Unpolarised TMD distributions and their extraction from experimental data

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Outline

- TMD factorisation and renormalisation.
- TMD evolution:
  - Collins-Soper-Sterman (CSS) approach,
  - Scimemi-Vladimirov (SV) approach.
- Landau-pole regularisation.

- Recent TMD extractions from data.
- Phenomenology at the LHC and the EIC.

Dialogue on factorisation (1/2)

Q: Why do we need factorisation for hadronic processes at large scales in QCD?

A: To separate, at the level of process *amplitude*, low-scale modes (of the order of the hadron mass) from high-scale modes of the order of the process hard scale.

Q: What's the advantage of factorisation in QCD?

A: Short answer: achieve *predictivity*. Longer answer:

1. exploit *asymptotic freedom* of QCD to compute high-scale contributions using perturbation theory,
2. "measure" the low-scale contribution from data of some processes,
3. exploit *universality* to predict some other processes.

B: When do we need *TMD* factorisation in QCD?

A: When *collinear* factorisation breaks down.

Q: Ok, then when does collinear factorisation break down?

A: When there are *two or more* hard scales ($\gg \Lambda_{QCD}$) widely different from each other.

Dialogue on factorisation (2/2)

Q: What is the problem with having, say, *two* very different hard scales?

A: Logs of the ratio are *large* and make low-order perturbation theory inapplicable.

Q: What is the origin of these logarithms in QCD?

A: They "interpolate" between different importance regions in momentum space.

Q: How does TMD factorisation solve the problem?

A: It gives us a recipe to *resum* these logs to all orders in perturbation theory.

Q: How about TMD distributions? What's their role?

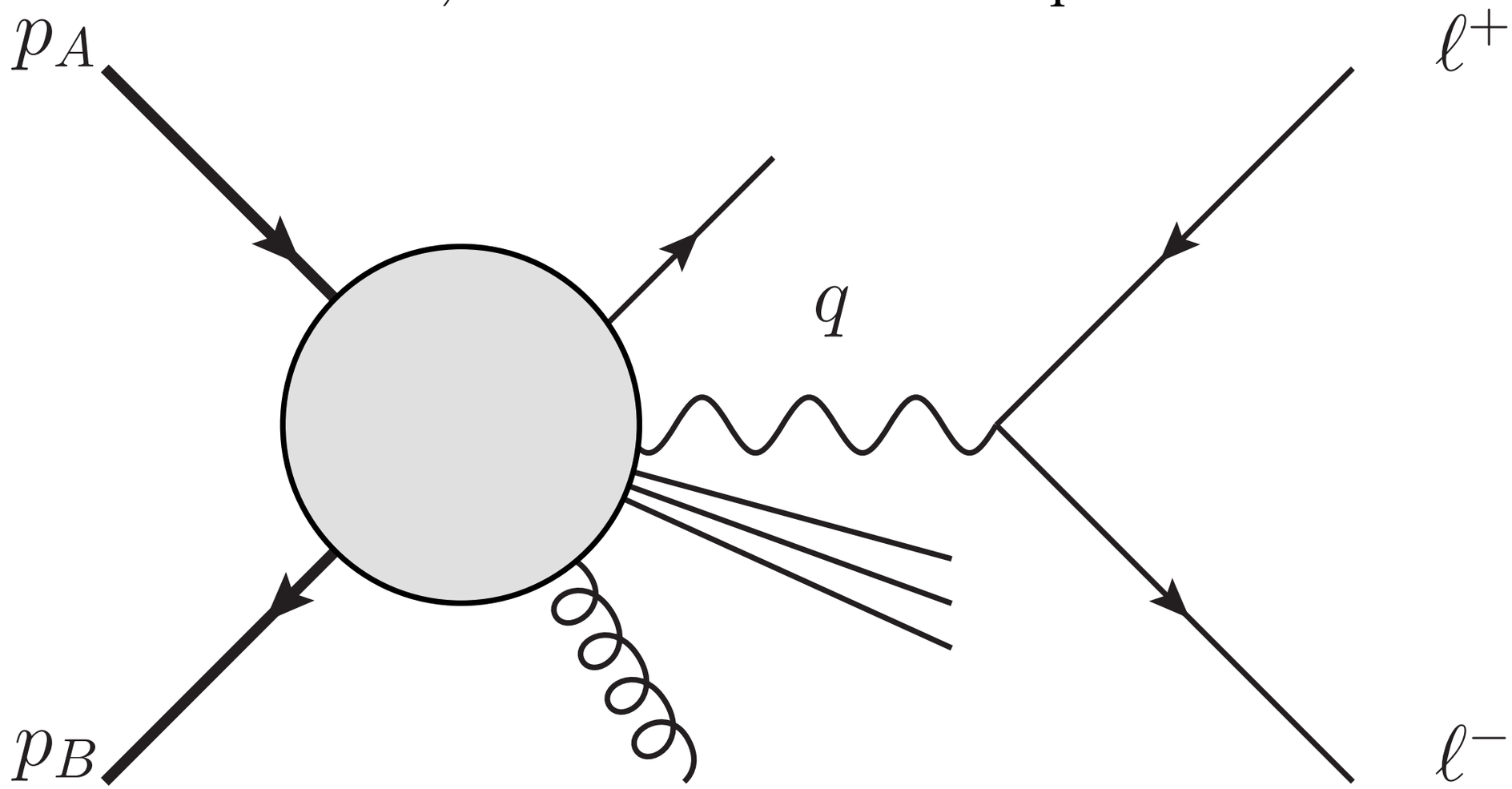
A: They are a "byproduct" of TMD factorisation. But importantly they carry information on the longitudinal- *and* transverse-momentum structure of hadrons.

Q: Does (TMD) factorisation hold for all processes in QCD?

A: Unfortunately not. In fact, factorising processes are to be considered more an exception than a rule… but perhaps this is a subject for another dialogue.

TMD factorisation

- Let us use Drell-Yan production (*i.e. inclusive* production of a lepton pair in hadron-hadron collisions) to sketch the main steps of factorisation:



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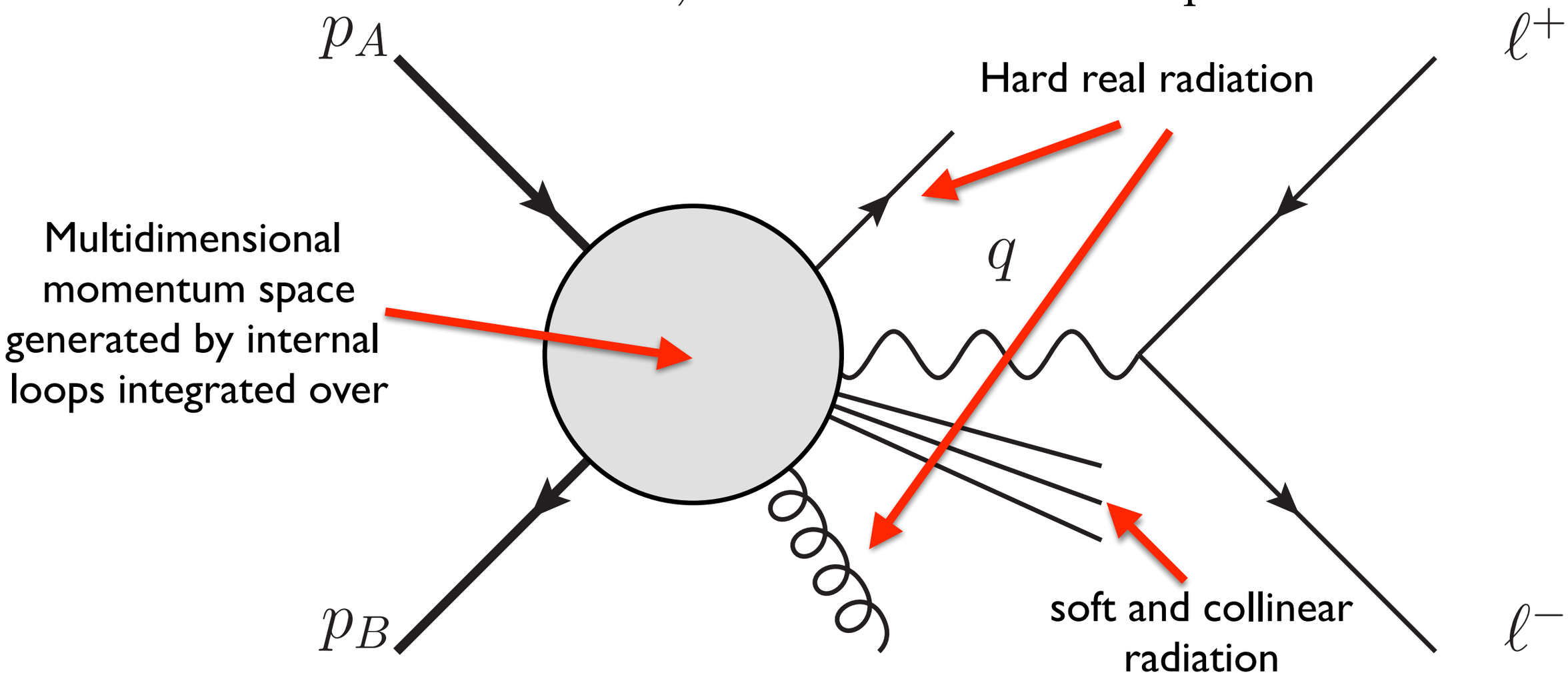


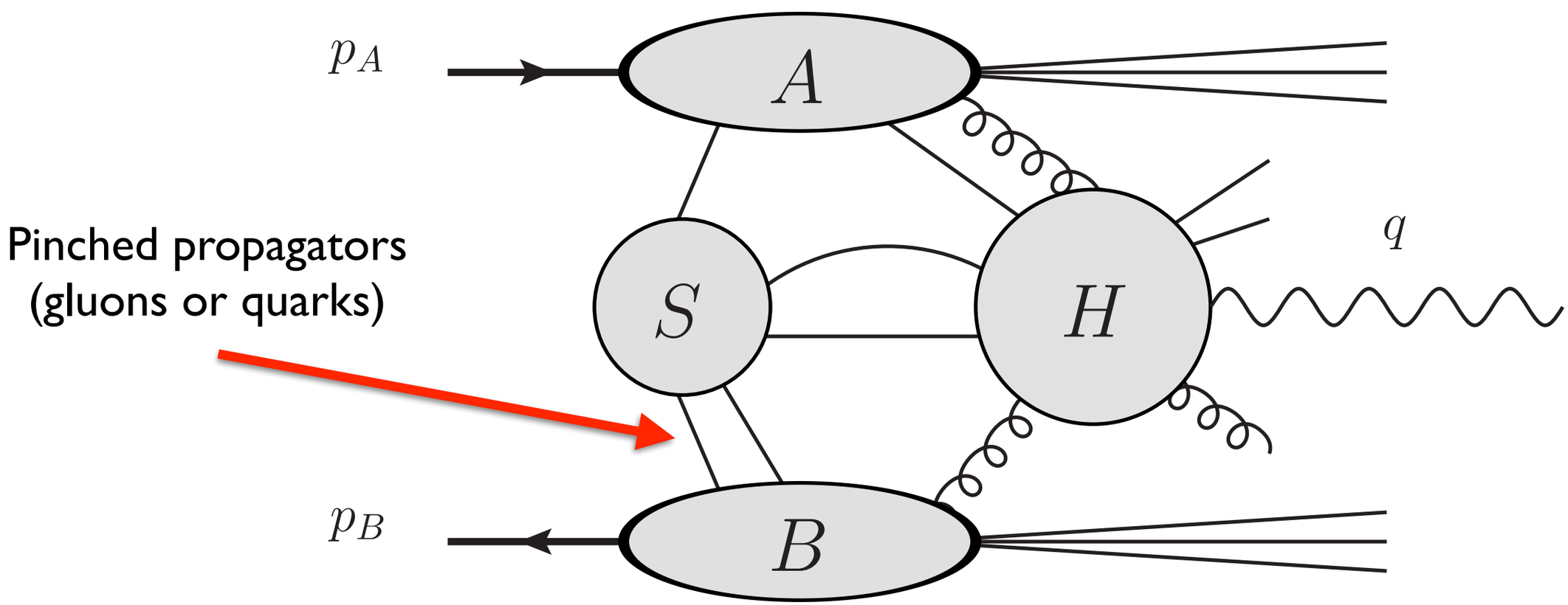
- Relevant scales and (light-cone) momenta in the Breit frame:

$$Q = \sqrt{q^2}\,, \qquad \lambda \sim \frac{\Lambda_{\rm QCD}}{Q} \ll 1$$

$$p_A \sim \frac{Q}{2}(1, \lambda^2, \lambda)\,, \qquad p_B \sim \frac{Q}{2}(\lambda^2, 1, \lambda)\,, \qquad q = Q(1, 0, 0)$$

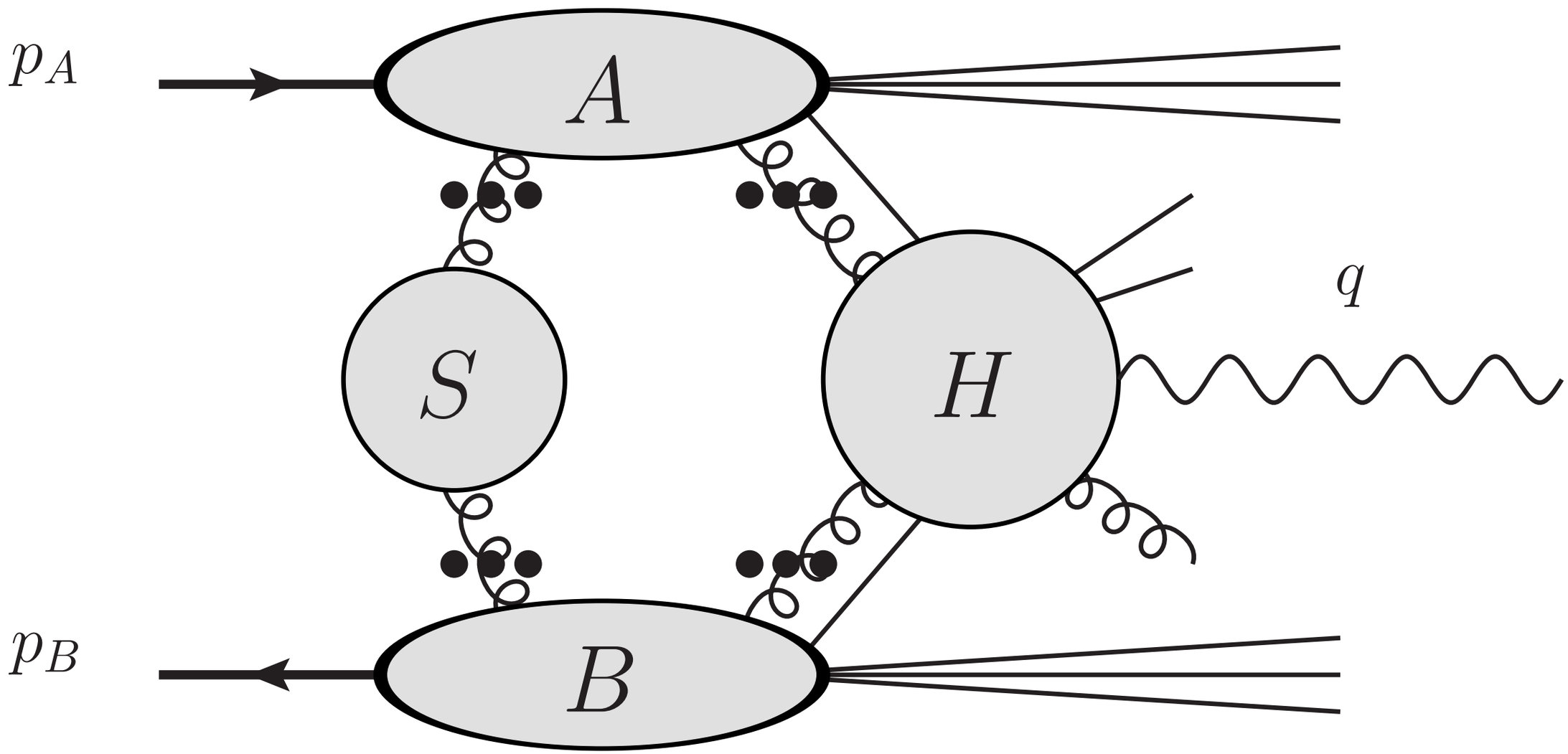
TMD factorisation

- For each *single* graph, identify the regions in the integration momentum space where the integrand is large:
  - *pinched* propagators (massless theory) $\Rightarrow$ Landau criterion and reduced graphs,
  - soft modes (**S**): $\boldsymbol{k \sim Q(\lambda^2, \lambda^2, \lambda^2)}$
  - (anti)collinear modes (**A** and **B**): $\boldsymbol{k \sim \begin{array}{l} Q(1, \lambda^2, \lambda) \\ Q(\lambda^2, 1, \lambda) \end{array}}$
  - hard modes (**H**): $\boldsymbol{k \sim Q(1, 1, 1)}$



TMD factorisation

- Apply Libby-Sterman power counting to identify the **asymptote**:
  - at **leading power** this means all reduced graphs that scale like $Q^{4-N_{\text{ext}}}$
  - possible super-leading graphs must cancel for the process to be factorisable.
- The (well-known) result (in a **covariant gauge**) is:



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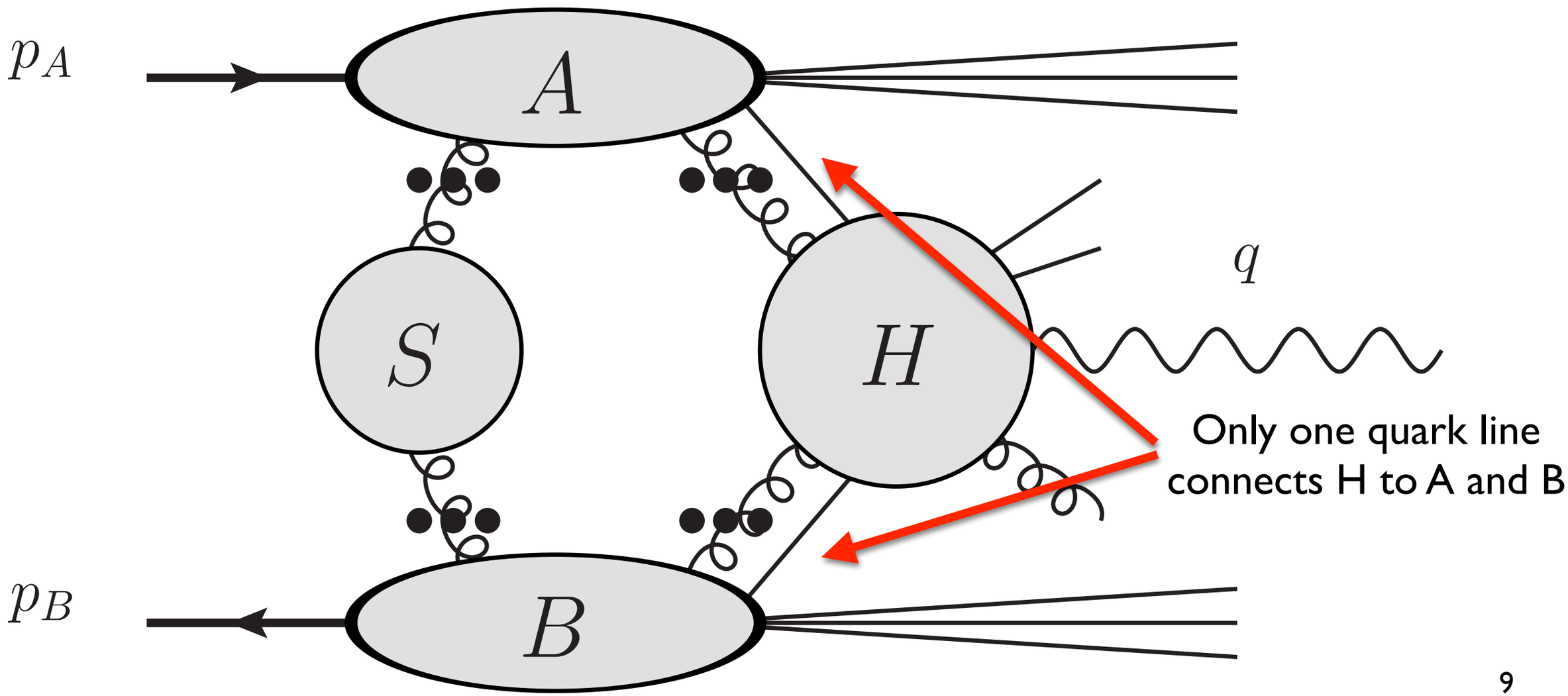

