Gauge-invariant TMD factorization for Drell-Yan hadronic tensor at small x

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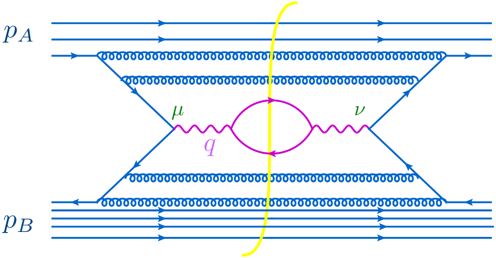
DY hadronic tensor for electromagnetic current

DY cross section is given by the product of leptonic tensor and hadronic tensor. The hadronic tensor $W_{\mu\nu}$ is defined as

$$W_{\mu\nu}(p_A, p_B, q) \;=\; \frac{1}{(2\pi)^4}\int d^4x\; e^{-iqx}\langle p_A, p_B|J_\mu(x)J_\nu(0)|p_A, p_B\rangle$$



p_A, p_B = hadron momenta, q = the momentum of DY pair, and J_μ is the electromagnetic or Z-boson current.

DY hadronic tensor for electromagnetic current

For unpolarized hadrons, the hadronic tensor $W_{\mu\nu}$ for EM current is parametrized by 4 functions, for example in Collins-Soper frame

$$W_{\mu\nu} = -\Big(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\Big)(W_T + W_{\Delta\Delta}) - 2X_\mu X_\nu W_{\Delta\Delta} + Z_\mu Z_\nu (W_L - W_T - W_{\Delta\Delta}) - (X_\mu Z_\nu + X_\nu Z_\mu) W_\Delta$$

where X, Z are unit vectors orthogonal to q and to each other

TMD representation for W_i

The hadronic tensor in the Sudakov region $q^2 \equiv Q^2 \gg q_\perp^2$ can be studied by TMD factorization. For example, functions W_T and $W_{\Delta\Delta}$ can be represented as

$$W_i = \sum_{\text{flavors}} e_f^2 \int d^2k_\perp \mathcal{D}^{(i)}_{f/A}(x_A, k_\perp) \mathcal{D}^{(i)}_{f/B}(x_B, q_\perp - k_\perp) C_i(q, k_\perp) + \text{power corrections} + \text{Y} - \text{terms} \qquad (1)$$

- $\mathcal{D}_{f/A}(x_A, k_\perp)$ is the TMD density of a parton f in hadron A with fraction of momentum x_A and transverse momentum $k_\perp$,
- $\mathcal{D}_{f/B}(x, q_\perp - k_\perp)$ is a similar quantity for hadron B,
- $C_i(q, k)$ are determined by the cross section $\sigma(f\!f \to \mu^+\mu^-)$ of production of DY pair of invariant mass q^2 in the scattering of two partons.

There is, however, a problem with Eq. (1) for the functions W_L and W_Δ.

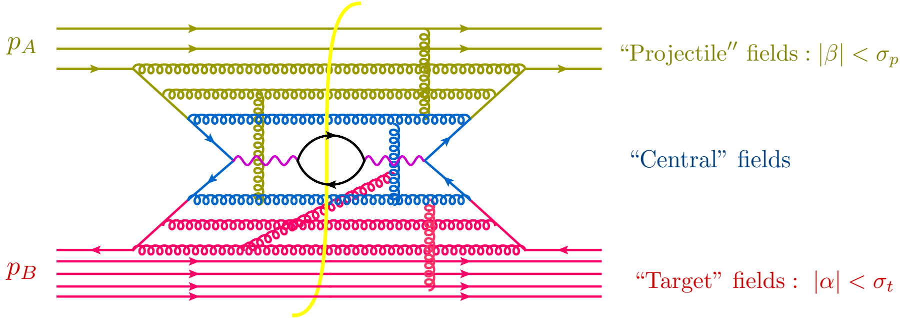
W_T and $W_{\Delta\Delta}$ are determined by leading-twist quark TMDs, but W_Δ and W_L start from terms $\sim \frac{q_\perp}{Q}$ and $\sim \frac{q_\perp^2}{Q^2}$ determined by quark-quark-gluon TMDs.

The power corrections $\sim \frac{q_\perp}{Q}$ were found more than two decades ago but there was no calculation of power corrections $\sim \frac{q_\perp^2}{Q^2}$ until recently.

Also, the leading-twist contribution is not EM gauge invariant.

