A spectator-model approach to TMD gluon distribution functions

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Università degli Studi di Pavia & Sezione INFN di Pavia

under the "**3DGLUE**" MIUR FARE grant (n. **R16XKPHL3N**)

Based on 🔗 [A. Bacchetta, F.G.C., M. Radici, P. Taels [arXiv:2005.02288]]

HAS QCD
HADRONIC STRUCTURE AND QUANTUM CHROMODYNAMICS

3DSPIN
MAPPING THE PROTON IN 3D

INFN
Istituto Nazionale di Fisica Nucleare
Sezione di Pavia

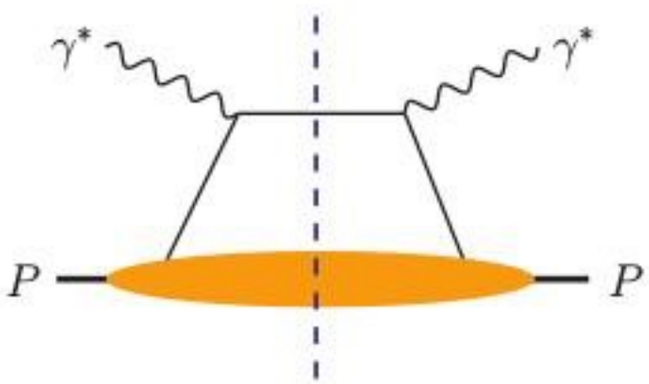
UNIVERSITÀ DI PAVIA

Parton densities: an overview

p_T - integrated

Collinear PDFs

- Inclusive processes
- p_T ~ hardest scale



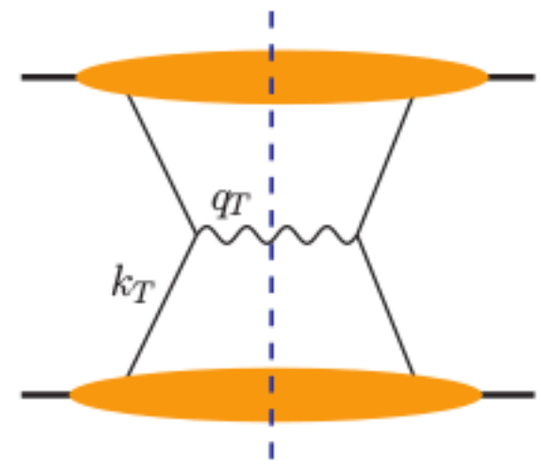
GPDs

- Exclusive processes
- *Skewness* effects

p_T - unintegrated

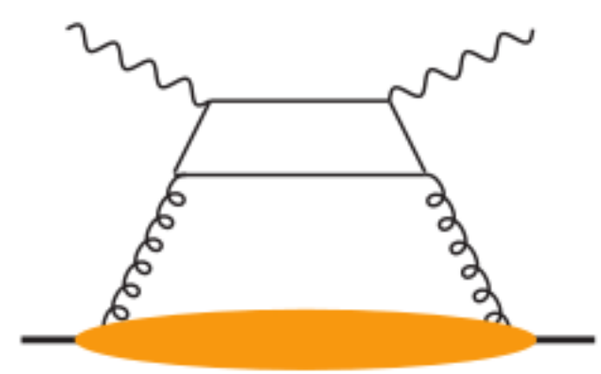
TMDs

- (Semi-)inclusive processes
- p_T << hardest scale



UGD

- **BFKL** 🔗 🔗 🔗 🔗
- Small x, large p_T



An incomplete family tree

Wigner distributions

$$\rho\left(x, \mathbf{k}_T, \mathbf{b}_T\right)$$

$$\int d^2\mathbf{k}_T$$

$$\int d^2\mathbf{b}_T$$

Generalized Parton Distributions

GPDs

Transverse Momentum Distributions

TMDs

$$\int d^2\mathbf{k}_T$$

Parton Distribution Functions

PDFs

An incomplete family tree

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Generalized Parton Distributions

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TMDs

$$\int d^2\mathbf{k}_T$$

Parton Distribution Functions

PDFs

...but wait...

where is the

UGD?

Speaks language of

Reggeized gluons

Total spin carried by quarks and gluons does not amount to 1/2, one needs orbital angular momentum, then a 3D description...

🔗 (proton spin crisis) [EMC Collaboration, CERN (1987)]

Total spin carried by quarks and gluons does not amount to 1/2, one needs orbital angular momentum, then a 3D description...

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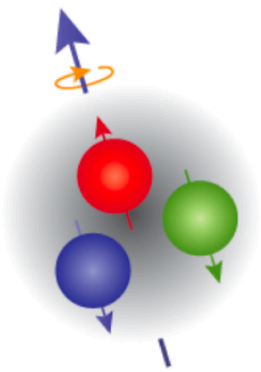
...many other effects in hadronic interactions cannot be understood in the purely collinear approach

PDF and TMD distributions



quarks helicity

~ 30 %

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

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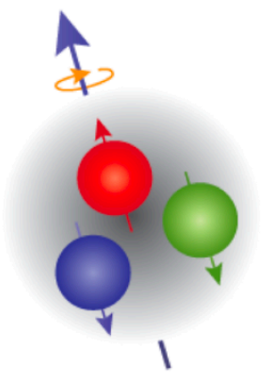

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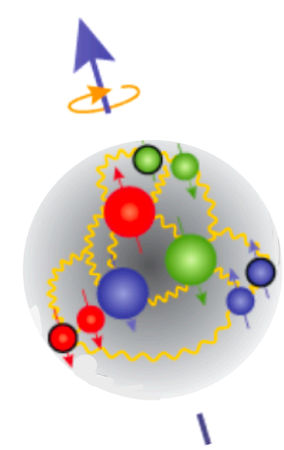
gluons helicity

~ 20% – 50 % **?**



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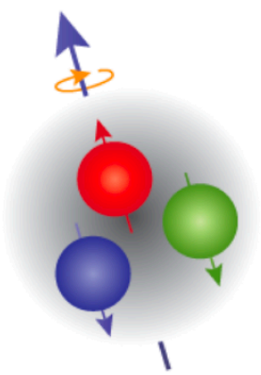

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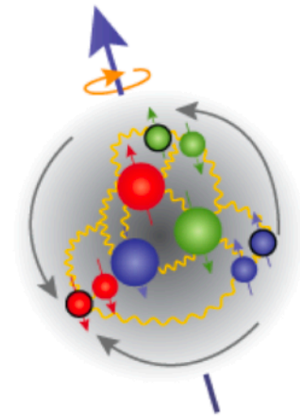


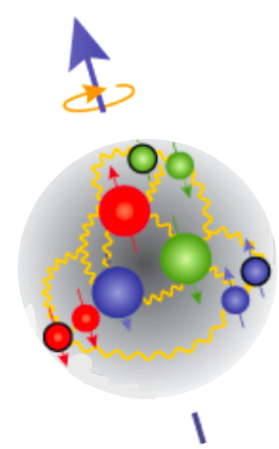
orbital angular momentum

unknown!

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

gluons helicity

~ 20% – 50 % **?**



Quark TMDs: current status

Pavia 2017 NLLsc

global fit: Drell-Yan, SIDIS and Z boson

Bacchetta, Delcarro, Pisano, Radici, Signori

🔗 arXiv:1703.10157

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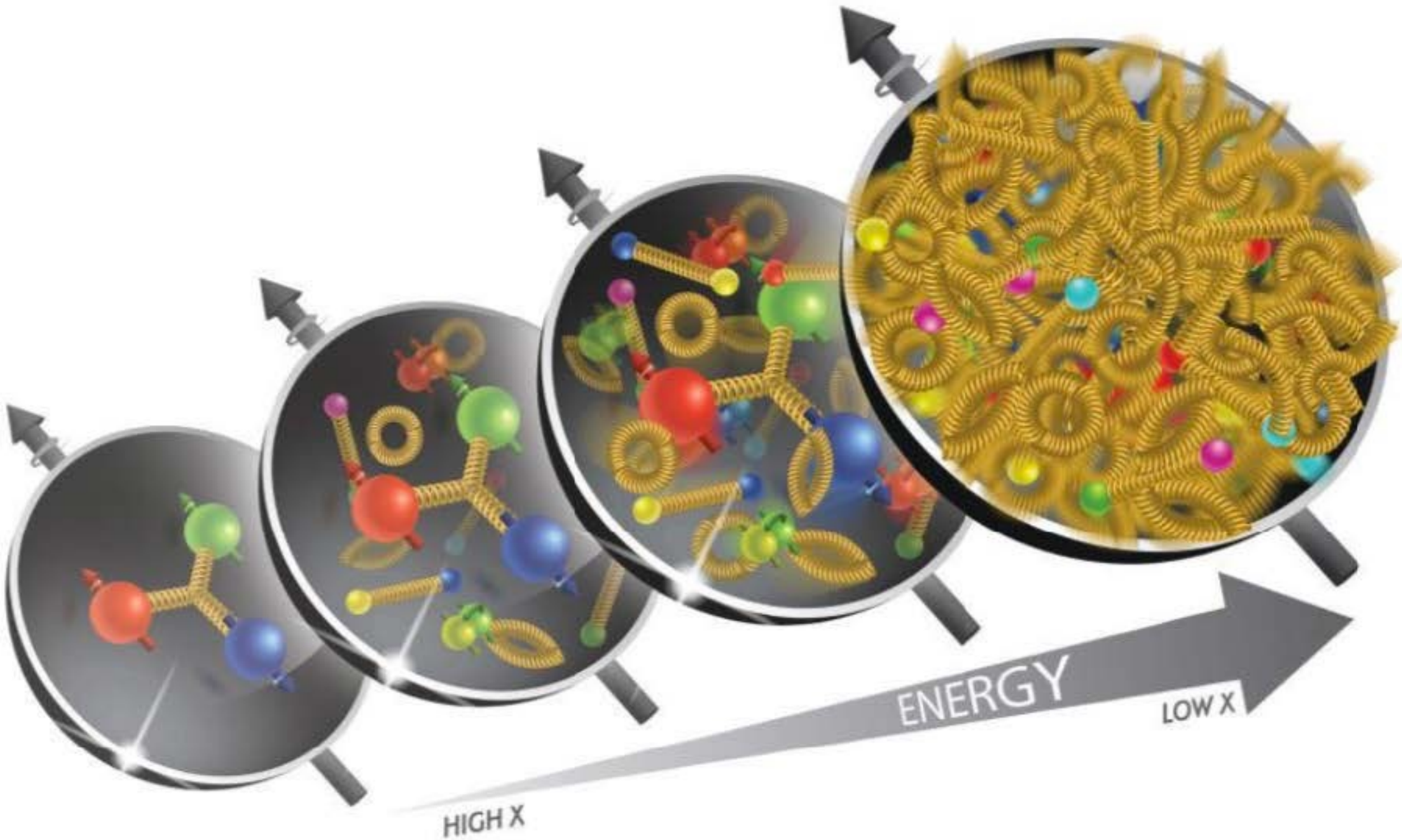
Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici

🔗 arXiv:1912.07550

Gluon TMDs:
a largely unexplored territory



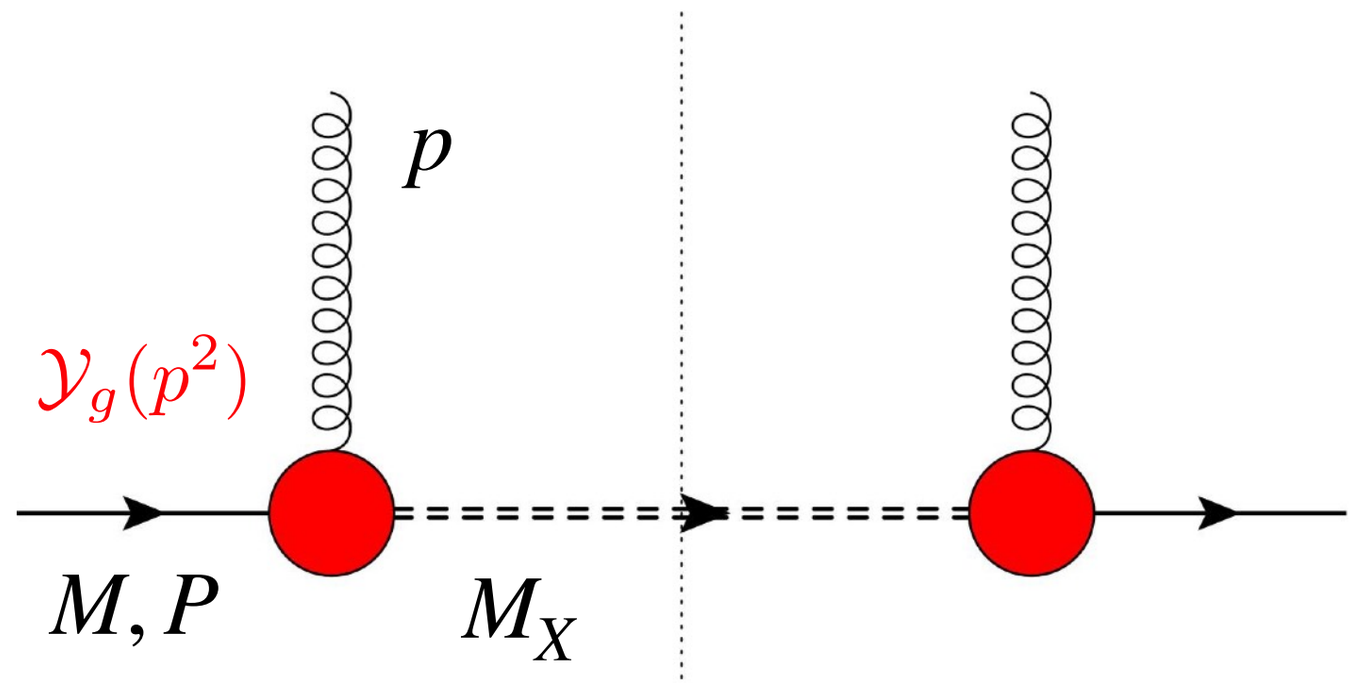
Motivation

- Gluon-TMD PDFs: a *core* sector of **EIC** studies
- Need for a *flexible* model, suited to *phenomenology*
- **Unpolarized** and **polarized gluon TMDs**
- *Consistent* framework for quark TMDs

Assumptions of the model

- **Effective vertex**

  Lowest Fock state:
  tri-quark spectator
  on-shell and
  with mass M_X



- **Spin-1/2 spectator (gluon)**

$$\Phi_g = \frac{1}{2(2\pi)^3(1-x)P^+} Tr\left[(\not{P}+M)\frac{1+\gamma^5\not{S}}{2}G^*_{\mu\rho}(p)G^{\nu\sigma}(p)\mathcal{Y}_g^{\rho *}\mathcal{Y}_{g\sigma}(\not{P}-\not{p}+M)\right]$$

$$\mathcal{Y}_g^\mu = g_1(p^2)\gamma^\mu + i\frac{g_2(p^2)}{2M}\sigma^{\mu\nu}p_\nu$$

mimics proton form factors
(conserved EM current
of a free nucleon)

Assumptions of the model

Link with collinear factorization

p_T-integrated TMDs **have to** reproduce PDFs at the lowest scale (Q_0) *before* evolution

Dipolar form factor(s)

$$g_{1,2}(p^2) = \kappa_{1,2} \frac{p^2}{|p^2 - \Lambda_X^2|^2}$$

1. Cancels singularity of gluon propagator
2. Suppresses effects of high p_T
3. Compensates log divergences arising from p_T-integration
4. Adds three more parameters: $\kappa_{1,2}$ and Λ_X

T-even gluon TMDs at twist-2

gluon pol.

nucleon pol.	U	circ.	lin.
U	f_1^g		$h_1^{\perp g}$
L		g_1^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	h_1^g, $h_{1T}^{\perp g}$

T-even

T-odd

T-even gluon TMDs at twist-2

unpolarized TMD

Boer-Mulders

gluon pol.

nucleon pol.

	U	circ.	lin.
U	f_1^g		$h_1^{\perp g}$
L		g_1^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	h_1^g, $h_{1T}^{\perp g}$

T-even

T-odd

helicity TMD

worm-gear

State of the art

- First calculation of leading-twist T-even quark TMDs with scalar and axial-vector di-quarks

  [R. Jakob, P. J. Mulders, J. Rodrigues (1997)]

- Gluon TMD PDFs and FFs

  [P.J. Mulders, J. Rodrigues (2001)]
  [J. Rodrigues, PhD thesis (2001)]

- Complete calculation of all the leading-twist TMDs with scalar di-quarks

  [S. Meissner, A. Metz, K. Goeke (2007)]

- Inclusion of different axial-vector di-quark polarization states and nucleon-parton-spectator form factors

  (fit to PDF parametrizations) [A. Bacchetta, F. Conti, M. Radici (2008)]
  (application on azimuthal asymmetries) [A. Bacchetta, M. Radici, F. Conti, M. Guagnelli (2010)]

How to improve the description?

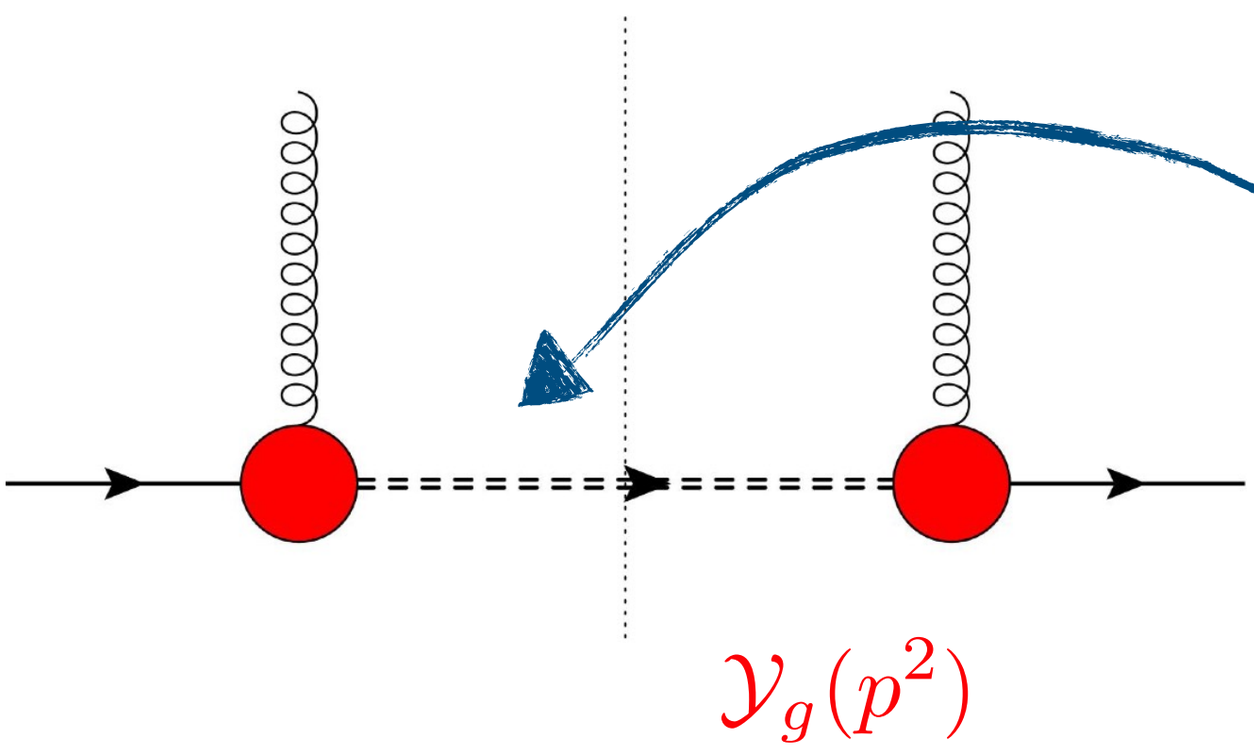
Spectator-system spectral-mass function

spectral-mass function

$$F(x, \boldsymbol{p}_T^2) = \int_M^\infty \mathrm{d}M_X \, \rho_X(M_X) \, \hat{F}(x, \boldsymbol{p}_T^2; M_X)$$

spectator-model TMD

[Inspired by G.R. Goldstein, J.O.G. Hernandez, S. Liuti (2011)]



Instead of a single on-shell spectator, a continuum of spectators

Spectator-system spectral-mass function

spectral-mass function

$$F(x, \boldsymbol{p}_T^2) = \int_M^\infty \mathrm{d}M_X \, \rho_X(M_X) \, \hat{F}(x, \boldsymbol{p}_T^2; M_X)$$

