Drell-Yan angular lepton distributions at small x from TMD factorization

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QCD @ Moriond, 30 March 2023

Long-term project: TMD factorization valid at small and moderate x

Particle production at $q_\perp \ll Q$ is described by TMD factorization

$$\frac{d\sigma}{d\eta d^2q_\perp} = \sum_{\text{flavors}} e_f^2 \int d^2k_\perp \mathcal{D}_{f/A}(x_A, k_\perp)\mathcal{D}_{f/B}(x_B, q_\perp - k_\perp)C(q, k_\perp)$$
$$+ \text{ power corrections } + \text{ ``Y} - \text{terms''}$$

The quantities $\mathcal{D}_{f/A}(x_A, k_\perp)$, $\mathcal{D}_{f/B}(x_B, q_\perp - k_\perp)$, and $C(q, k_\perp)$ are defined with cutoffs. The dependence on the cutoffs cancels in their product order by order in α_s.

At moderate x_A, x_B: CSS approach. The TMDs $\mathcal{D}_{f/A}(x_A, k_\perp)$ are defined with a combination of UV and rapidity cutoffs.

At $x_A, x_B \ll 1$: k_T-factorization approach. The TMDs are defined with rapidity-only cutoffs.

It is impossible to extend CSS approach to small x ($\Leftrightarrow$ nobody tried)

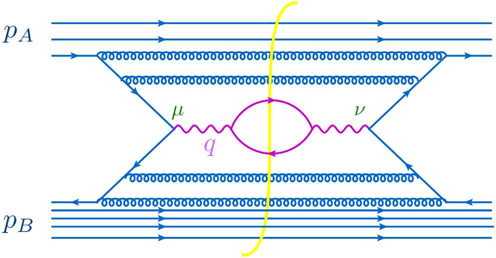
It *is* possible to study TMD factorization at moderate x using small-x methods (rapidity-only factorization etc.) (A. Tarasov, G. Chirilli, I.B, 2015-2023)

Visible success: power corrections $\sim \frac{1}{Q^2}$ for DY hadronic tensor $\Rightarrow$ EM gauge invariance of DY tensor. It is not obtained (yet?) using CSS or SCET

DY hadronic tensor

DY cross section is given by the product of leptonic tensor and hadronic tensor. The hadronic tensor $W_{\mu\nu}$ is defined as

$$W_{\mu\nu}(p_A, p_B, q) \ = \ \frac{1}{(2\pi)^4}\int d^4x \ e^{-iqx}\langle p_A, p_B|J_\mu(x)J_\nu(0)|p_A, p_B\rangle$$



p_A, p_B = hadron momenta, q = the momentum of DY pair, and J_μ is the electromagnetic or Z-boson current.

DY hadronic tensor for electromagnetic current

For unpolarized hadrons, the EM hadronic tensor $W_{\mu\nu}$ is parametrized by 4 functions, for example in Collins-Soper frame

$$
\begin{aligned}
W_{\mu\nu} \;=\; & -\Big(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\Big)(W_T + W_{\Delta\Delta}) - 2X_\mu X_\nu W_{\Delta\Delta} \\
& + Z_\mu Z_\nu (W_L - W_T - W_{\Delta\Delta}) - (X_\mu Z_\nu + X_\nu Z_\mu) W_\Delta
\end{aligned}
$$

where X, Z are unit vectors orthogonal to q and to each other

TMD representation for W_i

The hadronic tensor in the Sudakov region $q^2 \equiv Q^2 \gg q_\perp^2$ can be studied by TMD factorization. For example, functions W_T and $W_{\Delta\Delta}$ can be represented as

$$W_i = \sum_{\text{flavors}} e_f^2 \int d^2k_\perp \mathcal{D}^{(i)}_{f/A}(x_A, k_\perp)\mathcal{D}^{(i)}_{f/B}(x_B, q_\perp - k_\perp)C_i(q, k_\perp)$$
$$+ \text{power corrections} + \text{Y} - \text{terms} \quad (1)$$

- $\mathcal{D}_{f/A}(x_A, k_\perp)$ is the TMD density of a parton f in hadron A with fraction of momentum x_A and transverse momentum $k_\perp$,
- $\mathcal{D}_{f/B}(x, q_\perp - k_\perp)$ is a similar quantity for hadron B,
- $C_i(q, k)$ are determined by the cross section $\sigma(ff \to \mu^+\mu^-)$ of production of DY pair of invariant mass q^2 in the scattering of two partons.

