Pion Parton Distributions Using Threshold Resummation

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APS DNP Meeting 10/30/2020

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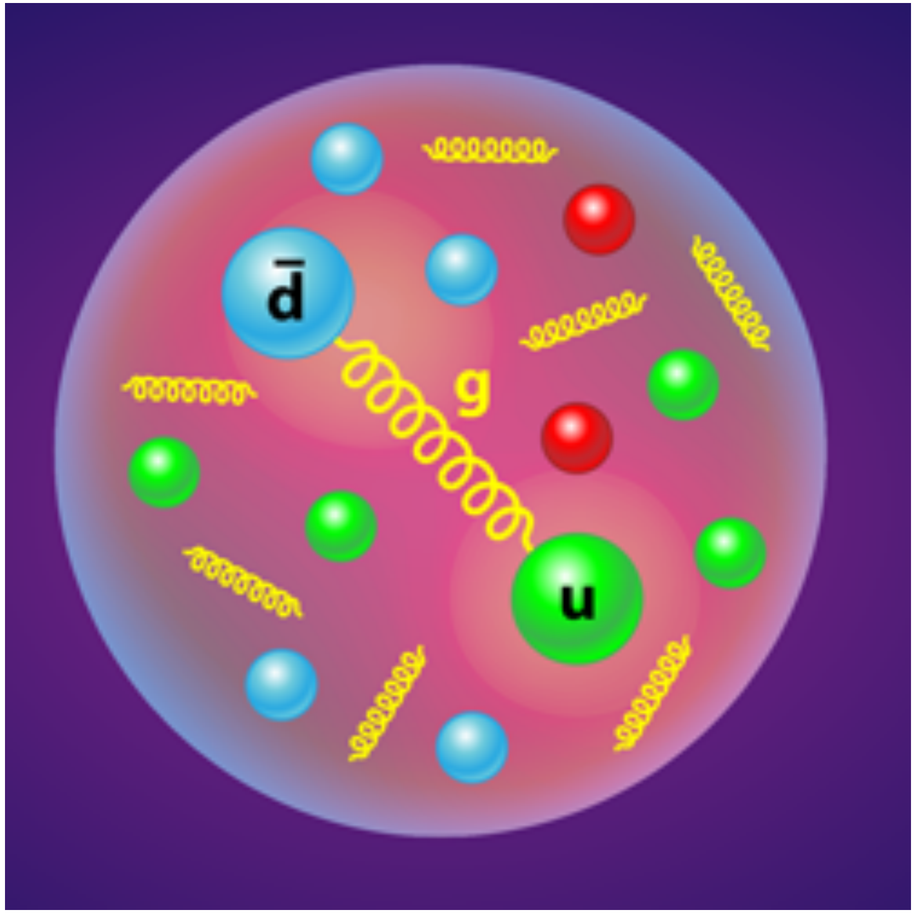
This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Workforce Development for Teachers and Scientists, Office of Science Graduate Student Research (SCGSR) program. The SCGSR program is administered by the Oak Ridge Institute for Science and Education for the DOE under contract number DE-SC0014664.

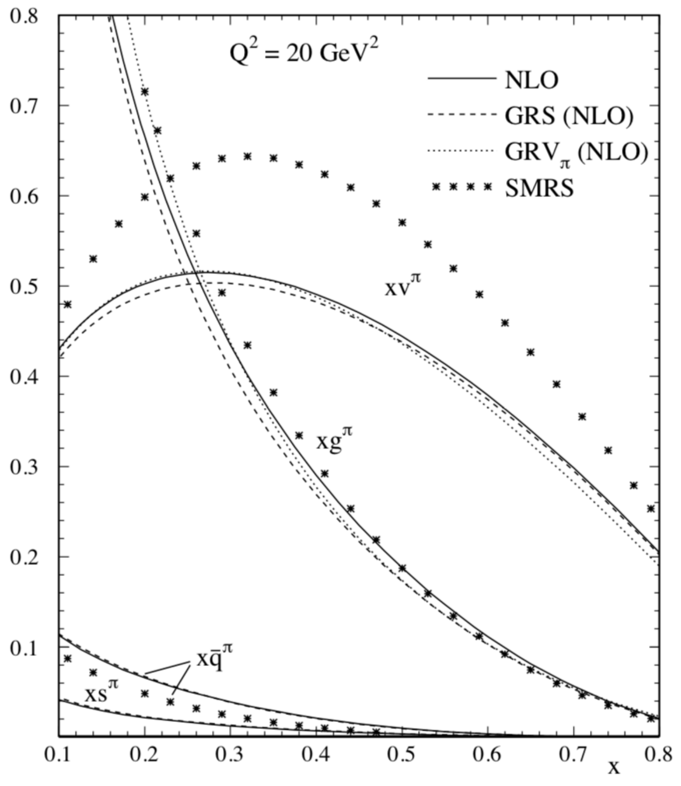
Pions

- Pion is the Goldstone boson associated with spontaneous symmetry breaking for chiral $SU(2)_L \times SU(2)_R$ symmetry
- Lightest hadron as $\frac{m_\pi}{M_N} \ll 1$ and dictates the nature of hadronic interactions at low energies
- Simultaneously a pseudoscalar meson made up of q and $\bar{q}$ constituents