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$$\rho_X\left(M_X; \{X^{(\mathrm{pars})}\} \equiv \{A, B, a, b, C, D, \sigma\}\right) = \mu^{2a}\left[\frac{A}{B + \mu^{2b}} + \frac{C}{\pi\sigma} e^{-\frac{(M_X - D)^2}{\sigma^2}}\right]$$

low-x (high-μ^2) tail $\propto (a - b)$

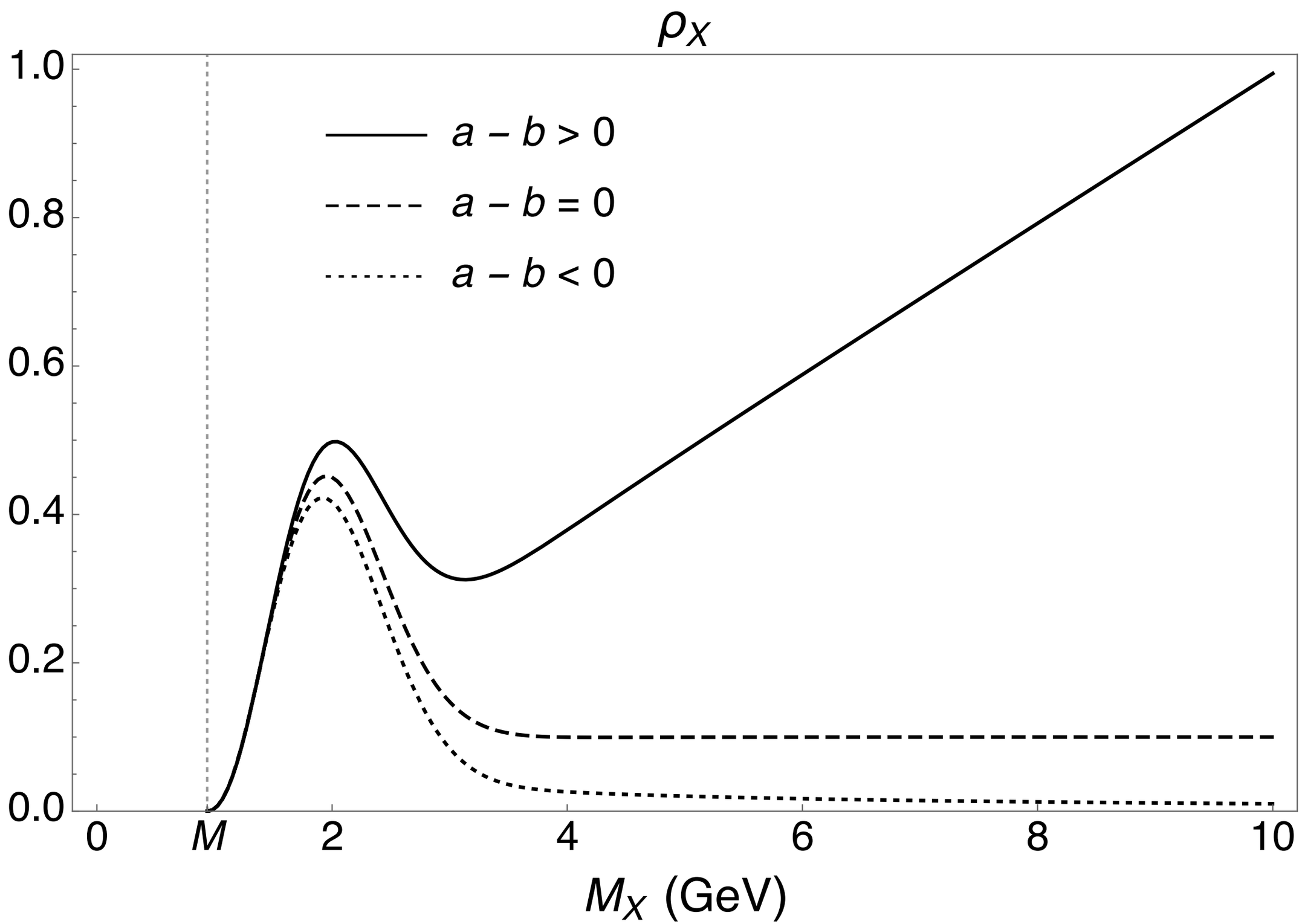
$q\bar{q}$ contributions energetically available at large M_X

$$\mu^2 = M_X^2 - M^2$$

moderate-x trend

pure tri-quark contribution at low M_X

Spectral function vs $(a - b)$



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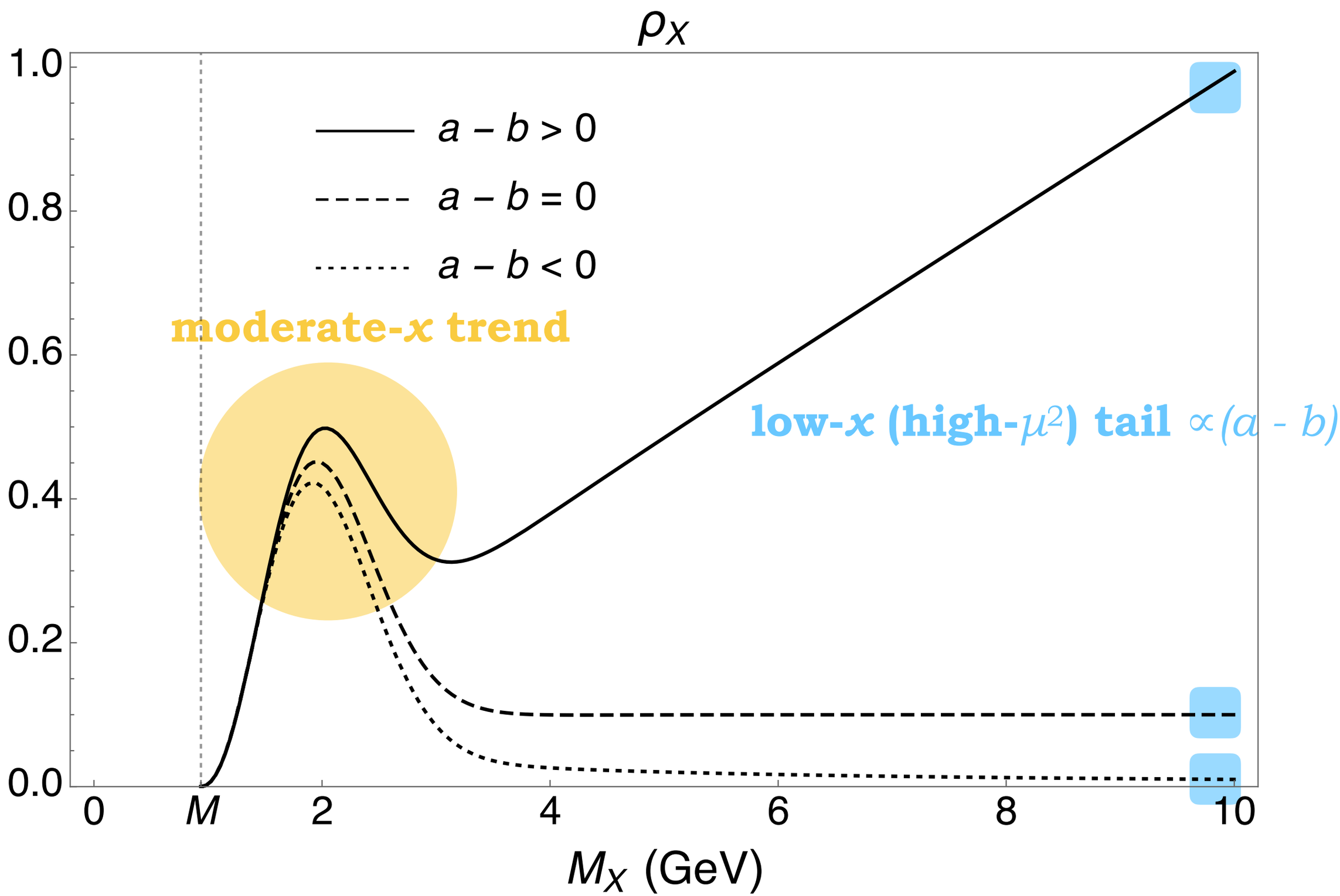
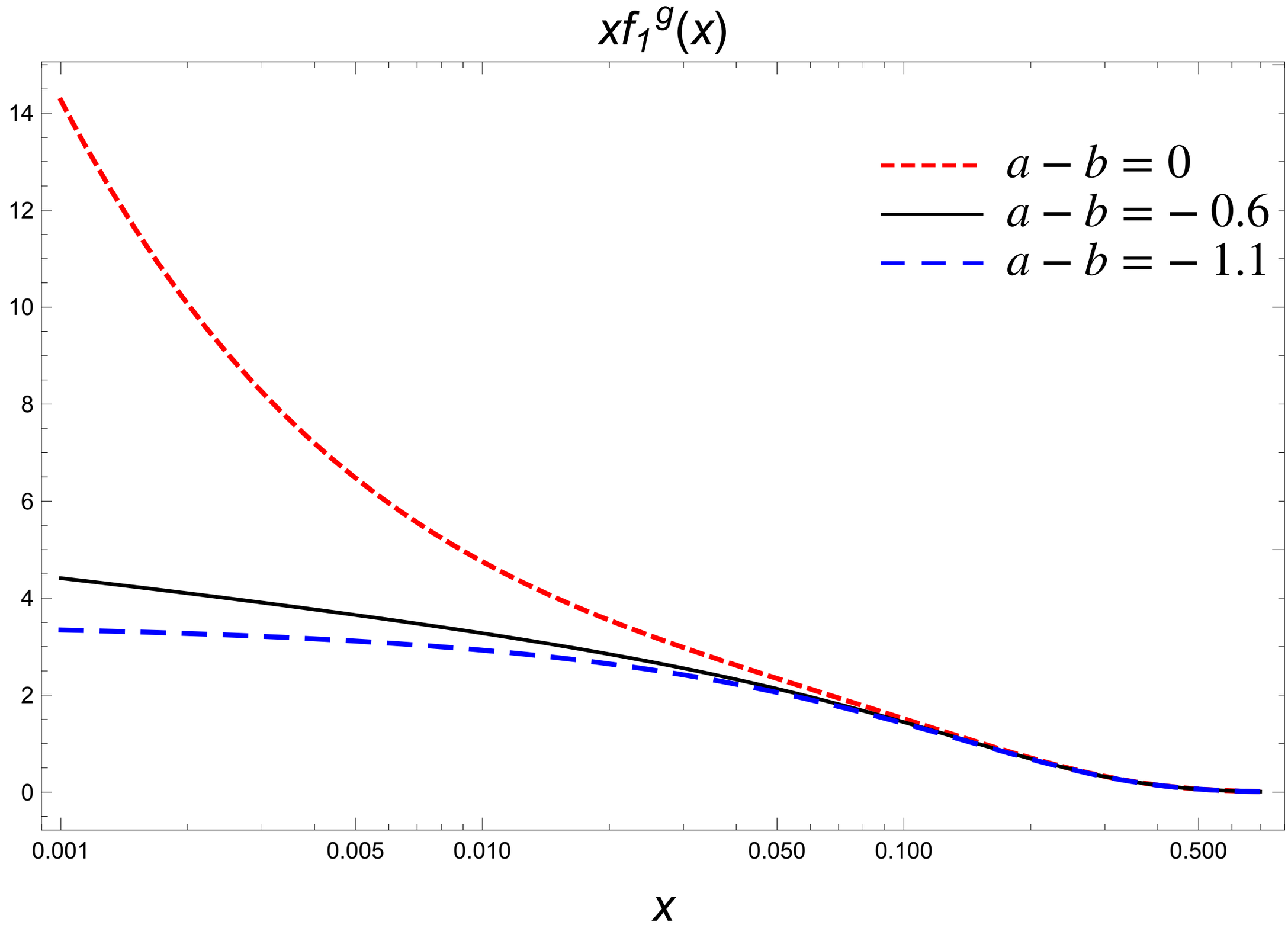
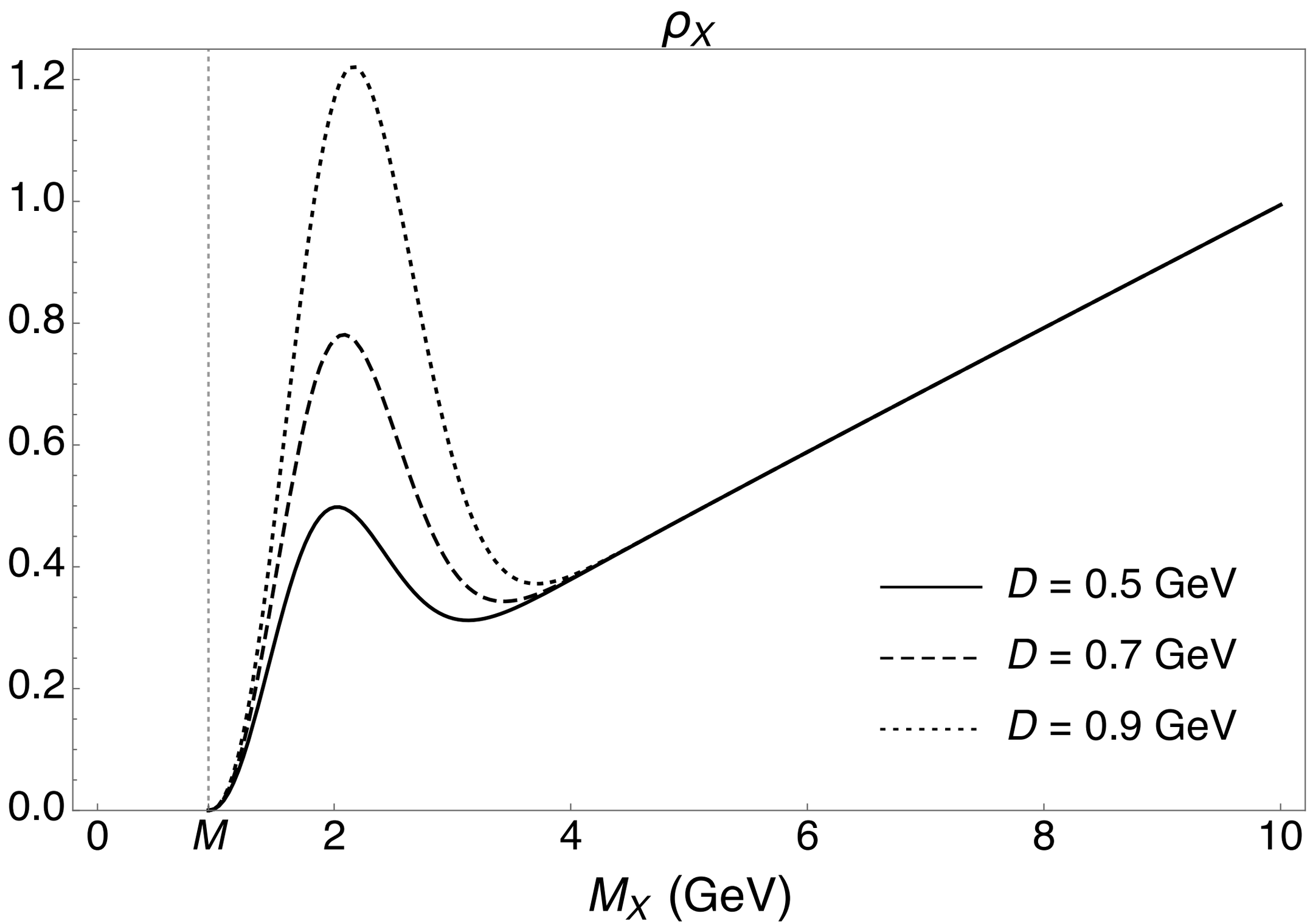


xf_1 collinear PDF vs $(a - b)$



Spectral function vs D



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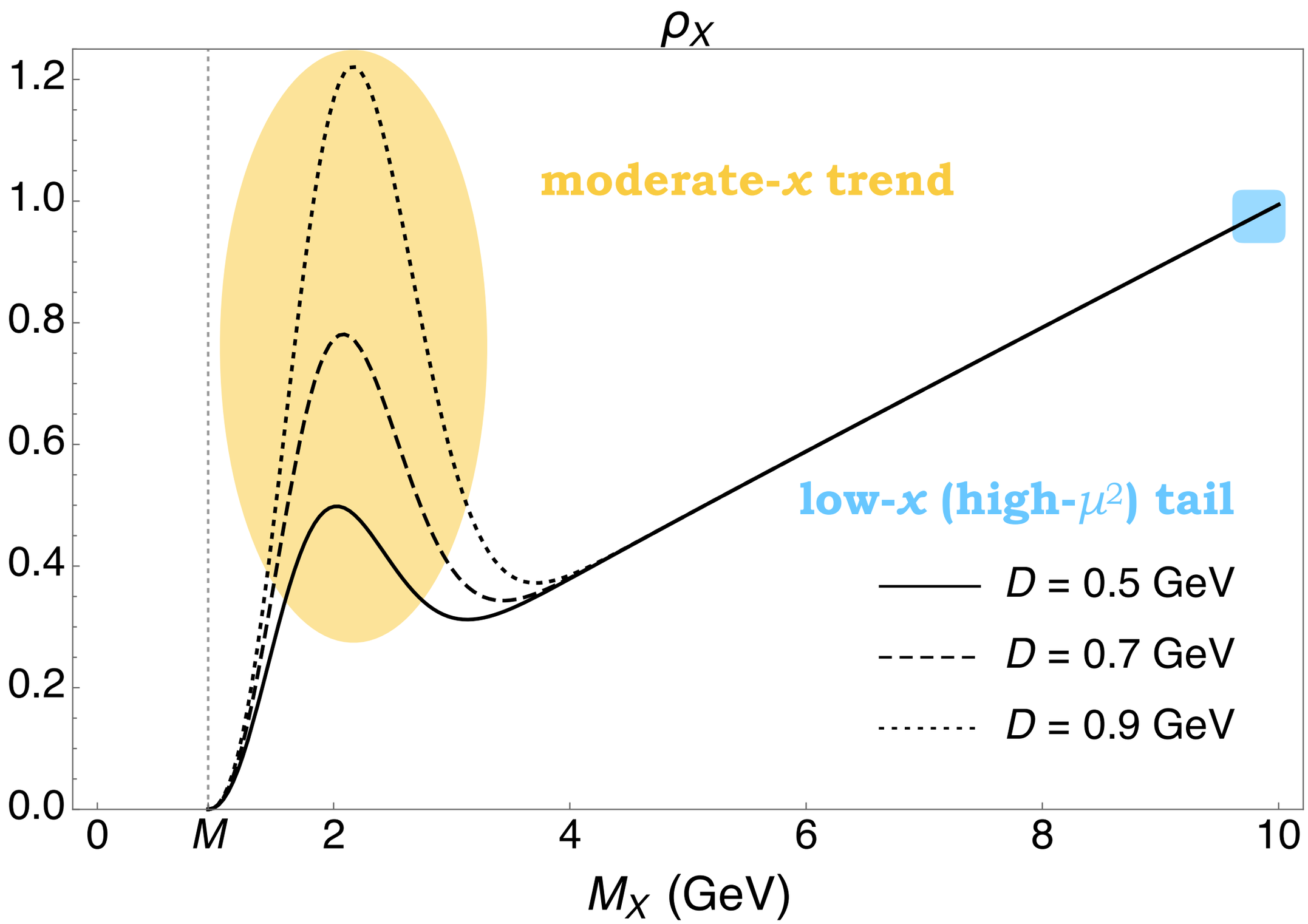
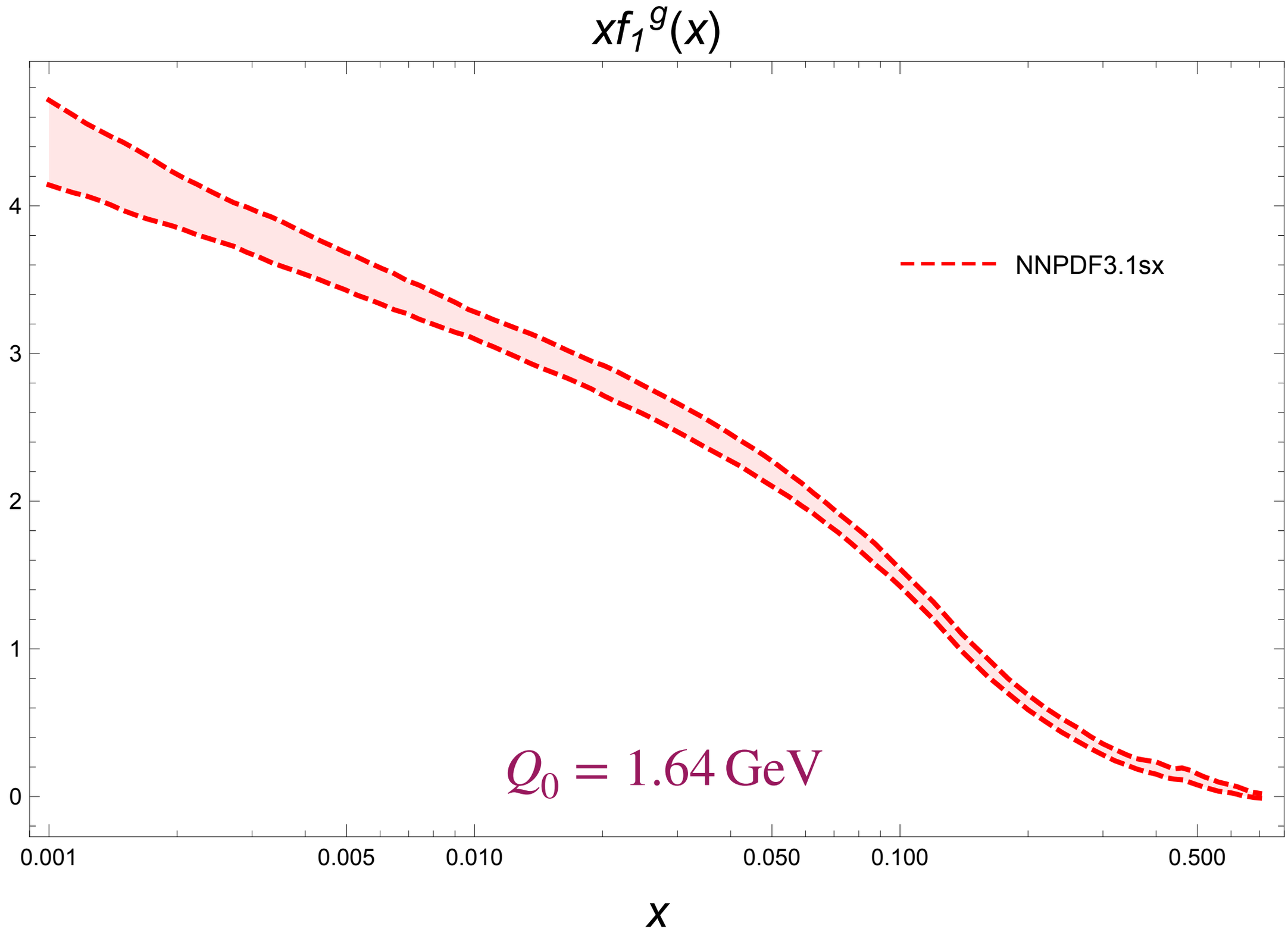


