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SPIN PHYSICS
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Tomography of pions and protons from transverse momentum dependent distributions

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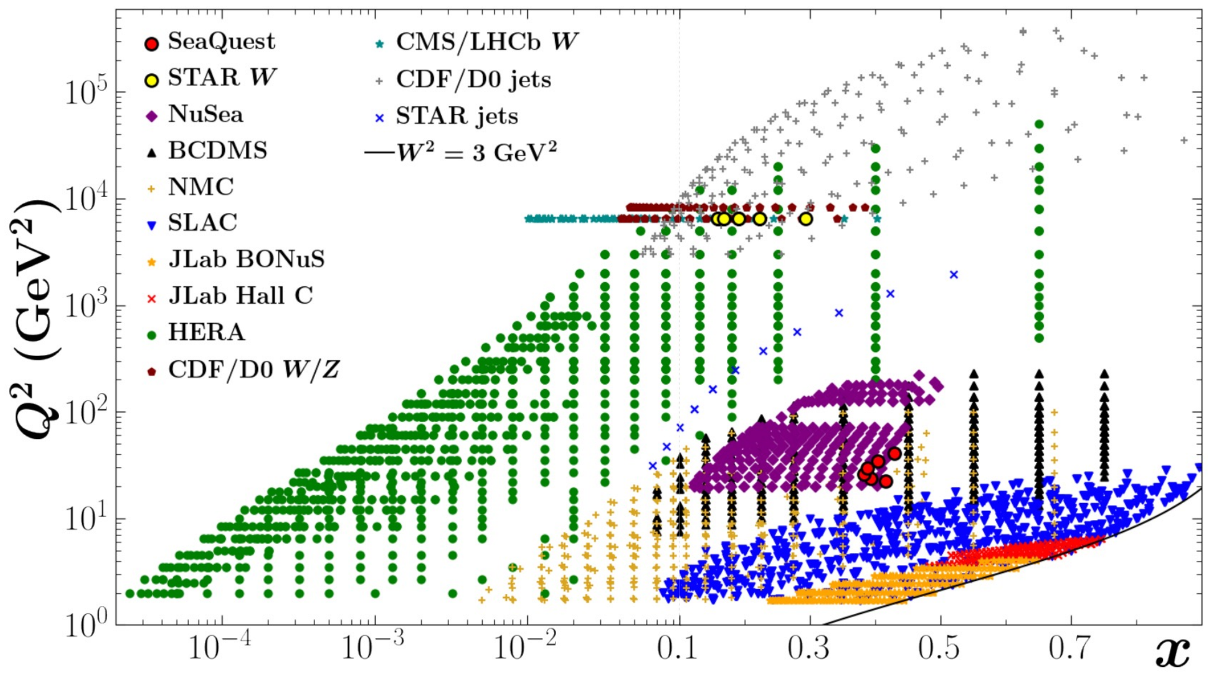
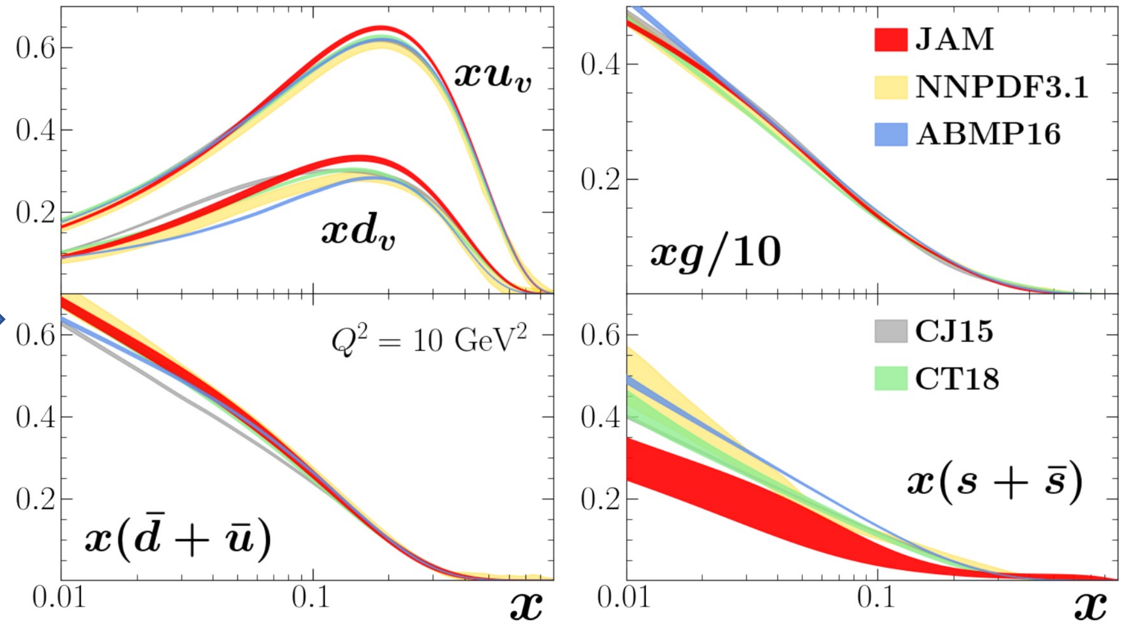
Based on: arXiv:2302.01192

Jefferson Lab
Exploring the Nature of Matter

What do we know about structures?

- Most well-known structure is through longitudinal structure of hadrons, particularly protons

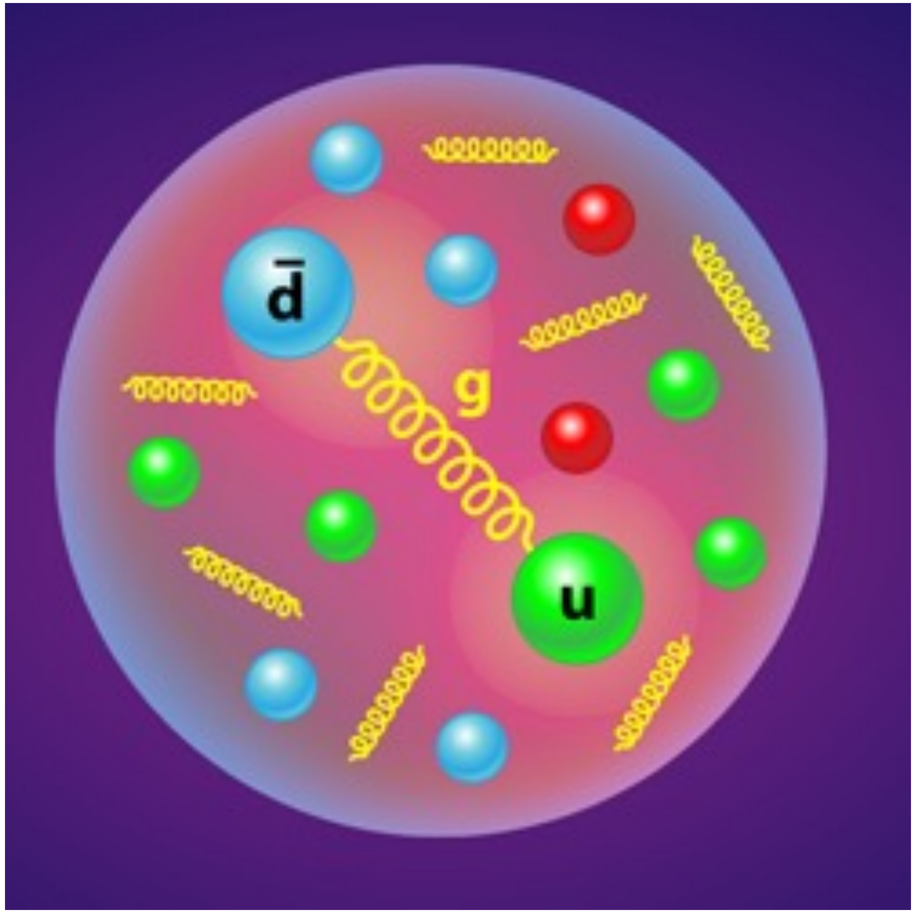




C. Cocuzza, et al., Phys. Rev. D **104**, 074031 (2021)

Other structures?

