Tomography of pions and protons from transverse momentum dependent distributions

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Based on: arXiv:2302.01192
What do we know about structures?

- Most well-known structure is through longitudinal structure of hadrons, particularly protons.

Other structures?

• To give deeper insights into color confined systems, we shouldn’t limit ourselves to proton structures

• Pions are also important because of their Goldstone-boson nature while also being made up of quarks and gluons
Available datasets for pion structures

- Much less available data than in the proton case
- Still valuable to study
Available datasets for pion structures

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- Still valuable to study
Pion PDFs in JAM

Drell-Yan (DY)

Leading Neutron (LN)

Threshold resummation in DY

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3D structures of hadrons

- Even more challenging is the 3D structure through GPDs and TMDs.
Unpolarized TMD PDF

\[ \tilde{f}_{q/N}(x, b_T) = \int \frac{db^-}{4\pi} e^{-ixP^+b^-} \text{Tr} \left[ \langle N | \bar{\psi}_q(b)\gamma^+ \mathcal{W}(b, 0)\psi_q(0) | N \rangle \right] \]

\[ b \equiv (b^-, 0^+, b_T) \]

- \( b_T \) is the Fourier conjugate to the intrinsic transverse momentum of quarks in the hadron, \( k_T \)
- We can learn about the coordinate space correlations of quark fields in hadrons
- Modification needed for UV and rapidity divergences; acquire regulators: \( \tilde{f}_{q/N}(x, b_T) \rightarrow \tilde{f}_{q/N}(x, b_T; \mu, \zeta) \)
Factorization for low-$q_T$ Drell-Yan

• Like collinear observable, a **hard part** with two functions that describe structure of beam and target

• So called “$W$”-term, valid only at low-$q_T$
TMD PDF within the $b_*$ prescription

\[ b_*(b_T) \equiv \frac{b_T}{\sqrt{1 + \frac{b_T^2}{b_{\text{max}}^2}}} . \]

\[
\tilde{f}_{q/\mathcal{N}(A)}(x, b_T, \mu_Q, Q^2) = (C \otimes f)_{q/\mathcal{N}(A)}(x; b_*) \\
\times \exp \left\{ -g_{q/\mathcal{N}(A)}(x, b_T) - g_K(b_T) \ln \frac{Q}{Q_0} - S(b_*, Q_0, Q, \mu_Q) \right\}
\]

- Low-$b_T$: perturbative
- High-$b_T$: non-perturbative

\[ g_{q/\mathcal{N}(A)}: \text{intrinsic non-perturbative structure of the TMD} \]
\[ g_K: \text{universal non-perturbative Collins-Soper kernel} \]

Relates the TMD at small-$b_T$ to the **collinear** PDF
\[ \Rightarrow \text{TMD is sensitive to collinear PDFs} \]

Controls the perturbative evolution of the TMD
A few details

• Nuclear TMD model linear combination of bound protons and neutrons
  • Include an additional $A$-dependent nuclear parameter
• We use the MAP collaboration’s parametrization for non-perturbative TMDs
  • Only tested parametrization flexible enough to capture features of $Q$ bins
• Perform a **simultaneous global analysis** of pion TMD and collinear PDFs, with proton (nuclear) TMDs
Note about E615 $\pi A$ Drell-Yan data

- Provides both $\frac{d\sigma}{dx_F d\sqrt{\tau}}$ ($p_T$-integrated) and $\frac{d\sigma}{dx_F dp_T}$ ($p_T$-dependent)
  - Large constraints on $\pi$ collinear PDFs from $p_T$-integrated
  - Large constraints on $\pi$ TMD PDFs from $p_T$-dependent
- Projections of same events $\Rightarrow$ correlated measurements
- They have the same luminosity uncertainty, so they have the same overall normalization uncertainty
- To account for this, we *equate* the fitted normalizations of the two otherwise independent measurements
  - No other guidance from experiment how the uncertainties are correlated
Note on collinear DY theory

• When equating the normalizations, we found
  • Agreement when using NLO theory on the collinear observables
  • Tension when using NLO+NLL threshold resummed theory on the collinear observables

• We note that in the OPE part of the TMD formalism, we use NLO accuracy
  • We do not use any threshold enhancements on the $p_T$-dependent observables
Data and theory agreement

• Fit both $pA$ and $\pi A$ DY data and achieve good agreement to both

<table>
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<tr>
<th>Process</th>
<th>Experiment</th>
<th>$\sqrt{s}$ (GeV)</th>
<th>$\chi^2/N$</th>
<th>Z-score</th>
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<td>$q_T$-dep. $pA$</td>
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<td>$pA \rightarrow \mu^+\mu^- X$</td>
<td>E288 [90]</td>
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<td>E605 [91]</td>
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<td>1.22</td>
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<td>(Fe/Be)</td>
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<td>$ep \rightarrow e n X$</td>
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<td>Total</td>
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<td>1.12</td>
<td>1.86</td>
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The small-$q_T$ data do not constrain much the PDFs.
Resulting TMD PDFs of proton and pion

\[ \tilde{f}_{q/N}(b_T|x; Q, Q^2) \equiv \frac{\tilde{f}_{q/N}(x, b_T; Q, Q^2)}{\int d^2b_T \tilde{f}_{q/N}(x, b_T; Q, Q^2)} \]