Unpolarized gluon PDF



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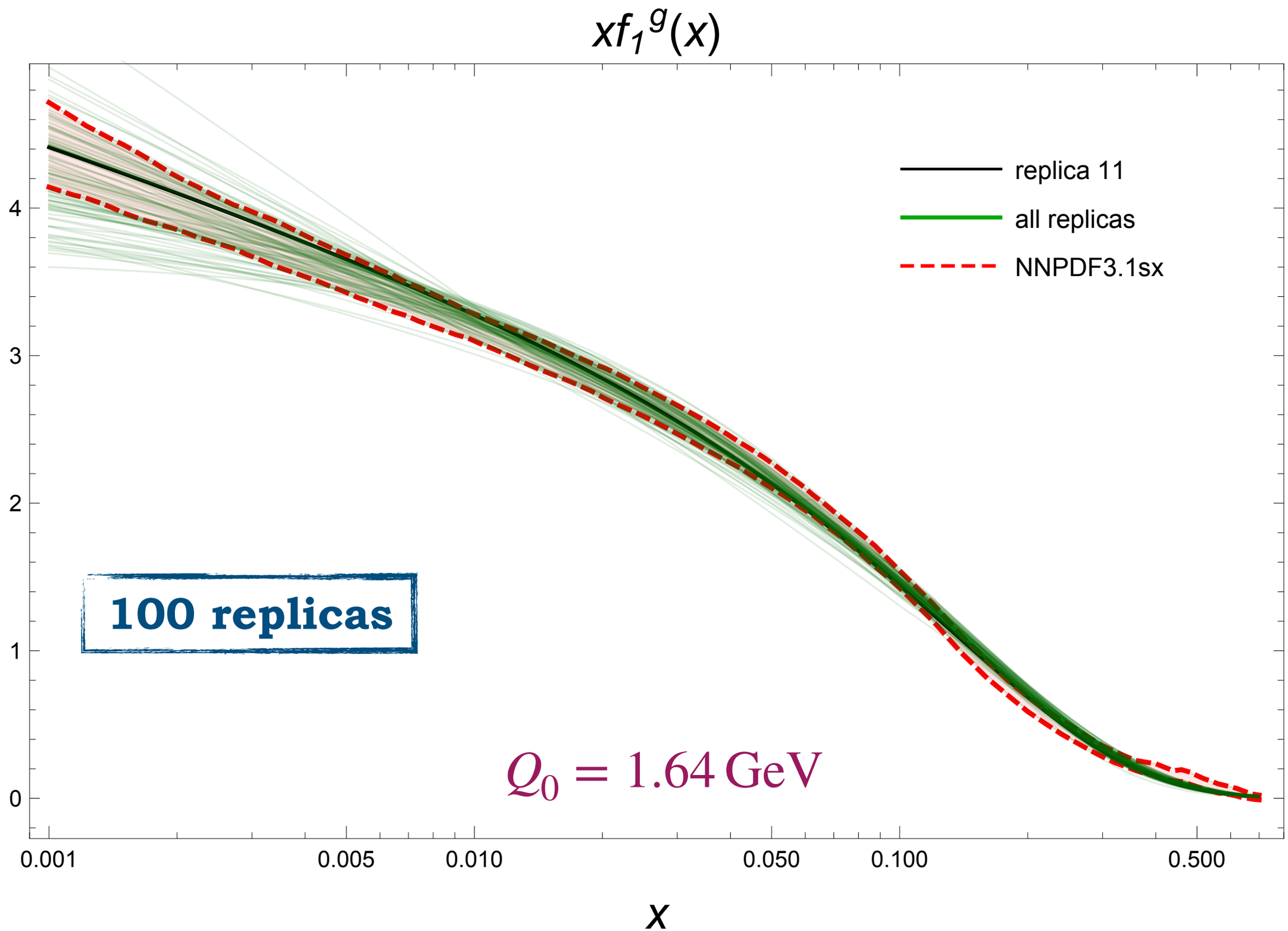
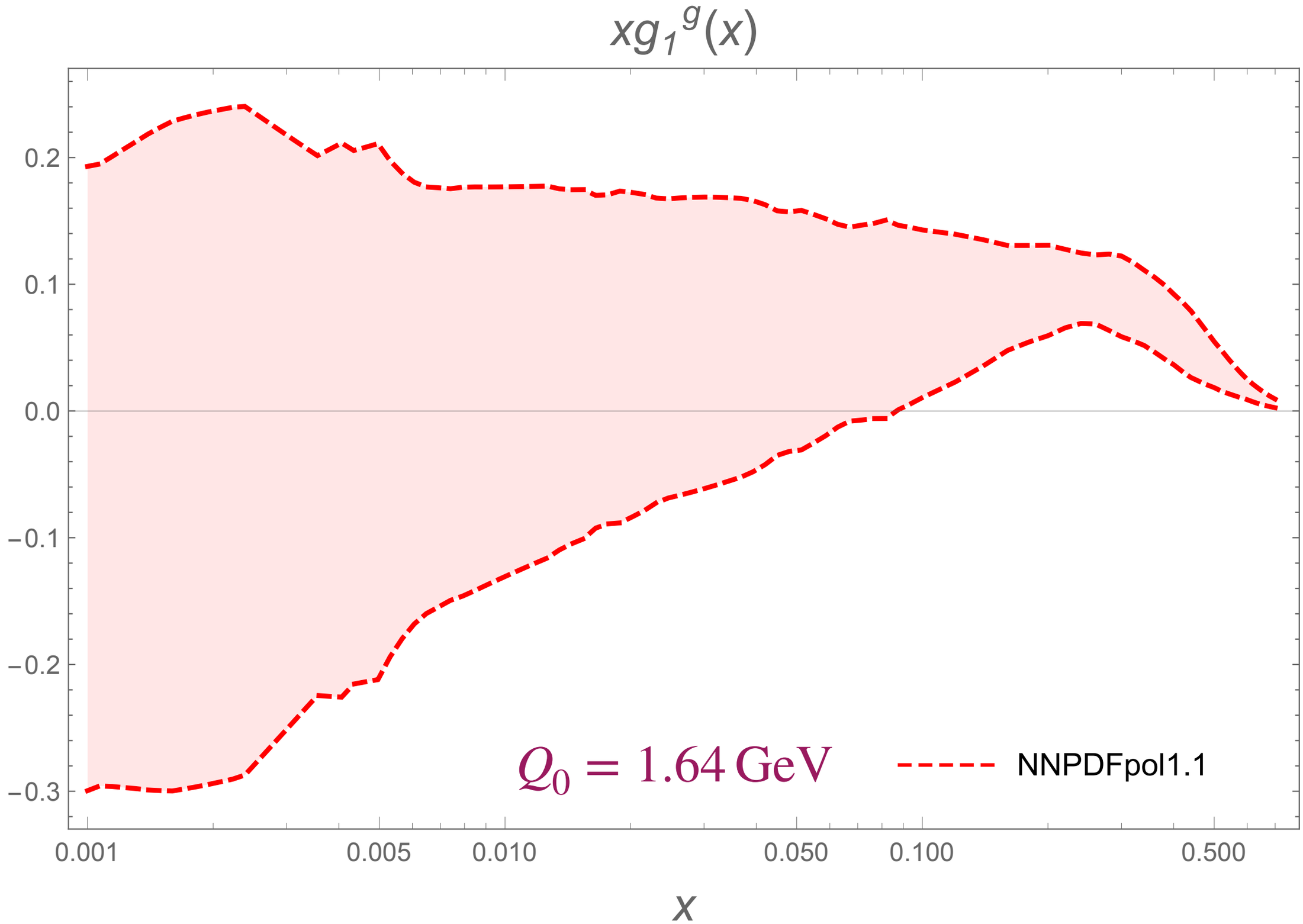


Helicity gluon PDF



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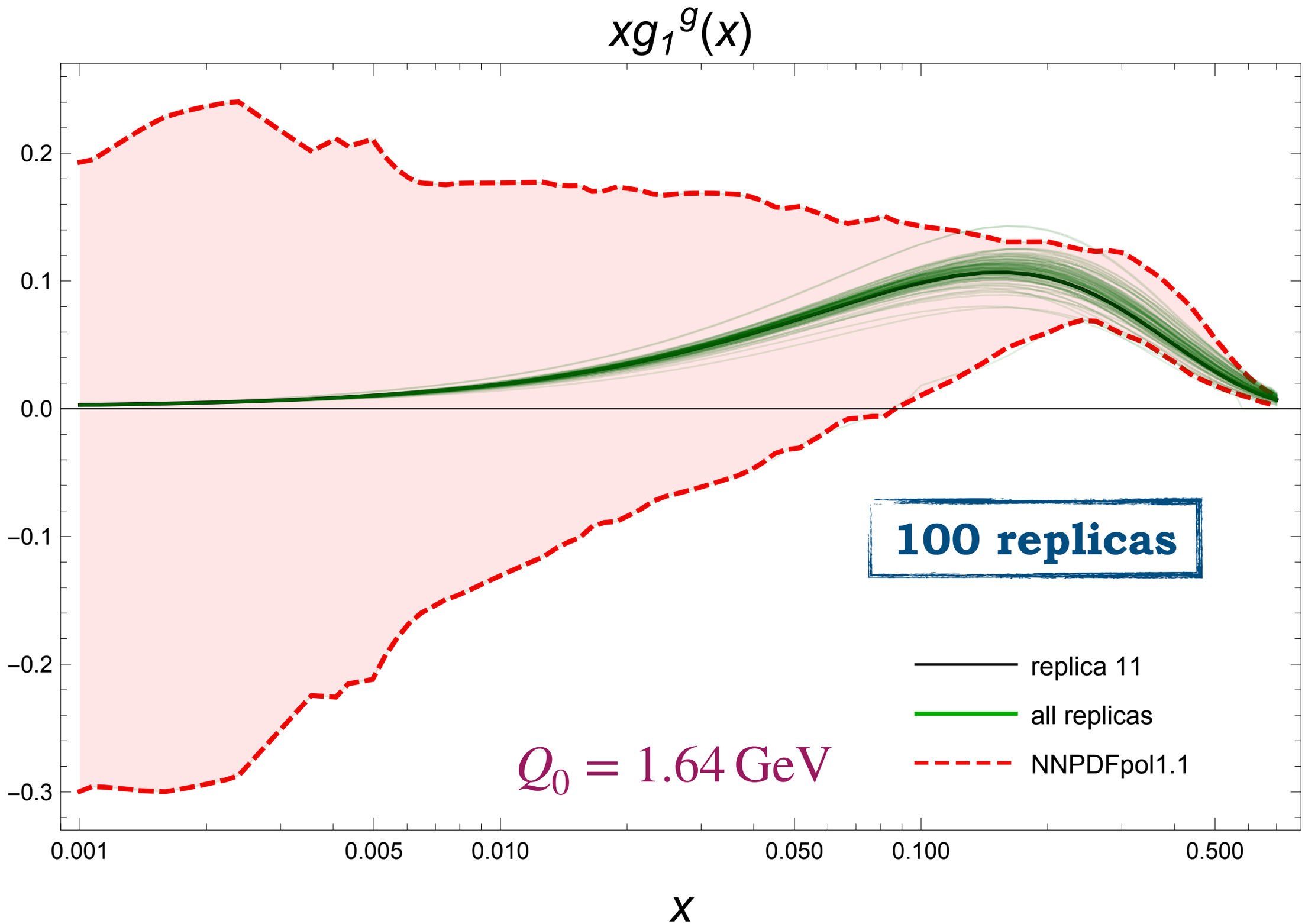

Fit specifics

$$\chi^2/\text{d.o.f.} = 0.54 \pm 0.38$$

no **overlearning**, just large errors for g_1

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$$\langle x \rangle_g = \int_0^1 dx\, x f_1^g(x, Q_0) \qquad S_g = \frac{1}{2}\langle 1 \rangle_{\Delta g} = \int_0^1 dx\, g_1^g(x, Q_0)$$

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Our model @ $Q_0 = 1.64$ GeV

$$\langle x \rangle_g = 0.424(9)$$

$$\langle S \rangle_g = 0.159(11)$$

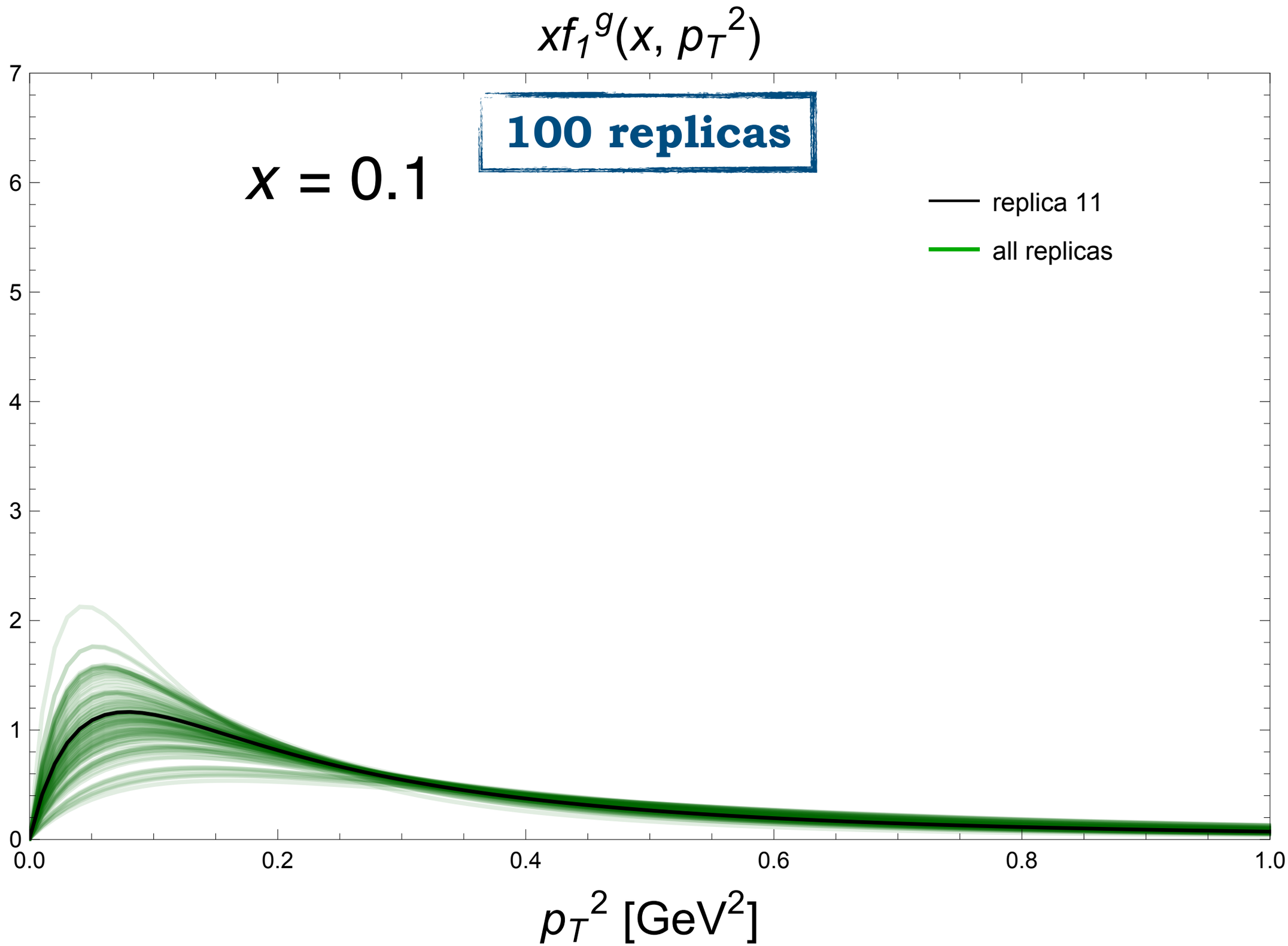
Lattice @ $Q_0 = 2$ GeV

$$\langle x \rangle_g = 0.427(92)$$

$$\langle J \rangle_g = 0.187(46)$$

[C. Alexandrou *et al.* (2020)]

Unpolarized gluon TMD



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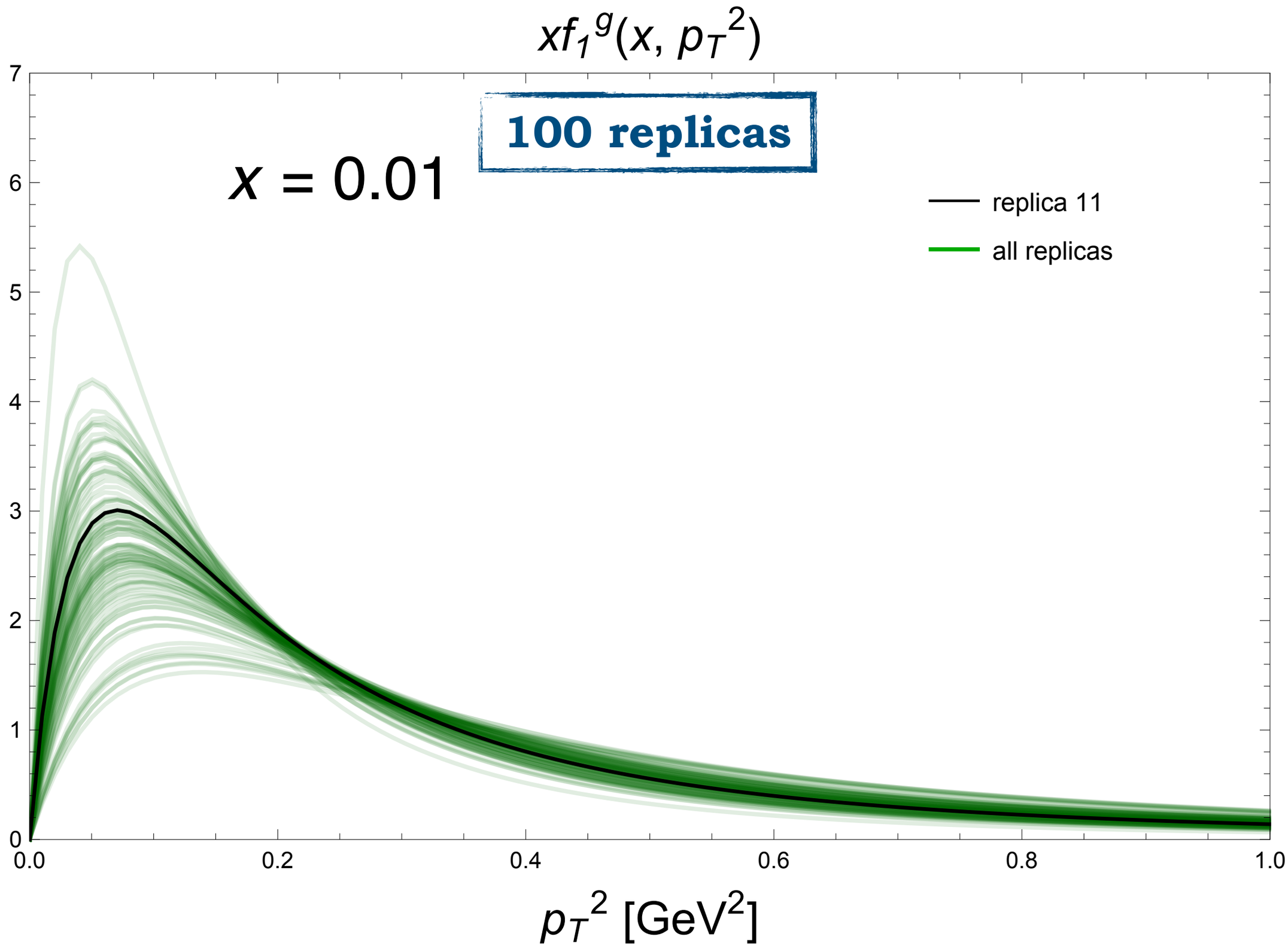

