

Transversely polarized Lambda production within a TMD approach

from e^+e^- to SIDIS and hadronic processes

Marco Zaccheddu

Cake Seminar 04/24/2024

Motivations and Contents

Lambda Transverse Polarization: longstanding problem!

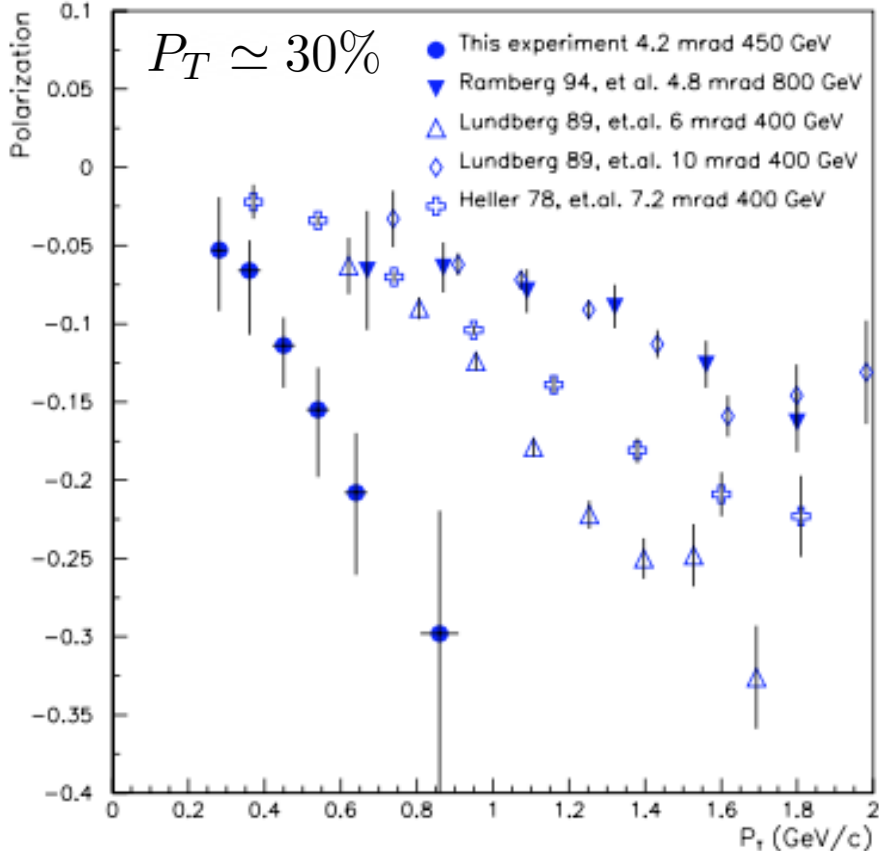
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Discovered in unpolarized pN collisions

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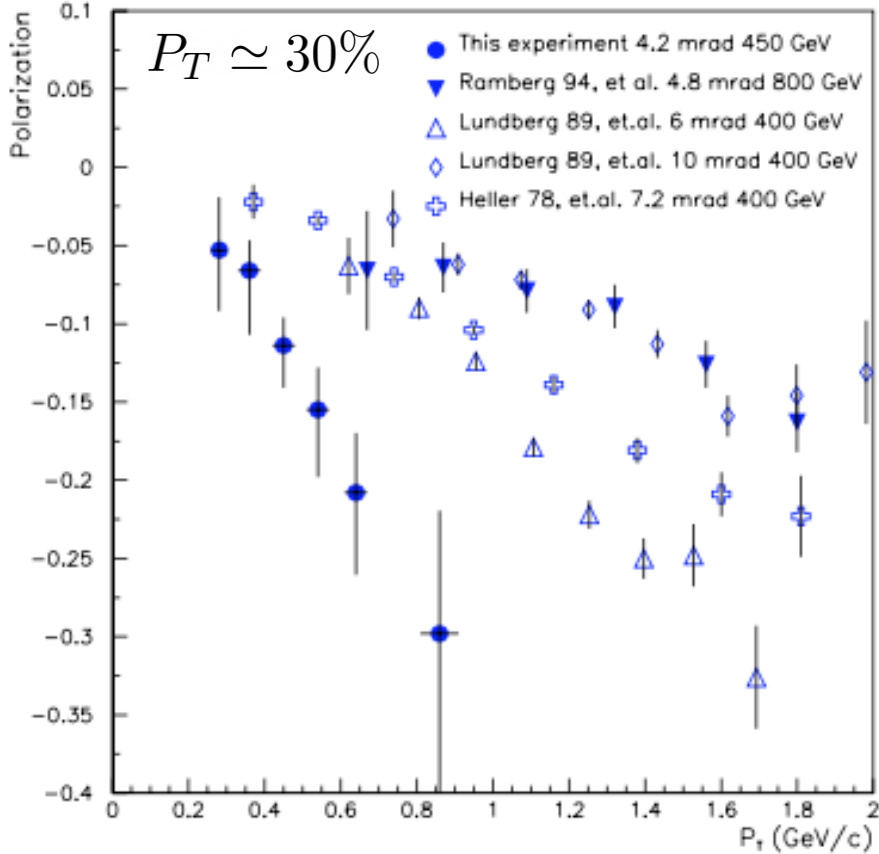


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Lambda Transverse Polarization: longstanding problem!

Collinear pQCD not able to describe the size of the Polarization (at leading-twist)
 $P_T \simeq 1 - 2 \%$

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Lambda Transverse Polarization: longstanding problem!

Collinear pQCD not able to describe the size of the Polarization (at leading-twist)

$$P_T \simeq 1 - 2 \%$$

Extension of the Collinear pQCD with the introduction of transverse momentum dependent fragmentation function (TMD-FFs)

Polarizing TMD-FF introduced

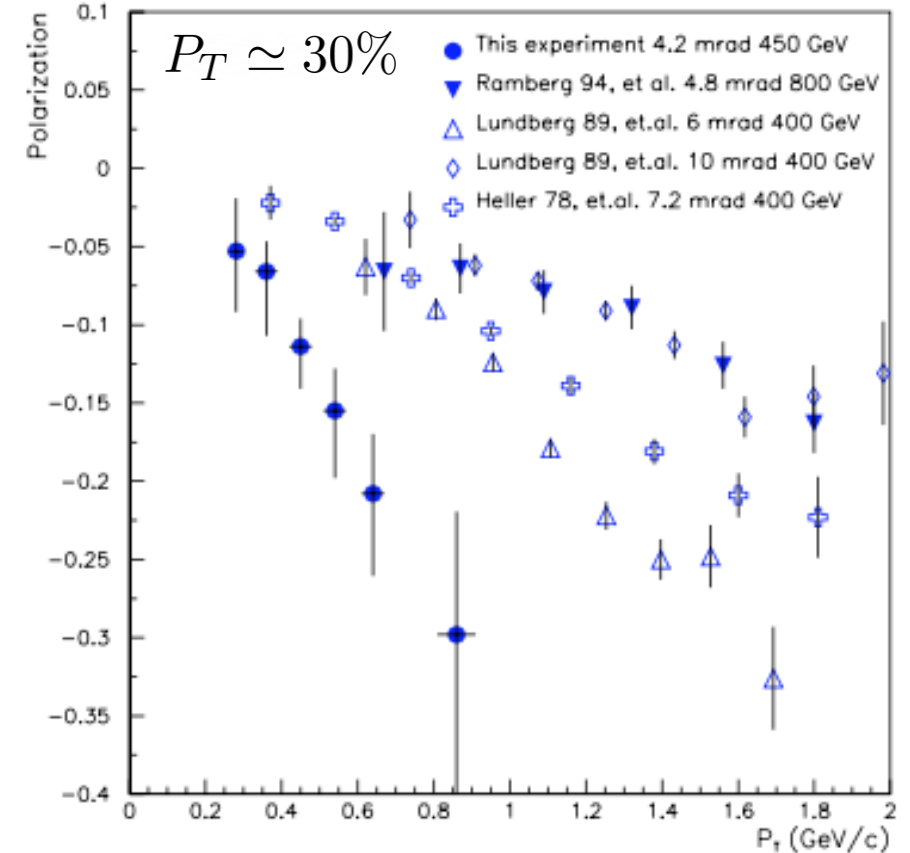
[P.J. Mulders and R.D. Tangerman, Nucl. Phys. B461, 197 (1996)]

Studied phenomenologically in a simplified TMD approach and with collinear twist 3 PDFs.

Discovered in unpolarized pN collisions

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Motivations and Contents

Observation of Transverse $\Lambda/\bar{\Lambda}$ Hyperon Polarization in e^+e^- Annihilation at Belle

- 2 data set @ $\sqrt{s} = 10.58$ GeV

[Y. Guan et al., Phys. Rev. Lett. 122. 042001 (2019)]

Double hadron production:

- $e^+e^- \rightarrow \Lambda\pi/K + X$: 128 points - bins of the energy fractions $z_\Lambda - z_{\pi,K}$

Single-inclusive hadron production:

- $e^+e^- \rightarrow \Lambda(\text{jet}) + X$: 32 points - $\Lambda(\text{jet})$, in bins of $z_\Lambda - p_\perp$

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Phenomenological analysis:

Fixed scale:

- D'Alesio, Murgia, MZ; *Phys.Rev.D* 102 (2020) 5, 054001
- Callos, Kang, Terry; *Phys.Rev.D* 102 (2020) 9, 096007
- Chen, Liang, Pan, Song, Wei; *Phys.Lett.B* 816 (2021) 136217

Motivations and Contents

Reanalysis with TMD evolution:

Transverse Λ polarization in e^+e^- annihilations and in SIDIS processes at the EIC within TMD factorization

- D'Alesio, Gamberg, Murgia, MZ; *JHEP* 12 (2022) 074

Transverse Λ polarization in unpolarized $pp \rightarrow \text{jet } \Lambda \uparrow X$

- D'Alesio, Gamberg, Murgia, MZ; *Phys.Rev.D* 108 (2023) 9, 094004

Contents:

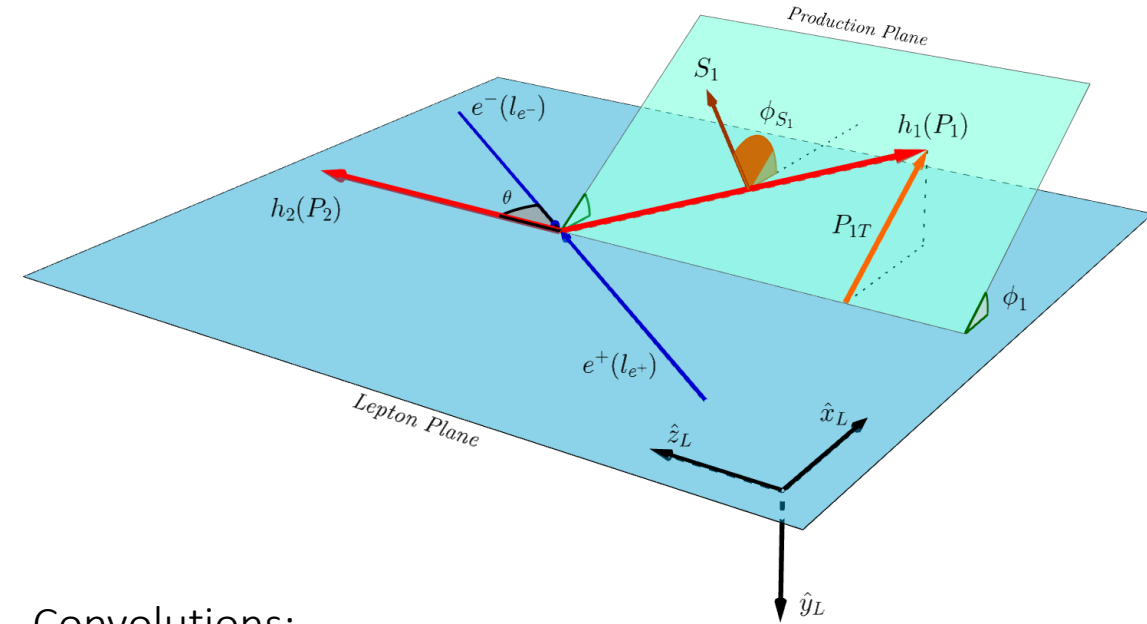
- TMD Fragmentation Functions
- Convolutions and Polarization: $e^+e^- \rightarrow h_1^\uparrow h_2 X$
- Fit results: Belle 2-h
- SIDIS predictions and Intrinsic Charm
- Lambda in jet: $pp \rightarrow \Lambda \text{ jet } X$
- Conclusions

TMD Fragmentation Functions for quarks

8 independent TMD Fragmentation Functions

		Hadron		
		U	L	T
Pol. States		U	L	T
Q u a r k	U	D_1 Unpolarized FF		D_{1T}^\perp Polarizing FF
	L		G_{1L}	G_{1T}
	T	H_1^\perp Collins FF	H_{1L}^\perp	H_1 / H_{1T}^\perp

Double hadron production in e^+e^- processes



Polarization:

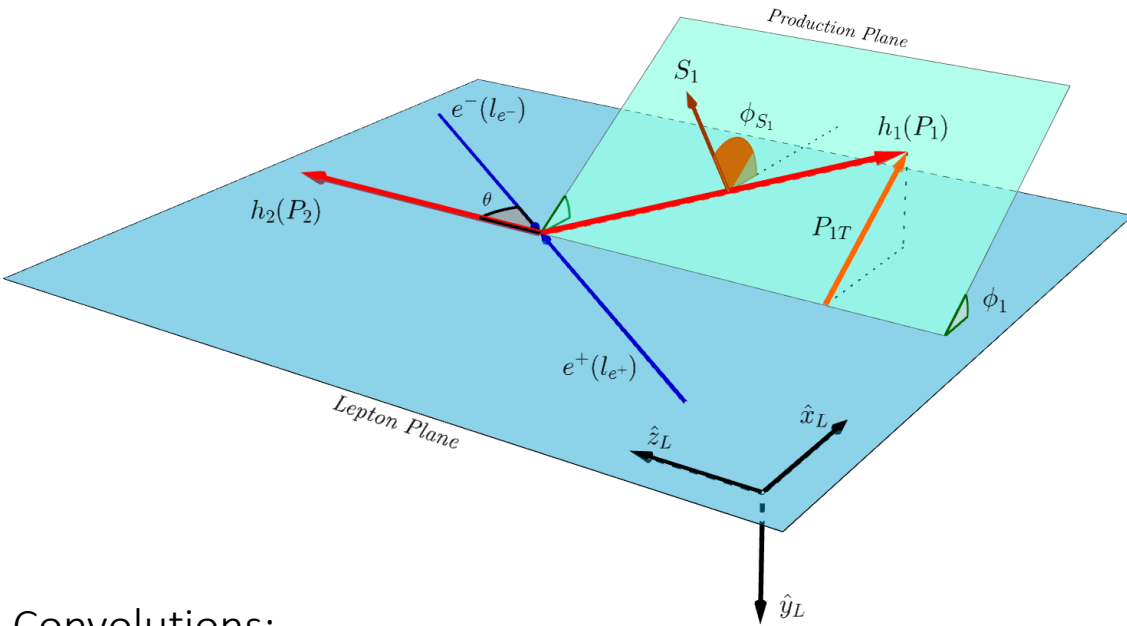
$$P_n^{h_1}(z_{h_1}, z_{h_2}) = \frac{M_1 \int dq_T q_T d\phi_1 \mathcal{B}_1 \left[\tilde{D}_{1T}^{\perp(1)} \tilde{\bar{D}}_1 \right]}{\int dq_T q_T d\phi_1 \mathcal{B}_0 \left[\tilde{\bar{D}}_1 \tilde{\bar{D}}_1 \right]}$$

Convolutions:

$$\mathcal{B}_0 \left[\tilde{\bar{D}} \tilde{\bar{D}} \right] = \frac{1}{z_1^2 z_2^2} \sum_q e_q^2 \int \frac{db_T}{2\pi} b_T J_0(b_T q_T) d_{q/h_1}(z_1; \bar{\mu}_b) d_{\bar{q}/h_2}(z_2; \bar{\mu}_b) \\ \times M_{D_1}(b_c(b_T), z_1) M_{D_2}(b_c(b_T), z_2) e^{-g_K(b_c(b_T); b_{\max}) \ln \left(\frac{Q^2 z_1 z_2}{M_1 M_2} \right) - S_{\text{pert}}(b_*; \bar{\mu}_b)}$$

$$\mathcal{B}_1 \left[\tilde{D}_{1T}^{\perp(1)} \tilde{\bar{D}}_1 \right] = \frac{1}{z_1^2 z_2^2} \sum_q e_q^2 \int \frac{db_T}{(2\pi)} b_T^2 J_1(b_T q_T) D_{1T}^{\perp(1)}(z_1; \bar{\mu}_b) d_{\bar{q}/h_2}(z_2; \bar{\mu}_b) \\ \times M_{D_1}^{\perp}(b_c(b_T), z_1) M_{D_2}(b_c(b_T), z_2) e^{-g_K(b_c(b_T); b_{\max}) \ln \left(\frac{Q^2 z_1 z_2}{M_1 M_2} \right) - S_{\text{pert}}(b_*; \bar{\mu}_b)},$$

Double hadron production in e^+e^- processes



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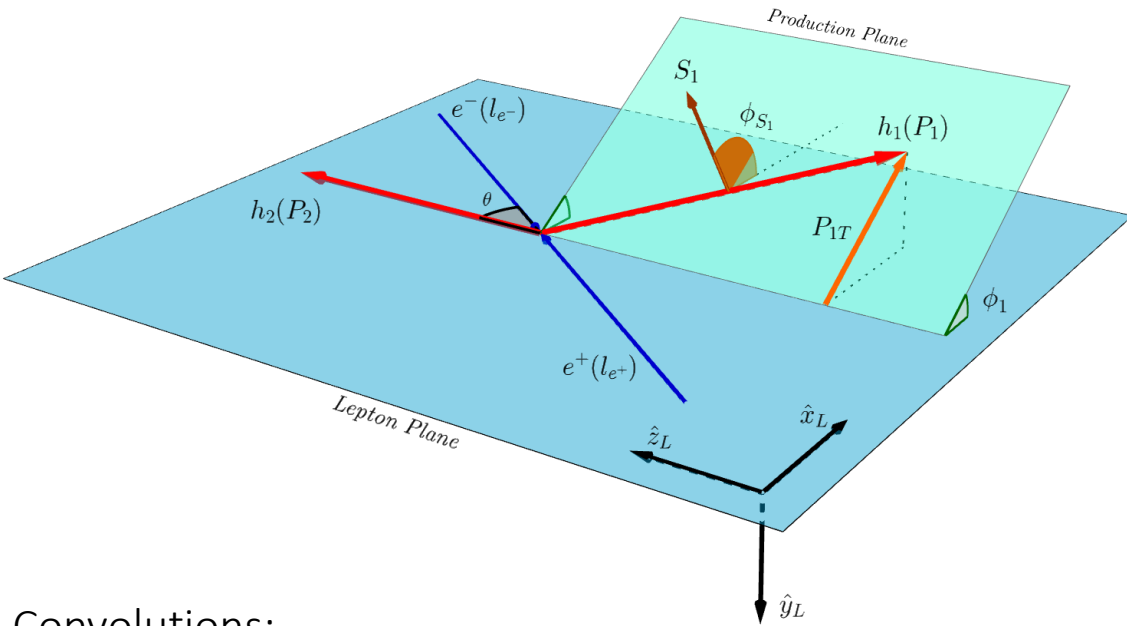
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Unpolarized FFs:
DSS set for π/K
AKK set for Λ

