PDFs and qPDFs in the Covariant Parton Model

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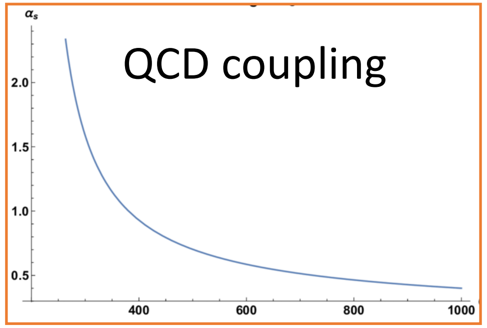
Outline

- Factorization
- Feynman’s parton model
- Covariant parton model (CPM)
- History of the CPM
- Current formulation of CPM
- Success of the model

- Quasi parton distribution functions (qPDFs)
- qPDFs in quark models
- qPDFs in CPM

- Summary and outlook

Factorization



Related to the target (intrinsic properties)

<u>Soft part: The PDF</u>

- Process independent
- Non-perturbative

$$F(x_{bj}, Q) = \int_{x_{bj}}^{1} \frac{d\xi}{\xi} \hat{H}\left(\frac{x_{bj}}{\xi}, \frac{\mu}{Q}\right) f(\xi, \mu) + \mathcal{O}\left(\frac{m}{Q}\right)$$

<u>The physical observable:</u>

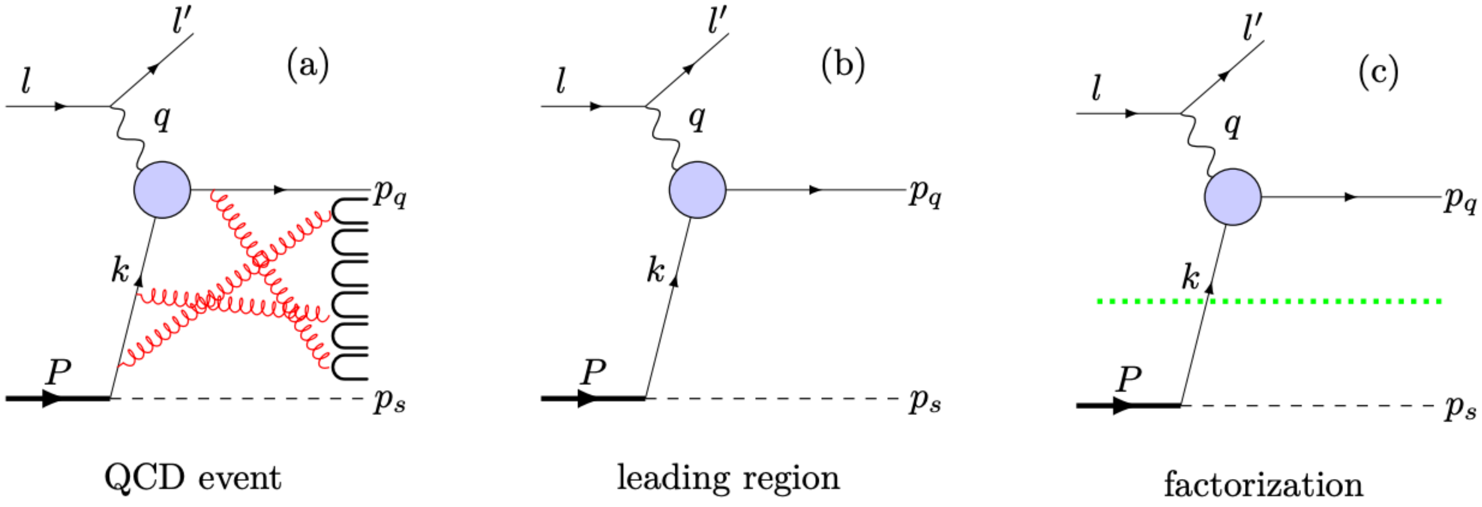
Structure Function

- *Expansion over Q*

Related to the parton (collision)

<u>Hard part: Partonic coefficient function</u>

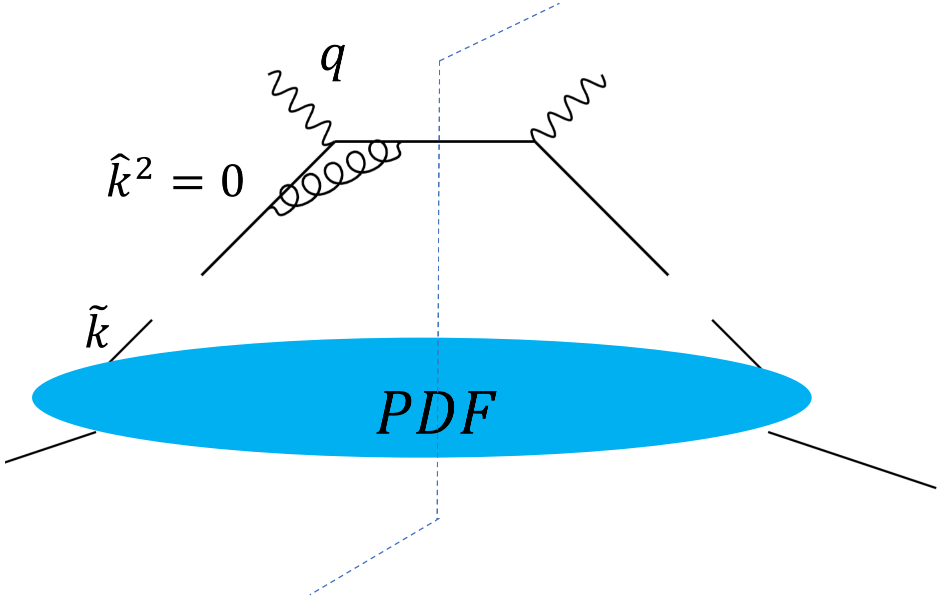
- Process dependent
- Perturbative



Feynman's Parton Model

Perturbative expansion
$A + B\alpha(\mu)^2 + \cdots$

$$F(x_{bj}, Q) = \int_{x_{bj}}^{1} \frac{d\xi}{\xi}\, \hat{H}\left(\frac{x_{bj}}{\xi}, \frac{\mu}{Q}\right) f(\xi, \mu) + \mathcal{O}\left(\frac{m}{Q}\right)$$



