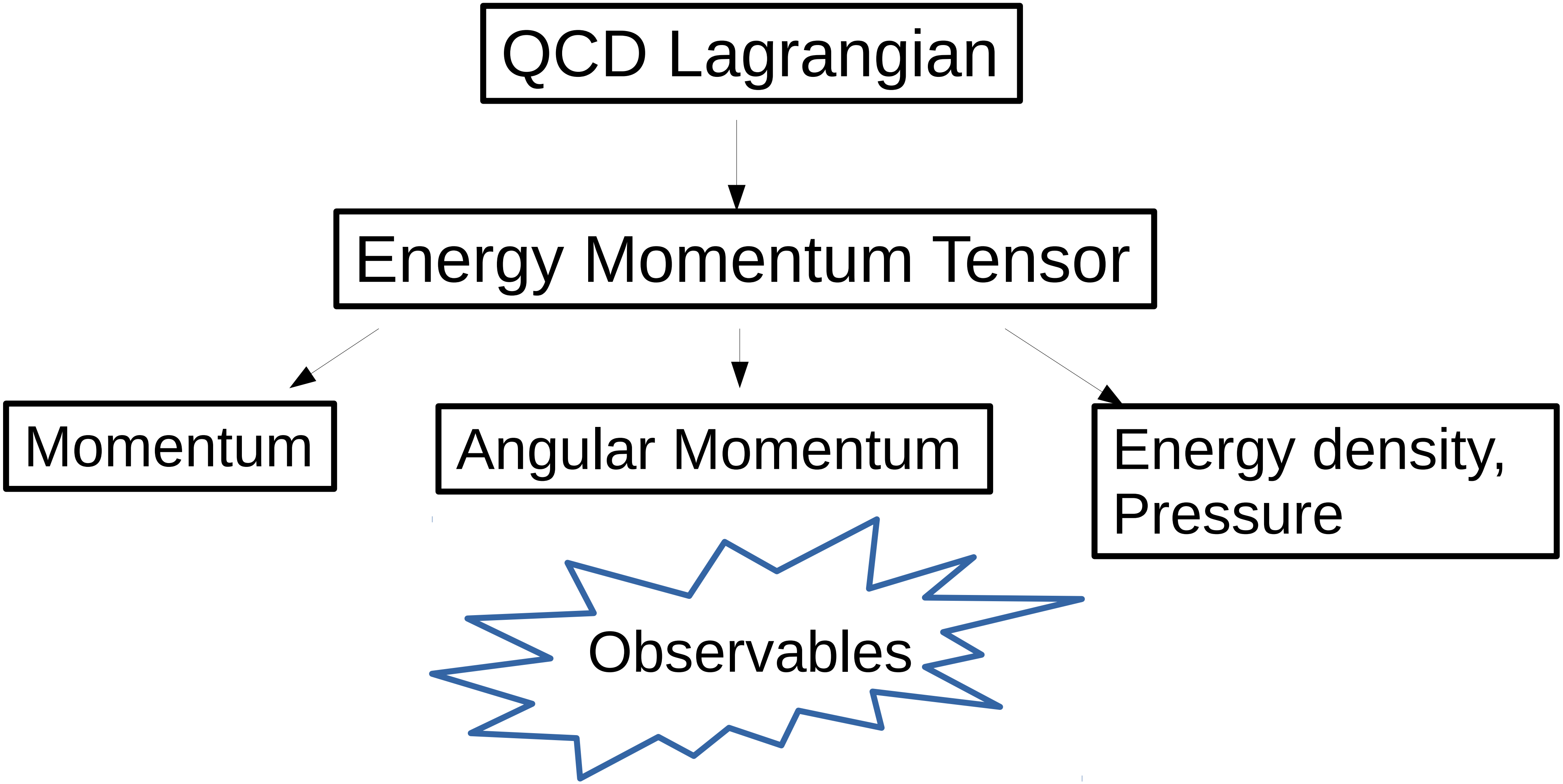
$$\frac{1}{2M}\int_{-\infty}^{\infty}\frac{d\lambda}{2\pi}e^{-i\lambda x}\left\langle P,S\left|\bar{\psi}\left(\frac{\lambda n}{2}\right)\gamma_\mu\gamma_5\psi\left(-\frac{\lambda n}{2}\right)\right|P,S\right\rangle = \Lambda g_1(x)p_\mu + g_T(x)S_{\perp_\mu}$$

Measured by EMC experiment in 1980s to be small, present values about 30% of total !!

What are other sources ?

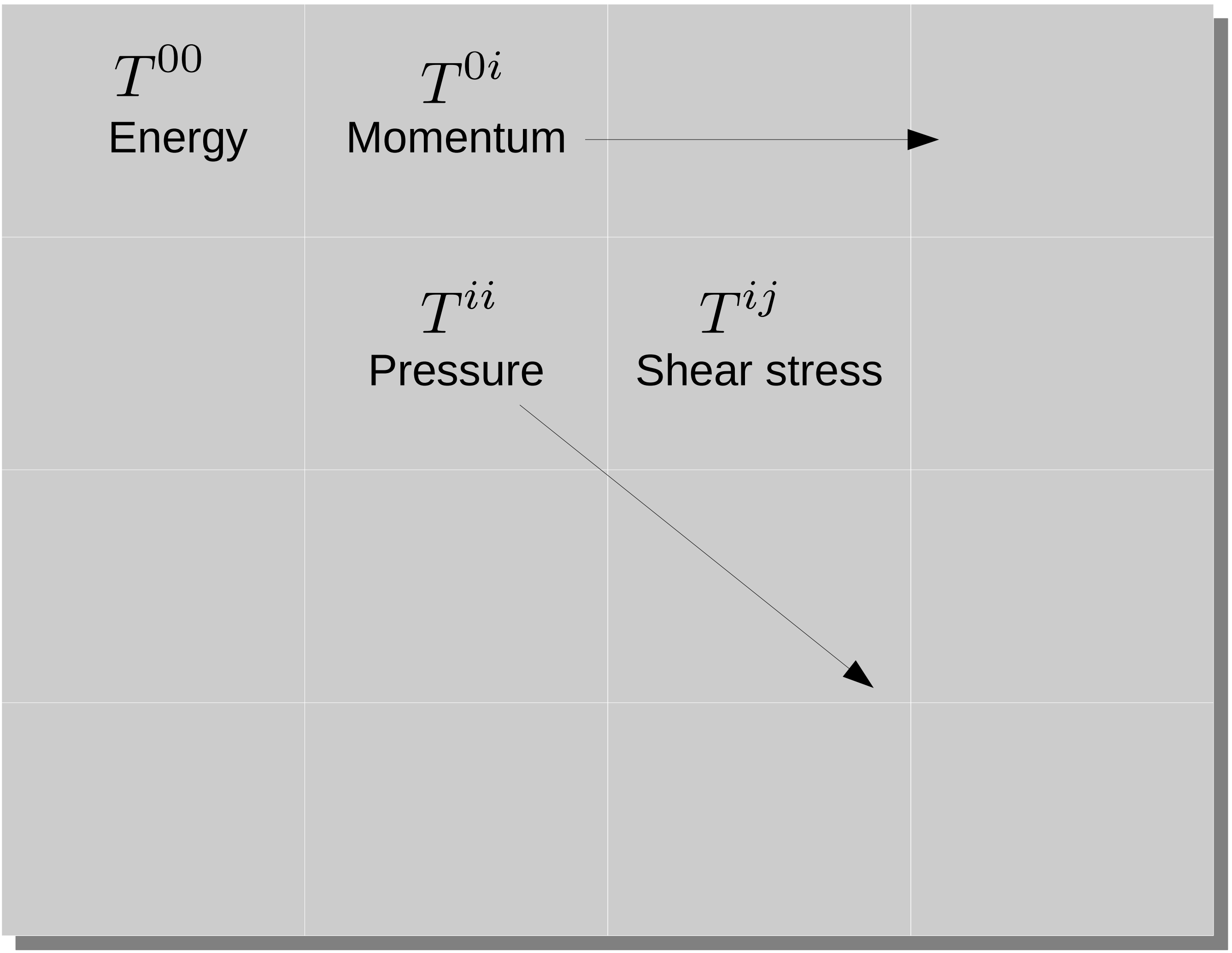
Partonic Orbital Angular Momentum

QCD Energy Momentum Tensor



Deeply Virtual Compton Scattering, moments of GPDs etc.

