Quasi Parton Distribution Functions in the Covariant Parton Model

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QGT Temple Meeting
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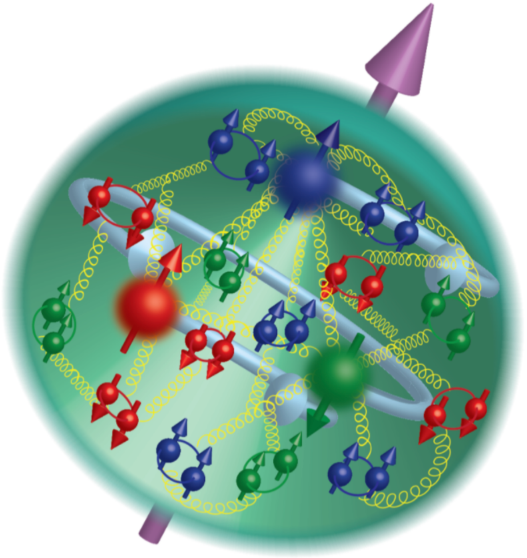
Covariant Parton Model

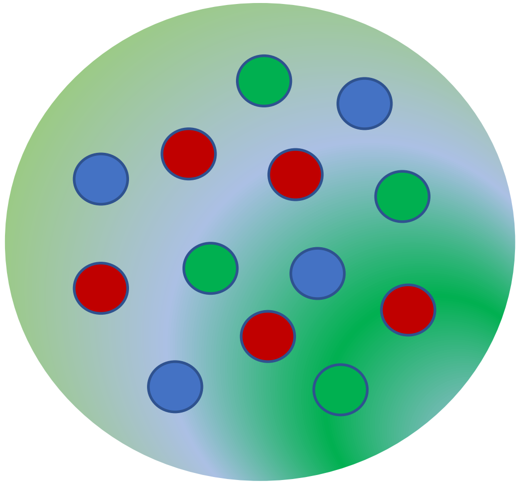
Zavada, 1996 Phys. Rev. D 55 4290

QCD Structure



Valence quarks, sea quarks, sea antiquarks and gluons all of which are spinning and also orbiting each other, bounded in the nucleon

Covariant Parton Model



Non-interacting quarks are on mass shell

$$k^2 = m^2.$$

Spherical phase space in the rest frame

$$\sqrt{k_z^2 + k_T^2} \leq k_m$$

Covariant Parton Model

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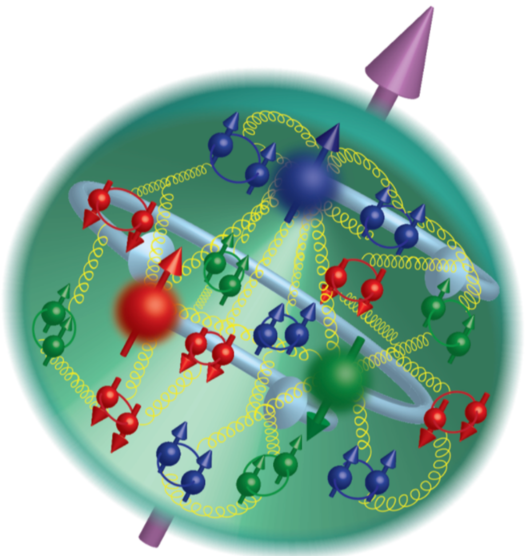

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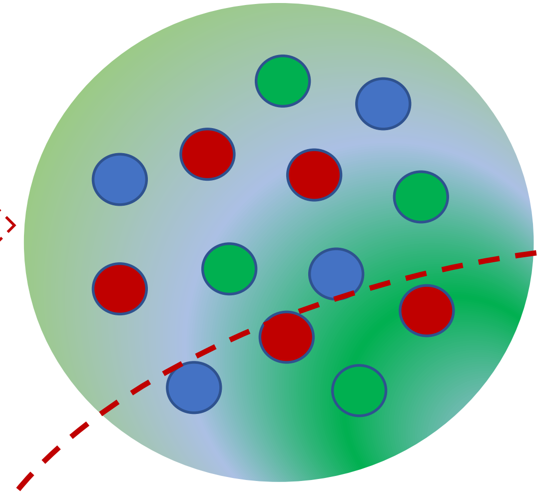