There is, however, a problem with Eq. (1) for the functions W_L and W_Δ.

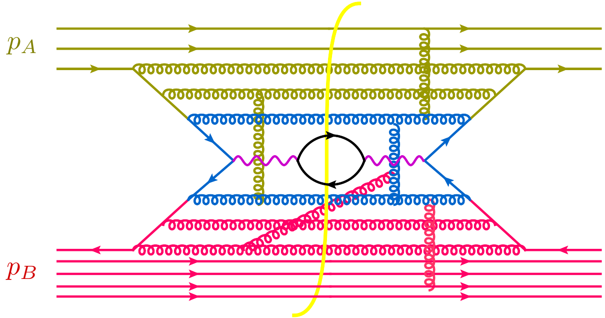
W_T and $W_{\Delta\Delta}$ are determined by leading-twist quark TMDs, but W_Δ and W_L start from terms $\sim \frac{q_\perp}{Q}$ and $\sim \frac{q_\perp^2}{Q^2}$ determined by quark-quark-gluon TMDs.

The power corrections $\sim \frac{q_\perp}{Q}$ were found more than two decades ago but there was no calculation of power corrections $\sim \frac{q_\perp^2}{Q^2}$ until recently. Also, the leading-twist contribution is not EM gauge invariant.

TMD factorization from rapidity factorization (A. Tarasov and I.B.)

Sudakov variables:

$$p \;=\; \alpha p_1 + \beta p_2 + p_\perp, \qquad p_1 \simeq p_A,\;\; p_2 \simeq p_B,\;\; p_1^2 = p_2^2 = 0$$



"Projectile″ fields : $|\beta| < \sigma_p$

"Central" fields

"Target" fields : $|\alpha| < \sigma_t$

The result of the integration over "central" fields in the background of projectile and target fields is a series of TMD operators made from projectile (or target) fields multiplied by powers of $\frac{1}{Q^2} \Rightarrow$ power corrections

Power corrections to DY hadronic tensor

Using rapidity-only factorization for particle production, I've calculated the power corrections to $W_{\mu\nu}(q)$

Power corrections are $\sim$ leading twist $\times\left(\frac{q_\perp}{Q} \text{ or } \frac{q_\perp^2}{Q^2}\right) \times \left(1+\frac{1}{N_c}+\frac{1}{N_c^2}\right)$.

A surprise: terms not suppressed by $\frac{1}{N_c}$ are determined by the leading-twist TMDs due to QCD equations of motion

Result at $x_A, x_B \ll 1$ (with $\frac{1}{N_c}$ accuracy):

$$
\begin{aligned}
W_{\mu\nu}(q) &= W^{1F}_{\mu\nu}(q) + W^{1H}_{\mu\nu}(q),\\
W^{1F}_{\mu\nu}(q) &= \sum_f e_f^2 W^{fF}_{\mu\nu}(q), \quad W^{fF}_{\mu\nu}(q) = \frac{1}{N_c}\int d^2k_\perp F^f(q,k_\perp)\mathcal{W}^F_{\mu\nu}(q,k_\perp),\\
W^{1H}_{\mu\nu}(q) &= \sum_f e_f^2 W^{fH}_{\mu\nu}(q), \quad W^{fH}_{\mu\nu}(q) = \frac{1}{N_c}\int d^2k_\perp H^f(q,k_\perp)\mathcal{W}^H_{\mu\nu}(q,k_\perp)
\end{aligned}
$$

where F^f and H^f are

$$
\begin{aligned}
F^f(q,k_\perp) &= f_1^f(x_A,k_\perp)\bar{f}_1^f(x_B,(q-k)_\perp) \; + \; f_1^f \leftrightarrow \bar{f}_1^f\\
H^f(q,k_\perp) &= h^\perp_{1f}(x_A,k_\perp)\bar{h}^\perp_{1f}(x_B,(q-k)_\perp) \; + \; h^\perp_{1f} \leftrightarrow \bar{h}^\perp_{1f}
\end{aligned}
$$

(recall $q = x_A p_A + x_B p_B + q_\perp$)

