Quarkonium production and polarization in the color evaporation model

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Overview

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   - Polarized calculations in collinear ICEM at $\mathcal{O}(\alpha_s^3)$
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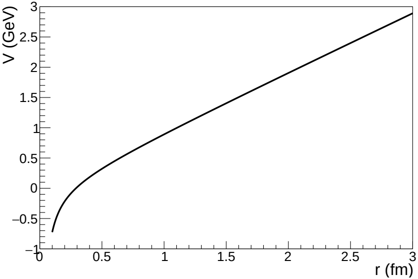
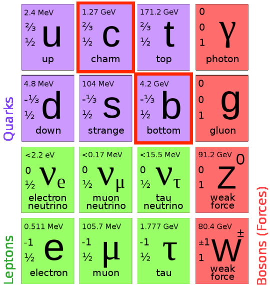
Quarkonium: A Bound State of $Q\overline{Q}$

Bound by the interquark potental: $V(r) = \sigma r - \alpha_c / r$[1]

- linear term refers to the confinement
- $1/r$ term refers to the Coulomb-like short distance behavior
- $\sigma = 0.192\ \text{GeV}^2$, $\alpha_c = 0.471$[2]

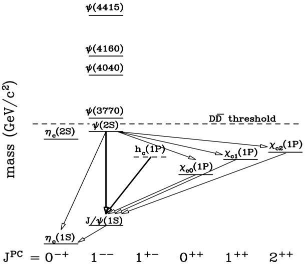




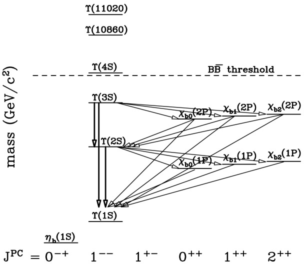
[1] E. Eichten *et al.*, Phys. Rev. D **17**, 3090 (1978).
[2] F. Karsch, M. T. Mehr, and H. Satz, Z. Phys. C **37**, 617 (1988).

Quarkonium Families – Charmonium and Bottomonium

Charmonium

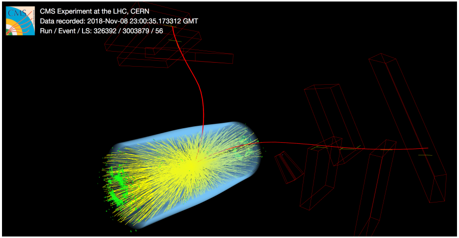


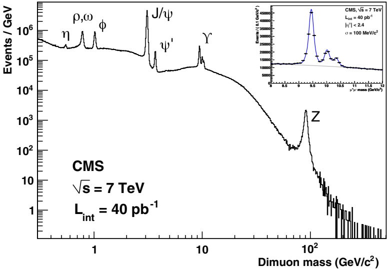
Bottomonium



- A spectrum of states come from combination of two spin $1/2$ particles and orbital angular momentum $\rightarrow$ different spin states ${}^{2S+1}L_J$
- All physical states are color singlets: ${}^{2S+1}L_J^{[1]}$
- The S states below the $H\overline{H}$ $(H = D, B)$ threshold decay electromagnetically into $\ell^+\ell^-$

Detection

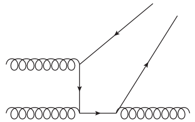


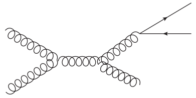


- S states ($J^{PC} = 1^{--}$) decay to $\ell^+\ell^-$, so they can be observed as peaks in dilepton mass spectra
- $\chi(n\mathrm{P})$ states ($J^{PC} = J^{++}$) can be reconstructed by matching an S state with a low momentum photon
- η_c and η_b states ($J^{PC} = 0^{-+}$) decay hadronically

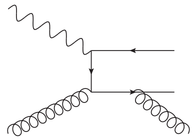
Some Production Diagrams in Different Systems

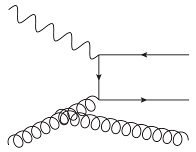
hh (RHIC, Tevatron, LHC)



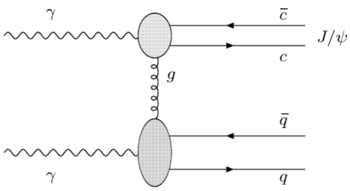


γp (HERA)