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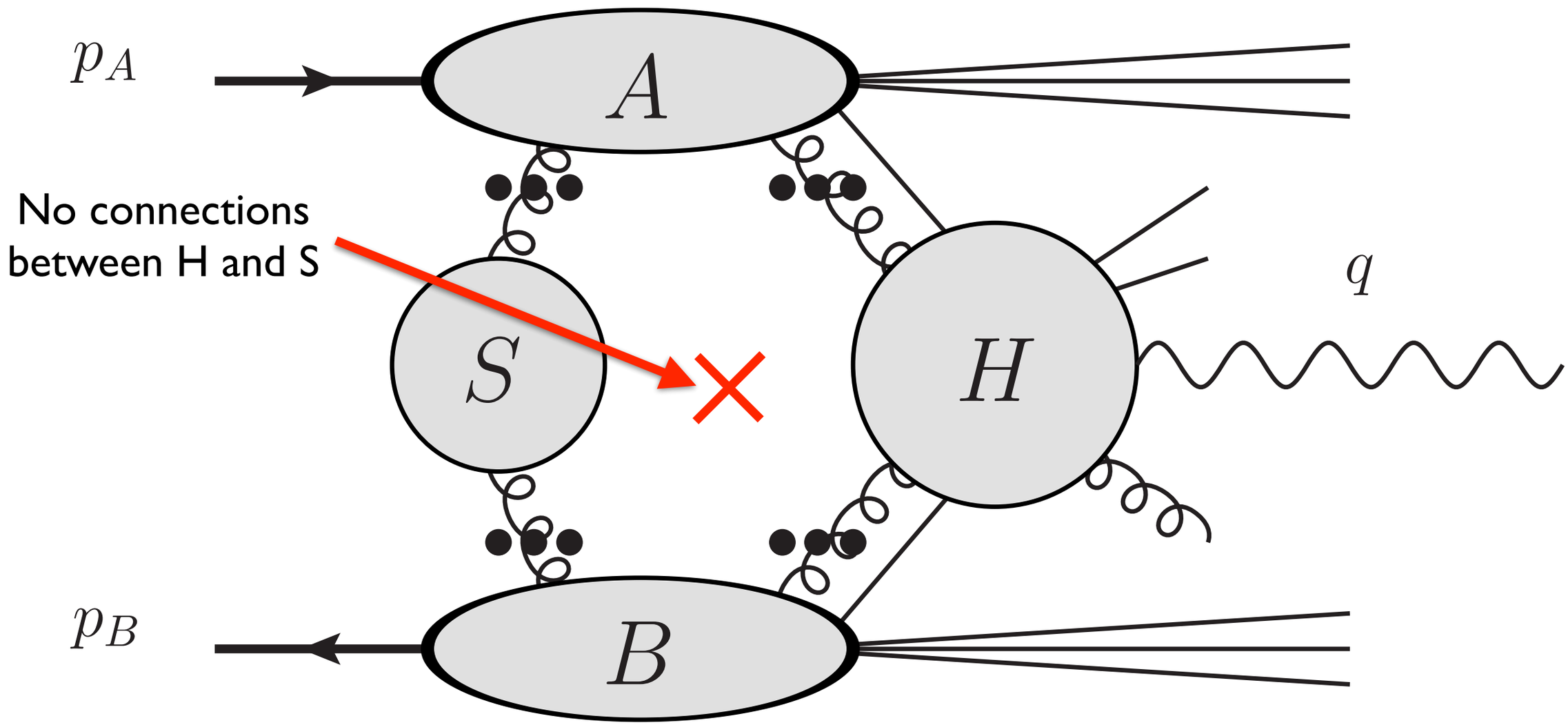

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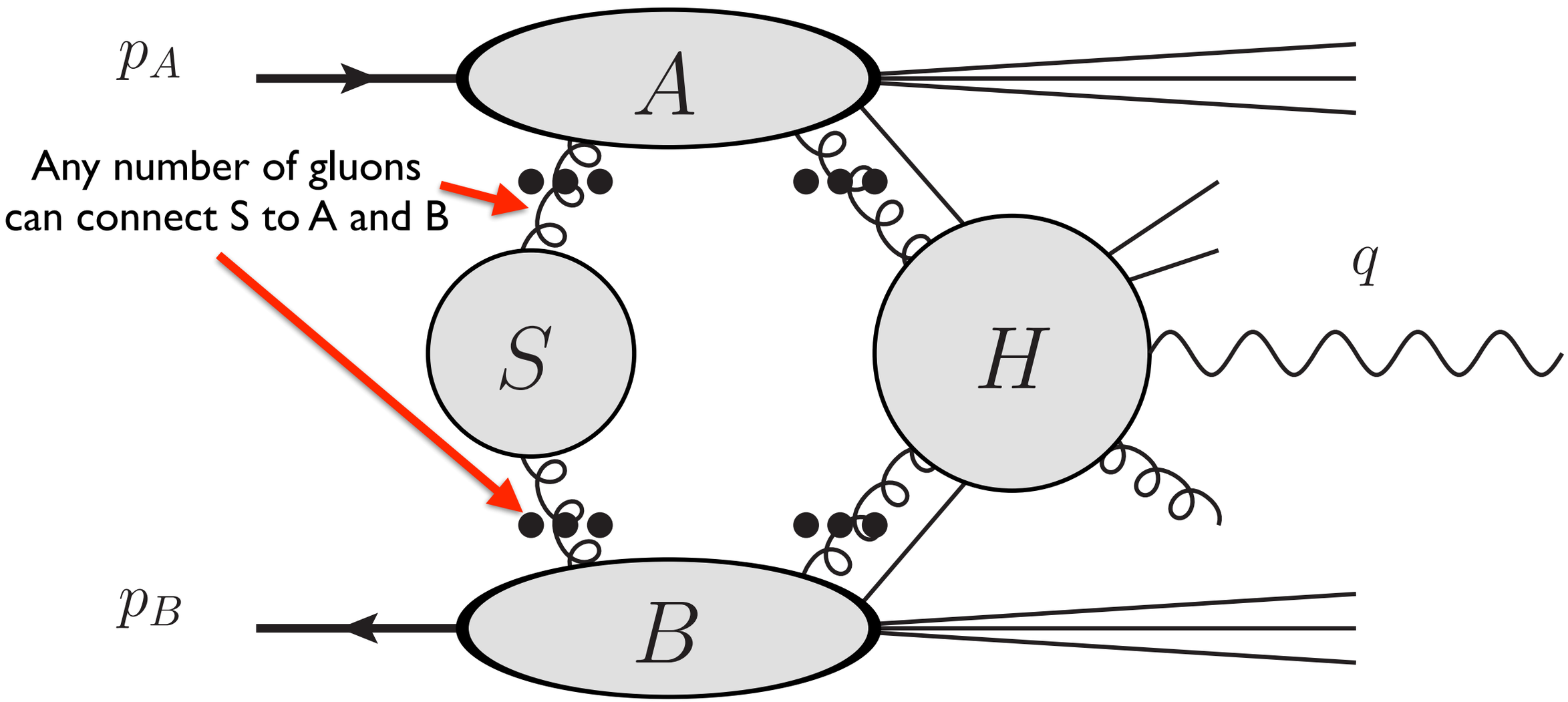

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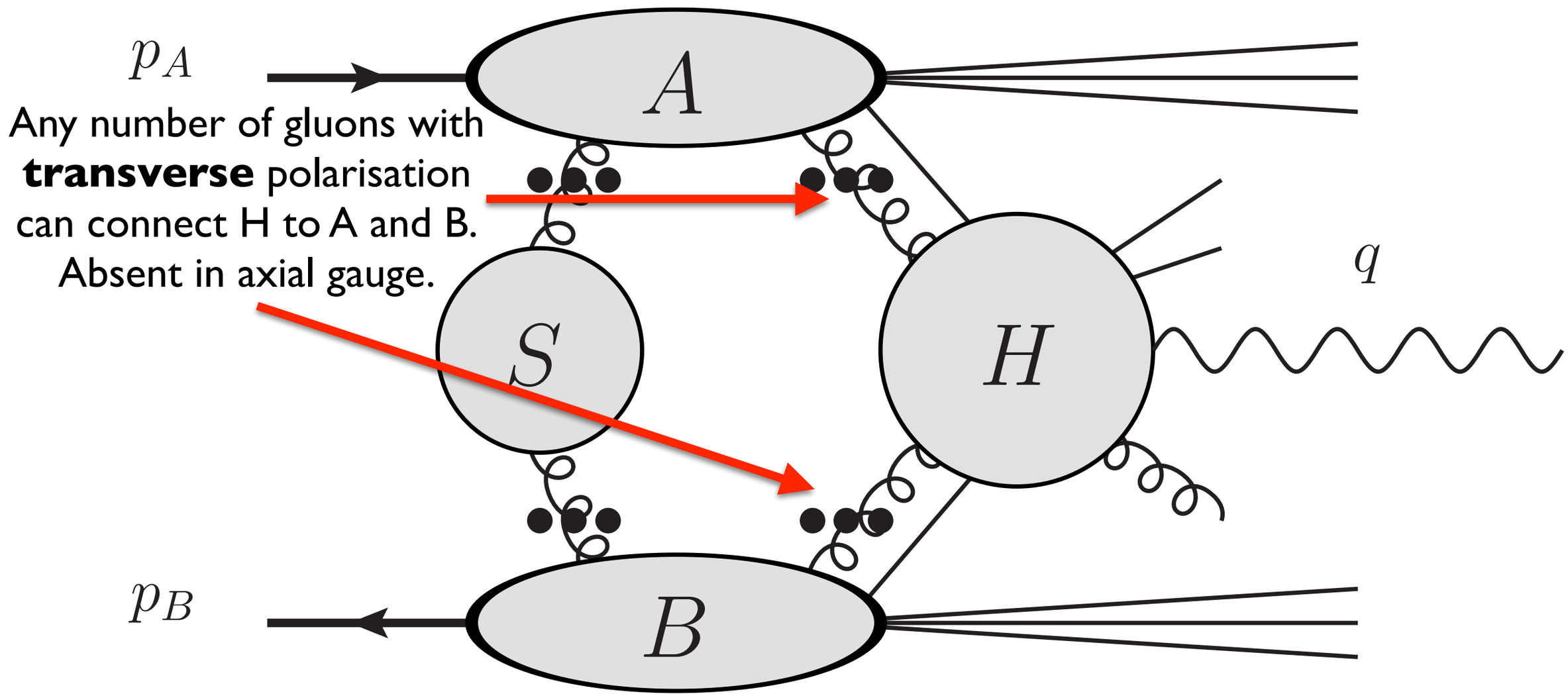

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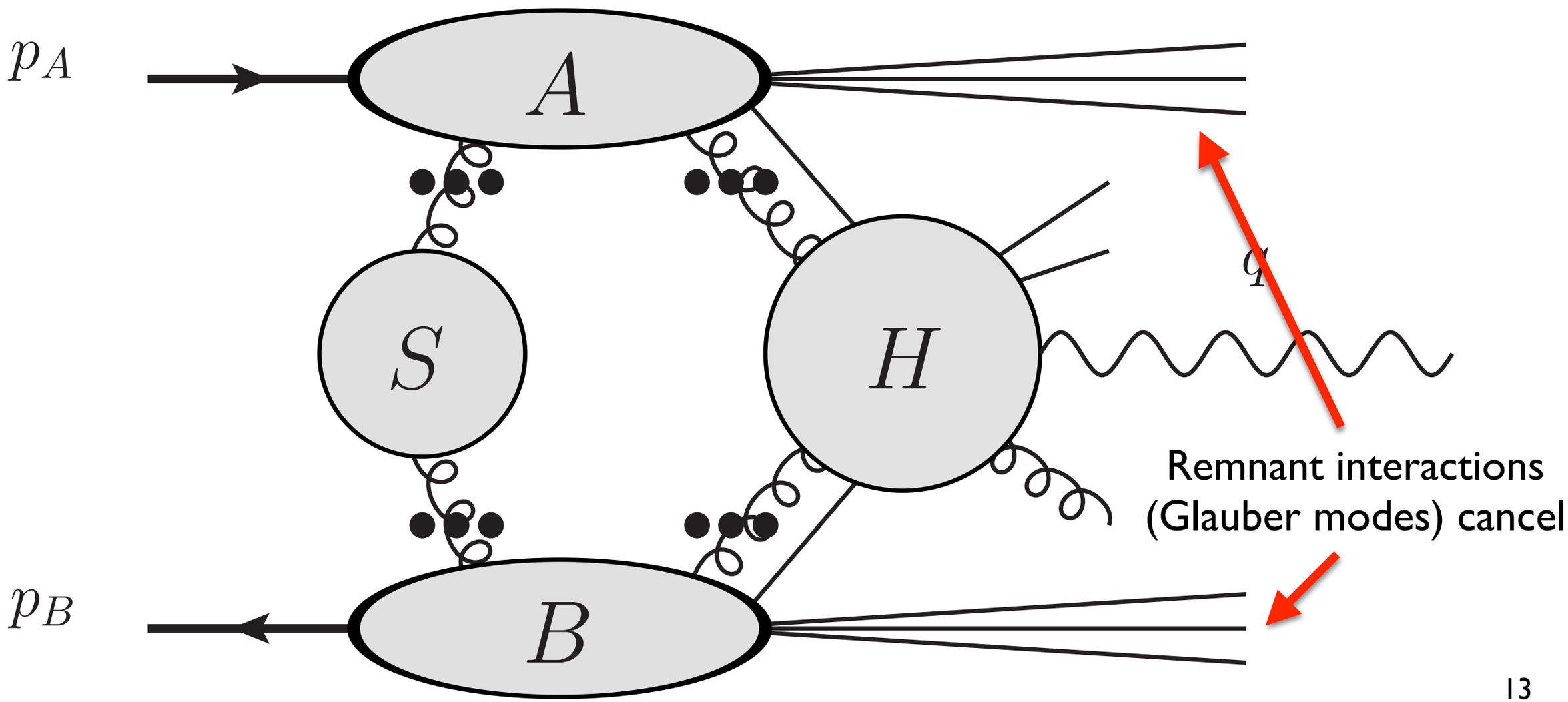


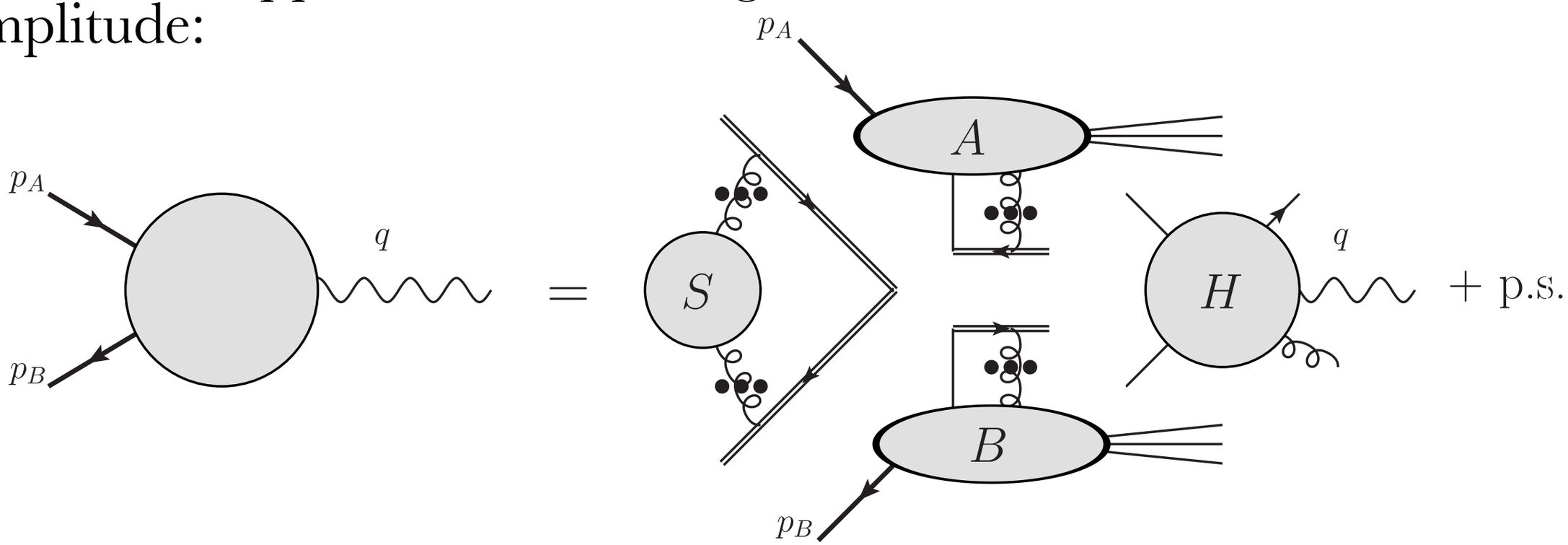
TMD factorisation

- In brief, factorisation is finally achieved by:
  - use Grammer-Yennie-like approximations for each soft/collinear gluon with momentum k to write, for example:

$$S^{\mu\cdots} g_{\mu\nu} A^{\nu\cdots} \sim (k \cdot S^{\cdots}) \frac{1}{n \cdot k} (n \cdot A^{\cdots})$$

  - this allows one to use **Ward identities** and introduces **Wilson lines**.