Gauge-invariant structures $\quad q^\mu \mathcal{W}^F_{\mu\nu} = q^\mu \mathcal{W}^H_{\mu\nu} = 0$

$$
\begin{aligned}
&\mathcal{W}^F_{\mu\nu}(q,k_\perp)\\
&= -g^\perp_{\mu\nu} + \frac{1}{Q_\parallel^2}(q^\parallel_\mu q^\perp_\nu + q^\parallel_\nu q^\perp_\mu) + \frac{q_\perp^2}{Q_\parallel^4} q^\parallel_\mu q^\parallel_\nu + \frac{\tilde{q}_\mu \tilde{q}_\nu}{Q_\parallel^2}[q_\perp^2 - 4(k,q-k)_\perp]\\
&- \Big[\frac{\tilde{q}_\mu}{Q_\parallel^2}\Big(g^\perp_{\nu i} - \frac{q^\parallel_\nu q_i}{Q_\parallel^2}\Big)(q-2k)^i_\perp + \mu \leftrightarrow \nu\Big]
\end{aligned}
$$

$$\tilde{q} \equiv x_A p_1 - x_B p_2$$

$$
\begin{aligned}
&m^2 \mathcal{W}^H_{\mu\nu}(q,k_\perp)\\
&= -k^\perp_\mu (q-k)^\perp_\nu - k^\perp_\nu (q-k)^\perp_\mu - g^\perp_{\mu\nu}(k,q-k)_\perp + 2\frac{\tilde{q}_\mu \tilde{q}_\nu - q^\parallel_\mu q^\parallel_\nu}{Q_\parallel^4} k_\perp^2 (q-k)_\perp^2\\
&- \Big(\frac{q^\parallel_\mu}{Q_\parallel^2}\big[k_\perp^2 (q-k)^\perp_\nu + k^\perp_\nu (q-k)^2_\perp\big] + \frac{\tilde{q}_\mu}{Q_\parallel^2}\big[k_\perp^2 (q-k)^\perp_\nu - k^\perp_\nu (q-k)^2_\perp\big] + \mu \leftrightarrow \nu\Big)\\
&- \frac{\tilde{q}_\mu \tilde{q}_\nu + q^\parallel_\mu q^\parallel_\nu}{Q_\parallel^4}\big[q_\perp^2 - 2(k,q-k)_\perp\big](k,q-k)_\perp - \frac{q^\parallel_\mu \tilde{q}_\nu + \tilde{q}_\mu q^\parallel_\nu}{Q_\parallel^4}(2k-q,q)_\perp (k,q-k)_\perp
\end{aligned}
$$

Logarithmic estimates of angular coefficients

Take $s = 8$ TeV, $Q = 90$ GeV and $q_\perp = 20$ GeV where $x_A, x_B \sim 0.1$ and power corrections are small but sizable.

The differential cross section of DY process is parametrized as

$$\Big(\frac{d\sigma}{d^4q}\Big)^{-1}\frac{d\sigma}{d\Omega d^4q} \;=\; \frac{3}{4\pi(\lambda+3)}\big(1+\lambda\cos^2\theta+\mu\sin 2\theta\cos\phi+\frac{\nu}{2}\sin^2\theta\cos 2\phi\big)$$

Logarithmic estimates of angular coefficients

$$1-\lambda \;=\; 2\frac{W_L}{W_T+W_L} \;\simeq\; 2\frac{1+2\frac{\ln Q^2/q_\perp^2}{\ln q_\perp^2/m^2}}{\frac{Q^2}{q_\perp^2}-\frac{1}{2}+2\frac{\ln Q^2/q_\perp^2}{\ln q_\perp^2/m^2}} \;\simeq\; 0.19$$

$$\nu \;=\; \frac{2W_{\Delta\Delta}}{W_T+W_L} \;\simeq\; \frac{1}{\frac{Q^2}{q_\perp^2}-\frac{1}{2}+2\frac{\ln Q^2/q_\perp^2}{\ln q_\perp^2/m^2}} \;\simeq\; 0.05$$

$\mu \;=\; \frac{W_\Delta}{W_T+W_L}, = 0$ if we use factorization models for TMDs.