Sudakov variables:

$$p = \alpha p_1 + \beta p_2 + p_\perp, \qquad p_1 \simeq p_A, \;\; p_2 \simeq p_B, \;\; p_1^2 = p_2^2 = 0$$



The result of the integration over "central" fields in the background of projectile and target fields is a series of TMD operators made from projectile (or target) fields multiplied by powers of $\frac{1}{Q^2} \Rightarrow$ power corrections

Leading-N_c power corrections

Power corrections are $\sim$ leading twist $\times\left(\frac{q_\perp}{Q} \text{ or } \frac{q_\perp^2}{Q^2}\right)\times\left(1+\frac{1}{N_c}+\frac{1}{N_c^2}\right)$.

(Pleasant) surprise: most of the terms not suppressed by $\frac{1}{N_c}$ are determined by the leading-twist TMDs due to QCD equations of motion

Leading twist:

$$\frac{1}{8\pi^3 s}\int dx_- d^2x_\perp\ e^{-i\alpha sqrt\frac{s}{2}x_+ + i(k,x)_\perp}\ \langle p_A|\hat{\bar{\psi}}_f(x_-,x_\perp)\ \not{p}_2\hat{\psi}_f(0)|p_A\rangle\ =\ f_{1f}(\alpha,k_\perp^2)$$

Power correction:

$$\frac{1}{8\pi^3 s}\int dx_- dx_\perp\ e^{-i\alpha_q\sqrt{\frac{s}{2}}x_- + i(k,x)_\perp}$$

$$\times\ \langle p_A|\hat{\bar{\psi}}^f(x_-,x_\perp)\ \not{p}_2[\hat{A}_i(x_-,x_\perp)-i\gamma_5\hat{\tilde{A}}_i(x_-,x_\perp)]\hat{\psi}^f(0)|p_A A\rangle$$

$$=\ -k_i f_1(\alpha_q,k_\perp)+\alpha_q k_i\big[f_\perp(\alpha_q,k_\perp)+g^\perp(\alpha_q,k_\perp)\big],$$

(Mulders & Tangerman, 1996)

At small $\alpha_q\equiv x_A$ one can drop the second term

Result for $W_{\mu\nu}$ for unpolarized hadrons

Result:

$$W_{\mu\nu}(q) \;=\; W^1_{\mu\nu}(q) + W^2_{\mu\nu}(q)$$

The first, gauge-invariant, part is given by

$$\begin{aligned}
W^1_{\mu\nu}(q) \;&=\; W^{1F}_{\mu\nu}(q) + W^{1H}_{\mu\nu}(q),\\
W^{1F}_{\mu\nu}(q) \;&=\; \sum_f e_f^2 W^{fF}_{\mu\nu}(q), \quad W^{fF}_{\mu\nu}(q) \;=\; \frac{1}{N_c}\int d^2k_\perp F^f(q,k_\perp)\mathcal{W}^F_{\mu\nu}(q,k_\perp),\\
W^{1H}_{\mu\nu}(q) \;&=\; \sum_f e_f^2 W^{fH}_{\mu\nu}(q), \quad W^{fH}_{\mu\nu}(q) \;=\; \frac{1}{N_c}\int d^2k_\perp H^f(q,k_\perp)\mathcal{W}^H_{\mu\nu}(q,k_\perp)
\end{aligned}$$

where F^f and H^f are ($\alpha_q \equiv x_A$, $\beta_q \equiv x_B$)

$$\begin{aligned}
F^f(q,k_\perp) \;&=\; f_1^f(\alpha_q,k_\perp)\bar{f}_1^f(\beta_q,(q-k)_\perp) \;+\; f_1^f \leftrightarrow \bar{f}_1^f\\
H^f(q,k_\perp) \;&=\; h_{1f}^\perp(\alpha_q,k_\perp)\bar{h}_{1f}^\perp(\beta_q,(q-k)_\perp) \;+\; h_{1f}^\perp \leftrightarrow \bar{h}_{1f}^\perp
\end{aligned}$$

Gauge-invariant structures $\quad q^\mu\mathcal{W}^F_{\mu\nu}=q^\mu\mathcal{W}^H_{\mu\nu}=0$

$$
\begin{aligned}
&\mathcal{W}^F_{\mu\nu}(q,k_\perp)\\
&=\ -g^\perp_{\mu\nu}+\frac{1}{Q_\parallel^2}(q^\parallel_\mu q^\perp_\nu+q^\parallel_\nu q^\perp_\mu)+\frac{q_\perp^2}{Q_\parallel^4}q^\parallel_\mu q^\parallel_\nu+\frac{\tilde q_\mu\tilde q_\nu}{Q_\parallel^2}[q_\perp^2-4(k,q-k)_\perp]\\
&-\Big[\frac{\tilde q_\mu}{Q_\parallel^2}\Big(g^\perp_{\nu i}-\frac{q^\parallel_\nu q_i}{Q_\parallel^2}\Big)(q-2k)^i_\perp+\mu\leftrightarrow\nu\Big]
\end{aligned}
$$