- A recursive application of this argument leads to factorisation of the amplitude:

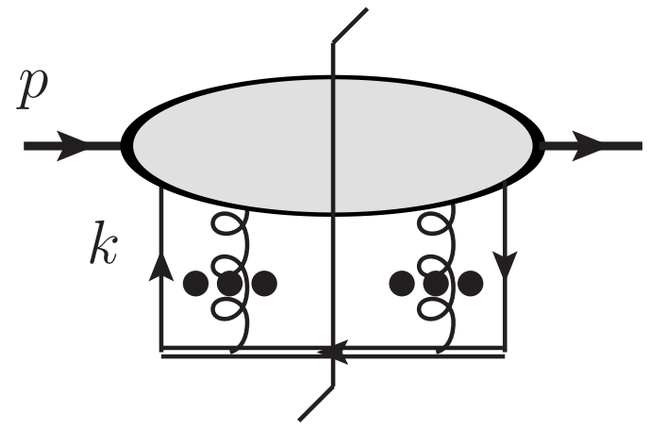


$$\boxed{\mathcal{A} \sim H \cdot A \cdot B \cdot S}$$

TMD factorisation

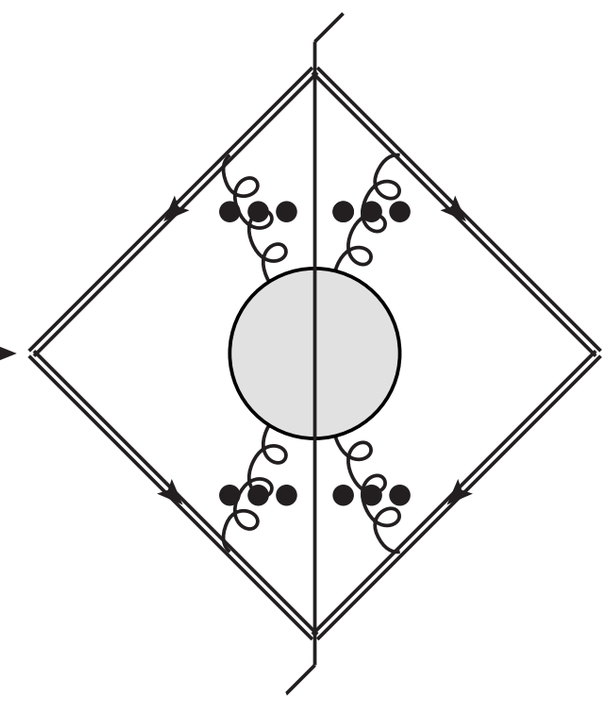
- Upon squaring, factorisation leads to the operator definition of:
  - gauge invariant (unsubtracted) parton-distribution function (**PDF**):

$$f^{(0)}(x, \mathbf{k}_T) \propto \int dk^-$$



  - and **soft function:**

$$S^{(0)}(\mathbf{k}_T) \propto \int dk^+ dk^- \xrightarrow{k}$$



- If $q_T \ll Q$ we *cannot* integrate over k_T because this would mean neglecting transverse momenta in the *other* blobs that are of the order or larger than q_T. Therefore, we need to **treat k_T exactly**: this is the very essence of TMD factorisation.

TMD renormalisation

- Both PDF and soft function are affected by **UV** divergencies that are removed by renormalising strong coupling α_s and wave functions:
  - the UV renormalisation causes the introduction of the **scale μ**.
- PDF and soft function are singularly affected by **rapidity** divergences:
  - whose *ad hoc* regularisation introduces the **scale ζ**.
- However, the following combination is free of divergencies:

$$f(x, k_T, \mu, \zeta) = Z^{\mathrm{UV}}(\mu) f^{(0)}(x, k_T, \mu, \zeta) \sqrt{S^{(0)}(k_T, \mu, \zeta)}$$

- The final **LP** factorised formula for the cross sections takes the form:

$$\frac{d\sigma}{dq_T} \propto H(Q,\mu) \int d^2k_{TA} d^2k_{TB} f_A(x, k_{TA}, \mu, \zeta_A) f_B(x, k_{TB}, \mu, \zeta_B) \delta^{(2)}(q_T - k_{TA} - k_{TB})$$

$$\propto H(Q,\mu) \int d^2b_T e^{ib_T \cdot q_T} f_A(x, b_T, \mu, \zeta_A) f_B(x, b_T, \mu, \zeta_B)$$

- Important constraints on the scales: $\mu \sim Q$ and $\zeta_A \zeta_B = Q^4$.

TMD evolution

- The removal of UV and rapidity divergencies allows one to write **two** evolution equations for the TMD, along with the cross derivative:

$$\begin{cases} \dfrac{d\ln f}{d\ln\mu} = \gamma(\mu,\zeta) \\[2ex] \dfrac{d\ln f}{d\ln\sqrt{\zeta}} = K(\mu) \end{cases} , \quad \frac{d^2\ln f}{d\ln\mu\, d\ln\sqrt{\zeta}} = \begin{cases} \dfrac{d\gamma}{d\ln\sqrt{\zeta}} \\[2ex] \dfrac{dK}{d\ln\mu} \end{cases} = \gamma_K(\alpha_s(\mu))$$

- To solve these equations we need to fix **two pairs of (*i.e.* four) scales**:
  - **initial** scales: (μ_0, ζ_0)
  - **final** scales: (μ, ζ)
- The final solution reads:

$$f(\mu,\zeta) = R\left[(\mu,\zeta) \leftarrow (\mu_0,\zeta_0)\right] f(\mu_0,\zeta_0)$$

$$R\left[(\mu,\zeta) \leftarrow (\mu_0,\zeta_0)\right] = \exp\left\{K(\mu_0)\ln\frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} + \int_{\mu_0}^{\mu}\frac{d\mu'}{\mu'}\left[\gamma_F(\alpha_s(\mu')) - \gamma_K(\alpha_s(\mu'))\ln\frac{\sqrt{\zeta}}{\mu'}\right]\right\}$$

- We know how to choose (μ, ζ) to compute a cross section.
- A question remains: **how do we *sensibly* choose** (μ_0, ζ_0)**?**

TMD evolution *à la* CSS

- A sensible choice of the scales is important to **allow truncated perturbation theory to be reliable**:
  - **no large unresummed logarithms** should be introduced,
  - each scale has to be set in the **vicinity of its "natural" value**.
- In TMD factorisation ($q_T \ll Q$) for DY the relevant scales are $\boldsymbol{q_T}$ and $\boldsymbol{Q}$:
  - natural to expect $\mu_0 \sim \sqrt{\zeta_0} \sim q_T \sim b_T^{-1}$ and $\mu \sim \sqrt{\zeta} \sim Q$
- In the $\overline{\text{MS}}$ scheme **central scales** are usually chosen to be:

$$\mu_0 = \sqrt{\zeta_0} = \frac{2e^{-\gamma_E}}{b_T} \equiv \mu_b \quad \text{and} \quad \mu = \sqrt{\zeta} = Q$$

- This particular choice **nullifies** all unresummed logs.
- Modest variations around these values give an estimate of higher-order corrections.

TMD evolution *à la* SV

- In [1803.11089] SV introduced the concept of **optimal TMD**.

- Motivation: within **truncated perturbation theory**, the **path-independence** of the TMD evolution is **violated**.

- This derives from the violation by *subleading* terms of the equation:

$$\frac{dK}{d\ln\mu} \neq \gamma_K(\alpha_s(\mu))$$

given to the choice to truncate to order N γ_K (as necessary) and also K:

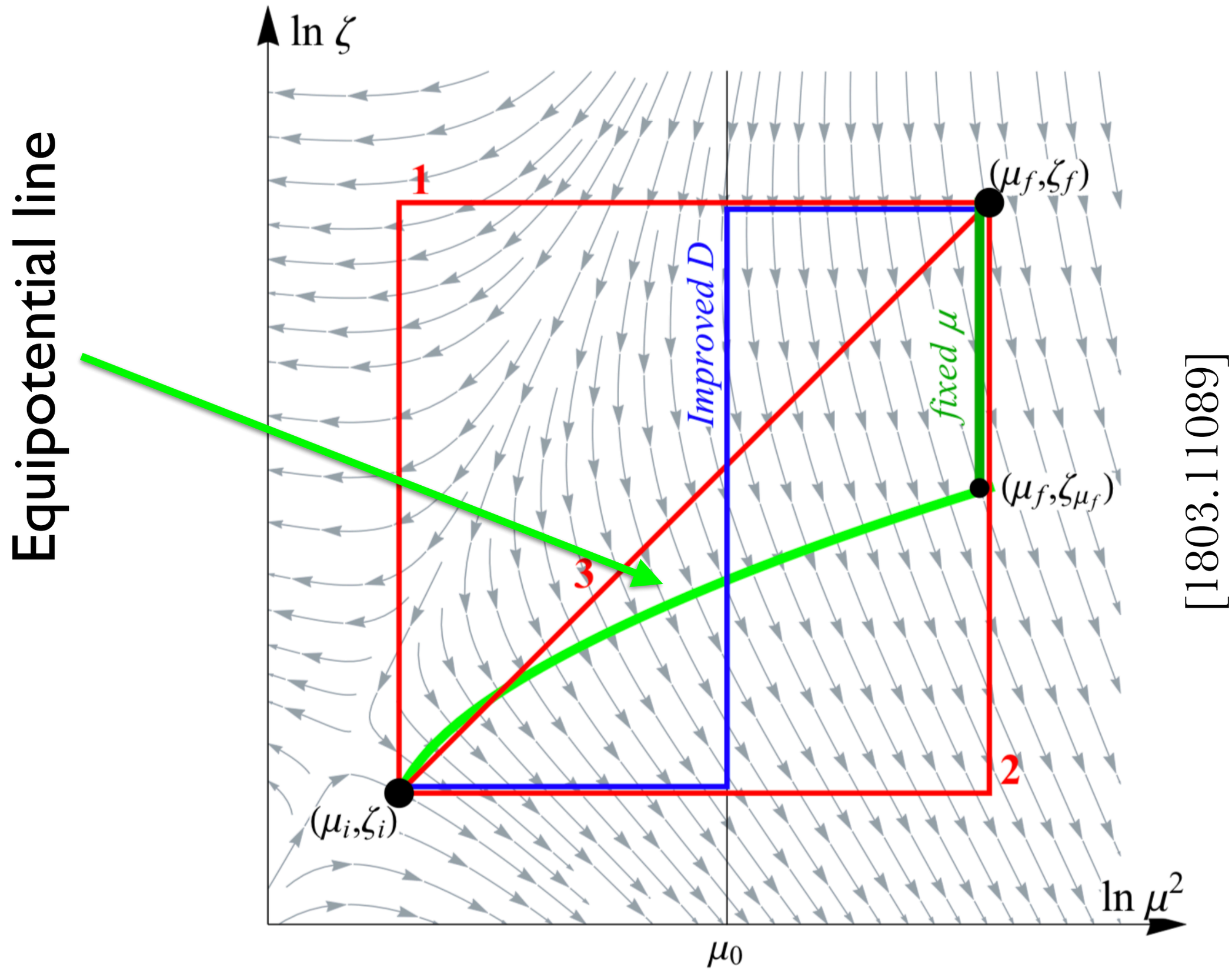
$$\gamma_K(\alpha_s(\mu)) = \sum_{n=0}^{N} \alpha_s^{n+1}(\mu)\gamma_K^{(0)} \quad \text{and} \quad K(\mu) = \sum_{n=0}^{N} \alpha_s^{n+1}(\mu) \sum_{k=0}^{n} \ln^k(\mu b_T)\, d^{(n,k)}$$

- This observation led SV to define the **optimal TMD** as the TMD on the plane (μ^2, ζ) at the saddle point of the vector field $E = (\gamma, K)$.

  - for any given value of b_T, the optimal TMD is scale independent.

  - The TMD at any scale can be computed as a shift along an equipotential line together with a single evolution evolution in ζ.

TMD evolution *à la* SV



TMD evolution *à la* SV

- Another ingredient of the TMD evolution *à la* SV is the **ζ-prescription**.
- Purpose: eliminate **double logs** from the ***matching functions*** C:

$$f(\mu_0, \zeta_0) = C(\mu_0, \zeta_0) \otimes f_{\rm coll}(\mu_0)$$

by exploiting the arbitrariness of ζ_0 assuming $\zeta_0 \doteq \zeta_0(\mu_0)$ and imposing:

$$\frac{d}{d\ln\mu_0} f(\mu_0, \zeta_0(\mu_0)) = 0$$

- This differential equation is the solved for $\ln(\zeta_0)$ order-by-order in α_s:

quark:

$$\mathbf{l}_{\zeta_\mu} = \frac{\mathbf{L}_\mu}{2} - \frac{3}{2} + a_s\Big[\frac{11C_A - 4T_F N_f}{36}\mathbf{L}_\mu^2 + C_F\left(-\frac{3}{4} + \pi^2 - 12\zeta_3\right) + C_A\left(\frac{649}{108} - \frac{17\pi^2}{12} + \frac{19}{2}\zeta_3\right) + T_F N_f\left(-\frac{53}{27} + \frac{\pi^2}{3}\right)\Big] + \mathcal{O}(a_s^3).$$

gluon:

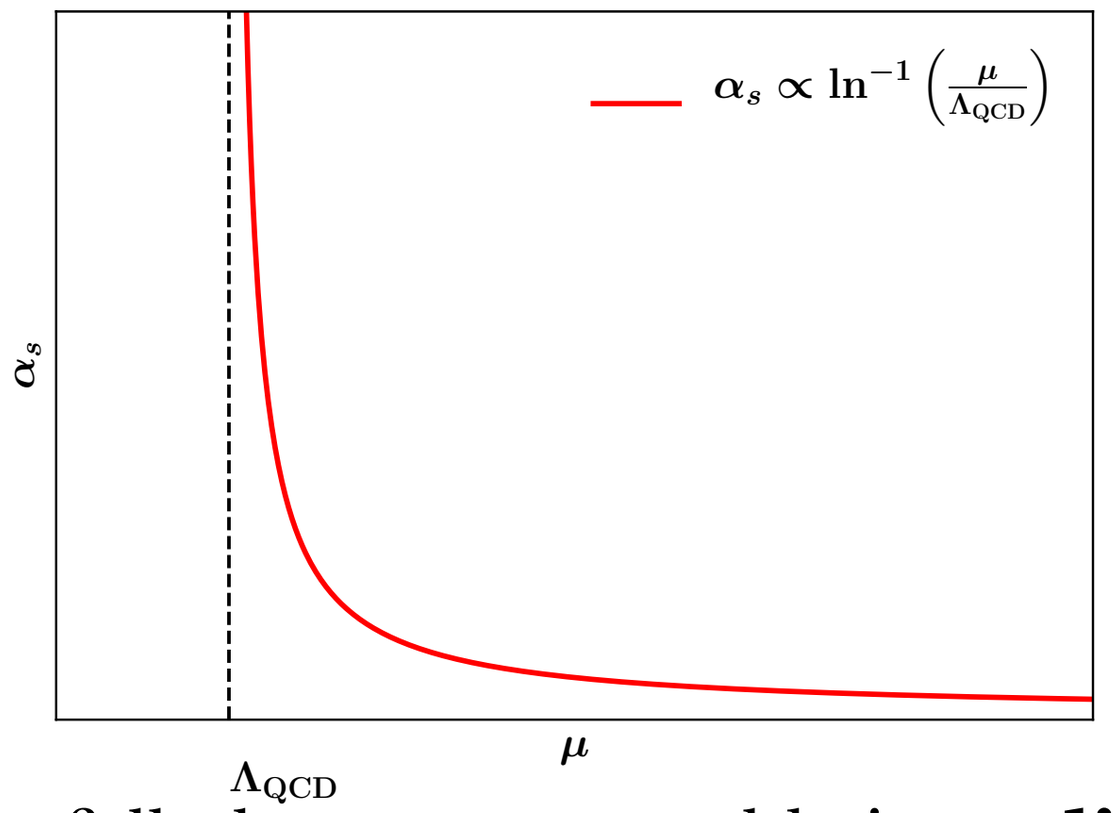
$$\mathbf{l}_{\zeta_\mu} = \frac{\mathbf{L}_\mu}{2} - \frac{11}{6} + \frac{2}{3}\frac{T_F N_f}{C_A} + a_s\Big[\frac{11C_A - 4T_F N_f}{36}\mathbf{L}_\mu^2 + C_A\left(\frac{247}{54} - \frac{11\pi^2}{36} - \frac{5\zeta_3}{2}\right) + T_F N_f\left(-\frac{16}{3} + \frac{\pi^2}{9}\right) + \left(2C_F + \frac{40}{27}T_F N_f\right)\frac{T_f N_f}{C_A}\Big] + \mathcal{O}(a_s^3).$$

Scimemi, Vladimirov [1706.01473]

Landau pole regularisation

*Origin of **TMD** non-perturbative corrections*

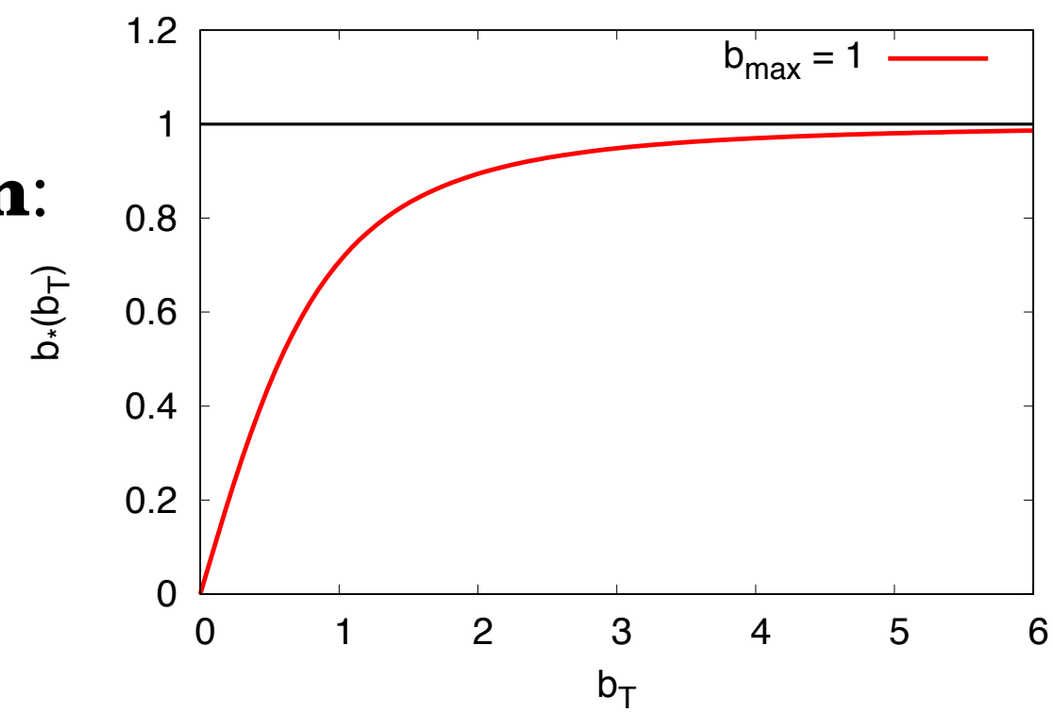
$$\sigma \propto \int_0^\infty db_{\mathrm{T}}\, \alpha_s^p \left(\frac{1}{b_{\mathrm{T}}}\right) \cdots \sim \int_0^Q dk_{\mathrm{T}}\, \alpha_s^p\,(k_{\mathrm{T}}) \ldots$$



- Integrating over the full phase space would give a **divergent** result.
- **Prescriptions** to avoid integrating over the **Landau pole** exist:
  - they all introduce **power corrections** $(\Lambda_{\mathrm{QCD}} / Q)^n$.
- TMD non-perturbative effects thus connected with **low-energy** region:
  - no matter how large Q is.

Non-perturbative component

- When integrating over b_T, **large values of** b_T give raise to low scales in the **non-perturbative** region.



- Introduce the so-called $\boldsymbol{b_*}$**- prescription**:

$$\text{e.g.}\quad b_*(b_T) = \frac{b_T}{\sqrt{1 + b_T^2/b_{\max}^2}}$$

- with $\boldsymbol{b_{\max}}$ such that $\mathbf{1/}\boldsymbol{b_{\max}} \gg \boldsymbol{\Lambda_{\mathrm{QCD}}}$.

