Quantum Monte Carlo Approaches to Lepton-Nucleus Scattering

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• Motivations
• Nuclear Interactions/Currents
• Inclusive Electron Scattering
• Two-Nucleon Structure/dynamics
• Short-time Evolution and two-nucleon dynamics
• Quantum computing approaches
• Summary / Outlook
Why study Leptons and Nuclei (nuclear scale and beyond)

Nuclear structure and dynamics at scale of inter-nucleon spacing

Quasi-elastic scattering: electrons and neutrinos (even 0+ to 0+)

Neutrino Properties: hierarchy, CP violation, double beta decay

Astrophysical Environments: neutron star mergers, supernovae

NP vs PP back-to-back pairs

Atmospheric Neutrinos

Accelerator Neutrinos

EMC effect (nuclear dependence)

Quasi-Elastic scattering, resonance region and deep inelastic scattering

References:

Oscillation Probabilities for Atmospheric Neutrinos

In Fig. 1, we present several oscillation probability as function of neutrino energy for \( E < 1 \) GeV, oscillations are induced largely by the aptly-named atmospheric neutrinos, and as in searches for new physics in the neutrino sector. We turn our attention to them now.

From Nuclear to Hadronic Scales


Pastore, et al; PRC 2018
Accelerator Neutrino Experiments

wide range of neutrino energies

importance of oscillations/cross sections for energies ~1-3 GeV

![Graph showing neutrino flux and oscillation probabilities for different experiments.](image)
Nuclear Scale: Quasi-elastic scattering

\[ \frac{d^2\sigma}{d\Omega_e'dE_{e'}} = \left( \frac{d\sigma}{d\Omega_e} \right)_M \left[ \frac{Q^4}{|q|^4} R_L(|q|,\omega) + \left( \frac{1}{2} \frac{Q^2}{|q|^2} + \tan^2 \frac{\theta}{2} \right) R_T(|q|,\omega) \right] \]

- Scaling with momentum transfer: ‘y’-scaling incoherent sum over scattering from single nucleons - scaling of 1st kind
- Target independence: Cross section nearly independent of nuclear target (after counting nucleons) - ‘superscaling’
Quasi-Elastic Scattering and Plane Wave Impulse Approximation

Incorporates incoherent scattering of single nucleons: $n(k)$ or spectral function $S(k,w)$ and single-nucleon form factors
Single-Nucleon Momentum Distributions

### Chiral Interactions

**4He: AV6'**

**16O: AV6'**

**4He: N^2LO R_0=1.0 fm**

**16O: N^2LO R_0=1.0 fm**

**4He: N^2LO R_0=1.2 fm**

**16O: N^2LO R_0=1.2 fm**

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### Different Nuclei

**AFDMC:**

- **4He AV6'**
- **16O AV6'**

**CVMC:**

- **40Ca AV18**

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**Integrated Strength:**

- 15-20% above k_F
- Amplitude ~ 0.3-0.4

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**Scaling of the 1st kind (w/ p)**

Donnelly & Sick (1999)
But, scattering from a single nucleon not the whole story

Scaled longitudinal vs. transverse scattering from $^{12}\text{C}$

from Benhar, Day, Sick, RMP 2008
data Finn, et al 1984

Distances probed at various $q$

$q$  
0.3 GeV/c 2.1 fm  
0.5 GeV/c 1.2 fm  
1 GeV/c 0.6 fm

Nearest-neighbor distances in nuclear matter

Nearest neighbor nucleons at
$\rho = 0.16 \text{ fm}^{-1} = 1 / (4/3 \pi r^3)$

$r = 1.14 \text{ fm}$
$d = 2.28 \text{ fm}$

$l/m_\pi \sim 1.5 \text{ fm}$
Electron Scattering:
Longitudinal and Transverse Response

Transverse (current) response:
\[ R_T(q, \omega) = \sum_f \langle 0 | j^\dagger(q) | f \rangle \langle f | j(q) | 0 \rangle \delta(\omega - (E_f - E_0)) \]

Longitudinal (charge) response:
\[ R_L(q, \omega) = \sum_f \langle 0 | \rho^\dagger(q) | f \rangle \langle f | \rho(q) | 0 \rangle \delta(\omega - (E_f - E_0)) \]