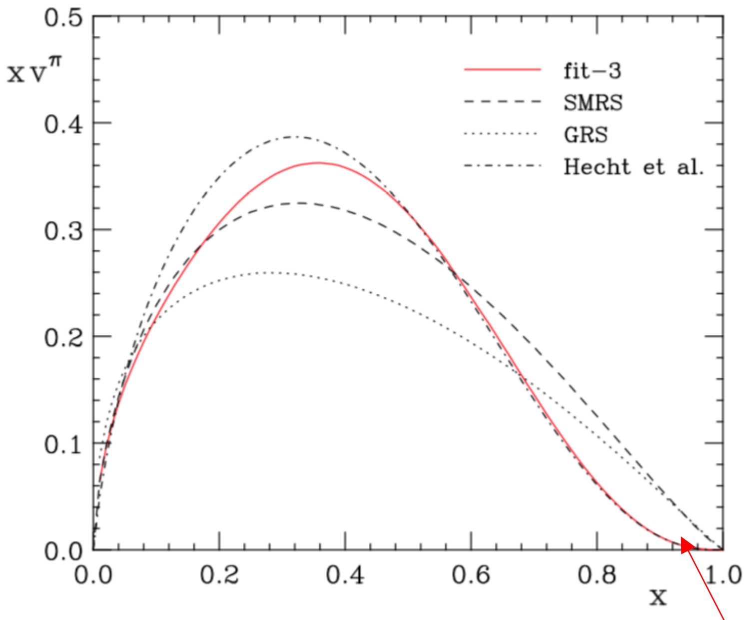


Previous Pion PDFs

- Fits to Drell-Yan, prompt photon, or both

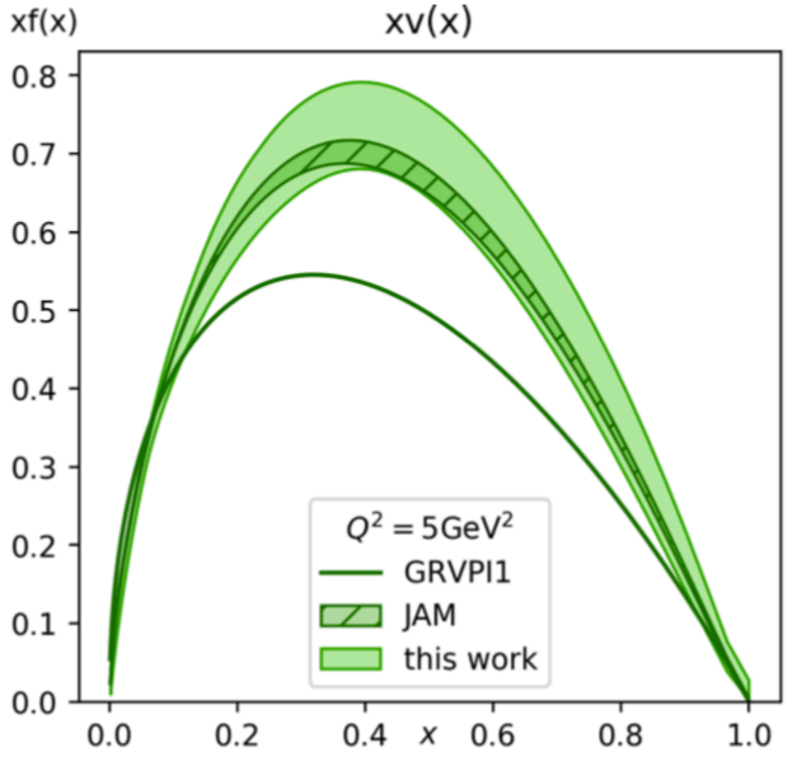


GRS, GRV, and SMRS



Aicher's valence PDF

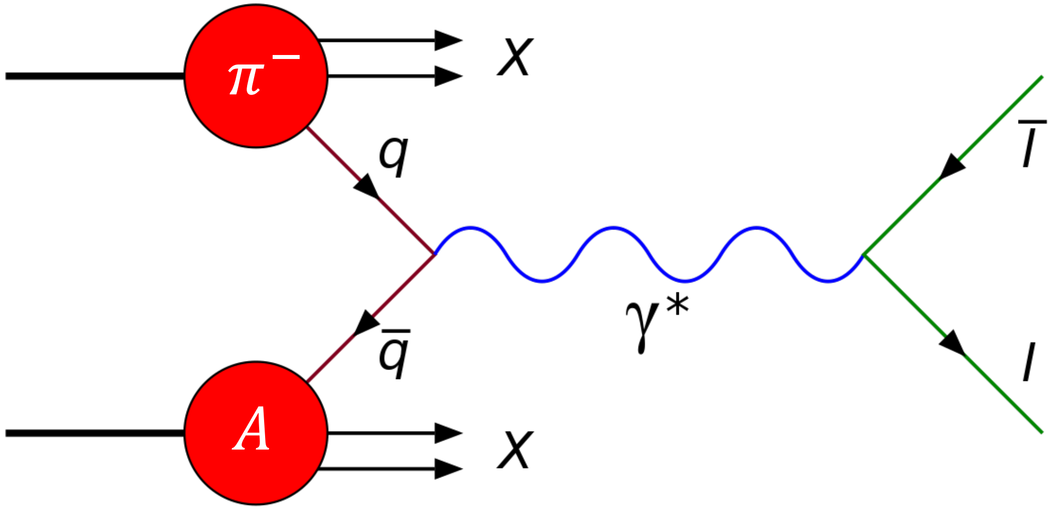
Uses threshold resummation



xFitter

Experiments to Probe Pion Structure

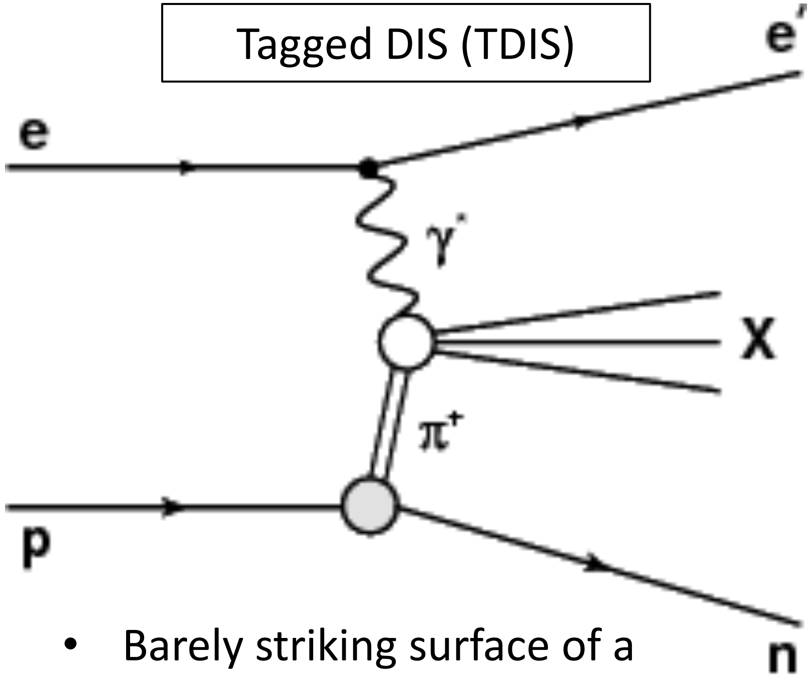
- Drell-Yan (DY)



- Accelerating pion allows for time dilation and longer lifetime

- Leading Neutron (LN)