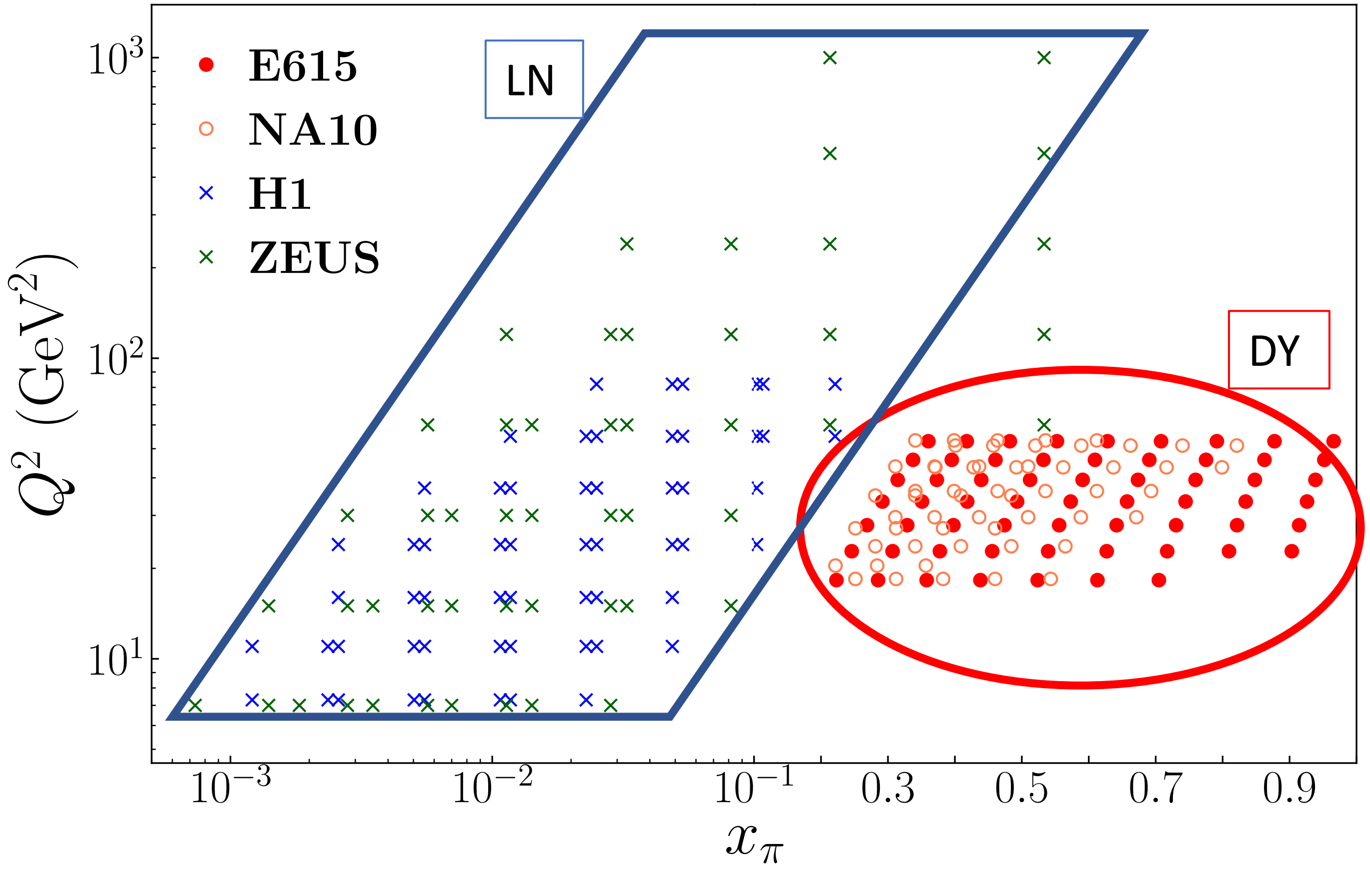
- To give deeper insights into color confined systems, we shouldn’t limit ourselves to proton structures
- Pions are also important because of their Goldstone-boson nature while also being made up of quarks and gluons



Available datasets for pion structures

- Much less available data than in the proton case

- Still valuable to study



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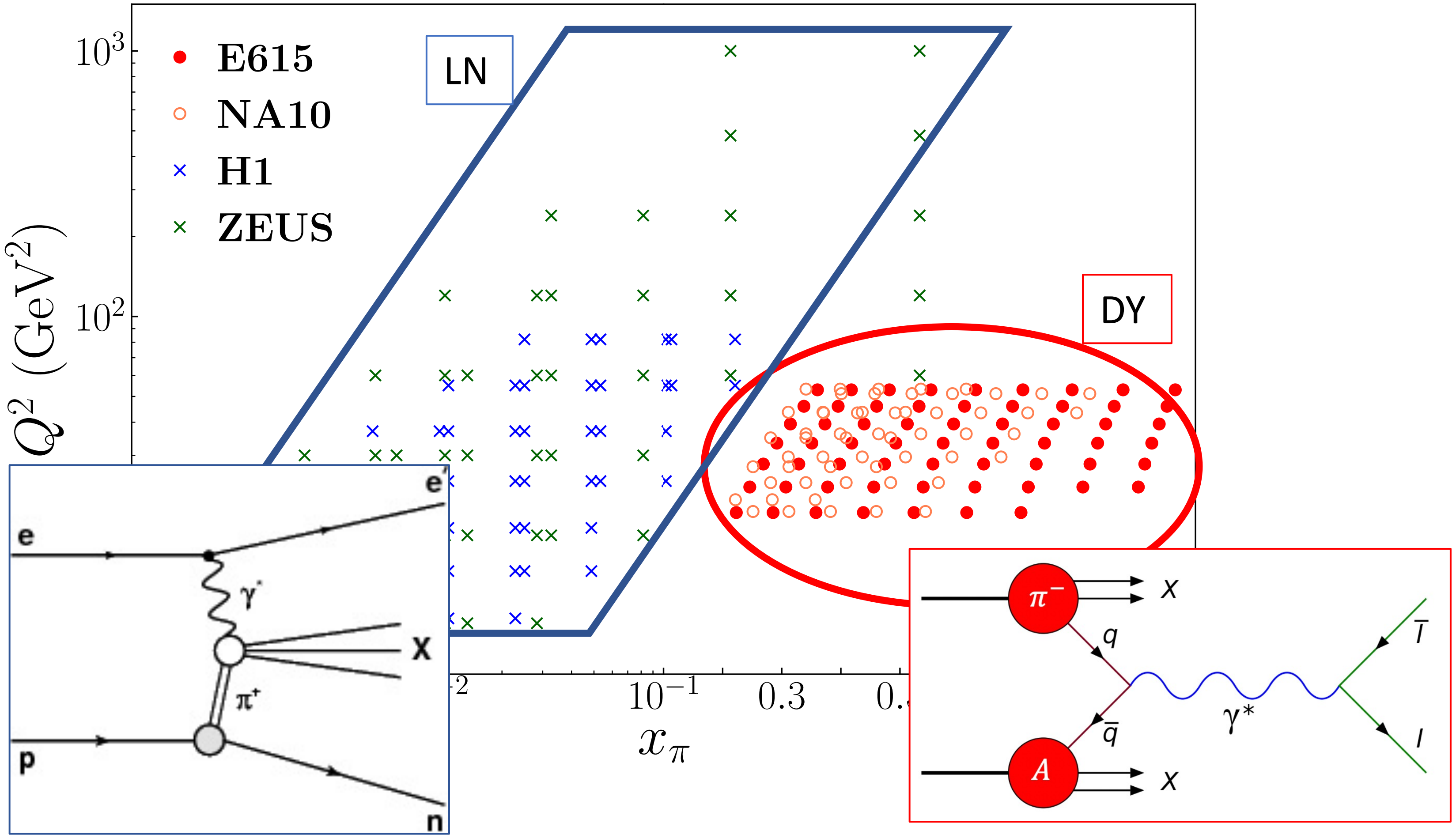
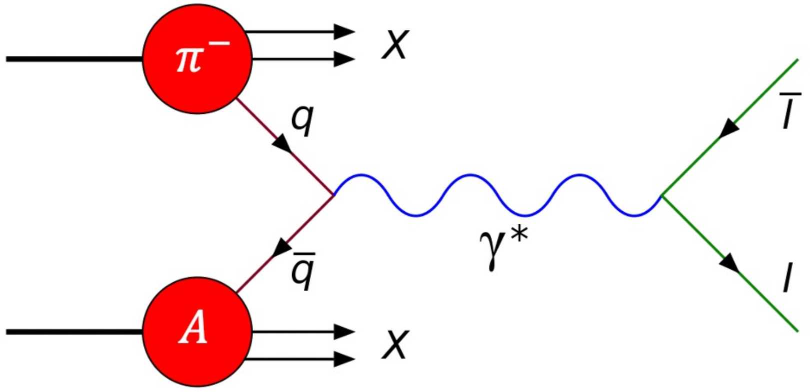
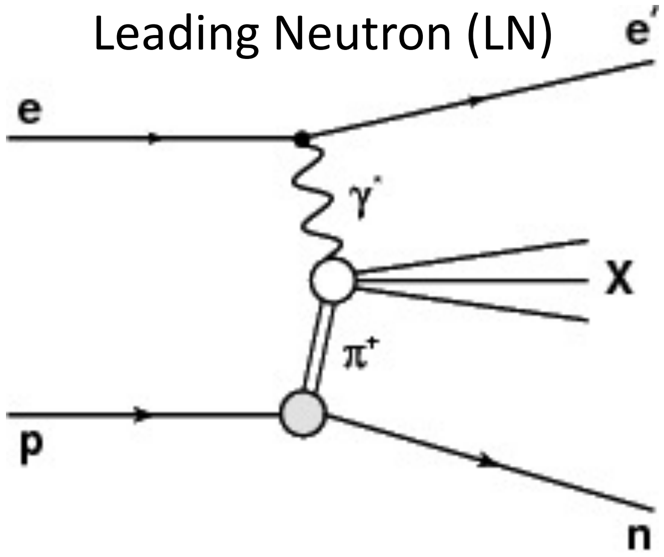


