Systematic Extraction of Pion Parton Distributions Using Threshold Resummation and Applications to Pion TMDs

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DIS 2021: Structure function and parton densities 4/15/2021
Motivation

What to do:

• **Define** a structure of hadrons in terms of quantum field theories
• **Identify** theoretical observables that factorize into non-perturbative objects and perturbatively calculable physics
• **Perform** global QCD analysis as structures are universal and are the same in all subprocesses
Pions

• Pion is the **Goldstone boson** associated with spontaneous symmetry breaking of chiral $SU(2)_L \times SU(2)_R$ symmetry

• **Lightest hadron** as $\frac{m_\pi}{M_N} \ll 1$ and dictates the nature of hadronic interactions at low energies

• Simultaneously a pseudoscalar meson made up of $q$ and $\bar{q}$ constituents
Experiments to Probe Pion Structure

• Drell-Yan (DY)

• Accelerating pion allows for time dilation and longer lifetime

• Leading Neutron (LN)

• Barely striking surface of a target proton knocks out an almost on-shell pion to probe

Tagged DIS (TDIS)
Datasets -- Kinematics

• Large $x_\pi$ -- Drell-Yan (DY)
• Small $x_\pi$ -- Leading Neutron (LN)
• Not much data overlap
• In DY:
  \[ x_\pi = \frac{1}{2} \left( x_F + \sqrt{x_F^2 + 4\tau} \right) \]
• In LN:
  \[ x_\pi = x_B / \bar{x}_L \]
JAM18 Pion PDFs

• Lightly shaded bands – only Drell-Yan data
• Darkly shaded bands – fit to both Drell-Yan and LN data


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Soft Gluon Resummation

• Fixed-target Drell-Yan notoriously has large-$x_F$ contamination of higher orders

• Large logarithms may spoil perturbation

• Focus on corrections to the most important $q\bar{q}$ channel

• Resum contributions to all orders of $\alpha_s$
Issues with Perturbative Calculations

• If $\tau$ is large, can potentially spoil the perturbative calculation

• Improvements can be made by resumming $\log(1 - z)_+$ terms

\[
\hat{\sigma} \sim \delta(1 - z) + \alpha_s(\log(1 - z))_+ \quad \rightarrow \quad \hat{\sigma} \sim \delta(1 - z)[1 + \alpha_s\log(1 - \tau)]
\]
An NLO calculation gathers the $O(\alpha_s)$ terms

### LL  |  NLL  |  ...  |  N^pLL

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<td>$\alpha_s^k \log(N)^{2k-2p} + ...$</td>
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Next-to-Leading + Next-to-Leading Logarithm Order Calculation

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Add the columns to the rows
Make sure only counted once!
- Subtract the matching

## Next-to-Leading + Next-to-Leading Logarithm Order Calculation

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Origin of Landau Pole

\[
\alpha_S C_{\text{soft}}^{(1)} (N) = 2 \frac{C_F}{\pi} \int_0^1 dz \frac{z^{N-1} - 1}{1 - z} \int_{Q^2} (1-z)^2 Q^2 \frac{dk_\perp^2}{k_\perp^2} \alpha_S (k_\perp^2)
\]

• Upper Limits imply that \( k_\perp^2 \) will go to 0
• \( \alpha_S (\mu^2 = 0) \) is NOT well-defined
• Ambiguities on how to deal with this provide needs for prescriptions
Methods of Resummation

• Make use of the Minimal Prescription to avoid Landau Pole

• Rapidity distribution \( \frac{d\sigma}{dQ^2dY} \) adds more complications

• We can perform a Mellin-Fourier transform to account for the rapidity
  • A cosine appears while doing Fourier transform; options:
    1) Take first order expansion, \( \cosine \approx 1 \)
    2) Keep cosine intact

• Can additionally perform a Double Mellin transform

• Explore the different methods and analyze effects
Data and Theory Comparison – Drell-Yan

- Cosine method tends to overpredict the data at very large $x_F$
- Double Mellin method is qualitatively very similar to NLO
- Resummation is largely a high-$x_F$ effect
PDF Results

• Large $x$ behavior in valence depends on prescription

Large $x$ momentum fraction in the valence shifts to low $x$ gluon momentum fraction
Effective $\beta_v$ parameter

- $q_v(x) \sim (1 - x)^{\beta_v}$ as $x \to 1$
- Threshold resummation does not give universal behavior of $\beta_v$
- NLO and double Mellin give $\beta_v \approx 1$
- Cosine and Expansion give $\beta_v > 2$
Transverse Momentum-dependent Drell-Yan
$p_T$-dependent spectrum for pion data

- Small-$p_T$ data – TMD factorization – partonic transverse momentum
- Large-$p_T$ data – collinear factorization – recoil transverse momentum