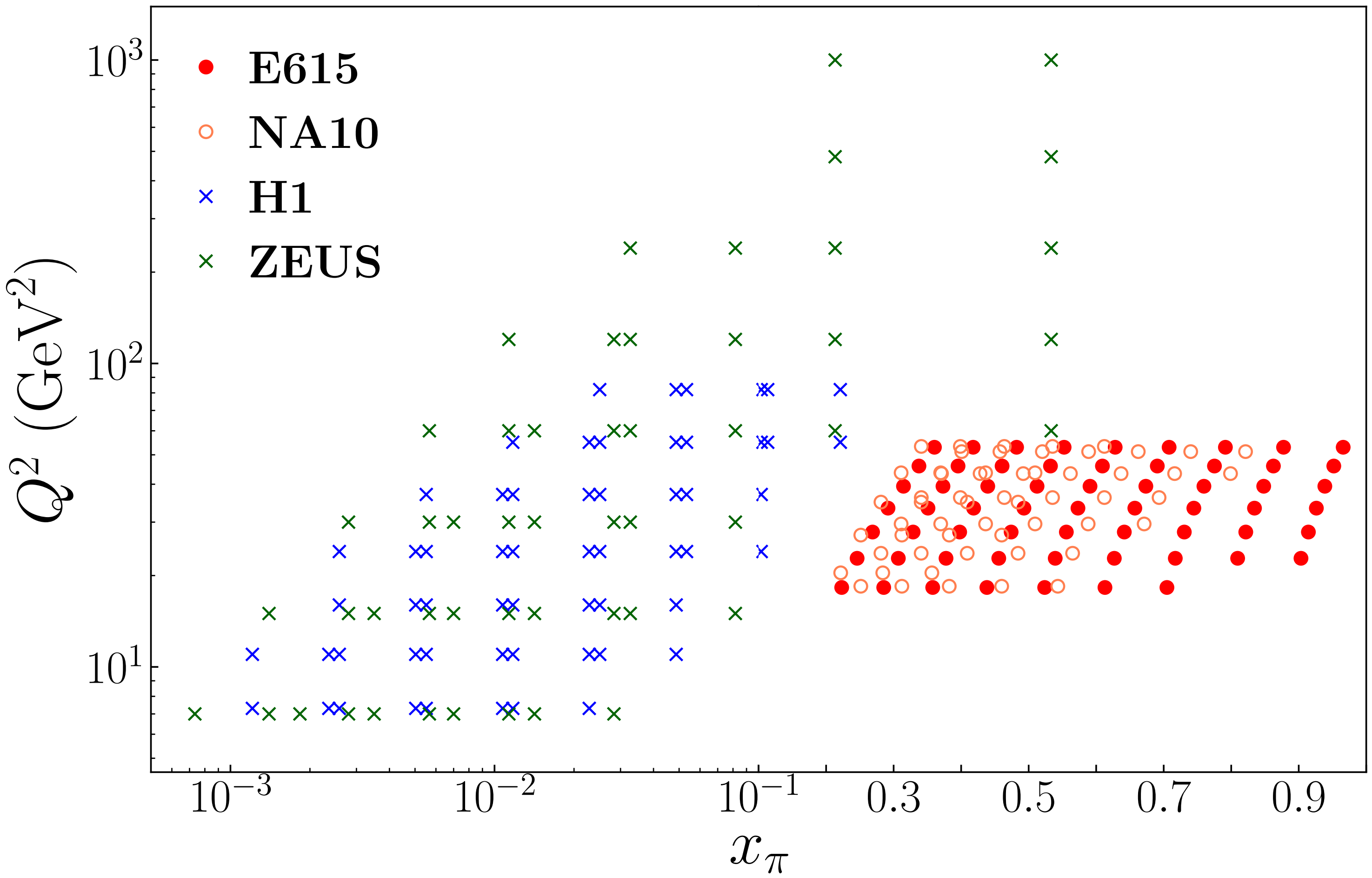
Tagged DIS (TDIS)



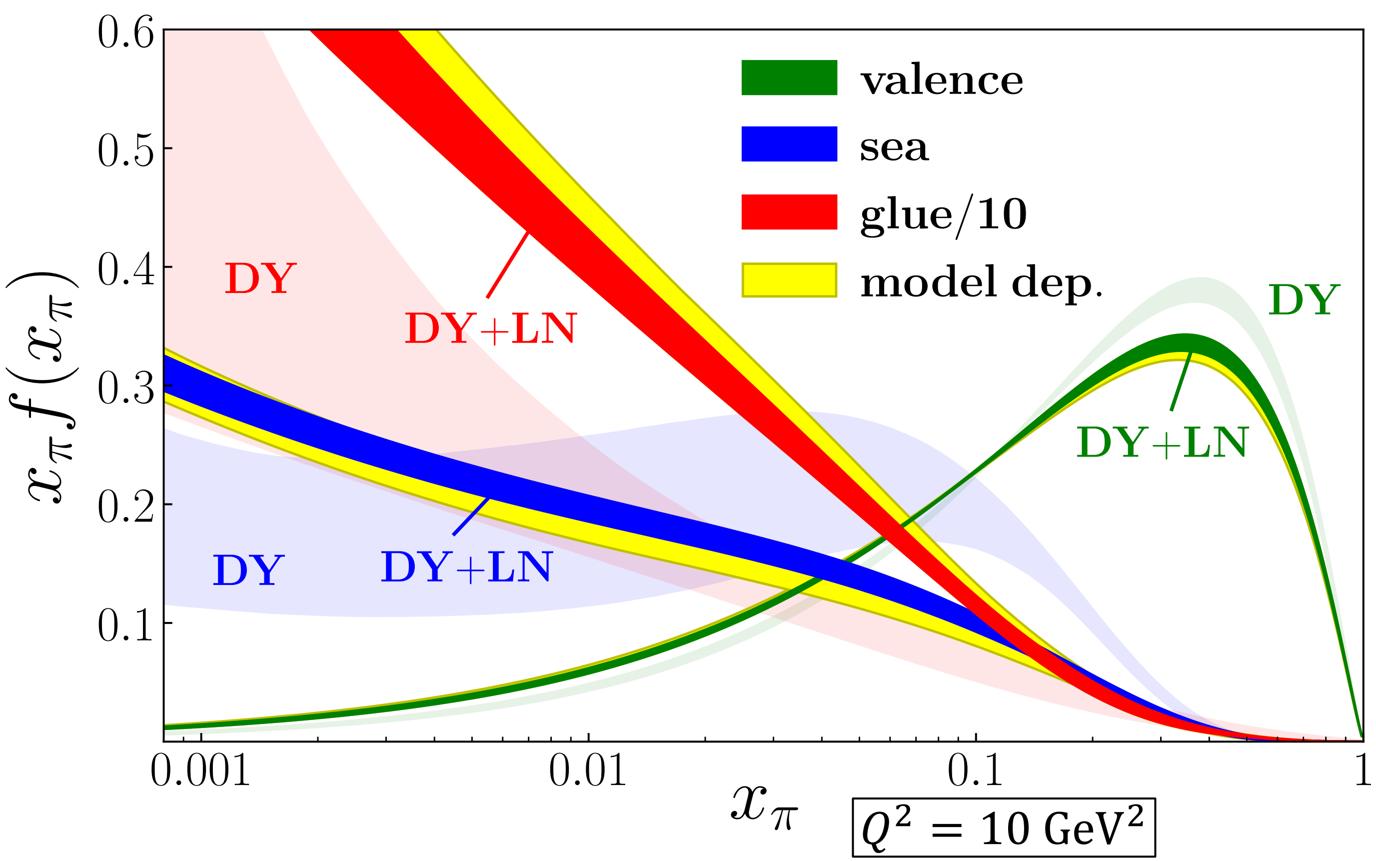
- Barely striking surface of a target proton knocks out an almost on-shell pion to probe