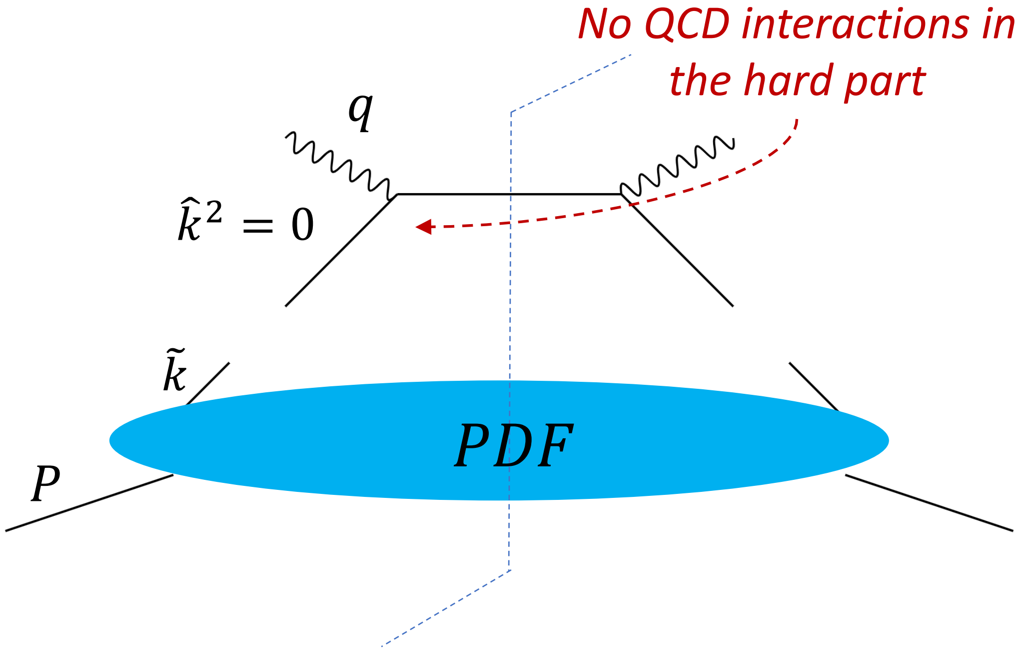
$Q \to \infty$

No evolution of the PDF

$$F(x_{bj}) = \int_{x_{bj}}^{1} \frac{d\xi}{\xi}\, \hat{H}\left(\frac{x_{bj}}{\xi}\right) f(\xi)$$

Bjorken scaling
No Q dependence

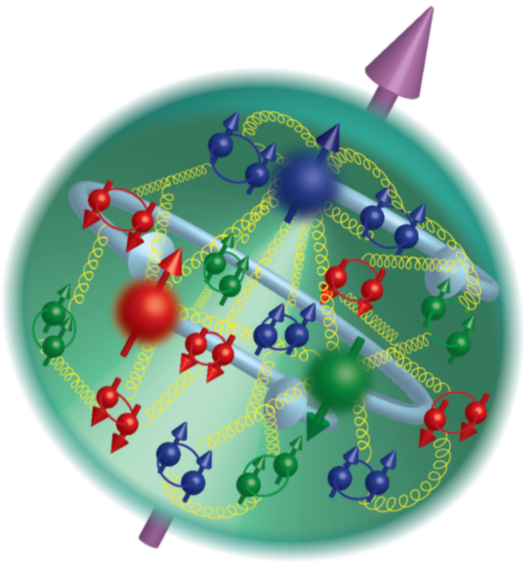
No QCD interactions in the hard part



Covariant Parton Model

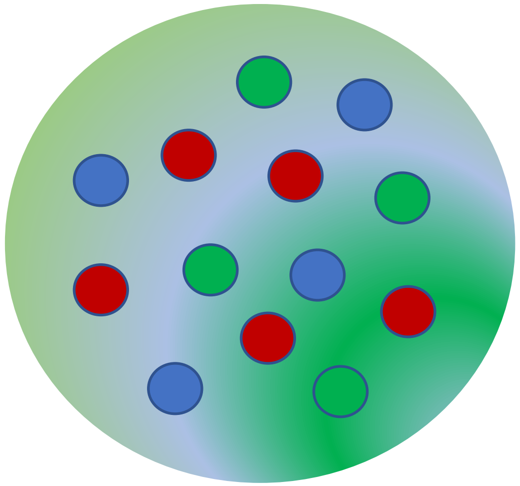
Zavada, 1996 Phys. Rev. D 55 4290

QCD Structure



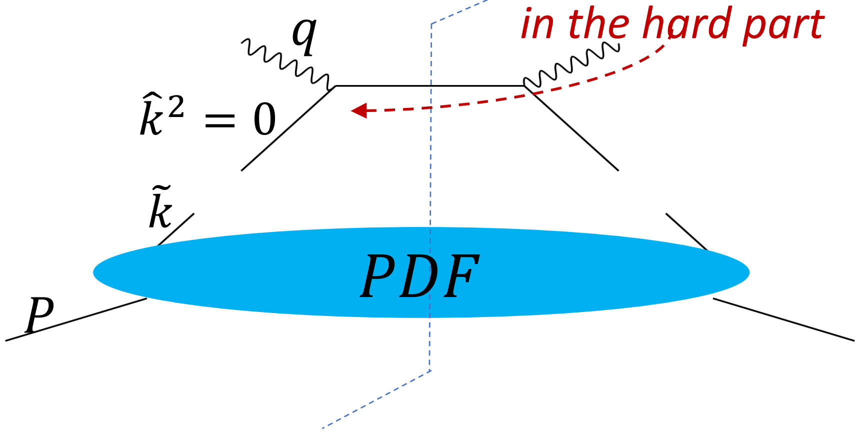
Valence quarks, sea quarks, sea antiquarks and gluons all of which are spinning and also orbiting each other, bounded in the nucleon

Covariant Parton Model



Non-interacting quarks are on mass shell
$k^2 = m^2$.
Spherical phase space in the rest frame
$\sqrt{k_z^2 + k_T^2} \leq k_m$