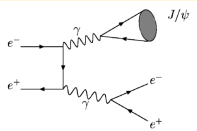


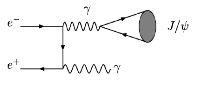


$\gamma\gamma$ (LEP)

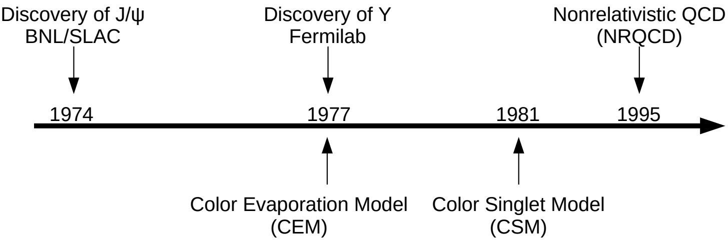


e^+e^- (KEKB)





Discovery and Production Models



Color Evaporation Model [Fritzsch 77; Halzen 77; Glück, Owens, Reya 78]

- spins and colors are averaged

Color Singlet Model [Berger, Jones 81; Baier, Rückl 81, Schuler 94, Lansberg 11]

- only color singlet contribution is considered

Nonrelativistic QCD (NRQCD) [Bodwin, Braaten, Lepage 95]

- separate all spin and color states

Quarkonium Production Models

We are not able to accurately describe every observable associated with quarkonium production using one production model with one set of model parameters.

Observables

- Yields and distributions of the S state quarkonia
- Yields and distributions of η's and χ's
- Production of one state realative to another (e.g. $\psi(2S)$ to J/ψ)
- Production of one spin state relative to another (i.e. polarization)

Production models are still unsettled

- J/ψ and Υ are discovered in 1974 and 1977 respectively
- The quarkonium production mechanism has not been solved
- Different models were developed to describe the observables

Quarkonium Production Models

Color Evaporation Model (CEM) [Fritzsch 77; Halzen 77; Glück, Owens, Reya 78; Gavai *et al.* 95; Schuler, Vogt 95]

Leading order cross section:

$$\sigma = F_{\mathcal{Q}} \sum_{i,j} \int_{4m_Q^2}^{4m_H^2} d\hat{s} \int dx_1 dx_2 f_{i/p}(x_1, \mu^2) f_{j/p}(x_2, \mu^2) \hat{\sigma}_{ij}(\hat{s}) \delta(\hat{s} - x_1 x_2 s) \, ,$$

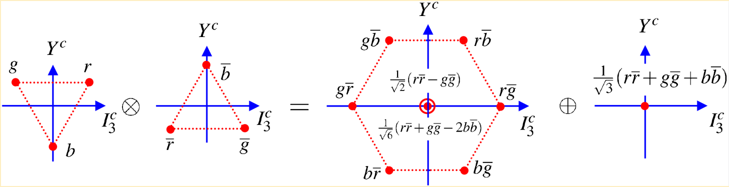
$F_{\mathcal{Q}}$ is a universal factor for the quarkonium state ($\mathcal{Q}$) and is independent of the projectile, target, and energy.

- all Quarkonium states are treated like $Q\overline{Q}$ ($Q = c, b$) below $H\overline{H}$ ($H = D, B$) threshold
- all diagrams for $Q\bar{Q}$ production included, independent of color
- fewer parameters than NRQCD (one $F_{\mathcal{Q}}$ for each Quarkonium state)
- $F_{\mathcal{Q}}$ is fixed by comparison of NLO calculation of $\sigma_{\mathcal{Q}}^{CEM}$ to $\sqrt{s}$ for J/ψ and Υ, $\sigma(x_F > 0)$ and $Bd\sigma/dy|_{y=0}$ for J/ψ, $Bd\sigma/dy|_{y=0}$ for Υ

Color Singlet Model (CSM) [Berger, Jones 81; Baier, Rückl 81, Schuler 94, Lansberg 11]