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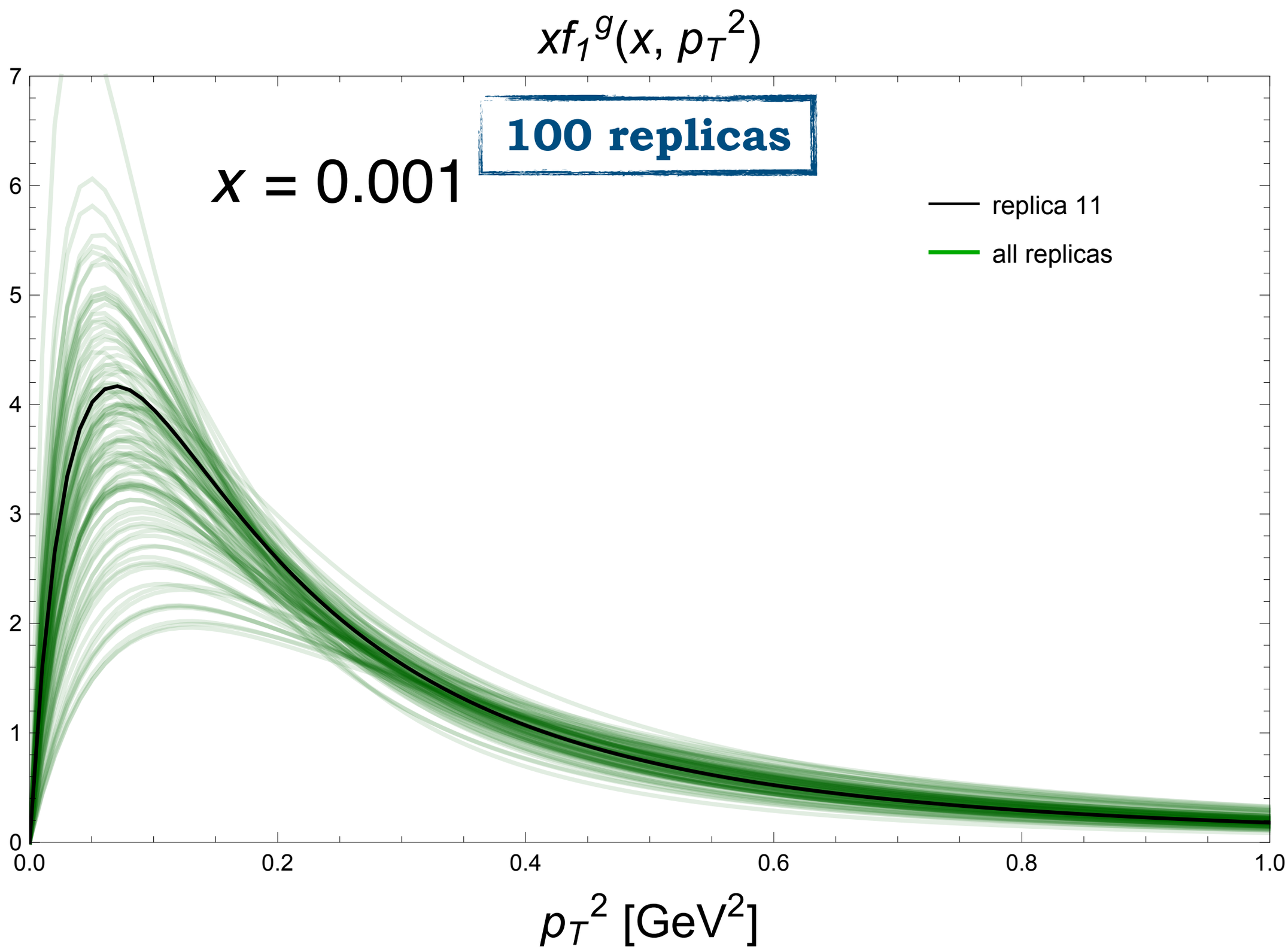
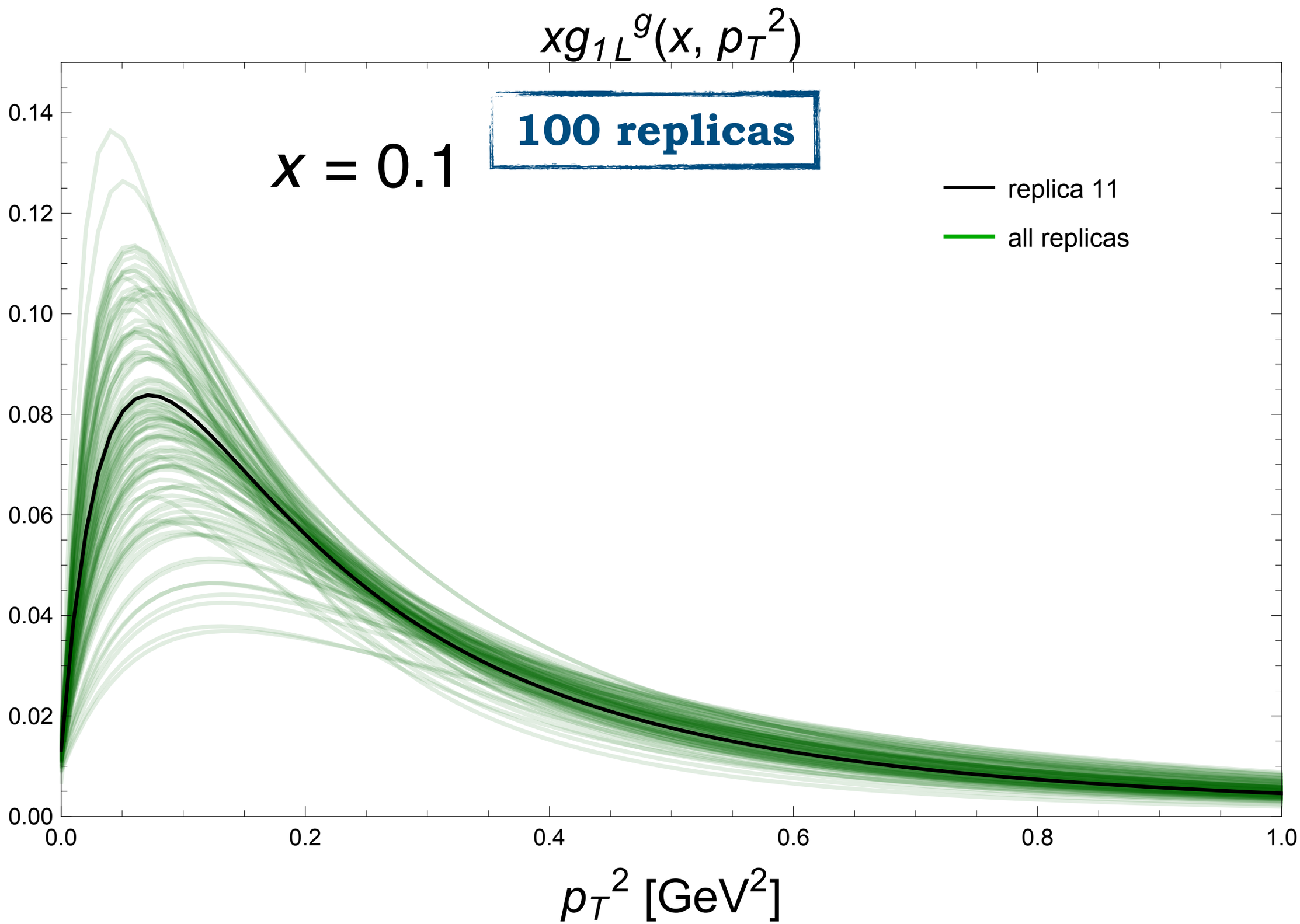


Helicity gluon TMD



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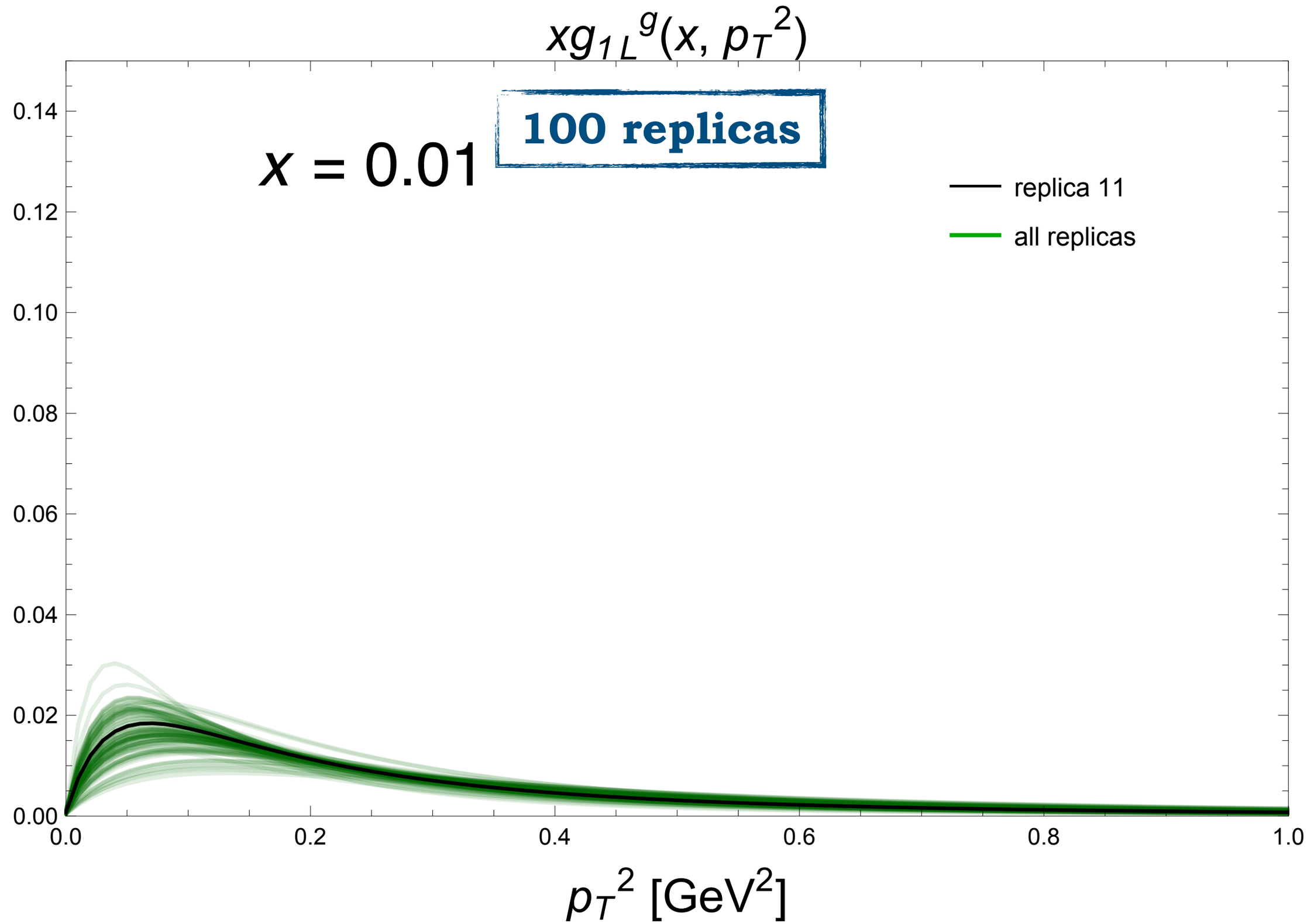

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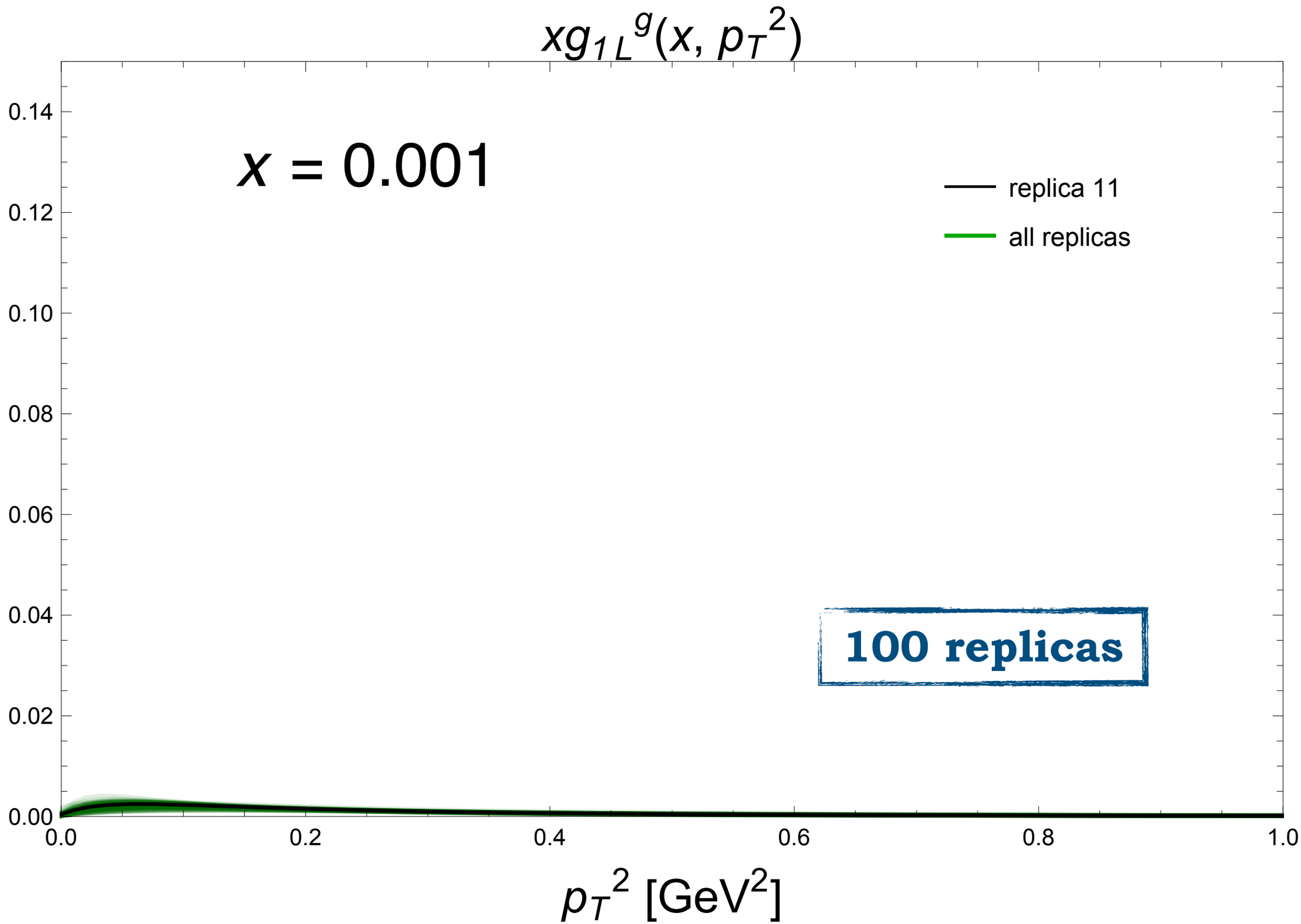
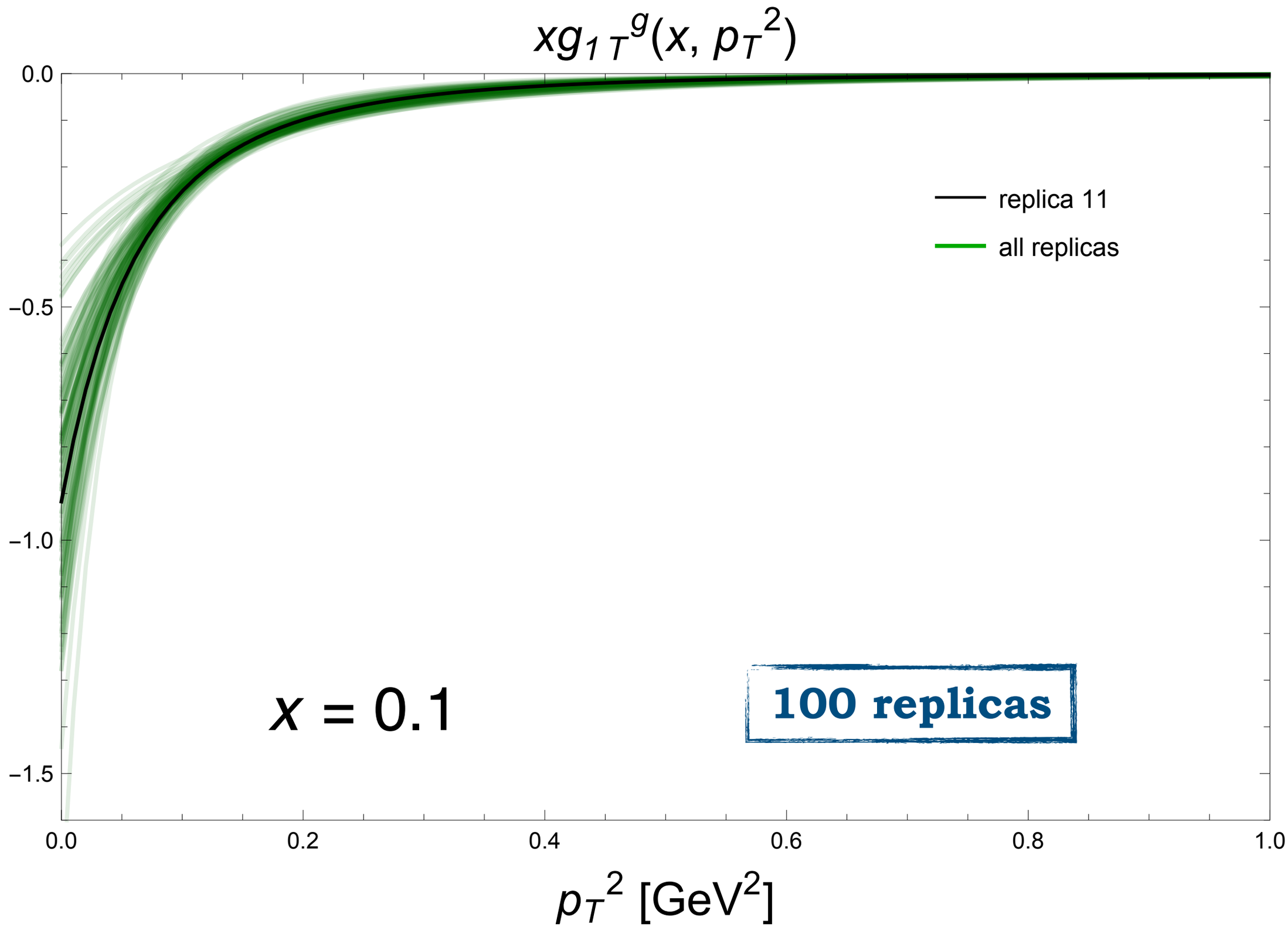


Worm-gear gluon TMD



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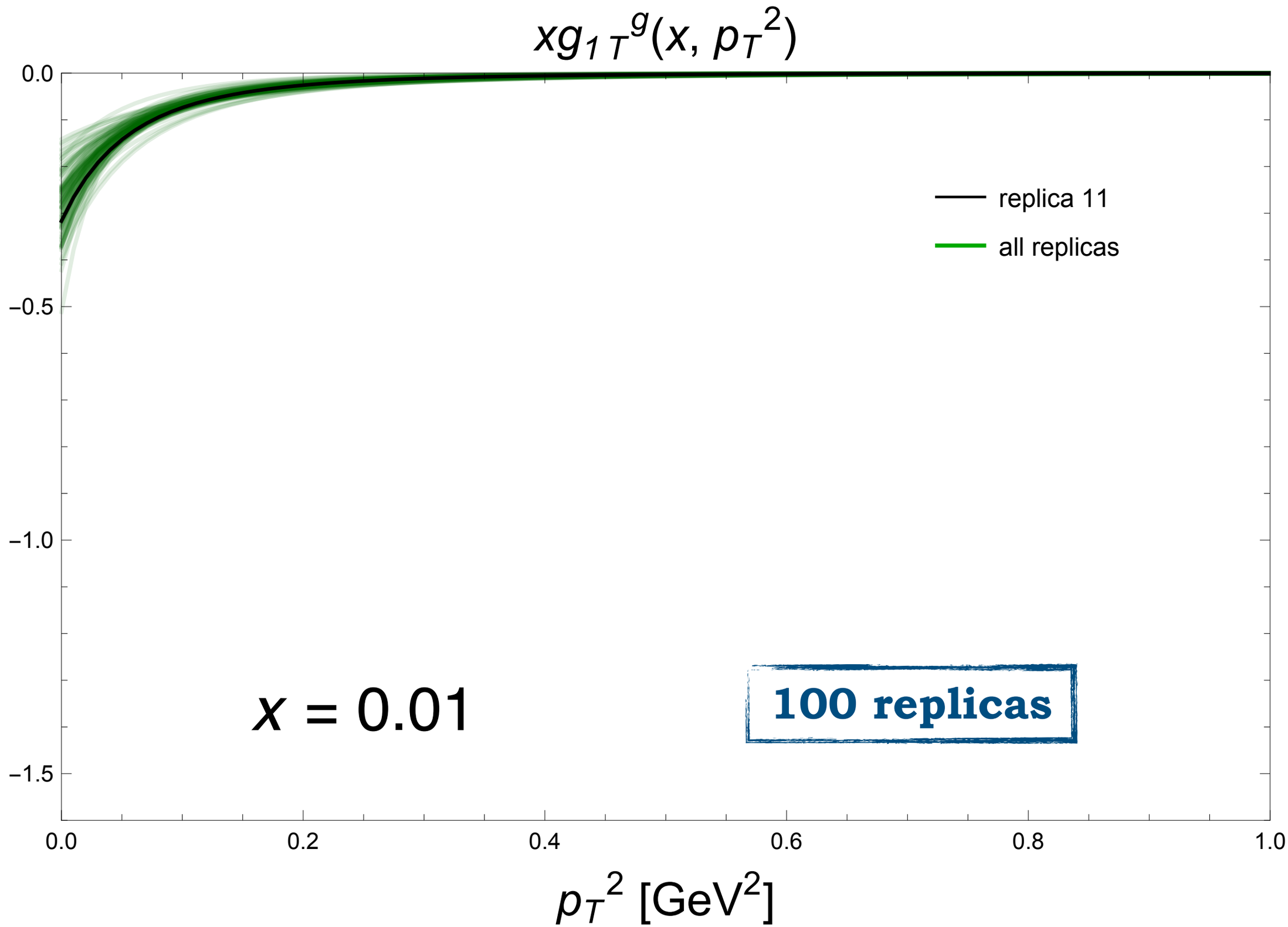