No QCD interactions in the hard part



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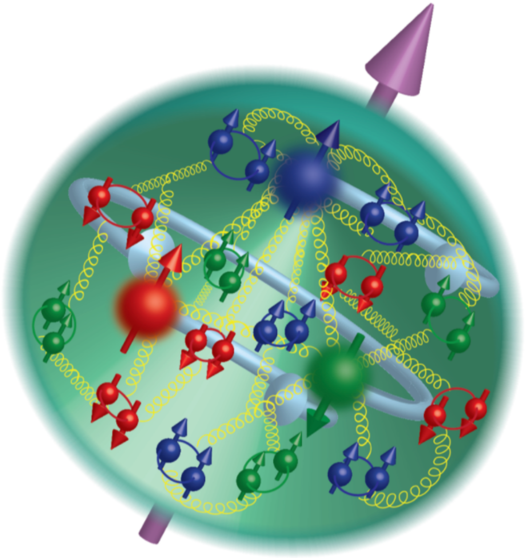
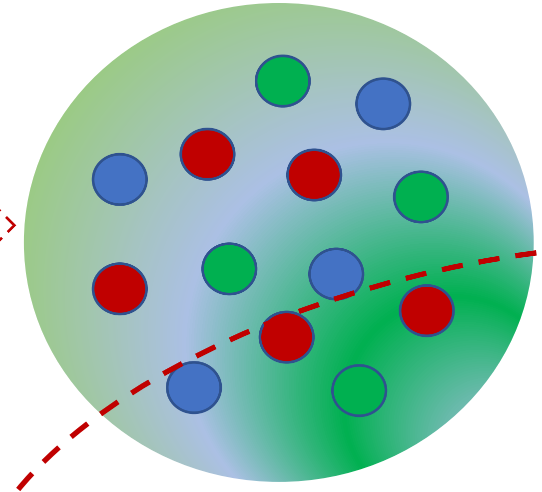


Valence quarks, sea quarks, sea antiquarks and gluons all of which are spinning and also orbiting each other, bounded in the nucleon

Covariant Parton Model



Non-interacting quarks are on mass shell

$$k^2 = m^2.$$

Spherical phase space in the rest frame

$$\sqrt{k_z^2 + k_T^2} \leq k_m$$

Now we can define a polarization vector for partons:

$$\omega_\mu = AP_\mu + BS_\mu + Ck_\mu$$

Covariant Parton Model

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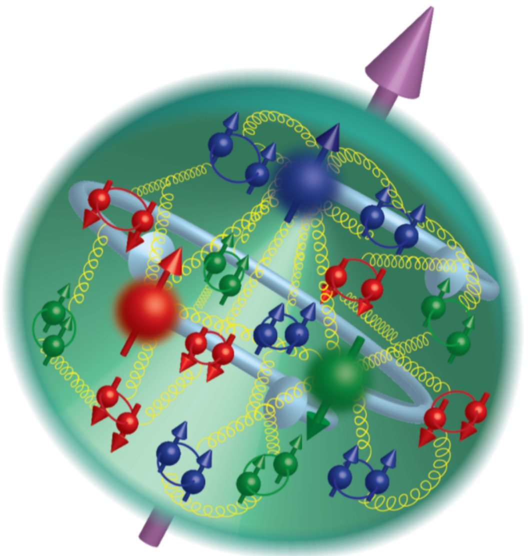
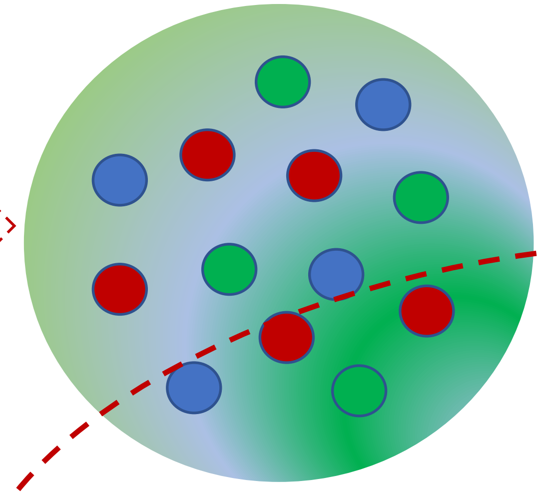
QCD Structure



Valence quarks, sea quarks, sea antiquarks and gluons all of which are spinning and also orbiting each other, bounded in the nucleon

Covariant Parton Model



Non-interacting quarks are on mass shell

$$k^2 = m^2.$$

Spherical phase space in the rest frame

$$\sqrt{k_z^2 + k_T^2} \leq k_m$$

Now we can define a polarization vector for partons:

$$\omega_\mu = AP_\mu + BS_\mu + Ck_\mu$$

$$0 \leq k_T^2 \leq M^2\left(x - \frac{m^2}{M^2}\right)(1 - x)$$

$$\frac{m_q^2}{M_N^2} < x < 1$$

Covariant Parton Model - History

- Description of the hadronic tensor

P. Zavada, Phys. Rev. D 55, 4290 (1997) [hep-ph/9609372]
P. Zavada, Phys. Rev. D 65, 054040 (2002) [hep-ph/0106215]
P. Zavada, Phys. Rev. D 67, 014019 (2003) [hep-ph/0210141]

$\Big\} \; f_1(x),\; g_1(x),\; g_T(x)$

- Auxiliary polarized process due to the interference of vector and scalar currents

V. Efremov, O. V. Teryaev and P. Zavada, Phys. Rev. D 70, 054018 (2004) [hep-ph/0405225].

$\Big\} \; \ldots + h_1(x)$

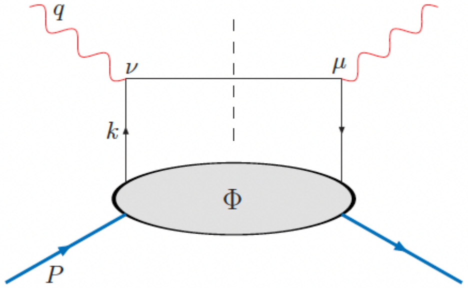
- Unintegrated structure functions," to describe twist-2 T-even TMDs.

A. V. Efremov, P. Schweitzer, O. V. Teryaev and P. Zavada,
Phys. Rev. D 80, 014021 (2009) [arXiv:0903.3490 [hep-ph]].

$\Big\} \; \ldots + f_1(x,k_T), g_1(x,k_T), h_1(x,k_T), g_{1T}^{\perp}(x,k_T), h_{1L}^{\perp}(x,k_T), h_{1T}^{\perp}(x,k_T)$

Still no access to the twist-3 TMDs !