Pion PDFs in JAM

Drell-Yan (DY)



Leading Neutron (LN)

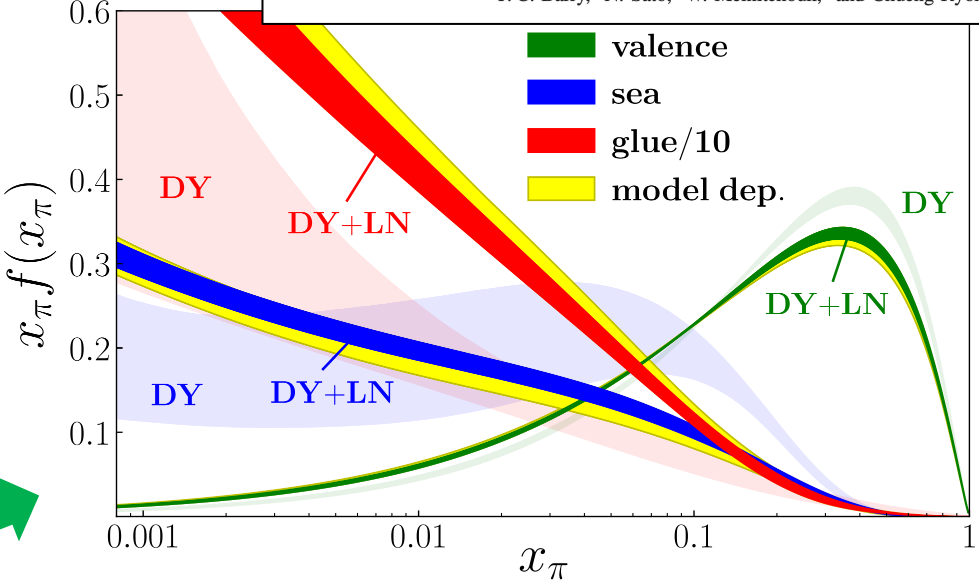


PHYSICAL REVIEW LETTERS 121, 152001 (2018)

Featured in Physics

First Monte Carlo Global QCD Analysis of Pion Parton Distributions

P. C. Barry,[1] N. Sato,[2] W. Melnitchouk,[3] and Chueng-Ryong Ji[1]

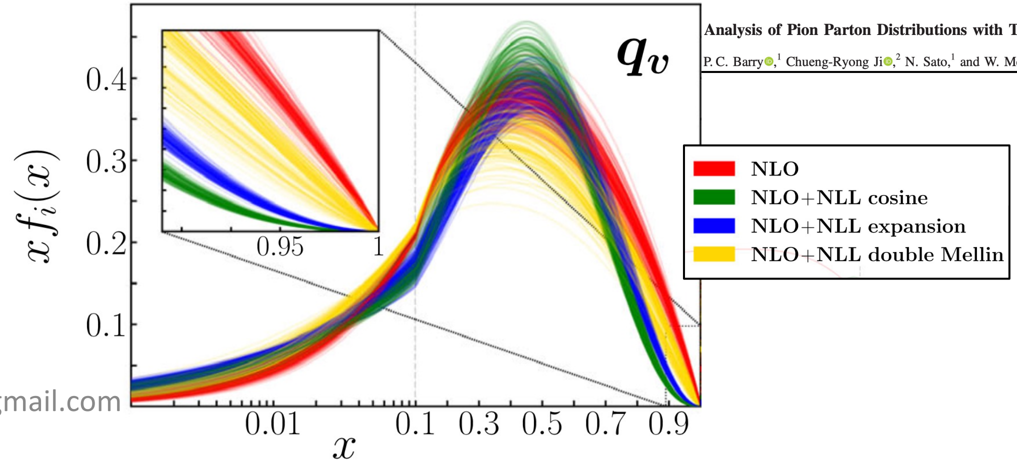


Threshold resummation in DY

PHYSICAL REVIEW LETTERS 127, 232001 (2021)

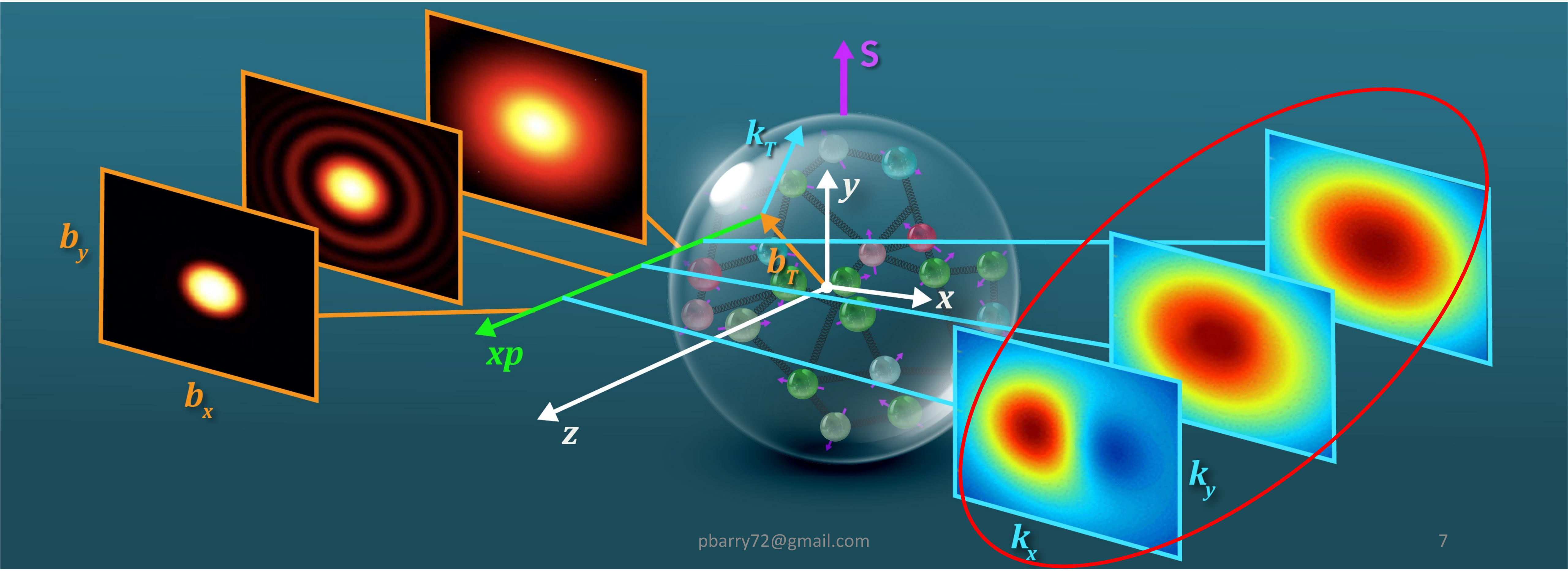
Analysis of Pion Parton Distributions with Threshold Resummation

P. C. Barry,[1] Chueng-Ryong Ji,[2] N. Sato,[1] and W. Melnitchouk[1]



3D structures of hadrons

- Even more challenging is the 3d structure through GPDs and TMDs



Unpolarized TMD PDF

$$\tilde{f}_{q/\mathcal{N}}(x, b_T) = \int \frac{\mathrm{d}b^-}{4\pi} e^{-ixP^+b^-} \mathrm{Tr}\left[\langle \mathcal{N} \,|\, \bar{\psi}_q(b)\gamma^+ \mathcal{W}(b,0)\psi_q(0) \,|\, \mathcal{N} \rangle\right]$$

$$b \equiv (b^-, 0^+, \boldsymbol{b_T})$$

- $\boldsymbol{b_T}$ is the Fourier conjugate to the intrinsic transverse momentum of quarks in the hadron, $\boldsymbol{k_T}$
- We can learn about the coordinate space correlations of quark fields in hadrons
- Modification needed for UV and rapidity divergences; acquire regulators: $\tilde{f}_{q/\mathcal{N}}(x, b_T) \to \tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, \zeta)$