Kinematic Coverage

- DY data (E615, NA10) exist at large x_π, while LN (H1, ZEUS) data have small x_π
- Not much overlap, need more data



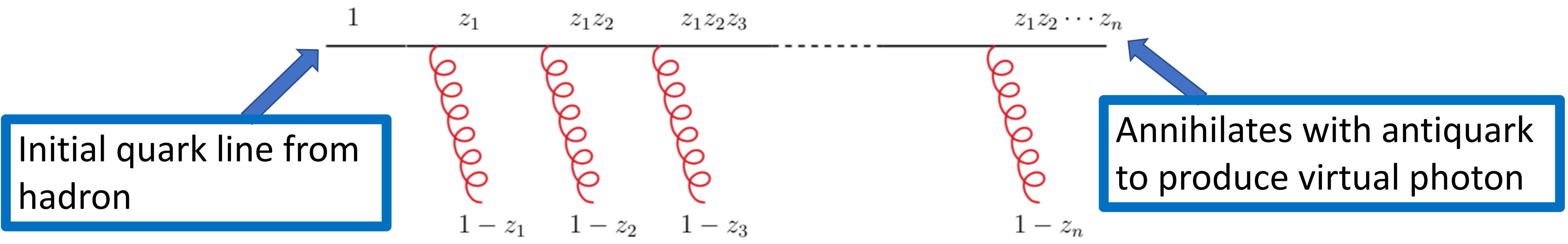
The First Global QCD Pion PDFs – Inclusion of LN



- All fixed order calculations
- The lightly shaded bands are fits to only DY data
- Uncertainties dramatically decrease with inclusion of LN data

We can see that the gluon distribution increases at low-x_π when adding in the LN data

Threshold Resummation



- Significant contributions to cross section occur in soft gluon emissions
- Terms are predictable to all orders of α_S
- Resummation should be performed in Mellin space to factorize the phase space

Next-to-Leading + Next-to-Leading Logarithm Order Calculation

Make sure only counted once!
- Subtract the matching

	LL	NLL	…	N^pLL
LO	1	--	…	--
NLO	$\alpha_s \log(N)^2$	$\alpha_s \log(N)$	…	--
NNLO	$\alpha_S^2 \log(N)^4$	$\alpha_S^2 (\log(N)^2, \log(N)^3)$	…	--
…	…	…	…	…
N^kLO	$\alpha_S^k \log(N)^{2k}$	$\alpha_S^k \left(\log(N)^{2k-1}, \log(N)^{2k-2}\right)$	…	$\alpha_S^k \log(N)^{2k-2p} + \cdots$