Covariant Parton Model - The correlator



$$\Phi(P,k,S,\eta) = MA_1 + \slashed{P}A_2 + \slashed{k}A_3 + \frac{1}{2M}[\slashed{P},\slashed{k}]A_4 + i(k\cdot S)\gamma_5 A_5 + M\slashed{S}\gamma_5 A_6 + \frac{k\cdot S}{M}\slashed{P}\gamma_5 A_7 + \frac{k\cdot S}{M}\slashed{k}\gamma_5 A_8$$
$$+\frac{[\slashed{P},\slashed{S}]}{2}\gamma_5 A_9 + \frac{[\slashed{k},\slashed{S}]}{2}\gamma_5 A_{10} + \frac{k\cdot S}{2M^2}[\slashed{P},\slashed{k}]\gamma_5 A_{11} + \frac{1}{M}\epsilon^{\mu\nu\rho\sigma}\gamma_\mu P_\nu k_\rho S_\sigma A_{12} + \mathcal{O}(B_i)$$

There are 32 amplitudes in this most general case

no gauge field + on mass shell quarks + pure spin states

$$\Phi(k,P,S) = (\slashed{k}+m)A_3 + \frac{(\slashed{k}+m)}{M^2}\Big[(P\cdot k) + mM\Big]A_{11}\slashed{S}\gamma_5$$

Drops down to 2 amplitudes after applying model's constraints

CPM correlator

$$\Phi(k,P,S)_{ij} = 2P^0\Theta(k^0)\delta(k^2-m^2)\bar{u}_j(k)u_i(k) \times \begin{cases} \mathcal{G}(kP) & \text{unpolarized partons} \\ \mathcal{H}(kP) & \text{polarized partons.} \end{cases}$$

$$\mathcal{G}^q(P\cdot k) = -\frac{1}{\pi M^3}\left[\frac{d}{dx}\frac{f_1^a(x)}{x}\right]$$
$$\mathcal{H}^q(P\cdot k) = \frac{1}{\pi x^2 M^3}\left[3g_1^a(x) + 2\int_x^1 \frac{dy}{y}\, g_1^a(y) - x\frac{dg_1^a(x)}{dx}\right]$$

Bastami, Efremov, Schweitzer, Teryaev, Zavada
Phys.Rev.D 103 (2021)

What can be calculated with the Covariant Parton Model? T-even Twist-2 and 3 distributions

Nucleon polarization	Quark polarization		
	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}^\perp$	h_1 $h_{1T}^\perp$

T-even TMDs (in blue color) can be computed in models based on quark degrees of freedom only. T-odd TMDs (in red color) require explicit gauge field degrees of freedom, and cannot be modeled in the approach used in this model.

$$\phi^{[1]} = \frac{M}{P^+}\left[e - \frac{\varepsilon^{jk} k_T^j S_T^k}{M} e_T^\perp\right],$$

$$\phi^{[i\gamma^5]} = \frac{M}{P^+}\left[S_L e_L + \frac{\mathbf{k_T}.\mathbf{S_T}}{M} e_T\right],$$

$$\phi^{[\gamma^j]} = \frac{M}{P^+}\left[\frac{k_T^j}{M} f^\perp + \varepsilon^{jk} S_T^k f_T + S_L \frac{\varepsilon^{jk} k_T^k}{M} f_L^\perp - \frac{\kappa^{jk}\varepsilon^{kl} S_T^l}{M^2} f_T^\perp\right],$$

$$\phi^{[\gamma^j\gamma^5]} = \frac{M}{P^+}\left[S_T^j g_T + S_L \frac{k_T^j}{M} g_L^\perp + \frac{\kappa^{jk} S_T^k}{M^2} g_T^\perp + \frac{\varepsilon^{jk} k_T^k}{M} g^\perp\right],$$

$$\phi^{[i\sigma^{jk}\gamma^5]} = \frac{M}{P^+}\left[\frac{S_T^j k_T^k - S_T^k k_t^j}{M} h_T^\perp - \varepsilon^{jk} h\right],$$

$$\phi^{[\gamma^j]} = \frac{M}{P^+}\left[S_L h_L + \frac{\mathbf{k_T}.\mathbf{S_T}}{M} h_T\right].$$

$e_T^\perp, e_L, e_T, f_T, f_L^\perp, f_T^\perp, g_T$, and h cannot be calculated in CPM because they are T-odd

Consistency of the covariant parton model – Lorentz invariance relations

Lorentz invariance relations (LIRs) **connect** the twist-2 and twist-3 parton distribution functions (PDFs) and weighted moments of transverse momentum dependent (TMD) correlation functions

LIRs are satisfied in the covariant parton model. ✓

$$g_T^q(x) \stackrel{\text{LIR}}{=} g_1^q(x) + \frac{\mathrm{d}}{\mathrm{d}x} g_{1T}^{\perp(1)q}(x)\,,$$

$$h_L^q(x) \stackrel{\text{LIR}}{=} h_1^q(x) - \frac{\mathrm{d}}{\mathrm{d}x} h_{1L}^{\perp(1)q}(x)\,,$$

$$h_T^q(x) \stackrel{\text{LIR}}{=} -\frac{\mathrm{d}}{\mathrm{d}x} h_{1T}^{\perp(1)q}(x)\,,$$

$$g_L^{\perp q}(x) + \frac{\mathrm{d}}{\mathrm{d}x} g_T^{\perp(1)q}(x) \stackrel{\text{LIR}}{=} 0\,,$$

$$h_T^q(x,p_T) - h_T^{\perp q}(x,p_T) \stackrel{\text{LIR}}{=} h_{1L}^{\perp q}(x,p_T)\,,$$

$$g_{1T}^{(1)}(x) = \int d^2\mathbf{k}_T \frac{\mathbf{k}_T^2}{2M^2} g_{1T}(x,\mathbf{k}_T^2), \quad \text{etc.}$$

P.J. Mulders and R.D. Tangerman, Nucl. Phys. B 461, 197-237 (1996).
D. Boer and P.J. Mulders, Phys. Rev. D 57, 5780 (1998).