Factorization for low-q_T Drell-Yan

- Like collinear observable, a hard part with two functions that describe structure of beam and target
- So called "W"-term, valid only at low-q_T

$$\frac{d^3\sigma}{d\tau dY dq_T^2} = \frac{4\pi^2\alpha^2}{9\tau S^2} \sum_q H_{q\bar{q}}(Q^2, \mu) \int d^2 b_T \, e^{i b_T \cdot q_T} \times \tilde{f}_{q/\pi}(x_\pi, b_T, \mu, Q^2) \, \tilde{f}_{\bar{q}/A}(x_A, b_T, \mu, Q^2) \,,$$

TMD PDF within the b_* prescription

$$\mathbf{b}_*(\mathbf{b}_T) \equiv \frac{\mathbf{b}_T}{\sqrt{1 + b_T^2/b_{\max}^2}}.$$

Low-b_T: perturbative

high-b_T: non-perturbative

$$\tilde{f}_{q/\mathcal{N}(A)}(x, b_T, \mu_Q, Q^2) = (C \otimes f)_{q/\mathcal{N}(A)}(x; b_*)$$

$$\times \exp\left\{-g_{q/\mathcal{N}(A)}(x, b_T) - g_K(b_T) \ln\frac{Q}{Q_0} - S(b_*, Q_0, Q, \mu_Q)\right\}$$

Relates the TMD at small-b_T to the **collinear** PDF

⇒ TMD is sensitive to collinear PDFs

$g_{q/\mathcal{N}(A)}$: intrinsic non-perturbative structure of the TMD

g_K: universal non-perturbative Collins-Soper kernel

Controls the perturbative evolution of the TMD

A few details

- Nuclear TMD model linear combination of bound protons and neutrons
  - Include an additional A-dependent nuclear parameter
- We use the MAP collaboration's parametrization for non-perturbative TMDs
  - Only tested parametrization flexible enough to capture features of Q bins
- Perform a **simultaneous global analysis** of pion TMD and collinear PDFs, with proton (nuclear) TMDs

Note about E615 πA Drell-Yan data

- Provides both $\frac{d\sigma}{dx_F d\sqrt{\tau}}$ (p_T-integrated) **and** $\frac{d\sigma}{dx_F dp_T}$ (p_T-dependent)
  - Large constraints on π collinear PDFs from p_T-integrated
  - Large constraints on π TMD PDFs from p_T-dependent
- Projections of same events ⇒ correlated measurements
- They have the **same luminosity** uncertainty, so they have the **same** overall **normalization** uncertainty
- To account for this, we *equate* the fitted normalizations of the two otherwise independent measurements
  - No other guidance from experiment how the uncertainties are correlated

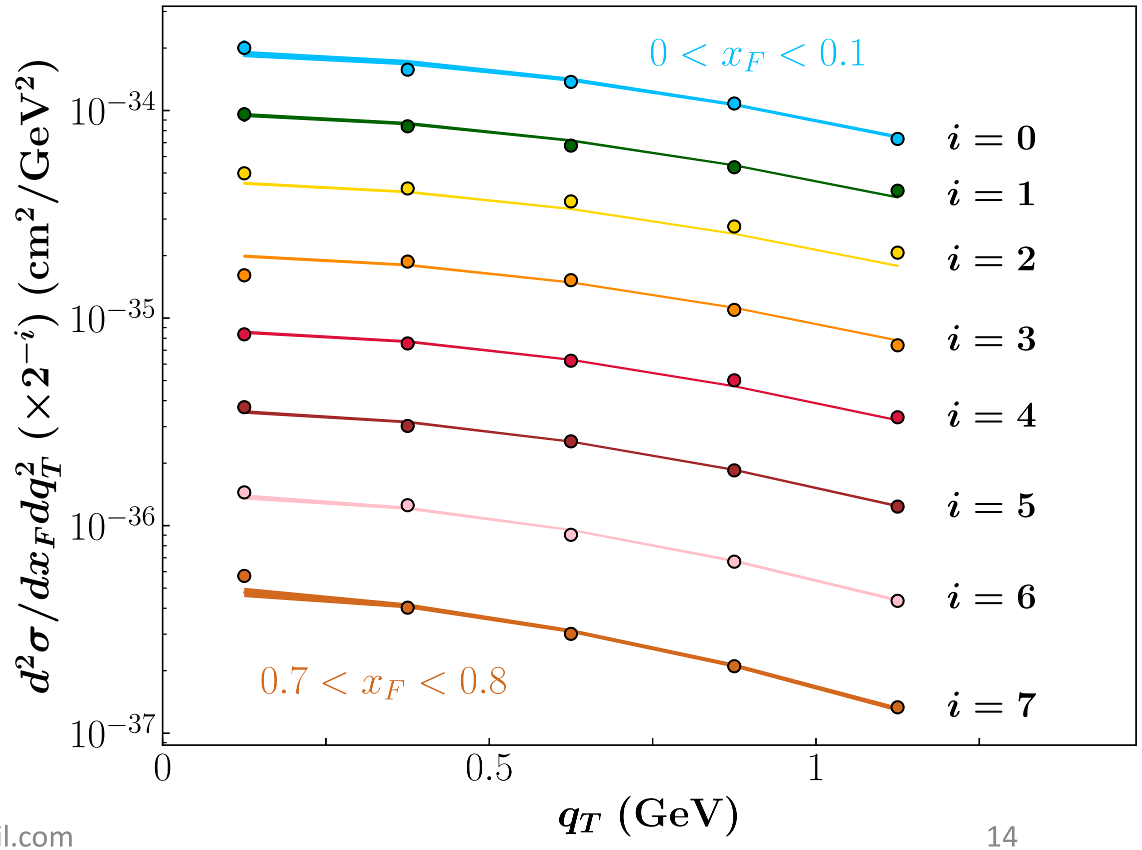
Note on collinear DY theory

- When equating the normalizations, we found
  - ***Agreement*** when using **NLO** theory on the collinear observables
  - ***Tension*** when using **NLO+NLL** threshold resummed theory on the collinear observables
- We note that in the OPE part of the TMD formalism, we use **NLO** accuracy
  - We do not use any *threshold enhancements* on the p_T-dependent observables

