Systematic Extraction of Pion Parton Distributions Using Threshold Resummation

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Motivation

• Perform a **global QCD analysis** on available pion data using **factorization theorems** and parametrizing universal **PDFs**

• Pion is the **Goldstone boson** associated with spontaneous symmetry breaking of chiral $SU(2)_L \times SU(2)_R$ symmetry

• Lightest hadron made up of $q$ and $\bar{q}$ constituents

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Experiments to Probe Pion Structure

• Drell-Yan (DY)

- Accelerating pion allows for time dilation and longer lifetime

• Leading Neutron (LN)

- Barely striking surface of a target proton knocks out an almost on-shell pion to probe

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Datasets -- Kinematics

- Large $x_\pi$ -- Drell-Yan (DY)
- Small $x_\pi$ -- Leading Neutron (LN)
- Not much data overlap
- In DY:
  \[ x_\pi = \frac{1}{2} \left( x_F + \sqrt{x_F^2 + 4\tau} \right) \]
- In LN:
  \[ x_\pi = x_B / \bar{x}_L \]
JAM18 Pion PDFs

- Lightly shaded bands – only Drell-Yan data
- Darkly shaded bands – fit to both Drell-Yan and LN data

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JAM20 Pion PDFs

- For the first time, we included large $p_T$-dependent Drell-Yan data, which follows collinear factorization.
- Large $p_T$ does not dramatically affect the PDF.

N. Cao, PCB, N. Sato, and W. Melnitchouk
Soft Gluon Resummation

- Fixed-target Drell-Yan notoriously has large-$x_F$ contamination of higher orders
- Large logarithms may spoil perturbation
- Focus on corrections to the most important $q\bar{q}$ channel
- Resum contributions to all orders of $\alpha_s$
Issues with Perturbative Calculations

• If $\tau$ is large, can potentially spoil the perturbative calculation

• Improvements can be made by resumming $\log(1 - z)_+$ terms

\[
\hat{\sigma} \sim \delta(1 - z) + \alpha_S(\log(1 - z))_+ \quad \rightarrow \quad \hat{\sigma} \sim \delta(1 - z)[1 + \alpha_S \log (1 - \tau)]
\]
An NLO calculation gathers the $\mathcal{O}(\alpha_s)$ terms

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<thead>
<tr>
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<th>NLL</th>
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<th>N^pLL</th>
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# Next-to-Leading + Next-to-Leading Logarithm Order Calculation

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Make sure only counted once!
- Subtract the matching

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Origin of Landau Pole

- Upper Limits imply that $k^2_{\perp}$ will go to 0
- $\alpha_S(\mu^2 = 0)$ is NOT well-defined
- Ambiguities on how to deal with this provide needs for prescriptions

$$\alpha_S C_{\text{soft}}^{(1)}(N) = 2 \frac{C_F}{\pi} \int_0^1 dz \frac{z^{N-1} - 1}{1-z} \int_{Q^2}^{(1-z)^2 Q^2} \frac{dk^2_{\perp}}{k^2_{\perp}} \alpha_S(k^2_{\perp})$$
Methods of Resummation

• Make use of the Minimal Prescription to avoid Landau Pole

• Rapidity distribution \( \frac{d\sigma}{dQ^2 dY} \) adds more complications

• We can perform a Mellin-Fourier transform to account for the rapidity
  • A cosine appears while doing Fourier transform; options:
    1) Take first order expansion, cosine \( \approx 1 \)
    2) Keep cosine intact

• Can additionally perform a Double Mellin transform

• Explore the different methods and analyze effects

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Data and Theory Comparison – Drell-Yan

• Cosine method tends to overpredict the data at very large $x_F$
• Double Mellin method is qualitatively very similar to NLO
• Resummation is largely a high-$x_F$ effect
PDF Results

• Large $x$ behavior in valence depends on prescription

Large $x$ momentum fraction in the valence shifts to low $x$ gluon momentum fraction
Effective $\beta_v$ parameter

- $q_v(x) \sim (1 - x)^{\beta_v}$ as $x \to 1$
- Threshold resummation does not give universal behavior of $\beta_v$
- NLO and double Mellin give $\beta_v \approx 1$
- Cosine and Expansion give $\beta_v > 2$

\begin{figure}
\centering
\includegraphics[width=\textwidth]{effectiv-beta-v-parameter.png}
\caption{Graph showing the effective $\beta_v$ parameter for different theoretical approaches.}
\end{figure}
Future Work

• Investigate high-$x$ behavior of valence PDF through constraints from the lattice data

• Confront the small-$p_T$ Drell-Yan data in terms of CSS formulations and extract pion TMDs

• Investigate impacts of future experiments on pion and kaon PDFs
Backup
Previous Pion PDFs

• Fits to Drell-Yan, prompt photon, or both

GRS, GRV, and SMRS

Aicher’s valence PDF

xFitter

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Drell-Yan (DY)

\[ \sigma \propto \sum_{i,j} f_i^{\pi}(x_\pi, \mu) \otimes f_j^A(x_A, \mu) \otimes \hat{\sigma}_{i,j}(x_\pi, x_A, Q/\mu) \]
Drell-Yan (DY) Definitions

Hadronic variable

\[ \tau = \frac{Q^2}{\hat{S}} \]