Consistency of the covariant parton model – Equation of motion relations

Twist-3 TMDs = Contribution from (Genuine twist 3 TMDs + Twist-2 TMDs + Mass terms)

Equation of motion relations are satisfied in the covariant parton model when genuine twist-3 terms are set to zero. ✓

$$xe = x\tilde{e} + \frac{m}{M} f_1$$

$$xf^{\perp} = x\tilde{f}^{\perp} + f_1$$

$$xg_L^{\perp} = x\tilde{g}_L^{\perp} + g_1 + \frac{m}{M} h_{1L}^{\perp}$$

$$xg_T = \tilde{g}_T + g_{1T}^{\perp(1)} + \frac{m}{M} h_1$$

$$xg_T^{\perp} = x\tilde{g}_T^{\perp} + g_{1T}^{\perp} + \frac{m}{M} h_{1T}^{\perp}$$

$$xh_L = x\tilde{h}_L - 2h_{1L}^{\perp(1)} + \frac{m}{M} g_1$$

$$xh_T = x\tilde{h}_T - h_1 - h_{1T}^{\perp(1)} + \frac{m}{M} g_{1T}^{\perp}$$

$$xh_T^{\perp} = x\tilde{h}_T^{\perp} + h_1 - h_{1T}^{\perp(1)}$$

Consistency of the covariant parton model – WW relations

$$g_T^q(x) \overset{\text{WW}}{=} \int_x^1 \frac{\mathrm{d}y}{y}\, g_1^q(y) + \frac{m}{M}\left[-\frac{h_1^q(x)}{x} + \int_x^1 \frac{\mathrm{d}y}{y^2}\, h_1^q(y)\right],$$

$$h_L^q(x) \overset{\text{WW}}{=} 2x \int_x^1 \frac{\mathrm{d}y}{y^2}\, h_1^q(y) + \frac{m}{M}\left[\frac{g_1^q(x)}{x} - 2x \int_x^1 \frac{\mathrm{d}y}{y^3}\, g_1^q(y)\right].$$

✓

Quark model relations in Covariant Parton Model

$$\begin{aligned}
g_{1T}^{\perp q}(x,p_T) &= -h_{1L}^{\perp q}(x,p_T),\\
g_T^{\perp q}(x,p_T) &= -h_{1T}^{\perp q}(x,p_T),\\
g_L^{\perp q}(x,p_T) &= -h_T^{q}(x,p_T),\\
g_1^q(x,p_T) - h_1^q(x,p_T) &= h_{1T}^{\perp(1)q}(x,p_T),\\
g_T^q(x,p_T) - h_L^q(x,p_T) &= h_{1T}^{\perp(1)q}(x,p_T),\\
h_T^q(x,p_T) - h_T^{\perp q}(x,p_T) &= h_{1L}^{\perp q}(x,p_T).
\end{aligned}$$

These relations are valid in a large class of quark models, including spectator models, bag model, light-front constituent quark model ✓

Antiquark correlator

Independent amplitudes in the quark and antiquark correlators and TMDs in the covariant parton model. Aslan, Bastami, Schweitzer (2020)

$$\bar{\Phi}(k,P,S,\eta) = M\bar{A}_1 + \not{P}\bar{A}_2 + \not{k}\bar{A}_3 + \frac{[\not{P},\not{k}]}{2M}\bar{A}_4 + i(k.S)\gamma_5\bar{A}_5 + M\not{S}\gamma_5\bar{A}_6 + \frac{k.S}{M}\not{P}\gamma_5\bar{A}_7 + \frac{k.S}{M}\not{k}\gamma_5\bar{A}_8 + \frac{[\not{P},\not{S}]}{2}\gamma_5\bar{A}_9 + \frac{[\not{k},\not{S}]}{2}\gamma_5\bar{A}_{10} + \frac{k.S}{2M^2}[\not{P},\not{k}]\gamma_5\bar{A}_{11} + \frac{1}{M}\epsilon^{\mu\nu\rho\sigma}\gamma_\mu P_\nu k_\rho S_\sigma \bar{A}_{12} + \mathcal{O}(\bar{B}_i)$$

I. In models without gauge field degrees of freedom T-odd amplitudes, $\bar{A}_4$, $\bar{A}_5$, $\bar{A}_{12}$ and all the $\bar{B}_i$ amplitudes are absent

II. Assuming the partons are on-shell, $Tr[\Phi\Gamma(\gamma.k-m)]=0$, leads to the relations between some amplitudes

III. Assuming pure spin states, $\omega^2=-1$, leads to $\bar{A}_8=\mp\bar{A}_{11}$

$$\bar{\Phi}(k,P,S) = (\not{k}-m)\bar{A}_3 + \frac{(\not{k}-m)}{M^2}\Big[(P\cdot k)-mM\Big]\bar{A}_{11}\bar{\not{\omega}}\gamma_5$$

$$\bar{\phi}(k,P,S)_{ij} = 2M\delta(k^2-m^2)\Theta(-k^0)\Theta[(P+k)^2]v_i(k)\bar{v}_j(k)\times\begin{cases}\bar{\mathcal{G}}(kP) & \text{unpolarized partons}\\ \bar{\mathcal{H}}(kP) & \text{polarized partons.}\end{cases}$$

Quasi PDFs

PDFs are calculated using light cone coordinates $(+, -, \perp)$

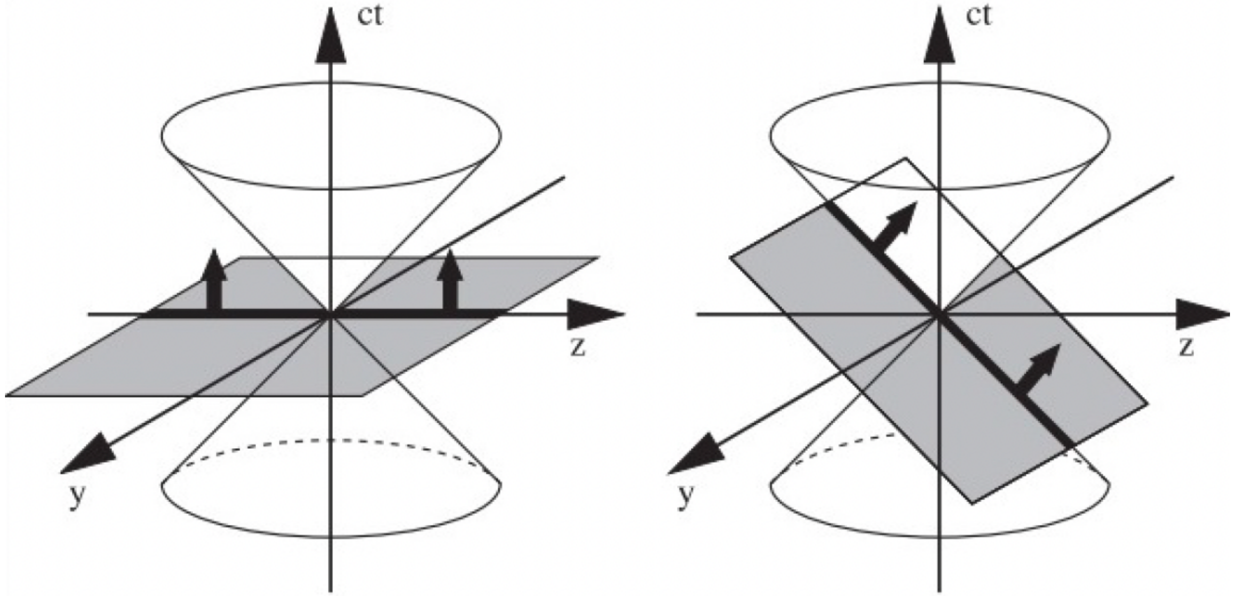
$$q(x,\mu) \equiv \int \frac{d\xi^-}{4\pi} e^{-ixP^+\xi^-} \langle P|\bar{\psi}(\xi^-)\gamma^+ \mathcal{U}(\xi^-,0)\psi(0)|P\rangle,$$