- Then rewrite the TMD as:

Perturbative

$$f(x, b_T, \mu, \zeta) = \left[\frac{f(x, b_T, \mu, \zeta)}{f(x, b_*(b_T), \mu, \zeta)}\right] f(x, b_*(b_T), \mu, \zeta) \equiv f_{\mathrm{NP}}(x, b_T, \zeta)\, f(x, b_*(b_T), \mu, \zeta)$$

Non-perturbative

- Properties of $\boldsymbol{f_{\mathrm{NP}}}$:
  - dependence on $\boldsymbol{\mu}$ drops,
  - dependence on $\boldsymbol{\zeta}$ is know: $f_{\mathrm{NP}}(x, b_T, \zeta) = \widetilde{f}_{\mathrm{NP}}(x, b_T) \exp\left[g_K(b_T) \ln\left(\frac{\sqrt{\zeta}}{Q_0}\right)\right]$
  - non-perturbative origin but strictly connected to the particular choice of $\boldsymbol{b_*}$

TMD factorisation

The operational version (for unpolarised DY) à la CSS

- At small values of $q_{\rm T}$, the DY cross section in TMD factorisation reads:

$$\frac{d\sigma}{dq_T} = \sigma_0 H(Q,Q) \int_0^\infty db_T\, b_T J_0(q_T b_T) f_A(x_1, b_T, Q, Q^2) f_B(x_2, b_T, Q, Q^2)$$

- Defining $\mu_{b_*} = \dfrac{2e^{-\gamma_E}}{b_*(b_T)}$, the single TMD distributions are given by:

$$\begin{aligned} f(x, b_T, Q, Q^2) \;=\; & C(x, \mu_{b_*}, \mu_{b_*}^2) \otimes f_{\rm coll}(x, \mu_{b_*}) \\ & \times \exp\left\{ K(\mu_{b_*}) \ln \frac{Q}{\mu_{b_*}} + \int_{\mu_{b_*}}^{Q} \frac{d\mu'}{\mu'} \left[\gamma_F(\alpha_s(\mu')) - \gamma_K(\alpha_s(\mu')) \ln \frac{Q}{\mu'} \right] \right\} \\ & \times \widetilde{f}_{\rm NP}(x, b_T) \exp\left[g_K(b_T) \ln \left(\frac{Q}{Q_0} \right) \right] \end{aligned}$$

Matching onto collinear distributions

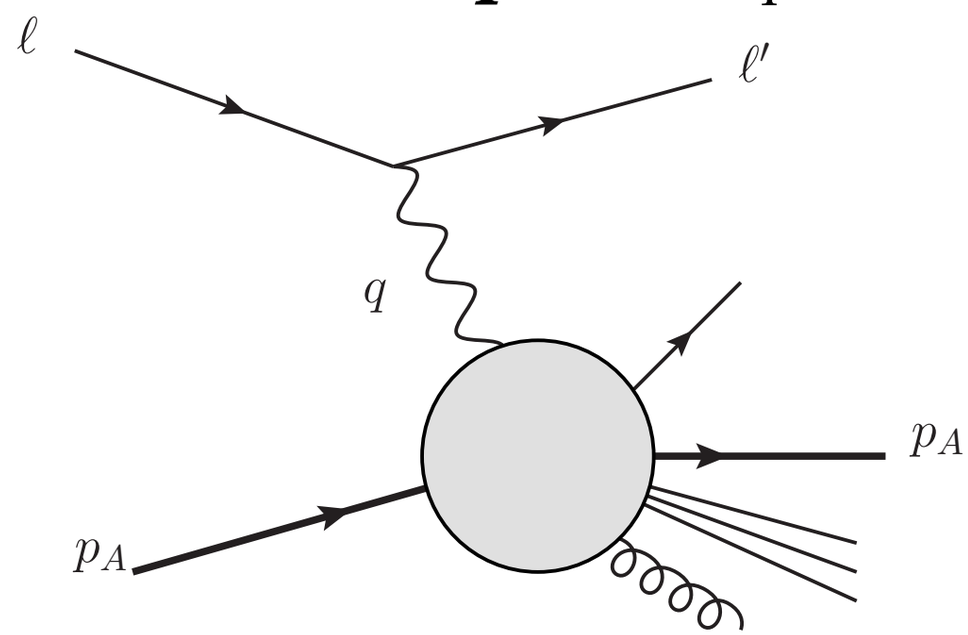
Perturbative evolution

TMD non-perturbative contribution

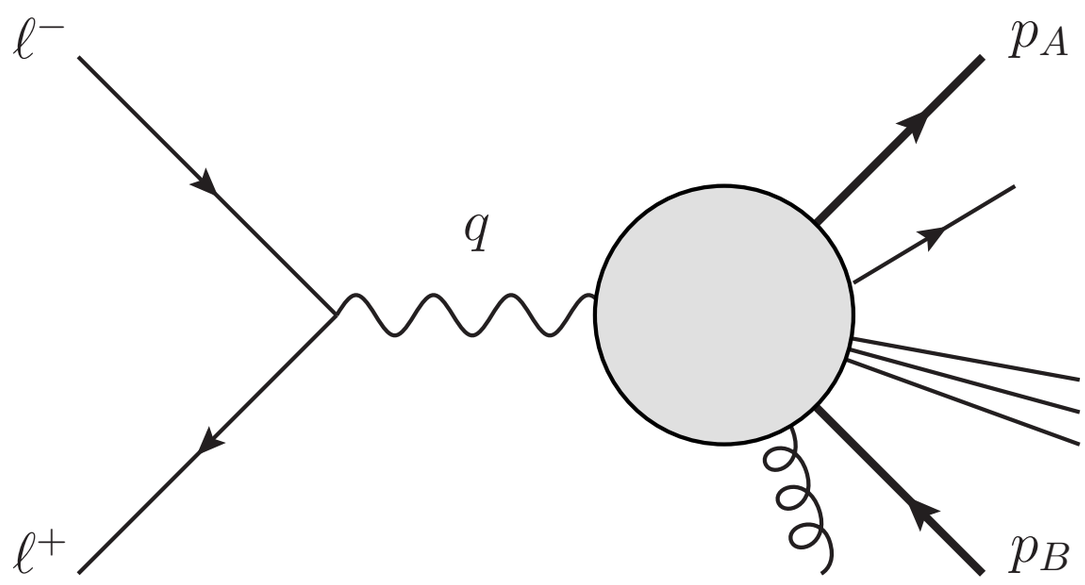
- Matching functions and anomalous dimensions are know from perturbation theory,
- collinear distributions are (accurately) known from dedicated fits,
- the TMD non-pert. contribution has to be **determined from data** at small $q_{\rm T}$.

TMD factorisation

- Leading-power TMD factorisation has been **proven** to hold also for:
  - Semi-inclusive DIS al low values of the $\boldsymbol{p_T}$ of the produced hadron:



  - double-inclusive e^+e^- annihilation where the two hadrons are almost back-to-back:



- Necessary to replace PDFs with **fragmentation functions** (FFs) where appropriate.

Phenomenology

Most recent TMD extractions

- **Pavia2019** [1912.07550]:
  - based on the CSS formalism

Transverse-momentum-dependent parton distributions up to N^{3}LL from Drell-Yan data

Alessandro Bacchetta,[a,b] Valerio Bertone,[a,b] Chiara Bissolotti,[a,b] Giuseppe Bozzi,[a,b] Filippo Delcarro,[c] Fulvio Piacenza,[a,b] and Marco Radici[b]

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- **SV2019** [1912.06532]:
  - based on the SV formalism

Non-perturbative structure of semi-inclusive deep-inelastic and Drell-Yan scattering at small transverse momentum

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Pavia2019

Dataset

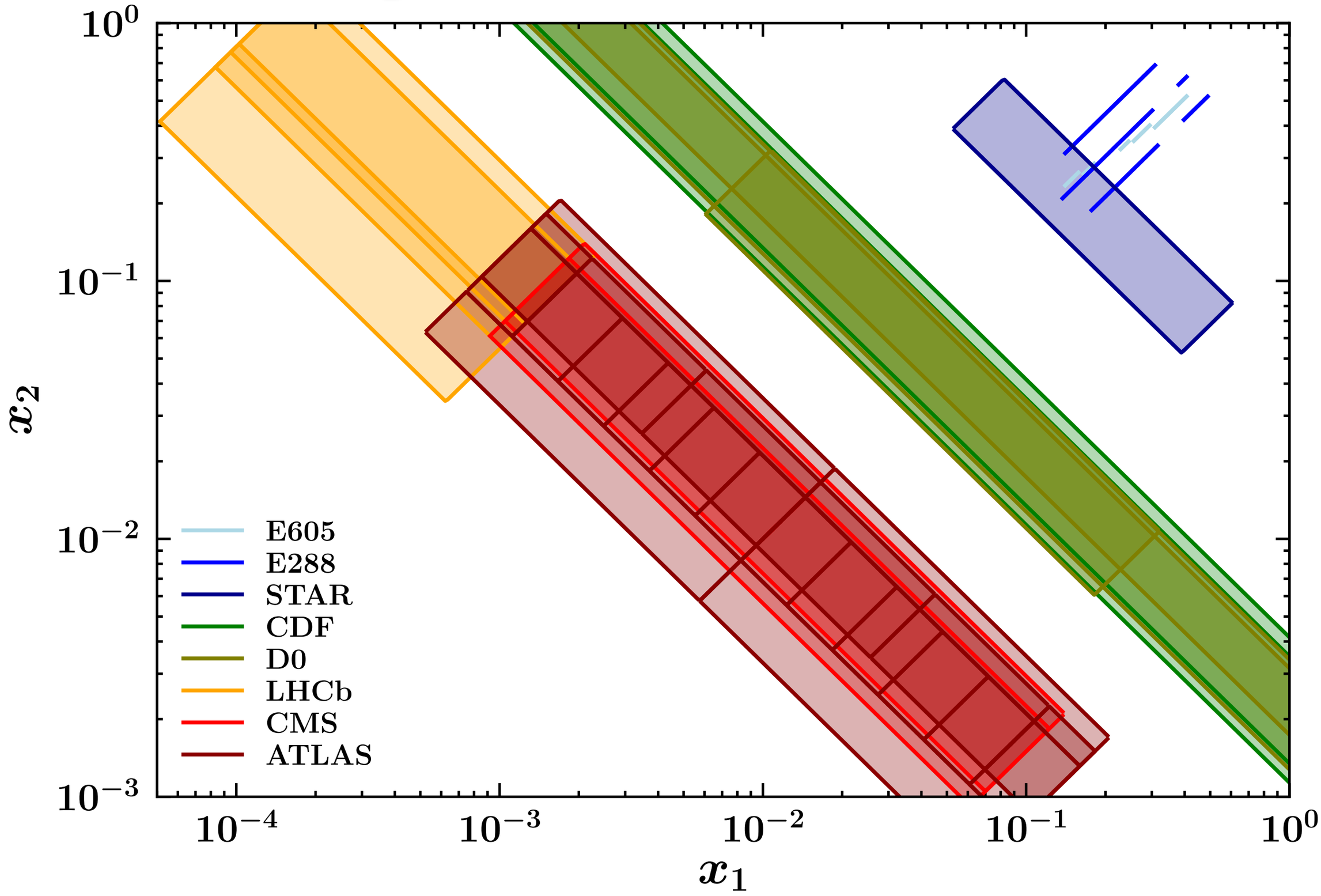
- DY data only:
  - fixed-target low-energy DY,
  - STAR data
  - LHC and Tevatron data,
  - 353 data points,
  - selection cut q_T / Q < 0.2.

Experiment	$N_{\rm dat}$	Observable	$\sqrt{s}$ [GeV]	Q [GeV]	y or x_F	Lepton cuts	Ref.
E605	50	$Ed^3\sigma/d^3q$	38.8	7 - 18	$x_F = 0.1$	-	[79]
E288 200 GeV	30	$Ed^3\sigma/d^3q$	19.4	4 - 9	$y = 0.40$	-	[80]
E288 300 GeV	39	$Ed^3\sigma/d^3q$	23.8	4 - 12	$y = 0.21$	-	[80]
E288 400 GeV	61	$Ed^3\sigma/d^3q$	27.4	5 - 14	$y = 0.03$	-	[80]
STAR 510	7	$d\sigma/dq_T$	510	73 - 114	$\|y\| < 1$	$p_{T\ell} > 25$ GeV $\|\eta_\ell\| < 1$	-
CDF Run I	25	$d\sigma/dq_T$	1800	66 - 116	Inclusive	-	[81]
CDF Run II	26	$d\sigma/dq_T$	1960	66 - 116	Inclusive	-	[82]
D0 Run I	12	$d\sigma/dq_T$	1800	75 - 105	Inclusive	-	[83]
D0 Run II	5	$(1/\sigma)d\sigma/dq_T$	1960	70 - 110	Inclusive	-	[84]
D0 Run II (μ)	3	$(1/\sigma)d\sigma/dq_T$	1960	65 - 115	$\|y\| < 1.7$	$p_{T\ell} > 15$ GeV $\|\eta_\ell\| < 1.7$	[85]
LHCb 7 TeV	7	$d\sigma/dq_T$	7000	60 - 120	$2 < y < 4.5$	$p_{T\ell} > 20$ GeV $2 < \eta_\ell < 4.5$	[86]
LHCb 8 TeV	7	$d\sigma/dq_T$	8000	60 - 120	$2 < y < 4.5$	$p_{T\ell} > 20$ GeV $2 < \eta_\ell < 4.5$	[87]
LHCb 13 TeV	7	$d\sigma/dq_T$	13000	60 - 120	$2 < y < 4.5$	$p_{T\ell} > 20$ GeV $2 < \eta_\ell < 4.5$	[92]
CMS 7 TeV	4	$(1/\sigma)d\sigma/dq_T$	7000	60 - 120	$\|y\| < 2.1$	$p_{T\ell} > 20$ GeV $\|\eta_\ell\| < 2.1$	[88]
CMS 8 TeV	4	$(1/\sigma)d\sigma/dq_T$	8000	60 - 120	$\|y\| < 2.1$	$p_{T\ell} > 15$ GeV $\|\eta_\ell\| < 2.1$	[89]
ATLAS 7 TeV	6 6 6	$(1/\sigma)d\sigma/dq_T$	7000	66 - 116	$\|y\| < 1$ $1 < \|y\| < 2$ $2 < \|y\| < 2.4$	$p_{T\ell} > 20$ GeV $\|\eta_\ell\| < 2.4$	[93]
ATLAS 8 TeV on-peak	6 6 6 6 6 6	$(1/\sigma)d\sigma/dq_T$	8000	66 - 116	$\|y\| < 0.4$ $0.4 < \|y\| < 0.8$ $0.8 < \|y\| < 1.2$ $1.2 < \|y\| < 1.6$ $1.6 < \|y\| < 2$ $2 < \|y\| < 2.4$	$p_{T\ell} > 20$ GeV $\|\eta_\ell\| < 2.4$	[90]
ATLAS 8 TeV off-peak	4 8	$(1/\sigma)d\sigma/dq_T$	8000	46 - 66 116 - 150	$\|y\| < 2.4$	$p_{T\ell} > 20$ GeV $\|\eta_\ell\| < 2.4$	[90]
Total	353	-	-	-	-	-	-

Pavia2019

Kinematic coverage

$$x_{1,2} = \frac{Q}{\sqrt{s}} e^{\pm y}$$

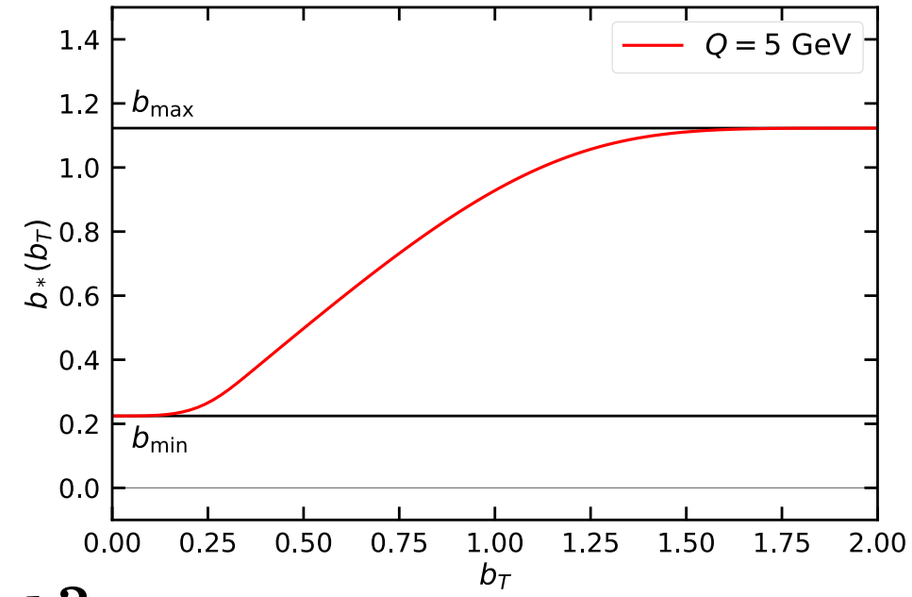


Pavia2019

Main settings

- b_* prescription:

$$b_*(b_T) = b_{\max}\left(\frac{1-e^{-b_T^4/b_{\max}^4}}{1-e^{-b_T^4/b_{\min}^4}}\right)^{1/4} \quad \text{with} \quad \begin{cases} b_{\max} = 2e^{-\gamma_E} \\ b_{\min} = b_{\max}/Q \end{cases}$$



- Non-perturbative function $f_{\rm NP}$:
  - evolution:

$$g_K(b_T) = -\left(g_2 + g_{2B}b_T^2\right)\frac{b_T^2}{2}$$

  - PDFs:

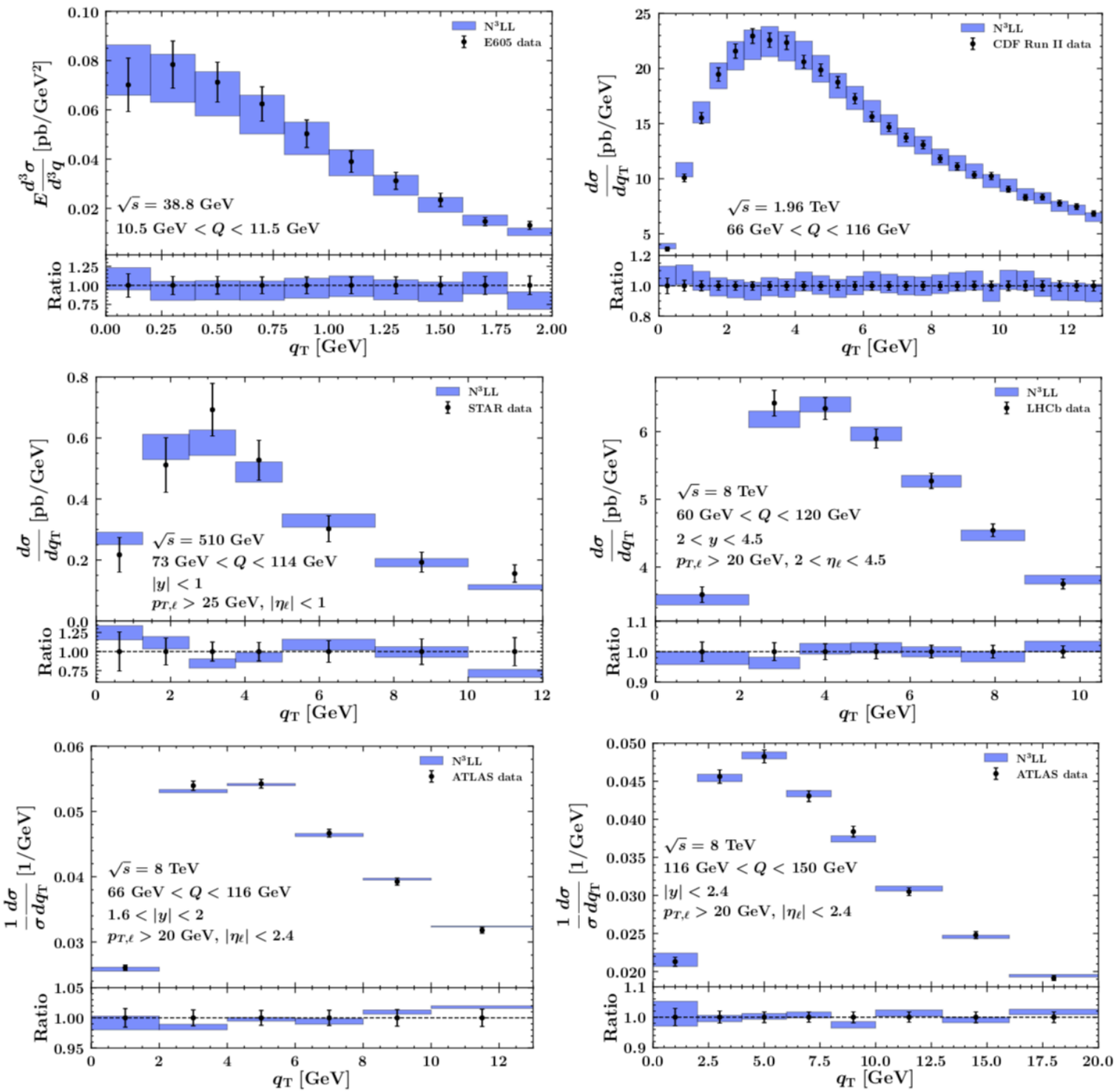
$$\widetilde{f}_{\rm NP}(x,b_T) = \left[\frac{1-\lambda}{1+g_1(x)\frac{b_T^2}{4}} + \lambda\exp\left(-g_{1B}(x)\frac{b_T^2}{4}\right)\right]$$

$$g_1(x) = \frac{N_1}{x\sigma}\exp\left[-\frac{1}{2\sigma^2}\ln^2\left(\frac{x}{\alpha}\right)\right] \qquad g_{1B}(x) = \frac{N_{1B}}{x\sigma_B}\exp\left[-\frac{1}{2\sigma_B^2}\ln^2\left(\frac{x}{\alpha_B}\right)\right]$$

- **9 free parameters** to fit to data.
- Perturbative accuracies: **NLL', NNLL, NNLL', N³LL**
- **Monte Carlo** method for the experimental error propagation.