Double hadron production in e^+e^- processes



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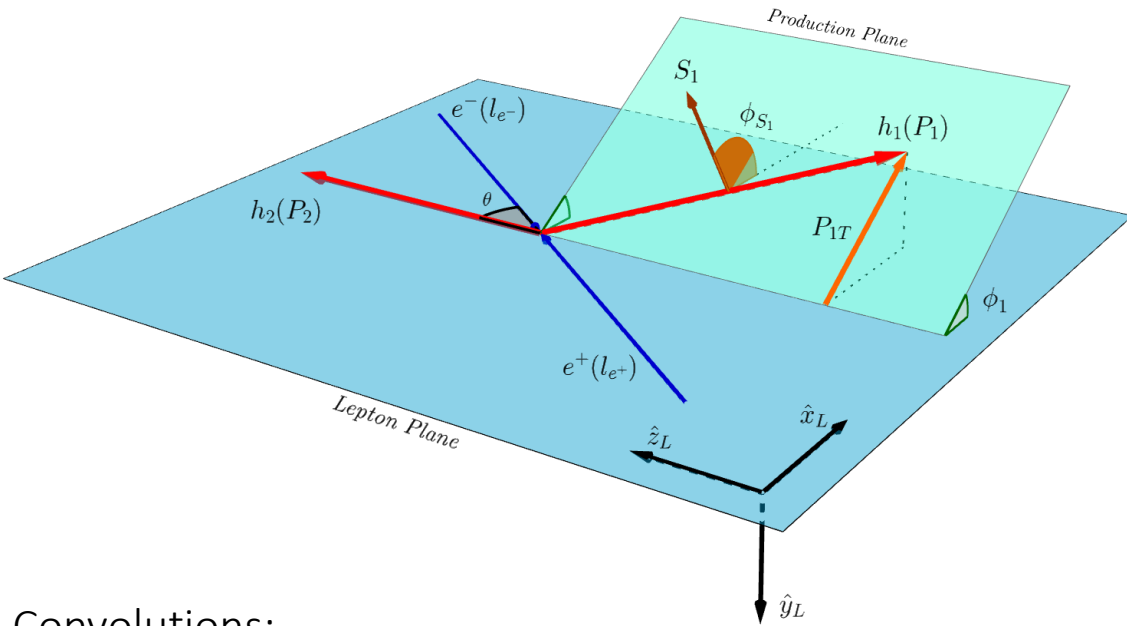
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Non-perturbative functions from

Bacchetta et al., *JHEP* 06 (2017) 081

Double hadron production in e^+e^- processes



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Double hadron production in e^+e^- processes

Data selection:

- $\Lambda + \pi/K$: $z_{\pi,K} = [0.5 - 0.9]$ bin excluded \rightarrow 96 data points

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Functions to be extracted:

Polarizing FF first moment

$$\tilde{D}_{1T, \Lambda/q}^{\perp(1)}(z; \mu_b) = \mathcal{N}_q^p(z) d_{q/\Lambda}(z; \mu_b)$$

$$\mathcal{N}_q^p(z) = N_q z^{a_q} (1-z)^{b_q} \frac{(a_q + b_q)^{(a_q + b_q)}}{a_q^{a_q} b_q^{b_q}}$$

Polarizing FF non-perturbative function

$$M_{D, \Lambda}^{\perp}(b_T, z) = \exp\left(-\frac{\langle p_{\perp}^2 \rangle_P b_T^2}{4z_p^2}\right)$$

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u, d, s

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u, d, s + charm

Isospin symmetry

- No SU(2): N_u, N_d, N_s, N_{sea}
- SU(2): $N_u = N_d, N_{\bar{u}} = N_{\bar{d}}, N_s, N_{\bar{s}}$

See also:

Chen, Liang, Pan, Song, Wei;
Phys.Lett.B 816 (2021) 136217

Double hadron production in e^+e^- processes

Scenarios considered:

1. No Charm, No SU(2) sym.
pFFs for: *up, down strange and sea*;
2. Charm, No SU(2) sym.
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3. Charm, SU(2) sym.
pFFs for: *up/down, $\overline{u\bar{p}/d\bar{down}}$, strange, $\overline{strange}$*

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χ^2_{dof}
96 points
1,174
1,259
1,361

Double hadron production in e^+e^- processes

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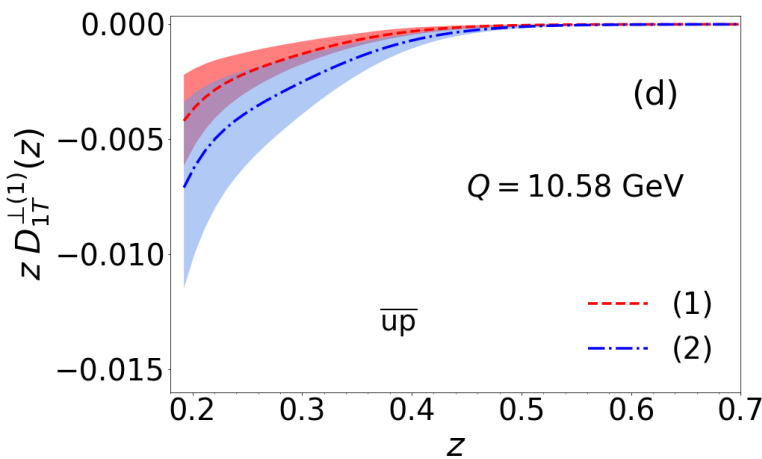
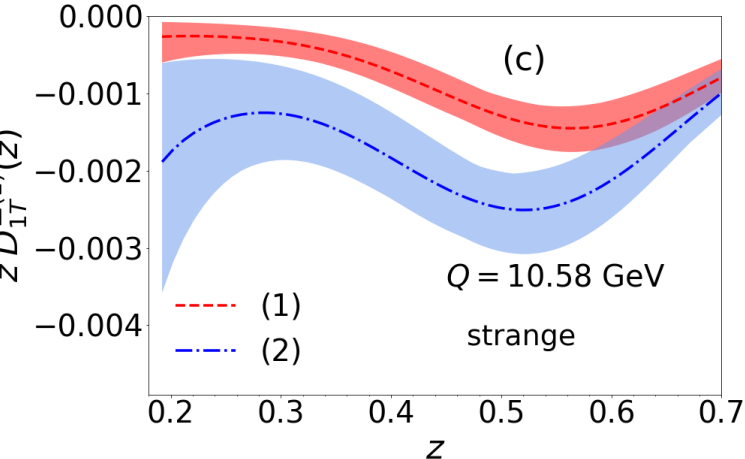
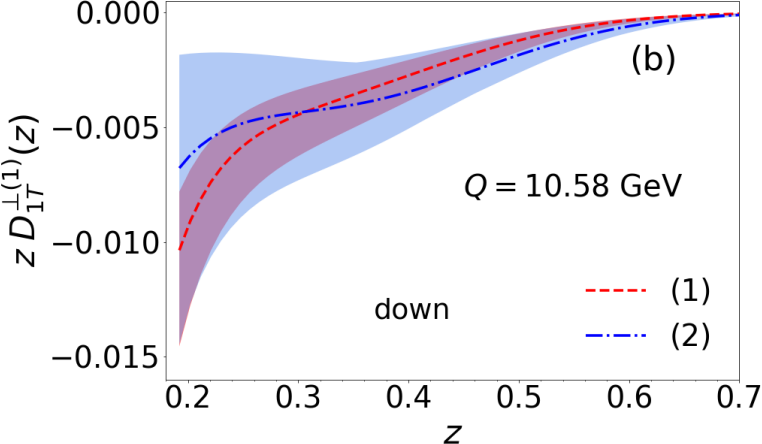
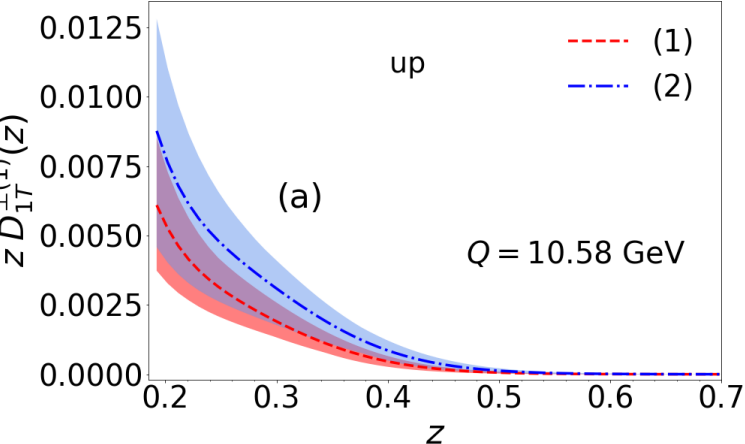
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pFFs for: *up/down, $\overline{up}/\overline{down}$, strange, $\overline{strange}$*

χ_{dof}^2	χ_{dof}^2
96 points	128 points
1,174	1,903
1,259	1,622
1,361	1,645

Double hadron production in e^+e^- processes

First moments: (1) & (2) scenarios

- (1) No Charm, No SU2
- (2) Charm, No SU2

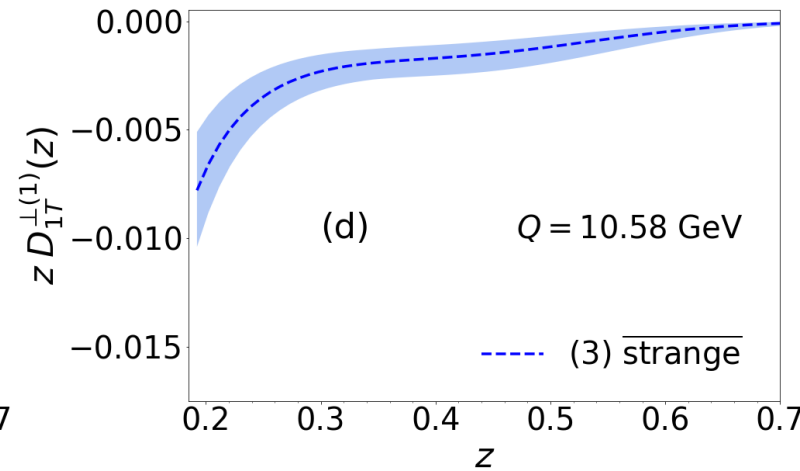
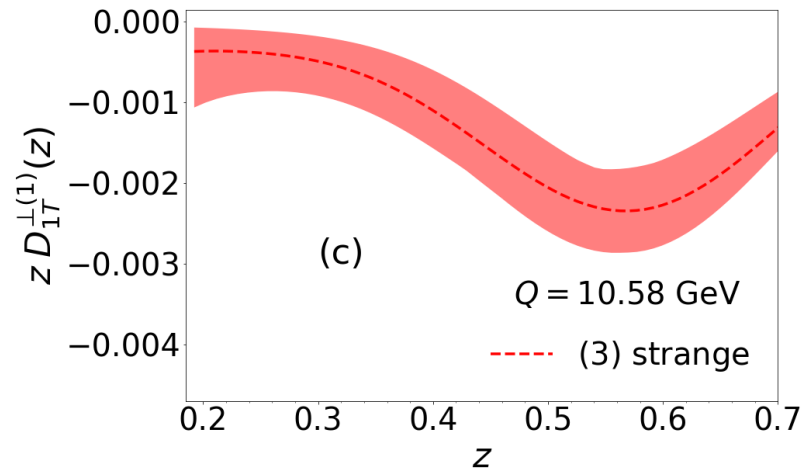
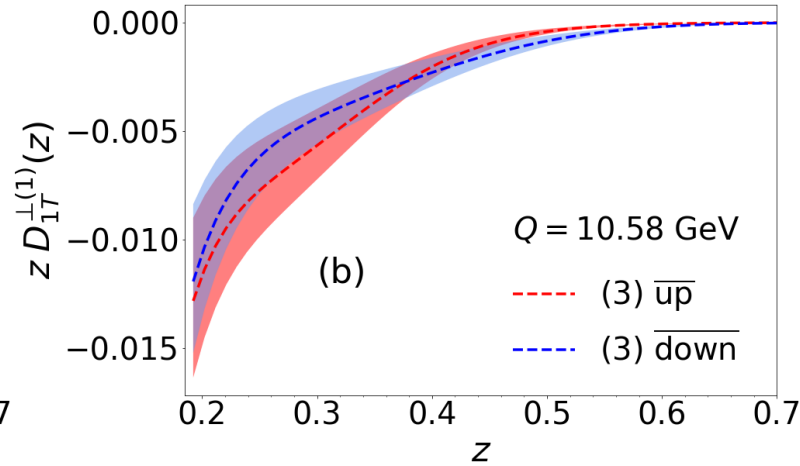
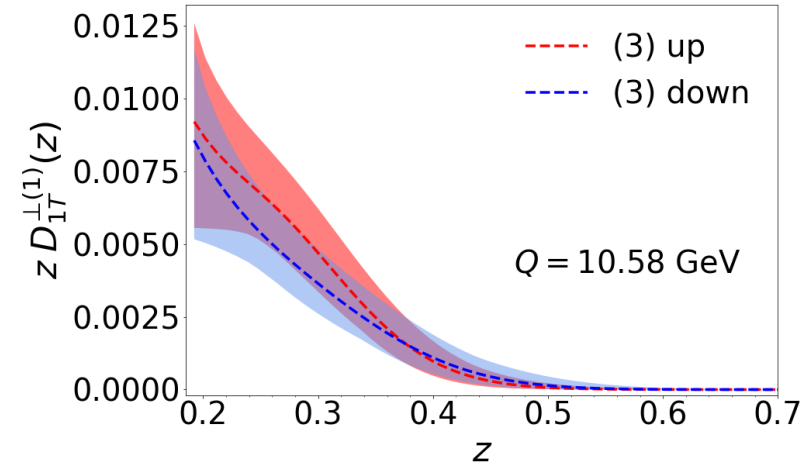


- pFFs are different in magnitude due to the charm contribution;
- up pFF is positive;
- First moments are compatible, except for the strange f.m.
- Similar size for the Gaussian width.

Double hadron production in e^+e^- processes

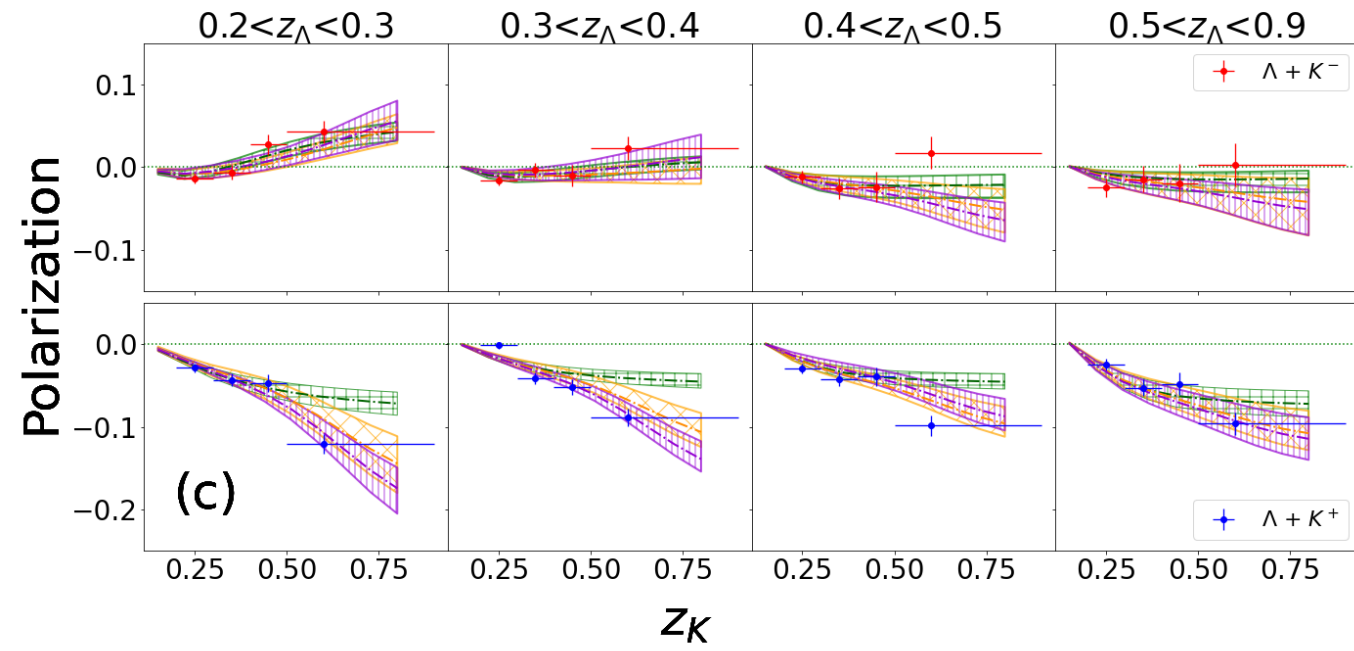
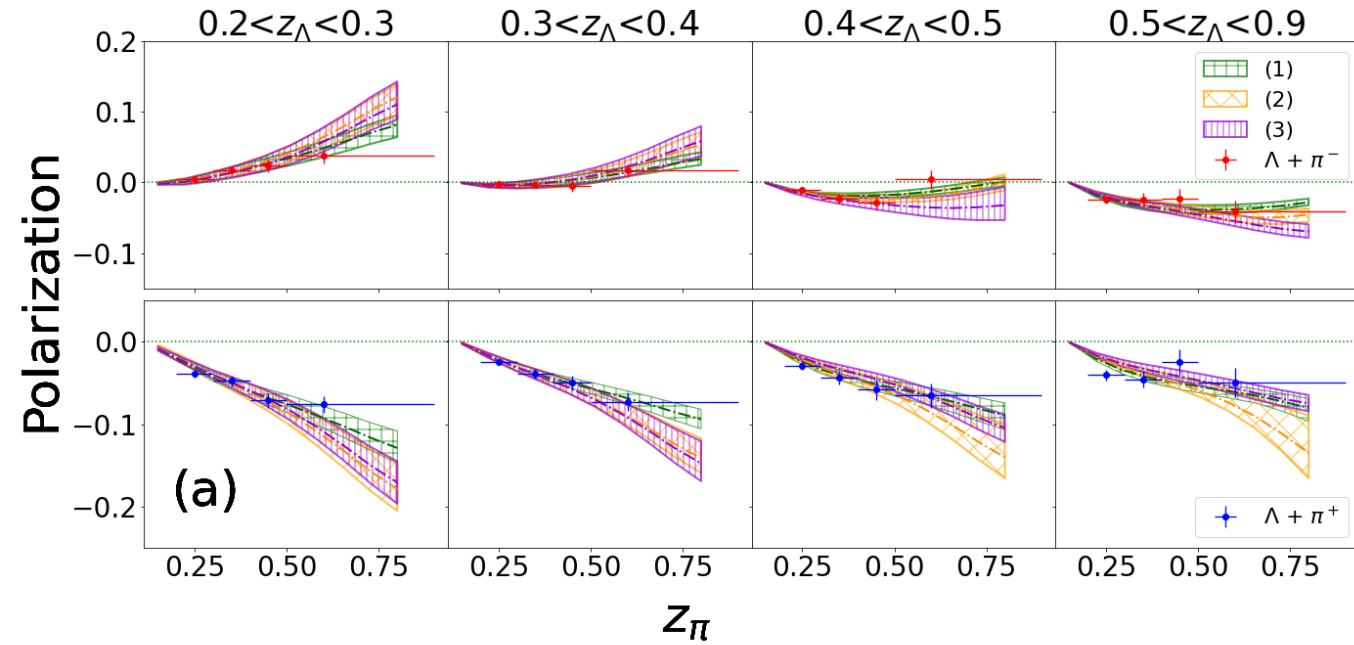
First moments: (3) scenario