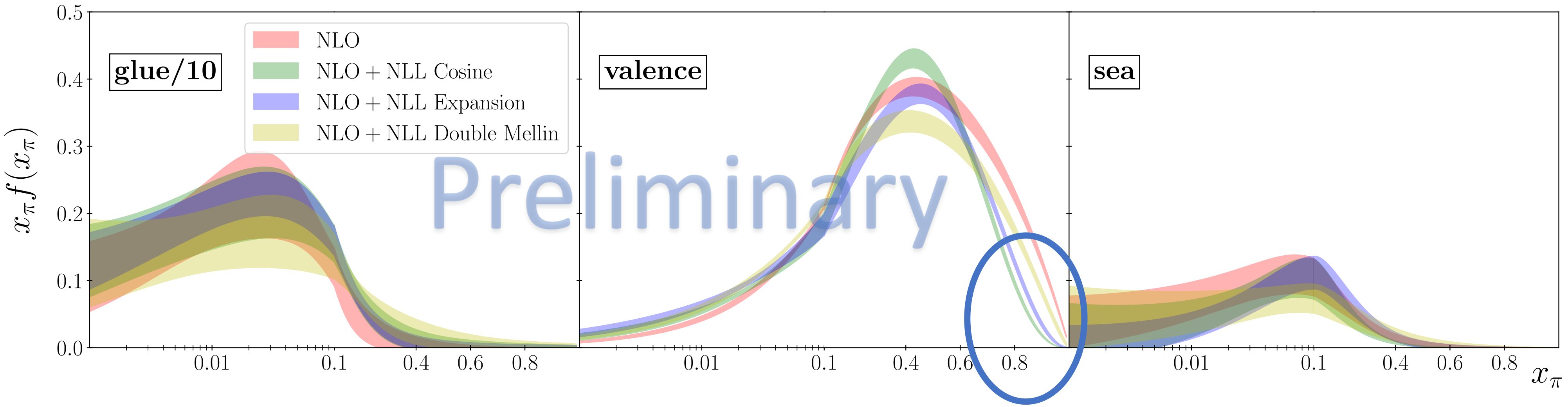
Methods of Resummation

- Rapidity distribution $\frac{d\sigma}{dQ^2 dY}$ adds more complications
- We can perform a Mellin-Fourier transform to account for the rapidity
  - A cosine appears while doing Fourier transform; options:
    1) Take first order expansion, cosine ≈ 1
    2) Keep cosine intact
- Can additionally perform a Double Mellin transform
- Explore the different methods and analyze effects

Fit results

- Performing threshold resummation on Drell-Yan process using various approximations and methods of calculation
- Fit to DY and LN data

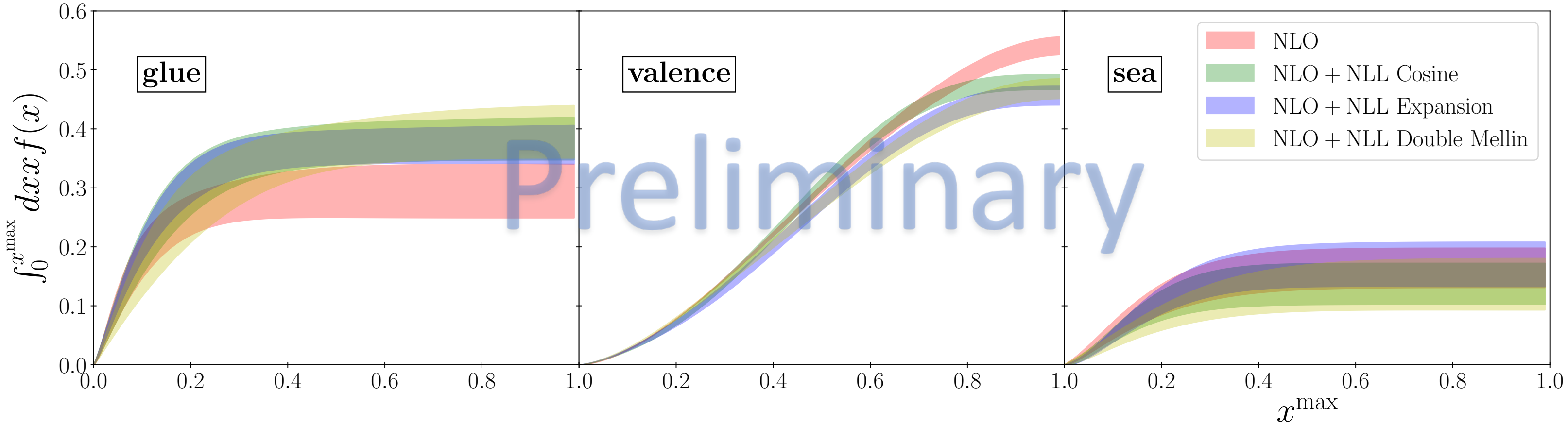
Parton distributions for gluon, valence, and sea at the input scale $Q_0^2 = m_c^2$