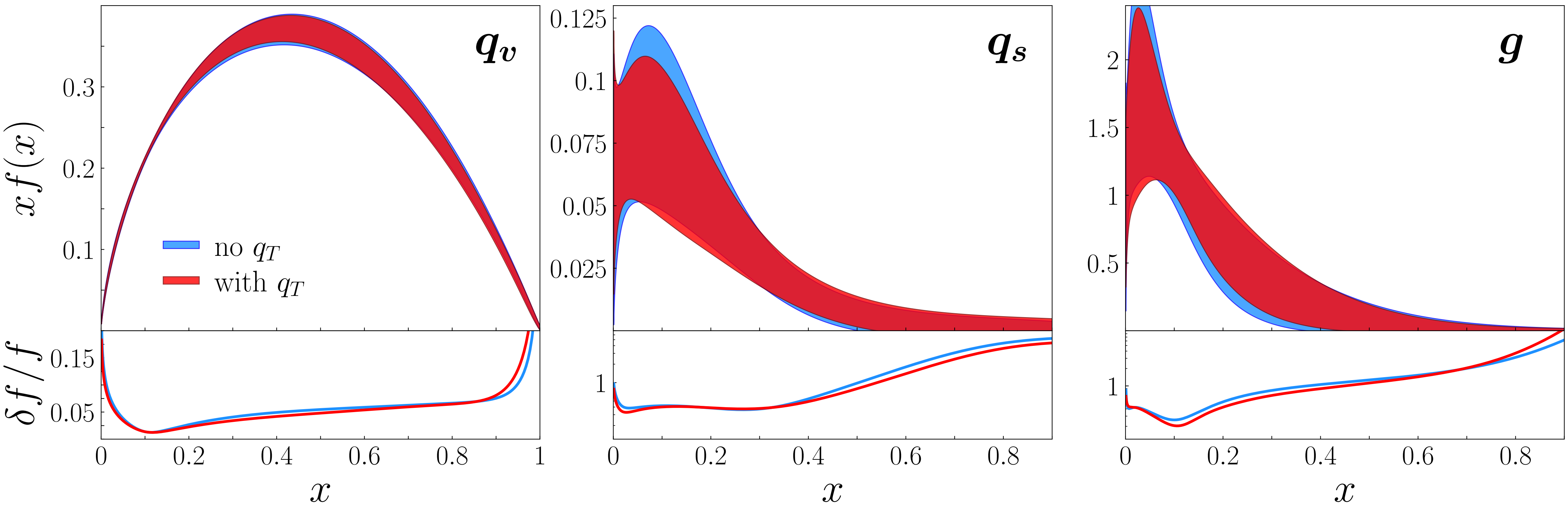
Data and theory agreement

- Fit both pA and πA DY data and achieve good agreement to both

Process	Experiment	$\sqrt{s}$ (GeV)	χ^2/N	Z-score
TMD				
q_T-dep. pA DY	E288 [90]	19.4	1.07	0.34
$pA \to \mu^+\mu^- X$	E288 [90]	23.8	0.99	0.05
	E288 [90]	24.7	0.82	0.99
	E605 [91]	38.8	1.22	1.03
	E772 [92]	38.8	2.54	5.64
(Fe/Be)	E866 [93]	38.8	1.10	0.36
(W/Be)	E866 [93]	38.8	0.96	0.15
q_T-dep. πA DY	E615 [94]	21.8	1.45	1.85
$\pi W \to \mu^+\mu^- X$	E537 [95]	15.3	0.97	0.03
collinear				
q_T-integr. DY	E615 [94]	21.8	0.90	0.48
$\pi W \to \mu^+\mu^- X$	NA10 [96]	19.1	0.59	1.98
	NA10 [96]	23.2	0.92	0.16
leading neutron	H1 [97]	318.7	0.36	4.59
$ep \to enX$	ZEUS [98]	300.3	1.48	2.15
Total			1.12	1.86



Extracted pion PDFs

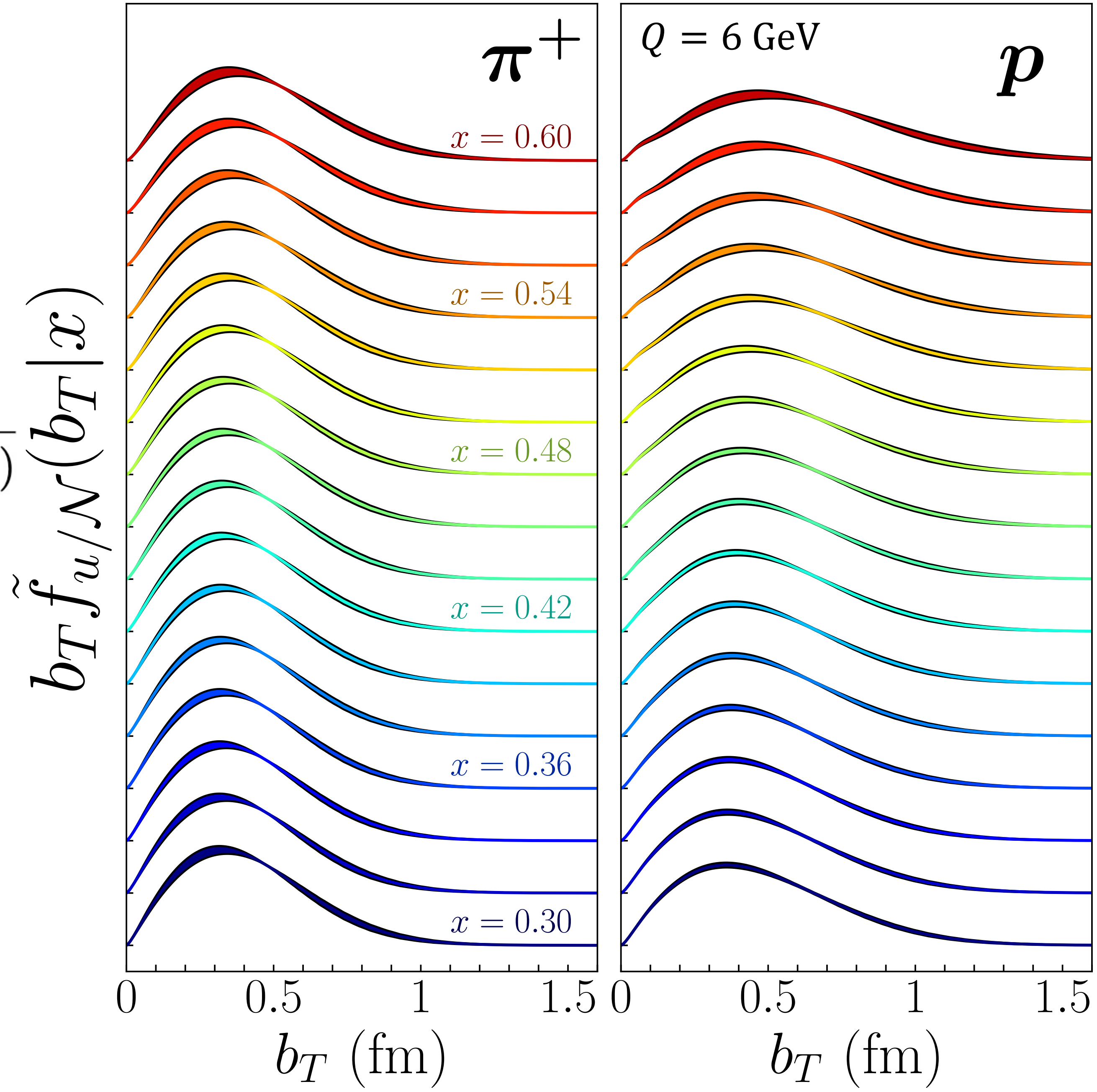


- The small-q_T data do not constrain much the PDFs

Resulting TMD PDFs of proton and pion

$$\tilde{f}_{q/\mathcal{N}}(b_T|x;Q,Q^2) \equiv \frac{\tilde{f}_{q/\mathcal{N}}(x,b_T;Q,Q^2)}{\int \mathrm{d}^2\boldsymbol{b}_T \tilde{f}_{q/\mathcal{N}}(x,b_T;Q,Q^2)}$$

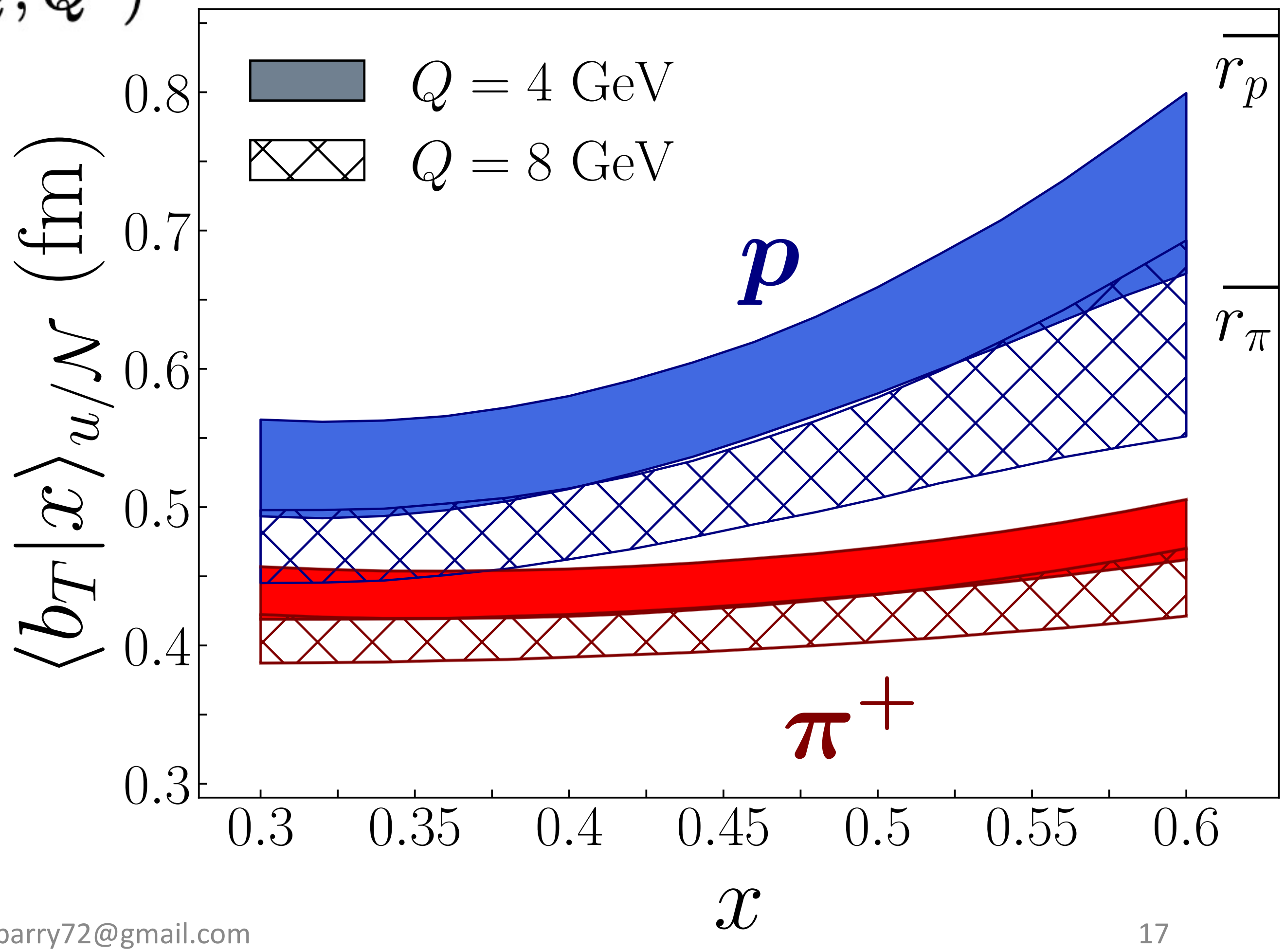
- Broadening appearing as x increases
- Up quark in pion is narrower than up quark in proton