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Now we can define a polarization vector for partons:

$$\omega_\mu = AP_\mu + BS_\mu + Ck_\mu$$

Covariant Parton Model

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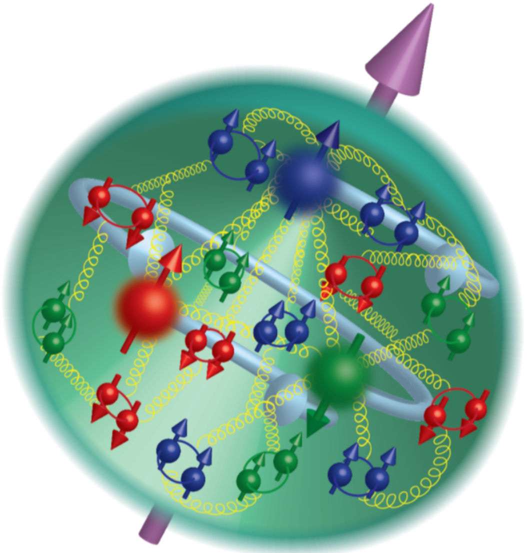

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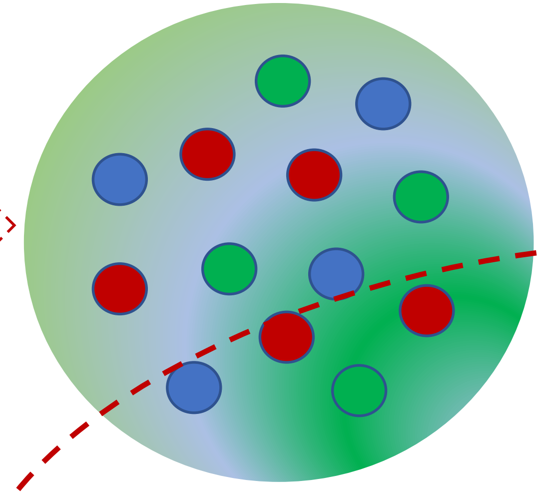

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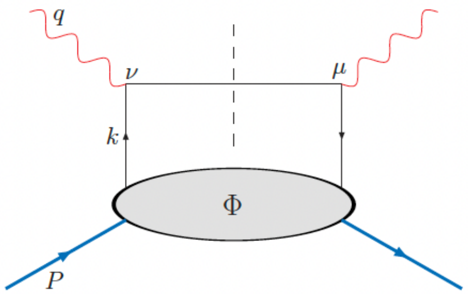
$$0 \leq k_T^2 \leq M^2\left(x - \frac{m^2}{M^2}\right)(1 - x)$$

$$\frac{m_q^2}{M_N^2} < x < 1$$

Covariant Parton Model



$$\Phi(P,k,S,\eta) = MA_1 + \not{P}A_2 + \not{k}A_3 + \frac{1}{2M}[\not{P},\not{k}]A_4 + i(k\cdot S)\gamma_5 A_5 + M\not{S}\gamma_5 A_6 + \frac{k\cdot S}{M}\not{P}\gamma_5 A_7 + \frac{k\cdot S}{M}\not{k}\gamma_5 A_8$$
$$+\frac{[\not{P},\not{S}]}{2}\gamma_5 A_9 + \frac{[\not{k},\not{S}]}{2}\gamma_5 A_{10} + \frac{k\cdot S}{2M^2}[\not{P},\not{k}]\gamma_5 A_{11} + \frac{1}{M}\epsilon^{\mu\nu\rho\sigma}\gamma_\mu P_\nu k_\rho S_\sigma A_{12} + \mathcal{O}(B_i)$$

There are 32 amplitudes in this most general case

no gauge field + on mass shell quarks + pure spin states

$$\Phi(k,P,S) = (\not{k}+m)A_3 + \frac{(\not{k}+m)}{M^2}\Big[(P\cdot k) + mM\Big]A_{11}\not{n}\gamma_5$$