(3) Charm, SU2



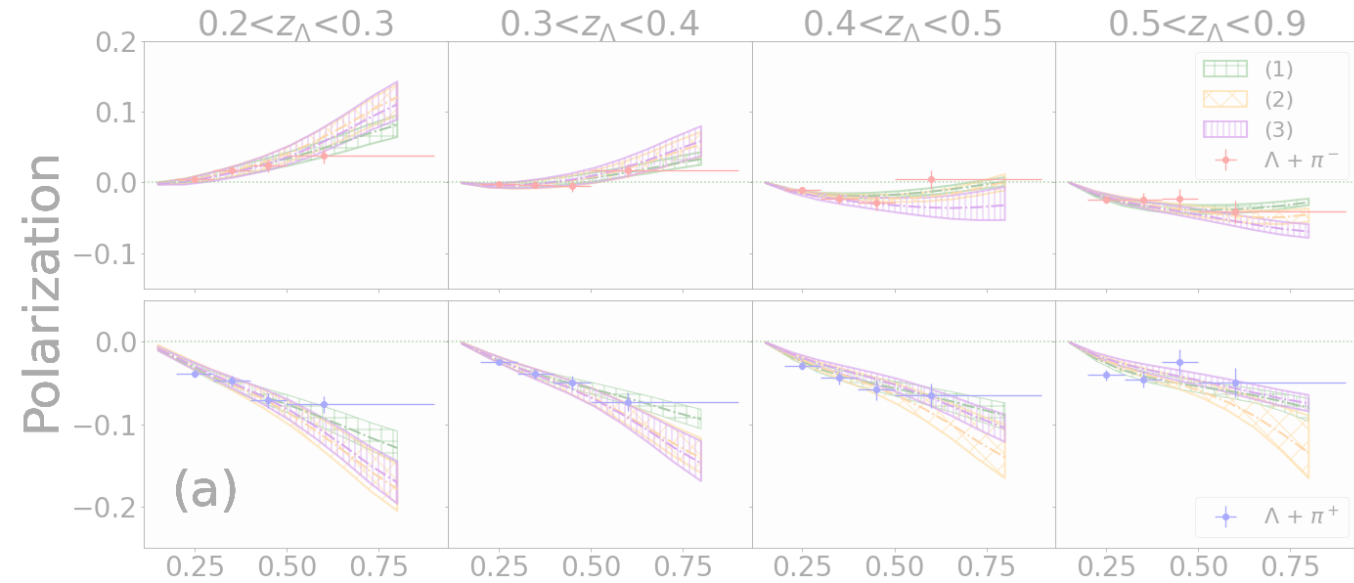
- $up/down$ pFFs are positive;
- $\overline{up}/\overline{down}$ pFFs are negative;
- $strange/\overline{strange}$ pFFs are negative;
- up & $\overline{strange}$ compatible with (1,2) scn.
- The negative sea contribution is larger in size;
- Similar size for the Gaussian width.

Double hadron production in e^+e^- processes

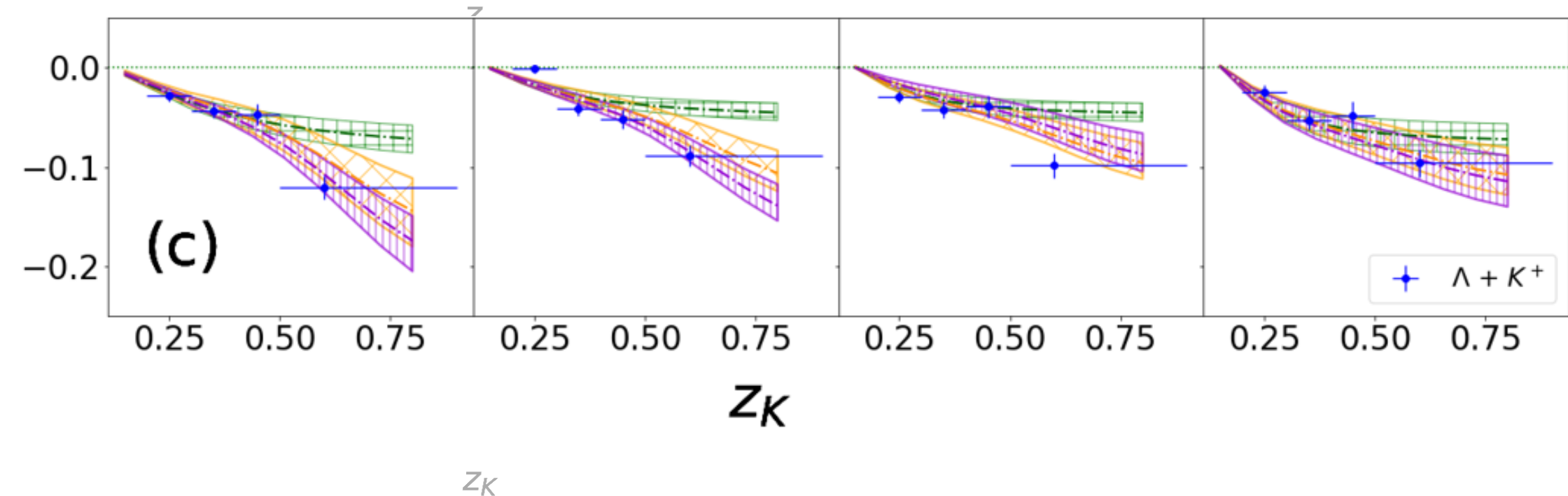


- All scenarios can describe $\Lambda\pi^\pm, \bar{\Lambda}\pi^\pm, \Lambda K^-, \bar{\Lambda}K^+$ data;
- Scenario (1) cannot describe $\Lambda K^+, \bar{\Lambda}K^-$ data with $z_K > 0,5$;
- With the Charm contribution we obtain similar good fits and description

Double hadron production in e^+e^- processes



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Double hadron production in e^+e^- processes

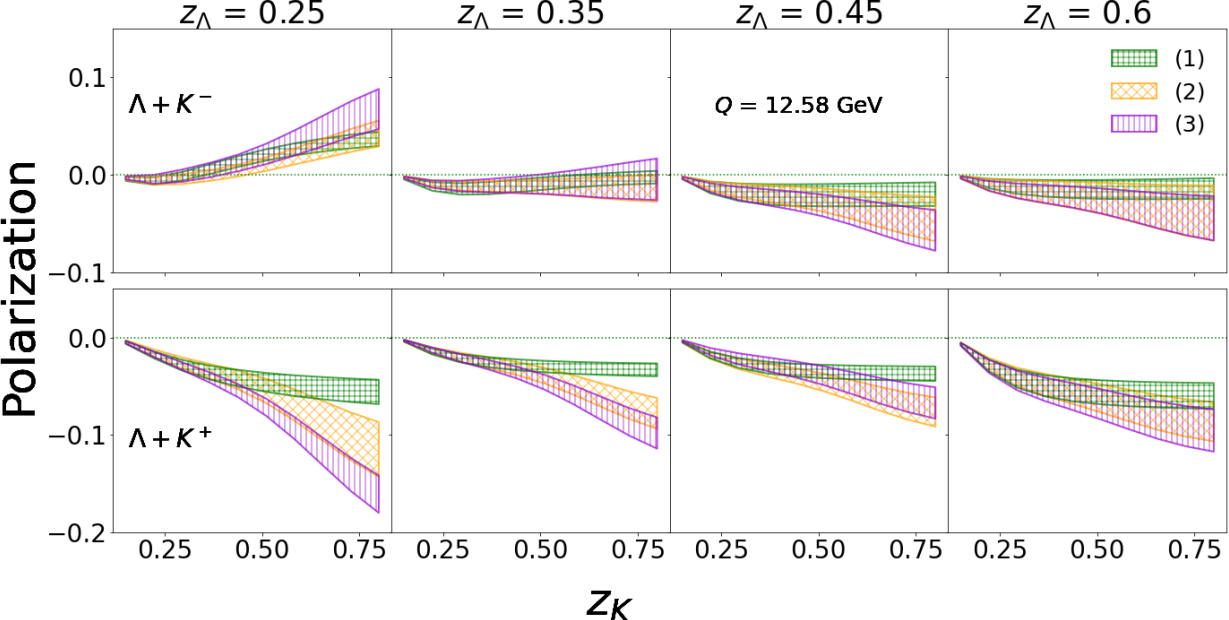
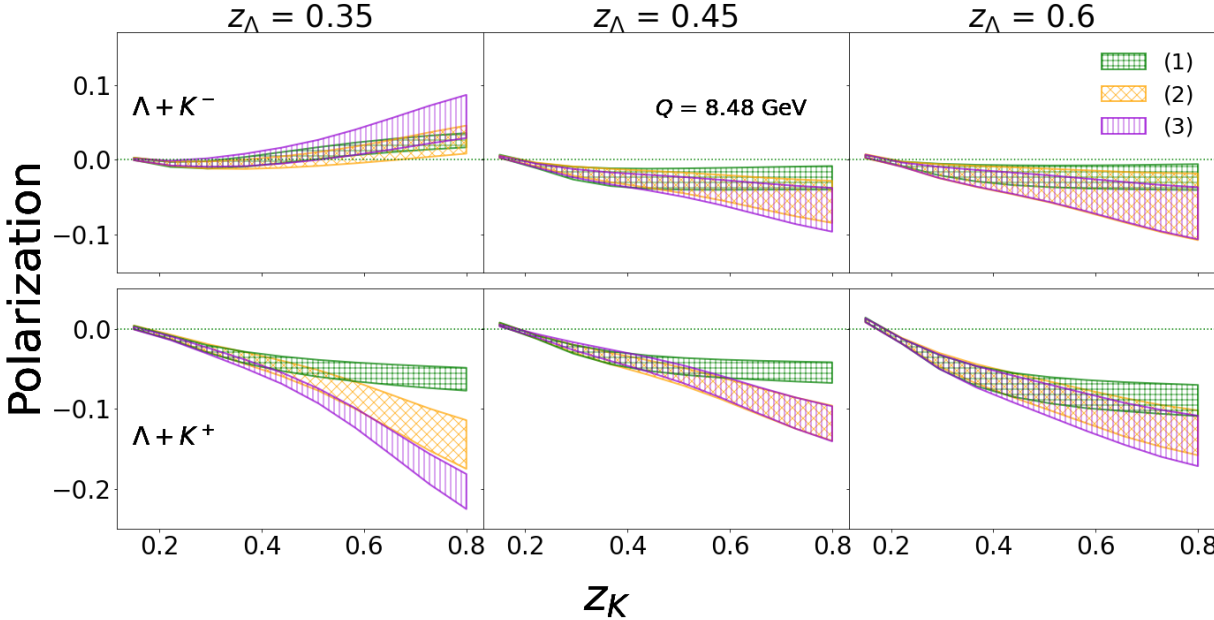
Some remarks:

- Charm contribution in the unpolarized C.S. is necessary;
Attempts were made to include this contribution also in the polarized c.s.
- We cannot distinguish between the (2) and (3) scenarios.
If Normalization factors are free,
up & down come out opposite, violating the SU(2) symmetry.

Double hadron production in e^+e^- processes

Investigate the polarization in:

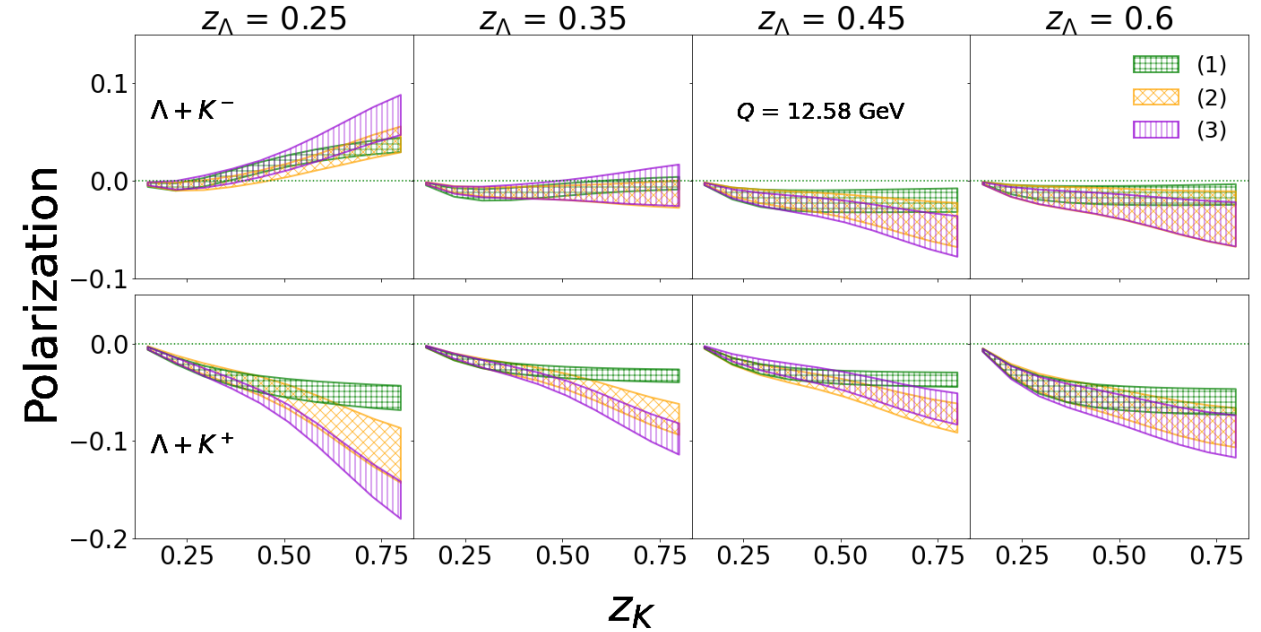
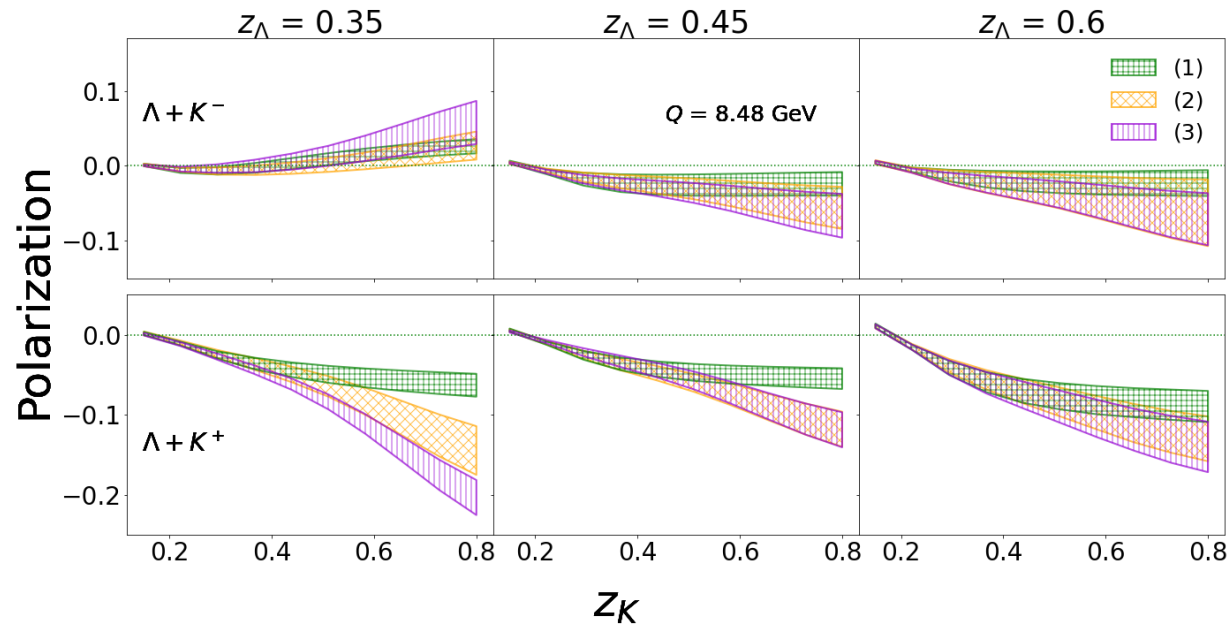
e^+e^- at different energies \rightarrow we cannot distinguish between (2) & (3) scenarios



Double hadron production in e^+e^- processes

Investigate the polarization in:

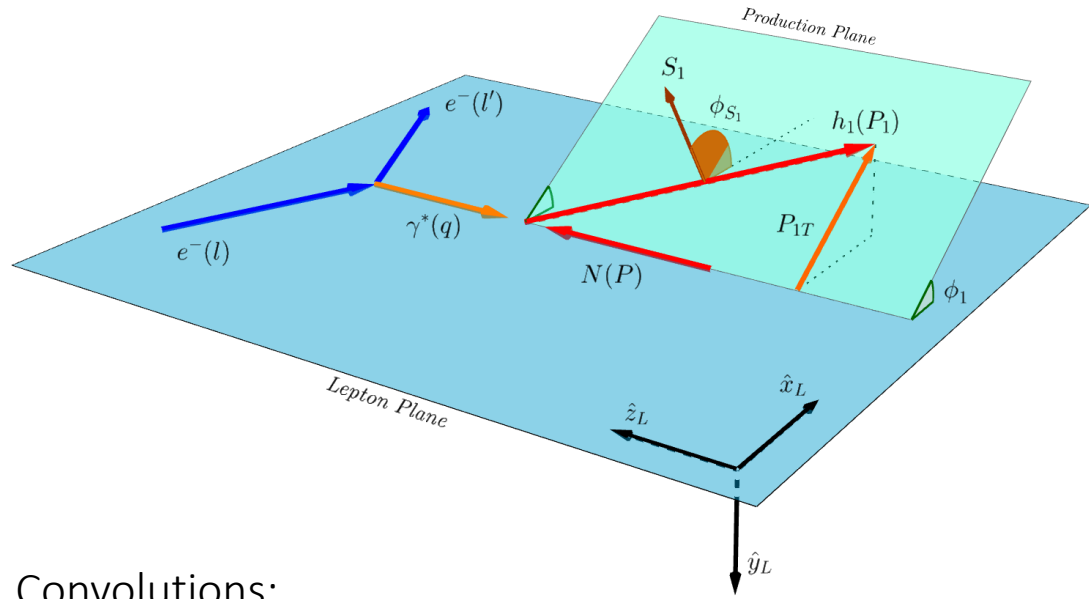
e^+e^- at different energies \rightarrow we cannot distinguish between (2) & (3) scenarios