- constrains the production of $Q\bar{Q}$ to the color singlet state only
- the produced $Q\bar{Q}$ pair does not change its color and spin between production and hadronization

$$
\begin{aligned}
d\sigma[\mathcal{Q}+X] &= \sum_{i,j}\int dx_i dx_j f_i(x_i,\mu_F) fj(x_j,\mu_F) d\hat{\sigma}_{i+j\to(Q\bar{Q})+x}(\mu_R,\mu_F) \\
&\times \quad |R(0)|^2 \, .
\end{aligned}
$$



Quarkonium Production Models

Non Relativistic QCD (NRQCD) [Bodwin, Braaten, Lepage 95]

- an Effective Field Theory where production is described as an expansion in powers of α_s and the heavy quark velocity, v/c
- At each order, the production is further factorized into perturbative Short Distance Coefficients and non-perturbative Long Distance Matrix Elements (LDMEs); e.g. for J/ψ, $\sigma_{J/\psi} = \sum_n \sigma_{c\bar{c}[n]} \langle \mathcal{O}^{J/\psi}[n] \rangle$
- $\sigma_{c\bar{c}[n]}$ are cross sections in a particular color and spin state n calcuated by perturbative QCD
- including ${}^3S_1^{[1]}$ (singlet), and ${}^3P_J^{[8]}, {}^3S_1^{[8]}$ and ${}^1S_0^{[8]}$ (octets)
- $\langle \mathcal{O}^{J/\psi}[n] \rangle$ are the LDMEs that describe the conversion of $c\bar{c}[n]$ state into final state J/ψ, assuming that the hadronization does not change the momentum
- LDMEs are conjectured to be universal and the mixing of LDMEs are determined by fitting to data

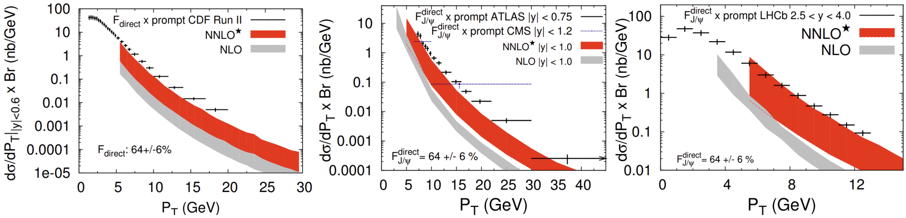
Quarkonium Production Models

Improved CEM (ICEM) [Ma, Vogt 16]

$$\sigma = F_{\mathcal{Q}} \sum_{i,j} \int_{M_\psi}^{2m_H} dM \int dx_i dx_j f_i(x_i, \mu_F) f_j(x_j, \mu_F) d\hat{\sigma}_{ij \to c\bar{c}+X}(p_{c\bar{c}}, \mu_R)|_{p_{c\bar{c}} = \frac{m}{M_\psi} p_\psi},$$

where M_ψ is the mass of the charmonium state, ψ.

- first new advance in the basic CEM model since 1990s
- able to describe relative production of ψ(2S) to J/ψ, where the ratio is flat in the traditional CEM
- distinction between the momentum of the $c\bar{c}$ pair and that of charmonium so that the p_T spectra will be softer and thus may explain the high p_T data better
- employed to calculate production and polarization of all S states, and relative production of χ states

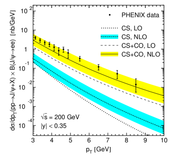


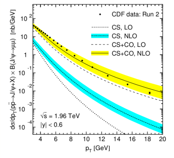
- LO and NLO calculations underestimate the Tevatron p_T distributions
- Recent advancements in CSM show that by adding real-emission contribution at NNLO, CSM can describe the distributions[3] (NNLO*)

[3] J.P. Lansberg, J. Phys. G **38**, 124110 (2011).

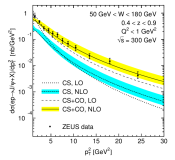
Results in NRQCD - A global fit of LDMEs[4]

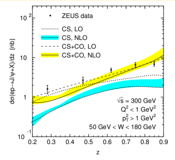
hh ($p_T > 3$ GeV)





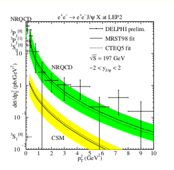
γp ($p_T > 3$ GeV)