Drops down to 2 amplitudes after applying model's constraints

CPM correlator

$$\Phi(k,P,S)_{ij} = 2P^0\Theta(k^0)\delta(k^2-m^2)\bar{u}_j(k)u_i(k)\times\begin{cases}\mathcal{G}(kP) & \text{unpolarized partons}\\ \mathcal{H}(kP) & \text{polarized partons.}\end{cases}$$

$$\mathcal{G}^q(P\cdot k) = -\frac{1}{\pi M^3}\left[\frac{d}{dx}\frac{f_1^a(x)}{x}\right]$$
$$\mathcal{H}^q(P\cdot k) = \frac{1}{\pi x^2 M^3}\left[3g_1^a(x) + 2\int_x^1\frac{dy}{y}g_1^a(y) - x\frac{dg_1^a(x)}{dx}\right]$$

Bastami, Efremov, Schweitzer, Teryaev, Zavada
Phys.Rev.D 103 (2021)

What can be calculated with the Covariant Parton Model? T-even Twist-2 and 3 distributions

		Quark polarization		
Nucleon polarization		U	L	T
	U	f_1		$h_1^\perp$
	L		g_1	$h_{1L}^\perp$
	T	$f_{1T}^\perp$	$g_{1T}^\perp$	h_1 $h_{1T}^\perp$

T-even TMDs (in blue color) can be computed in models based on quark degrees of freedom only. T-odd TMDs (in red color) require explicit gauge field degrees of freedom, and cannot be modeled in the approach used in this model.

$$
\begin{aligned}
\phi^{[1]} &= \frac{M}{P^+}\left[e - \frac{\varepsilon^{jk}k_T^j S_T^k}{M} e_T^\perp\right],\\
\phi^{[i\gamma^5]} &= \frac{M}{P^+}\left[S_L e_L + \frac{\mathbf{k_T}.\mathbf{S_T}}{M} e_T\right],\\
\phi^{[\gamma^j]} &= \frac{M}{P^+}\left[\frac{k_T^j}{M} f^\perp + \varepsilon^{jk} S_T^k f_T + S_L \frac{\varepsilon^{jk}k_T^k}{M} f_L^\perp - \frac{\kappa^{jk}\varepsilon^{kl}S_T^l}{M^2} f_T^\perp\right],\\
\phi^{[\gamma^j\gamma^5]} &= \frac{M}{P^+}\left[S_T^j g_T + S_L \frac{k_T^j}{M} g_L^\perp + \frac{\kappa^{jk}S_T^k}{M^2} g_T^\perp + \frac{\varepsilon^{jk}k_T^k}{M} g^\perp\right],\\
\phi^{[i\sigma^{jk}\gamma^5]} &= \frac{M}{P^+}\left[\frac{S_T^j k_T^k - S_T^k k_t^j}{M} h_T^\perp - \varepsilon^{jk} h\right],\\
\phi^{[\gamma^j]} &= \frac{M}{P^+}\left[S_L h_L + \frac{\mathbf{k_T}.\mathbf{S_T}}{M} h_T\right].
\end{aligned}
$$

$e_T^\perp, e_L, e_T, f_T, f_L^\perp, f_T^\perp, g_T$, and h cannot be calculated in CPM because they are T-odd

Consistency of the covariant parton model – Lorentz invariance relations

Lorentz invariance relations (LIRs) **connect** the twist-2 and twist-3 parton distribution functions (PDFs) and weighted moments of transverse momentum dependent (TMD) correlation functions