Processes to explore:

- SIDIS
- Lambda in jet production in proton-proton collisions

Semi-inclusive Deep Inelastic Scattering



Polarization:

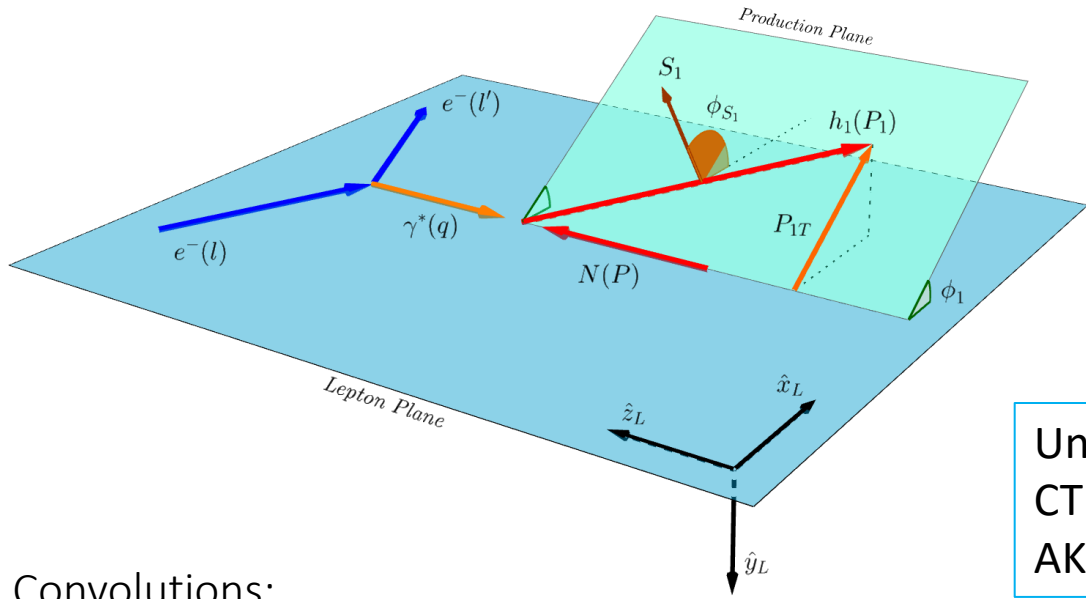
$$P_n^{h_1}(x_B, z_h) = \frac{M_1 \int dq_T q_T d\phi_1 \mathcal{B}_1 \left[\tilde{f}_1 \tilde{D}_{1T}^{\perp(1)} \right]}{\int dq_T q_T d\phi_1 \mathcal{B}_0 \left[\tilde{f}_1 \tilde{D}_1 \right]}$$

Convolutions:

$$\mathcal{B}_0 \left[\tilde{f}_1 \tilde{D}_1 \right] = \frac{1}{z^2} \sum_q e_q^2 \int \frac{db_T}{(2\pi)} b_T J_0(b_T q_T) f_{N/q}(x; \bar{\mu}_b) d_{q/h}(z; \bar{\mu}_b) \\ \times M_{f_1}(b_c(b_T), x) M_{D_h}(b_c(b_T), z) e^{-g_K(b_c(b_T); b_{\max}) \ln \left(\frac{Q^2 z}{x M_P M_h} \right) - S_{\text{pert}}(b_*; \bar{\mu}_b)}$$

$$\mathcal{B}_1 \left[\tilde{f}_1 \tilde{D}_{1T}^{\perp(1)} \right] = \frac{1}{z^2} \sum_q e_q^2 \int \frac{db_T}{(2\pi)} b_T^2 J_1(b_T q_T) f_{N/q}(x; \bar{\mu}_b) D_{1T,q}^{\perp(1)}(z; \bar{\mu}_b) \\ \times M_{f_1}(b_c(b_T), x) M_{D_1}^{\perp}(b_c(b_T), z) e^{-g_K(b_c(b_T); b_{\max}) \ln \left(\frac{Q^2 z}{x M_P M_h} \right) - S_{\text{pert}}(b_*; \bar{\mu}_b)}$$

Semi-inclusive Deep Inelastic Scattering



Polarization:

$$P_n^{h_1}(x_B, z_h) = \frac{M_1 \int dq_T q_T d\phi_1 \mathcal{B}_1 \left[\tilde{f}_1 \tilde{D}_{1T}^{\perp(1)} \right]}{\int dq_T q_T d\phi_1 \mathcal{B}_0 \left[\tilde{f}_1 \tilde{D}_1 \right]}$$

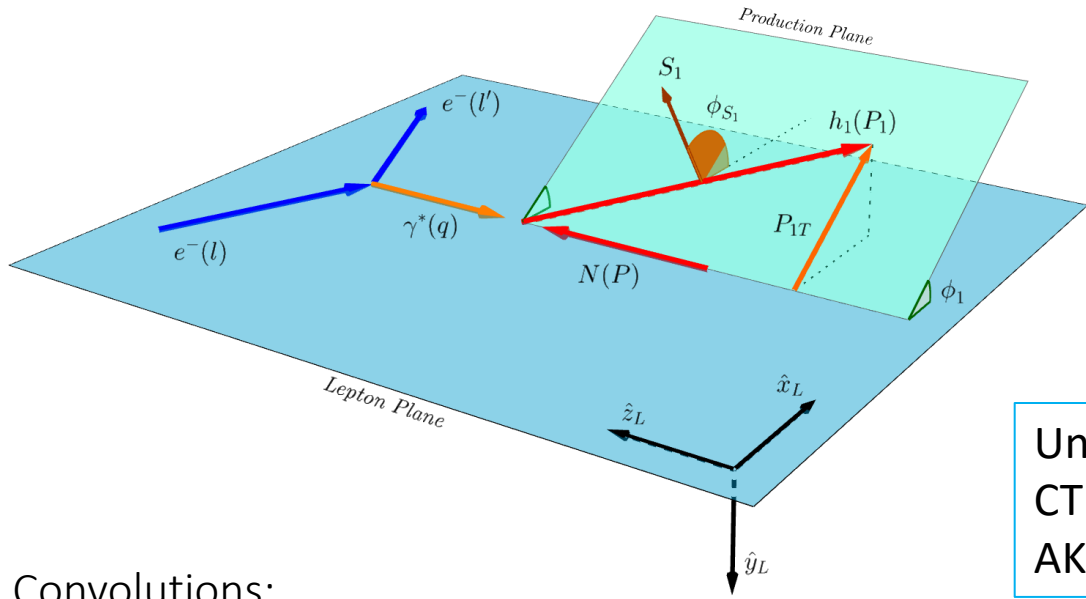
Unpolarized functions:
CT14nnlo set for proton
AKK set for Λ

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Semi-inclusive Deep Inelastic Scattering



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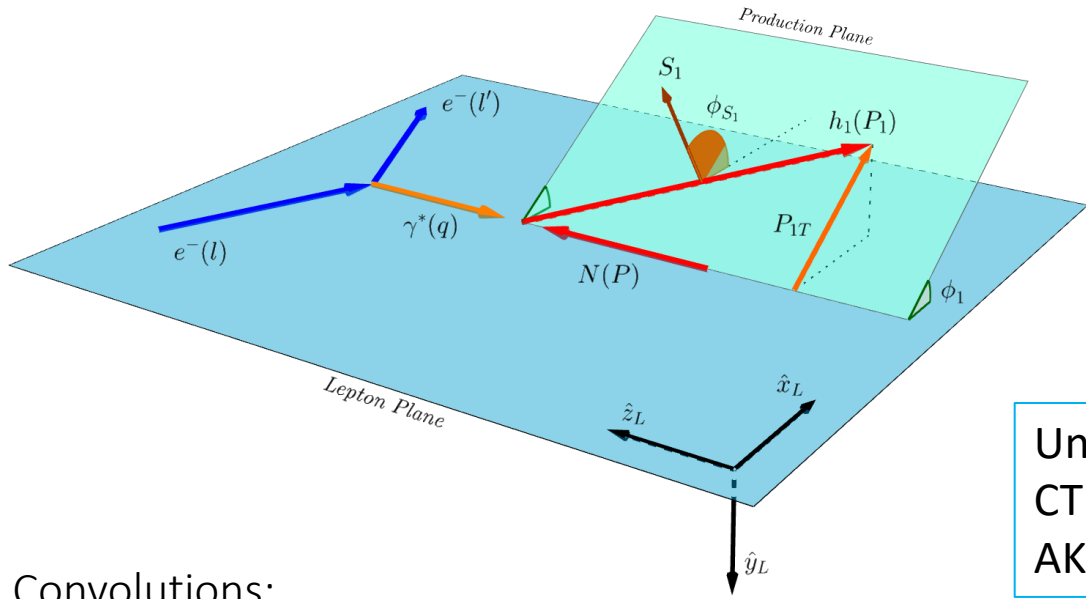
Non-perturbative functions from
Bacchetta et al., *JHEP* 06 (2017) 081

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Semi-inclusive Deep Inelastic Scattering



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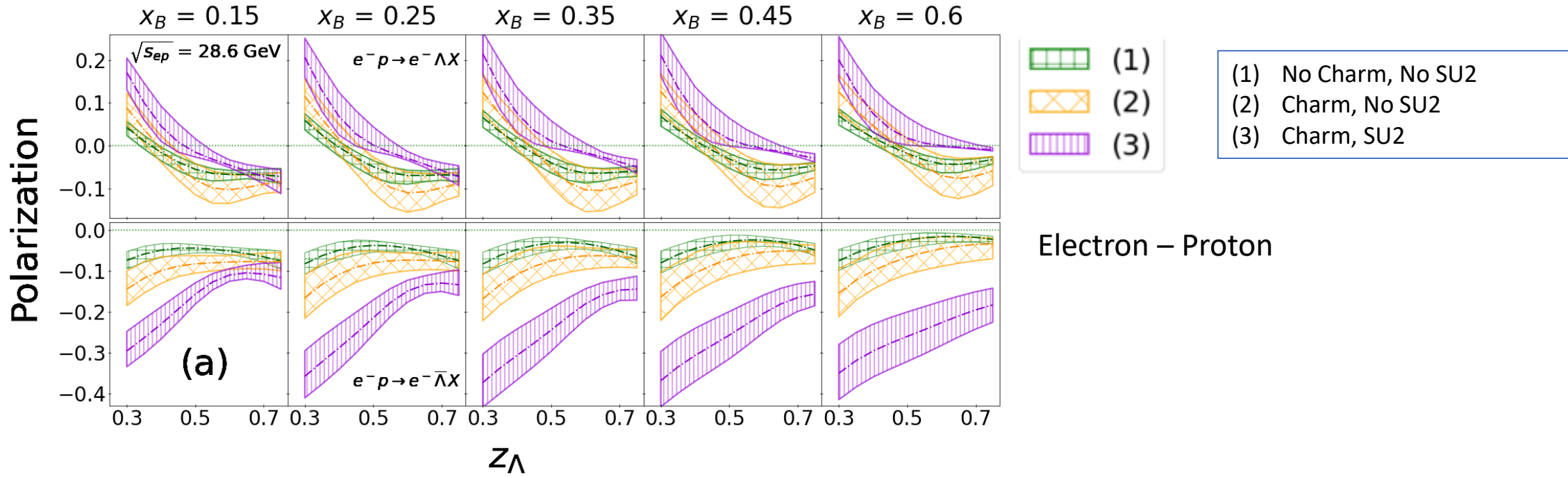
$$\mathcal{B}_1 \left[\tilde{f}_1 \tilde{D}_{1T}^{\perp(1)} \right] = \frac{1}{z^2} \sum_q e_q^2 \int \frac{db_T}{(2\pi)} b_T^2 J_1(b_T q_T) f_{N/q}(x; \bar{\mu}_b) D_{1T,q}^{\perp(1)}(z; \bar{\mu}_b) \times M_{f_1}(b_c(b_T), x) M_{D_1}^{\perp}(b_c(b_T), z) e^{-g_K(b_c(b_T); b_{\max}) \ln \left(\frac{Q^2 z}{x M_P M_h} \right) - S_{\text{pert}}(b_*; \bar{\mu}_b)}$$

Predictions are given at different energies:

E_N (GeV)	E_{e^-} (GeV)	$\sqrt{s_{eN}}$ (GeV)
41	5	28.6
100	10	63.2

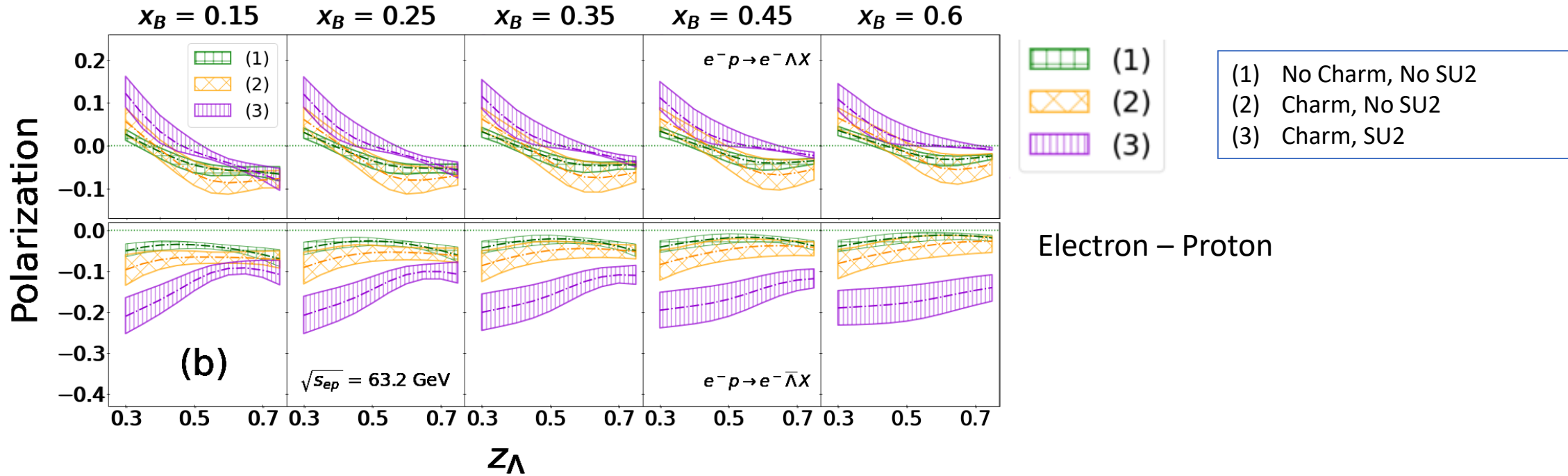
$$s = 4E_N E_e, \quad Q^2 = x_B y s$$

Semi-inclusive Deep Inelastic Scattering



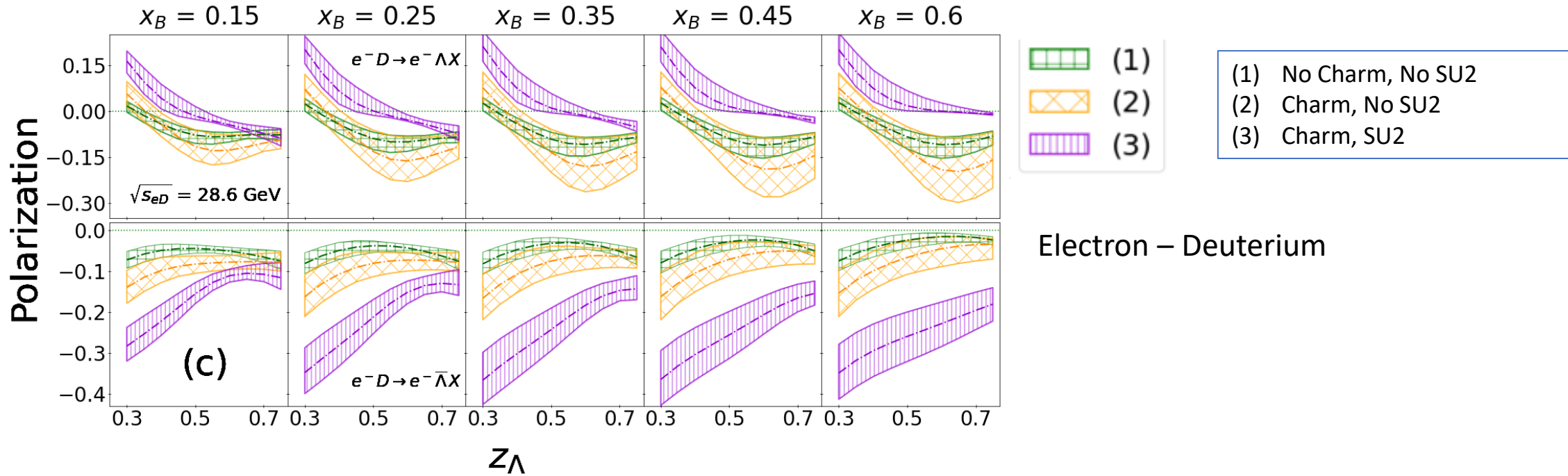
- (1) & (2) scenarios: polarization of similar size and behavior;
- (3) scenario: similar size;
- Λ pol. decreases and becomes negative;
- $\bar{\Lambda}$ is always negative;
- $\bar{\Lambda}$ pol. similar or slightly greater size;
- $\bar{\Lambda}$ most significant difference;
- $\sqrt{s_{ep}}=28,6$ pol. has the same size, for greater values there is a general reduction as x_B grows.