E615 $\pi W$ Drell-Yan

$p_T$-dependent spectrum for pion data

- Various factorization theorems break down in certain regions of $p_T$
- Errors are related with $\mathcal{O}(p_T/Q)$ (low-$p_T$) or $\mathcal{O}(m/p_T)$ (large-$p_T$)

E615 $\pi W$ Drell-Yan

JAM20 Pion PDFs

- For the first time, we included large $p_T$-dependent Drell-Yan data, which follows collinear factorization
- Large $p_T$ does not dramatically affect the PDF
- Successfully describe data with a scale $\mu = p_T/2$

N.Cao, PCB, N. Sato, and W. Melnitchouk
TMD factorization

- In small-$p_T$ region, Use the CSS formalism for TMD evolution

\[
\frac{d\sigma}{dQ^2\,dy\,dq^2_T} = \frac{4\pi^2\alpha^2}{9Q^2s} \sum_{j,j_A,j_B} H_{jj}^{DY}(Q,\mu_Q,a_s(\mu_Q)) \int \frac{d^2b_T}{(2\pi)^2} e^{iq_T\cdot b_T} \times e^{\left[ -g_{j/A}(x_A,b_T;b_{max}) \right]} \int_{x_A}^{1} \frac{d\xi_A}{\xi_A} f_{j_A/A}(\xi_A;\mu_{b_*}) \tilde{C}_{j/A}^{PDF} \left( \frac{x_A}{\xi_A},b_*;\mu_{b_*},\mu_{b_*},a_s(\mu_{b_*}) \right) \\
\times e^{\left[ -g_{j/B}(x_B,b_T;b_{max}) \right]} \int_{x_B}^{1} \frac{d\xi_B}{\xi_B} f_{j_B/B}(\xi_B;\mu_{b_*}) \tilde{C}_{j/B}^{PDF} \left( \frac{x_B}{\xi_B},b_*;\mu_{b_*},\mu_{b_*},a_s(\mu_{b_*}) \right) \\
\times \exp \left\{ -g_K(b_T;b_{max}) \ln \frac{Q^2}{Q_0^2} + \tilde{K}(b_*;\mu_{b_*}) \ln \frac{Q^2}{\mu_{b_*}^2} + \int_{\mu_{b_*}}^{\mu} \frac{d\mu'}{\mu'} \left[ 2\gamma_j(a_s(\mu')) - \ln \frac{Q^2}{(\mu')^2} \gamma_K(a_s(\mu')) \right] \right\}
\]

- Approach with leading order to diagnose current situation

Non-perturbative TMDs to extract
Results – E288 $pp$ data

- Perform simultaneous fit of small-$p_T$ TMDs to E288 ($pp$) and E615 ($\pi A$) data
- FO is prediction using collinear PDFs and scale $\mu = p_T/2$
Results – E615 $\pi W$ data

- Proton and pion TMDs are different, but $g_K$ are same
- Perform simultaneous fit of small-$p_T$ TMDs to E288 ($pp$) and E615 ($\pi A$) data
Future Work

• Investigate high-$\chi$ behavior of valence PDF through constraints from the lattice data
• Perform a simultaneous extraction of pion PDFs and TMDs using available low- and high-$p_T$ data
• Explore matching procedure between $W$ and $FO$ terms in $p_T$ spectrum
Backup
Previous Pion PDFs

- Fits to Drell-Yan, prompt photon, or both

GRS, GRV, and SMRS


Aicher’s valence PDF


xFitter


barryp@jlab.org
Drell-Yan (DY)

\[
\sigma \propto \sum_{i,j} f_i^\pi (x_\pi, \mu) \otimes f_j^A (x_A, \mu) \otimes \hat{\sigma}_{i,j} (x_\pi, x_A, Q/\mu)
\]
Drell-Yan (DY) Definitions

Hadronic variable

\[ \tau = \frac{Q^2}{S} \]

Partonic variable

\[ z \equiv \frac{Q^2}{\hat{S}} = \frac{\tau}{x_1 x_2} \]

\( \hat{S} \) is the center of mass momentum squared of incoming partons
LO

\[ \mathcal{O}(1) \]

- \( z = 1 \) corresponds to partonic threshold
- All \( \hat{S} \) is equal to \( Q^2 \)
- All energy of hard partons turns into virtuality of photon

\[ C_{q\bar{q}} = \delta(1 - z) \frac{\delta(y) + \delta(1 - y)}{2} \]
NLO Virtual

• Virtual corrections at NLO are proportional to $\delta(1 - z)$
  • Exhibit Born kinematics

$$C_{q\bar{q}}^{\text{virtual}} = \delta(1 - z) \frac{\delta(y) + \delta(1 - y)}{2} \left[ \frac{C_F \alpha_S}{\pi} \left( \frac{3}{2} \ln \frac{Q^2}{\mu^2} + \frac{2\pi^2}{3} - 6 \right) \right]$$
NLO Real Emission