Resulting average b_T

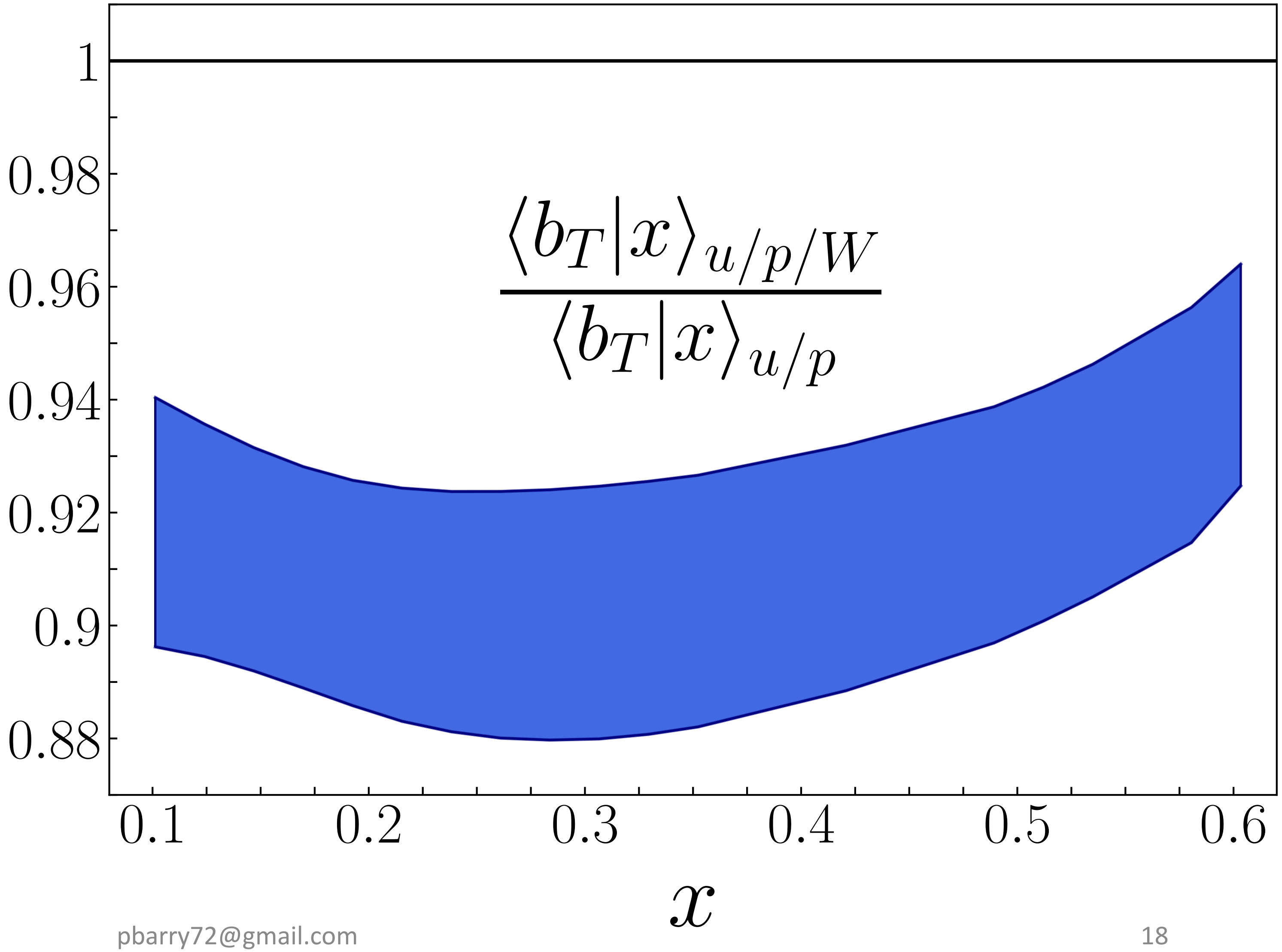
$$\langle b_T|x\rangle_{q/\mathcal{N}} = \int \mathrm{d}^2\boldsymbol{b}_T\, b_T\, \tilde{f}_{q/\mathcal{N}}(b_T|x;Q,Q^2)$$

- Average transverse spatial correlation of the up quark in proton is ~ 1.2 times bigger than that of pion
- Pion's $\langle b_T|x\rangle$ is $4-5.2\sigma$ smaller than proton in this range
- Decreases as x decreases



Transverse EMC effect

- Compare the average b_T given x for the up quark in the bound proton to that of the free proton
- Less than 1 by $\sim 5-12\%$ over the x range



Outlook

- Future studies needed for theoretical explanations of these phenomena
- Look into threshold corrections in the OPE formalism

- Lattice QCD can in principle calculate any hadronic state – look to kaons, rho mesons, etc.
- Future tagged experiments such as at EIC and JLab 22 GeV can provide measurements for neutrons, pions, and kaons

Backup

Small b_T operator product expansion

- At small b_T, the TMDPDF can be described in terms of its OPE:

$$\tilde{f}_{f/h}(x, b_T; \mu, \zeta_F) = \sum_j \int_x^1 \frac{d\xi}{\xi} \tilde{\mathcal{C}}_{f/j}(x/\xi, b_T; \zeta_F, \mu) f_{j/h}(\xi; \mu) + \mathcal{O}((\Lambda_{\rm QCD} b_T)^a)$$

- where $\tilde{C}$ are the Wilson coefficients, and $f_{j/h}$ is the collinear PDF
- Breaks down when b_T gets large

b_* prescription

- A common approach to regulating large b_T behavior

$$\mathbf{b}_*(\mathbf{b}_T) \equiv \frac{\mathbf{b}_T}{\sqrt{1 + b_T^2/b_{\rm max}^2}}.$$

Must choose an appropriate value; a transition from perturbative to non-perturbative physics

- At small b_T, $b_*(b_T) = b_T$
- At large b_T, $b_*(b_T) = b_{\rm max}$

Introduction of non-perturbative functions

- Because $b_* \neq b_T$, have to non-perturbatively describe large b_T behavior

Completely general – independent of quark, hadron, PDF or FF

$$g_K(b_T; b_{\max}) = -\tilde{K}(b_T, \mu) + \tilde{K}(b_*, \mu)$$

Non-perturbative function dependent in principle on flavor, hadron, etc.

$$e^{-g_{j/H}(x, \boldsymbol{b}_\mathrm{T}; b_{\max})} = \frac{\tilde{f}_{j/H}(x, \boldsymbol{b}_\mathrm{T}; \zeta, \mu)}{\tilde{f}_{j/H}(x, \boldsymbol{b}_*; \zeta, \mu)} e^{g_K(b_\mathrm{T}; b_{\max}) \ln(\sqrt{\zeta}/Q_0)}.$$

TMD factorization in Drell-Yan

- In small-q_{T} region, use the Collins-Soper-Sterman (CSS) formalism and b_* prescription

Can these data constrain the pion collinear PDF?