- Broadening appearing as \( x \) increases
- Up quark in pion is narrower than up quark in proton

\[ Q = 6 \text{ GeV} \]

\[ x = 0.60, 0.54, 0.48, 0.42, 0.36, 0.30 \]
Resulting average $b_T$

$$\langle b_T | x \rangle_{q/N} = \int d^2 b_T \; b_T \; \tilde{f}_{q/N}(b_T | x; Q, Q^2)$$

- Average transverse spatial correlation of the up quark in proton is $\sim 1.2$ times bigger than that of pion
- Pion’s $\langle b_T | x \rangle$ is $4 - 5.2\sigma$ smaller than proton in this range
- Decreases as $x$ decreases
Transverse EMC effect

• Compare the average $b_T$ given $x$ for the up quark in the bound proton to that of the free proton

• Less than 1 by $\sim 5 - 12\%$ over the $x$ range
Outlook

• Future studies needed for theoretical explanations of these phenomena

• Look into threshold corrections in the OPE formalism

• Lattice QCD can in principle calculate any hadronic state – look to kaons, rho mesons, etc.

• Future tagged experiments such as at EIC and JLab 22 GeV can provide measurements for neutrons, pions, and kaons
Backup
Small $b_T$ operator product expansion

• At small $b_T$, the TMDPDF can be described in terms of its OPE:

$$\tilde{f}_{f/h}(x, b_T; \mu, \zeta_F) = \sum_j \int_x^1 \frac{d\xi}{\xi} \tilde{C}_{f/j}(x/\xi, b_T; \zeta_F, \mu) f_{j/h}(\xi; \mu) + \mathcal{O}((\Lambda_{QCD} b_T)^a)$$

• where $\tilde{C}$ are the Wilson coefficients, and $f_{j/h}$ is the collinear PDF

• Breaks down when $b_T$ gets large
\( b_* \) prescription

• A common approach to regulating large \( b_T \) behavior

\[
b_*(b_T) \equiv \frac{b_T}{\sqrt{1 + b_T^2 / b_{\text{max}}^2}}.
\]

• At small \( b_T \), \( b_*(b_T) = b_T \)

• At large \( b_T \), \( b_*(b_T) = b_{\text{max}} \)

Must choose an appropriate value; a transition from perturbative to non-perturbative physics
Introduction of non-perturbative functions

• Because $b_* \neq b_T$, have to non-perturbatively describe large $b_T$ behavior

$$g_K(b_T; b_{\text{max}}) = -\tilde{K}(b_T, \mu) + \tilde{K}(b_*, \mu)$$

Non-perturbative function dependent in principle on flavor, hadron, etc.

$$e^{-g_j/H(x, b_T; b_{\text{max}})} = \frac{\tilde{f}_{j/H}(x, \zeta, \mu)}{f_{j/H}(x, b_*; \zeta, \mu)} e^{g_K(b_T; b_{\text{max}}) \ln(\sqrt{\zeta}/Q_0)}.$$
TMD factorization in Drell-Yan

- In small-$q_T$ region, use the Collins-Soper-Sterman (CSS) formalism and $b_*$ prescription

\[
\frac{d\sigma}{dQ^2 \, dy \, dq_T^2} = \frac{4\pi^2\alpha^2}{9Q^2 s} \sum_{j, j_A, j_B} H^{DY}_{jj}(Q, \mu_Q, a_s(\mu_Q)) \int \frac{d^2b_T}{(2\pi)^2} e^{i q_T \cdot b_T} 
\]

\[ \times e^{-g_{j/A}(x_A, b_T; b_{\text{max}})} \int_{x_A}^{1} \frac{d\xi_A}{\xi_A} f_{j/A}(\xi_A; \mu_{b_*}) \tilde{C}_{j/A}^{\text{PDF}} \left( \frac{x_A}{\xi_A}, b_*; \mu_{b_*}^2, a_s(\mu_{b_*}) \right) \]

\[ \times e^{-g_{j/B}(x_B, b_T; b_{\text{max}})} \int_{x_B}^{1} \frac{d\xi_B}{\xi_B} f_{j/B}(\xi_B; \mu_{b_*}) \tilde{C}_{j/B}^{\text{PDF}} \left( \frac{x_B}{\xi_B}, b_*; \mu_{b_*}^2, a_s(\mu_{b_*}) \right) \]

\[ \times \exp \left\{ -g_K(b_T; b_{\text{max}}) \ln \frac{Q^2}{Q_0^2} + \tilde{K}(b_*; \mu_{b_*}) \ln \frac{Q^2}{\mu_{b_*}^2} + \int_{\mu_{b_*}}^{\mu} \frac{d\mu'}{\mu'} \left[ 2\gamma_j(a_s(\mu')) - \ln \frac{Q^2}{(\mu')^2} \gamma_K(a_s(\mu')) \right] \right\} \]

Can these data constrain the pion collinear PDF?