qPDFs are calculated using cartesian coordinates $(0, \perp, z)$

$$\tilde{q}(\tilde{x},\mu,P_z) \equiv \int \frac{dz}{4\pi} e^{-i\tilde{x}P_z z} \langle P|\bar{\psi}(z)\gamma^z \mathcal{U}(z,0)\psi(0)|P\rangle,$$

PDFs and qPDFs are related through matching

$$\tilde{q}(\tilde{x},P_z,\mu) = \int_{-1}^{1} \frac{dx}{x}\, \mathcal{C}(\tilde{x},x,\mu)\; q(x,\mu) + \mathcal{O}(1/P_z)$$



The instant form

$$\tilde{x}^0 = ct,\quad \tilde{x}^1 = x,\quad \tilde{x}^2 = y,\quad \tilde{x}^3 = z$$

$$\tilde{g}_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

The front form

$$\tilde{x}^0 = ct+z,\quad \tilde{x}^1 = x,\quad \tilde{x}^2 = y,\quad \tilde{x}^3 = ct-z$$

$$\tilde{g}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$$

qPDFs in quark models

Point splitting in + direction

Twist 2 PDF: $$f_1^q(x) = \frac{1}{2P^+}\int d^4k \,\mathrm{tr}\left[\Phi^q(k,P,S)\gamma^+\right]\delta\left(x-\frac{k^+}{P^+}\right),$$

Its quasi counterpart: $$D^q(x_v,\Gamma,v) = \frac{1}{2P^3}\int d^4k \,\mathrm{tr}\left[\Phi^q(k,P,S)\,\Gamma\,\right]\delta\left(x-\frac{k^3}{P^3}\right).$$

Point splitting in z direction

$\Gamma = \gamma^0 \; or \; \gamma^3$

$$\Phi(P,k,S,\eta) = MA_1 + \not{P}A_2 + \not{k}A_3 + \frac{1}{2M}[\not{P},\not{k}]A_4 + i(k\cdot S)\gamma_5 A_5 + M\not{S}\gamma_5 A_6 + \frac{k\cdot S}{M}\not{P}\gamma_5 A_7 + \frac{k\cdot S}{M}\not{k}\gamma_5 A_8$$
$$+\frac{[\not{P},\not{S}]}{2}\gamma_5 A_9 + \frac{[\not{k},\not{S}]}{2}\gamma_5 A_{10} + \frac{k\cdot S}{2M^2}[\not{P},\not{k}]\gamma_5 A_{11} + \frac{1}{M}\epsilon^{\mu\nu\rho\sigma}\gamma_\mu P_\nu k_\rho S_\sigma A_{12} + \mathcal{O}(B_i)$$

Quark models- no gauge field degrees of freedom- T-odd amplitudes, $\bar{A}_4$, $\bar{A}_5$, $\bar{A}_{12}$ and all the $\bar{B}_i$ amplitudes are absent

$$f_1^q(x) = 2\int d^4k \left(A_2^q + x\,A_3^q\right)\delta\left(x-\frac{k^+}{P^+}\right),$$

$$D^q(x_v,\gamma^\mu,v) = 2\int d^4k\left(\frac{P^\mu}{P^3}A_2^q + \frac{k^\mu}{P^3}A_3^q\right)\delta\left(x-\frac{k^3}{P^3}\right),\quad \mu = 0,\,3\,.$$

Sum rules

Number of valence quarks of flavor q

Flavor sum rule: $$\int_0^1 dx \left(f_1^q(x) - f_1^{\bar{q}}(x) \right) \equiv \int_{-1}^1 dx\, f_1^q(x) = N^q \,,$$

Momentum sum rule: $$\int_0^1 dx\, x \left(f_1^q(x) + f_1^{\bar{q}}(x) \right) \equiv \int_{-1}^1 dx\, x\, f_1^q(x) = A^q(0) \,,$$

A form factor of the energy momentum tensor at zero momentum transfer

$\sum_a A^a(0) = 1$

Flavor sum rules:

$$\int_0^\infty dx \left(D^q(x_v, \gamma^0, v) - D^{\bar{q}}(x_v, \gamma^0, v) \right) = \int_{-\infty}^\infty dx\, D^q(x_v, \gamma^0, v) = \frac{N^q}{v} \,,$$

$$\int_0^\infty dx \left(D^q(x_v, \gamma^3, v) - D^{\bar{q}}(x_v, \gamma^3, v) \right) = \int_{-\infty}^\infty dx\, D^q(x_v, \gamma^3, v) = N^q \,,$$

Momentum sum rule:

$$\int_0^\infty dx\, x \left(D^q(x_v, \gamma^0, v) + D^{\bar{q}}(x_v, \gamma^0, v) \right) = \int_{-\infty}^\infty dx\, x\, D^q(x_v, \gamma^0, v) = \frac{A^q(0)}{v} \,,$$

$$\int_0^\infty dx\, x \left(D^q(x_v, \gamma^3, v) + D^{\bar{q}}(x_v, \gamma^3, v) \right) = \int_{-\infty}^\infty dx\, x\, D^q(x_v, \gamma^3, v) = A^q(0) - \frac{1 - v^2}{v^2}\, \bar{c}^q(0) \,,$$