$\tilde q\equiv x_Ap_1-x_Bp_2$

$$
\begin{aligned}
&m^2\mathcal{W}^H_{\mu\nu}(q,k_\perp)\\
&=\ -k^\perp_\mu(q-k)^\perp_\nu-k^\perp_\nu(q-k)^\perp_\mu-g^\perp_{\mu\nu}(k,q-k)_\perp+2\frac{\tilde q_\mu\tilde q_\nu-q^\parallel_\mu q^\parallel_\nu}{Q_\parallel^4}k_\perp^2(q-k)_\perp^2\\
&-\Big(\frac{q^\parallel_\mu}{Q_\parallel^2}\big[k_\perp^2(q-k)^\perp_\nu+k^\perp_\nu(q-k)^2_\perp\big]+\ \frac{\tilde q_\mu}{Q_\parallel^2}\big[k_\perp^2(q-k)^\perp_\nu-k^\perp_\nu(q-k)^2_\perp\big]+\mu\leftrightarrow\nu\Big)\\
&-\ \frac{\tilde q_\mu\tilde q_\nu+q^\parallel_\mu q^\parallel_\nu}{Q_\parallel^4}\big[q_\perp^2-2(k,q-k)_\perp\big](k,q-k)_\perp-\ \frac{q^\parallel_\mu\tilde q_\nu+\tilde q_\mu q^\parallel_\nu}{Q_\parallel^4}(2k-q,q)_\perp(k,q-k)_\perp
\end{aligned}
$$

In CMS and ATLAS experiments $s = 8$ TeV, $Q = 80 - 100$ GeV and $Q_\perp$ varies from 0 to 120 GeV.

Our analysis is valid at $Q_\perp = 10 - 30$ GeV and $Y \simeq 0$ ($x_A \sim x_B \sim 0.1$) so that power corrections are small but sizable.

Angular distribution of DY leptons in the Collins-Soper frame ($c_\phi \equiv \cos\phi$, $s_\phi \equiv \sin\phi$ etc.)

$$\frac{d\sigma}{dQ^2 dy d\Omega_l} = \frac{3}{16\pi}\frac{d\sigma}{dQ^2 dy}\Big[(1 + c_\theta^2) + \frac{A_0}{2}(1 - 3c_\theta^2) + A_1 s_{2\theta} c_\phi + \frac{A_2}{2} s_\theta^2 c_{2\phi}$$
$$+ A_3 s_\theta c_\phi + A_4 c_\theta + A_5 s_\theta^2 s_{2\phi} + A_6 s_{2\theta} s_\phi + A_7 s_\theta s_\phi\Big]$$

Result with $\frac{1}{Q^2}$, large-N_c and "f_1" accuracy

$$
\begin{aligned}
\mathbb{W}(q,l,l') \;=\;& c_e^2 c_f^2 \frac{Q^4}{|m_Z^2 - Q^2|^2 + \Gamma_Z^2 m_Z^2} \\
\times & \sum_f \Big\{ (a_e^2+1)(a_f^2+1)\Big(\Big[\mathcal{W}^{\rm Ff} - \frac{Q_\perp^2}{2Q^2}(\mathcal{W}^{\rm Ff} - \mathcal{W}_L^{\rm Ff}) \Big](1+\cos^2\theta) \\
+ & \frac{Q_\perp^2}{2Q^2}\mathcal{W}_L^{\rm Ff}(1-3\cos^2\theta) + \frac{Q_\perp}{Q}\mathcal{W}_1^{\rm Ff}\sin 2\theta\cos\phi + \frac{Q_\perp^2}{2Q^2}\mathcal{W}^{\rm Ff}\sin^2\theta\cos 2\phi \Big] \Big) \\
+ & 8a_e a_f \Big[\frac{Q_\perp}{Q}\mathcal{W}_3^{\rm Ff}\sin\theta\cos\phi + \mathcal{W}_4^{\rm Ff}\cos\theta \Big] \Big\}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{W}^{\rm Ff}(q) \;&=\; \int d^2k_\perp F^f(q,k_\perp), \qquad \mathcal{W}_L^{\rm Ff}(q) \;=\; \int dk_\perp \frac{(q-2k)_\perp^2}{q_\perp^2} F^f(q,k_\perp) \\
\mathcal{W}_1^{\rm Ff}(q) \;&=\; \int d^2k_\perp \frac{(q,q-2k)_\perp}{q_\perp^2} F^f(q,k_\perp) \\
\mathcal{W}_3^{\rm Ff}(q) \;&=\; \int d^2k_\perp \frac{(q,q-2k)_\perp}{q_\perp^2} \mathcal{F}^f(q,k_\perp), \quad \mathcal{W}_4^{\rm Ff}(q) \;=\; \int d^2k_\perp \mathcal{F}^f(q,k_\perp),
\end{aligned}
$$