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}y\,\mathrm{d}q_{\mathrm{T}}^2} = \frac{4\pi^2\alpha^2}{9Q^2 s} \sum_{j,j_A,j_B} H_{j\bar{j}}^{\mathrm{DY}}(Q,\mu_Q,a_s(\mu_Q)) \int \frac{\mathrm{d}^2\boldsymbol{b}_{\mathrm{T}}}{(2\pi)^2} e^{i\boldsymbol{q}_{\mathrm{T}}\cdot\boldsymbol{b}_{\mathrm{T}}}$$

$$\times\, e^{-g_{j/A}(x_A,b_{\mathrm{T}};b_{\max})} \int_{x_A}^1 \frac{\mathrm{d}\xi_A}{\xi_A} f_{j_A/A}(\xi_A;\mu_{b_*})\, \tilde{C}_{j/j_A}^{\mathrm{PDF}}\left(\frac{x_A}{\xi_A}, b_*;\mu_{b_*}^2,\mu_{b_*},a_s(\mu_{b_*})\right)$$

$$\times\, e^{-g_{\bar{j}/B}(x_B,b_{\mathrm{T}};b_{\max})} \int_{x_B}^1 \frac{\mathrm{d}\xi_B}{\xi_B} f_{j_B/B}(\xi_B;\mu_{b_*})\, \tilde{C}_{\bar{j}/j_B}^{\mathrm{PDF}}\left(\frac{x_B}{\xi_B}, b_*;\mu_{b_*}^2,\mu_{b_*},a_s(\mu_{b_*})\right)$$

$$\times \exp\left\{-g_K(b_{\mathrm{T}};b_{\max}) \ln\frac{Q^2}{Q_0^2} + \tilde{K}(b_*;\mu_{b_*}) \ln\frac{Q^2}{\mu_{b_*}^2} + \int_{\mu_{b_*}}^{\mu_Q} \frac{\mathrm{d}\mu'}{\mu'}\left[2\gamma_j(a_s(\mu')) - \ln\frac{Q^2}{(\mu')^2}\gamma_K(a_s(\mu'))\right]\right\}$$

Perturbative pieces

Non-perturbative pieces

Non-perturbative piece of the CS kernel

MAP parametrization

- A recent work from the MAP collaboration (arXiv:2206.07598) used a complicated form for the non-perturbative function

$$f_{1\,NP}(x, \boldsymbol{b}_T^2; \zeta, Q_0) = \frac{g_1(x)\, e^{-g_1(x)\frac{\boldsymbol{b}_T^2}{4}} + \lambda^2\, g_{1B}^2(x) \left[1 - g_{1B}(x)\frac{\boldsymbol{b}_T^2}{4}\right] e^{-g_{1B}(x)\frac{\boldsymbol{b}_T^2}{4}} + \lambda_2^2\, g_{1C}(x)\, e^{-g_{1C}(x)\frac{\boldsymbol{b}_T^2}{4}}}{g_1(x) + \lambda^2\, g_{1B}^2(x) + \lambda_2^2\, g_{1C}(x)} \left[\frac{\zeta}{Q_0^2}\right]^{g_K(\boldsymbol{b}_T^2)/2}, \quad (38)$$

$$g_{\{1,1B,1C\}}(x) = N_{\{1,1B,1C\}} \frac{x^{\sigma_{\{1,2,3\}}} (1-x)^{\alpha^2_{\{1,2,3\}}}}{\hat{x}^{\sigma_{\{1,2,3\}}} (1-\hat{x})^{\alpha^2_{\{1,2,3\}}}},$$

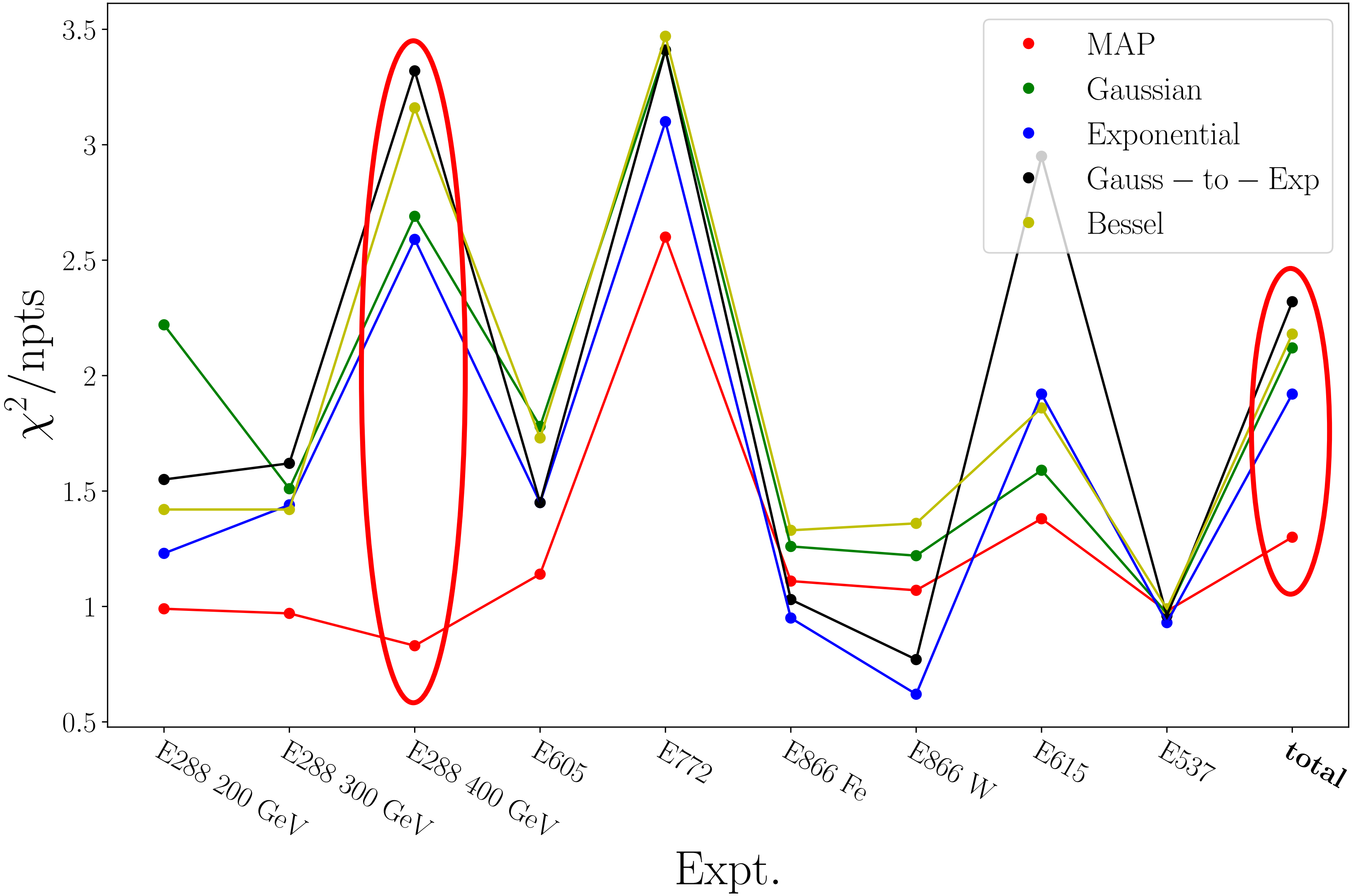
$$g_K(\boldsymbol{b}_T^2) = -g_2^2 \frac{\boldsymbol{b}_T^2}{2}$$

Universal CS kernel

- 11 free parameters for each hadron! (flavor dependence not necessary) (12 if we include the nuclear TMD parameter)

Resulting χ^2 for each parametrization

- Tried multiple parametrizations for non-perturbative TMD structures
- MAP parametrization is able to describe better all the datasets



Nuclear TMD PDFs – working hypothesis

- We must model the nuclear TMD PDF from proton

$$\tilde{f}_{q/A}(x, b_T, \mu, \zeta) = \frac{Z}{A}\tilde{f}_{q/p/A}(x, b_T, \mu, \zeta) + \frac{A-Z}{A}\tilde{f}_{q/n/A}(x, b_T, \mu, \zeta)$$

- Each object on the right side independently obeys the CSS equation
  - **Assumption** that the bound proton and bound neutron follow TMD factorization
- Make use of isospin symmetry in that $u/p/A \leftrightarrow d/n/A$, etc.