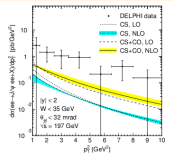




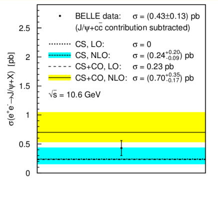
$\gamma\gamma$ (Right: $p_T > 1$ GeV)

[6]





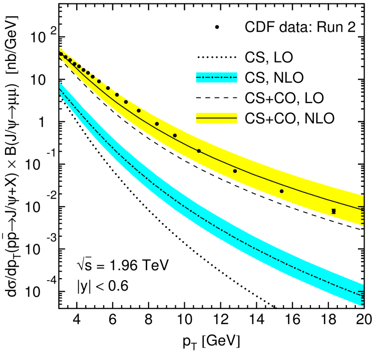
e^+e^-

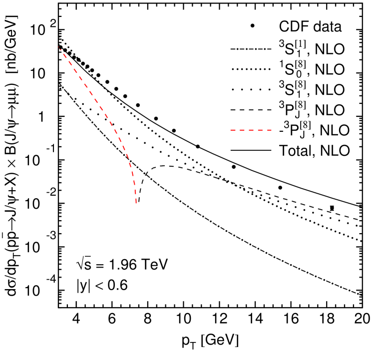


[4] M. Butenschoen and B. A. Kniehl, Nucl. Phys. Proc. Suppl. **222-224**, 151 (2012).

[5] M. Klasen *et. al*, DESY 01-202.

Decomposition of NRQCD[4]



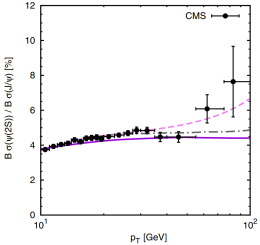


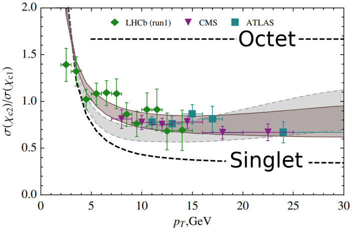
- CS alone is well below data
- Sizeable NLO CO corrections

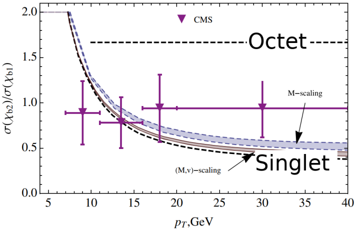
[4]M. Butenschoen and B. A. Kniehl, Nucl. Phys. Proc. Suppl. **222-224**, 151 (2012).

Relative production in NRQCD

- $\psi(2S)$ to J/ψ ratio agrees with data at most p_T[6]
- relative production of χ_c and χ_b are dominated by CSM contribution[7]



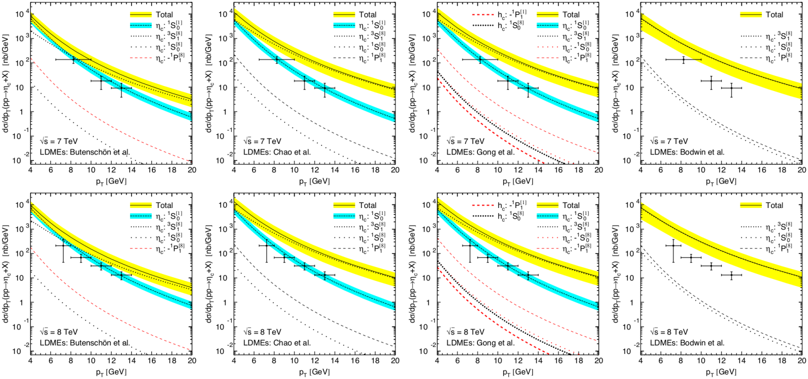




[6] S. P. Baranov and A. V. Lipatov, Phys. Rev. D **96**, 034019 (2017).
[7] A. K. Likhoded *et al.*, Phys. Rev. D **90**, 074021 (2014).