High-x_π behavior of valence distribution quite different

Momentum Fractions

- The truncated momentum fraction $\int_0^{x^{max}} dx\, x\, f(x)$ shown as a function of x^{max}



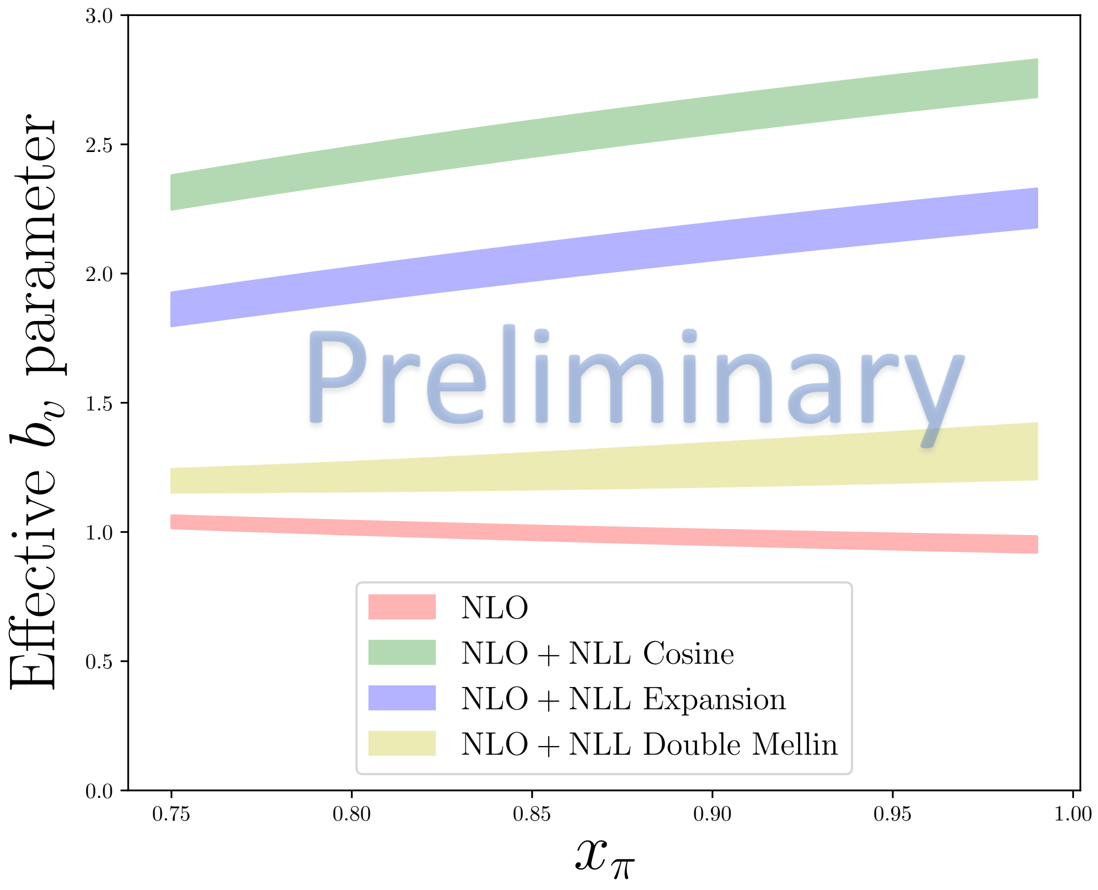
- Using resummation, less $\langle x_\pi \rangle_v$ and more $\langle x_\pi \rangle_g$
- Valence deviates from fixed order at large x^{max}

Values of the total momentum fractions by flavor and resummation method

Flavor	Cosine	Expansion	Double Mellin	NLO
valence	0.48(1)	0.46(2)	0.47(2)	0.54(2)
sea	0.14(3)	0.17(4)	0.14(4)	0.16(3)
gluon	0.38(4)	0.37(3)	0.40(4)	0.30(5)

Effective b_v Parameter

- The effective large x_π behavior in $q_v(x_\pi, \mu_0^2) \sim (1 - x_\pi)^{b_v}$
- NLO gives $b_v \sim 1$
- All resummation methods give effective b_v parameter > 1



Conclusions

- Major effect in the large x_π behavior of the valence quark distribution
- Indirectly affects the correlated low-x_π gluon and sea quark distributions
- We find a larger momentum fraction of the gluon when using threshold resummation than using fixed order
- Different methods of threshold resummation describe well the data, but give different PDFs

Backup Slides

Issues with Perturbative Calculations

$$\hat{\sigma} \sim \delta(1-z) + \alpha_S(\log(1-z))_+ \longrightarrow \hat{\sigma} \sim \delta(1-z)[1 + \alpha_S \log(1-\tau)]$$