Approximately the same λ and ν values as in analysis of LHC data by Lambertsen and Vogelsang

Z-boson production at LHC

The relevant terms of the Lagrangian for quark fields ψ^f are

$$\mathcal{L}_Z \;=\; e\int d^4x\; \mathcal{J}_\mu Z^\mu(x), \qquad \mathcal{J}_\mu \;=\; c_e\bar{e}(a_e-\gamma_5)e - \sum_{\text{flavors}} c_f\bar{\psi}^f\gamma_\mu(a_f-\gamma_5)\psi^f$$

where

$$c_{u,c} \;=\; \frac{1}{4c_W s_W}, \quad a_{u,c} \;=\; 1-\frac{8}{3}s_W^2, \quad c_{d,s} \;=\; -\frac{1}{4c_W s_W}, \quad a_{d,s} \;=\; 1-\frac{4}{3}s_W^2,$$
$$c_e \;=\; \frac{1}{4c_W s_W}, \quad a_e \;=\; 1-4s_W^2, \qquad c_W\equiv\cos\theta_W, \quad s_W\equiv\sin\theta_W.$$

$$d\sigma \;=\; \frac{e^4}{16\pi^2 sN_c}\frac{dQ^2}{Q^2}dYd^2q_\perp d\Omega_l\;\mathbb{W}(q,l,l')$$

l, l' - lepton momenta

Angular coefficients of Z-boson production

In CMS and ATLAS experiments $s = 8$ TeV, $Q = 80 - 100$ GeV and $Q_\perp$ varies from 0 to 120 GeV.

Our analysis is valid at $Q_\perp = 10 - 30$ GeV and $Y \simeq 0$ ($x_A \sim x_B \sim 0.1$) so that power corrections are small but sizable.

Angular distribution of DY leptons in the Collins-Soper frame ($c_\phi \equiv \cos\phi$, $s_\phi \equiv \sin\phi$ etc.)

$$\frac{d\sigma}{dQ^2 dy d\Omega_l} = \frac{3}{16\pi}\frac{d\sigma}{dQ^2 dy}\Big[(1+c_\theta^2) + \frac{A_0}{2}(1-3c_\theta^2) + A_1 s_{2\theta} c_\phi + \frac{A_2}{2} s_\theta^2 c_{2\phi} + A_3 s_\theta c_\phi + A_4 c_\theta + A_5 s_\theta^2 s_{2\phi} + A_6 s_{2\theta} s_\phi + A_7 s_\theta s_\phi\Big]$$

Result with $\frac{1}{Q^2}$, large-N_c and "f_1" accuracy

$$\mathbb{W}(q,l,l') = c_e^2 c_f^2 \frac{Q^4}{|m_Z^2 - Q^2|^2 + \Gamma_Z^2 m_Z^2}$$
$$\times \sum_f \Big\{ (a_e^2+1)(a_f^2+1)\Big(\Big[\mathcal{W}^{\mathrm{Ff}} - \frac{Q_\perp^2}{2Q^2}(\mathcal{W}^{\mathrm{Ff}} - \mathcal{W}_L^{\mathrm{Ff}})\Big](1+\cos^2\theta)$$
$$+ \frac{Q_\perp^2}{2Q^2}\mathcal{W}_L^{\mathrm{Ff}}(1-3\cos^2\theta) + \frac{Q_\perp}{Q}\mathcal{W}_1^{\mathrm{Ff}}\sin 2\theta\cos\phi + \frac{Q_\perp^2}{2Q^2}\mathcal{W}^{\mathrm{Ff}}\sin^2\theta\cos 2\phi \Big] \Big)$$
$$+ 8a_e a_f \Big[\frac{Q_\perp}{Q}\mathcal{W}_3^{\mathrm{F}f}\sin\theta\cos\phi + \mathcal{W}_4^{\mathrm{F}f}\cos\theta \Big] \Big\}$$

$$\mathcal{W}^{\mathrm{F}f}(q) = \int d^2k_\perp F^f(q,k_\perp), \qquad \mathcal{W}_L^{\mathrm{F}f}(q) = \int dk_\perp \frac{(q-2k)_\perp^2}{q_\perp^2} F^f(q,k_\perp)$$
$$\mathcal{W}_1^{\mathrm{F}f}(q) = \int d^2k_\perp \frac{(q,q-2k)_\perp}{q_\perp^2} F^f(q,k_\perp)$$
$$\mathcal{W}_3^{\mathrm{F}f}(q) = \int d^2k_\perp \frac{(q,q-2k)_\perp}{q_\perp^2} \mathcal{F}^f(q,k_\perp), \qquad \mathcal{W}_4^{\mathrm{F}f}(q) = \int d^2k_\perp \mathcal{F}^f(q,k_\perp),$$

$$\mathcal{F}^f(q,k_\perp) = f_1^f(\alpha_q,k_\perp)\bar{f}_1^f(\beta_q,(q-k)_\perp) - f_1^f \leftrightarrow \bar{f}_1^f$$