Building of the nuclear TMD PDF

- Then taking into account the intrinsic non-perturbative, we model the flavor-dependent pieces of the TMD PDF as

$$(C \otimes f)_{u/A}(x) e^{-g_{u/A}(x,b_T)} \to \frac{Z}{A}(C \otimes f)_{u/p/A}(x) e^{-g_{u/p/A}(x,b_T)} + \frac{A-Z}{A}(C \otimes f)_{d/p/A}(x) e^{-g_{d/p/A}(x,b_T)}$$

and

$$(C \otimes f)_{d/A}(x) e^{-g_{d/A}(x,b_T)} \to \frac{Z}{A}(C \otimes f)_{d/p/A}(x) e^{-g_{d/p/A}(x,b_T)} + \frac{A-Z}{A}(C \otimes f)_{u/p/A}(x) e^{-g_{u/p/A}(x,b_T)}.$$

Nuclear TMD parametrization

- Specifically, we include a parametrization similar to Alrashed, et al., Phys. Rev. Lett **129**, 242001 (2022).

$$g_{q/\mathcal{N}/A} = g_{q/\mathcal{N}}\left(1 - a_{\mathcal{N}}\left(A^{1/3} - 1\right)\right)$$

- Where $a_{\mathcal{N}}$ is an additional parameter to be fit

Bayesian Inference

- Minimize the χ^2 for each replica

Normalization parameter

$$\chi^2(\boldsymbol{a}, \mathrm{data}) = \sum_e \left(\sum_i \left[\frac{d_i^e - \sum_k r_k^e \beta_{k,i}^e - t_i^e(\boldsymbol{a})/n_e}{\alpha_i^e} \right]^2 + \left(\frac{1 - n_e}{\delta n_e} \right)^2 + \sum_k \left(r_k^e \right)^2 \right)$$

- Perform N total χ^2 minimizations and compute statistical quantities

Expectation value $$\mathrm{E}[\mathcal{O}] = \frac{1}{N} \sum_k \mathcal{O}(\boldsymbol{a}_k),$$

Variance $$\mathrm{V}[\mathcal{O}] = \frac{1}{N} \sum_k \left[\mathcal{O}(\boldsymbol{a}_k) - \mathrm{E}[\mathcal{O}] \right]^2,$$

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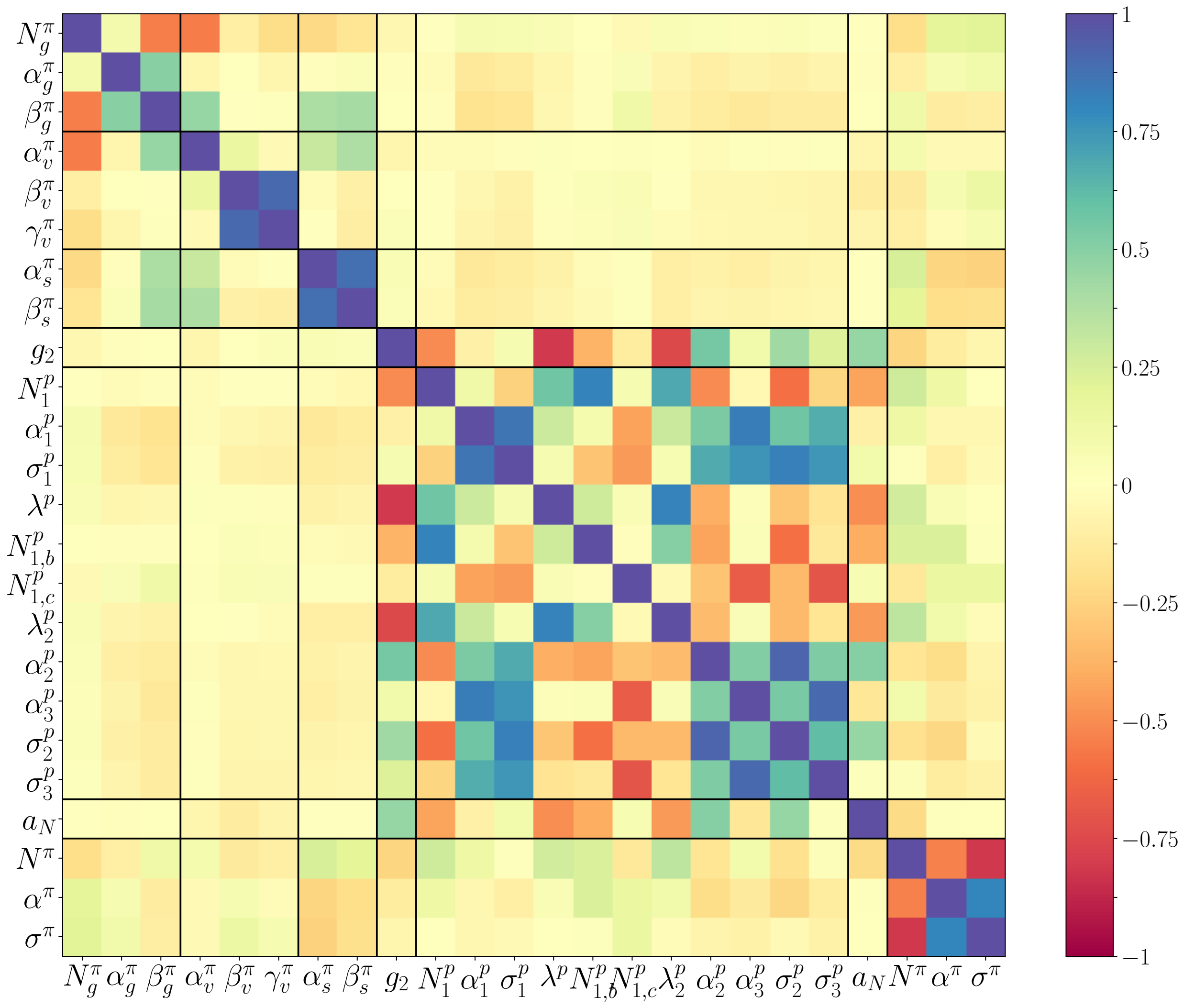
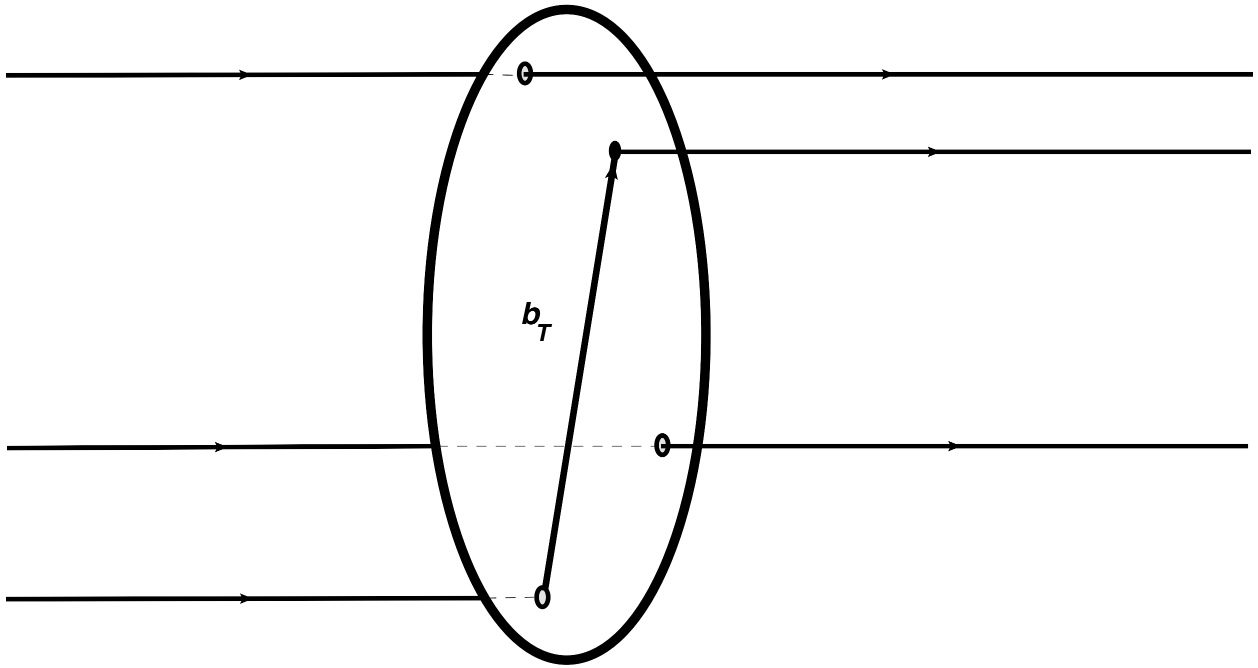
Correlations

- Level at which the distributions are correlated with each other
- Different distributions are largely correlated only within themselves



Possible explanation

- At large x, we are in a valence region, where only the valence quarks are populating the momentum dependence of the hadron



Possible explanation

- At small x, sea quarks and potential $q\bar{q}$ bound states allowing only for a smaller bound system