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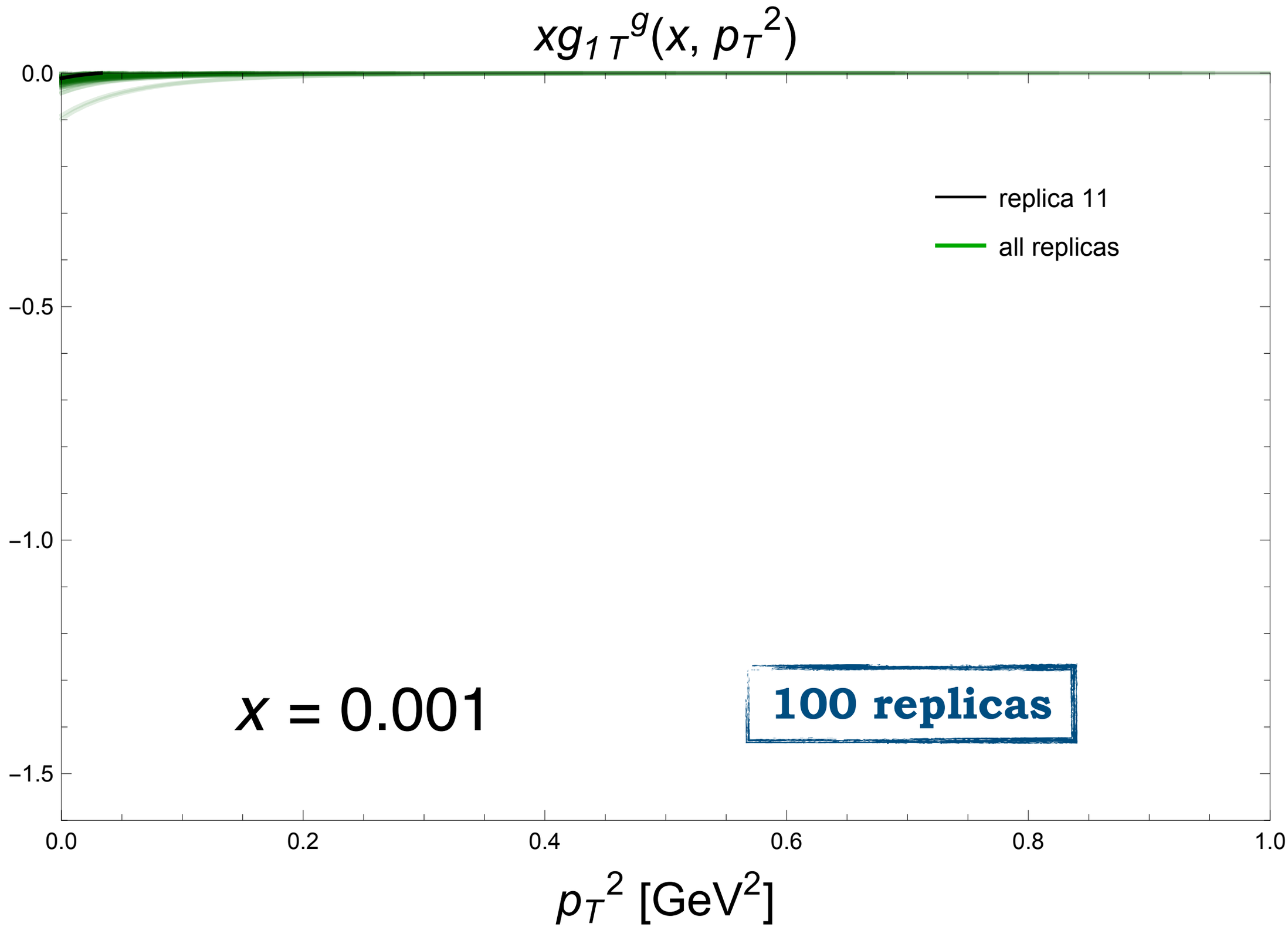
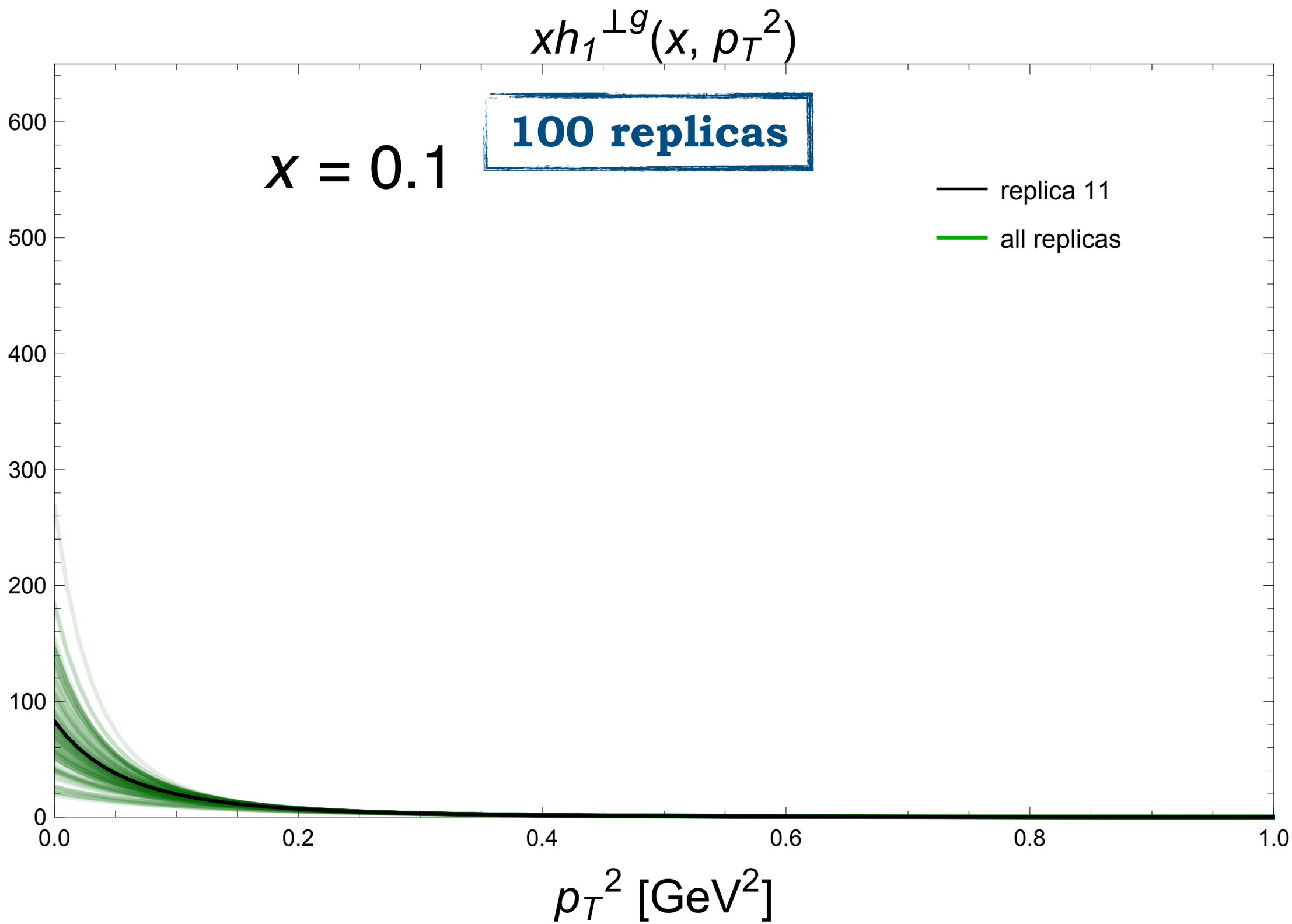


Boer-Mulders gluon TMD



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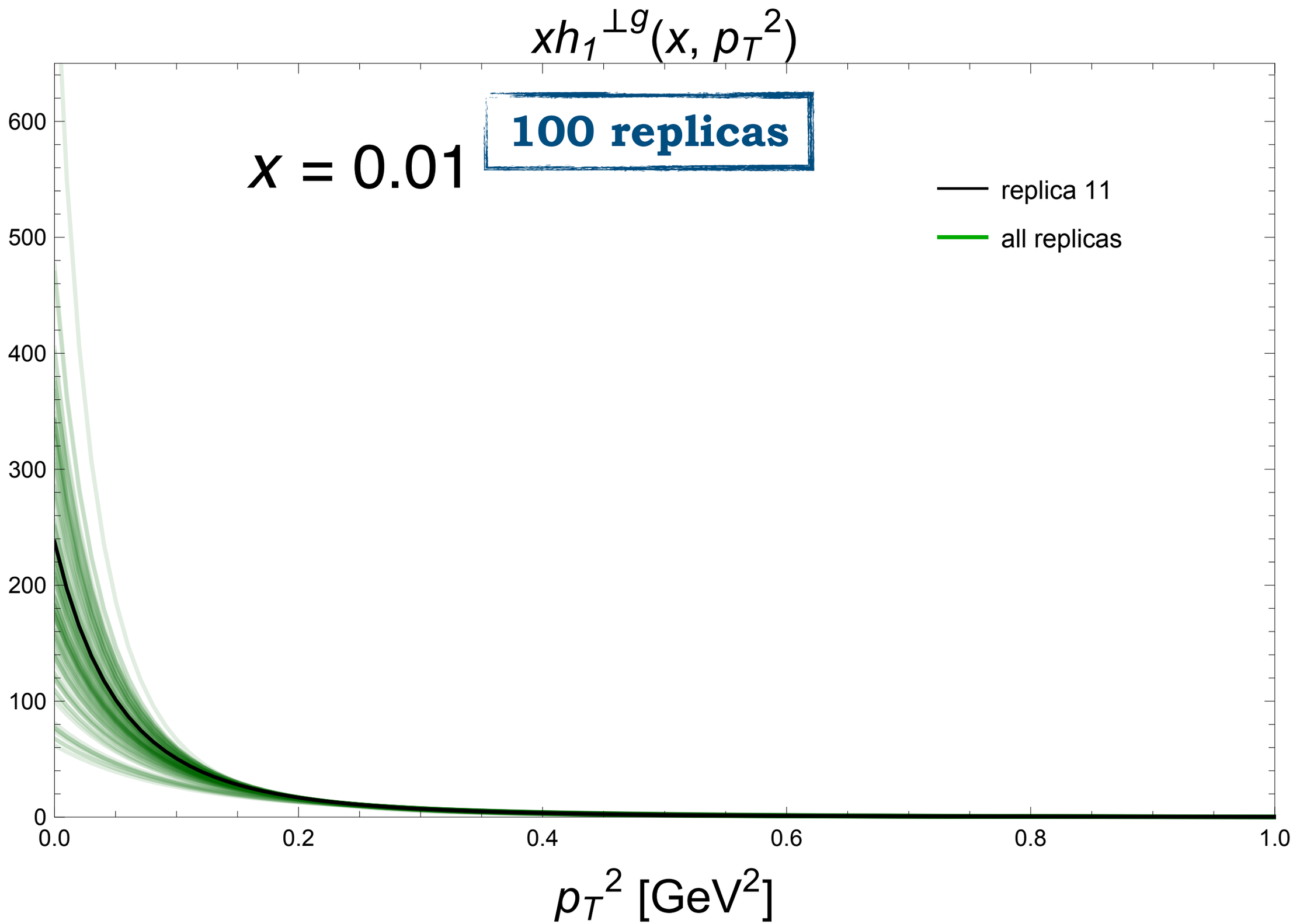

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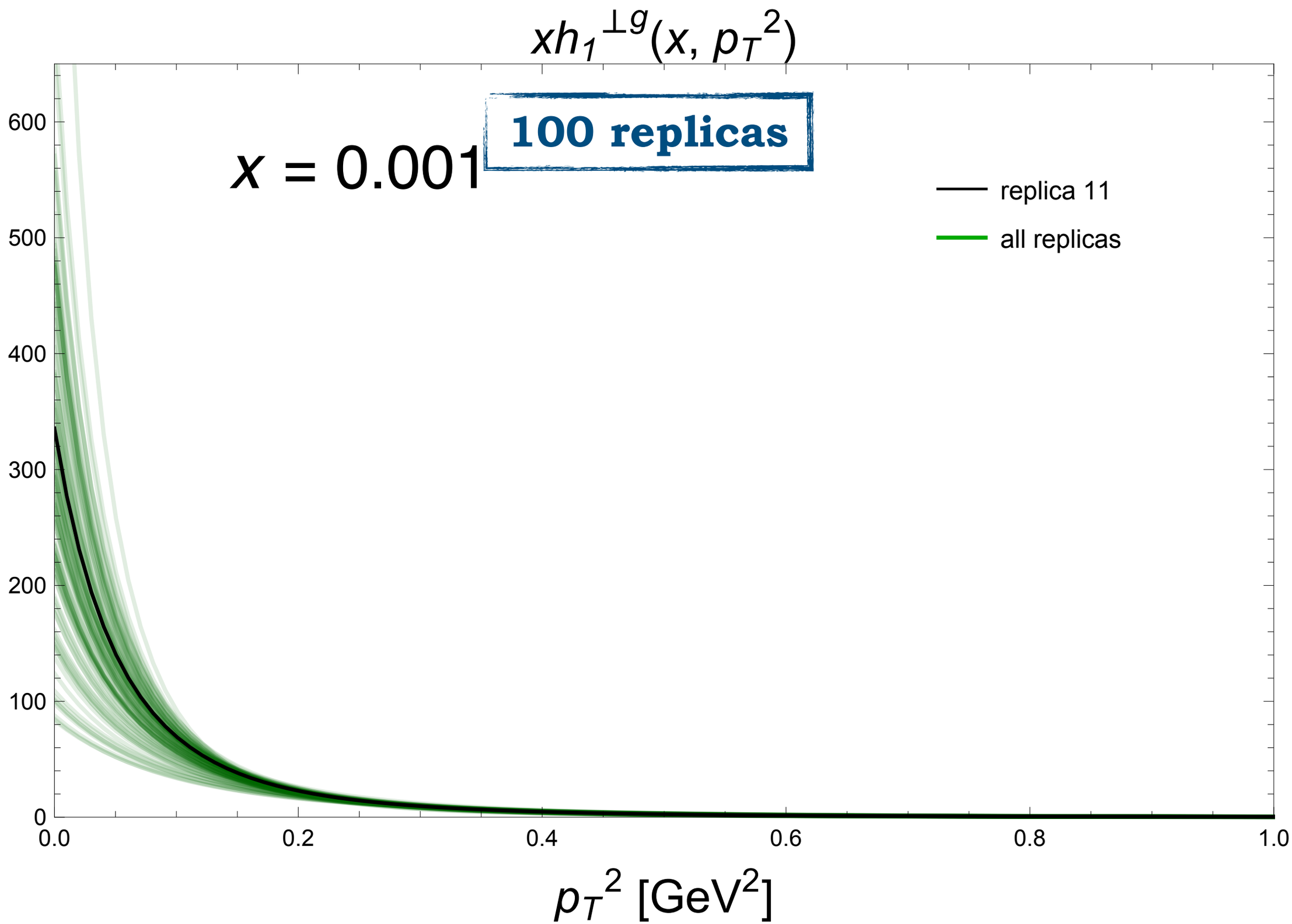


Bottom line

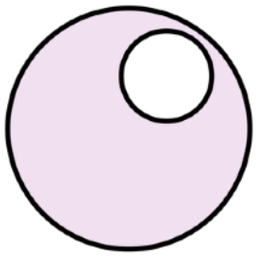
- Each TMD shows a distinctive x- and p_T-behavior

- Data on gluon TMDs will exclude many replicas and constrain parameters not yet so well constrained by collinear PDFs

- Simultaneous fit on two distinct PDFs provides with *corroborating evidence* of reliability of our model

- Standard CSS 🔗 🔗 *evolution* can be turned on

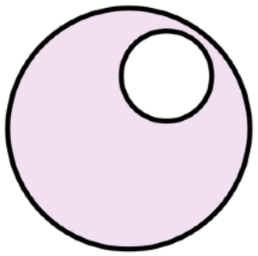
ρ-densities



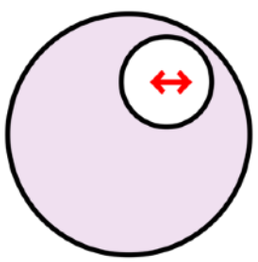
Unpolarized [u/u]

$$f_1(x, p_x, p_y)$$

ρ-densities

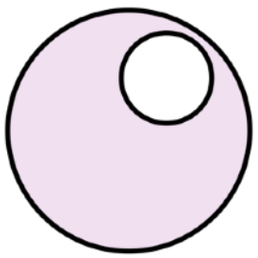


Unpolarized [u/u] $f_1(x, p_x, p_y)$

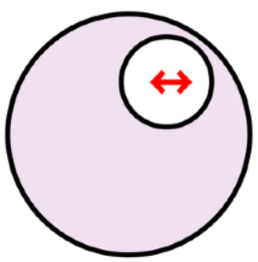


Boer-Mulders [↔/u] $f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x, p_x, p_y)$

ρ-densities



Unpolarized [u/u] $f_1(x, p_x, p_y)$



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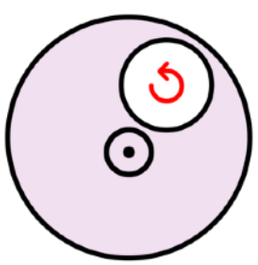
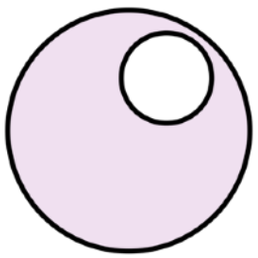
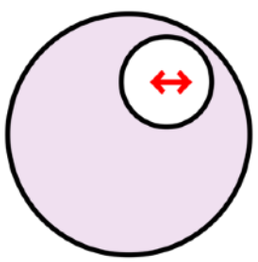


Helicity [↺/+] $\frac{1}{2}\left[f_1(x, p_x, p_y) + g_{1L}(x, p_x, p_y)\right]$

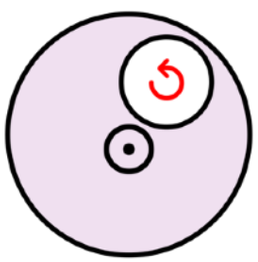
ρ-densities



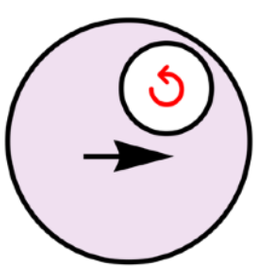
Unpolarized [u/u] $f_1(x, p_x, p_y)$



Boer-Mulders [↔/u] $f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x, p_x, p_y)$



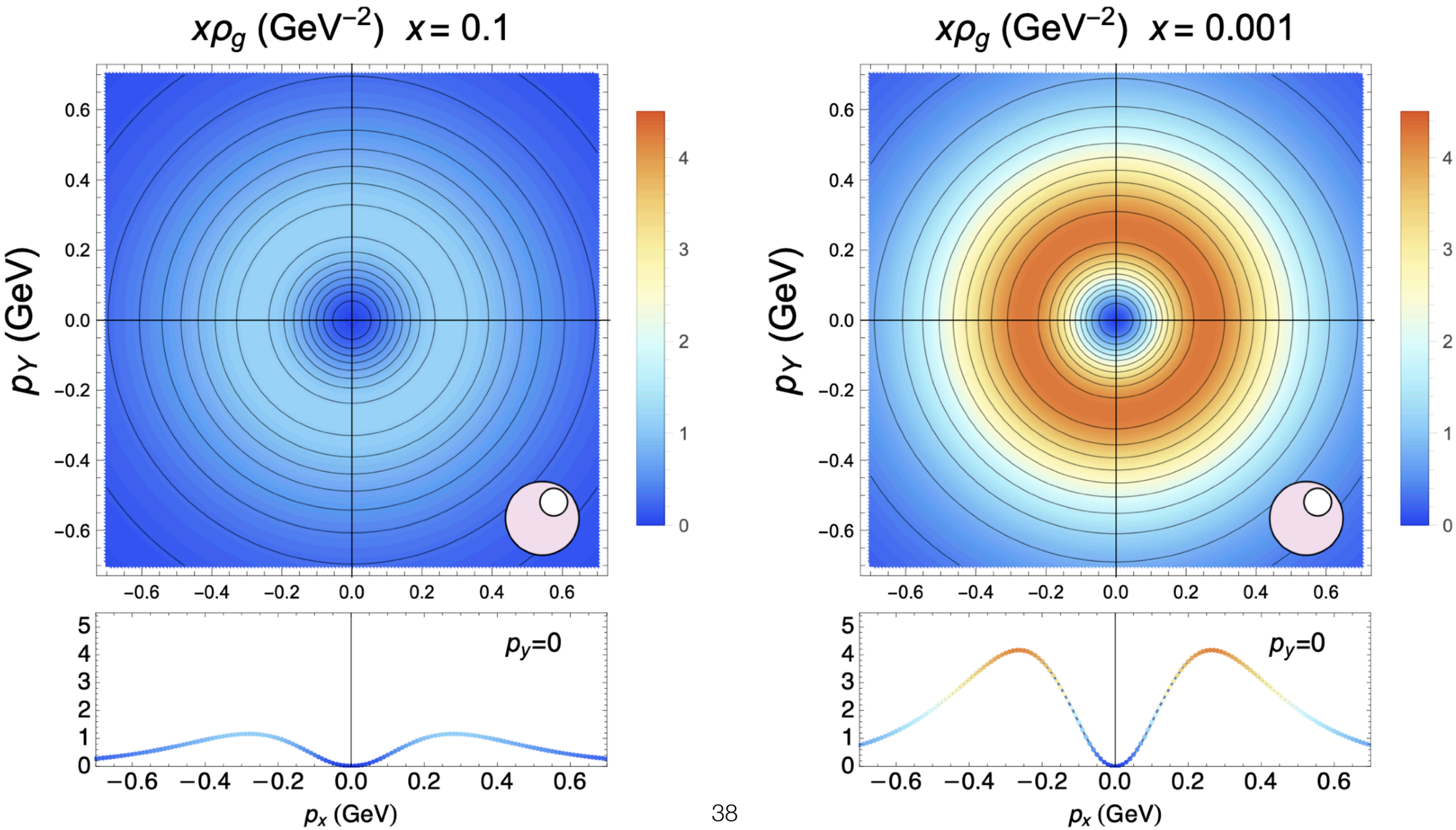
Helicity [↻/+] $\frac{1}{2}\left[f_1(x, p_x, p_y) + g_{1L}(x, p_x, p_y)\right]$