Comparison with LHC results

$$\mathbb{W} \sim \sum_f \mathcal{W}^{\rm Ff}\Big\{(a_e^2+1)(a_f^2+1)\Big(\Big[1-\frac{Q_\perp^2}{2m_Z^2}+\frac{Q_\perp^2}{2m_Z^2}\frac{\mathcal{W}_L^{\rm Ff}}{\mathcal{W}^{\rm Ff}}\Big](1+\cos^2\theta)$$
$$+\frac{Q_\perp^2}{2m_Z^2}\frac{\mathcal{W}_L^{\rm Ff}}{\mathcal{W}^{\rm Ff}}(1-3\cos^2\theta)+\frac{Q_\perp}{m_Z}\frac{\mathcal{W}_1^{\rm Ff}}{\mathcal{W}^{\rm Ff}}\sin 2\theta\cos\phi+\frac{Q_\perp^2}{2m_Z^2}\sin^2\theta\cos 2\phi]\Big)$$
$$+8a_ea_f\Big[\frac{Q_\perp}{m_Z}\frac{\mathcal{W}_3^{\rm Ff}}{\mathcal{W}^{\rm Ff}}\sin\theta\cos\phi+\frac{\mathcal{W}_4^{\rm Ff}}{\mathcal{W}^{\rm Ff}}\cos\theta\Big]\Big\}$$

We can easily estimate A_0 and A_2 which depend on $\frac{\mathcal{W}_L^{\rm Ff}}{\mathcal{W}^{\rm Ff}}$.

Logarithmic estimate of $\frac{\mathcal{W}_L^{\rm Ff}}{\mathcal{W}^{\rm Ff}}$: if $k_\perp^2 \gg m_N^2$ we can approximate

$$f_1(x,k_\perp^2) \simeq \frac{f(x)}{k_\perp^2} \quad\Rightarrow\quad F(q,k_\perp) \simeq \frac{f(\alpha_q)\bar{f}(\beta_q)+f\leftrightarrow\bar{f}}{k_\perp^2(q-k)_\perp^2}$$

Performing integration over $k_\perp$ in logarithmical approximation, one obtains

$$\frac{\mathcal{W}_L^{\rm Ff}}{\mathcal{W}^{\rm Ff}} \simeq 1+2\frac{\ln m_z^2/Q_\perp^2}{\ln Q_\perp^2/m^2}$$

Comparison of A_0 with LHC results

Logarithmic estimate of A_0

$$\frac{\mathcal{W}_L^{\rm Ff}}{\mathcal{W}^{\rm Ff}} \simeq 1+2\frac{\ln m_z^2/Q_\perp^2}{\ln Q_\perp^2/m^2} \quad \Rightarrow \quad A_0 = \frac{Q_\perp^2}{m_z^2}\frac{1+2\frac{\ln m_z^2/Q_\perp^2}{\ln Q_\perp^2/m^2}}{1+\frac{Q_\perp^2}{m_z^2}\frac{\ln m_z^2/Q_\perp^2}{\ln Q_\perp^2/m^2}} \qquad (*)$$

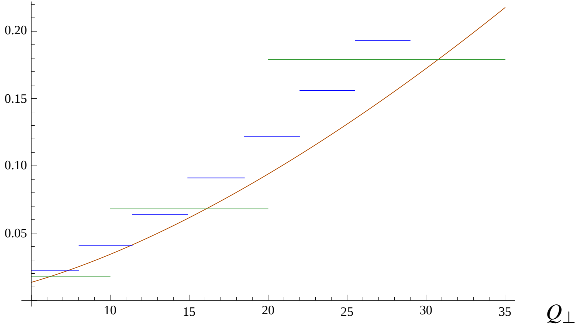


Figure: Comparison of prediction $(*)$ with lines depicting angular coefficient A_0 in bins of $Q_\perp$ and $Y < 1$ from CMS (arXiv:1504.03512) and ATLAS (arXiv1606.00689)