Pavia2019

Fit quality

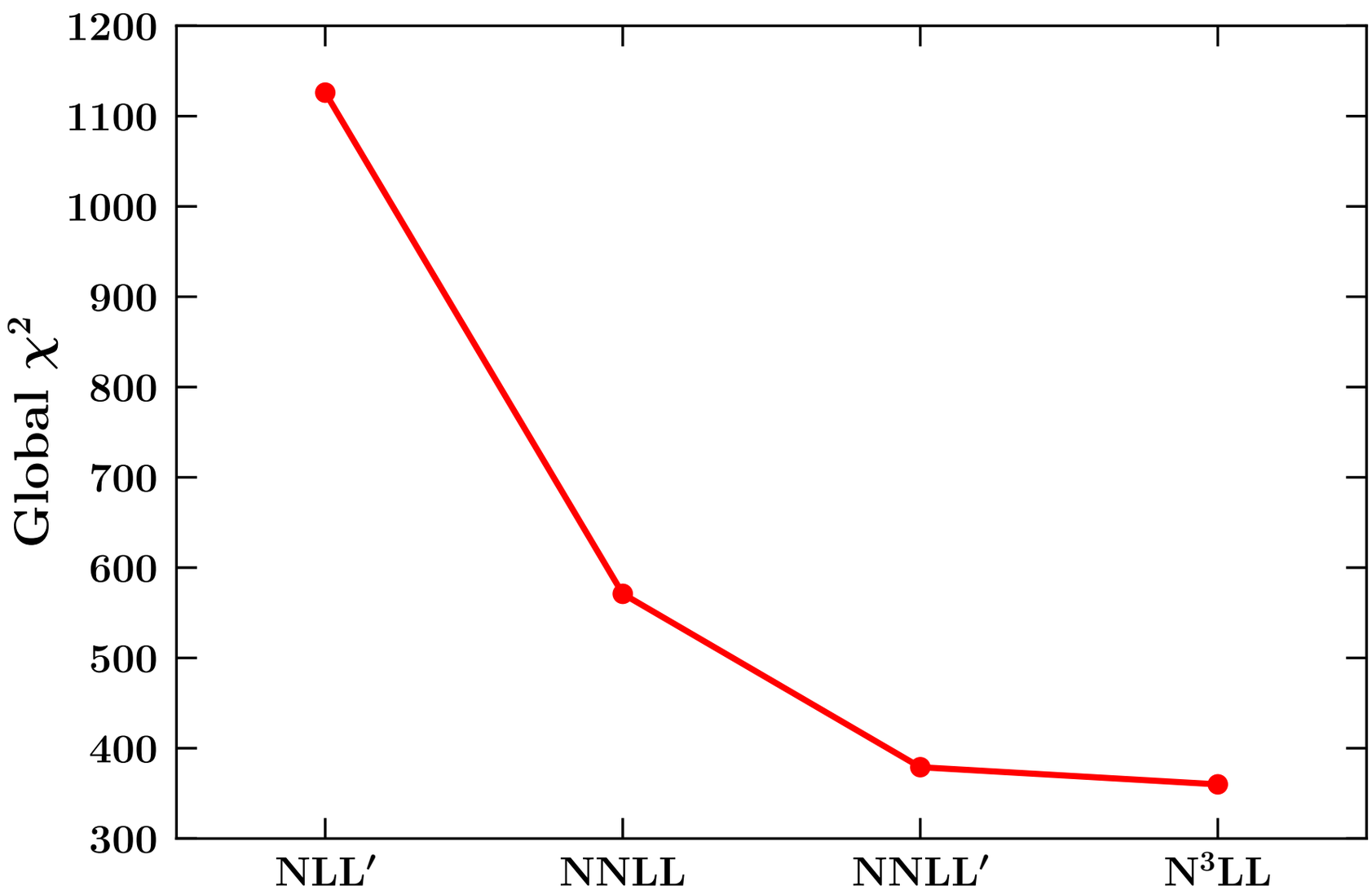


Experiment		$\chi^2_D/N_{\rm dat}$	$\chi^2_\lambda/N_{\rm dat}$	$\chi^2/N_{\rm dat}$
E605	7 GeV < Q < 8 GeV	0.419	0.068	0.487
	8 GeV < Q < 9 GeV	0.995	0.034	1.029
	10.5 GeV < Q < 11.5 GeV	0.191	0.137	0.328
	11.5 GeV < Q < 13.5 GeV	0.491	0.284	0.775
	13.5 GeV < Q < 18 GeV	0.491	0.385	0.877
E288 200 GeV	4 GeV < Q < 5 GeV	0.213	0.649	0.862
	5 GeV < Q < 6 GeV	0.673	0.292	0.965
	6 GeV < Q < 7 GeV	0.133	0.141	0.275
	7 GeV < Q < 8 GeV	0.254	0.014	0.268
	8 GeV < Q < 9 GeV	0.652	0.024	0.676
E288 300 GeV	4 GeV < Q < 5 GeV	0.231	0.555	0.785
	5 GeV < Q < 6 GeV	0.502	0.204	0.706
	6 GeV < Q < 7 GeV	0.315	0.063	0.378
	7 GeV < Q < 8 GeV	0.056	0.030	0.086
	8 GeV < Q < 9 GeV	0.530	0.017	0.547
	11 GeV < Q < 12 GeV	1.047	0.167	1.215
E288 400 GeV	5 GeV < Q < 6 GeV	0.312	0.065	0.377
	6 GeV < Q < 7 GeV	0.100	0.005	0.105
	7 GeV < Q < 8 GeV	0.018	0.011	0.029
	8 GeV < Q < 9 GeV	0.437	0.039	0.477
	11 GeV < Q < 12 GeV	0.637	0.036	0.673
	12 GeV < Q < 13 GeV	0.788	0.028	0.816
	13 GeV < Q < 14 GeV	1.064	0.044	1.107
STAR		0.782	0.054	0.836
CDF Run I		0.480	0.058	0.538
CDF Run II		0.959	0.001	0.959
D0 Run I		0.711	0.043	0.753
D0 Run II		1.325	0.612	1.937
D0 Run II (μ)		3.196	0.023	3.218
LHCb 7 TeV		1.069	0.194	1.263
LHCb 8 TeV		0.460	0.075	0.535
LHCb 13 TeV		0.735	0.020	0.755
CMS 7 TeV		2.131	0.000	2.131
CMS 8 TeV		1.405	0.007	1.412
ATLAS 7 TeV	0 < |y| < 1	2.581	0.028	2.609
	1 < |y| < 2	4.333	1.032	5.365
	2 < |y| < 2.4	3.561	0.378	3.939
ATLAS 8 TeV on-peak	0 < |y| < 0.4	1.924	0.337	2.262
	0.4 < |y| < 0.8	2.342	0.247	2.590
	0.8 < |y| < 1.2	0.917	0.061	0.978
	1.2 < |y| < 1.6	0.912	0.095	1.006
	1.6 < |y| < 2	0.721	0.092	0.814
	2 < |y| < 2.4	0.932	0.348	1.280
ATLAS 8 TeV off-peak	46 GeV < Q < 66 GeV	2.138	0.745	2.883
	116 GeV < Q < 150 GeV	0.501	0.003	0.504
Global		**0.88**	**0.14**	**1.02**

Pavia2019

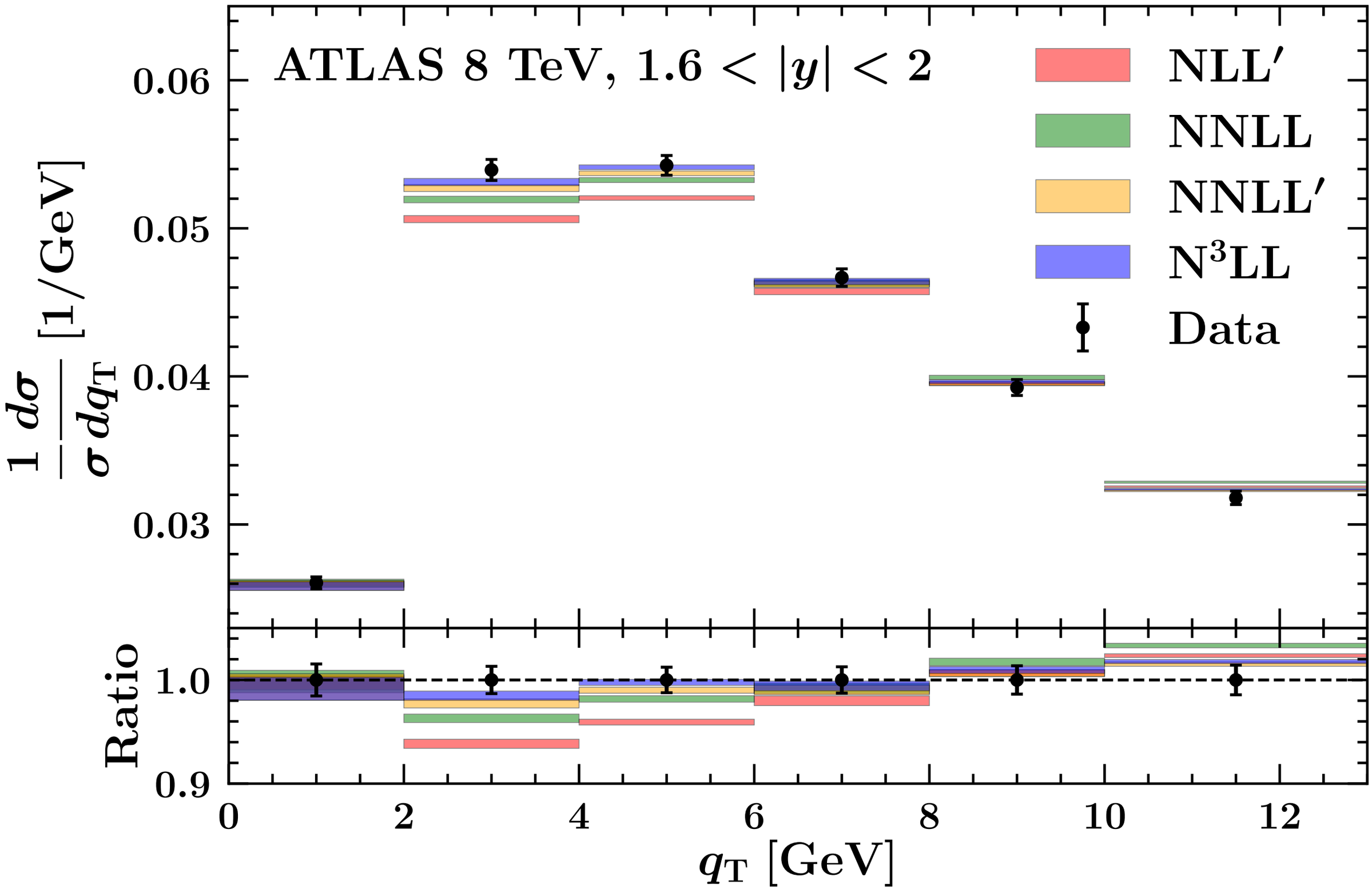
Perturbative convergence

	NLL′	NNLL	NNLL′	N^{3}LL
Global χ^2	1126	571	379	360



Pavia2019

Perturbative convergence



SV2019

Dataset

- Both DY and SIDIS data:
  - fixed-target low-energy DY,
  - PHENIX data,
  - LHC and Tevatron data,
  - HERMES and COMPASS,
  - 457 + 582 = 1039 data points.

SIDIS $\langle Q \rangle \geq 2\text{GeV}$ $\quad \delta \equiv \dfrac{\langle q_T \rangle}{\langle Q \rangle} < 0.25$

Experiment	Reaction	ref.	Kinematics	$N_{\rm pt}$ after cuts
HERMES	$p \to \pi^+$	[58]	0.023<x<0.6 (6 bins) 0.2<z<0.8 (6 bins) 1.0<Q<$\sqrt{20}$GeV $W^2 > 10\text{GeV}^2$ 0.1<y<0.85	24
	$p \to \pi^-$			24
	$p \to K^+$			24
	$p \to K^-$			24
	$D \to \pi^+$			24
	$D \to \pi^-$			24
	$D \to K^+$			24
	$D \to K^-$			24
COMPASS	$d \to h^+$	[59]	0.003<x<0.4 (8 bins) 0.2<z<0.8 (4 bins) 1.0<Q$\simeq$ 9GeV (5 bins)	195
	$d \to h^-$			195
Total				582

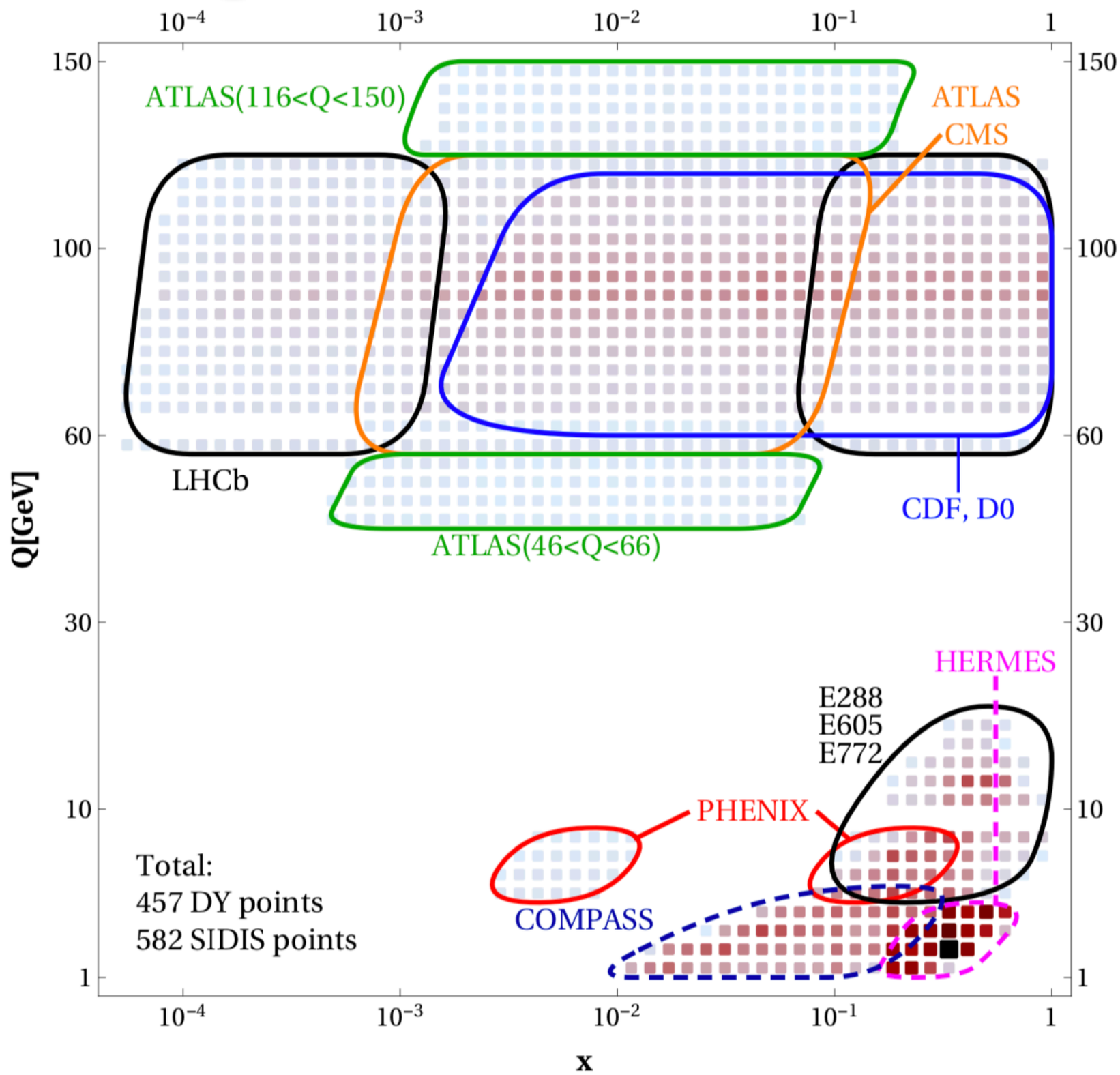
DY $\delta \equiv \dfrac{\langle q_T \rangle}{\langle Q \rangle} < 0.1 \qquad \delta < 0.25 \quad \text{if} \quad \delta^2 < \sigma$

Experiment	ref.	$\sqrt{s}$ [GeV]	Q [GeV]	y/x_F	fiducial region	$N_{\rm pt}$ after cuts
E288 (200)	[64]	19.4	4 - 9 in 1 GeV bins*	$0.1 < x_F < 0.7$	-	43
E288 (300)	[64]	23.8	4 - 12 in 1 GeV bins*	$-0.09 < x_F < 0.51$	-	53
E288 (400)	[64]	27.4	5 - 14 in 1 GeV bins*	$-0.27 < x_F < 0.33$	-	76
E605	[65]	38.8	7 - 18 in 5 bins*	$-0.1 < x_F < 0.2$	-	53
E772	[66]	38.8	5 - 15 in 8 bins*	$0.1 < x_F < 0.3$	-	35
PHENIX	[67]	200	4.8 - 8.2	$1.2 < y < 2.2$	-	3
CDF (run1)	[68]	1800	66 - 116	-	-	33
CDF (run2)	[69]	1960	66 - 116	-	-	39
D0 (run1)	[70]	1800	75 - 105	-	-	16
D0 (run2)	[71]	1960	70 - 110	-	-	8
D0 (run2)$_\mu$	[72]	1960	65 - 115	$\lvert y\rvert < 1.7$	$p_T > 15$ GeV $\lvert\eta\rvert < 1.7$	3
ATLAS (7TeV)	[45]	7000	66 - 116	$\lvert y\rvert < 1$ $1 < \lvert y\rvert < 2$ $2 < \lvert y\rvert < 2.4$	$p_T > 20$ GeV $\lvert\eta\rvert < 2.4$	15
ATLAS (8TeV)	[46]	8000	66 - 116	$\lvert y\rvert < 2.4$ in 6 bins	$p_T > 20$ GeV $\lvert\eta\rvert < 2.4$	30
ATLAS (8TeV)	[46]	8000	46 - 66	$\lvert y\rvert < 2.4$	$p_T > 20$ GeV $\lvert\eta\rvert < 2.4$	3
ATLAS (8TeV)	[46]	8000	116 - 150	$\lvert y\rvert < 2.4$	$p_T > 20$ GeV $\lvert\eta\rvert < 2.4$	7
CMS (7TeV)	[47]	7000	60 - 120	$\lvert y\rvert < 2.1$	$p_T > 20$ GeV $\lvert\eta\rvert < 2.1$	8
CMS (8TeV)	[48]	8000	60 - 120	$\lvert y\rvert < 2.1$	$p_T > 20$ GeV $\lvert\eta\rvert < 2.1$	8
LHCb (7TeV)	[73]	7000	60 - 120	$2 < y < 4.5$	$p_T > 20$ GeV $2 < \eta < 4.5$	8
LHCb (8TeV)	[74]	8000	60 - 120	$2 < y < 4.5$	$p_T > 20$ GeV $2 < \eta < 4.5$	7
LHCb (13TeV)	[75]	13000	60 - 120	$2 < y < 4.5$	$p_T > 20$ GeV $2 < \eta < 4.5$	9
Total						457

*Bins with $9 \lesssim Q \lesssim 11$ are omitted due to the Υ resonance.