LIRs are satisfied in the covariant parton model. ✓

$$g_T^q(x) \overset{\text{LIR}}{=} g_1^q(x) + \frac{\mathrm{d}}{\mathrm{d}x} g_{1T}^{\perp(1)q}(x)\,,$$

$$h_L^q(x) \overset{\text{LIR}}{=} h_1^q(x) - \frac{\mathrm{d}}{\mathrm{d}x} h_{1L}^{\perp(1)q}(x)\,,$$

$$h_T^q(x) \overset{\text{LIR}}{=} -\frac{\mathrm{d}}{\mathrm{d}x} h_{1T}^{\perp(1)q}(x)\,,$$

$$g_L^{\perp q}(x) + \frac{\mathrm{d}}{\mathrm{d}x} g_T^{\perp(1)q}(x) \overset{\text{LIR}}{=} 0\,,$$

$$h_T^q(x,p_T) - h_T^{\perp q}(x,p_T) \overset{\text{LIR}}{=} h_{1L}^{\perp q}(x,p_T)\,,$$

$$g_{1T}^{(1)}(x) = \int d^2\mathbf{k}_T \frac{\mathbf{k}_T^2}{2M^2} g_{1T}(x,\mathbf{k}_T^2), \quad \text{etc.}$$

P.J. Mulders and R.D. Tangerman, Nucl. Phys. B 461, 197-237 (1996).
D. Boer and P.J. Mulders, Phys. Rev. D 57, 5780 (1998).

Consistency of the covariant parton model – Equation of motion relations

Twist-3 TMDs = Contribution from (Genuine twist 3 TMDs + Twist-2 TMDs + Mass terms)

Equation of motion relations <u>are satisfied</u> in the covariant parton model when genuine twist-3 terms are set to zero. ✓

$$xe = x\tilde{e} + \frac{m}{M} f_1$$

$$xf^{\perp} = x\tilde{f}^{\perp} + f_1$$

$$xg_L^{\perp} = x\tilde{g}_L^{\perp} + g_1 + \frac{m}{M} h_{1L}^{\perp}$$

$$xg_T = \tilde{g}_T + g_{1T}^{\perp(1)} + \frac{m}{M} h_1$$

$$xg_T^{\perp} = x\tilde{g}_T^{\perp} + g_{1T}^{\perp} + \frac{m}{M} h_{1T}^{\perp}$$

$$xh_L = x\tilde{h}_L - 2h_{1L}^{\perp(1)} + \frac{m}{M} g_1$$

$$xh_T = x\tilde{h}_T - h_1 - h_{1T}^{\perp(1)} + \frac{m}{M} g_{1T}^{\perp}$$

$$xh_T^{\perp} = x\tilde{h}_T^{\perp} + h_1 - h_{1T}^{\perp(1)}$$

Consistency of the covariant parton model – WW relations

$$g_T^q(x) \overset{\rm WW}{=} \int_x^1 \frac{dy}{y}\, g_1^q(y) + \frac{m}{M}\left[-\frac{h_1^q(x)}{x} + \int_x^1 \frac{dy}{y^2}\, h_1^q(y)\right],$$

$$h_L^q(x) \overset{\rm WW}{=} 2x \int_x^1 \frac{dy}{y^2}\, h_1^q(y) + \frac{m}{M}\left[\frac{g_1^q(x)}{x} - 2x \int_x^1 \frac{dy}{y^3}\, g_1^q(y)\right].$$

✓

Quark model relations in Covariant Parton Model

$$\begin{aligned}
g_{1T}^{\perp q}(x,p_T) &= -h_{1L}^{\perp q}(x,p_T),\\
g_T^{\perp q}(x,p_T) &= -h_{1T}^{\perp q}(x,p_T),\\
g_L^{\perp q}(x,p_T) &= -h_T^q(x,p_T),\\
g_1^q(x,p_T) - h_1^q(x,p_T) &= h_{1T}^{\perp(1)q}(x,p_T),\\
g_T^q(x,p_T) - h_L^q(x,p_T) &= h_{1T}^{\perp(1)q}(x,p_T),\\
h_T^q(x,p_T) - h_T^{\perp q}(x,p_T) &= h_{1L}^{\perp q}(x,p_T).
\end{aligned}$$

These relations are valid in a large class of quark models, including spectator models, bag model, light-front constituent quark model ✓

qPDFs in quark models

Point splitting in + direction

Twist 2 PDF: $$f_1^q(x) = \frac{1}{2P^+}\int d^4k \, \mathrm{tr}\left[\Phi^q(k,P,S)\gamma^+\right]\delta\left(x - \frac{k^+}{P^+}\right),$$