• Next to leading order, real gluon emissions

\[
C_{q\bar{q}}^{\text{real}} = \frac{C_F \alpha_s}{\pi} \left[ \frac{\delta(y) + \delta(1 - y)}{2} \right] \left( 1 + z^2 \right) \left( \frac{1}{1 - z} \ln \frac{Q^2(1 - z)^2}{\mu^2 z} \right) + 1 - z \\
+ \frac{1}{2} \left[ \frac{(1 - z)^2}{z} y(1 - y) \right] \left[ \frac{1 + z^2}{1 - z} \left( \left[ \frac{1}{y} \right]_+ + \left[ \frac{1}{1 - y} \right]_+ \right) - 2(1 - z) \right]
\]
NLO Real Emission

- Real quark emissions
- $C_{qg} = C_{gq} \mid_{y \rightarrow 1-y}$

\[ C_{qg}^{\text{real}} = \frac{T_F \alpha_S}{2\pi} \left[ \delta(y) \left( z^2 + (1-z)^2 \right) \ln \frac{Q^2(1-z)^2}{\mu^2 z} + 2z(1-z) \right] \]

\[ + \left[ 1 + \frac{(1-z)^2}{z} y(1-y) \right] \left[ (z^2 + (1-z)^2) \left( \frac{1}{y} \right) + 2z(1-z) + (1-z)^2 y \right] \]
Leading Neutron (LN)

\[
\frac{d\sigma}{dx dQ^2 dy} \sim f_{p \rightarrow \pi^+ n}(y) \times \sum_q \int_{x/y}^1 \frac{d\xi}{\xi} C(\xi) q \left(\frac{x/y}{\xi}, \mu^2\right)
\]
Large $x_L$

- $x_L$ is fraction of longitudinal momentum carried by neutron relative to initial proton
- For $t$ to be close to pion pole, has to go near 0 – happens at large $x_L$
- In this region, one pion exchange dominates
Splitting Function and Regulators

Amplitude for proton to dissociate into a $\pi^+$ and neutron:

$$f_{\pi N}(\bar{x}_L) = \frac{g_A^2 M^2}{(4\pi f_\pi)^2} \int dk_{\perp}^2 \frac{\bar{x}_L \left[ k_{\perp}^2 + \bar{x}_L^2 M^2 \right]}{x_L^2 D_{\pi N}^2} |\mathcal{F}|^2,$$

$$D_{\pi N} \equiv t - m_\pi^2 = -\frac{1}{1 - y} \left[ k_{\perp}^2 + y^2 M^2 + (1 - y)m_\pi^2 \right]$$

\[ \mathcal{F} = \begin{cases} 
(i) & \exp \left( \frac{(M^2 - s)}{\Lambda^2} \right) \quad \text{s-dep. exponential} \\
(ii) & \exp \left( \frac{D_{\pi N}}{\Lambda^2} \right) \quad \text{t-dep. exponential} \\
(iii) & \frac{(\Lambda^2 - m_\pi^2)}{(\Lambda^2 - t)} \quad \text{t-dep. monopole} \\
(iv) & \bar{x}_L^{-\alpha(x(t))} \exp \left( \frac{D_{\pi N}}{\Lambda^2} \right) \quad \text{Regge} \\
v) & \left[ 1 - D_{\pi N}^2 / (\Lambda^2 - t)^2 \right]^{1/2} \quad \text{Pauli-Villars} 
\end{cases} \]

- We examine five regulators, and we fit $\Lambda$
- $\mathcal{F}$ is a UV regulator, which the data chooses
Bayesian Inference

• Minimize the $\chi^2$ for each replica

$$\chi^2(a, \text{data}) = \sum_e \left( \sum_i \left[ \frac{d_i^e - \sum_k r_k^e \beta_k^e \alpha_i^e}{n_e} - t_i^e(a)/n_e \right]^2 + \left( \frac{1 - n_e}{\delta n_e} \right)^2 + \sum_k (r_k^e)^2 \right)$$

• Perform $N$ total $\chi^2$ minimizations and compute statistical quantities

Expectation value

$$E[\mathcal{O}] = \frac{1}{N} \sum_k \mathcal{O}(a_k),$$

Variance

$$V[\mathcal{O}] = \frac{1}{N} \sum_k \left[ \mathcal{O}(a_k) - E[\mathcal{O}] \right]^2,$$
TMD Non-Perturbative Parametrization

• Parametrize $g_j$ for each hadron and $g_K$ as in Slide 21

$$g_j/h(x, b_T^2) = -b_T^2 g_1 \left( \frac{1}{2} + g_3 \log(10x) \right)$$

$$g_K(b_T) = -b_T^2 \frac{g_2}{2}$$
$\chi^2$ as a function of $x_F^{\text{max}}$

- Phenomenologically determine where TMD factorization breaks down in large $x_F$ data
**E288 pp**

- Calculate large-\( p_T \) FO term with \( \mu = Q \)
- Not a good description