η_c production in NRQCD

M. Butenschoen, Z-G He, and B. A. Kniehl, Phys. Rev. Lett. **114**, 092004 (2015).

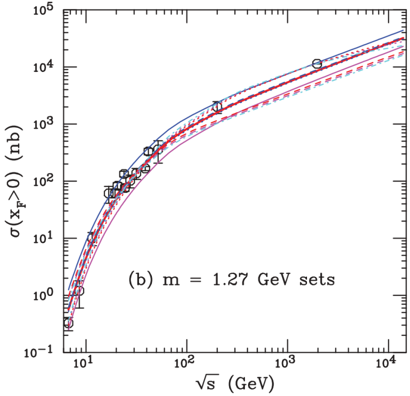


- all results so far overpredict LHCb η_c yields
- results can be described by CSM alone
- PRL 114, 092005 (2015) and PRL 114, 092006 (2015) describe the η_c results but not the J/ψ polarization

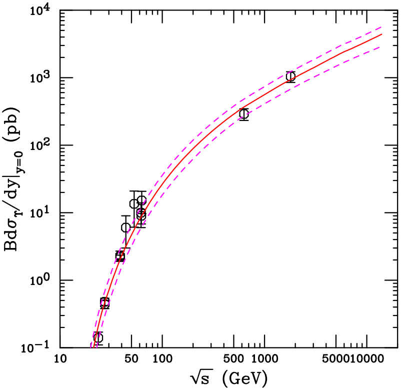
Results in the CEM[8]

- one fitting factor ($F_{\mathcal{Q}}$) for each quarkonium state ($\mathcal{Q}$)
- great consistency with experimental results over large range of $\sqrt{s}$

J/ψ

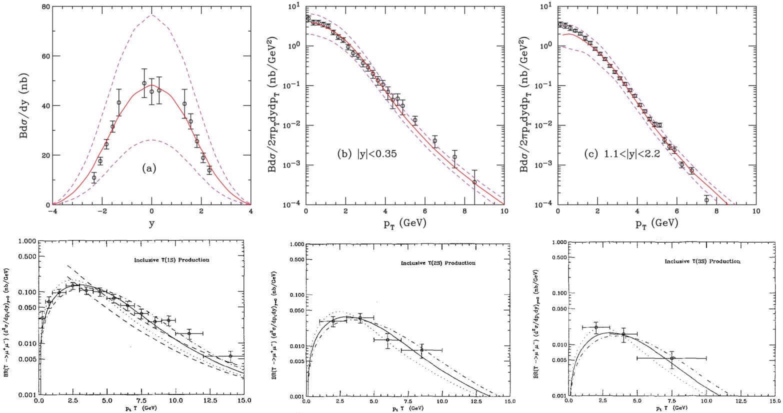


$\sum \Upsilon$'s



[8] R. E. Nelson, R. Vogt and A. D. Frawley, Phys. Rev. C **87**, 014908 (2013).

Results in the CEM[8,9]

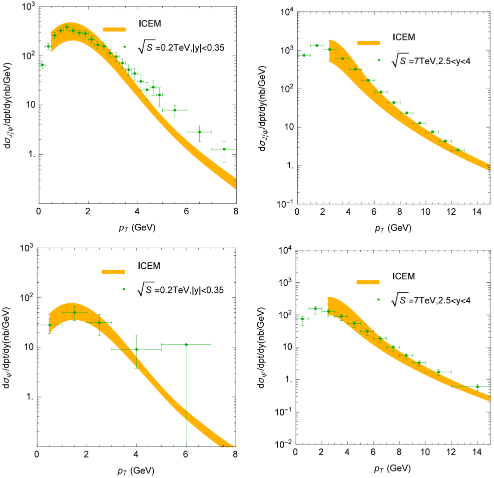


- overall less rigorous, but accurate predictions
- no advances in the basic model since 1990s

[8] R. E. Nelson, R. Vogt and A. D. Frawley, Phys. Rev. C **87**, 014908 (2013).
[9] G. A. Schuler and R. Vogt, Phys. Lett. B **387**, 181 (1996).