- If τ is large, can potentially spoil the perturbative calculation
- Improvements can be made by resumming $\log(1-z)_+$ terms

Threshold Resummation

- Phase space needs to be broken up and factorized
- A convenient way to do this is by Mellin transforms

$$\log(1-z) \rightarrow \log N$$

- Kernels will exponentiate in Mellin space

Resummed Kernel

$$\ln C_{\mathrm{NLL}}^{\mathrm{res}}(N, \alpha_S) = C_q + 2h^{(1)}(\lambda) \ln \bar{N} + 2h^{(2)}(\lambda, \frac{Q^2}{\mu^2})$$

$$\bar{N} = N e^{\gamma_E}$$

$$\lambda = b_0 \alpha_S(\mu^2) \ln \bar{N}$$

$$C_q = \frac{\alpha_S}{\pi} C_F \Big(-4 + \frac{2\pi^2}{3} + \frac{3}{2} \ln \frac{Q^2}{\mu^2} \Big)$$

Resummed Kernel

$$h^{(1)}(\lambda) = \frac{A_q^{(1)}}{2\pi b_0 \lambda}[2\lambda + (1-2\lambda)\ln(1-2\lambda)]$$

$$h^{(2)}(\lambda) = (\pi A_q^{(1)} b_1 - b_0 A_q^{(2)}) \frac{2\lambda + \ln(1-2\lambda)}{2\pi^2 b_0^3}$$

$$+\frac{A_q^{(1)} b_1}{4\pi b_0^3} \ln^2(1-2\lambda) + \frac{A_q^{(1)}}{2\pi b_0} \ln(1-2\lambda) \ln\frac{Q^2}{\mu^2}$$

Landau Pole

- Leading logarithm (LL) resummation term

$$h^{(1)}(\lambda) = \frac{A_q^{(1)}}{2\pi b_0 \lambda}[2\lambda + (1-2\lambda)\ln(1-2\lambda)]$$

- The argument of a logarithm cannot be ≤ 0
- The value of N which this occurs is the Landau pole

$$N_{\text{Landau}} = \exp\left(\frac{1}{2b_0\alpha_S} - \gamma_E\right)$$

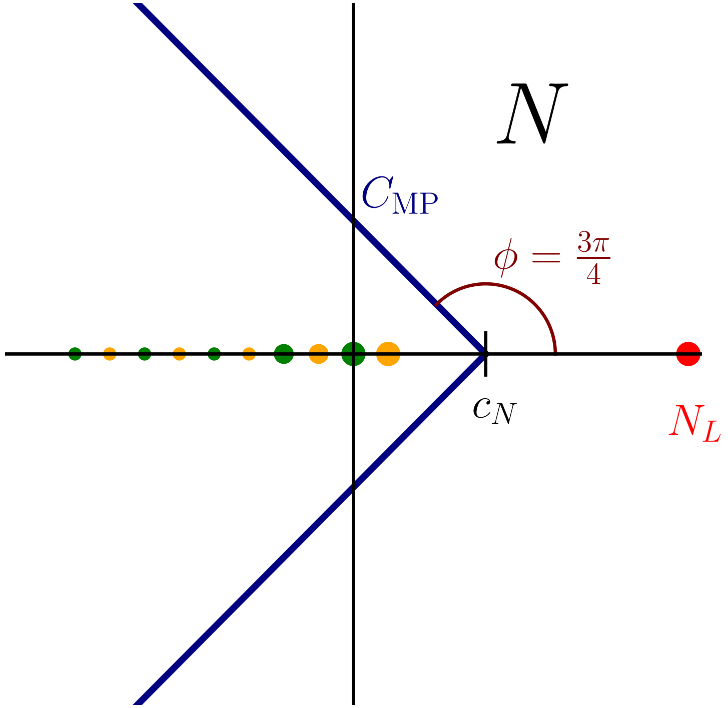
Origin of Landau Pole

$$\alpha_S C^{(1)}_{\text{soft}}(N) = 2\frac{C_F}{\pi}\int_0^1 dz \frac{z^{N-1}-1}{1-z}\int_{Q^2}^{(1-z)^2Q^2}\frac{dk_\perp^2}{k_\perp^2}\alpha_S(k_\perp^2)$$

- Upper Limits imply that $k_\perp^2$ will go to 0
- $\alpha_S(\mu^2 = 0)$ is NOT well-defined
- Ambiguities on how to deal with this provide needs for prescriptions such as Minimal Prescription (MP) and Borel Prescription(BP)
- Focus on MP

Minimal Prescription (MP)

- Need to Mellin invert to z space to compare with data
- The MP makes use of a contour that does not enclose Landau Pole



K-factors

- Ratios of calculated NLO+NLL to the NLO cross section
- Various methods gather different amounts of higher order corrections