A form factor of the energy momentum tensor at zero momentum transfer

$\sum_a \bar{c}^a(t) = 0$

Bhattacharya, Cocuzza, Metz, Phys.Rev.D 102 (2020) 5, 054021

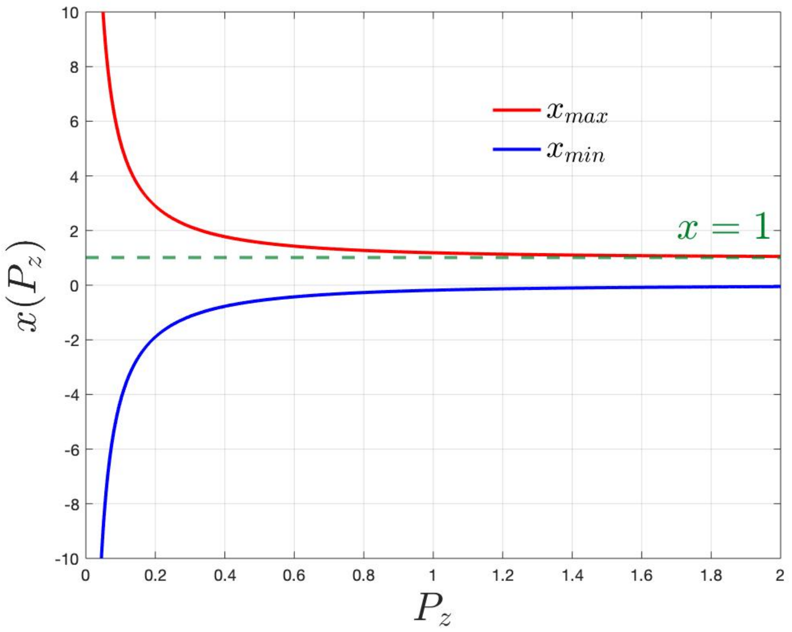
qPDFs in the Covariant Parton Model

In renormalizable theories

- PDFs : -1$\leq x \leq 1$
- qPDFs: -$\infty \leq x \leq \infty$

In CPM

- PDFs : $\frac{m_q^2}{M_N^2} \leq x \leq 1$
- qPDFs: $x_{min} < x < x_{max}$



$$\tilde{x}_{max} = \frac{1}{2}(1+\frac{m_q^2}{M^2}) + \frac{1}{2}\sqrt{(1+\frac{m_q^2}{M^2})^2 - 4\frac{m_q^2}{M^2}(1+\frac{M^2}{P_z^2}) + \frac{M^2}{P_z^2}(1+\frac{m_q^2}{M^2})^2}$$

$$\tilde{x}_{min} = \frac{1}{2}(1+\frac{m_q^2}{M^2}) - \frac{1}{2}\sqrt{(1+\frac{m_q^2}{M^2})^2 - 4\frac{m_q^2}{M^2}(1+\frac{M^2}{P_z^2}) + \frac{M^2}{P_z^2}(1+\frac{m_q^2}{M^2})^2}$$

As $P^z \to 0$ $\quad \tilde{x}_{max} = \frac{M}{2P^z}(1-\frac{m_q^2}{M^2}) \to \infty$

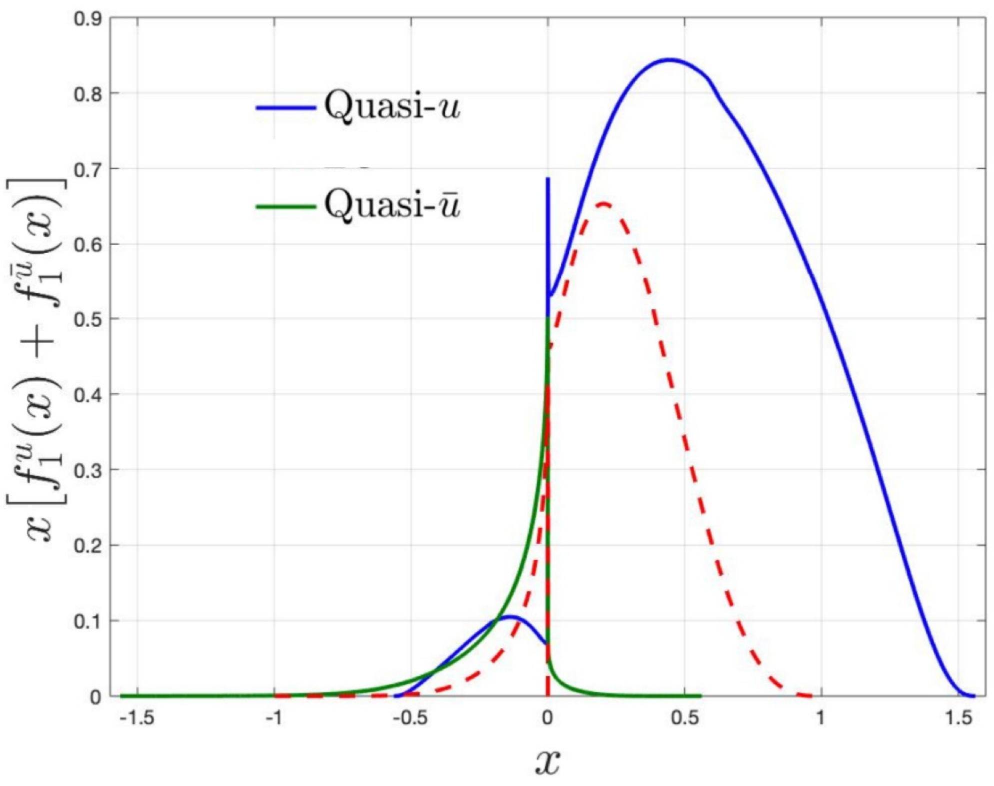
$$\tilde{x}_{min} = -\frac{M}{2P^z}(1-\frac{m_q^2}{M^2}) \to -\infty$$