Semi-inclusive Deep Inelastic Scattering



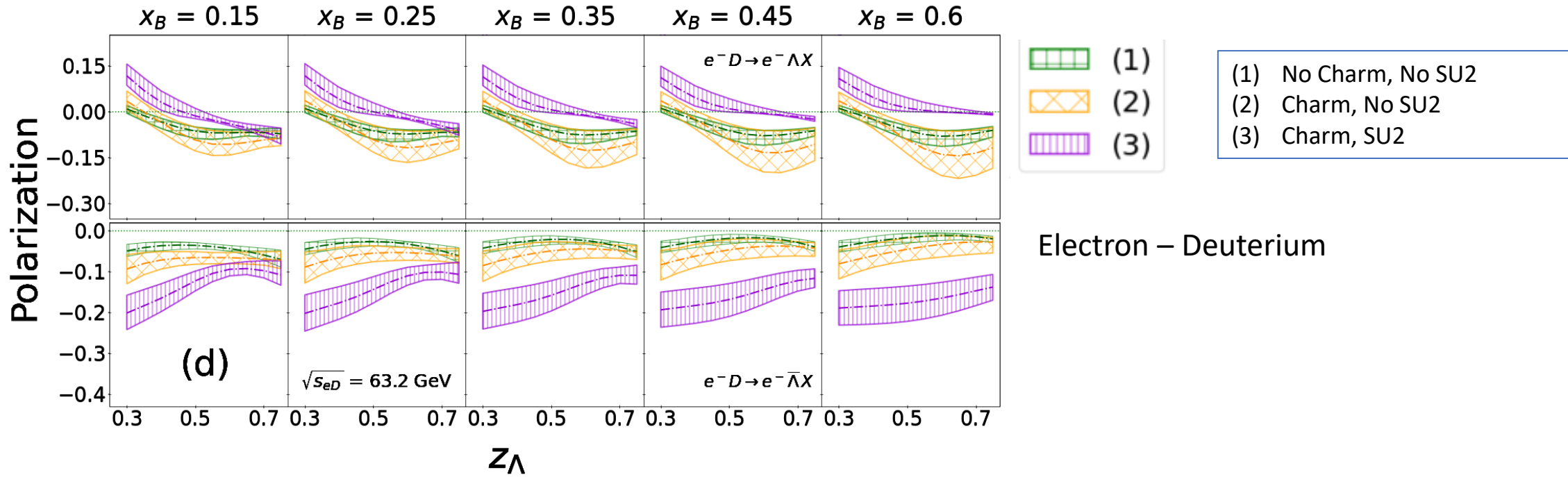
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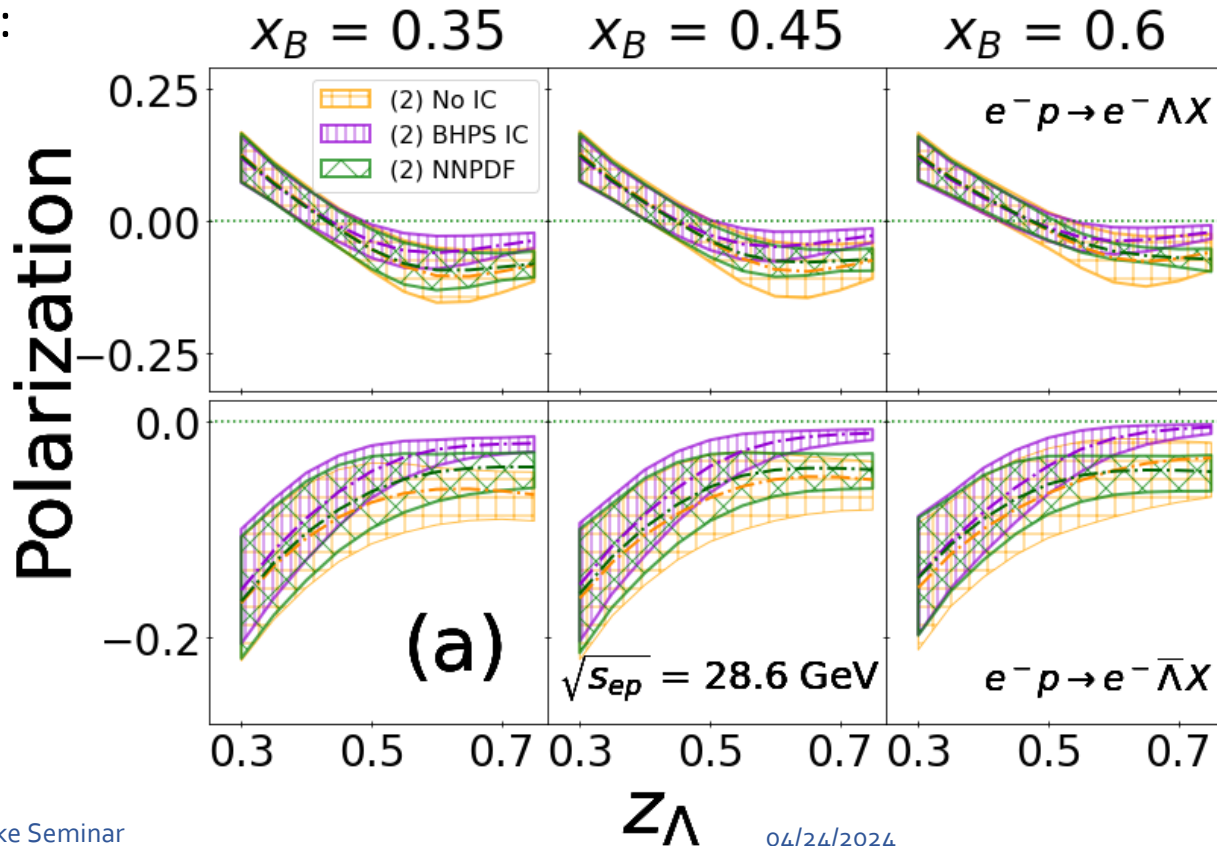
Semi-inclusive Deep Inelastic Scattering

The charm contribution in the fragmentation process is relevant

Intrinsic Charm (IC) component in the proton:

- CT14nnloIC set with BHPS model [T.-J. Hou et al., *JHEP* 02 (2018) 059]
- NNPDF4.0nnlo set [NNPDF Coll., *Eur.Phys.J.C* 82 (2022) 5, 428]

(2) Scenario:

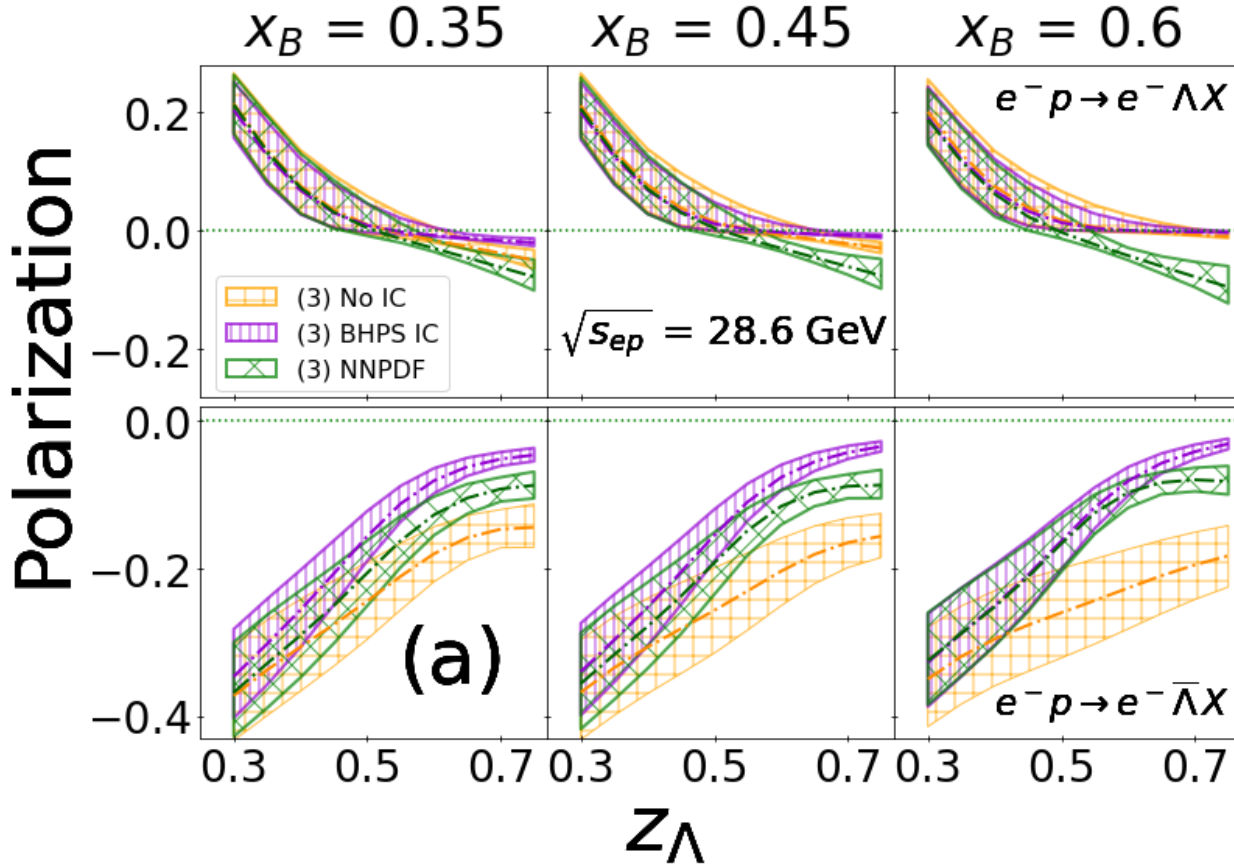


(2) Charm, No SU2

- BHPS and NNPDF: similar polarization of previous predictions
- Same behavior is present for greater values of the c.m. energy.

Semi-inclusive Deep Inelastic Scattering

(3) Scenario:



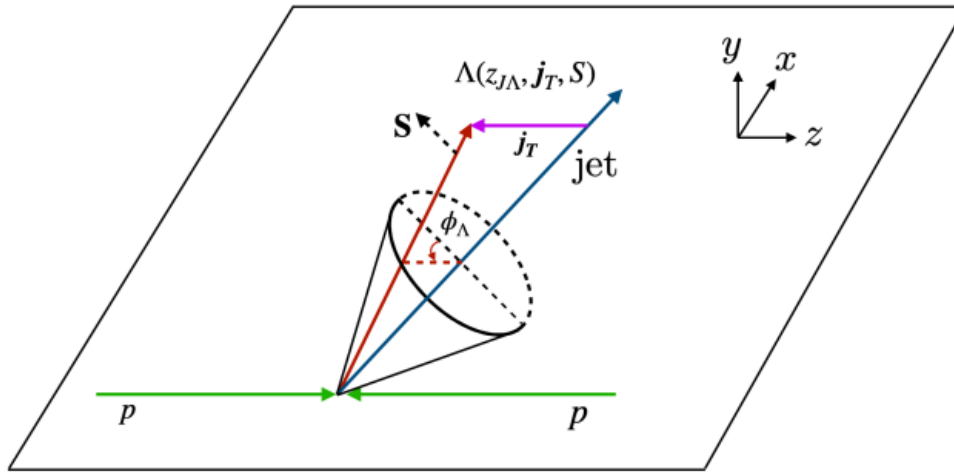
(3) Charm, SU2

- Estimates vary significantly as x_B increases;
- $\bar{\Lambda}$ estimates with BHPS and NNPDF different from the previous ones;
- Λ : decreases to zero
- Λ : NNPDF become negative

Unpolarized proton – proton collisions: $pp \rightarrow \Lambda \text{ jet } X$

Preliminary STAR data: arxiv/2402.01168

$$A(p_A) B(p_B) \rightarrow \text{jet}(p_j) \Lambda^\uparrow(p_\Lambda) X \quad \sqrt{s} = 200 \text{ GeV}$$



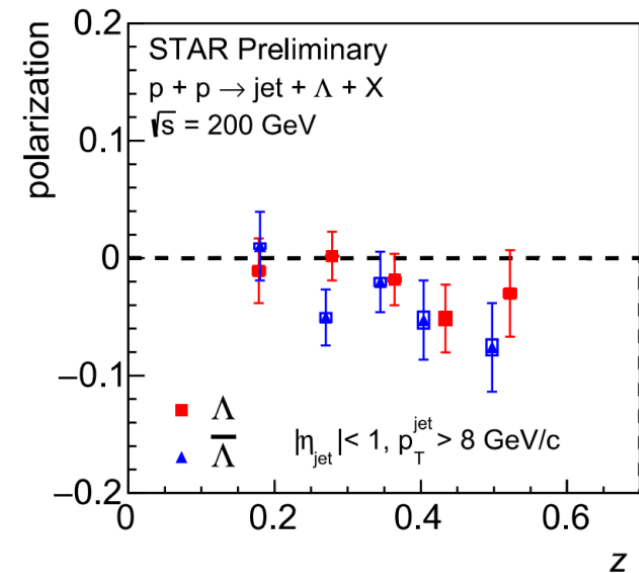
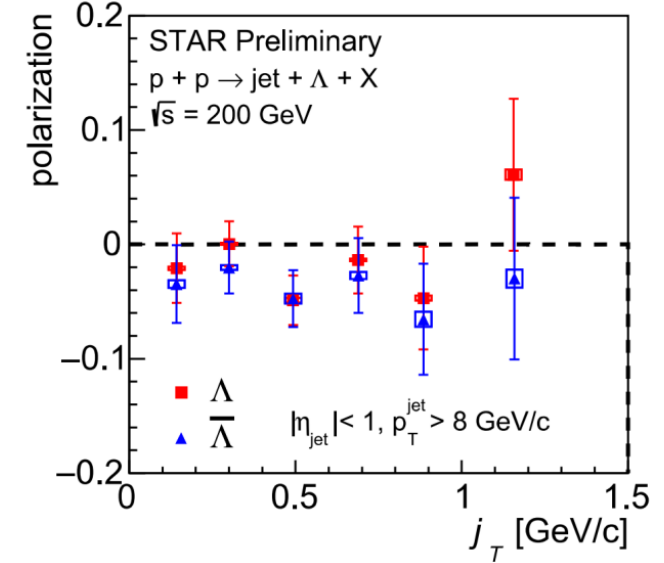
$$p_{\perp\Lambda} \leq 1.6 \text{ GeV}/c, \quad 0 \leq z \leq 1,$$

Kinematic cuts:

$$8 \leq p_{jT} \leq 25 \text{ GeV}/c \text{ with } \langle p_{jT} \rangle = 11 \text{ GeV}/c,$$

$$|\eta_j| \leq 1.0, \quad p_{T\Lambda} \leq 10 \text{ GeV}/c, \quad |\eta_\Lambda| \leq 1.5$$

Anti- k_T algorithm with cone radius $R = 0.6$



Unpolarized proton – proton collisions: $pp \rightarrow \Lambda jet X$

$$\text{Polarization: } P_T^\Lambda(p_j, \xi, p_{\perp\Lambda}) = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} = \frac{d\Delta\sigma}{d\sigma_{\text{unp}}}$$

$$d\Delta\sigma = \sum_{a,b,c,d} \int dx_a dx_b \frac{\alpha_s^2}{\hat{s}} f_{a/A}(x_a) f_{b/B}(x_b) \\ \times |\overline{M}|^2 \delta(\hat{s} + \hat{t} + \hat{u}) \Delta D_{\Lambda^\uparrow/c}(\xi, p_{\perp\Lambda})$$

$$d\sigma_{\text{unp}} = \sum_{a,b,c,d} \int dx_a dx_b \frac{\alpha_s^2}{\hat{s}} f_{a/A}(x_a) f_{b/B}(x_b) \\ \times |\overline{M}|^2 \delta(\hat{s} + \hat{t} + \hat{u}) D_{\Lambda/c}(\xi, p_{\perp\Lambda})$$

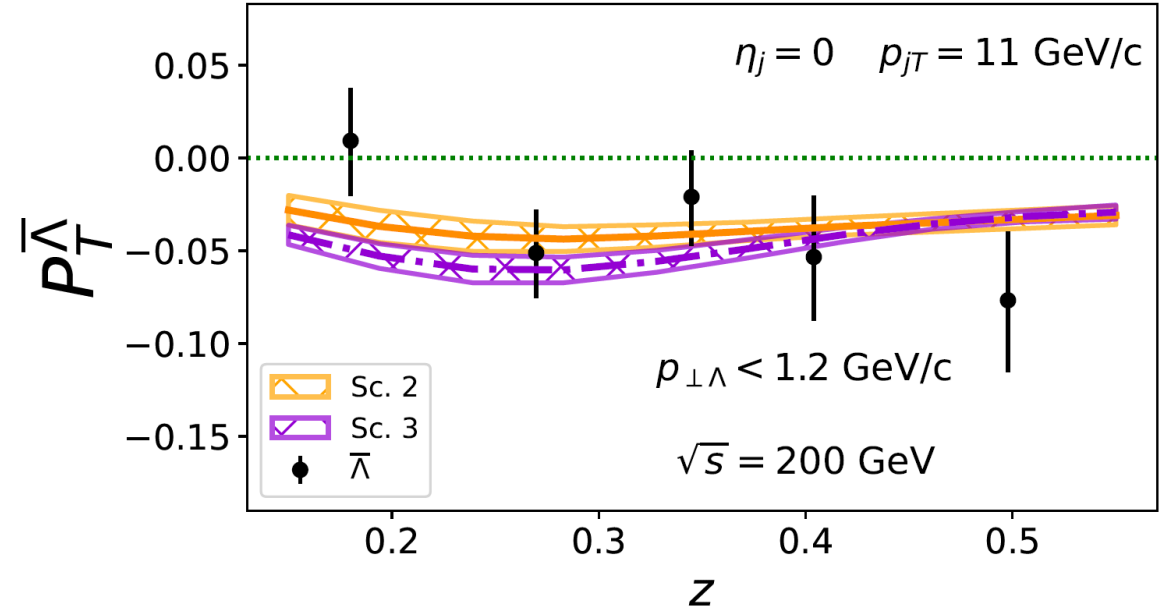
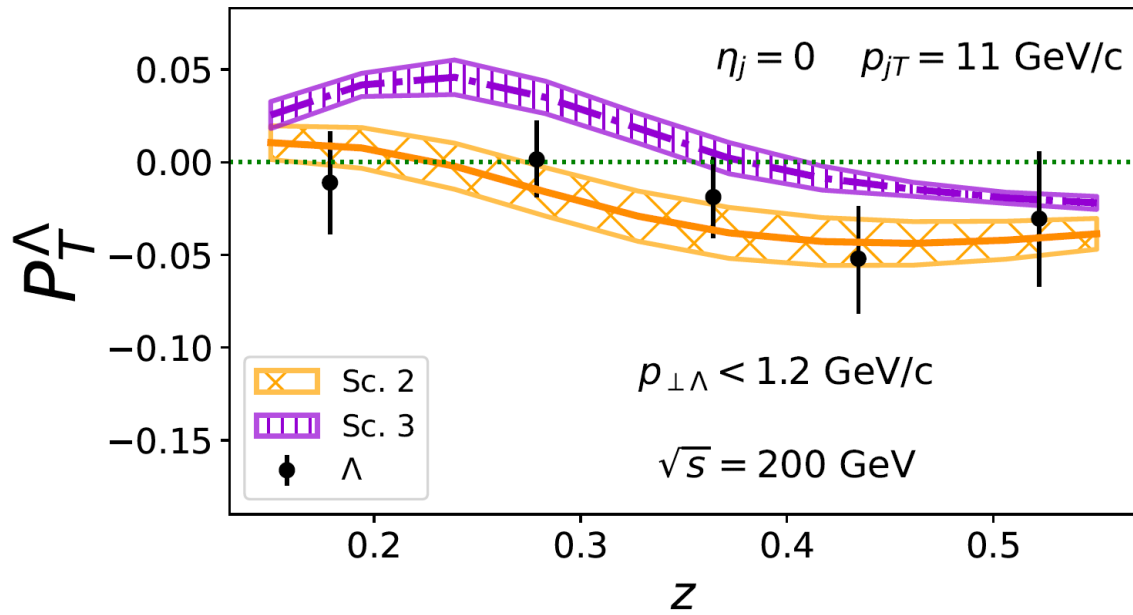
Unpolarized functions:
CT14nnlo set for proton
AKK set for Λ

Polarizing FF

$$\Delta D_{\Lambda^\uparrow/c}(\xi, p_{\perp}) = \frac{p_{\perp\Lambda}}{\xi m_\Lambda} D_{1T}^{\perp c}(\xi, p_{\perp})$$