SV2019

Kinematic coverage



SV2019

Main settings

- b_* prescription: $$b_*(b_T) = \sqrt{\frac{b_T^2 B_{\rm NP}^2}{b_T^2 + B_{\rm NP}^2}}$$
- Non-perturbative function $f_{\rm NP}$:
  - evolution:
    $$g_K(b_T) = -c_0 b_T b_*(b_T) \quad \rightarrow \quad \begin{cases} -c_0 b_T^2 & \text{for } b_T \to 0 \\ -c_0 B_{\rm NP} b_T & \text{for } b_T \to \infty \end{cases}$$
  - PDFs and FFs:

$$f_{NP}(x,b) = \exp\left(-\frac{\lambda_1(1-x)+\lambda_2 x + x(1-x)\lambda_5}{\sqrt{1+\lambda_3 x^{\lambda_4}\boldsymbol{b}^2}}\boldsymbol{b}^2\right)$$

$$D_{NP}(x,b) = \exp\left(-\frac{\eta_1 z + \eta_2(1-z)}{\sqrt{1+\eta_3(\boldsymbol{b}/z)^2}}\frac{\boldsymbol{b}^2}{z^2}\right)\left(1+\eta_4\frac{\boldsymbol{b}^2}{z^2}\right)$$

- **11 free parameters** to fit to data.
- Perturbative accuracies: **NNLL'(NNLO), N³LL (N³L0)**
- **Monte Carlo** method for the experimental error propagation.

SV2019

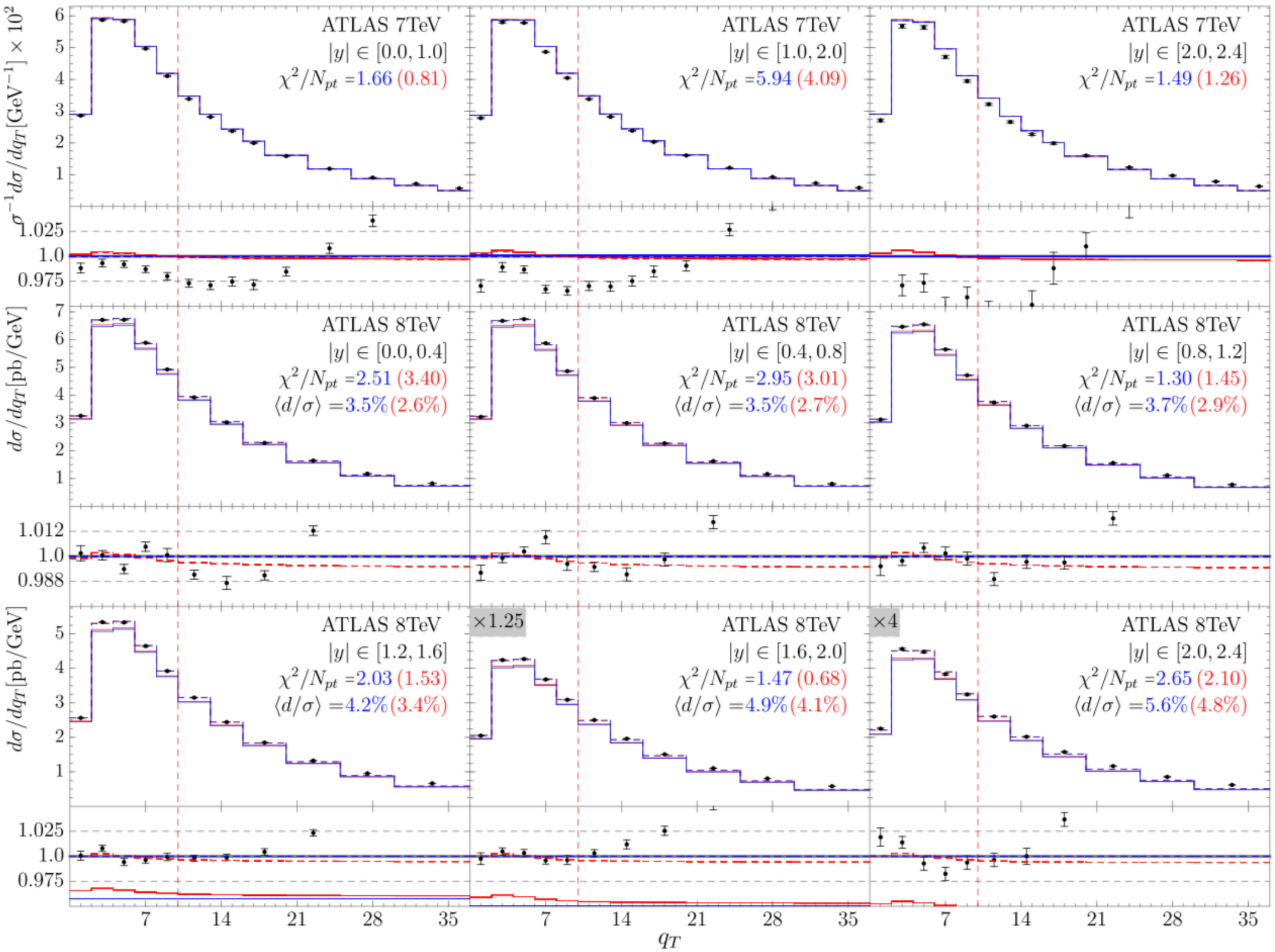
Fit quality

- Remarkably good total χ^2,
- DY and SIDIS data are separately well described,
- **Important achievement**:
  - simultaneous description of SIDIS and DY data within the same fit at high perturbative order.

		NNLO		N^{3}LO	
Data set	N_{pt}	χ^2/N_{pt}	$\langle d/\sigma \rangle$	χ^2/N_{pt}	$\langle d/\sigma \rangle$
CDF run1	33	0.66	8.4%	0.67	7.8%
CDF run2	39	1.28	2.8%	1.41	2.1%
D0 run1	16	0.72	0.1%	0.78	-0.5%
D0 run2	8	1.38	-	1.64	-
D0 run2 (μ)	3	0.62	-	0.69	-
Tevatron total	99	0.97		1.06	
ATLAS 7TeV 0.0<|y|<1.0	5	1.66	-	0.81	-
ATLAS 7TeV 1.0<|y|<2.0	5	5.94	-	4.09	-
ATLAS 7TeV 2.0<|y|<2.4	5	1.49	-	1.26	-
ATLAS 8TeV 0.0<|y|<0.4	5	2.51	3.5%	3.40	2.8%
ATLAS 8TeV 0.4<|y|<0.8	5	2.95	3.5%	3.03	2.7%
ATLAS 8TeV 0.8<|y|<1.2	5	1.30	3.7%	1.45	2.9%
ATLAS 8TeV 1.2<|y|<1.6	5	2.03	4.2%	1.53	3.4%
ATLAS 8TeV 1.6<|y|<2.0	5	1.47	4.9%	0.70	4.1%
ATLAS 8TeV 2.0<|y|<2.4	5	2.64	5.6%	2.10	4.8%
ATLAS 8TeV 46<Q<66GeV	3	0.31	1.1%	0.31	0.2%
ATLAS 8TeV 116<Q<150GeV	7	0.84	1.9%	0.97	1.2%
ATLAS total	55	2.12		1.82	
CMS 7TeV	8	1.25	-	1.24	-
CMS 8TeV	8	0.77	-	0.76	-
CMS total	16	1.01		1.00	
LHCb 7TeV	8	2.68	5.8%	2.37	5.2%
LHCb 8TeV	7	4.81	5.8%	4.16	5.1%
LHCb 13TeV	9	0.91	6.4%	0.81	5.7%
LHCb total	24	2.63		2.31	
High energy DY total	194	1.51		1.42	
PHE200	3	0.28	0.2%	0.29	-0.3%
E228-200	43	1.00	35.7%	1.12	35.0%
E228-300	53	0.90	29.2%	1.01	28.3%
E228-400	76	0.86	20.6%	0.96	19.5%
E772	35	1.84	9.5%	1.91	8.5%
E605	53	0.57	21.3%	0.60	20.1%
Low energy DY total	263	0.96		1.04	
HERMES ($p \to \pi^+$)	24	2.20	1.7%	3.06	2.2%
HERMES ($p \to \pi^-$)	24	1.12	0.6%	1.45	0.9%
HERMES ($p \to K^+$)	24	0.71	-0.1%	0.66	0.0%
HERMES ($p \to K^-$)	24	0.69	0.0%	0.66	0.0%
HERMES ($d \to \pi^+$)	24	0.57	0.3%	0.78	0.8%
HERMES ($d \to \pi^-$)	24	0.74	0.5%	0.96	0.7%
HERMES ($d \to K^+$)	24	0.52	-0.1%	0.53	0.0%
HERMES ($d \to K^-$)	24	1.27	0.0%	1.17	0.1%
HERMES total	192	0.98		1.16	
COMPASS ($d \to h^+$)	195	0.61	3.3%	0.76	5.1%
COMPASS ($d \to h^-$)	195	0.68	-2.3%	0.92	-0.5%
COMPASS total	390	0.65		0.84	
SIDIS total	582	0.76		0.95	
Total	**1039**	**0.95**		**1.06**	

SV2019

Fit quality



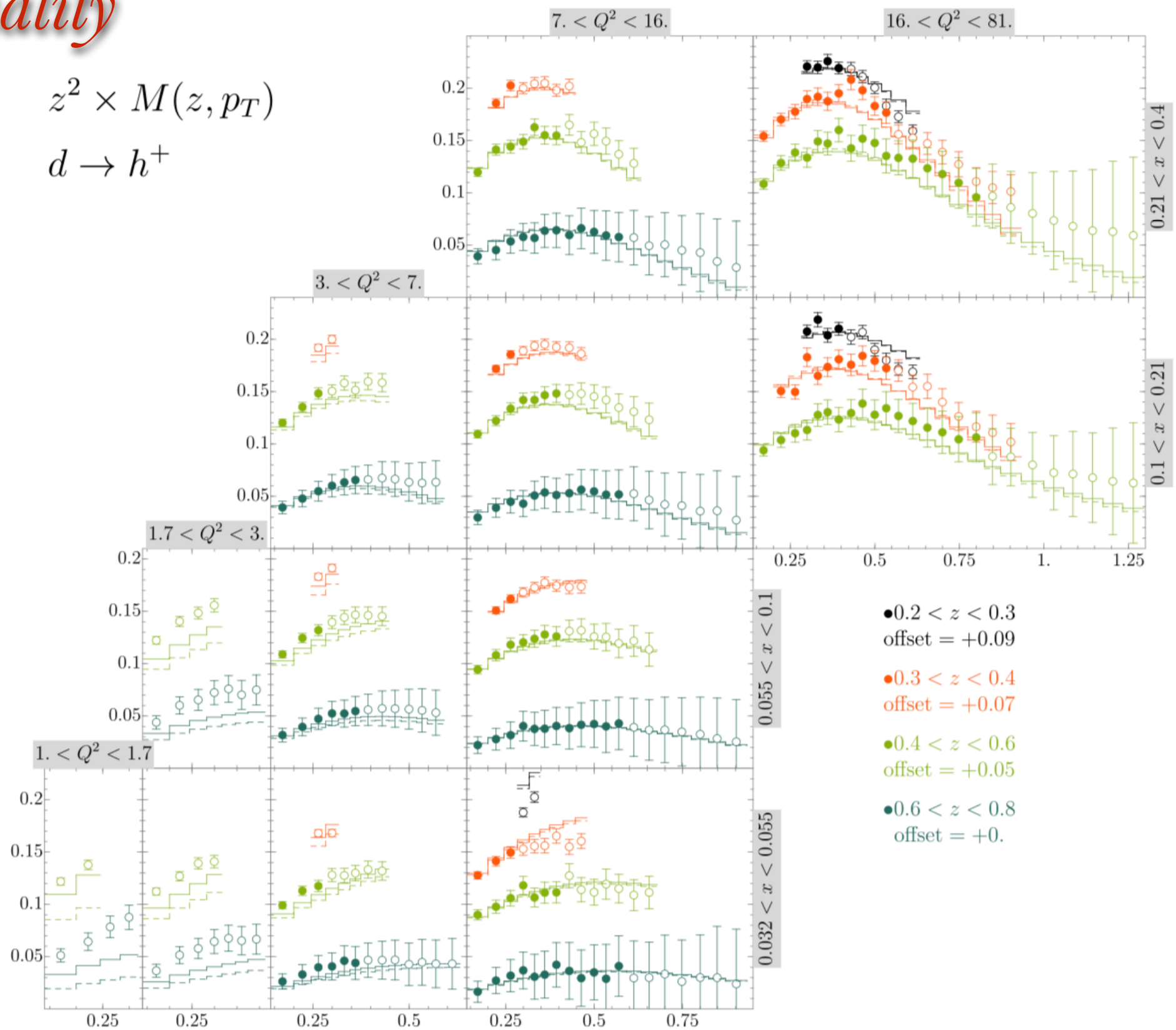
SV2019

Fit quality

$z^2 \times M(z, p_T)$

$d \to h^+$

COMPASS



TMDs at the LHC

The W mass

- A precise determination of the ***W* mass** plays an important role in testing the Standard Model and thus for **BSM** physics.

- This is a central task of the **LHC physics**.

- In order to minimise experimental systematic effects, the most promising procedure relies on the measurement of the W/Z ratio cross section:

  - the W mass is basically determined through template fits of:

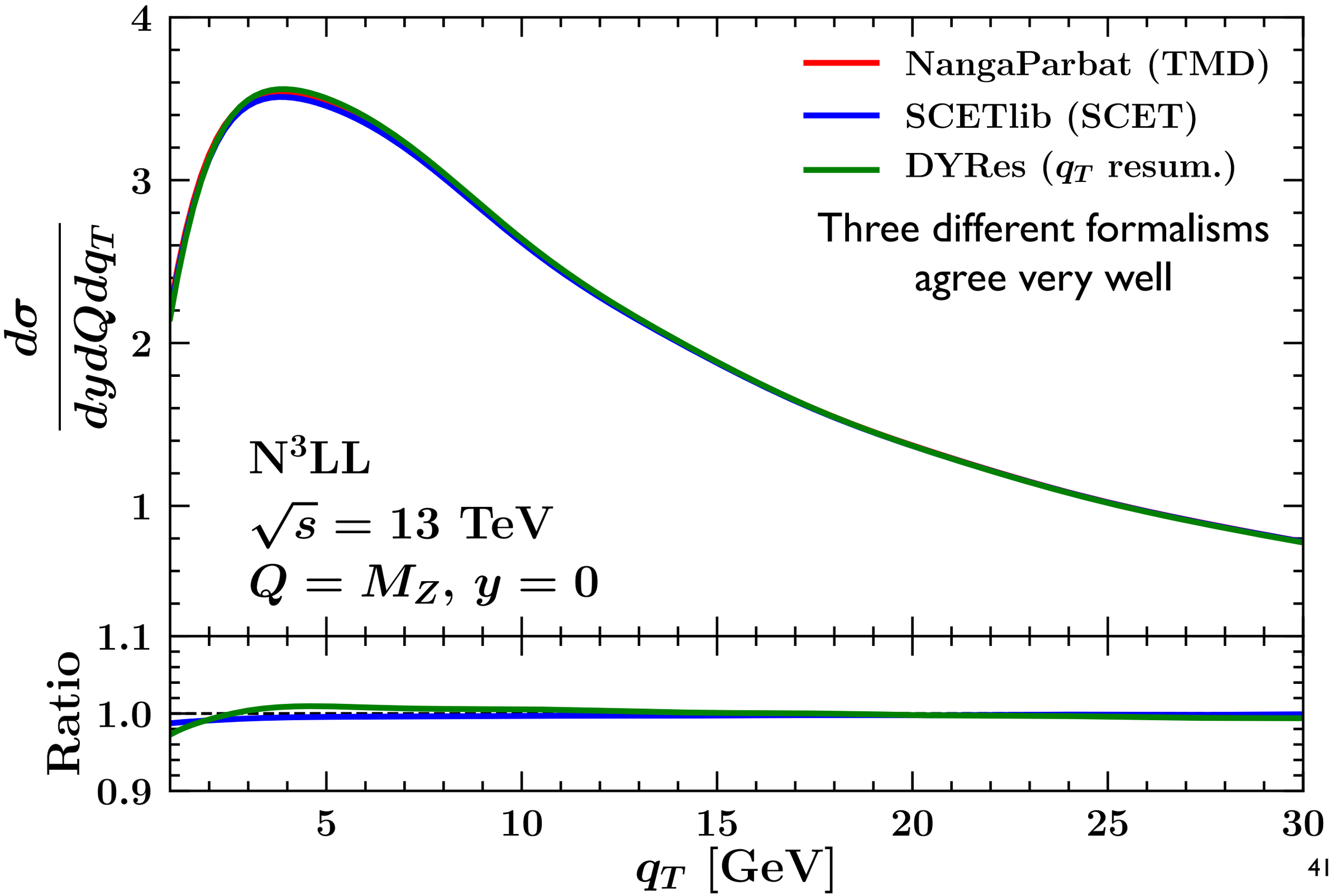
$$\frac{d\sigma^W}{dq_T} = \left(\frac{d\sigma^W/dq_T}{d\sigma^Z/dq_T}\right)_{\mathbf{exp.}} \left(\frac{d\sigma^Z}{dq_T}\right)_{\mathbf{th.}}$$

- Therefore, an accurate and reliable prediction of the ***Z* spectrum** is essential.