QCD Energy Momentum Tensor



GPD based definition of Angular Momentum

$$J^i_{q,g} = \frac{1}{2}\epsilon^{ijk}\int d^3x \left(T^{0k}_{q,g}x^j - T^{0j}_{q,g}x^k\right)$$

$$\vec{J}_q = \int d^3x\psi^\dagger\left[\vec{\gamma}\gamma_5 + \vec{x}\times i\vec{D}\right]\psi \qquad \vec{J}_g = \int d^3x\left(\vec{x}\times\left(\vec{E}\times\vec{B}\right)\right)$$

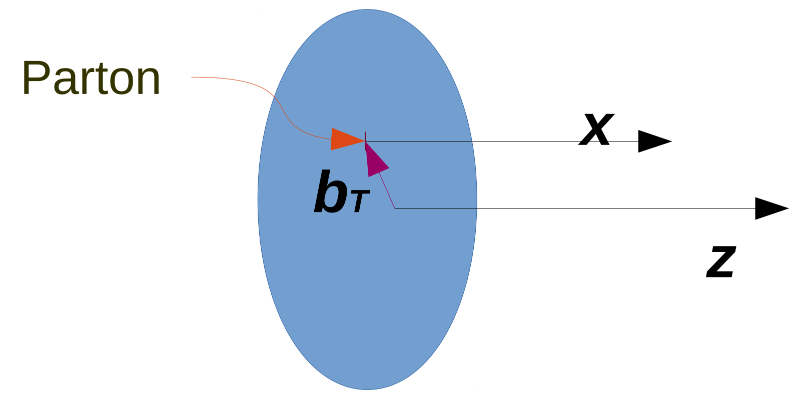
$$J_q = \frac{1}{2}\int dx x(H_q(x,0,0) + E_q(x,0,0))$$

Xiangdong Ji, PRL 78.610,1997

To access OAM, we take the difference between total angular momentum and spin

$$\mathcal{L}_q = J_q - \frac{1}{2}\Delta\Sigma$$

OAM Total Spin



Direct description of OAM

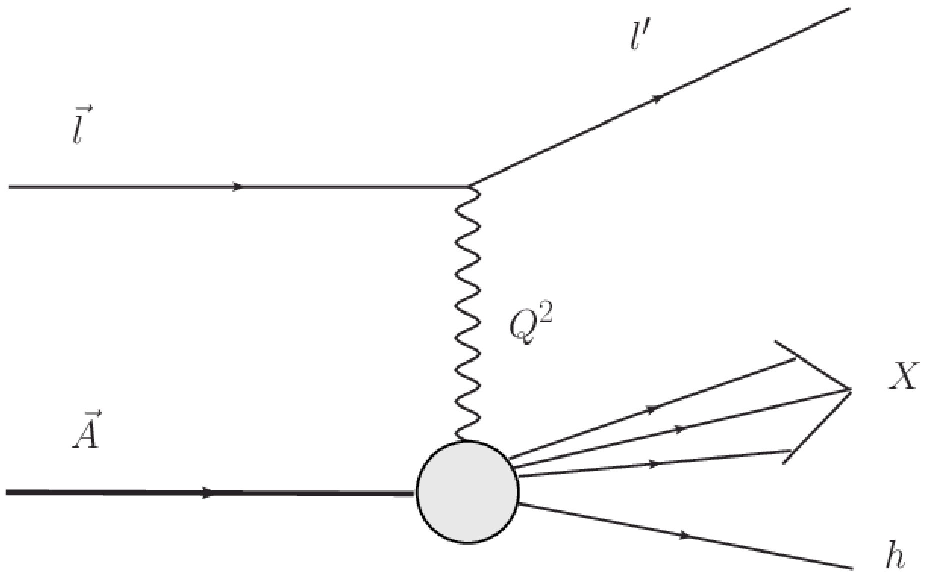
$$\int dx\, x G_2 = \int dx\, x(H+E) - \int dx\, \tilde{H}$$

$$G_2 \equiv \tilde{E}_{2T} + H + E$$

Kiptily and Polyakov, Eur Phys J C 37 (2004)
Hatta and Yoshida, JHEP (1210), 2012

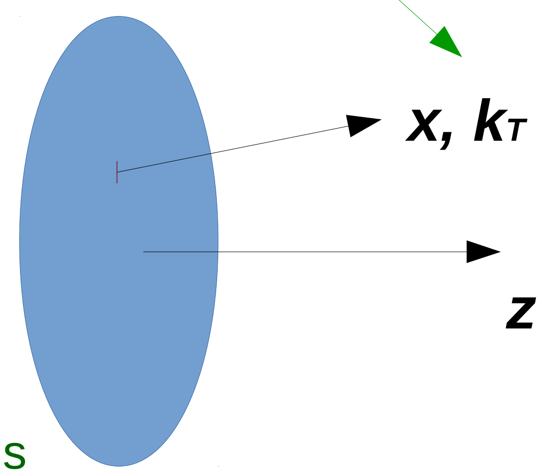
- The moment in x of the GPD G_2 shown to be OAM

Intrinsic Transverse Momentum



The partonic transverse momentum is correlated with the momentum of the observed hadron

Semi inclusive Deep Inelastic Scattering

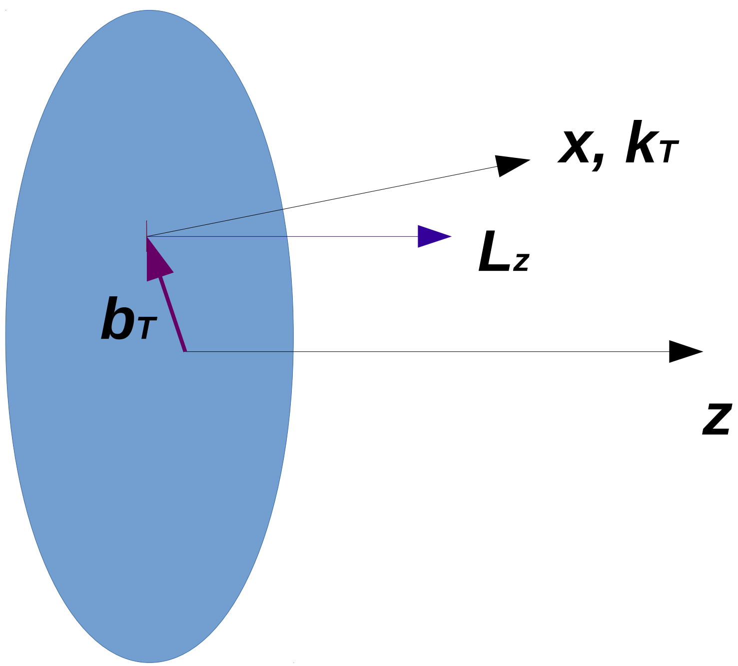


However, the target does not remain intact, no access to the spatial distribution of partons

Transverse Momentum Distributions

Partonic Orbital Angular Momentum II

- Consider measuring both the intrinsic transverse momentum and the spatial distribution of partons
- $L_{q,z}$ = $\mathbf{b_T} \times \mathbf{k_T}$



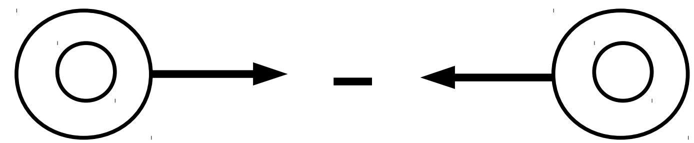
$$W^{[\gamma^+]}_{\Lambda,\Lambda'} = \frac{1}{2M}\bar{U}(p',\Lambda')[F_{11} + \frac{i\sigma^{i+}k_T^i}{\bar{p}_+}F_{12} + \frac{i\sigma^{i+}\Delta_T^i}{\bar{p}_+}F_{13} + \frac{i\sigma^{ij}k_T^i\Delta_T^j}{M^2}F_{14}]U(p,\Lambda)$$

Generalized Transverse Momentum Distributions (related by Fourier transform to Wigner Distributions)

Meissner Metz and Schlegel, JHEP 0908 (2009)

GTMDs that describe OAM

- How does F_{14} connect to OAM ?



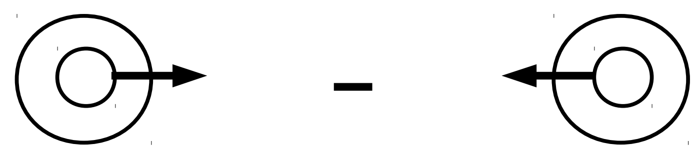
Unpolarized quark in a longitudinally polarized proton

$$\mathcal{W}(x, \mathbf{k}_T, \mathbf{b}) = \int \frac{d^2\Delta_T}{(2\pi)^2} e^{ib.\Delta_T} \left[W^{\gamma^+}_{++} - W^{\gamma^+}_{--} \right]$$

$$L = \int dx \int d^2k_T \int d^2\mathbf{b} (\mathbf{b} \times \mathbf{k}_T) \mathcal{W}(x, \mathbf{k}_T, \mathbf{b}) = -\int dx \int d^2k_T \frac{k_T^2}{M^2} F_{14}$$

Lorce et al PRD84, (2011)

- Another GTMD relevant to OAM



G_{11} describes a longitudinally polarized quark in an unpolarized proton. Measures spin orbit correlation.

The Two Definitions

- Weighted average of b_T X k_T

GTMD

$$L_z = -\int dx \int d^2k_T \frac{k_T^2}{M^2} F_{14}$$

Lorce, Pasquini (2011)

- Difference of total angular momentum and spin

GPD

$$\mathcal{L}_q = J_q - \frac{1}{2}\Delta\Sigma$$

$$\frac{1}{2}\int_{-1}^{1} dx x(H_q + E_q) \qquad \frac{1}{2}\int_{-1}^{1} dx \tilde{H}_q$$

The Two Definitions

$$F_{14}^{(1)}$$

- Weighted average of b_T X k_T

GTMD

$$L_z = -\int dx \boxed{\int d^2 k_T \frac{k_T^2}{M^2} F_{14}}$$

Lorce, Pasquini (2011)

- Difference of total angular momentum and spin

GPD

$$\mathcal{L}_q = J_q - \frac{1}{2}\Delta\Sigma$$

$$\frac{1}{2}\int_{-1}^{1} dx x (H_q + E_q) \qquad \frac{1}{2}\int_{-1}^{1} dx \tilde{H}_q$$

Is there a connection ?