Requires models of nuclear interactions and currents
Basic building blocks: Nuclear interactions and currents

NN interactions

3N interactions

NN currents
Low Momenta - Beta Decay in Light Nuclei

Pastore, et al, 2017

- Contact fit to Tritium beta decay
- Substantial reduction due to two-body correlations
- Modest 2N current contribution
- Good description of experimental data, explains ‘quenching’
- Many calculations with larger nuclei underway
Back to Back Nucleons (total $Q \sim 0$)

np pairs dominate over nn and pp


2-nucleon momentum distributions

Bob Wiringa, Diego Lonardoni

Nucleon pairs at short distance and nuclear EMC

**EMC slope vs A**

- $\frac{dR_{EMC}}{dx} = m(a_2 - 1)$
- $m = 0.084(4)$
- $\chi^2$/datum = 0.98

**Ratios vs. $a_2$**

- $N^2LO\ E\tau\ R_0 = 1.0\ fm$
- $N^2LO\ E\tau\ R_0 = 1.2\ fm$
- AV18 + UX
- AV4 + UX
- Exp

Lynn, et al, 1903.12587
Electron Scattering: Longitudinal and Transverse Response

Transverse (current) response:
\[ R_T(q, \omega) = \sum_f \langle 0 | j^\dagger(q) | f \rangle \langle f | j(q) | 0 \rangle \delta(\omega - (E_f - E_0)) \]

Longitudinal (charge) response:
\[ R_L(q, \omega) = \sum_f \langle 0 | \rho^\dagger(q) | f \rangle \langle f | \rho(q) | 0 \rangle \delta(\omega - (E_f - E_0)) \]

Requires models of nuclear interactions and currents
Connections to Lattice QCD: one- and two-N matrix elements

- Elastic Nucleon form factors (particularly axial)
- Inelastic form factors:
  - Inclusive (sum over all hadronic final states): constrains hadronic input
  - Exclusive (e.g. specific π-N final state)
- Two-Nucleon matrix elements w/ current insertions (particularly for NN final state)

Solutions or advances on dealing with sign problem imaginary to real time response and dynamics ....
Nearly Static Property: Sum Rules (Longitudinal Response)

\[ S(q) = \langle 0 | \mathbf{j}^\dagger(q) \mathbf{j}(q) 0 \rangle \]

Gives an indication of total strength, but not energy dependence.

Energy dependence: pion exchange final state interaction.

Two-body dynamics broadens peak for modest q
S. Pastore, et al., 2019
2 Nucleon Currents also important: Vector Current Sum Rule

Sum Rule: Constructive Interference between 1- and 2-body currents w/ tensor correlations

Large enhancement from initial state correlations and two-nucleon currents similar in axial response

Note enhancement from final states have larger momenta

\[ \propto \sigma_i \cdot k \sigma_i \cdot q \ (\sigma_j \cdot k)^2 \ (\tau_i \cdot \tau_j)^2 \ v_\pi^2(k) \]
Sum rules in $^{12}$C: neutral current scattering

Lovato, et. al PRL 2014

Single Nucleon currents (open symbols) versus Full currents (filled symbols)
Full treatment of (inclusive) dynamics: Euclidean Response

Want to calculate

$$ R(q, \omega) = \int dt \langle 0 | j^\dagger \exp[i(H - \omega)t] j | 0 \rangle $$

Can calculate

$$ \tilde{R}(q, \tau) = \langle 0 | j^\dagger \exp[-(H - E_0 - q^2/(2m))\tau] j | 0 \rangle > $$

- Exact given a model of interactions, currents
- `Thermal' statistical average
- Full final-state interactions
- All contributions included - elastic, low-lying states, quasi elastic, ...

Excellent agreement w/ EM (L & T)
response in A=4,12
Lovato, 2015, PRL 2016

Sum rule → elastic FF^2 w/ increasing
Electron Scattering from $^{12}$C: Longitudinal Response

- We inverted the electromagnetic Euclidean response of $^{12}$C
- Good agreement with data without in-medium modifications of the nucleon form factors
- Small contribution from two-body currents.