Worm-gear [↻/→] $f_1(x, p_x, p_y) - \frac{p_x}{M} g_{1T}(x, p_x, p_y)$

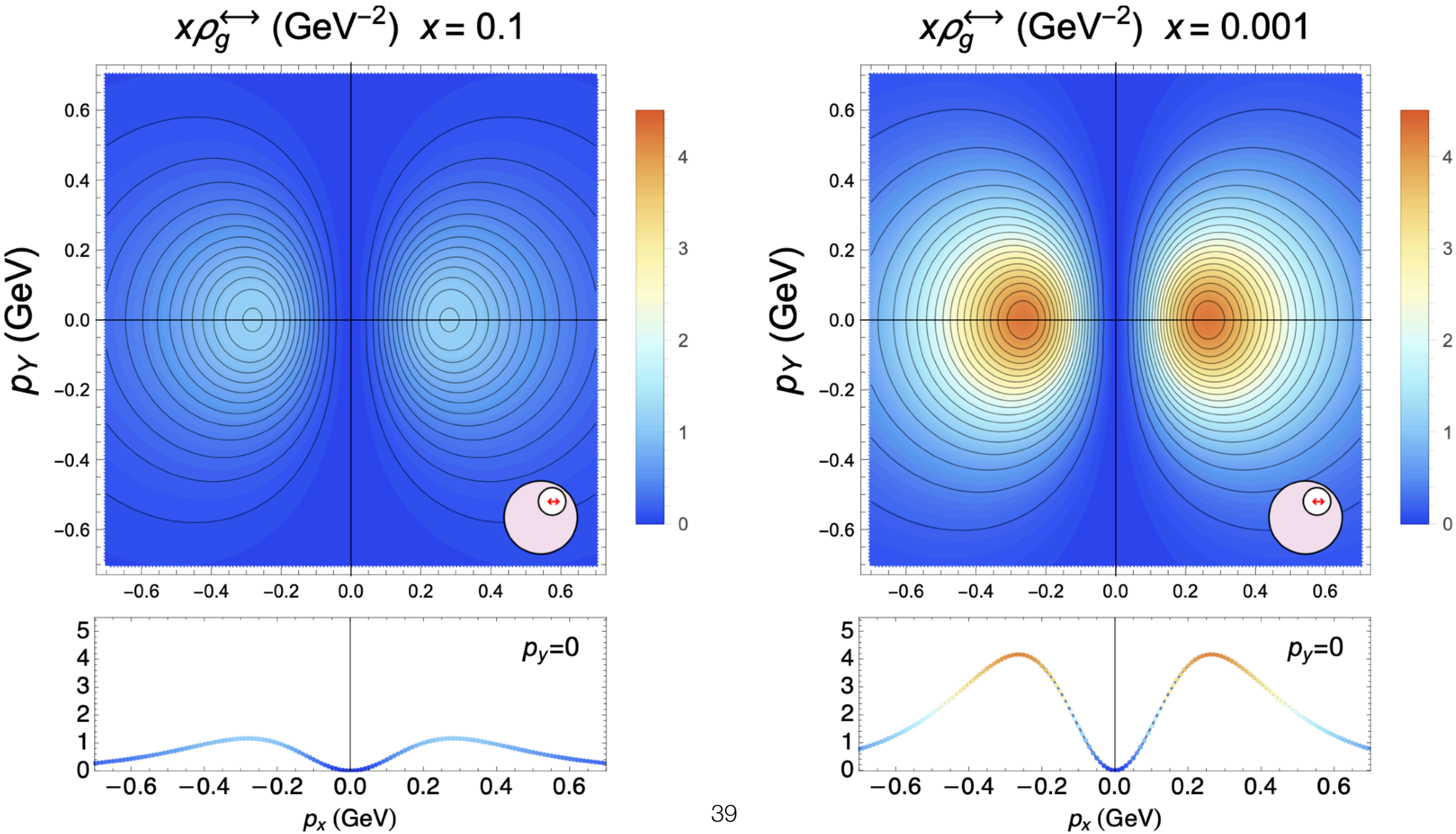
[u/u] ρ-density (Unpolarized)

$$x f_1(x, p_x, p_y)$$



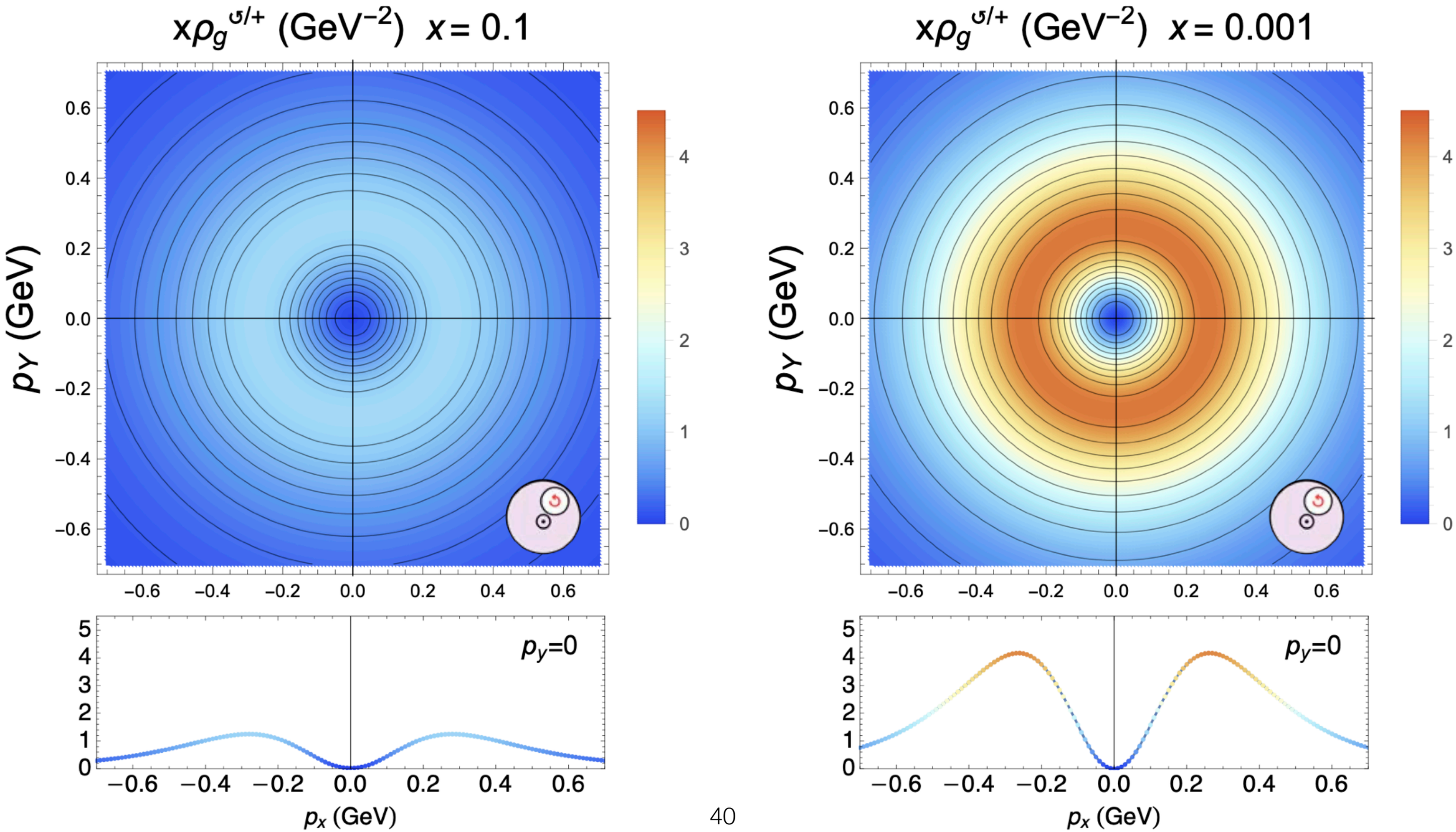
[↔/u] ρ-density (Boer-Mulders)

$$x f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} x\, h_1^{\perp}(x, p_x, p_y)$$



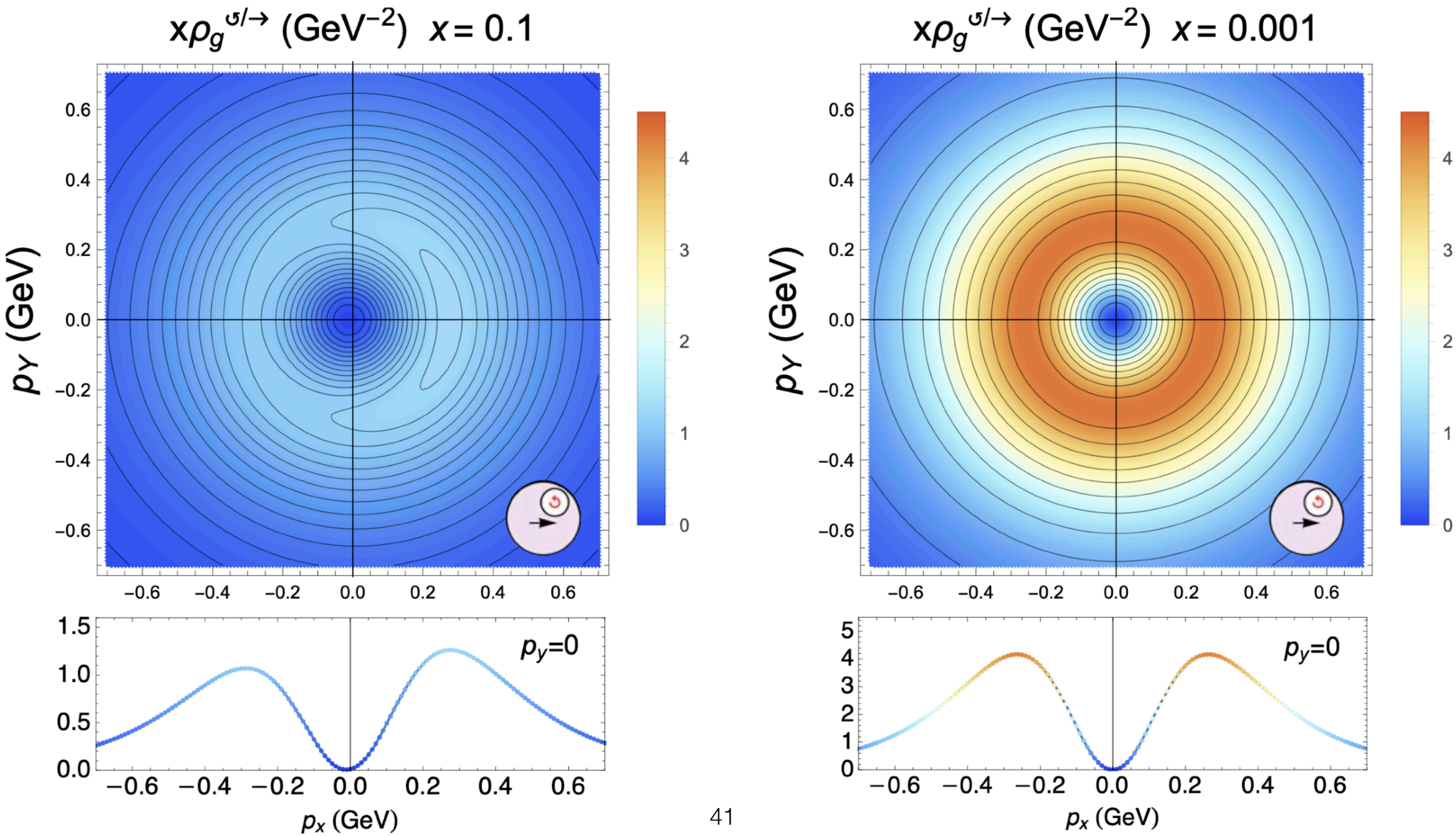
[↺/+] ρ-density (helicity)

$$\frac{1}{2}\left[x f_1(x, p_x, p_y) + x\, g_{1L}(x, p_x, p_y)\right]$$



[↺/→] ρ-density (worm-gear)

$$x f_1(x, p_x, p_y) - \frac{p_x}{M} x\, g_{1T}(x, p_x, p_y)$$



...towards twist-2 *T*-odd gluon TMDs

T-odd gluon TMDs at twist-2

gluon pol.

nucleon pol.	U	circ.	lin.
U	f_1^g		$h_1^{\perp g}$
L		g_1^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	h_1^g, $h_{1T}^{\perp g}$

T-even

T-odd

T-odd gluon TMDs at twist-2

gluon pol.

nucleon pol.	U	circ.	lin.
U	f_1^g		$h_1^{\perp g}$
L		g_1^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	h_1^g, $h_{1T}^{\perp g}$

worm-gear

T-even

T-odd

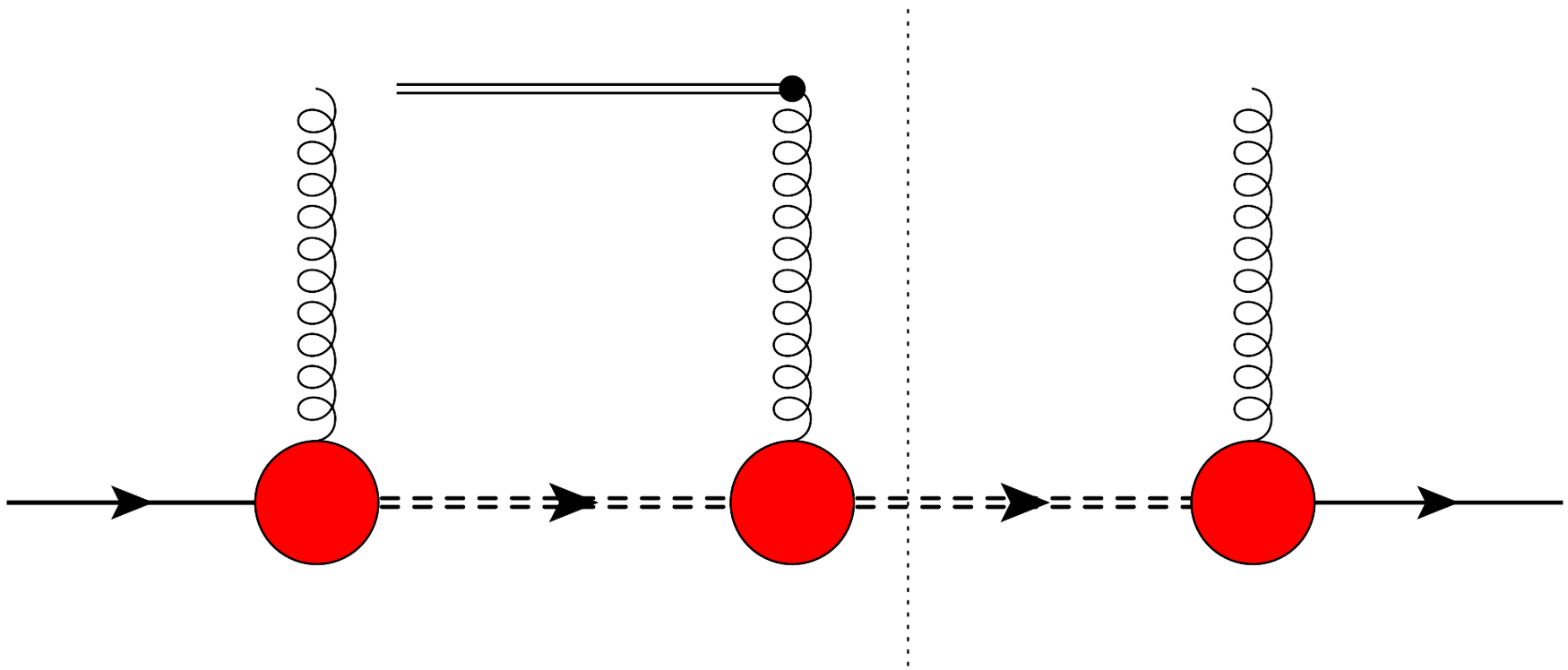
Sivers

linearity TMD

pretzelosity

T-odd gluon TMDs in the spectator model

- No residual gluon-spectator interaction at tree level
- *Interference* with one-gluon exchange (*eikonal*)



- Calculation of **Sivers** function *underway*!

T-odd gluon TMDs and semi-inclusive reactions

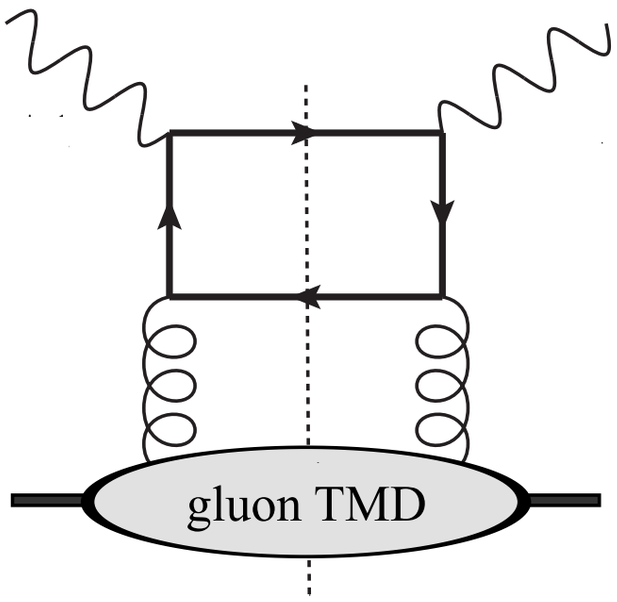
1. Almost back-to-back di-jet production

2. Open-charm (heavy-light meson) states

3. Almost back-to-back J/Ψ-plus-jet production

4. Inclusive J/Ψ production at low p_T

$ep \to e + \text{jet} + \text{jet} + X$



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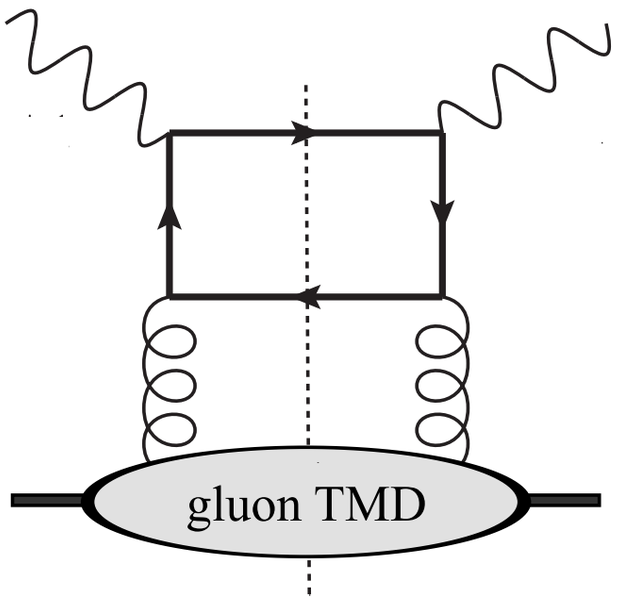

- Gluon-induced processes
- **Spin-asymmetry** studies feasible
- Small-x physics supported

Closing statements

- ☑ Systematic calculation of all twist-2 T-even gluon TMDs
- ☑ Spectral mass to catch small- and moderate-x effects
- ☑ **Simultaneous fit** of f_1 and g_1 PDFs via **replica method**