- We find that

$$F_{14}^{(1)}(x) = \int_x^1 dy \left(\tilde{E}_{2T}(y) + H(y) + E(y) \right)$$

AR, Engelhardt and Liuti PRD 98 (2018)

AR, Courtoy, Engelhardt and Liuti PRD 94 (2016)

- This is a form of Lorentz Invariant Relation (LIR)
- This is a distribution of OAM in x
- Derived for a straight gauge link

Derivation of Generalized LIRs

To derive these we look at the parameterization of the quark quark correlator function at different levels

$$\int \frac{d^4 z}{2\pi} e^{ik.z} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda\rangle$$

Generalized Parton Correlation Functions (GPCFS)

Integrate over k^-

Meissner Metz and Schlegel, JHEP 0908 (2009)

$$\int \frac{dz_- d^2 z_T}{2\pi} e^{ixP^+ z^- - k_T . z_T} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda\rangle_{z^+=0}$$

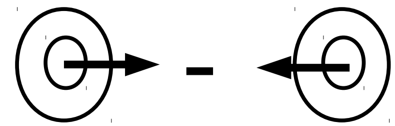
GTMDs

Integrate over k_T

$$\int \frac{dz_-}{2\pi} e^{ixP^+ z^-} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda\rangle_{z^+=z_T=0}$$

GPDs

Generalized Lorentz Invariance Relations



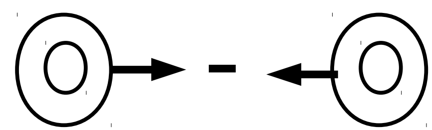
Axial Vector

$$\frac{dG_{11}^{e(1)}}{dx} = -\left(2\tilde{H}_{2T}' + E_{2T}'\right) - \tilde{H}$$

$$\frac{dG_{12}^{e(1)}}{dx} = H_{2T}' - \frac{\Delta_T^2}{4M^2}E_{2T}' - \left(1 + \frac{\Delta_T^2}{2M^2}\right)\tilde{H}$$

Vector

$$\frac{dF_{14}^{e(1)}}{dx} = \tilde{E}_{2T} + H + E$$



The GTMDs are complex in general.

$$X = X^e + iX^o$$

The imaginary part integrates to zero, on integration over k_T.

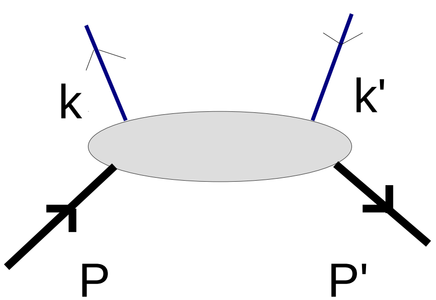
Higher Twist

$$\int \frac{dz_-}{2\pi} e^{ixP^+z^-} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda\rangle_{z^+=z_T=0}$$

$$\gamma^+, \gamma^+\gamma^5, \sigma^{i+}\gamma^5$$

Leading twist – twist 2

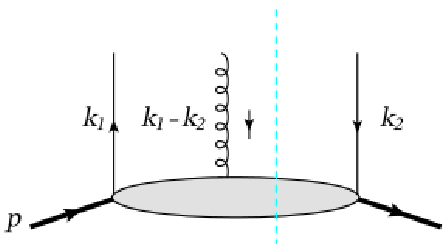
- Involve only good components
- Simple interpretation in terms of parton densities



$$\gamma^i, \gamma^i\gamma^5, \sigma^{ij}\gamma^5, 1, \gamma^5, \sigma^{+-}\gamma^5$$

Higher twist – twist 3

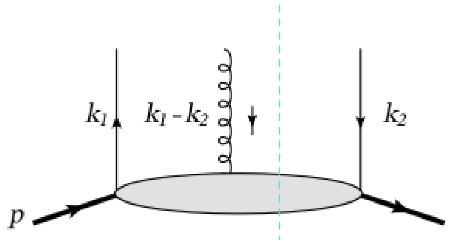
- Involve one good and one bad component
- The bad component represents a quark gluon composite



Collinear Picture : Transverse Quark Current, Higher Twist

$$\bar{\psi}(-z/2)\gamma^{+}\psi(z/2) \longrightarrow$$ Leading order quark current

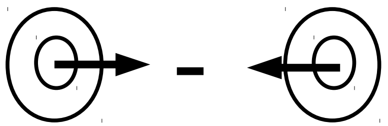
$$\bar{\psi}(-z/2)\gamma_T^{i}\psi(z/2) \longrightarrow$$ Transverse quark current, implicitly involves quark gluon interactions



Probabilistic parton model interpretation works well at leading order, with transverse quark projection operator need to include quark gluon interactions

Both in Collinear Picture

Generalized Lorentz Invariance Relations



Axial Vector

$$\frac{dG_{11}^{e(1)}}{dx} = -\left(2\tilde{H}'_{2T} + E'_{2T}\right) - \tilde{H}$$

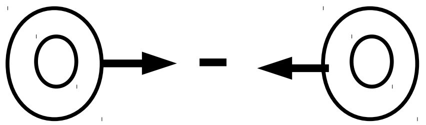
$$\frac{dG_{12}^{e(1)}}{dx} = H'_{2T} - \frac{\Delta_T^2}{4M^2}E'_{2T} - \left(1 + \frac{\Delta_T^2}{2M^2}\right)\tilde{H}$$

Twist two

Twist three

Vector

$$\frac{dF_{14}^{e(1)}}{dx} = \tilde{E}_{2T} + H + E$$



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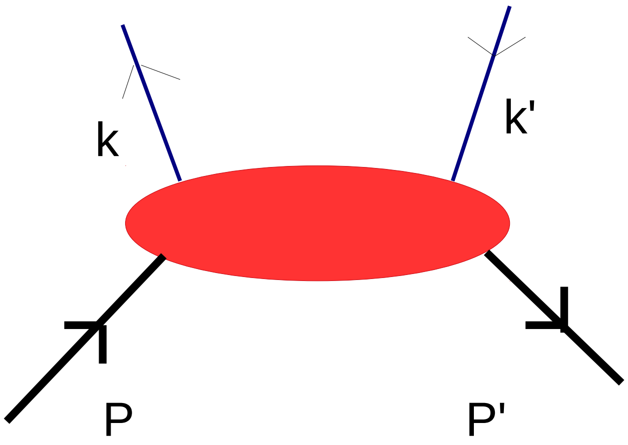
$$X = X^e + iX^o$$

The imaginary part integrates to zero, on integration over k_T.

Intrinsic Momentum vs Momentum Transfer Δ

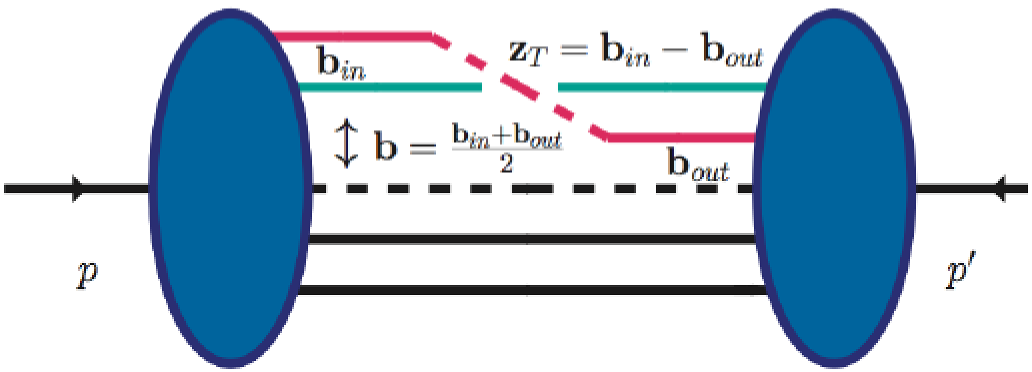


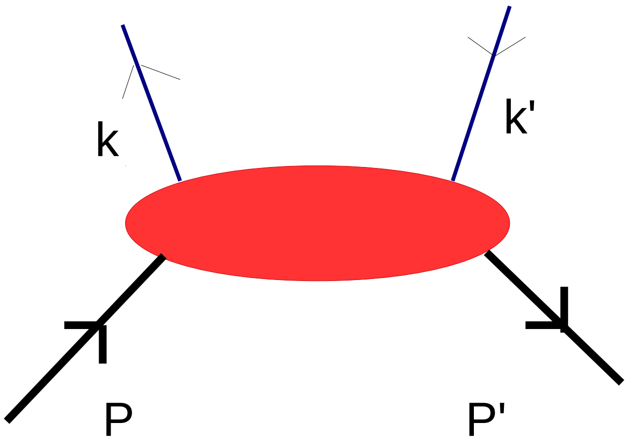
Courtoy et al PhysLett B731, 2013

Burkardt,Phys Rev D62, 2000

$$\int \frac{dz_-}{2\pi} e^{ixP^+z^-} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda\rangle_{z^+=z_T=0}$$

Intrinsic Momentum vs Momentum Transfer Δ





$$k \longleftrightarrow z$$

$$\Delta \longleftrightarrow b$$

Courtoy et al PhysLett B731, 2013

Burkardt,Phys Rev D62, 2000

$$\int \frac{dz_-}{2\pi} e^{ixP^+z^-} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda\rangle_{z^+=z_T=0}$$

Equations of Motion Relations

How do we obtain these ?

$$
\begin{aligned}
(i\not{D} - m)\psi(z_{out}) &= (i\not{\partial} + g\not{A} - m)\psi(z_{out}) = 0, \\
\bar{\psi}(z_{in})(i\overleftarrow{\not{D}} + m) &= \bar{\psi}(z_{in})(i\overleftarrow{\not{\partial}} - g\not{A} + m) = 0
\end{aligned}
$$

Equations of Motion Relations

How do we obtain these ?