![Graph showing the longitudinal response function of $^{12}$C, $q=570$ MeV, with curves for GFMC $O_{1b}$, GFMC $O_{1b+2b}$, PWIA, and world data. The graph includes error bars for the world data.](Image)

Caption: $^{12}$C, $q=570$ MeV

Legend:
- Red dashed line: GFMC $O_{1b}$
- Black solid line: GFMC $O_{1b+2b}$
- Green dashed line: PWIA
- Blue line with circles: World data

AL et al. PRL 117 082501 (2016)
Electron Scattering from $^{12}$C: Transverse Response

- We inverted the electromagnetic Euclidean response of $^{12}$C.
- Good agreement with the experimental data once two-body currents are accounted for.
- Need to include relativistic corrections in the kinematics.

![Graph](AL et al. PRL 117 082501 (2016))
Why Y-scaling?

Even though two-nucleon currents are important, main contribution comes from interference w/ one-body currents.

Overall strength enhanced, but through interference w/ same final states: similar shape.
Why superscaling?

For nuclei with $N \sim Z$, bulk density is very similar: nuclear saturation at $\sim 0.16$ fm$^{-3}$

also pair densities very similar for $A > 12$ nuclei

Inclusive Scattering at Quasi-Elastic energies and momenta is a nearly local operator

Free particle propagator:

$$\exp[-(r - r')^2 / (4\frac{\hbar^2}{2m}\delta\tau)]$$

at $\delta\tau = 1/100$ MeV; $r-r' \sim 1.1$ fm

at $\delta\tau = 1/50$ MeV; $r-r' \sim 1.6$ fm
We recently computed the charged-current response function of $^{12}$C

- Calculations from $q = 200 - 700$ MeV/c
We recently computed the charged-current response function of $^{12}$C.

Two-body currents have a sizable effect in the transverse response, both in the vector and in the axial contributions.

Calculations from $q = 200 - 700$ MeV/c.
Towards Exclusive Scattering and Larger Nuclei

Ground-state nuclei: doable with some approximations
Propagation: $^{12}\text{C}$ GFMC calculations to $\tau \sim 0.1 \text{ MeV}^{-1}$
Each particle propagates $\sim 3 \text{ fm}$

Sign problem much worse in Ar than Carbon
Any fermion interchange in the system contributes to the noise

How much information can we get from very short \textit{real} times?
Short Time Approximation: Towards real-time dynamics
Saori Pastore, et al, 2019

\[ R^O(q, \omega) = \frac{1}{4\pi} \int d\Omega_q \sum_f \langle \Psi_0 | \mathcal{O}^\dagger(q) | \Psi_f \rangle \langle \Psi_f | \mathcal{O}(q) | \Psi_0 \rangle \delta(E_f - E_0 - \omega), \]

\[ R^O(q, \omega) = \frac{1}{4\pi} \int d\Omega_q \int \frac{dt}{2\pi} \exp[i\omega t] \langle \Psi_0 | \mathcal{O}^\dagger(q, t') \exp[-iHt] \mathcal{O}(q, t = 0) \Psi_0 \rangle, \]

At short time evolution can be described as a product of NN propagators

\[ \langle \mathbf{R}', \sigma', \tau' | \exp[-iHt] | \mathbf{R}, \sigma, \tau \rangle \approx \langle \mathbf{R}', \sigma', \tau' | \prod_i \exp[-iH^0_i t] S \prod_{i<j} \exp[-iH_{ij}(t)] \prod_{i<j} \exp[-iH^0_{ij}(t)] | \mathbf{R}, \sigma, \tau \rangle \]

Evaluate as a sum of matrix elements of NN states embedded in the nucleus

Incoherent sum of single nucleon currents

\[ \sum_{q,Q,J,L,S,T} \langle \Psi_0 | j_i^\dagger | \psi_{NN}(q, Q) \rangle \langle \psi_{NN}(q, Q) | j_i | \Psi_0 \rangle \delta(E_f - E_i - \omega) \]

Interference of 1- and 2-nucleon currents

\[ \sum_{q,Q,J,L,S,T} \langle \Psi_0 | j_{ij}^\dagger | \psi_{NN}(q, Q) \rangle \langle \psi_{NN}(q, Q) | j_i | \Psi_0 \rangle \delta(E_f - E_i - \omega) \]