Its quasi counterpart: $$D^q(x_v,\Gamma,v) = \frac{1}{2P^3}\int d^4k \, \mathrm{tr}\left[\Phi^q(k,P,S)\,\Gamma\,\right]\delta\left(x - \frac{k^3}{P^3}\right).$$

Point splitting in z direction

$\Gamma = \gamma^0 \; or \; \gamma^3$

$$\Phi(P,k,S,\eta) = MA_1 + \not{P}A_2 + \not{k}A_3 + \frac{1}{2M}[\not{P},\not{k}]A_4 + i(k\cdot S)\gamma_5 A_5 + M\not{S}\gamma_5 A_6 + \frac{k\cdot S}{M}\not{P}\gamma_5 A_7 + \frac{k\cdot S}{M}\not{k}\gamma_5 A_8$$
$$+\frac{[\not{P},\not{S}]}{2}\gamma_5 A_9 + \frac{[\not{k},\not{S}]}{2}\gamma_5 A_{10} + \frac{k\cdot S}{2M^2}[\not{P},\not{k}]\gamma_5 A_{11} + \frac{1}{M}\epsilon^{\mu\nu\rho\sigma}\gamma_\mu P_\nu k_\rho S_\sigma A_{12} + \mathcal{O}(B_i)$$

Quark models- no gauge field degrees of freedom- T-odd amplitudes, $\bar{A}_4$, $\bar{A}_5$, $\bar{A}_{12}$ and all the $\bar{B}_i$ amplitudes are absent

$$f_1^q(x) = 2\int d^4k\left(A_2^q + x\,A_3^q\right)\delta\left(x - \frac{k^+}{P^+}\right),$$
$$D^q(x_v,\gamma^\mu,v) = 2\int d^4k\left(\frac{P^\mu}{P^3}A_2^q + \frac{k^\mu}{P^3}A_3^q\right)\delta\left(x - \frac{k^3}{P^3}\right),\quad \mu = 0,\,3\,.$$

Sum rules

Number of valence quarks of flavor q

Flavor sum rule: $$\int_0^1 dx \left(f_1^q(x) - f_1^{\bar{q}}(x) \right) \equiv \int_{-1}^1 dx\, f_1^q(x) = N^q \,,$$

Momentum sum rule: $$\int_0^1 dx\, x \left(f_1^q(x) + f_1^{\bar{q}}(x) \right) \equiv \int_{-1}^1 dx\, x\, f_1^q(x) = A^q(0) \,,$$

A form factor of the energy momentum tensor at zero momentum transfer

$\sum_a A^a(0) = 1$

Flavor sum rules:

$$\int_0^\infty dx \left(D^q(x_v, \gamma^0, v) - D^{\bar{q}}(x_v, \gamma^0, v) \right) = \int_{-\infty}^\infty dx\, D^q(x_v, \gamma^0, v) = \frac{N^q}{v} \,,$$

$$\int_0^\infty dx \left(D^q(x_v, \gamma^3, v) - D^{\bar{q}}(x_v, \gamma^3, v) \right) = \int_{-\infty}^\infty dx\, D^q(x_v, \gamma^3, v) = N^q \,,$$

Momentum sum rule:

$$\int_0^\infty dx\, x \left(D^q(x_v, \gamma^0, v) + D^{\bar{q}}(x_v, \gamma^0, v) \right) = \int_{-\infty}^\infty dx\, x\, D^q(x_v, \gamma^0, v) = \frac{A^q(0)}{v} \,,$$

$$\int_0^\infty dx\, x \left(D^q(x_v, \gamma^3, v) + D^{\bar{q}}(x_v, \gamma^3, v) \right) = \int_{-\infty}^\infty dx\, x\, D^q(x_v, \gamma^3, v) = A^q(0) - \frac{1 - v^2}{v^2} \bar{c}^q(0) \,,$$

A form factor of the energy momentum tensor at zero momentum transfer $\sum_a \bar{c}^a(t) = 0$