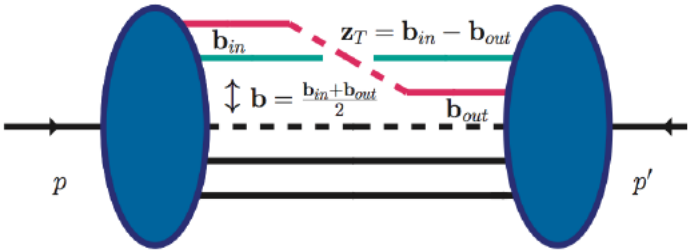
$$
\begin{aligned}
\mathcal{U} i\sigma^{i+}\gamma_5 (i\not{D} - m)\psi(z_{out}) &= \mathcal{U} i\sigma^{i+}\gamma_5 (i\not{\partial} + g\not{A} - m)\psi(z_{out}) = 0, \\
\bar{\psi}(z_{in})(i\overleftarrow{\not{D}} + m) i\sigma^{i+}\gamma_5 \mathcal{U} &= \bar{\psi}(z_{in})(i\overleftarrow{\not{\partial}} - g\not{A} + m) i\sigma^{i+}\gamma_5 \mathcal{U} = 0
\end{aligned}
$$

Equations of Motion Relations

How do we obtain these ?

$$
\begin{aligned}
\mathcal{U} i\sigma^{i+}\gamma_5(i\not{D}-m)\psi(z_{out}) &= \mathcal{U} i\sigma^{i+}\gamma_5(i\not{\partial}+g\not{A}-m)\psi(z_{out})=0,\\
\bar{\psi}(z_{in})(i\overleftarrow{\not{D}}+m)i\sigma^{i+}\gamma_5\mathcal{U} &= \bar{\psi}(z_{in})(i\overleftarrow{\not{\partial}}-g\not{A}+m)i\sigma^{i+}\gamma_5\mathcal{U}=0
\end{aligned}
$$

$$
b=\frac{z_{in}+z_{out}}{2}, \qquad z=z_{in}-z_{out}
$$



$$
\int db^- d^2b_T e^{-ib\cdot\Delta}\int dz^- d^2z_T e^{-ik\cdot z}\langle p',\Lambda'|\bar{\psi}\left[(i\overleftarrow{\not{D}}+m)i\sigma^{i+}\gamma^5 \pm i\sigma^{i+}\gamma^5(i\overrightarrow{\not{D}}-m)\right]\psi|p,\Lambda\rangle=0
$$

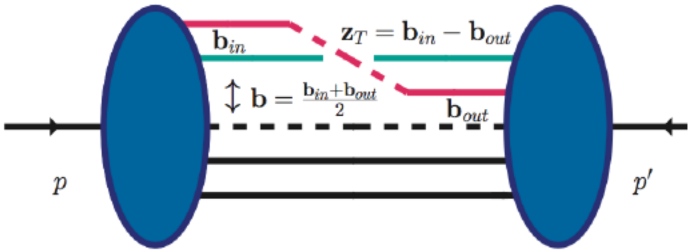
Equations of Motion R

Crucial for understanding qgq contribution to GPDs!!

How do we obtain these ?

$$\begin{aligned}\mathcal{U} i\sigma^{i+}\gamma_5(i\not{D}-m)\psi(z_{out}) &= \mathcal{U} i\sigma^{i+}\gamma_5(i\not{\partial}+g\not{A}-m)\psi(z_{out})=0,\\ \bar{\psi}(z_{in})(i\overleftarrow{\not{D}}+m)i\sigma^{i+}\gamma_5\mathcal{U} &= \bar{\psi}(z_{in})(i\overleftarrow{\not{\partial}}-g\not{A}+m)i\sigma^{i+}\gamma_5\mathcal{U}=0\end{aligned}$$

$$b=\frac{z_{in}+z_{out}}{2}, \qquad z=z_{in}-z_{out}$$



$$\int db^- d^2b_T e^{-ib\cdot\Delta}\int dz^- d^2z_T e^{-ik\cdot z}\langle p',\Lambda'|\bar{\psi}\left[(i\overleftarrow{\not{D}}+m)i\sigma^{i+}\gamma^5 \pm i\sigma^{i+}\gamma^5(i\overrightarrow{\not{D}}-m)\right]\psi|p,\Lambda\rangle=0$$

Use LIRs and Equation of Motion Relations to derive Wandzura Wilczek Relations

- The equations of motion connect k_T dependent quantities with collinear objects.
- These k_T dependent quantities are also connected to collinear objects by LIRs. This is independent of equation of motion relations.
- Use the LIR to eliminate the k_T dependent quantities in equation of motion relations. This results in the Wandzura Wilczek relations for twist 3 GPDs.

EoM relations for Orbital Angular Momentum

$$x\tilde{E}_{2T} = -\tilde{H} + \boxed{2\int d^2k_T \frac{k_T^2 sin^2\phi}{M^2} F_{14}} + \frac{\Delta^i}{\Delta_T^2}\int d^2k_T (\mathcal{M}^{i,S}_{++} - \mathcal{M}^{i,S}_{--})$$

Twist 3

Twist 2

Genuine Twist 3 (explicit gluon)

$$\boxed{\frac{dF_{14}^{(1)}}{dx} = \tilde{E}_{2T} + H + E}$$

$$\mathcal{M}^{i,S}_{\Lambda'\Lambda} = \frac{i}{4}\int \frac{dz^- d^2z_T}{(2\pi)^3} e^{ixP^+z^- - ik_T\cdot z_T} \langle p', \Lambda' \mid \overline{\psi}\left(-\frac{z}{2}\right)\left[(\overrightarrow{\not\partial} - ig\not{A})\mathcal{U}\Gamma\Big|_{-z/2} + \Gamma\mathcal{U}(\overleftarrow{\not\partial} + ig\not{A})\Big|_{z/2}\right]\psi\left(\frac{z}{2}\right) \mid p, \Lambda\rangle_{z^+=0}$$

$$\int dx \int d^2k_T\, \mathcal{M}^{i,S}_{\Lambda'\Lambda} = i\epsilon^{ij} g v^- \frac{1}{2P^+}\int_0^1 ds\, \langle p', \Lambda' | \bar{\psi}(0)\gamma^+ U(0,sv) F^{+j}(sv) U(sv,0)\psi(0) | p, \Lambda\rangle$$

Wandzura Wilczek Relations

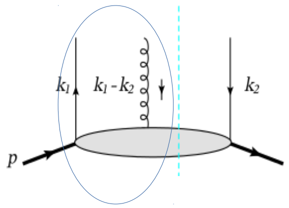
$$\tilde{E}_{2T} = -\int_x^1 \frac{dy}{y}(H+E) + \left[\frac{\tilde{H}}{x} - \int_x^1 \frac{dy}{y^2}\tilde{H}\right] + \left[\frac{1}{x}\mathcal{M}_{F_{14}} - \int_x^1 \frac{dy}{y^2}\mathcal{M}_{F_{14}}\right]$$

Twist three vector GPD

Twist two

Axial vector GPD contributes to a vector GPD

Genuine Tw 3



AR, Engelhardt and Liuti PRD 98 (2018)

Hatta and Yoshida, JHEP (1210), 2012

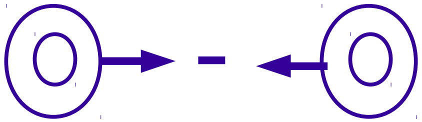
$$g_2(x) = -g_1(x) + \int_x^1 \frac{dy}{y} g_1(x) + \bar{g}_2(x)$$

Twist three PDF

Twist two

Genuine Tw 3

Moments of twist three GPDs
-Quark gluon structure



$$\int dx \widetilde{E}_{2T} \;=\; -\int dx (H+E) \qquad \Rightarrow \int dx \left(\widetilde{E}_{2T} + H + E\right) = 0$$

$$\int dx x \widetilde{E}_{2T} \;=\; -\frac{1}{2}\int dx x (H+E) - \frac{1}{2}\int dx \widetilde{H}$$

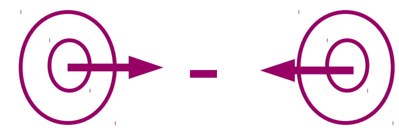
OAM

$$\int dx x^2 \widetilde{E}_{2T} \;=\; -\frac{1}{3}\int dx x^2 (H+E) - \frac{2}{3}\int dx x \widetilde{H} - \boxed{\frac{2}{3}\int dx x \, \mathcal{M}_{F_{14}}\Big|_{v=0}}$$

Genuine Twist Three

$$\int dx\, x \int d^2 k_T \, \mathcal{M}^{i,S}_{\Lambda'\Lambda} \;=\; \frac{ig}{4(P^+)^2} \langle p', \Lambda' | \bar{\psi}(0) \gamma^+ \gamma^5 F^{+i}(0) \psi(0) | p, \Lambda \rangle$$

Moments of twist three GPDs -Quark gluon structure



$$\int dx \left(E'_{2T} + 2\widetilde{H}'_{2T} \right) = -\int dx \widetilde{H} \qquad \Rightarrow \int dx \left(E'_{2T} + 2\widetilde{H}'_{2T} + \widetilde{H} \right) = 0$$

$$\int dx\underline{x} \left(E'_{2T} + 2\widetilde{H}'_{2T} \right) = -\frac{1}{2}\int dx x \widetilde{H} - \frac{1}{2}\int dx H + \boxed{\frac{m}{2M}\int dx (E_T + 2\widetilde{H}_T)}$$