- Collinear PDFs
- Transverse momentum dependence only in the Fragmentation Function

Unpolarized proton – proton collisions: $pp \rightarrow \Lambda \text{ jet } X$



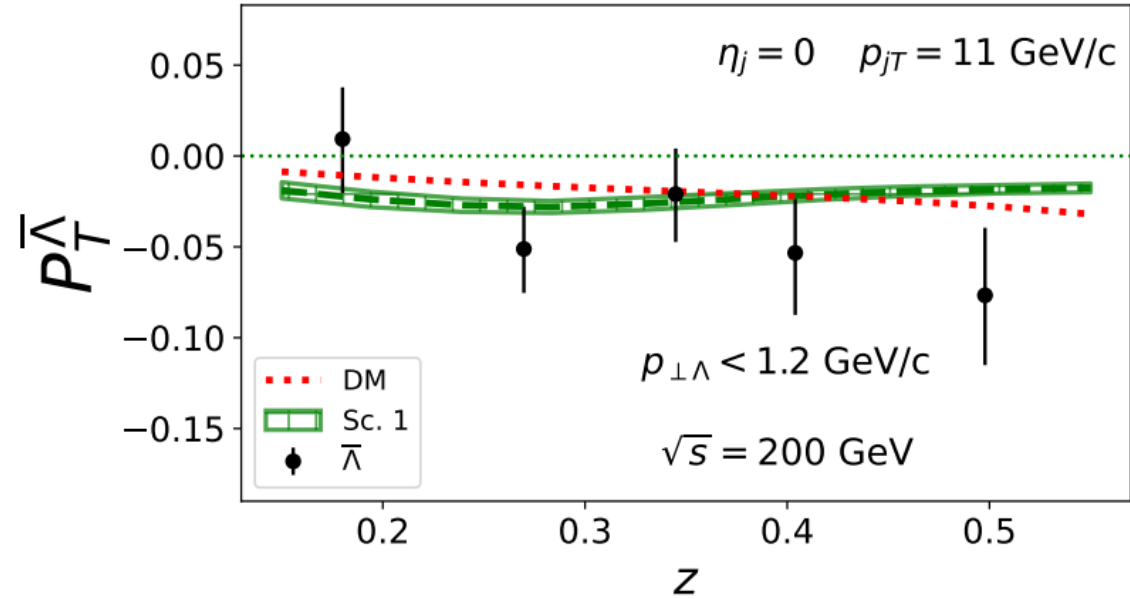
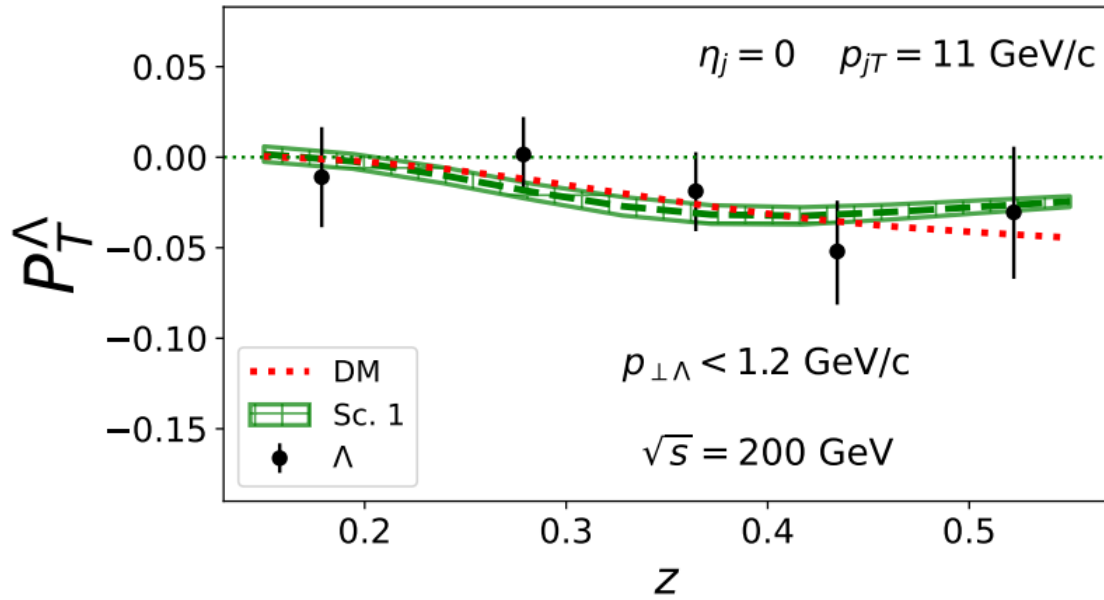
The behaviour in z is driven by the relative contribution of the polFFs:

- In Sc. 1 and 2 only the up is positive \rightarrow leads to a negative value of the polarization

In Sc. 3 both up and down are positive \rightarrow leads to positive value of the polarization at small z and negative at intermediate values.

- Lambda-bar: the polarization is negative and is driven by the negative sign of the sea polFFs.

Unpolarized proton – proton collisions: $pp \rightarrow \Lambda jet X$



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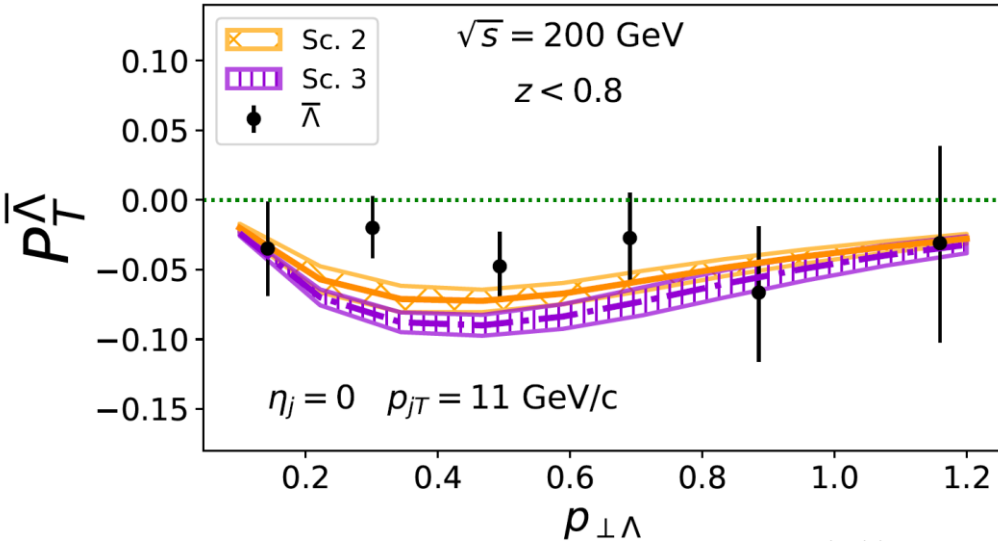
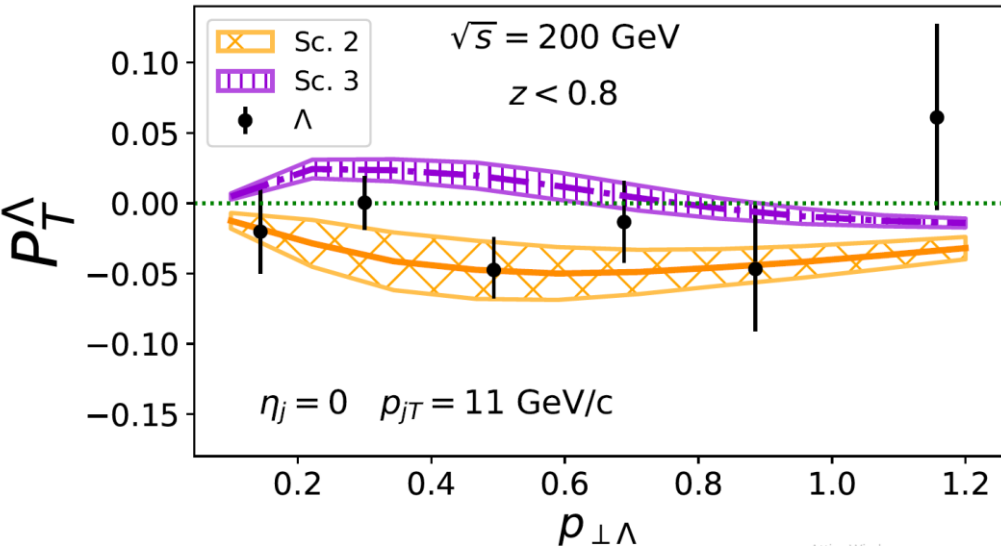
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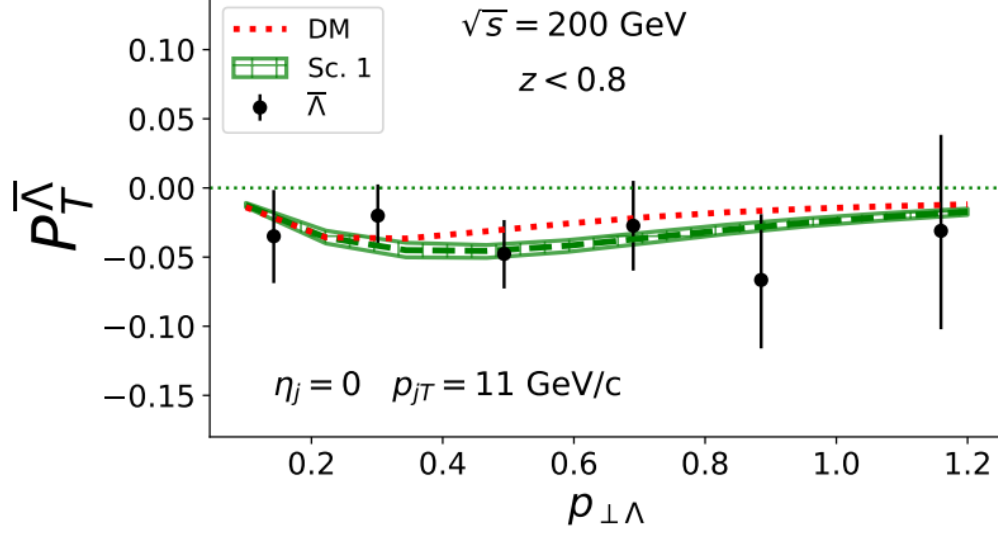
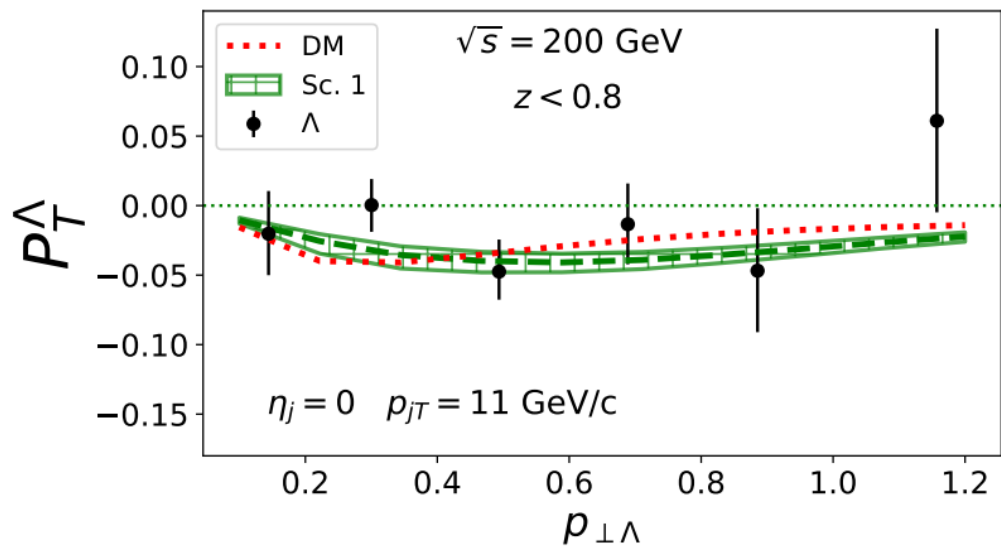
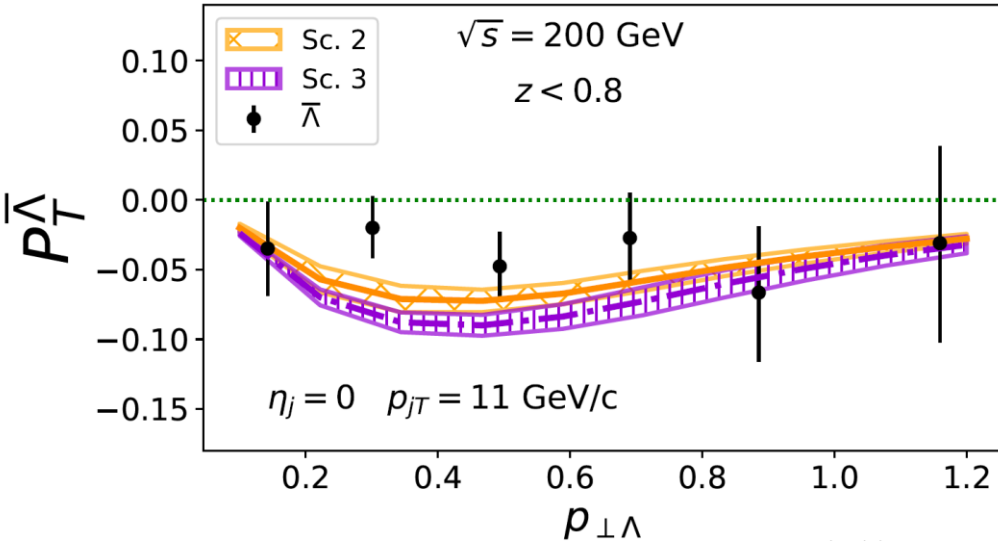
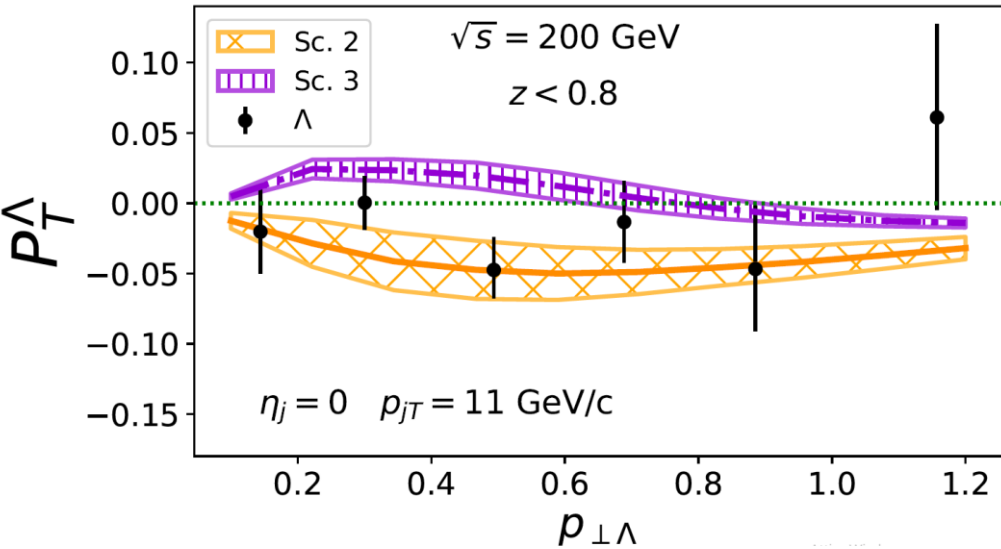
Unpolarized proton – proton collisions: $pp \rightarrow \Lambda \text{ jet } X$

Similar comments as for the z behavior.



Unpolarized proton – proton collisions: $pp \rightarrow \Lambda jet X$

Similar comments as for the z behavior.



Unpolarized proton – proton collisions: $pp \rightarrow \Lambda jet X$

Some remarks:

- No Intrinsic Charm effects;
- General agreement with data but errors bars prevent drawing a strong conclusion.

Potential role of the gluon polFF:

- UnpFF contribution to unpolarized cross sec. is about 50% ;
- Since quark contribution to polarization is about 5-8% ;
- Gluon polFF can be only around 10% of its positivity bound
- First hint on the size of the gluon polFF, at a qualitative level

In e^+e^- and SIDIS we cannot access directly the gluon FF, since it enters only at NLO

Conclusions

- **Double hadron production in e^+e^- :**
Fit results
Charm is necessary!
(2) and (3) scenarios cannot be distinguished \rightarrow open issues!
- **SIDIS:**
predictions for the transverse Lambda polarization
(2) and (3) scenarios predictions are different and can be distinguished
- **$pp \rightarrow \Lambda jet X$:**
estimates compared with STAR data
first hint on the size of the gluon polFF

Backup

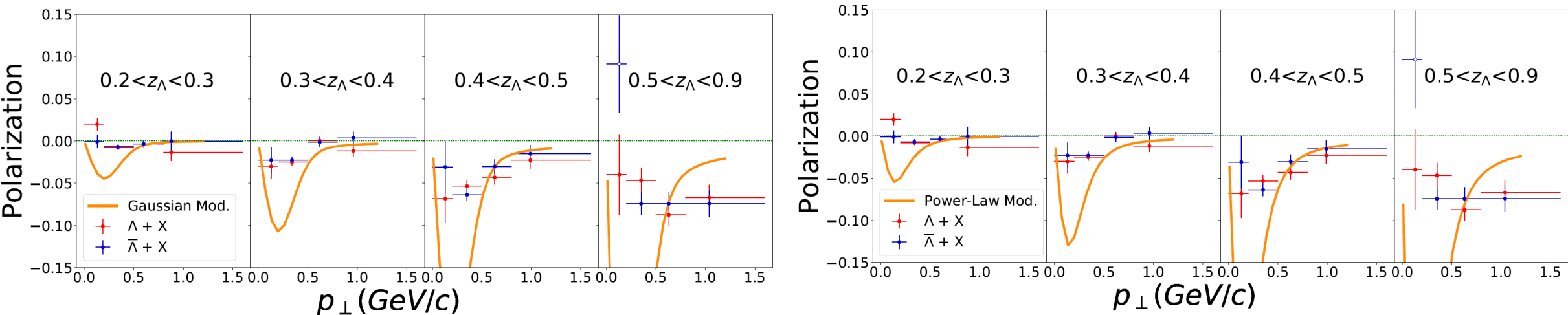
Unpolarized proton – proton collisions: $pp \rightarrow \Lambda jet X$

- Universality of the polarizing FF (provided that factorization holds)
- Role of the charm contribution and SU(2) isospin symmetry;
- Gluon TMD unpolarized and polarizing Fragmentation Function

$$g_K^q = g_2 \frac{b_T^2}{2}, g_K^g = \frac{C_A}{C_F} g_K^q$$

Single-inclusive Polarization

The parameters extracted in 2-h Fit cannot reproduce the 1-h data



If we include 1-h data

Polarizing	Unpolarized	g_K	$\chi_{dof}^2(2-h)$	$\chi_{dof}^2(2-h + 1-h)$
Gaussian	Power-Law	Logarithmic	1.192	2.813
Power-Law	Power-Law	Logarithmic	1.21	2.39

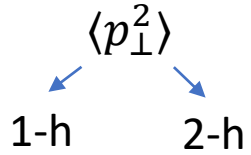
Different factorization or different hadronic model?