Comparison of A_2 with LHC results

Logarithmic estimate of A_2

$$\frac{\mathcal{W}_L^{\rm Ff}}{\mathcal{W}^{\rm Ff}} \simeq 1+2\frac{\ln m_z^2/Q_\perp^2}{\ln Q_\perp^2/m^2} \quad \Rightarrow \quad A_2 = \frac{Q_\perp^2}{m_z^2}\frac{1}{1+\frac{Q_\perp^2}{m_z^2}\frac{\ln m_z^2/Q_\perp^2}{\ln Q_\perp^2/m^2}} \qquad (**)$$

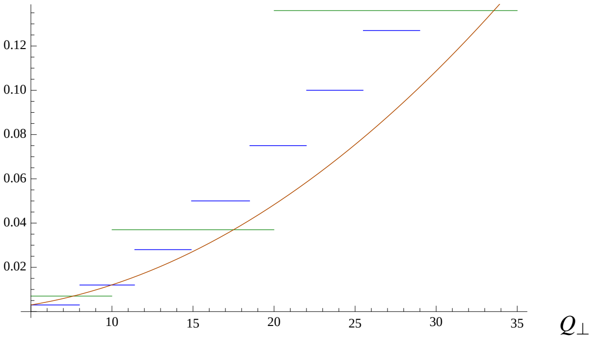


Figure: Comparison of prediction $(**)$ with lines depicting angular coefficient A_2 in bins of $Q_\perp$ and $Y < 1$ from CMS (arXiv:1504.03512) and ATLAS (arXiv1606.00689)

Qualitative checks

$$\mathbb{W} \sim \sum_f r^f \mathcal{W}^{\rm Ff}\Big\{1+\cos^2\theta + \frac{Q_\perp^2}{2m_Z^2}\frac{\mathcal{W}_L^{\rm Ff}}{\mathcal{W}^{\rm Ff} r^f}(1-3\cos^2\theta)$$
$$+ \frac{Q_\perp}{m_Z}\frac{\mathcal{W}_1^{\rm Ff}}{\mathcal{W}^{\rm Ff} r^f}\sin 2\theta\cos\phi + \frac{Q_\perp^2}{2m_Z^2 r^f}\sin^2\theta\cos 2\phi]$$
$$+ \frac{8a_e a_f}{(a_e^2+1)(a_f^2+1)}\Big[\frac{Q_\perp}{m_Z}\frac{\mathcal{W}_3^{\rm Ff}}{\mathcal{W}^{\rm Ff} r^f}\sin\theta\cos\phi + \frac{\mathcal{W}_4^{\rm Ff}}{\mathcal{W}^{\rm Ff} r^f}\cos\theta\Big]\Big\}$$

$r^f \equiv 1 - \frac{Q_\perp^2}{2m_Z^2} + \frac{Q_\perp^2}{2m_Z^2}\frac{\mathcal{W}_L^{\rm Ff}}{\mathcal{W}^{\rm Ff}}$

Qualitative checks:

- Factorization of TMD $f_1(x,k_\perp^2) \simeq f(x)f(k_\perp^2) \Rightarrow \mathcal{W}_1^{\rm Ff}(q) = 0$
  $\Rightarrow A_1$ is smaller than A_2
- A_4 does not depend on $Q_\perp$ and increases with rapidity
- A_3 is smaller than A_4
- A_5, A_6, A_7 are order of magnitude smaller than A_0, A_2, A_4

1. Conclusions
   - The Drell-Yan hadronic tensor is calculated in the Sudakov region $s \gg Q^2 \gg q_\perp^2$ in the tree approximation with $\frac{1}{Q^2}$ accuracy.
   - In the leading order in N_c the higher-twist quark-quark-gluon TMDs reduce to leading-twist TMDs due to QCD equation of motion.
   - The resulting hadronic tensor for unpolarized hadrons is (EM) gauge-invariant and depends on two leading-twist TMDs: f_1 responsible for total DY cross section, and Boer-Mulders function $h_1^\perp$.
   - Results for angular coefficients of Z-boson production seem to agree with LHC measurements at corresponding kinematics.
2. Outlook
   - Power corrections at moderate x_A and/or x_B.

Thank you for attention!