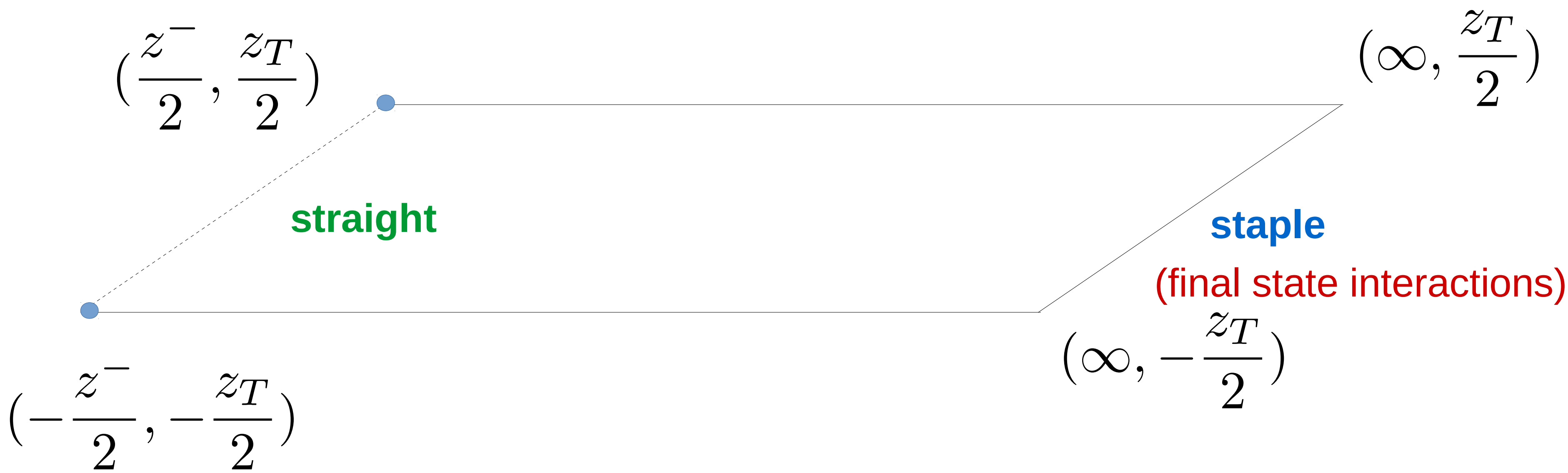
mass term

$$\int dx\, \underline{x}^2 \left(E'_{2T} + 2\widetilde{H}'_{2T} \right) = -\frac{1}{3}\int dx x^2 \widetilde{H} - \frac{2}{3}\int dx x H + \boxed{\frac{2m}{3M}\int dx x (E_T + 2\widetilde{H}_T)} - \boxed{\frac{2}{3}\int dx x \mathcal{M}_{G_{11}}\Big|_{v=0}}$$

Genuine Twist Three d_2

$$\int dx\, x \int d^2k_T\, \mathcal{M}^{i,A}_{\Lambda'\Lambda} = \frac{g}{4(P^+)^2}\epsilon^{ij}\langle p', \Lambda' | \bar{\psi}(0)\gamma^+ F^{+j}(0)\psi(0) | p, \Lambda \rangle$$

Quark gluon quark contributions



$$\int dx \int d^2k_T\, \mathcal{M}^{i,S}_{\Lambda'\Lambda} \quad = \quad i\epsilon^{ij} g v^- \frac{1}{2P^+} \int_0^1 ds\, \langle p', \Lambda' | \bar{\psi}(0)\gamma^+ U(0,sv) F^{+j}(sv) U(sv,0)\psi(0)|p,\Lambda\rangle$$

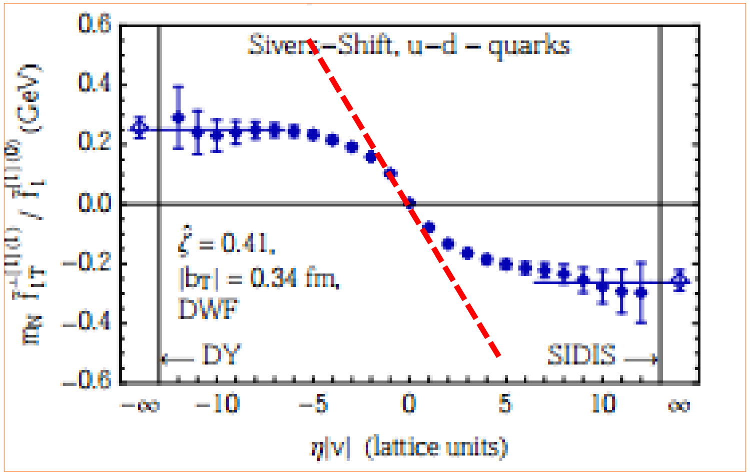
$$\int dx \int d^2k_T\, \mathcal{M}^{i,A}_{\Lambda'\Lambda} \quad = \quad -g v^- \frac{1}{2P^+} \int_0^1 ds\, \langle p', \Lambda' | \bar{\psi}(0)\gamma^+\gamma^5 U(0,sv) F^{+i}(sv) U(sv,0)\psi(0)|p,\Lambda\rangle$$

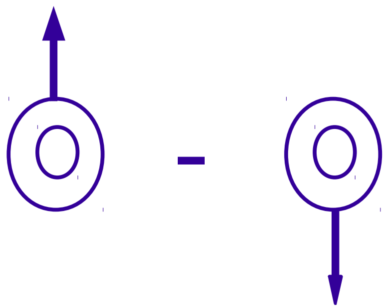
Calculating the force from Lattice data – Sivers function

$$\frac{d}{dv^-}\int dx F_{12}^{(1)}\Big|_{v^-=0} = \frac{d}{dv^-}\int dx \mathcal{M}_{F_{12}}\Big|_{v^-=0} = i(2P^+)\int dx\, x\frac{\Delta_i}{\Delta_T^2}\left(\mathcal{M}_{++}^{i,A} - \mathcal{M}_{--}^{i,A}\right) = \mathcal{M}_{G_{12}}^{n=3}$$

$$\frac{d}{dv^-}\int dx G_{12}^{(1)}\Big|_{v^-=0} = \frac{d}{dv^-}\int dx \mathcal{M}_{G_{12}}\Big|_{v^-=0} = i(2P^+)\int dx\, x\frac{\Delta_i}{\Delta_T^2}\left(\mathcal{M}_{++}^{i,S} + \mathcal{M}_{--}^{i,S}\right) = \mathcal{M}_{F_{12}}^{n=3}$$

The derivative with respect to the gauge link direction gives the force!



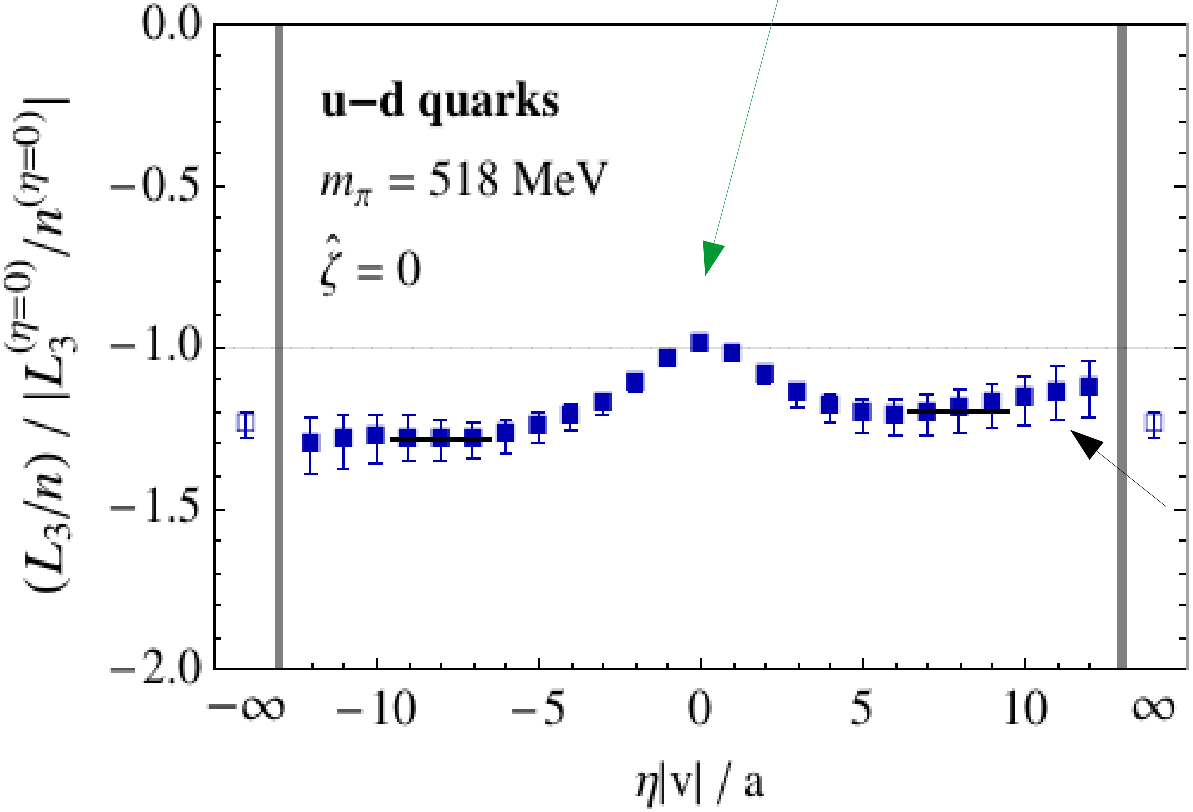


Transversely polarized proton

Calculating the torque from Lattice

Straight link

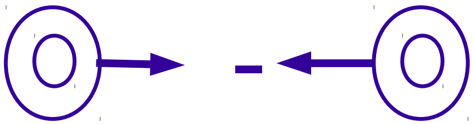
F_{14}



Staple

Michael Engelhardt

Phys. Rev. D95 (2017)



Longitudinally polarized proton

$$\mathcal{L}_{JM} - \mathcal{L}_{Ji} = \mathcal{T}$$

Torque!