Different combinations of NP functions fits give $\chi_{dof}^2 = [2.4 - 5.4]$

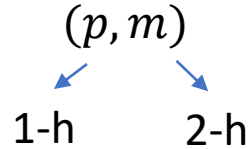
Combined Fit: Double Model

- Same parametrization for $D_{1T}^{\perp(1)}(b_T)$
- Two set of parameters for hadron models

Gaussian mod.

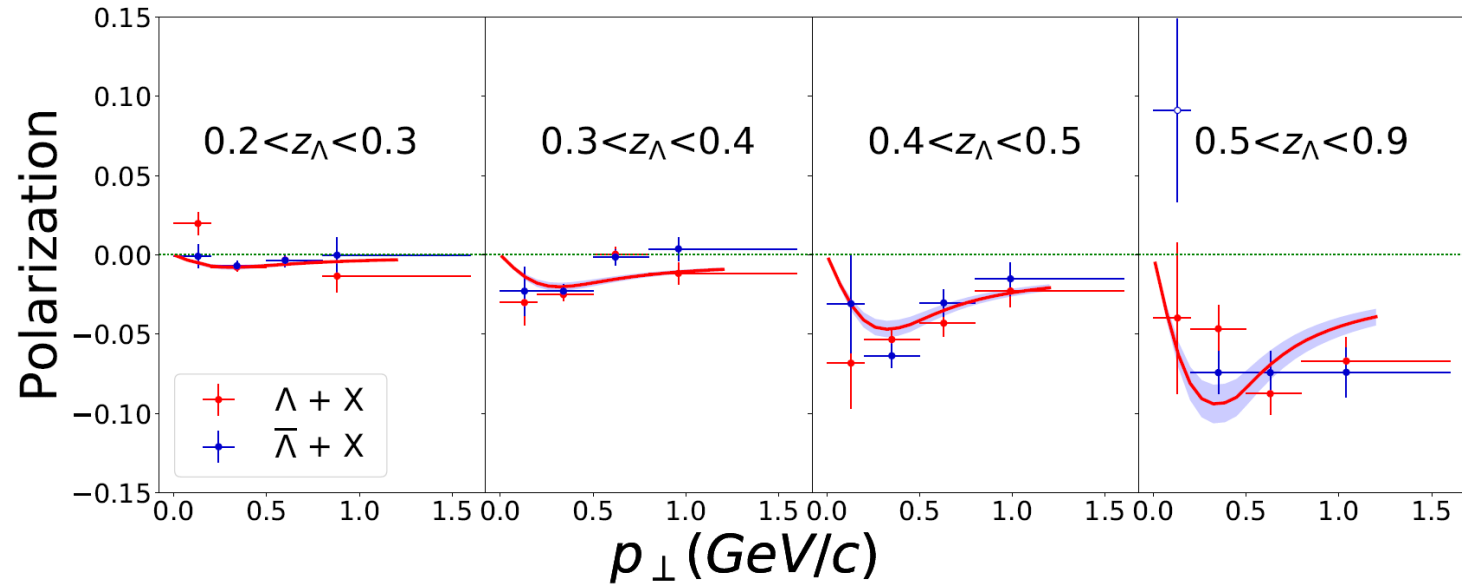


Power-Law mod.

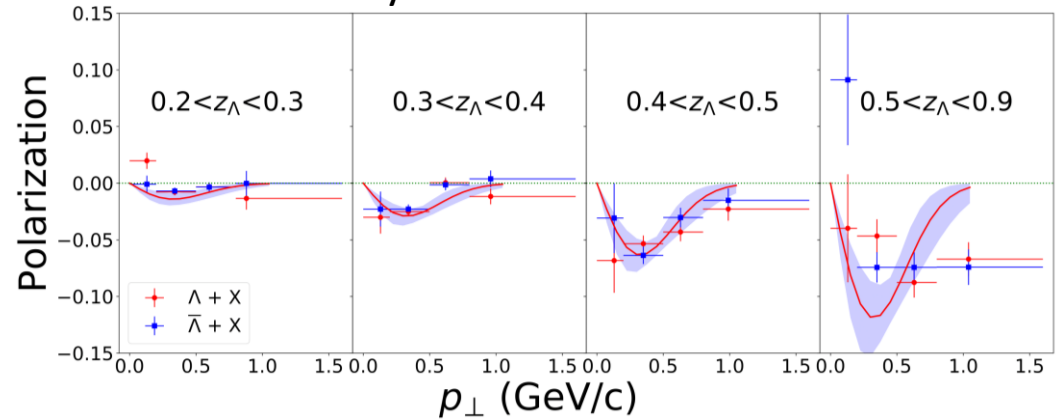


Gaussian		Power-Law	
$\chi_{dof}^2 = 1.801$		$\chi_{dof}^2 = 1.565$	
2-h	1-h	2-h	1-h
$\langle p_{\perp}^2 \rangle_p$	0.04 0.2	p	1.352 1.623
		m	0.151 0.48

With Power-Law model

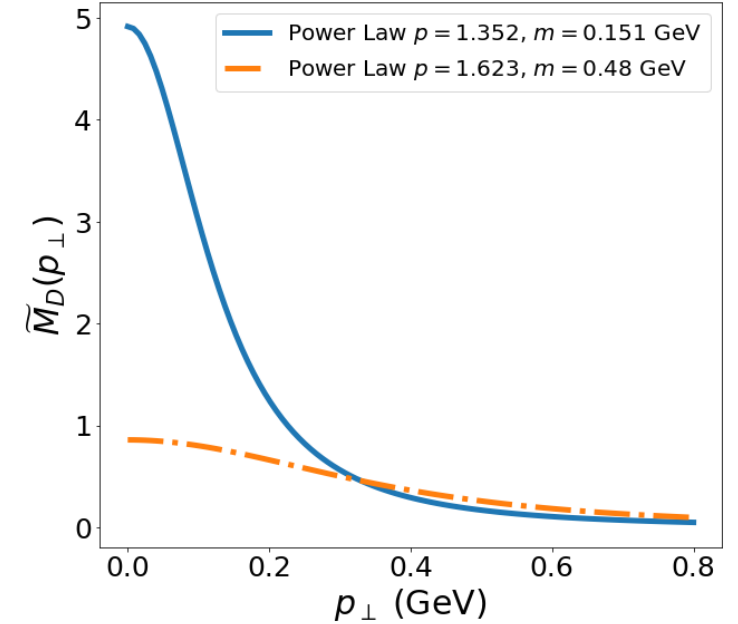
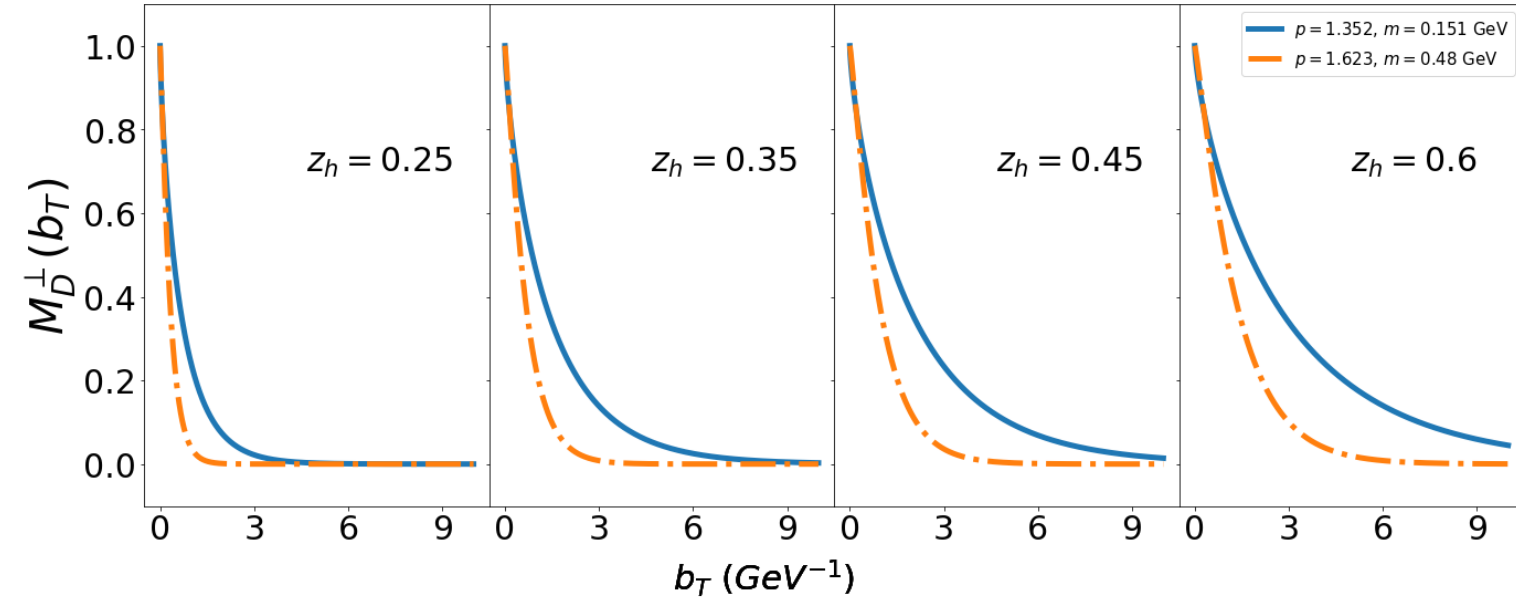


Previous analysis



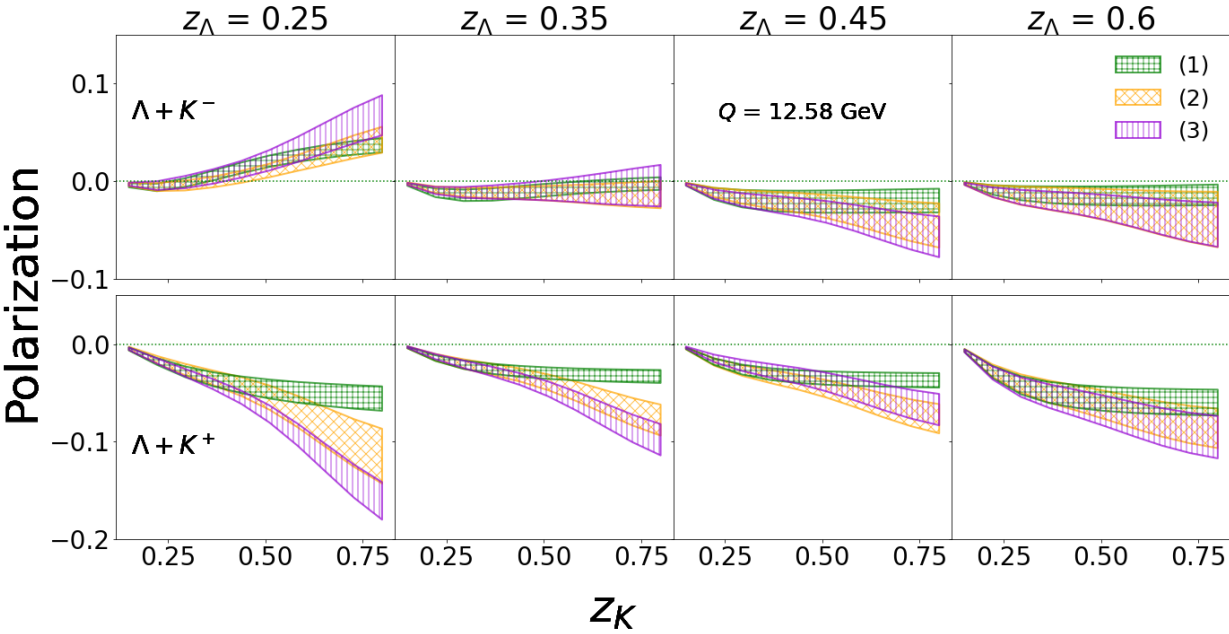
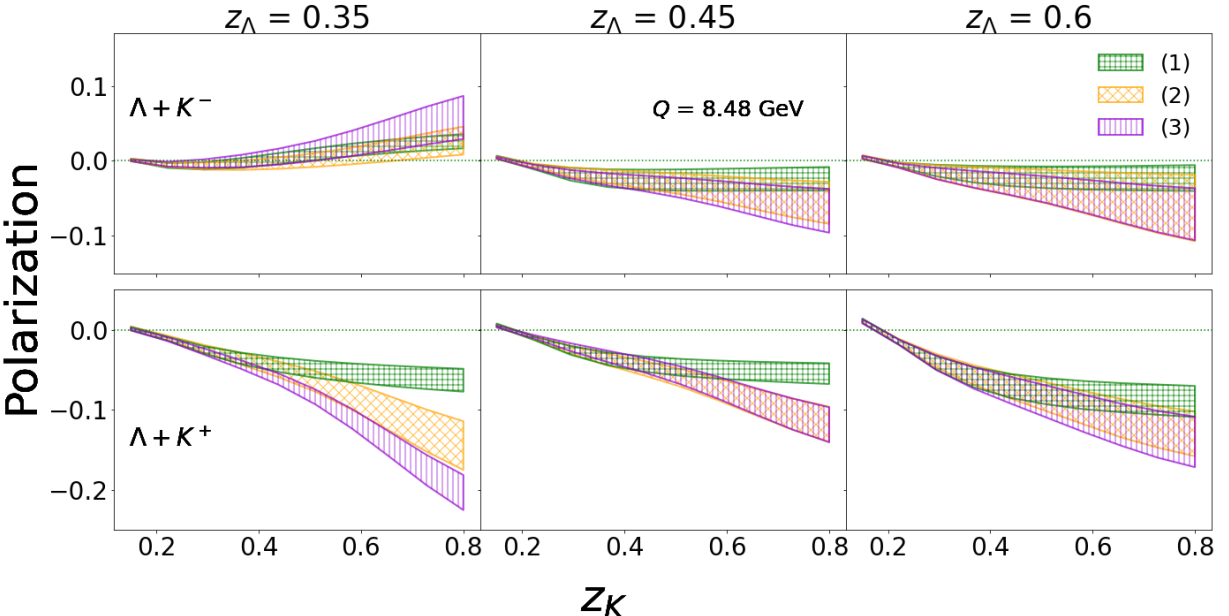
Combined Fit: Double Model

- 2-h Power-Law model
- 1-h Power-Law model

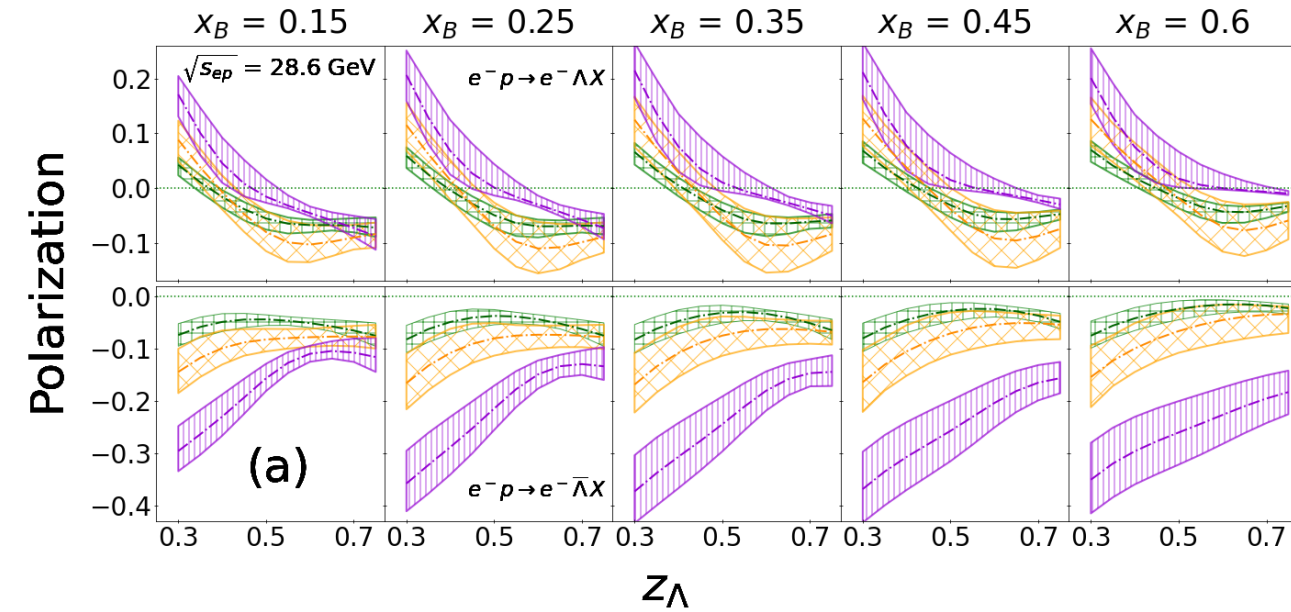


- Both models have same value at small b_T \longrightarrow collinear limit
- In p_{\perp} - space: same value at large p_{\perp}
- 2-h wider than 1-h: different behaviour at large b_T
- In p_{\perp} - space: different value at small p_{\perp}
- Possible different contribution from Soft gluons