Closing statements

- ☑ Systematic calculation of all twist-2 T-even gluon TMDs
- ☑ Spectral mass to catch small- and moderate-x effects
- ☑ **Simultaneous fit** of f_1 and g_1 PDFs via **replica method**
- ☐ Inclusion of **BFKL** input on small-x tail
- ☐ Twist-2 T-odd TMDs (**Sivers**, etc.) soon available!
- ☐ Relevant **spin asymmetries** to be identified
- ☐ **Predictions** as inputs to generate **pseudodata**
- ☐ Extension to quark TMDs in the same framework

Backup slides

TMD factorization

$$\frac{d\sigma}{dQ\,dy\,dq_T} = \frac{16\pi\alpha^2 q_T \mathcal{P}}{9Q^3} H(Q,\mu) \sum_q c_q(Q) \int \frac{d^2\mathbf{b}}{4\pi} e^{i\mathbf{b}\cdot\mathbf{q}_T} x_1 F_q(x_1,\mathbf{b};\mu,\zeta_1) x_2 F_{\bar{q}}(x_2,\mathbf{b};\mu,\zeta_2)$$

$$F_f(x,\mathbf{b};\mu,\zeta) = \sum_j \left(C_{f/j} \otimes f^j\right)(x,b_*;\mu_b) e^{R(b_*;\mu_b,\mu)} f_{\mathrm{NP}}(x,b,\zeta)$$

scales

$$\mu = Q \qquad\qquad \zeta_1 = \zeta_2 = Q^2$$

slide from C. Bissolotti

Logarithmic accuracy

$$\frac{d\sigma}{dq_T} \propto H(Q,\mu) \int \frac{d^2\mathbf{b}}{4\pi} e^{i\mathbf{b}\cdot\mathbf{q}_T} F_q(x_1,\mathbf{b};\mu) F_{\bar{q}}(x_2,\mathbf{b};\mu)$$

hard factor

matching coefficients

collinear PDF

$$F(x,\mathbf{b};\mu) = \sum_j \left(C_j \otimes f^j\right)(x,b_*;\mu_b) e^{R(b_*;\mu_b,\mu)} f_{\rm NP}(x,b)$$

Sudakov form factor

slide from C. Bissolotti

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hard factor

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collinear PDF

non perturbative function

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slide from C. Bissolotti

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hard factor

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collinear PDF

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Sudakov form factor

perturbative expansion in $\alpha_s(\mu)$

resummation of

$$L = \ln \frac{Q^2}{\mu_b^2}$$

define logarithmic ordering

slide from C. Bissolotti

TMDs

matching to the collinear region

factorizes as hard and longitudinal non-perturbative

$b_T \ll 1/\Lambda_{QCD}$

$$F_{f/P}(x, \mathbf{b}_T; \mu, \zeta) = \sum_j C_{f/j}(x, b_*; \mu_b, \zeta_F) \otimes f_{j/P}(x, \mu_b)$$

$$\times \exp\left\{ K(b_*; \mu_b) \ln \frac{\sqrt{\zeta_F}}{\mu_b} + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_F}}{\mu'} \right] \right\}$$

CS and RGE evolution

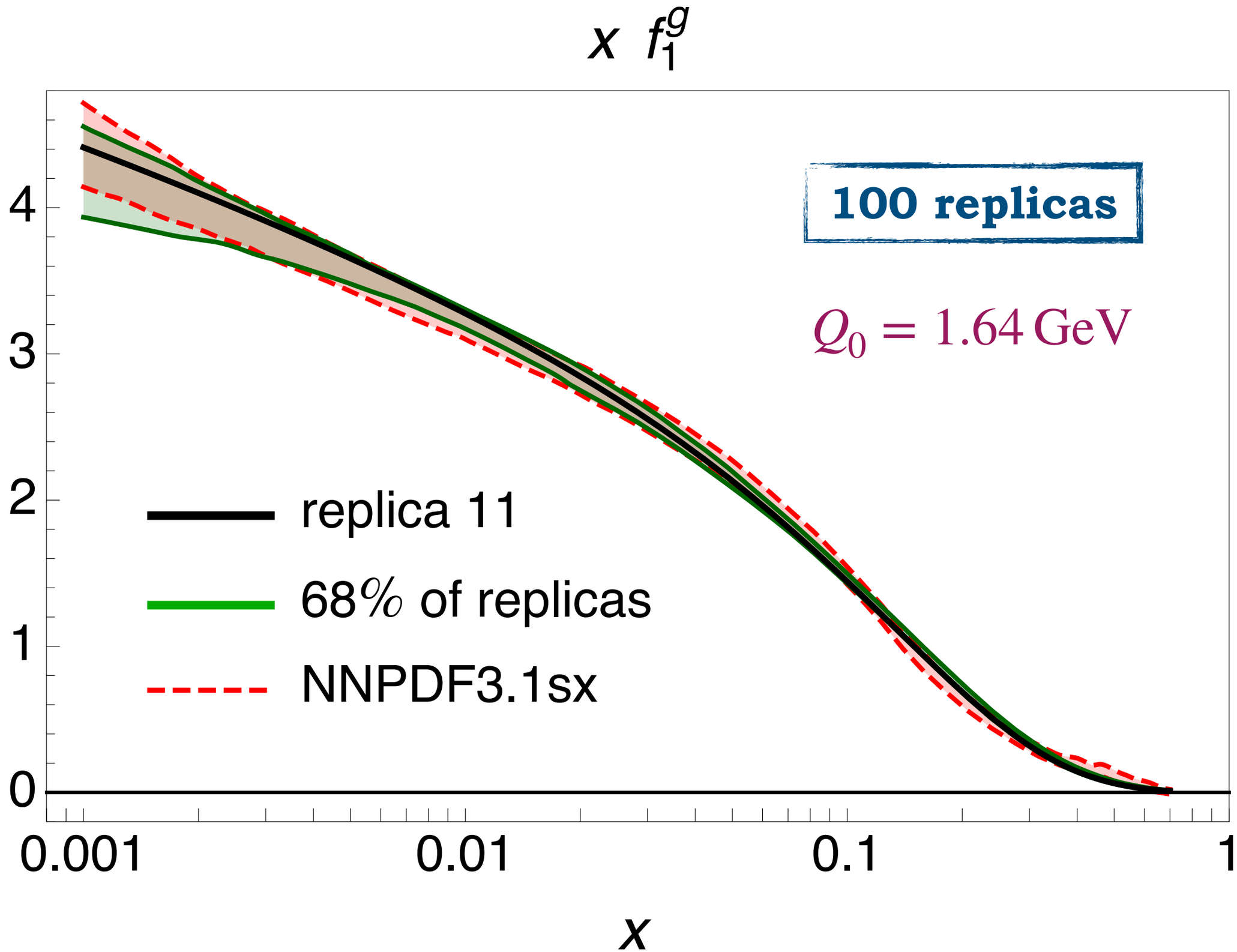
$$\times \exp\left\{ g_{j/P}(x, b_T) + g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F,0}}} \right\}$$

non perturbative transverse content

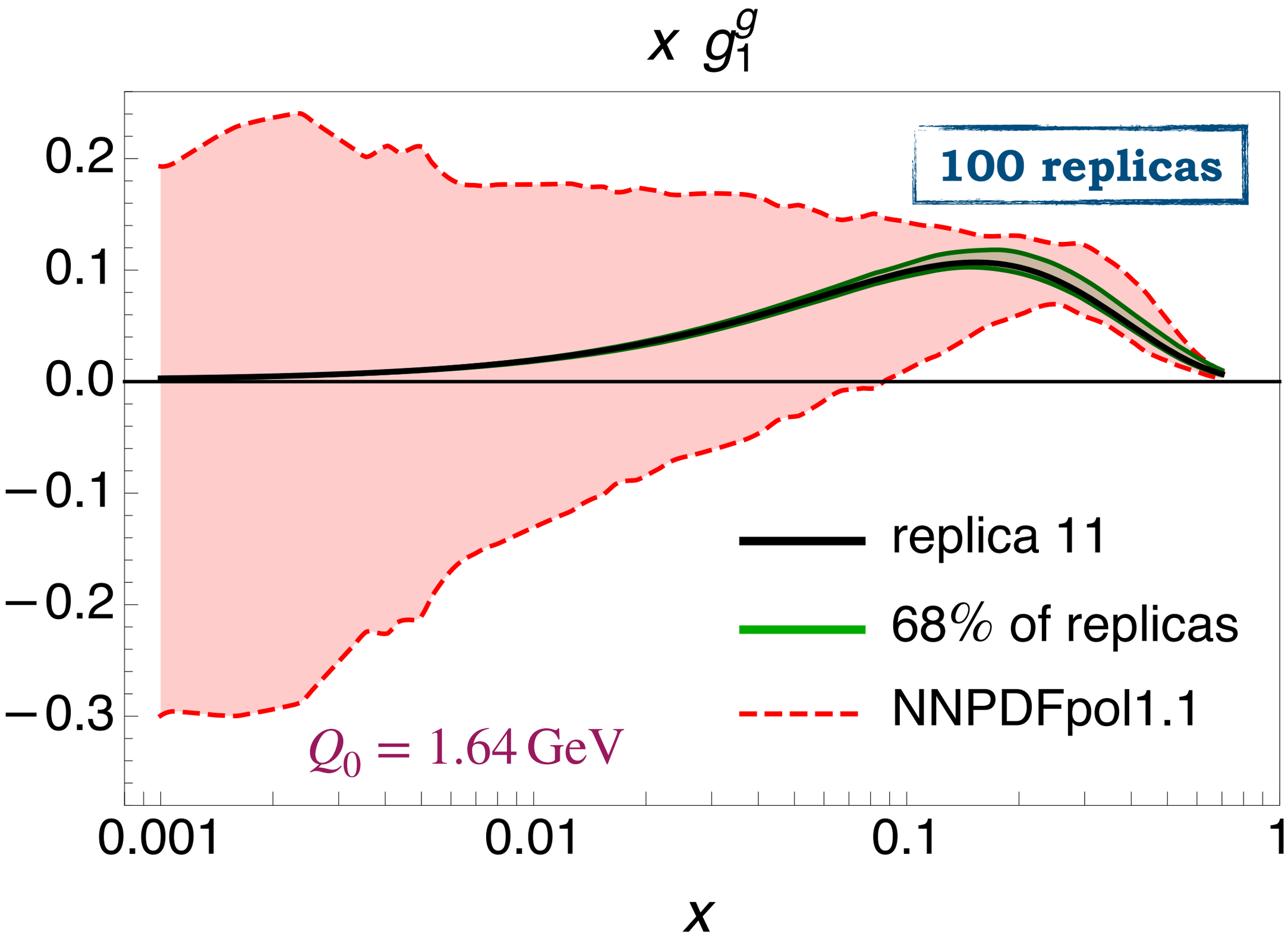
parametrized and **fitted to data**

slide from C. Bissolotti

Unpolarized gluon PDF



Helicity gluon PDF

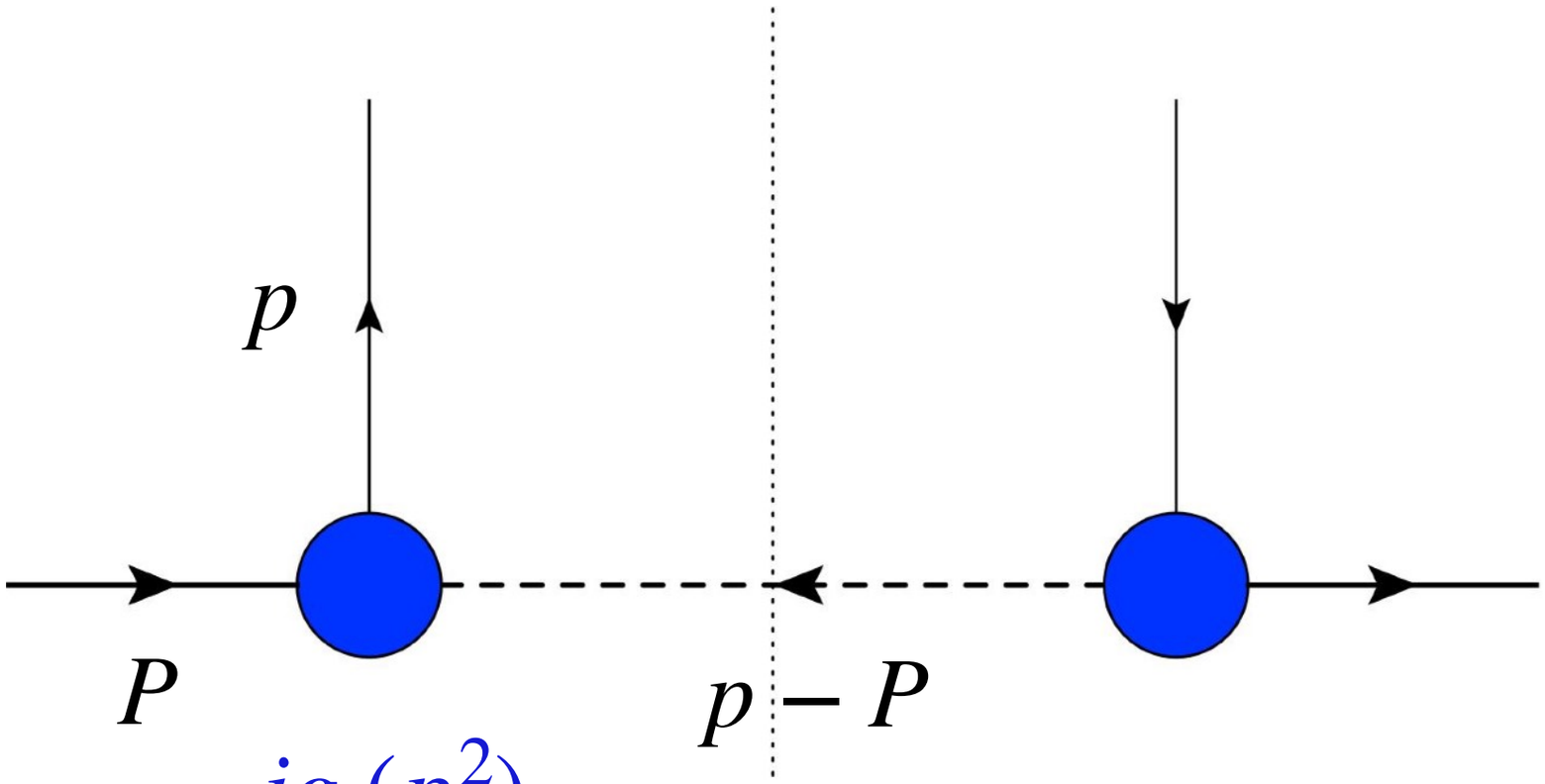


Fit parameters

parameter	mean	replica 11
A	6.1 ± 2.3	6.0
a	0.82 ± 0.21	0.78
b	1.43 ± 0.23	1.38
C	371 ± 58	346
D (GeV)	0.548 ± 0.081	0.548
σ (GeV)	0.52 ± 0.14	0.50
Λ_X (GeV)	0.472 ± 0.058	0.448
κ_1 (GeV^2)	1.51 ± 0.16	1.46
κ_2 (GeV^2)	0.414 ± 0.036	0.414

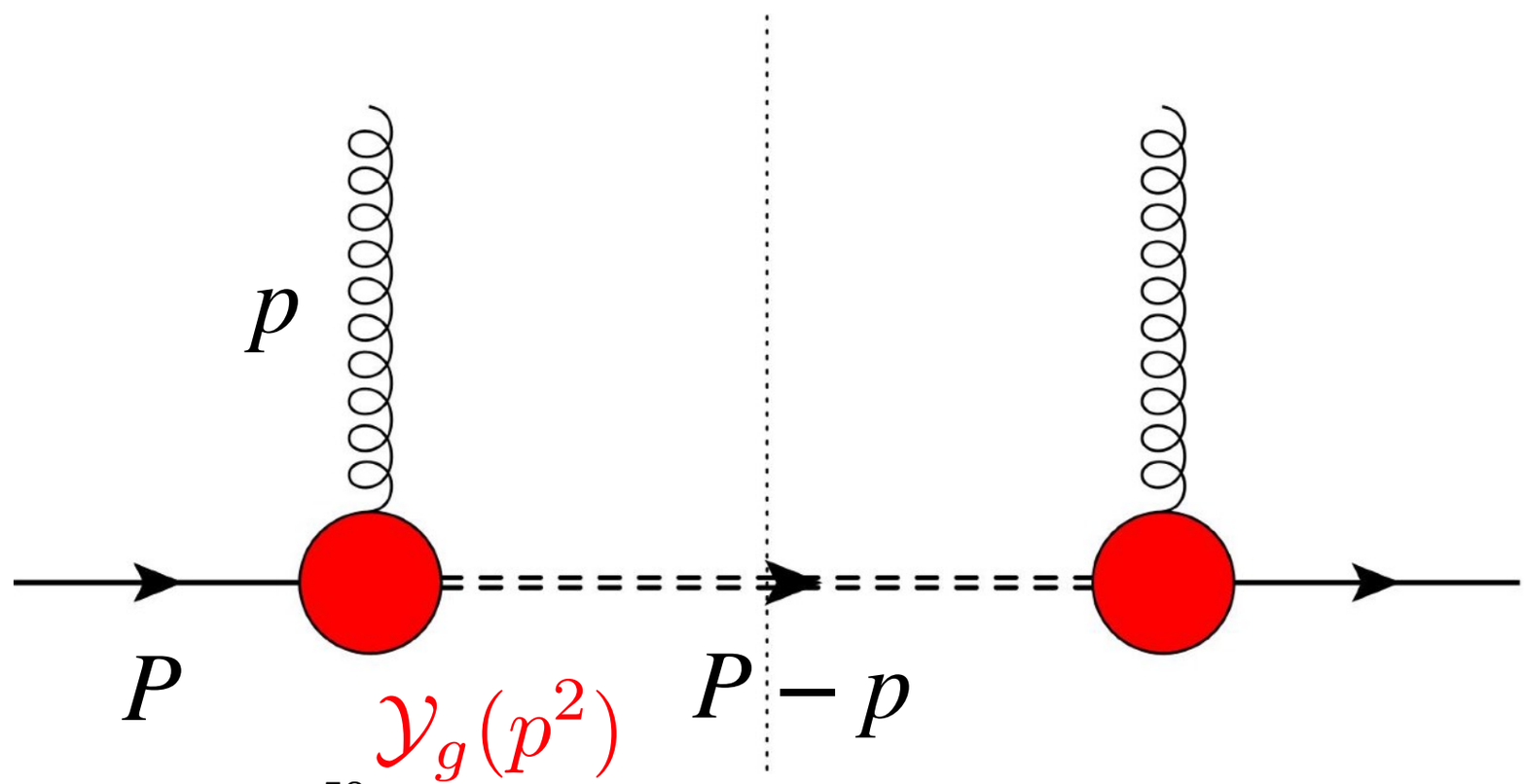
B fixed at 2.1

Effective vertices



$$i g_s(p^2)$$

$$i \frac{g_a(p^2)}{\sqrt{2}} \gamma^\mu \gamma^5$$



Quark and gluon correlators

Scalar-diquark spectator

$$\Phi_s = \frac{1}{2(2\pi)^3(1-x)P^+}\frac{g_s^2}{(p^2-m_q^2)^2}(\not p + m_q)\frac{1+\gamma^5\not S}{2}(\not P + M_H)(\not p + m_q)$$

Axial-vector-diquark spectator

$$\Phi_a = \frac{1}{2(2\pi)^3(1-x)P^+}\frac{g_a^2}{(p^2-m_q^2)^2}(\not p + m_q)\frac{\gamma^5\gamma_\mu}{\sqrt{2}}\frac{1+\gamma^5\not S}{2}(\not P + M_H)\frac{\gamma^5\gamma_\nu}{\sqrt{2}}(\not p + m_q)d_T^{\mu\nu}(P-p)$$

Spin-1/2 spectator (gluon)

$$\Phi_g = \frac{1}{2(2\pi)^3(1-x)P^+}Tr\left[(\not P + M_H)\frac{1+\gamma^5\not S}{2}G^*_{\mu\rho}(p)G^{\nu\sigma}(p)\mathcal{Y}_g^{\rho *}\mathcal{Y}_{g\sigma}(\not P - \not p + M_X)\right]$$

$$\mathcal{Y}_g^\mu = g_1(p^2)\gamma^\mu + i\frac{g_2(p^2)}{2M_H}\sigma^{\mu\nu}p_\nu$$

Quark and gluon correlators

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$$\Phi_s = \frac{1}{2(2\pi)^3(1-x)P^+}\frac{g_s^2}{(p^2-m_q^2)^2}(\not p + m_q)\frac{1+\gamma^5\not S}{2}(\not P + M_H)(\not p + m_q)$$

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$$\mathcal{Y}_g^\mu = g_1(p^2)\gamma^\mu + i\frac{g_2(p^2)}{2M_H}\sigma^{\mu\nu}p_\nu$$