Non-perturbative pieces

Non-perturbative piece of the CS kernel

Perturbative pieces
MAP parametrization

• A recent work from the MAP collaboration (arXiv:2206.07598) used a complicated form for the non-perturbative function

\[
f_{1NP}(x, b^2_T; \zeta, Q_0) = \frac{g_1(x) e^{-g_1(x) \frac{b^2_T}{4}} + \lambda^2 g^2_{1B}(x) \left[ 1 - g_{1B}(x) \frac{b^2_T}{4} \right] e^{-g_{1B}(x) \frac{b^2_T}{4}} + \lambda_2^2 g_{1C}(x) e^{-g_{1C}(x) \frac{b^2_T}{4}}}{g_1(x) + \lambda^2 g^2_{1B}(x) + \lambda_2^2 g_{1C}(x)} \left[ \frac{\zeta}{Q_0^2} \right]^{g_K(b^2_T)/2}, \tag{38}
\]

\[
g_{\{1,1B,1C\}}(x) = N_{\{1,1B,1C\}} \frac{x^{\sigma_{\{1,2,3\}}} (1 - x)^{\alpha^2_{\{1,2,3\}}}}{\hat{x}^{\sigma_{\{1,2,3\}}} (1 - \hat{x})^{\alpha^2_{\{1,2,3\}}}},
\]

\[
g_K(b^2_T) = -g_2^2 \frac{b_T^2}{2}
\]

• 11 free parameters for each hadron! (flavor dependence not necessary) (12 if we include the nuclear TMD parameter)
Resulting $\chi^2$ for each parametrization

- Tried multiple parametrizations for non-perturbative TMD structures
- MAP parametrization is able to describe better all the datasets
Nuclear TMD PDFs – working hypothesis

• We must model the nuclear TMD PDF from proton

\[ \tilde{f}_{q/A}(x, b_T, \mu, \zeta) = \frac{Z}{A} \tilde{f}_{q/p/A}(x, b_T, \mu, \zeta) + \frac{A - Z}{A} \tilde{f}_{q/n/A}(x, b_T, \mu, \zeta) \]

• Each object on the right side independently obeys the CSS equation
  • Assumption that the bound proton and bound neutron follow TMD factorization

• Make use of isospin symmetry in that \( u/p/A \leftrightarrow d/n/A \), etc.
Building of the nuclear TMD PDF

- Then taking into account the intrinsic non-perturbative, we model the flavor-dependent pieces of the TMD PDF as

\[
(C \otimes f)_{u/A}(x)e^{-g_{u/A}(x,b_T)} \rightarrow \frac{Z}{A}(C \otimes f)_{u/p/A}(x)e^{-g_{u/p/A}(x,b_T)}
\]

\[+ \frac{A-Z}{A}(C \otimes f)_{d/p/A}(x)e^{-g_{d/p/A}(x,b_T)}\]

and

\[
(C \otimes f)_{d/A}(x)e^{-g_{d/A}(x,b_T)} \rightarrow \frac{Z}{A}(C \otimes f)_{d/p/A}(x)e^{-g_{d/p/A}(x,b_T)}
\]

\[+ \frac{A-Z}{A}(C \otimes f)_{u/p/A}(x)e^{-g_{u/p/A}(x,b_T)}.
\]
Nuclear TMD parametrization

- Specifically, we include a parametrization similar to Alrashed, et al., Phys. Rev. Lett 129, 242001 (2022).

\[ g_{q/N/A} = g_{q/N} \left( 1 - a_N \left( A^{1/3} - 1 \right) \right) \]

- Where $a_N$ is an additional parameter to be fit
Bayesian Inference

• Minimize the $\chi^2$ for each replica

$$\chi^2(a, \text{data}) = \sum_e \left( \sum_i \left[ \frac{d_i^e - \sum_k r_k^e \beta_{k,i}^e - t_i^e(a)/n_e}{\alpha_i^e} \right]^2 + \left( \frac{1 - n_e}{\delta n_e} \right)^2 + \sum_k (r_k^e)^2 \right).$$

• Perform $N$ total $\chi^2$ minimizations and compute statistical quantities

\begin{align*}
\text{Expectation value} & \quad E[\mathcal{O}] = \frac{1}{N} \sum_k \mathcal{O}(a_k), \\
\text{Variance} & \quad V[\mathcal{O}] = \frac{1}{N} \sum_k \left[ \mathcal{O}(a_k) - E[\mathcal{O}] \right]^2.
\end{align*}
Correlations

• Level at which the distributions are correlated with each other

• Different distributions are largely correlated only within themselves
Possible explanation

• At large $x$, we are in a valence region, where only the valence quarks are populating the momentum dependence of the hadron
Possible explanation

• At small $x$, sea quarks and potential $q\bar{q}$ bound states allowing only for a smaller bound system