- **TMD-based predictions** are currently playing an important role within the LHC electroweak working group along **other formalisms**.

TMDs at the LHC

The W mass



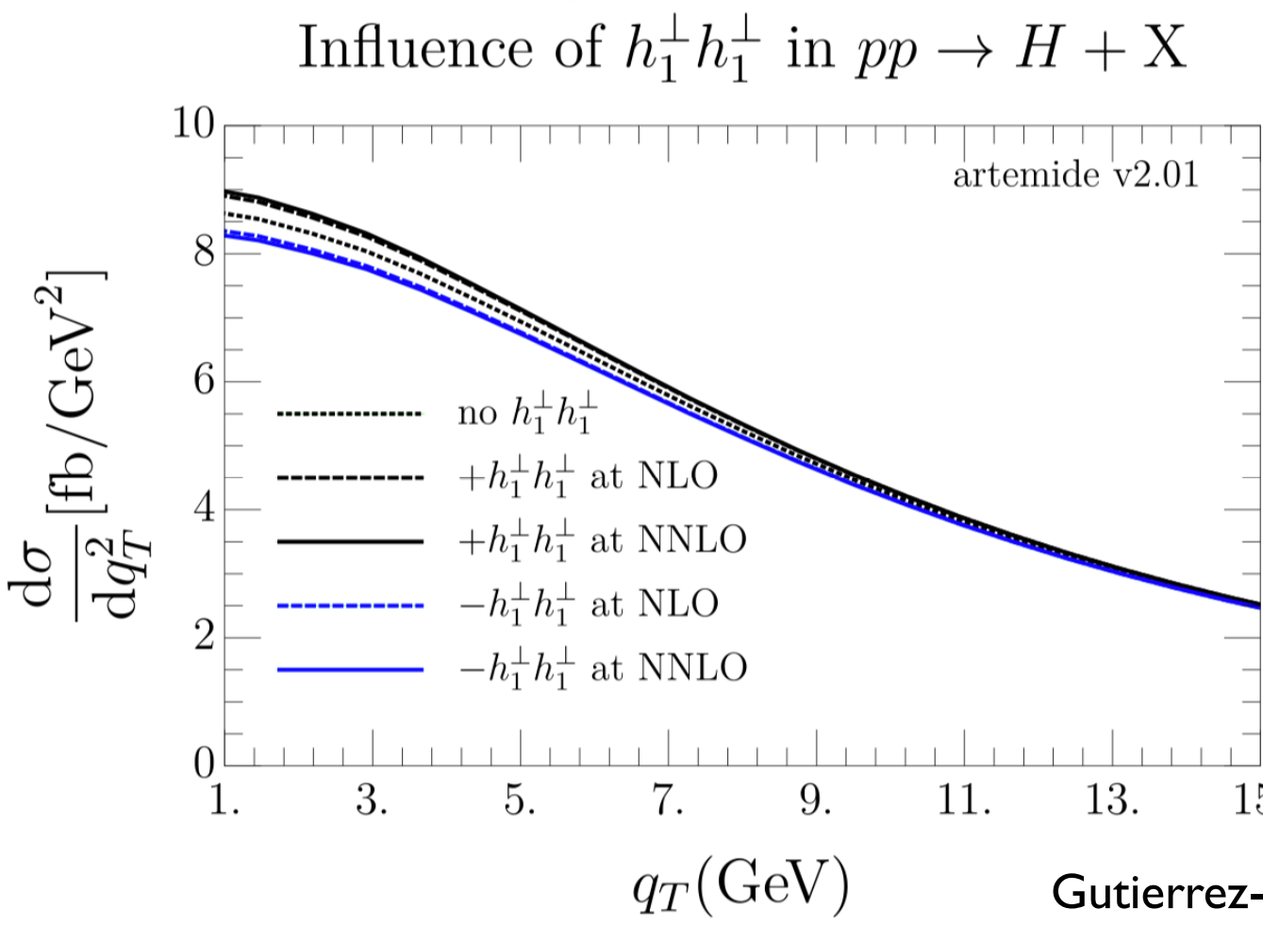
TMDs at the LHC

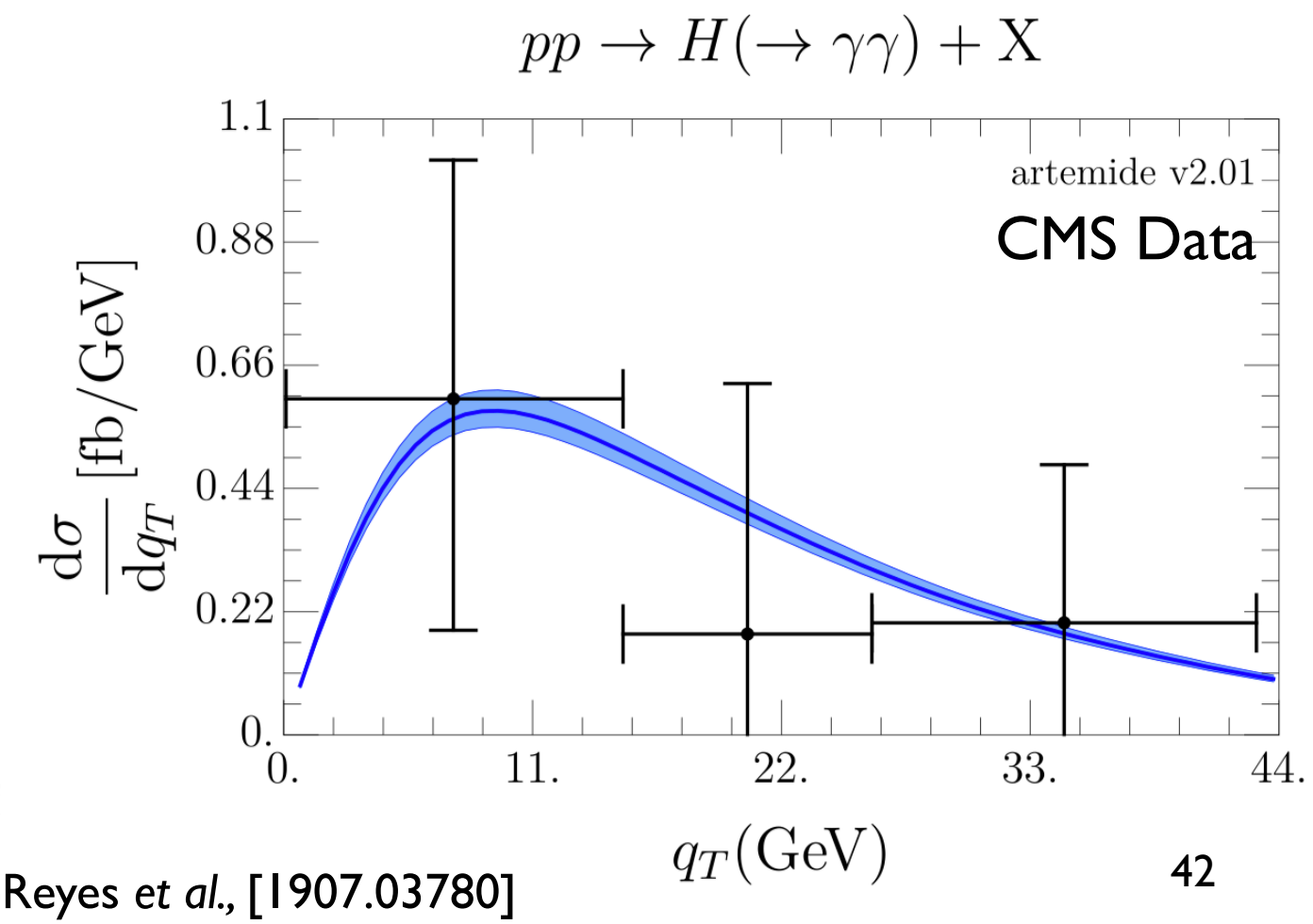
Linearly polarised gluon and the Higgs boson

- The *linearly polarised* gluon TMD PDF $h_{\rm T}$ contributes to the *unpolarised* low-$q_{\rm T}$ spectrum of Higgs production in gluon fusion:

$$\frac{d\sigma}{dy d^2\boldsymbol{q}_T} = \frac{\sigma_{gg\to H}}{(2\pi)^2}\int d^2\boldsymbol{b}\, e^{-i(\boldsymbol{b}\boldsymbol{q}_T)}\Big(f_{1,g}(x_A,\boldsymbol{b})f_{1,g}(x_B,\boldsymbol{b}) + h^{\perp}_{1,g}(x_A,\boldsymbol{b})h^{\perp}_{1,g}(x_B,\boldsymbol{b})\Big)$$

- Despite the linearly polarised gluon enters beyond NNLL, its effect on the cross section can be sizeable, particularly at low $q_{\rm T}$, but below current data accuracy:



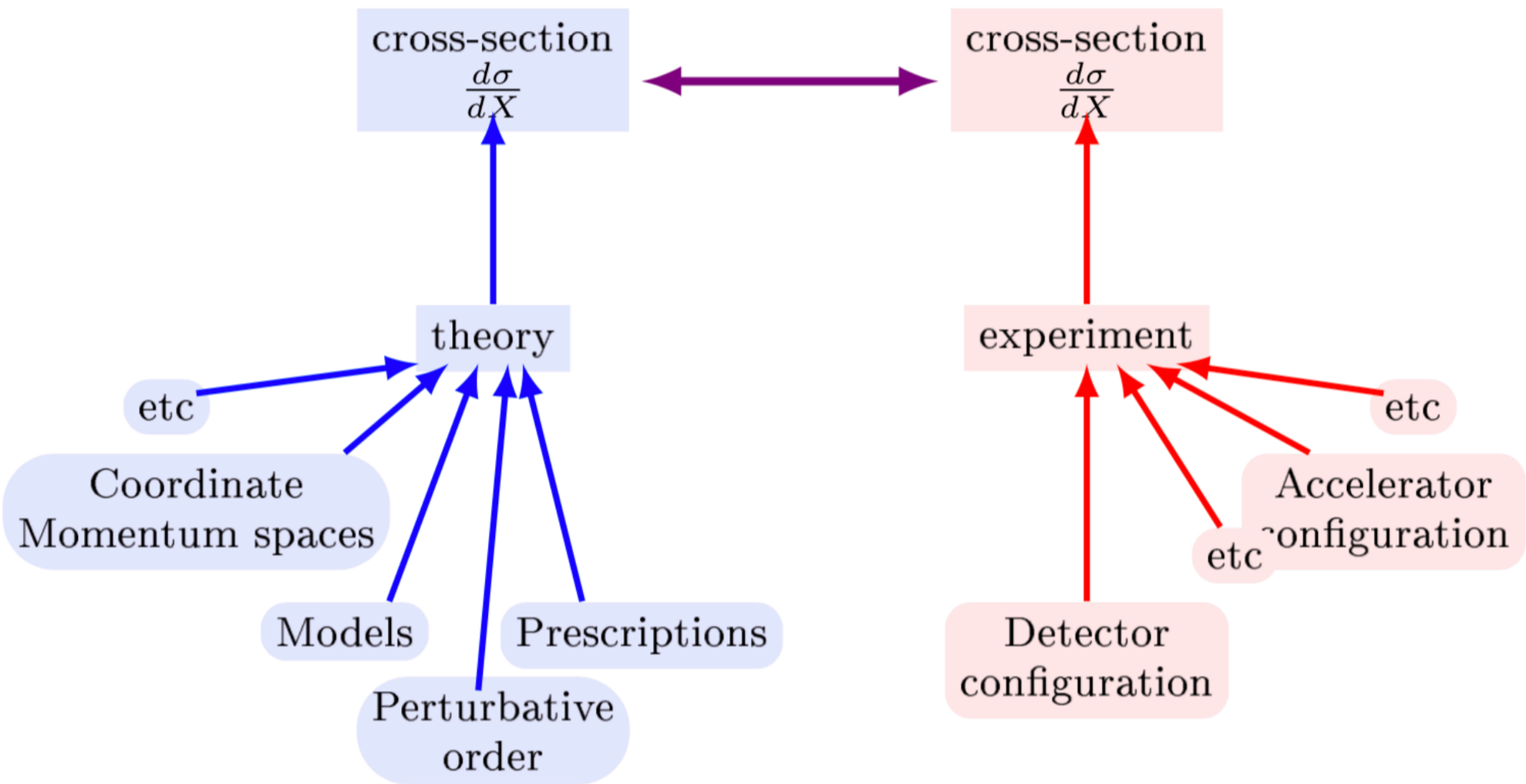


Gutierrez-Reyes *et al.*, [1907.03780]

TMDs at the EIC

- The EIC has now been **approved** and there is a lot of ferment aiming at defining the **basic physics goals** and the consequent **detector requirements**.

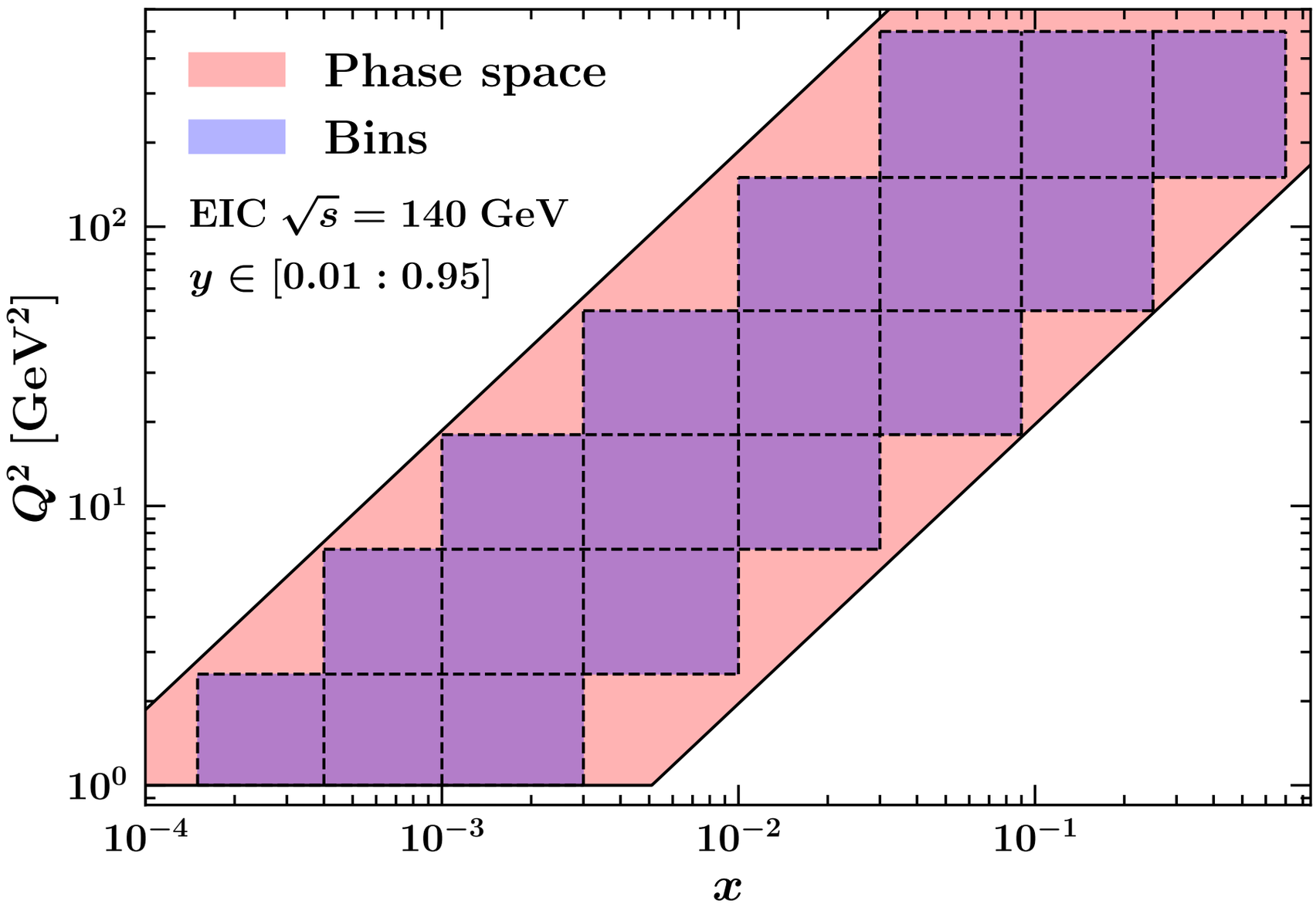
- An accurate knowledge of TMDs (polarised and unpolarised) is of extreme importance to this purpose.