$$
\mathcal{F}^f(q,k_\perp) \;=\; f_1^f(\alpha_q,k_\perp)\bar{f}_1^f(\beta_q,(q-k)_\perp) \;-\; f_1^f \leftrightarrow \bar{f}_1^f
$$

Comparison with LHC results

$$\mathbb{W} \sim \sum_f \mathcal{W}^{\mathrm{Ff}} \Big\{ (a_e^2+1)(a_f^2+1) \Big(\Big[1 - \frac{Q_\perp^2}{2m_Z^2} + \frac{Q_\perp^2}{2m_Z^2} \frac{\mathcal{W}_L^{\mathrm{Ff}}}{\mathcal{W}^{\mathrm{Ff}}} \Big] (1+\cos^2\theta)$$
$$+ \frac{Q_\perp^2}{2m_Z^2} \frac{\mathcal{W}_L^{\mathrm{Ff}}}{\mathcal{W}^{\mathrm{Ff}}} (1-3\cos^2\theta) + \frac{Q_\perp}{m_Z} \frac{\mathcal{W}_1^{\mathrm{Ff}}}{\mathcal{W}^{\mathrm{Ff}}} \sin 2\theta \cos\phi + \frac{Q_\perp^2}{2m_Z^2} \sin^2\theta \cos 2\phi] \Big)$$
$$+ 8a_e a_f \Big[\frac{Q_\perp}{m_Z} \frac{\mathcal{W}_3^{\mathrm{Ff}}}{\mathcal{W}^{\mathrm{Ff}}} \sin\theta\cos\phi + \frac{\mathcal{W}_4^{\mathrm{Ff}}}{\mathcal{W}^{\mathrm{Ff}}} \cos\theta \Big] \Big\}$$

We can easily estimate A_0 and A_2 which depend on $\frac{\mathcal{W}_L^{\mathrm{Ff}}}{\mathcal{W}^{\mathrm{Ff}}}$.

Logarithmic estimate of $\frac{\mathcal{W}_L^{\mathrm{Ff}}}{\mathcal{W}^{\mathrm{Ff}}}$: if $k_\perp^2 \gg m_N^2$ we can approximate

$$f_1(x, k_\perp^2) \simeq \frac{f(x)}{k_\perp^2} \quad \Rightarrow \quad F(q, k_\perp) \simeq \frac{f(\alpha_q)\bar{f}(\beta_q) + f \leftrightarrow \bar{f}}{k_\perp^2 (q-k)_\perp^2}$$

Performing integration over $k_\perp$ in logarithmical approximation, one obtains

$$\frac{\mathcal{W}_L^{\mathrm{Ff}}}{\mathcal{W}^{\mathrm{Ff}}} \simeq 1 + 2\frac{\ln m_z^2/Q_\perp^2}{\ln Q_\perp^2/m^2}$$

Comparison of A_0 with LHC results

Logarithmic estimate of A_0

$$\frac{\mathcal{W}_L^{\rm Ff}}{\mathcal{W}^{\rm Ff}} \simeq 1+2\frac{\ln m_z^2/Q_\perp^2}{\ln Q_\perp^2/m^2} \quad \Rightarrow \quad A_0 = \frac{Q_\perp^2}{m_z^2}\frac{1+2\frac{\ln m_z^2/Q_\perp^2}{\ln Q_\perp^2/m^2}}{1+\frac{Q_\perp^2}{m_z^2}\frac{\ln m_z^2/Q_\perp^2}{\ln Q_\perp^2/m^2}} \qquad (*)$$

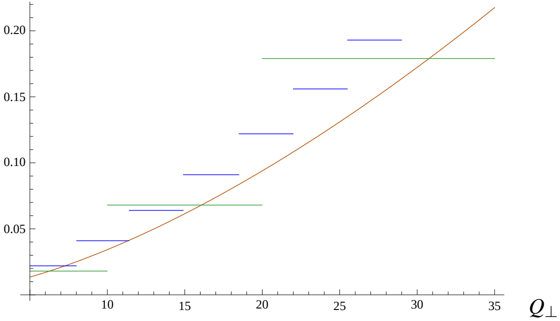


Figure: Comparison of prediction $(*)$ with lines depicting angular coefficient A_0 in bins of $Q_\perp$ and $Y < 1$ from CMS (arXiv:1504.03512) and ATLAS (arXiv1606.00689)