Results in the ICEM

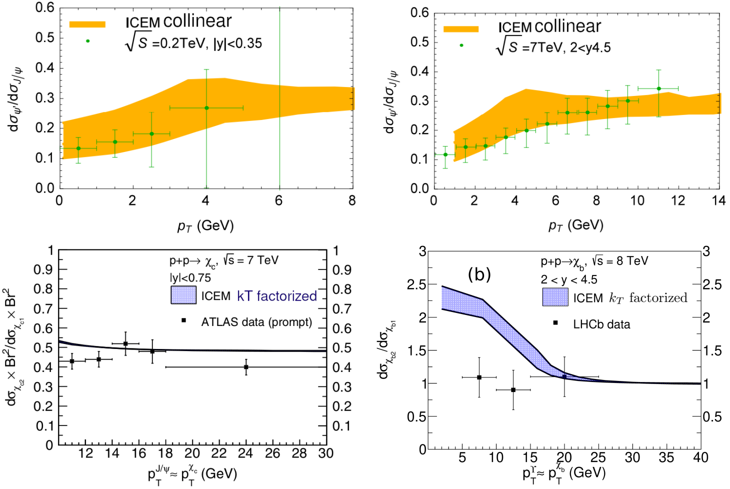
$$\frac{d\sigma_\psi(P)}{dp_T} = F_\psi \int_{\boxed{M_\psi}}^{2M_D} dM \frac{M}{M_\psi} \frac{d\sigma_{c\bar{c}}(M, P')}{dM\, dp'_T}\Bigg|_{\boxed{p'_T=(M/M_\psi)p_T}}$$



Ma and Vogt, PRD 94, 114029 (2016).

- explicit charmonium mass dependence $\rightarrow$ the ratio of cross sections is no longer p_T-independent
- distinction between the momentum of the $c\bar{c}$ pair and that of charmonium $\rightarrow$ p_T spectra will be softer and thus may explain the high p_T data better

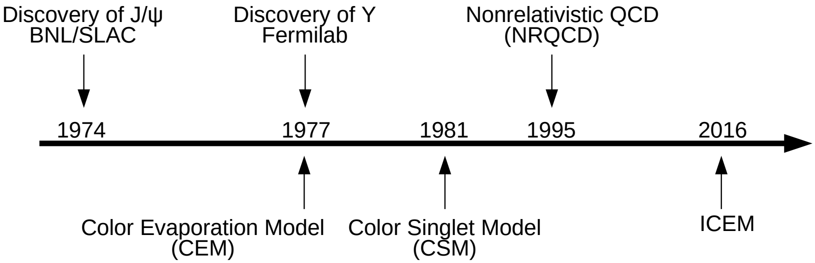
Relative production in the ICEM[10,11]



[10] Y. Q. Ma and R. Vogt, Phys. Rev. D **94**, 114029 (2016).

[11] V. Cheung and R. Vogt, Phys. Rev. D **98**, 114029 (2018) and **99**, 034007 (2019).

Model Development



Summary

- Start with a simple model by averaging over all color states (CEM).
- Make more sense by limiting to only color singlet production (CSM).
- Bring in the contribution from the color octet states through non-perturbative parameters (NRQCD)
- Improvements are made on the traditional CEM to give more and better descriptions (ICEM).

Tests of Models

- CEM and NRQCD remain the most commonly used models today.
- They can predict yields and relative production of different quarkonium states.
- What about the relative production of different spin projection states of the same quarkonium state? $\rightarrow$ Polarization

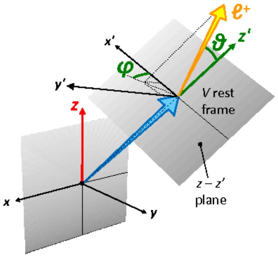
(I)CEM

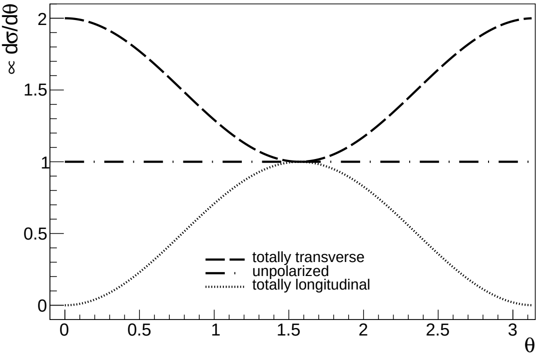
- Less rigorous
- Fewer fit parameters
- Applied extensively to only hadroproduction (so far)