Understanding the mass decomposition of the proton

Mass decomposition of the proton

$$T^{\mu\nu} = \boxed{T^{\mu\nu}_{q,kin} + T^{\mu\nu}_{g,kin}} + T^{\mu\nu}_{m} + T^{\mu\nu}_{a}$$

Traceless

X Ji (1995)

$$M = \frac{\langle P| \int d^3\mathbf{x} T^{00}(0,\mathbf{x})|P\rangle}{\langle P|P\rangle} \equiv \langle T^{00}\rangle$$

Rest frame

$$\langle \bar{T}^{00}\rangle = 3/4M$$

Traceless

$$\langle \hat{T}^{00}\rangle = 1/4M$$

Trace part

Energy Momentum Tensor Parameterization

$$T^{\mu\nu} = \frac{1}{2}\bar{\psi} i \overleftrightarrow{D}^{(\mu}\gamma^{\nu)} + \frac{1}{4} g^{\mu\nu} F^2 - F^{\mu\alpha} F^{\nu}_{\alpha}$$

- The full energy momentum tensor is a conserved quantity and is scale independent.
- The separate contributions from the quarks and gluons on the other hand are not and do depend on the renormalization scale.

$$\langle P' | (T^{\mu\nu})_R | P \rangle =$$
$$\bar{u}(P') \left[A_{q,g} \gamma^{(\mu} \bar{P}^{\nu)} + B_{q,g} \frac{\bar{P}^{(\mu} i\sigma^{\nu)\alpha} \Delta_\alpha}{2M} + C_{q,g} \frac{\Delta^\mu \Delta^\nu - \eta^{\mu\nu} \Delta^2}{M} + \bar{C}_{q,g} M \eta^{\mu\nu} \right] u(P)$$

Trace Anamoly

* $\langle P|T^{\mu\nu}|P\rangle = P^\mu P^\nu/M \longrightarrow M$ **Total**

Trace Anamoly

* $\langle P|T^{\mu\nu}|P\rangle = P^\mu P^\nu/M \longrightarrow M$ **Total**

* $\langle P'|(T^{\mu\nu}_{q,g})_R|P\rangle =$

$$\bar{u}(P')\left[A_{q,g}\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + C_{q,g}\frac{\Delta^\mu\Delta^\nu - \eta^{\mu\nu}\Delta^2}{M} + \bar{C}_{q,g}M\eta^{\mu\nu}\right]u(P)$$

$$\Big\downarrow \text{Trace}$$

Quark and gluon components separated

$$\langle P'|(T^R_{q,g})^\alpha_\alpha|P\rangle = 2M^2\left(A^R_{q,g}(\mu) + 4\bar{C}^R_{q,g}(\mu)\right)$$

Trace Anamoly

* $\langle P|T^{\mu\nu}|P\rangle = P^\mu P^\nu/M \longrightarrow M$ **Total**

* $\langle P'|(T^{\mu\nu}_{q,g})_R|P\rangle =$

$$\bar{u}(P')\left[A_{q,g}\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + C_{q,g}\frac{\Delta^\mu\Delta^\nu - \eta^{\mu\nu}\Delta^2}{M} + \bar{C}_{q,g}M\eta^{\mu\nu}\right]u(P)$$

$\big\downarrow$ Trace

Quark and gluon components separated

$$\langle P'|(T^R_{q,g})^\alpha_\alpha|P\rangle = 2M^2\left(A^R_{q,g}(\mu) + 4\bar{C}^R_{q,g}(\mu)\right)$$

* $$T^{\mu\nu} = \frac{1}{2}\bar{\psi}i\overleftrightarrow{D}^{(\mu}\gamma^{\nu)} + \frac{1}{4}g^{\mu\nu}F^2 - F^{\mu\alpha}F^\nu_\alpha$$

$\big\downarrow$ Trace

$$T^\mu_\mu = (1+\gamma_m)m\bar{\psi}\psi + \frac{\beta}{2g}F^2$$

By studying the Gravitational form factors A and C̄ we will know the quark and gluon contributions to the trace anomaly separately.

Quark and gluon contributions to the trace anomaly

$$T^{\mu\nu} = \frac{1}{2}\bar{\psi} i \overleftrightarrow{D}^{(\mu}\gamma^{\nu)} + \frac{1}{4}g^{\mu\nu}F^2 - F^{\mu\alpha}F^{\nu}_{\alpha}$$

$$\Big\downarrow \text{Trace}$$

$$T^{\mu}_{\mu} = (1+\gamma_m)m\bar{\psi}\psi + \frac{\beta}{2g}F^2$$

Quark and gluon contributions to the trace anomaly

$$T^{\mu\nu} = \frac{1}{2}\bar{\psi} i \overleftrightarrow{D}^{(\mu}\gamma^{\nu)} + \frac{1}{4} g^{\mu\nu} F^2 - F^{\mu\alpha} F^{\nu}_{\alpha}$$

Trace

$$T^{\mu}_{\mu} = (1+\gamma_m) m\bar{\psi}\psi + \frac{\beta}{2g} F^2$$

$$(T^{\alpha}_{g\alpha})_R = A^R_g(\mu) + 4\bar{C}^R_g(\mu)$$

$$= \frac{1}{2M^2}\langle P|\frac{\alpha_s}{2\pi}\left(-\frac{11C_A}{6}\underline{(F^2)_R} + \frac{14C_F}{3}\underline{(m\bar{\psi}\psi)_R}\right)|P\rangle$$

gluons

$$(T^{\alpha}_{q\alpha})_R = A^R_q(\mu) + 4\bar{C}^R_q(\mu)$$

$$= \frac{1}{2M^2}\langle P|(m\bar{\psi}\psi)_R + \frac{\alpha_s}{4\pi}\left(\frac{n_f}{3}\underline{(F^2)_R} + \frac{4C_F}{3}\underline{(m\bar{\psi}\psi)_R}\right)|P\rangle$$

quarks

Y Hatta, AR, K Tanaka arxiv:1810.05116

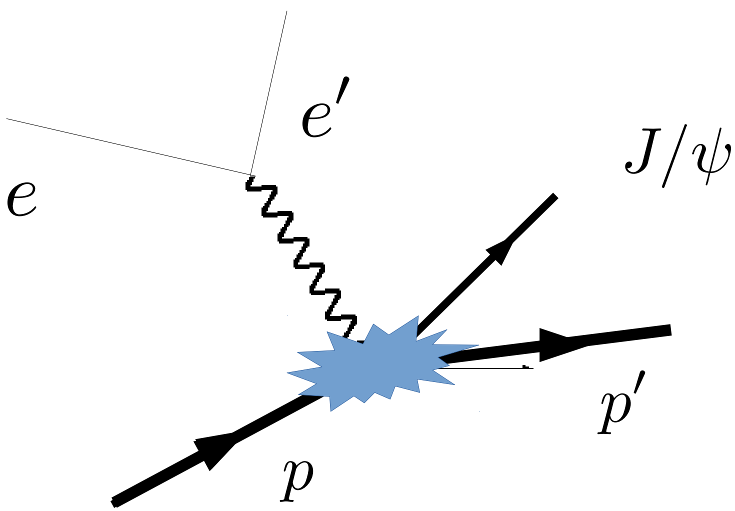
Experimental measurements

The production of a heavy quarkonium near threshold in electron-proton scattering is connected to the origin of the proton mass via the QCD trace anomaly.

D.E Kharzeev (1995)

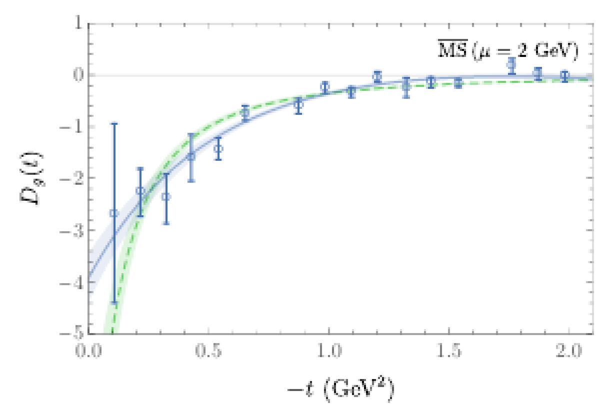
$$ep \to e'\gamma^* p \to e'p'J/\psi$$

Y Hatta, DL Yang PRD98 (2018)



In an actual experiment $p' - p \equiv t \neq 0$

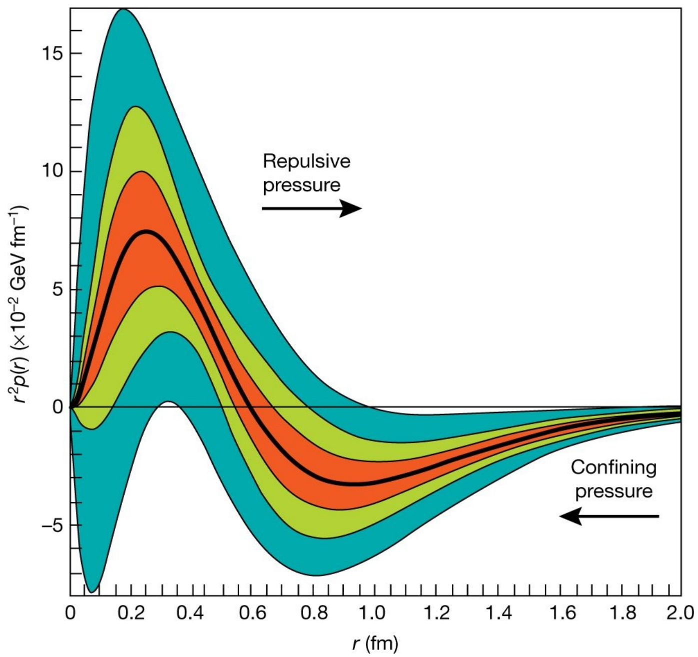
Calculate cross-section using input from latest lattice QCD calculations of gluon gravitational form factors.