As $P^z \to \infty$ $\quad \tilde{x}_{max} = 1$

$$\tilde{x}_{min} = \frac{m_q^2}{M^2}$$

qPDFs in the Covariant Parton Model

- Quark distribution leaks to anti-quark distribution and vice versa



qPDFs in the Covariant Parton Model

- $D^q(x_v, \gamma^0, v)$ and $D^q(x_v, \gamma^3, v)$ are related

$$D^q(x_v,\gamma^0,v) = v\,D^q(x_v,\gamma^3,v) + (1-v^2)\;2\pi\,M\int_{L(v)}^{\frac{1}{2}M} dk\;k\,\mathcal{G}^q(Mk)\,.$$

- EMT Form factors are calculated and found that

1) Sum rules are satisfied

$$\int_0^\infty dx\left(D^q(x_v,\gamma^0,v) - D^{\bar q}(x_v,\gamma^0,v)\right) = \int_{-\infty}^\infty dx\,D^q(x_v,\gamma^0,v) = \frac{N^q}{v}\,,$$
$$\int_0^\infty dx\left(D^q(x_v,\gamma^3,v) - D^{\bar q}(x_v,\gamma^3,v)\right) = \int_{-\infty}^\infty dx\,D^q(x_v,\gamma^3,v) = N^q\,,$$
$$\int_0^\infty dx\,x\left(D^q(x_v,\gamma^0,v) + D^{\bar q}(x_v,\gamma^0,v)\right) = \int_{-\infty}^\infty dx\,x\,D^q(x_v,\gamma^0,v) = \frac{A^q(0)}{v}\,,$$
$$\int_0^\infty dx\,x\left(D^q(x_v,\gamma^3,v) + D^{\bar q}(x_v,\gamma^3,v)\right) = \int_{-\infty}^\infty dx\,x\,D^q(x_v,\gamma^3,v) = A^q(0) - \frac{1-v^2}{v^2}\,\bar c^q(0)\,,$$

2)

$$\bar c^q(0) = -\,\tfrac{1}{4}\,A^q(0)\,,\quad \text{CPM}$$
$$\bar c^q(0) = -\,\tfrac{1}{4}\,A^q(0)\,,\quad \text{bag model}$$

This might a consequence of the fact that the quarks inside the bag obey the free Dirac equation as they do in the CPM.

Summary

Covariant parton model
- On mass shell quarks
- Spherical phase space in the rest frame
- On-shell partons in pure spin states

Quarks	Antiquarks
$\Phi = (\not{k}+m)A_3 + \frac{(\not{k}+m)}{M^2}\Big[(P\cdot k)+mM\Big]A_{11}\not{\omega}\gamma_5$	$\bar{\Phi} = (\not{k}-m)\bar{A}_3 + \frac{(\not{k}-m)}{M^2}\Big[(P\cdot k)-mM\Big]\bar{A}_{11}\bar{\not{\omega}}\gamma_5$
$\omega^\mu = \left\{S^\mu - \frac{(k.S)}{\Big[(P\cdot k)+mM\Big]}P^\mu - \frac{M}{m}\frac{(k\cdot S)}{\Big[(P\cdot k)+mM\Big]}k^\mu\right\}$	$\bar{\omega}^\mu = \left\{S^\mu - \frac{(k.S)}{\Big[(P\cdot k)-mM\Big]}P^\mu + \frac{M}{m}\frac{(k\cdot S)}{\Big[(P\cdot k)-mM\Big]}k^\mu\right\}$
$A_3(k.P) = P^0\delta_+(k^2-m^2)\mathcal{G}(k.P)$	$\bar{A}_3(k.P) = P^0\delta_-(k^2-m^2)\bar{\mathcal{G}}(k.P)$
$A_{11}(k.P) = P^0\delta_+(k^2-m^2)\mathcal{H}(k.P)\Big(-\frac{M^2}{k.P+mM}\Big)$	$\bar{A}_{11}(k.P) = P^0\delta_-(k^2-m^2)\bar{\mathcal{H}}(k.P)\Big(\frac{M^2}{k.P-mM}\Big)$

Table 1: The quark and antiquark correlators, polarization vectors and amplitudes for $A_8 = -A_{11}$ and $\bar{A}_8 = -\bar{A}_{11}$

- All polarized and unpolarized T-even TMDs are systematically obtained for quarks and antiquarks,
- TMD relations supported by other quark models are satisfied
- Quark distribution leaks to anti-quark distribution and vice versa
- $D^q(x_v, \gamma^0, v)$ and $D^q(x_v, \gamma^3, v)$ are related
- ……..

Outlook

- Polarized qPDFs to be completed
- Making the model more realistic by including off-shell-ness effects
- Wish to access T-odd TMDs
- Calculating other distributions and quasi distributions: GTMDs, GPDs, etc..
- .
- .
- .

THANK YOU

Quark correlator – no gauge field + on mass shell + pure spin states

Assuming pure spin states, $\omega^2 = -1$, leads to $A_8 = \mp A_{11}$

ω^2	
Mixed spin state	$-1 \leq \omega^2 \leq 0$
Pure spin state	$\omega^2 = -1$

$$Amplitudes: \quad A_3, A_{11}$$

Choosing $A_8 = -A_{11}$

$$\Phi(k,P,S) = (\not{k}+m)A_3 + \frac{(\not{k}+m)}{M^2}\Big[(P\cdot k)+mM\Big]A_{11}\not{\omega}\gamma_5$$

$$\omega^\mu(k,P,S) = \left\{S^\mu - \frac{(k.S)}{\Big[(P\cdot k)+mM\Big]}P^\mu - \frac{M}{m}\frac{(k\cdot S)}{\Big[(P\cdot k)+mM\Big]}k^\mu\right\}$$

Covariant Parton Model - The amplitudes for quarks

- Model

$$Tr[\Phi(P,k,S)\Gamma] = P^0\Theta(k^0)\delta(k^2-m^2)Tr\Big[(\not{k}+m)(\mathcal{G}(kP)+\mathcal{H}(kP)\gamma^5\not{w})\Gamma\Big]$$

- Quark correlator with no gauge field + on mass shell + pure spin states

$$\Phi(k,P,S) = (\not{k}+m)A_3 + \frac{(\not{k}+m)}{M^2}\Big[(P\cdot k)+mM\Big]A_{11}\not{w}\gamma_5$$

- Amplitudes obtained in terms of the covariant distribution functions

$$A_3(k.P) = P^0\delta_+(k^2-m^2)\mathcal{G}(k.P)$$

$$A_{11}(k.P) = P^0\delta_+(k^2-m^2)\mathcal{H}(k.P)\Big(-\frac{M^2}{k.P+mM}\Big)$$