Bhattacharya, Cocuzza, Metz, Phys.Rev.D 102 (2020) 5, 054021

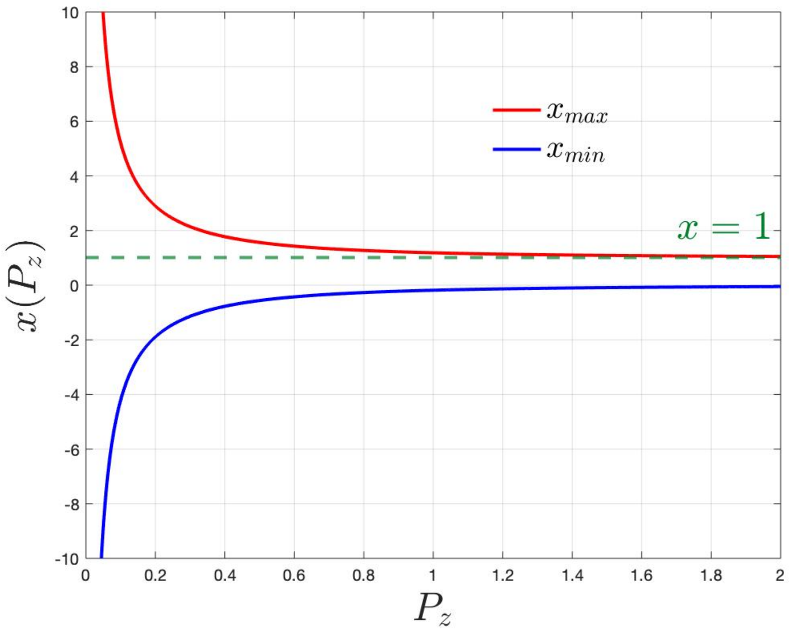
qPDFs in the Covariant Parton Model

In renormalizable theories

- PDFs : -1≤ $x \leq 1$
- qPDFs: -∞ ≤ $x \leq \infty$

In CPM

- PDFs : $\frac{m_q^2}{M_N^2} \leq x \leq 1$
- qPDFs: $x_{min} < x < x_{max}$



$$\tilde{x}_{max} = \frac{1}{2}(1+\frac{m_q^2}{M^2}) + \frac{1}{2}\sqrt{(1+\frac{m_q^2}{M^2})^2 - 4\frac{m_q^2}{M^2}(1+\frac{M^2}{P_z^2}) + \frac{M^2}{P_z^2}(1+\frac{m_q^2}{M^2})^2}$$

$$\tilde{x}_{min} = \frac{1}{2}(1+\frac{m_q^2}{M^2}) - \frac{1}{2}\sqrt{(1+\frac{m_q^2}{M^2})^2 - 4\frac{m_q^2}{M^2}(1+\frac{M^2}{P_z^2}) + \frac{M^2}{P_z^2}(1+\frac{m_q^2}{M^2})^2}$$

As $P^z \to 0$ $\quad \tilde{x}_{max} = \frac{M}{2P^z}(1-\frac{m_q^2}{M^2}) \to \infty$