Double hadron production in e^+e^- processes

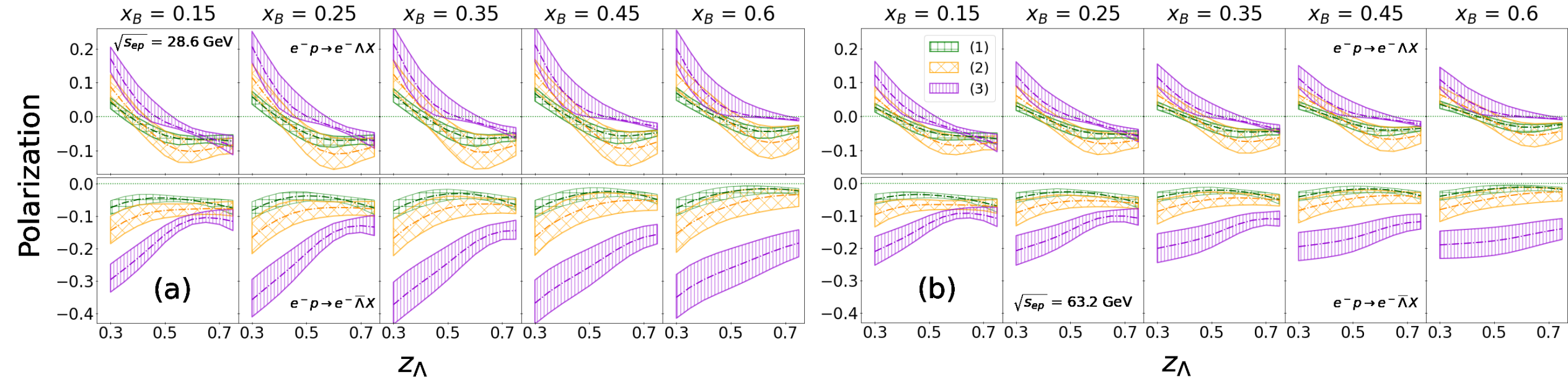


Semi-inclusive Deep Inelastic Scattering



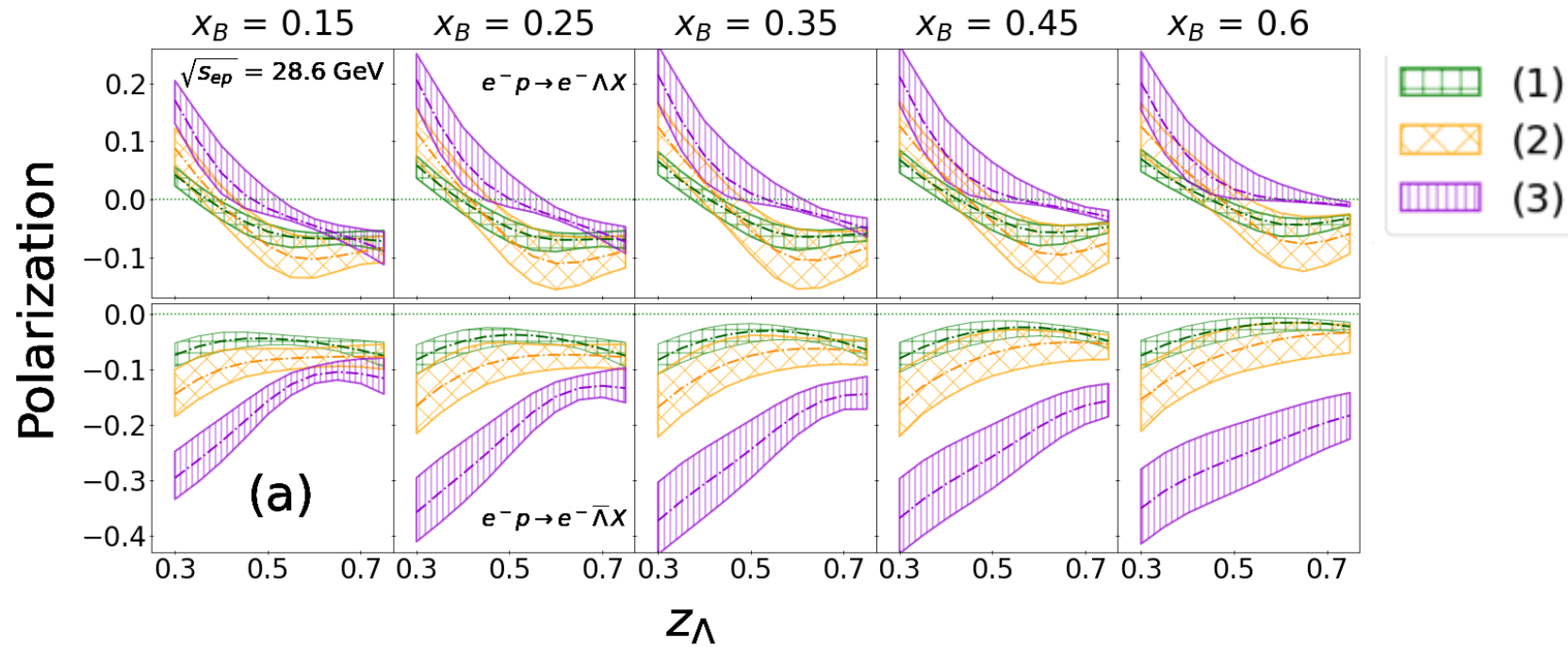
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- $\bar{\Lambda}$ most significant difference;

Semi-inclusive Deep Inelastic Scattering



- (1) & (2) scenarios: polarization of similar size and behavior;
- (3) scenario: similar size;
- Λ pol. decreases and becomes negative;
- $\bar{\Lambda}$ is always negative;
- $\sqrt{s_{ep}}=28,6$ pol. has the same size, for greater values there is a general reduction as x_B grows.
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Semi-inclusive Deep Inelastic Scattering

The charm contribution in the fragmentation process is relevant

Intrinsic Charm (IC) component in the proton:

- CT14nnloIC set with BHPS model [T.-J. Hou et al., *JHEP* 02 (2018) 059]
- NNPDF4.0nnlo set [NNPDF Coll., *Eur.Phys.J.C* 82 (2022) 5, 428]

Semi-inclusive Deep Inelastic Scattering

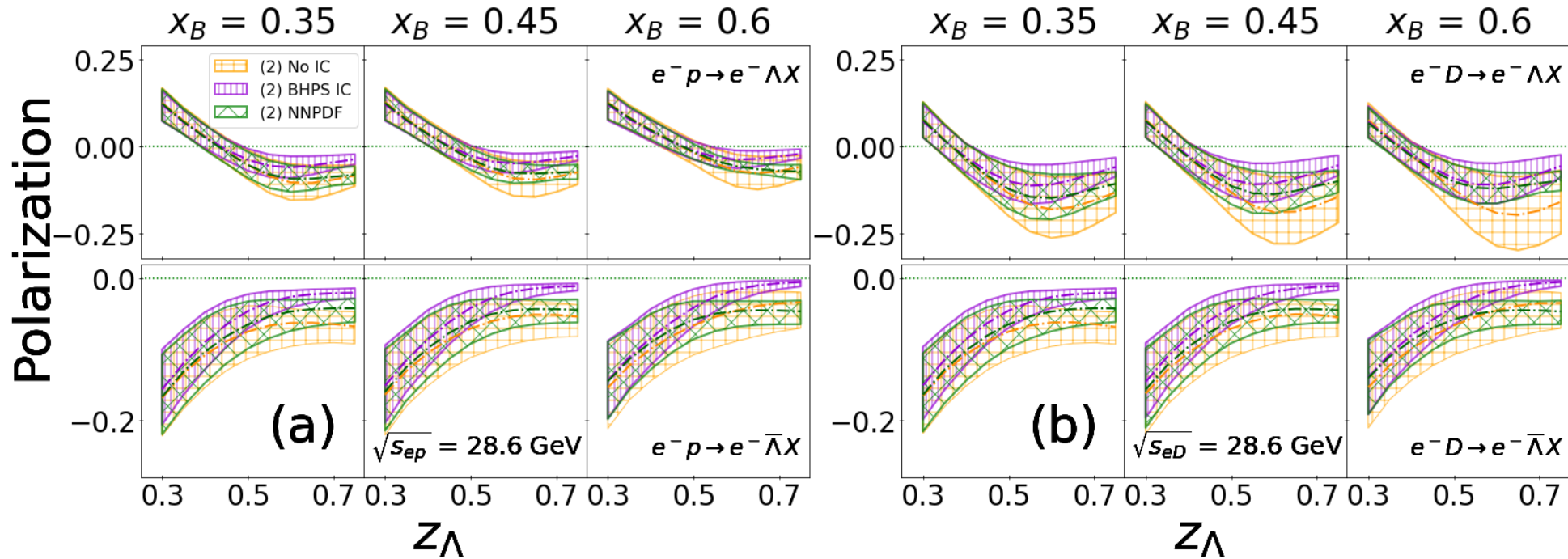
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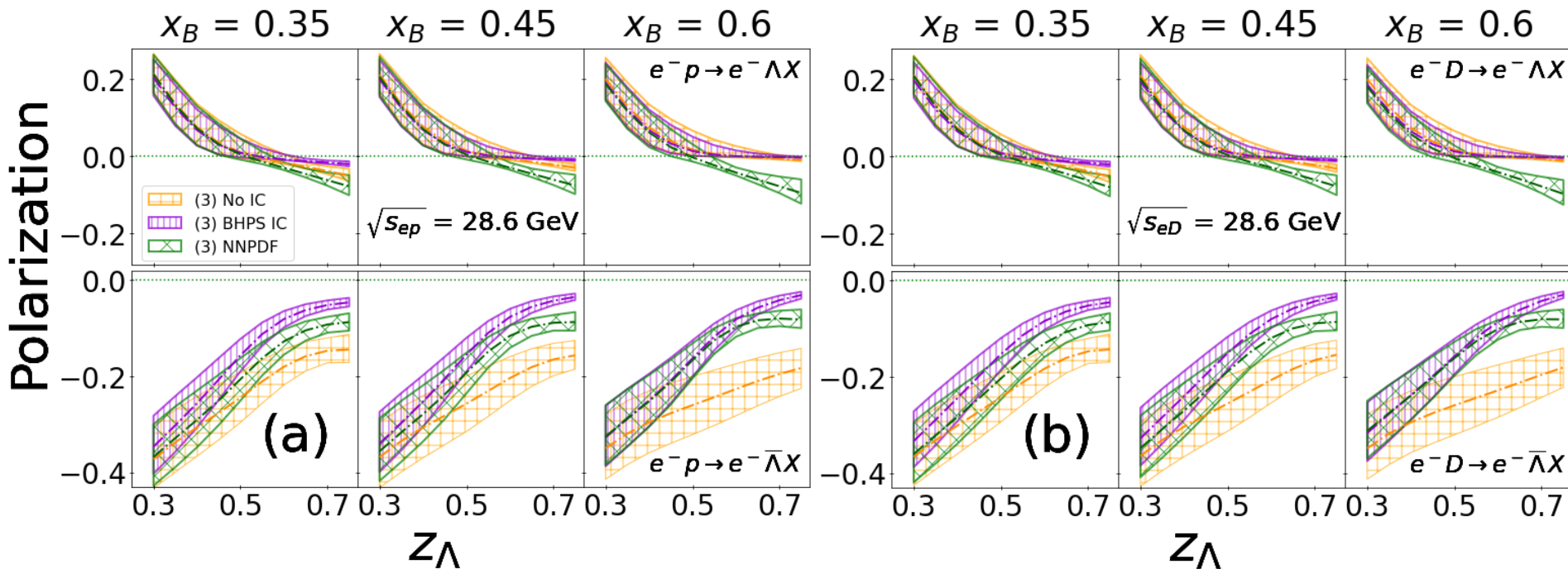
- BHPS and NNPDF: similar polarization of previous predictions
- Same behavior is present for greater values of the c.m. energy.

(2) Scenario:



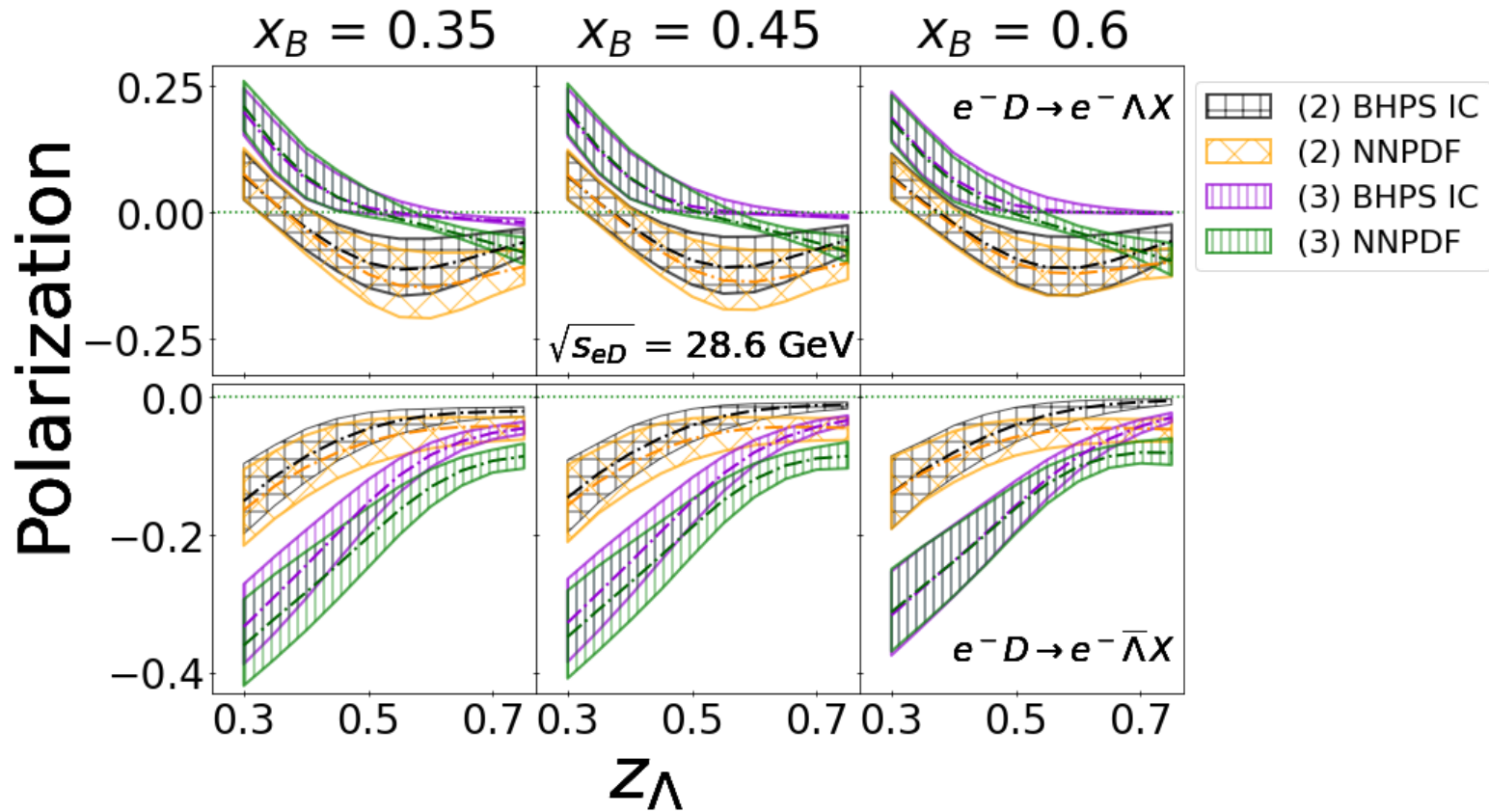
Semi-inclusive Deep Inelastic Scattering

(3) Scenario:



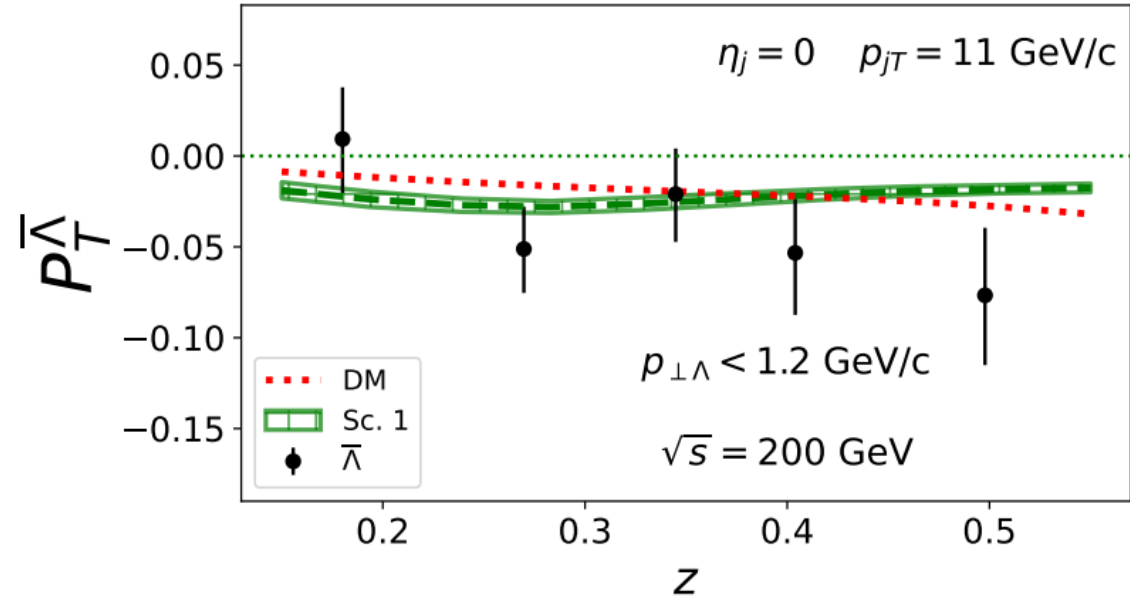
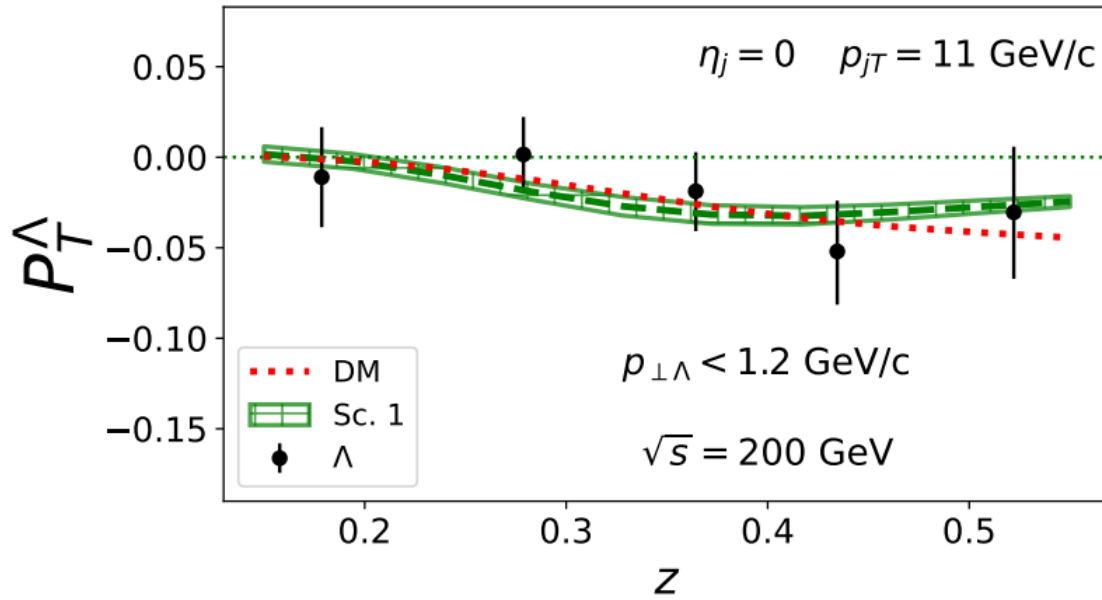
- Estimates vary significantly as x_B increases;
- $\bar{\Lambda}$ estimates with BHPS and NNPDF different from the previous ones;
- Λ : decreases to zero
- $\bar{\Lambda}$: NNPDF become negative

Semi-inclusive Deep Inelastic Scattering



- Λ : the predictions are compatible
- $\bar{\Lambda}$: within (2) and (3) are still different

Unpolarized proton – proton collisions: $pp \rightarrow \Lambda jet X$



The behaviour in z is driven by the relative contribution of the polFFs:

- In Sc. 2 only the up is positive
- In Sc. 3 both up and down are positive

This results into a positive value of the polarization in Sc. 3 at small z
Becoming negative at intermediate values.

For anti Lambda: in both scenarios the polarization is negative and is driven
By the negative sign of the sea polFFs.

No IC effects

General agreement with data

The large experimental error bars prevent
Us to draw strong conclusions