Partonic variable

\[ z \equiv \frac{Q^2}{\hat{S}} = \frac{\tau}{x_1 x_2} \]

\( \hat{S} \) is the center of mass momentum squared of incoming partons

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$C_{q\bar{q}} = \delta(1 - z) \frac{\delta(y) + \delta(1 - y)}{2}$

- $z = 1$ corresponds to partonic threshold
- All $\hat{S}$ is equal to $Q^2$
- All energy of hard partons turns into virtuality of photon
NLO Virtual

• Virtual corrections at NLO are proportional to $\delta(1 - z)$
  • Exhibit Born kinematics

\[
C_{q\bar{q}}^{\text{virtual}} = \delta(1 - z) \frac{\delta(y) + \delta(1 - y)}{2} \left[ \frac{C_F \alpha_S}{\pi} \left( \frac{3}{2} \ln \frac{Q^2}{\mu^2} + \frac{2\pi^2}{3} - 6 \right) \right]
\]
NLO Real Emission

• Next to leading order, real gluon emissions

\[ C^{\text{real}}_{q\bar{q}} = \frac{C_F \alpha_s}{\pi} \left[ \frac{\delta(y) + \delta(1 - y)}{2} \right] \left[ (1 + z^2) \left( \frac{1}{1 - z} \ln \frac{Q^2(1 - z)^2}{\mu^2 z} \right)_+ + 1 - z \right] + \frac{1}{2} \left[ \frac{(1 - z)^2}{z} y(1 - y) \right] \left[ \frac{1 + z^2}{1 - z} \left( \left[ \frac{1}{y} \right]_+ + \left[ \frac{1}{1 - y} \right]_+ \right) - 2(1 - z) \right] \]
NLO Real Emission

- Real quark emissions
- \( C_{qg} = C_{gq} \big|_{y \to 1-y} \)

\[
C_{qg}^{\text{real}} = \frac{T_F \alpha_S}{2\pi} \left[ \delta(y) \left[ (z^2 + (1 - z)^2) \ln \frac{Q^2(1 - z)^2}{\mu^2 z} + 2z(1 - z) \right] 
+ \left[ 1 + \frac{(1 - z)^2}{z} y(1 - y) \right] \left[ (z^2 + (1 - z)^2) \left( \frac{1}{y} \right) + 2z(1 - z) + (1 - z)^2 y \right] \right]
\]
Leading Neutron (LN)

\[
\frac{d\sigma}{dx dQ^2 dy} \sim f_{p \to \pi^+ n}(y) \times \sum_q \int_{x/y}^{1} \frac{d\xi}{\xi} C(\xi) q \left( \frac{x/y}{\xi}, \mu^2 \right)
\]
Large $x_L$

- $x_L$ is fraction of longitudinal momentum carried by neutron relative to initial proton.
- For $t$ to be close to pion pole, has to go near 0 – happens at large $x_L$.
- In this region, one pion exchange dominates.
Splitting Function and Regulators

We examine five regulators, and we fit $\Lambda$

$\mathcal{F}$ is a UV regulator, which the data chooses

Amplitude for proton to dissociate into a $\pi^+$ and neutron:

$$f_{\pi N}(x_L) = \frac{g_A^2 M^2}{(4\pi f_\pi)^2} \int dk_{\perp}^2 \frac{x_L}{x_L^2 D_{\pi N}^2} \left[ \frac{k_{\perp}^2 + x_L^2 M^2}{x_L^2 D_{\pi N}^2} \right] |\mathcal{F}|^2,$$

$$D_{\pi N} \equiv t - m_{\pi}^2 = -\frac{1}{1 - y} \left[ k_{\perp}^2 + y^2 M^2 + (1 - y)m_{\pi}^2 \right]$$

$$\mathcal{F} = \begin{cases} 
(i) \quad \exp \left( \frac{(M^2 - s)}{\Lambda^2} \right) &\text{s-dep. exponential} \\
(ii) \quad \exp \left( \frac{D_{\pi N}}{\Lambda^2} \right) &\text{t-dep. exponential} \\
(iii) \quad (\Lambda^2 - m_{\pi}^2)/(\Lambda^2 - t) &\text{t-dep. monopole} \\
(iv) \quad \tilde{x}_L^{-\alpha(t)} \exp \left( \frac{D_{\pi N}}{\Lambda^2} \right) &\text{Regge} \\
(v) \quad \left[ 1 - \frac{D_{\pi N}}{\Lambda^2 - t} \right]^{1/2} &\text{Pauli-Villars} 
\end{cases}$$

Best fit

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Bayesian Inference

- Minimize the $\chi^2$ for each replica

$$\chi^2(a, \text{data}) = \sum_e \left( \sum_i \left[ \frac{d_i^e - \sum_k r_k^e \beta_{k,i}^e - t_i^e(a)/n_e}{\alpha_i^e} \right]^2 + \left( \frac{1 - n_e}{\delta n_e} \right)^2 + \sum_k (r_k^e)^2 \right)$$

- Perform $N$ total $\chi^2$ minimizations and compute statistical quantities

Expectation value
$$E[\mathcal{O}] = \frac{1}{N} \sum_k \mathcal{O}(a_k),$$

Variance
$$V[\mathcal{O}] = \frac{1}{N} \sum_k \left[ \mathcal{O}(a_k) - E[\mathcal{O}] \right]^2,$$