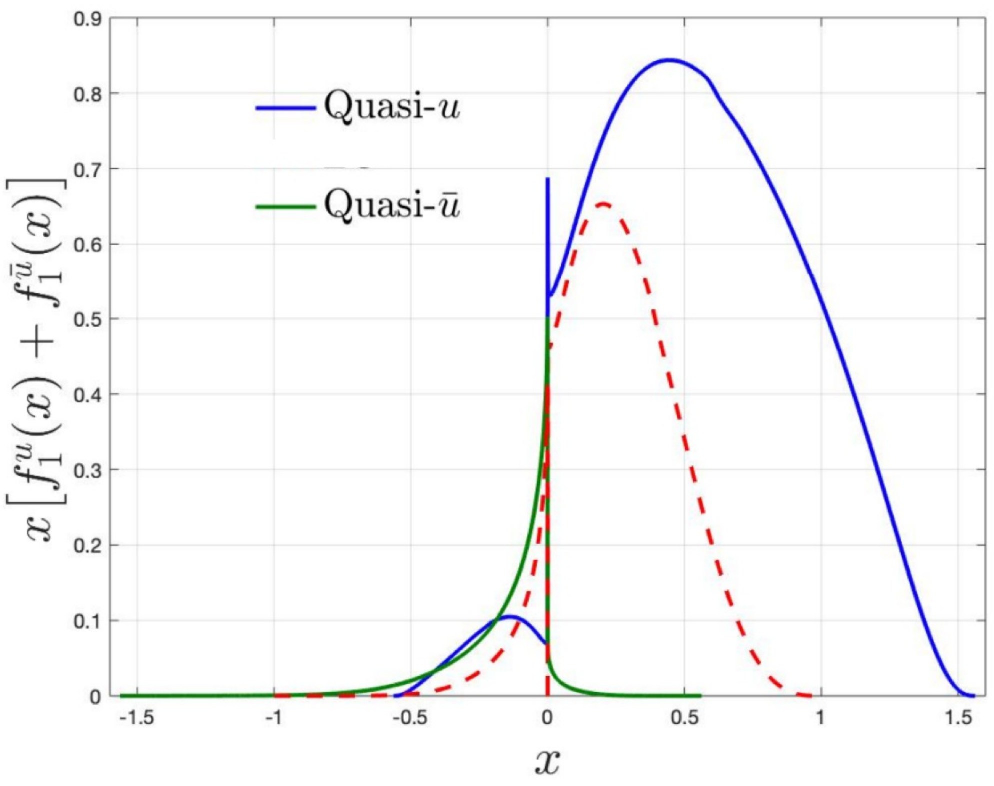
$$\tilde{x}_{min} = -\frac{M}{2P^z}(1-\frac{m_q^2}{M^2}) \to -\infty$$

As $P^z \to \infty$ $\quad \tilde{x}_{max} = 1$

$$\tilde{x}_{min} = \frac{m_q^2}{M^2}$$

qPDFs in the Covariant Parton Model

- Quark distribution leaks to anti-quark distribution and vice versa



qPDFs in the Covariant Parton Model

- $D^q(x_v, \gamma^0, v)$ and $D^q(x_v, \gamma^3, v)$ are related

$$D^q(x_v,\gamma^0,v) = v\,D^q(x_v,\gamma^3,v) + (1-v^2)\;2\,\pi\,M\int_{L(v)}^{\frac{1}{2}M} dk\;k\,\mathcal{G}^q(Mk)\,.$$

- EMT Form factors are calculated and found that

1) Sum rules are satisfied

$$\int_0^\infty dx\left(D^q(x_v,\gamma^0,v) - D^{\bar q}(x_v,\gamma^0,v)\right) = \int_{-\infty}^\infty dx\,D^q(x_v,\gamma^0,v) = \frac{N^q}{v}\,,$$
$$\int_0^\infty dx\left(D^q(x_v,\gamma^3,v) - D^{\bar q}(x_v,\gamma^3,v)\right) = \int_{-\infty}^\infty dx\,D^q(x_v,\gamma^3,v) = N^q\,,$$
$$\int_0^\infty dx\,x\left(D^q(x_v,\gamma^0,v) + D^{\bar q}(x_v,\gamma^0,v)\right) = \int_{-\infty}^\infty dx\,x\,D^q(x_v,\gamma^0,v) = \frac{A^q(0)}{v}\,,$$
$$\int_0^\infty dx\,x\left(D^q(x_v,\gamma^3,v) + D^{\bar q}(x_v,\gamma^3,v)\right) = \int_{-\infty}^\infty dx\,x\,D^q(x_v,\gamma^3,v) = A^q(0) - \frac{1-v^2}{v^2}\,\bar c^q(0)\,,$$

2)

$$\bar c^q(0) = -\,\tfrac{1}{4}\,A^q(0)\,,\quad \text{CPM}$$
$$\bar c^q(0) = -\,\tfrac{1}{4}\,A^q(0)\,,\quad \text{bag model}$$

This might a consequence of the fact that the quarks inside the bag obey the free Dirac equation as they do in the CPM.

Outlook

- Polarized qPDFs to be completed
- Making the model more realistic by including off-shell-ness effects
- Wish to access T-odd TMDs
- Calculating other distributions and quasi distributions: GTMDs, GPDs, etc..
- .
- .
- .

THANK YOU