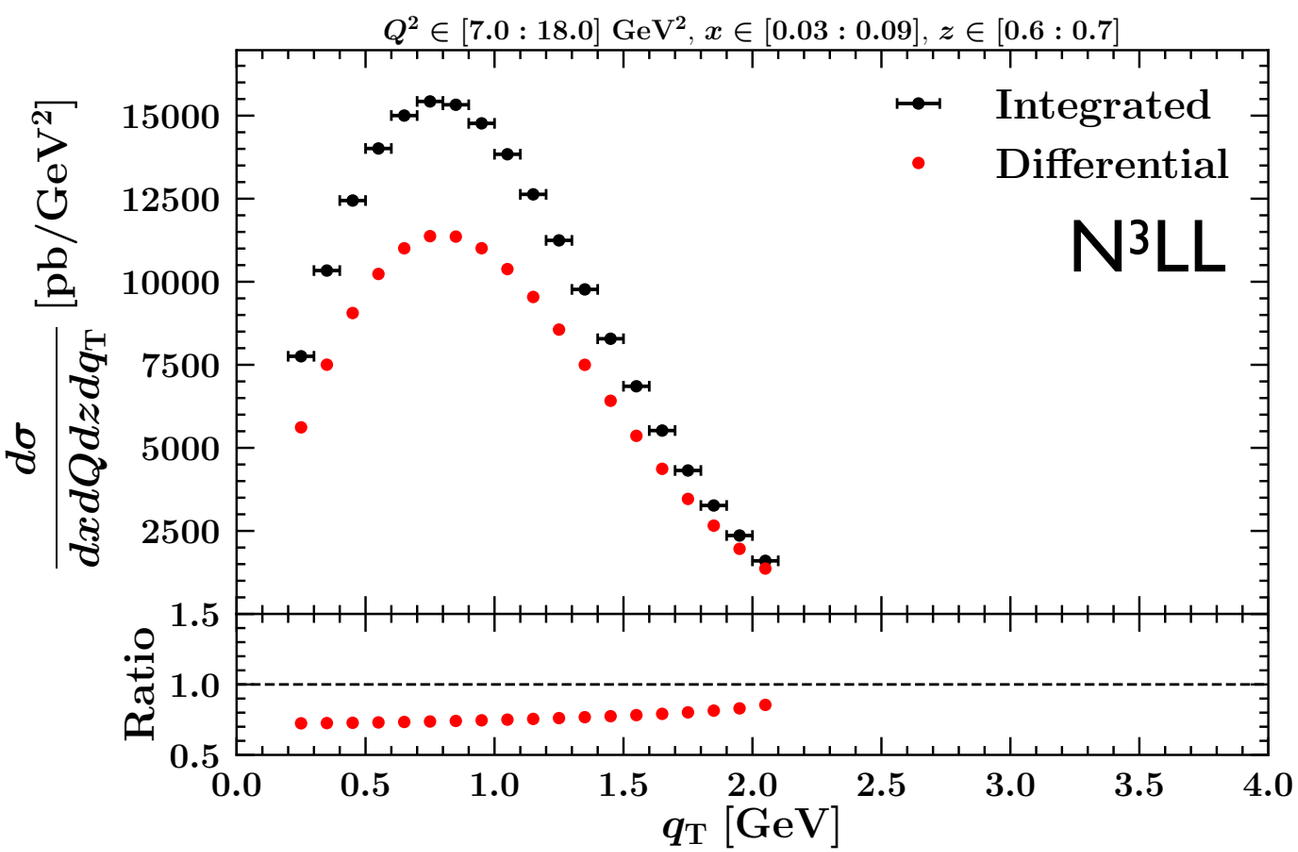
From A. Vladimirov's slides at the last YR SIDIS WG meeting

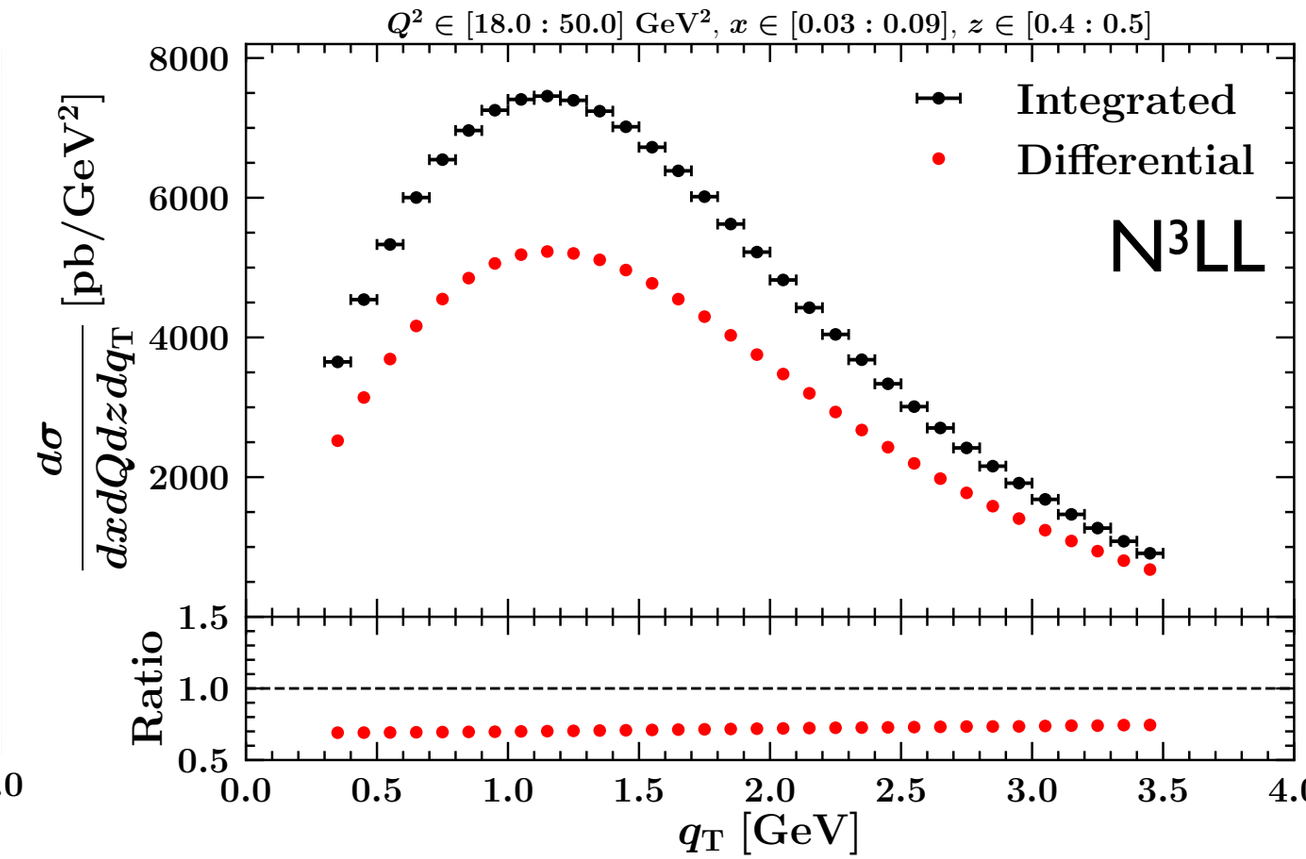
TMDs at the EIC

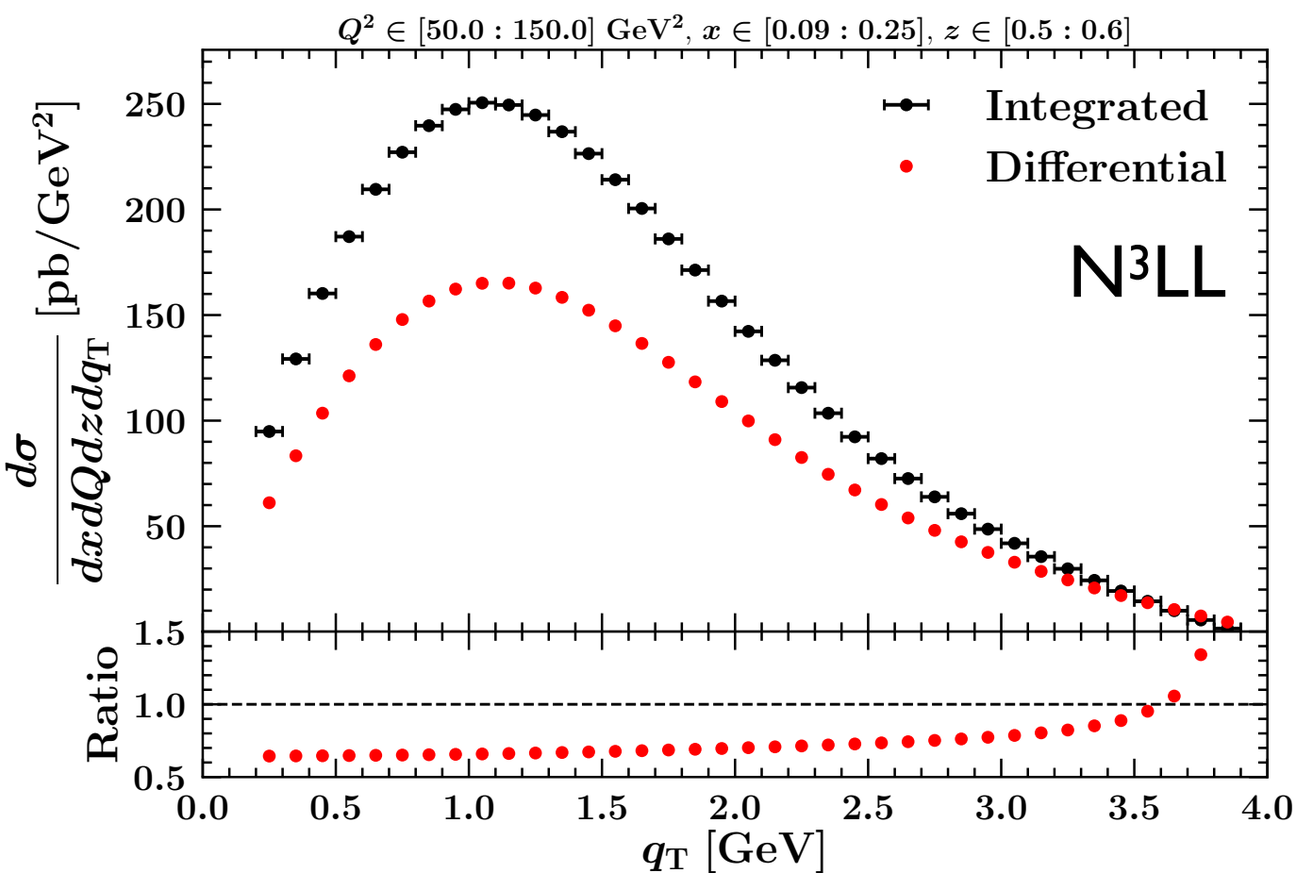
- An important step is the definition of the relevant **observables** to be measure and the respective **binning**.
- TMDs are crucial to take this step.
- Binning in x and Q^2 under discussion:

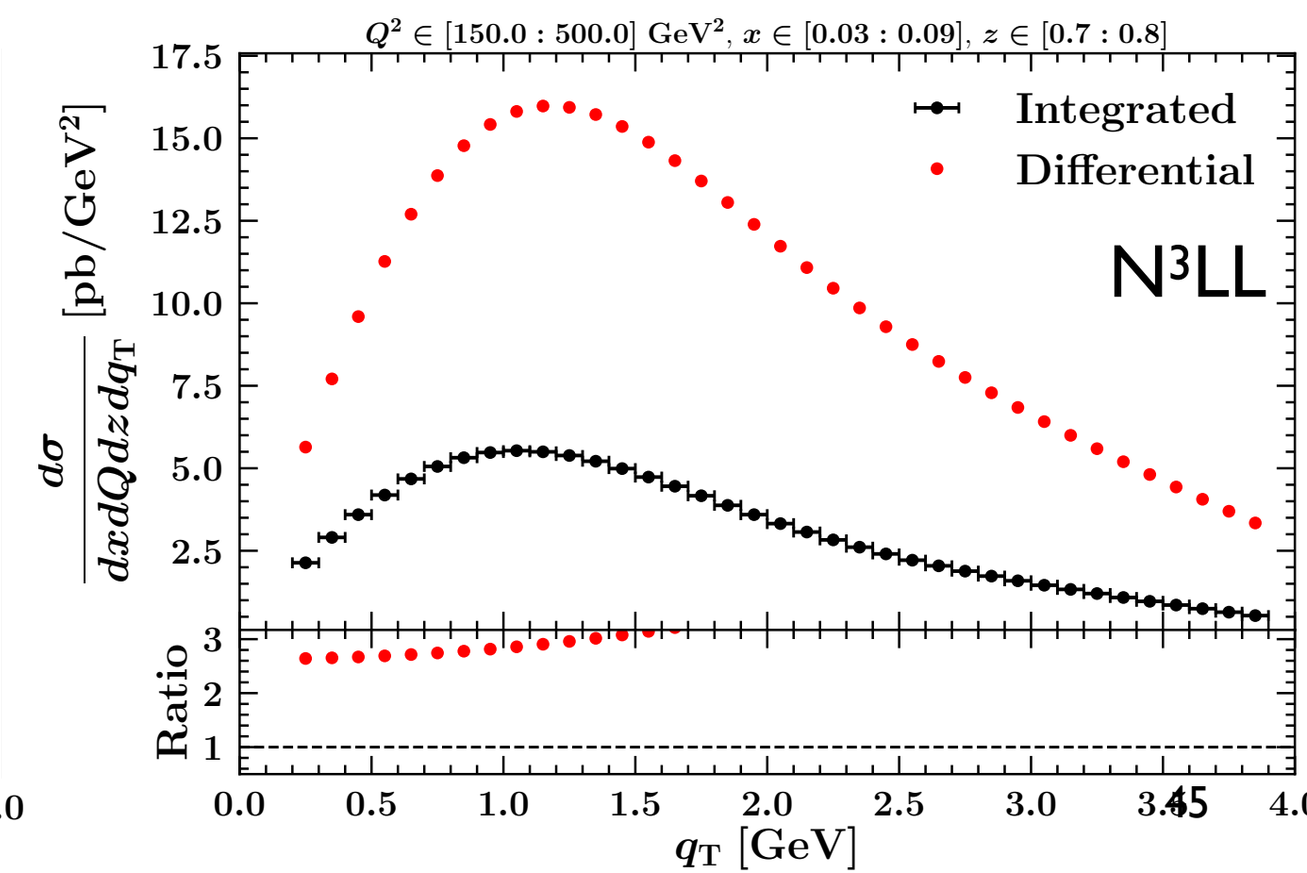


TMDs at the EIC









Thank you

Resummation formalisms

- Different formulations of the q_T spectrum:

$$\left(\frac{d\sigma}{dq_T}\right)_{\text{res.}} \propto \begin{cases} e^{2S}\left[f_1 \otimes \mathcal{H} \otimes f_2\right] & \text{: Resum.} \\ H \times F_1 \times F_2 & \text{: TMD} \\ H \times B_1 \times B_2 \times S & \text{: SCET} \end{cases} + \mathcal{O}\left[\left(\frac{q_T}{Q}\right)^m\right]$$

- Dictionary:

$$\mathcal{H} = HC_1C_2$$

$$F_i = e^S C_i \otimes f_i$$

$$F_i = \sqrt{S} \times B_i$$

- All **equivalent** for *exponentiating* processes.

Logarithmic counting

$$\left(\frac{d\sigma}{dq_T}\right)_{\text{res.}} \overset{\text{TMD}}{=} \sigma_0 H(Q) \int d^2\mathrm{b}_T e^{i\mathrm{b}_T\cdot\mathrm{q}_T} F_1(x_1,\mathrm{b}_T,Q,Q^2) F_2(x_2,\mathrm{b}_T,Q,Q^2)$$

$$\begin{aligned} F_f(x,\mathrm{b}_T,\mu,\zeta) &= \sum_j C_{f/j}(c,b_T;\mu_b,\zeta) \otimes f_j(x,\mu_b) \\ &\times \exp\left\{ K(b_T,\mu_b)\ln\frac{\sqrt{\zeta}}{\mu_b} + \int_{\mu_b}^{\mu}\frac{d\mu'}{\mu'}\left[\gamma_F - \gamma_K \ln\frac{\sqrt{\zeta}}{\mu'}\right]\right\} \end{aligned}$$

Accuracy	γ_K	γ_F	K	$C_{f/j}$	H
LL	α_s	-	-	1	1
NLL	α_s^2	α_s	α_s	1	1
NLL’	α_s^2	α_s	α_s	α_s	α_s
N^{2}LL	α_s^3	α_s^2	α_s^2	α_s	α_s
N^{2}LL’	α_s^3	α_s^2	α_s^2	α_s^2	α_s^2
N^{3}LL	α_s^4	α_s^3	α_s^3	α_s^2	α_s^2
N^{3}LL’	α_s^4	α_s^3	α_s^3	α_s^3	α_s^3

Additive matching and counting

- Accurate predictions for all q_T's by **additive matching**, order by order in perturbation theory, of collinear and TMD calculations:

$$\left(\frac{d\sigma}{dq_T}\right)_{\mathbf{add.match.}} = \left(\frac{d\sigma}{dq_T}\right)_{\mathbf{res.}} + \left(\frac{d\sigma}{dq_T}\right)_{\mathbf{f.o.}} - \left(\frac{d\sigma}{dq_T}\right)_{\mathbf{d.c.}}$$

- In order for the match to actually take place:

$$\left(\frac{d\sigma}{dq_T}\right)_{\mathbf{res.}} \xrightarrow[\mathbf{f.o.}]{} \left(\frac{d\sigma}{dq_T}\right)_{\mathbf{d.c.}} \xleftarrow[q_T \ll Q]{} \left(\frac{d\sigma}{dq_T}\right)_{\mathbf{f.o.}}$$

- Therefore, the "fixed-order" parts have to match in the relevant limits:

Log Accuracy	Minimal f.o. accuracy
NLL'	α_s (LO)
N^{2}LL	α_s (LO)
N^{2}LL'	α_s^2 (NLO)
N^{3}LL	α_s^2 (NLO)
N^{3}LL'	α_s^3 (NNLO)

Experimental uncertainties

$$m_i \pm \sigma_{i,\mathrm{stat}} \pm \sigma_{i,\mathrm{unc}} \pm \sigma^{(1)}_{i,\mathrm{corr}} \pm \cdots \pm \sigma^{(k)}_{i,\mathrm{corr}}$$

statistic

systematic

the central value of
the *i-th* measurement

Slide by C. Bissolotti

Experimental uncertainties

$$m_i \pm \sigma_{i,\mathrm{stat}} \pm \sigma_{i,\mathrm{unc}} \pm \sigma^{(1)}_{i,\mathrm{corr}} \pm \cdots \pm \sigma^{(k)}_{i,\mathrm{corr}}$$

uncorrelated **correlated**

additive **multiplicative**

$$\chi^2 = \sum_{i,j=1}^{n} (m_i - t_i) V^{-1}_{ij} (m_j - t_j)$$

$$\sigma^{(l)}_{i,\mathrm{corr}} \equiv \delta^{(l)}_{i,\mathrm{corr}} m_i$$

covariance matrix

$$V_{ij} = s_i^2 \delta_{ij} + \left(\sum_{l=1}^{k_a} \delta^{(l)}_{i,\mathrm{add}} \delta^{(l)}_{j,\mathrm{add}} + \sum_{l=1}^{k_m} \delta^{(l)}_{i,\mathrm{mult}} \delta^{(l)}_{j,\mathrm{mult}} \right) m_i m_j$$

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χ^2 chisquare

systematic shift

$$d_i = \sum_{\alpha=1}^{k} \lambda_\alpha \sigma_{i,\mathrm{corr}}^{(\alpha)}$$

shift

$$\frac{\partial \chi^2}{\partial \lambda_\alpha} = 0$$

nuisance parameters

$$\bar{t}_i = t_i + d_i$$

shifted prediction

$$\chi^2 = \sum_{i=1}^{n} \left(\frac{m_i - \bar{t}_i}{s_i} \right)^2 + \sum_{\alpha=1}^{k} \lambda_\alpha^2$$

penalty term

recover the form of
the uncorrelated definition

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Pavia 2019

Flavour dependence of TMD PDFs [arXiv:1807.02101]

- A possible **flavour dependence** of the intrinsic p_T should not be neglected:
  - possible effect on the determination of $\boldsymbol{M_W}$ (PDG: M_W=80.379 ± **0.012** GeV).
- Introduce a NP function that depends on the quark flavour:

$$f_{\rm NP}(b_T) \to f_{\rm NP}^{(a)}(b_T) = \exp\left[-\left(g_{\rm evo}\ln\frac{Q^2}{Q_0^2} + g_a\right)b_T^2\right]$$

  - a universal non-perturbative correction to the TMD evolution is also introduced.
- **Template** fits of M_W performed to sets of **pseudo-data** of p_{Tl} and M_T distributions with ATLAS and CDF kinematics and experimental uncertainties:
  - pseudo-data generated by varying $f_{\rm NP}$ in a range such that W-q_T distributions are all **stat. equivalent**.
  - Shifts comparable to the word-average uncertainty.
  - Flavour dependence of the intrinsic p_T may be required for a precise determination of M_W.
  - Data from flavour-sensitive processes such as **SIDIS** from **COMPASS** will shed new light on the flavour decomposition of the unpolarised TMD PDFs.

M_W shifts in MeV

	ΔM_{W^+}		ΔM_{W^-}	
Set	m_T	$p_{T\ell}$	m_T	$p_{T\ell}$
1	0	-1	-2	3
2	0	-6	-2	0
3	-1	9	-2	-4
4	0	0	-2	-4
5	0	4	-1	-3