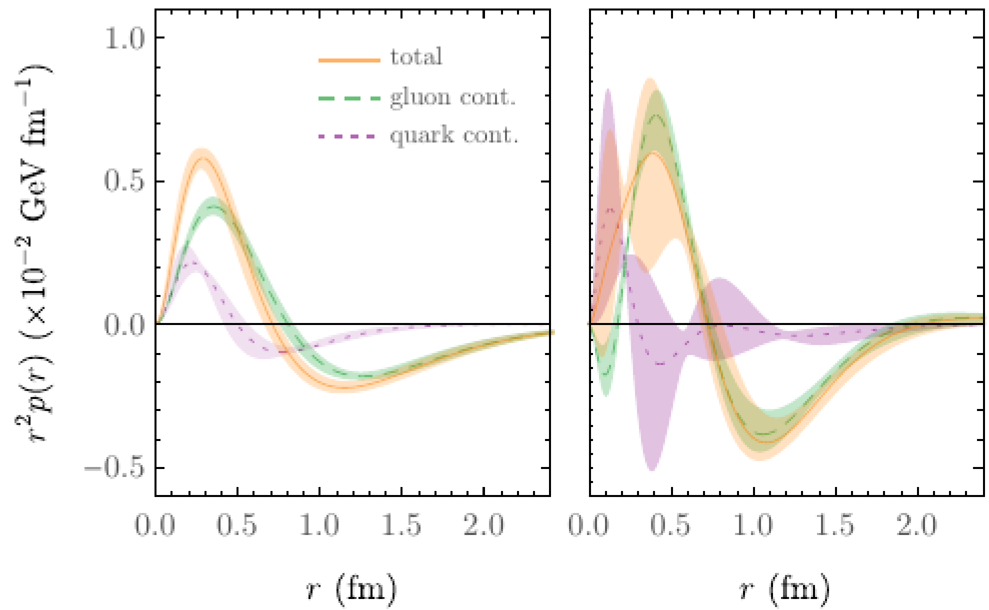
Detmold and Shanahan arxiv:1810:04626

Equation of state of Neutron Stars at Short Distances

Pressure Distribution inside the Proton

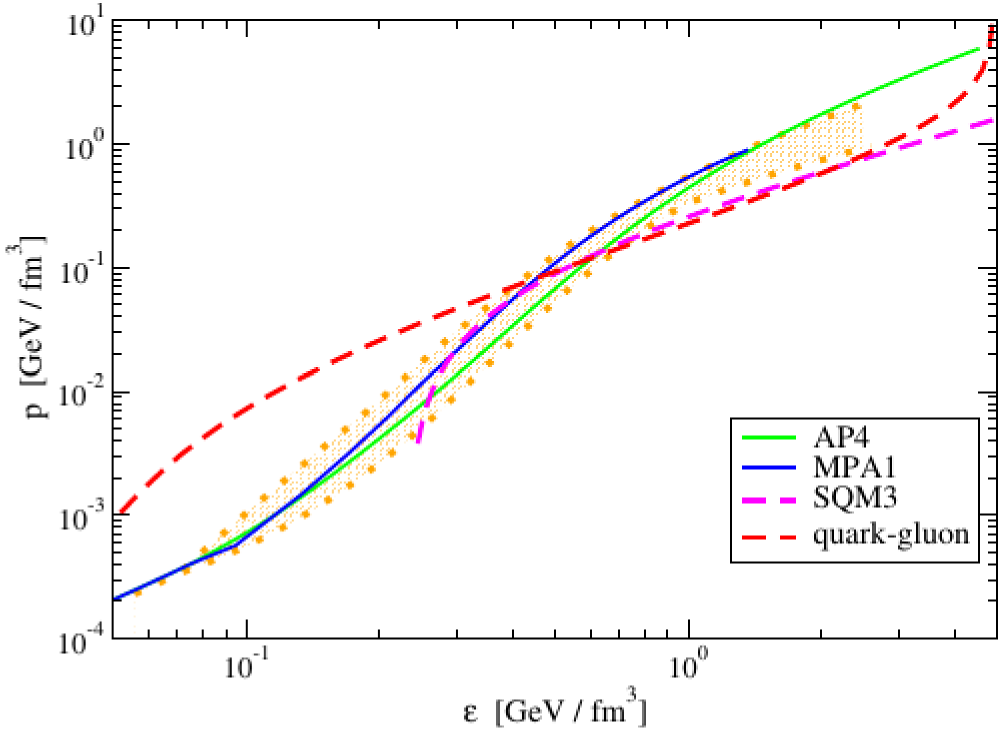


Burkert, Elouadrhiri, Girod (Nature, 2018)



Detmold and Shanahan (PRL, 2019)

Equation of State of Neutron Stars



S Liuti, AR, K Yagi arxiv:1812.01479

$$\epsilon_{q,g}(r) \;=\; \int \frac{d^2 \mathbf{\Delta}_T}{(2\pi)^2} e^{i\mathbf{\Delta}_T\cdot\mathbf{b}} A_2^{q,g}(t),$$

$$p_{q,g}(r) \;=\; \int \frac{d^2 \mathbf{\Delta}_T}{(2\pi)^2} e^{i\mathbf{\Delta}_T\cdot\mathbf{b}}\, 2\,t\, C_2^{q,g}(t)$$

$$\sum_{\Lambda,\lambda} \rho^q_{\Lambda\lambda}(\mathbf{b}) = H_q(\mathbf{b}^2) = \int \frac{d^2 \mathbf{\Delta}_T}{(2\pi)^2} e^{i\mathbf{\Delta}_T\cdot\mathbf{b}} A_1^q(t).$$

Conclusions

- Way of deriving the Wandzura Wilczek relations. Allows us to **write out precisely quark gluon contribution to twist 3. Study x dependence.**

- Gluons play a key role in the mass and pressure make up of the proton – even neutron stars!

- Quark gluon quark interactions are at the heart of unravelling the structure of the proton.

Thank you!

Thanks!

Quark and gluon contributions to the trace anomaly

$$T^{\mu\nu} = \frac{1}{2}\bar{\psi} i \overset{\leftrightarrow}{D}{}^{(\mu}\gamma^{\nu)} + \frac{1}{4} g^{\mu\nu} F^2 - F^{\mu\alpha} F^{\nu}_{\alpha}$$

Trace

$$T^{\mu}_{\mu} = (1+\gamma_m) m\bar{\psi}\psi + \frac{\beta}{2g} F^2$$

$$T^{\alpha}_{\alpha} = -2\epsilon \frac{F^2}{4} + m\bar{\psi}\psi$$

renormalization

$$-2\epsilon \frac{F^2}{4} = \frac{\beta}{2g} F^2_R + \gamma_m (m\bar{\psi}\psi)_R$$

$$T^{\alpha}_{g\alpha} = \frac{\beta}{2g} (F^2)_R + \gamma_m (m\bar{\psi}\psi)_R$$

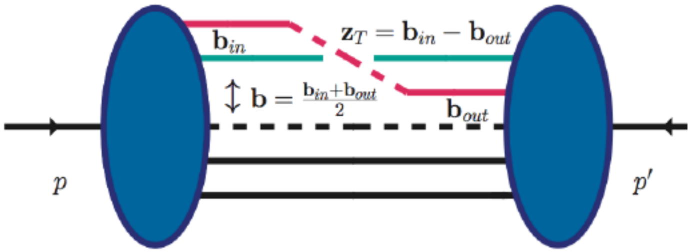
$$T^{\alpha}_{q\alpha} = m(\bar{\psi}\psi)_R$$

Equations of Motion Relations

How do we obtain these ?

$$\begin{aligned} \mathcal{U} i\sigma^{i+}\gamma_5 (i\not{D} - m)\psi(z_{out}) &= \mathcal{U} i\sigma^{i+}\gamma_5 (i\not{\partial} + g\not{A} - m)\psi(z_{out}) = 0, \\ \bar{\psi}(z_{in})(i\overleftarrow{\not{D}} + m) i\sigma^{i+}\gamma_5 \mathcal{U} &= \bar{\psi}(z_{in})(i\overleftarrow{\not{\partial}} - g\not{A} + m) i\sigma^{i+}\gamma_5 \mathcal{U} = 0 \end{aligned}$$

$$b = \frac{z_{in} + z_{out}}{2}, \qquad z = z_{in} - z_{out}$$



$$\int db^- d^2 b_T e^{-ib\cdot\Delta} \int dz^- d^2 z_T e^{-ik\cdot z} \langle p', \Lambda' | \bar{\psi} \left[(i\overleftarrow{\not{D}} + m) i\sigma^{i+}\gamma^5 \pm i\sigma^{i+}\gamma^5 (i\overrightarrow{\not{D}} - m) \right] \psi | p, \Lambda \rangle = 0$$

Derivation of Generalized LIRs

To derive these we look at the parameterization of the quark quark correlator function at different levels

$$\int \frac{d^4z}{2\pi} e^{ik.z} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda\rangle$$

Generalized Parton Correlation Functions (GPCFS)

Integrate over k^-

Meissner Metz and Schlegel, JHEP 0908 (2009)

$$\int \frac{dz_- d^2z_T}{2\pi} e^{ixP^+z^- - k_T.z_T} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda\rangle_{z^+=0}$$

GTMDs

Integrate over k_T

$$\int \frac{dz_-}{2\pi} e^{ixP^+z^-} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda\rangle_{z^+=z_T=0}$$

GPDs

Generalized Lorentz Invariance Relations

- The same set of As describe the whole vector sector.

$$F_{14}^{(1)} = \int d\sigma d\sigma' d\tau \frac{M^3}{2} J \left[A_8^F + x A_9^F\right] \qquad J = \sqrt{x\sigma - \tau - \frac{x^2P^2}{M^2} - \frac{\Delta_T^2 \sigma'^2}{M^2}}$$

γ^+

$$H + E = \int d\sigma d\sigma' d\tau \frac{M^3}{J} \sigma' A_5^F + A_6^F + \left(\frac{\sigma}{2} - \frac{xP^2}{M^2}\right)\left(A_8^F + x A_9^F\right)$$