Selection out of 12 Dirac structures

Spectator-model gluon TMDs (1)

$$\hat{f}_1^g(x, \boldsymbol{p}_T^2) = \big[4g_1^2 M_H^2 (x^2 (M_X - M_H(1-x))^2 + \boldsymbol{p}_T^2((x-2)x+2))$$
$$- 4g_1 g_2 M_H x^2 (M_H + M_X)((M_X - M_H(1-x))^2 + \boldsymbol{p}_T^2)$$
$$+ g_2^2 (\boldsymbol{p}_T^2 x (M_H^2 (3x-2) + 2M_H M_X x + M_X^2 (x+2))$$
$$+ x^2 (M_H + M_X)^2 (M_X - M_H(1-x))^2 + 2(\boldsymbol{p}_T^2)^2)\big]$$
$$/4(2\pi)^3 M_H^2 x (L_g^2(m_g^2) + \boldsymbol{p}_T^2)^2$$

$$\hat{g}_{1L}^g(x, \boldsymbol{p}_T^2) = \big[(2g_1 M_H - g_2(M_H + M_X))(2g_1 M_H (x(M_X - M_H(1-x))^2 - \boldsymbol{p}_T^2(x-2))$$
$$+ g_2(-M_H^2 x^3 (M_H + M_X) + \boldsymbol{p}_T^2 x (M_X - 3M_H) + 2\boldsymbol{p}_T^2 (M_H - M_X)$$
$$+ 2M_H x^2 (M_H - M_X)(M_H + M_X) - x(M_H - M_X)^2 (M_H + M_X)))\big]$$
$$/4(2\pi)^3 M_H^2 (L_g^2(m_g^2) + \boldsymbol{p}_T^2)^2$$

$$L_X^2(m^2) = xM_X^2 + (1-x)m^2 - x(1-x)M_H^2$$

Spectator-model gluon TMDs (2)

$$g_{1T}^{g}(x,\boldsymbol{p}_T^2) = \big[((2g_1 M_H - g_2(M_H + M_X))((M_X - M_H(1-x)) \\ (-2g_1 M_H(1-x) - g_2 M_X x) - g_2 \boldsymbol{p}_T^2))\big] \\ /(2(2\pi)^3 M_H (L_X^2(m_g^2) + \boldsymbol{p}_T^2)^2)$$

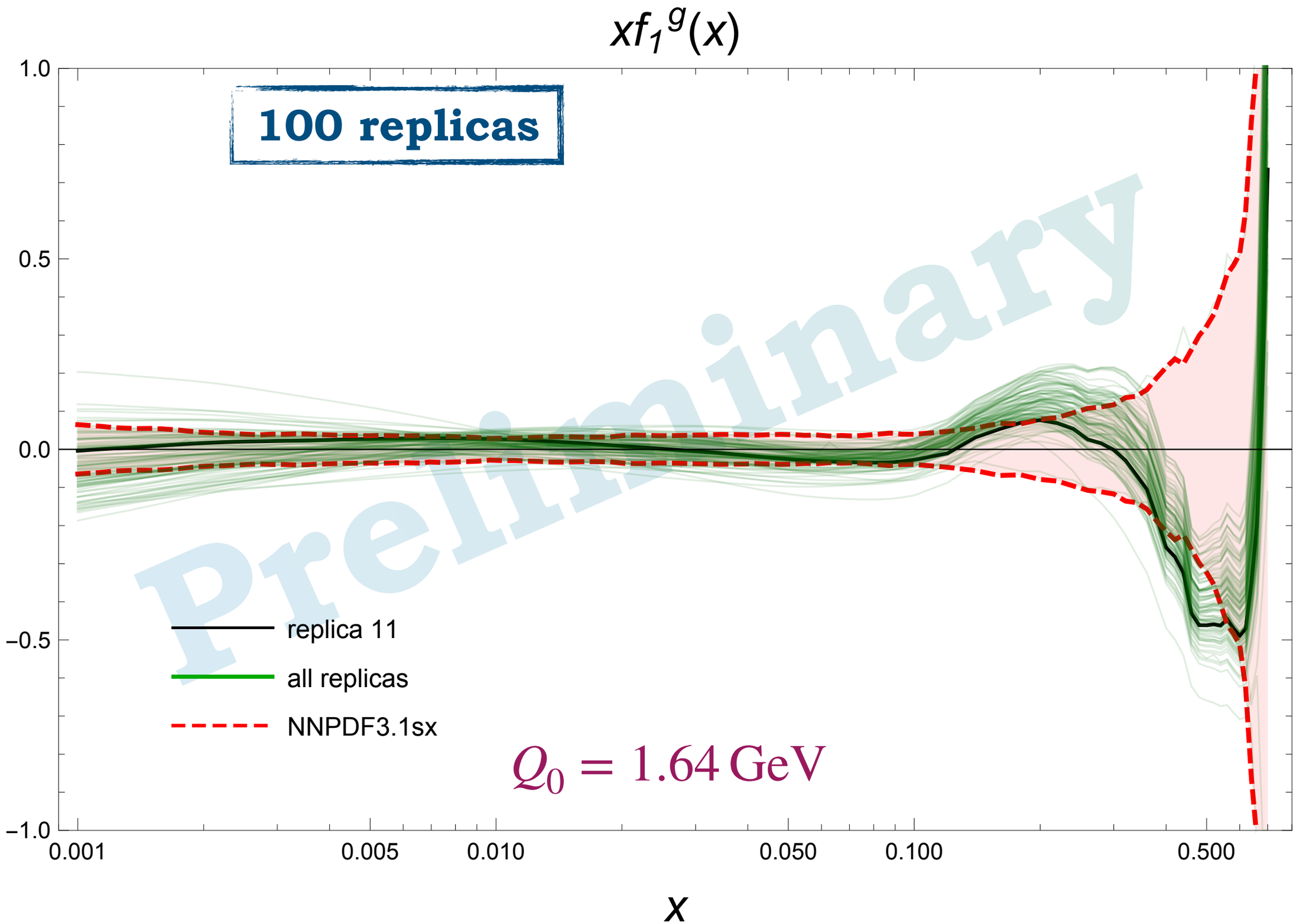
$$h_1^{\perp g}(x,\boldsymbol{p}_T^2) = \big[(4g_1^2 M_H^2(1-x) + g_2^2(x(M_X^2 - M_H^2(1-x)) + \boldsymbol{p}_T^2))\big] \\ /((2\pi)^3 x(L_X^2(m_g^2) + \boldsymbol{p}_T^2)^2)$$

$$L_X^2(m^2) = xM_X^2 + (1-x)m^2 - x(1-x)M_H^2$$

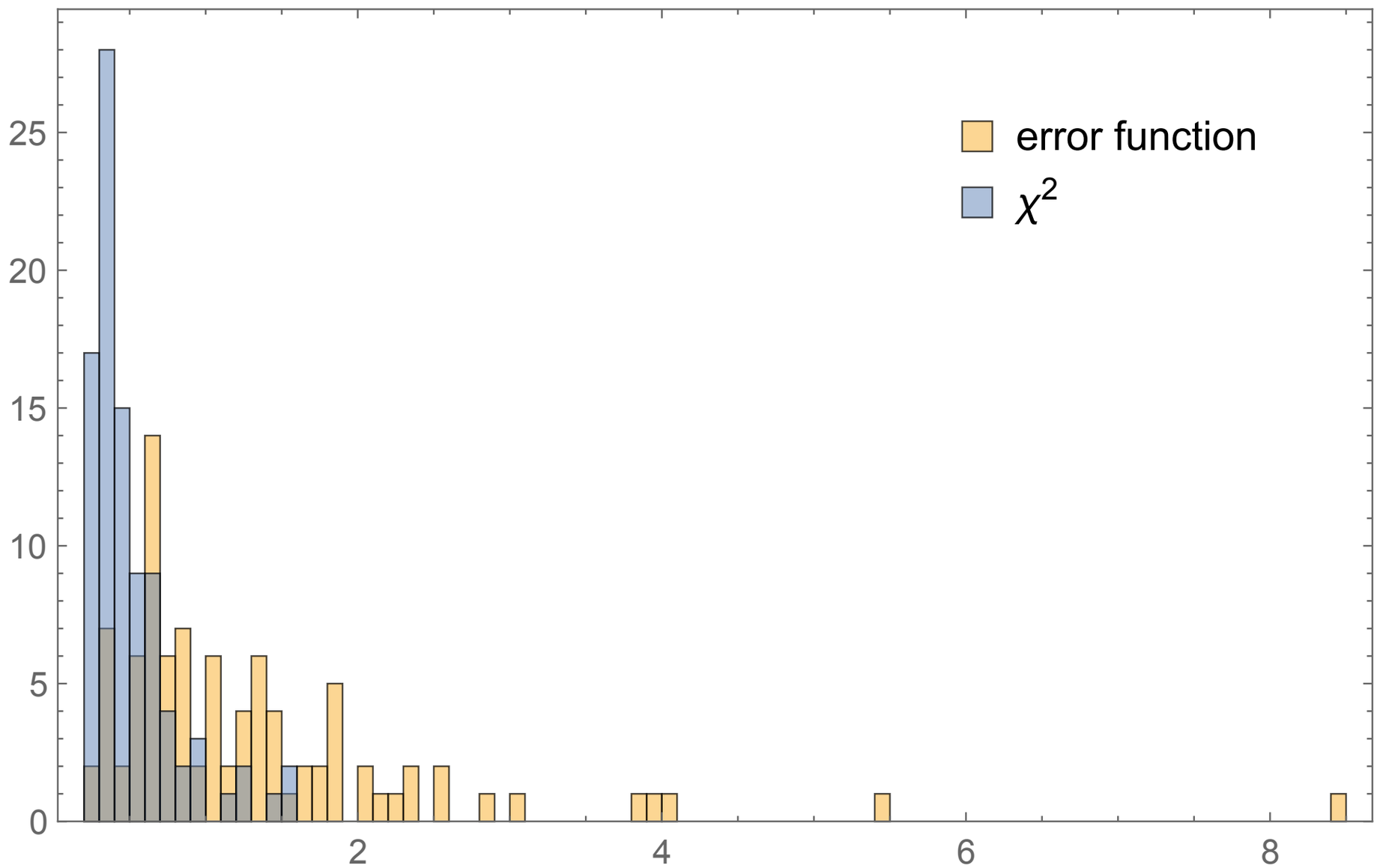
Nucleon-parton-spectator effective vertex

$$g_X(p^2) = \begin{cases} g_X^{p.l.} & \text{pointlike} \\ g_X^{dip} \dfrac{p^2 - m^2}{|p^2 - \Lambda_X^2|^2} & \text{dipolar} \\ g_X^{exp}\, e^{(p^2 - m^2)/\Lambda_X^2} & \text{exponential} \end{cases}$$

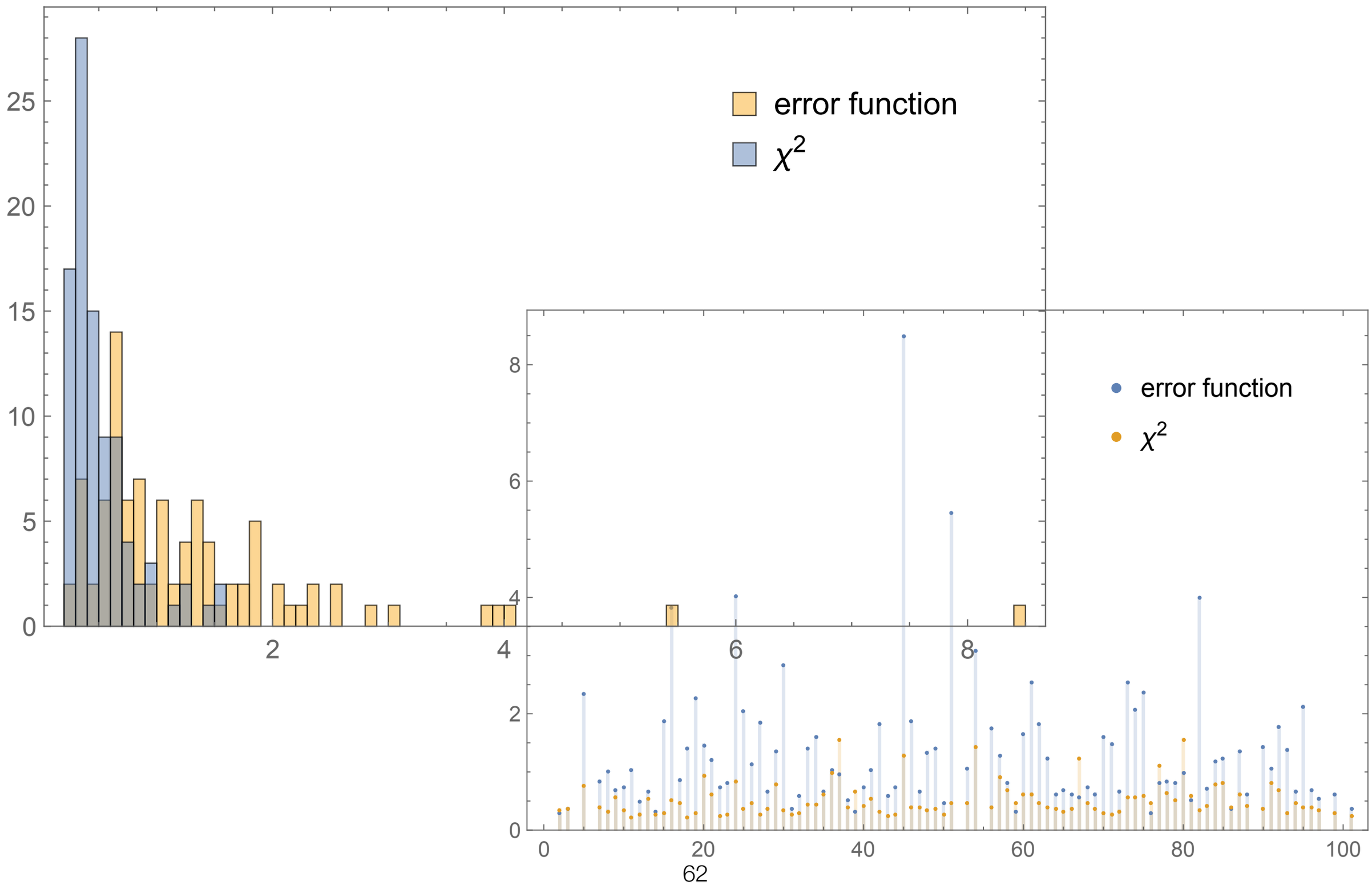
Unpolarized gluon PDF



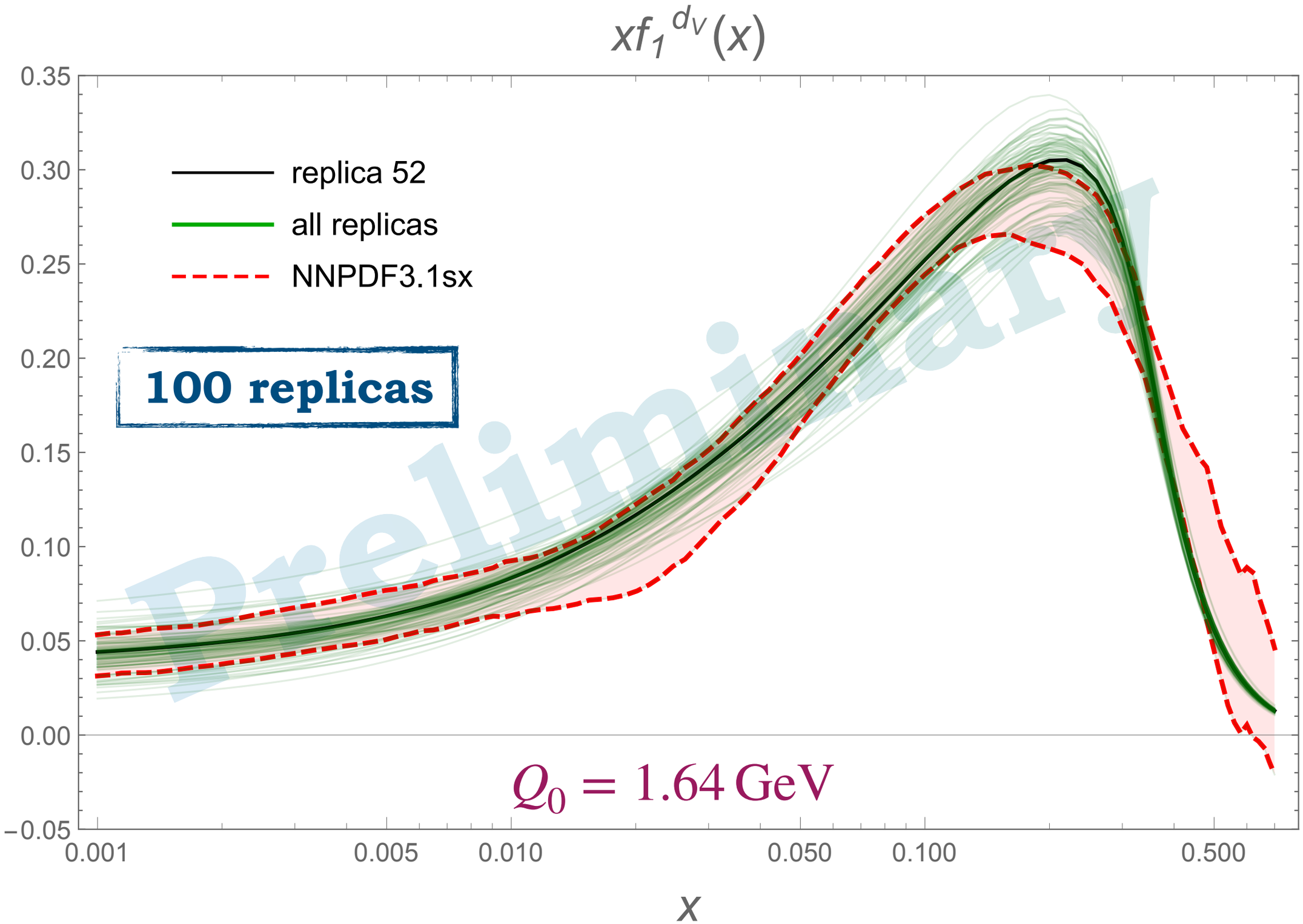
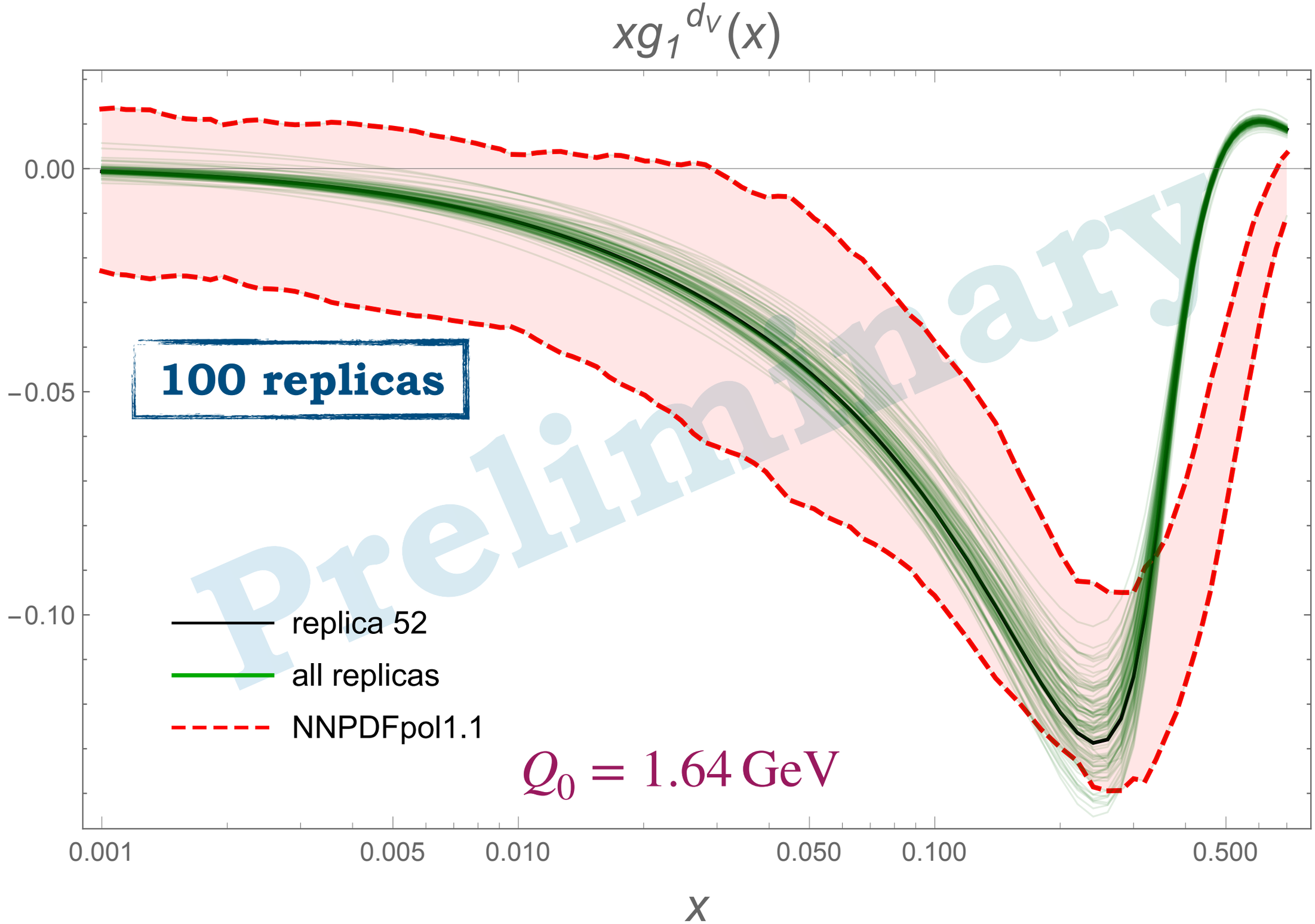
χ^2-analysis



χ^2-analysis



Unpolarized up-quark PDF



Helicity up-quark PDF