Diagonal 2-nucleon currents

\[ \sum_{q,Q,J,L,S,T} \langle \Psi_0 | j_{ij}^\dagger | \psi_{NN}(q, Q) \rangle \langle \psi_{NN}(q, Q) | j_{ij} | \Psi_0 \rangle \delta(E_f - E_i - \omega) \]
Short Time Approximation: Towards real-time dynamics

Saori Pastore, et al, 2019

\[ R^O(q, \omega) = \frac{\int d\Omega_q}{4\pi} \int \frac{dt}{2\pi} \exp[i\omega t] \langle \Psi_0 | \mathcal{O}^\dagger(q, t') \exp[-iHt] \mathcal{O}(q, t = 0) \Psi_0 \rangle, \]

At short time evolution can be described as a product of NN propagators

\[ \langle \mathbf{R}', \sigma', \tau' | \exp[-iHt] | \mathbf{R}, \sigma, \tau \rangle \approx \langle \mathbf{R}', \sigma', \tau' | \prod_i \exp[-iH_i^0t] \frac{S \prod_{i<j} \exp[-iH_{ij}t]}{\prod_{i<j} \exp[-iH_{ij}^0t]} | \mathbf{R}, \sigma, \tau \rangle \]

Evaluate as a sum of matrix elements of NN states embedded in the nucleus

A set of two-nucleon off-diagonal density matrix matrix elements:

- Calculate for each operator and each q
- Incorporates: Exact sum rule nearly exact energy-weighted sum rule
- Incorporates full Pauli principal (A-nucleon ME)
- Information on the 2-nucleon quantum state right after the vertex - couple with semi-classical event generators
component and the spectator nucleus, one can more easily incorporate relativistic kinematics and currents, pion production, and resonance production. Treating such effects at the two-nucleon level is vastly easier than calculating the same processes in a full A-nucleon treatment. We expect that interference processes, for example different processes leading to pion production, may be important here as well.

VI. ACKNOWLEDGEMENTS


Response Densities

Transverse Density $q = 300$ MeV

- Calculate individual response densities as a function of CM and relative energies of the struck pair
- The integral over surfaces w/ constant $e+E$ gives full response
Response Densities

Fraction of Transverse response that include a 2N current

\[ q = 500 \]

Large impact of 2-body currents at high relative energy np vs. pp, etc.

np vs pp in back-to-back kinematics
Comparison to Data (A=4)

FIG. 11. Transverse responses at $q = 300–600$ MeV/c compared with the world data [10] for specific electroweak two-nucleon current operators and Pauli blocking between the struck and spectator nucleons. The cost is that it must be evaluated explicitly in the ground state for each momentum transfer $q$ and each transition current operator. Additionally, the STA provides information about pairs of nucleons at the interaction vertex. This can be very valuable when trying to understand more exclusive processes like back-to-back nucleons that can be measured experimentally. It is also important in neutrino physics where final state information is used to help gain information on the initial neutrino energy, an important ingredient in neutrino oscillation analyses. For large nuclei this information about the vertex will have to be augmented by semi-classical event generators.

The STA is amenable to many improvements associated with calculating responses at higher energy. Since it factorizes the response into two-nucleon component and the spectator nucleus, one can more easily incorporate relativistic kinematics and currents, pion production, and resonance production. Treating such effects at the two-nucleon level is vastly easier than calculating the same processes in a full $A$-nucleon treatment. We expect...
Beyond the short-time approximation: Quantum Computing

Algorithm requires:
• ground-state preparation,
• coupling to current
• real time propagation (short) Roggero, JC; PRC 2019

Highly Simplified Lattice Problem

Scaling with problem size, highly simplified problem on actual QC Roggero, et al, arXiv: 1911.06368 STA is only one (nontrivial) time step
Conclusions

EW processes on nuclei at the $q \geq k_F$ are important, even sometimes at low energy.

electron/neutrino scattering

electron and neutrinos in astrophysics

beta decay and double beta decay

$0^+ \rightarrow 0^+$ beta decay

Good description w/ realistic nuclear interactions

and currents

Real-time dynamics is important
Future directions

• Larger Nuclei

• Relativistic few-nucleon dynamics

• Pion Production (Noemi Rocco, et al) requires NN inelastic processes can we match to lattice

• Quantum to Classical Transition can we match to generators

• Quantum Computing: even a short coherence time may be valuable.