γ_T^i

$$\tilde{E}_{2T} = \int d\sigma d\sigma' d\tau \frac{M^3}{J}\left[\left(x\sigma - \tau - \frac{x^2P^2}{M^2} - \frac{\Delta_T^2\sigma'^2}{M^2}\right) A_9^F - \sigma' A_5^F - A_6^F\right]$$

$$\sigma \equiv \frac{2k.P}{M^2}, \qquad \tau \equiv \frac{k^2}{M^2}, \qquad \sigma' \equiv \frac{k.\Delta}{\Delta^2} = \frac{k_T.\Delta_T}{\Delta_T^2}$$

$$-\frac{dF_{14}^{(1)}}{dx} = \tilde{E}_{2T} + H + E$$

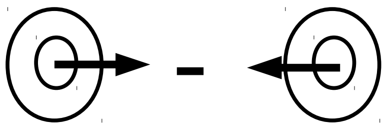
$$F_{14}^{(1)}(x) = \int_x^1 dy \left(\tilde{E}_{2T}(y) + H(y) + E(y)\right)$$

Distribution of OAM in x !

k_T^2 moment of a twist two function

Twist three function

Generalized Lorentz Invariance Relations



Axial Vector

$$\frac{dG_{11}^{e(1)}}{dx} = -\left(2\tilde{H}'_{2T} + E'_{2T}\right) - \tilde{H}$$

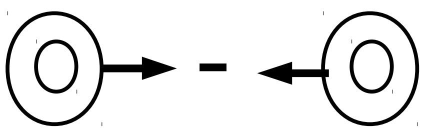
$$\frac{dG_{12}^{e(1)}}{dx} = H'_{2T} - \frac{\Delta_T^2}{4M^2}E'_{2T} - \left(1 + \frac{\Delta_T^2}{2M^2}\right)\tilde{H}$$

Twist two

Twist three

Vector

$$\frac{dF_{14}^{e(1)}}{dx} = \tilde{E}_{2T} + H + E$$



Intrinsic transverse momentum

Quark gluon interactions

EoM relations for Orbital Angular Momentum

$$x\tilde{E}_{2T} = -\tilde{H} + \boxed{2\int d^2k_T \frac{k_T^2 sin^2\phi}{M^2} F_{14}} + \frac{\Delta^i}{\Delta_T^2}\int d^2k_T (\mathcal{M}^{i,S}_{++} - \mathcal{M}^{i,S}_{--})$$

Twist 3

Twist 2

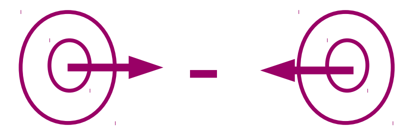
Genuine Twist 3 (explicit gluon)

$$\boxed{\frac{dF_{14}^{(1)}}{dx} = \tilde{E}_{2T} + H + E}$$

$$\mathcal{M}^{i,S}_{\Lambda'\Lambda} = \frac{i}{4}\int \frac{dz^- d^2z_T}{(2\pi)^3} e^{ixP^+z^- - ik_T\cdot z_T} \langle p', \Lambda' \mid \overline{\psi}\left(-\frac{z}{2}\right)\left[(\overrightarrow{\not\partial} - ig\not{A})\mathcal{U}\Gamma\Big|_{-z/2} + \Gamma\mathcal{U}(\overleftarrow{\not\partial} + ig\not{A})\Big|_{z/2}\right]\psi\left(\frac{z}{2}\right) \mid p, \Lambda\rangle_{z^+=0}$$

$$\int dx \int d^2k_T\, \mathcal{M}^{i,S}_{\Lambda'\Lambda} = i\epsilon^{ij} g v^- \frac{1}{2P^+}\int_0^1 ds\, \langle p', \Lambda' | \bar{\psi}(0)\gamma^+ U(0, sv) F^{+j}(sv) U(sv, 0)\psi(0) | p, \Lambda\rangle$$

Moments of twist three GPDs -Quark gluon structure



$$\int dx \left(E'_{2T} + 2\widetilde{H}'_{2T}\right) = -\int dx \widetilde{H} \qquad \Rightarrow \int dx \left(E'_{2T} + 2\widetilde{H}'_{2T} + \widetilde{H}\right) = 0$$

$$\int dx\underline{x} \left(E'_{2T} + 2\widetilde{H}'_{2T}\right) = -\frac{1}{2}\int dx x\widetilde{H} - \frac{1}{2}\int dx H + \boxed{\frac{m}{2M}\int dx (E_T + 2\widetilde{H}_T)}$$

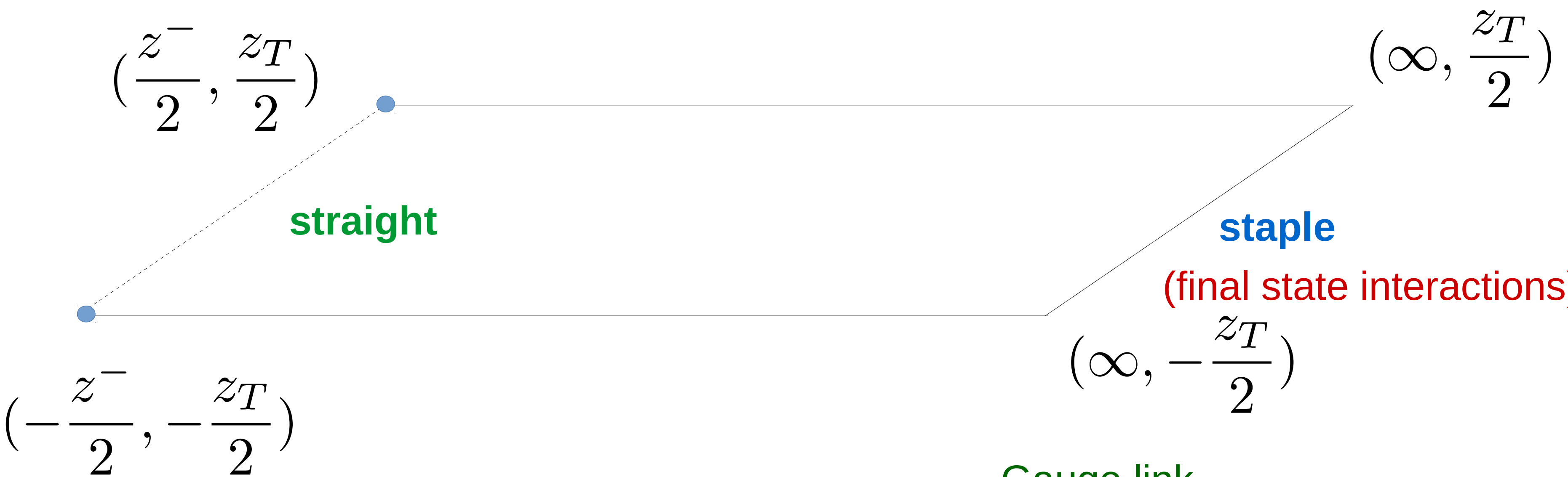
mass term

$$\int dx\, \underline{x}^2 \left(E'_{2T} + 2\widetilde{H}'_{2T}\right) = -\frac{1}{3}\int dx x^2 \widetilde{H} - \frac{2}{3}\int dx x H + \boxed{\frac{2m}{3M}\int dx x (E_T + 2\widetilde{H}_T)}$$
$$- \boxed{\frac{2}{3}\int dx x \mathcal{M}_{G_{11}}\Big|_{v=0}}$$

Genuine Twist Three d_2

$$\int dx\, x \int d^2k_T\, \mathcal{M}^{i,A}_{\Lambda'\Lambda} = \frac{g}{4(P^+)^2}\epsilon^{ij}\langle p', \Lambda' | \bar{\psi}(0)\gamma^+ F^{+j}(0)\psi(0)|p,\Lambda\rangle$$

Staple gauge link



Gauge link

$$\int \frac{d^4z}{2\pi} e^{ik.z} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\mathcal{U}\Gamma\psi(z/2) \mid p, \Lambda\rangle$$

Non local operator

$$\frac{dF_{14}^{e(1)}}{dx} = \tilde{E}_{2T} + H + E + \underline{\mathcal{A}_{F_{14}}}$$

LIR violating term

$$
\begin{aligned}
\mathcal{A}_{F_{14}}(x) &\equiv v^{-}\frac{(2P^{+})^{2}}{M^{2}}\int d^{2}k_{T}\int dk^{-}\left[\frac{k_T\cdot\Delta_T}{\Delta_T^2}(A_{11}+xA_{12})+A_{14}\right.\\
&+ \left.\frac{k_T^2\Delta_T^2-(k_T\cdot\Delta_T)^2}{\Delta_T^2}\left(\frac{\partial A_8}{\partial(k\cdot v)}+x\frac{\partial A_9}{\partial(k\cdot v)}\right)\right]\\
&= \frac{dF_{14}^{(1)}}{dx}-\left.\frac{dF_{14}^{(1)}}{dx}\right|_{v=0} \qquad (1)
\end{aligned}
$$

$$
F_{14}^{(1)}-\left.F_{14}^{(1)}\right|_{v=0}=\mathcal{M}_{F_{14}}-\mathcal{M}_{F_{14}}|_{v=0} \; (1)
$$

$$
\mathcal{A}_{F_{14}}(x)=\frac{d}{dx}\left(\mathcal{M}_{F_{14}}-\mathcal{M}_{F_{14}}|_{v=0}\right) (1)
$$

$$
\begin{aligned}
-\int dx\,\left(F_{14}^{(1)}-\left.F_{14}^{(1)}\right|_{v=0}\right)\Big|_{\Delta_T=0} &= \qquad (1)\\
-\frac{\partial}{\partial\Delta^i}i\epsilon^{ij}gv^{-}\frac{1}{2P^{+}}\int_0^1 ds\,\langle p',+|\bar{\psi}(0)\gamma^{+}U(0,sv)F^{+j}(sv)U(sv,0)\psi(0)|p,+\rangle\Big|_{\Delta_T=0}&,
\end{aligned}
$$