Comparison of A_2 with LHC results

Logarithmic estimate of A_2

$$\frac{\mathcal{W}_L^{\rm Ff}}{\mathcal{W}^{\rm Ff}} \simeq 1+2\frac{\ln m_z^2/Q_\perp^2}{\ln Q_\perp^2/m^2} \quad\Rightarrow\quad A_2 = \frac{Q_\perp^2}{m_z^2}\frac{1}{1+\frac{Q_\perp^2}{m_z^2}\frac{\ln m_z^2/Q_\perp^2}{\ln Q_\perp^2/m^2}} \qquad (**)$$

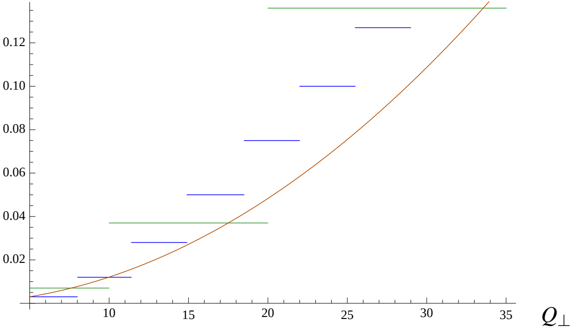


Figure: Comparison of prediction $(**)$ with lines depicting angular coefficient A_2 in bins of $Q_\perp$ and $Y < 1$ from CMS (arXiv:1504.03512) and ATLAS (arXiv1606.00689)

Qualitative checks

$$
\begin{aligned}
\mathbb{W} \sim \sum_f r^f \mathcal{W}^{\mathrm{Ff}} \Big\{ & 1 + \cos^2\theta + \frac{Q_\perp^2}{2m_Z^2} \frac{\mathcal{W}_L^{\mathrm{Ff}}}{\mathcal{W}^{\mathrm{Ff}} r^f} (1 - 3\cos^2\theta) \\
& + \frac{Q_\perp}{m_Z} \frac{\mathcal{W}_1^{\mathrm{Ff}}}{\mathcal{W}^{\mathrm{Ff}} r^f} \sin 2\theta \cos\phi + \frac{Q_\perp^2}{2m_Z^2 r^f} \sin^2\theta \cos 2\phi] \\
& + \frac{8a_e a_f}{(a_e^2+1)(a_f^2+1)} \Big[\frac{Q_\perp}{m_Z} \frac{\mathcal{W}_3^{\mathrm{Ff}}}{\mathcal{W}^{\mathrm{Ff}} r^f} \sin\theta\cos\phi + \frac{\mathcal{W}_4^{\mathrm{Ff}}}{\mathcal{W}^{\mathrm{Ff}} r^f} \cos\theta \Big] \Big\}
\end{aligned}
$$

$r^f \equiv 1 - \frac{Q_\perp^2}{2m_Z^2} + \frac{Q_\perp^2}{2m_Z^2} \frac{\mathcal{W}_L^{\mathrm{Ff}}}{\mathcal{W}^{\mathrm{Ff}}}$

Qualitative checks:

- Factorization of TMD $f_1(x, k_\perp^2) \simeq f(x) f(k_\perp^2) \Rightarrow \mathcal{W}_1^{\mathrm{F}f}(q) = 0$
  $\Rightarrow A_1$ is smaller than A_2
- A_4 does not depend on $Q_\perp$ and increases with rapidity
- A_3 is smaller than A_4
- A_5, A_6, A_7 are order of magnitude smaller than A_0, A_2, A_4

1. Conclusions
   - The Drell-Yan hadronic tensor is calculated in the Sudakov region $s \gg Q^2 \gg q_\perp^2$ in the tree approximation with $\frac{1}{Q^2}$ accuracy.
   - In the leading order in N_c the higher-twist quark-quark-gluon TMDs reduce to leading-twist TMDs due to QCD equation of motion.
   - The resulting hadronic tensor for unpolarized hadrons is (EM) gauge-invariant and depends on two leading-twist TMDs: f_1 responsible for total DY cross section, and Boer-Mulders function $h_1^\perp$.
   - Results for angular coefficients of Z-boson production seem to agree with LHC measurements at corresponding kinematics.
2. Outlook
   - Rapidity factorization at the one-loop level.

Thank you for attention!