NRQCD

- More rigorous
- More fit parameters
- Applied to all collision systems

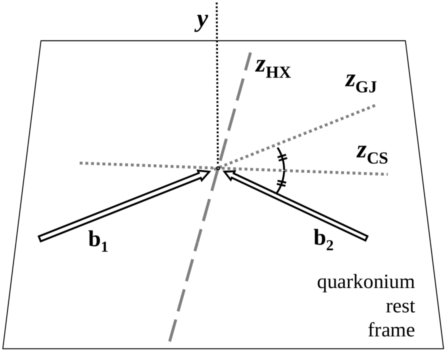
Polarization of Quarkonium

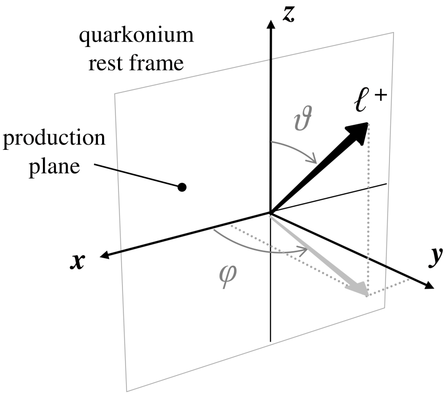




- defined as the tendency of quarkonium to be in a certain angular momentum state given its total angular momentum
- e.g. an unpolarized $J = 1$ production means J_z = -1, 0, +1 production is equally likely
- longitudinal $\rightarrow$ peak at $\vartheta = \pi/2$; transverse $\rightarrow$ peaks at $\vartheta = 0, \pi$

Polarization Measurement

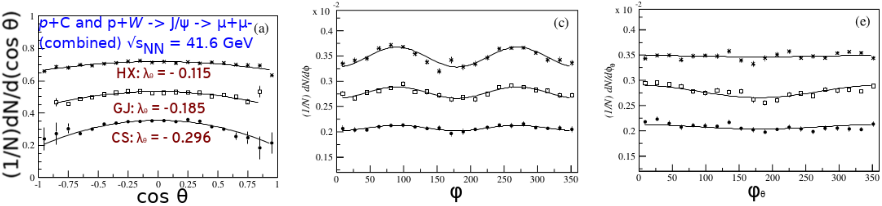




- There are three commonly used choices for the z-axis, namely z_{HX} (helicity), z_{CS} (Collins-Soper), and z_{GJ} (Gottfried-Jackson)
- ϑ is defined as the angle between the z-axis and the direction of travel for the ℓ^+ in the quarkonium rest frame

$$\frac{d\sigma}{d\Omega} \propto 1 + \lambda_\theta \cos^2\theta + \lambda_\phi \sin^2\theta \cos(2\phi) + \lambda_{\theta\phi} \sin(2\theta)\cos\phi$$

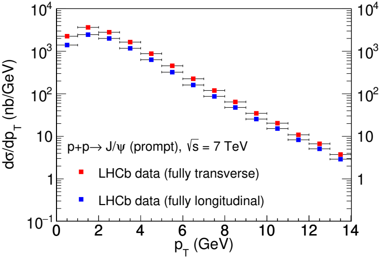
- Polarization parameters can be obtained by fitting the angular spectra as a function of θ and ϕ
- One can write $\phi_\theta = \phi - \frac{\pi}{2} \mp \frac{\pi}{4}$ for $\cos\theta \lessgtr 0$, then[12]
- $\frac{d\sigma}{d\phi_\theta} \propto 1 + \frac{\sqrt{2}\lambda_{\theta\phi}}{3+\lambda_\theta} \cos\phi_\theta$



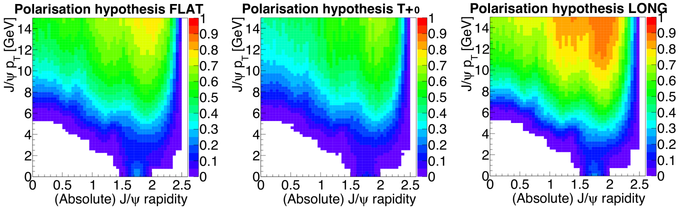
[12] I. Abt *et al.* (HERA-B Collaboration), Eur. Phys. J. C **60**, 517 (2009).

Importance of Polarization

- Polarization predictions are strong tests of production models
- Detector acceptance depends on polarization hypothesis
- Understanding polarization helps narrow systematic uncertainties



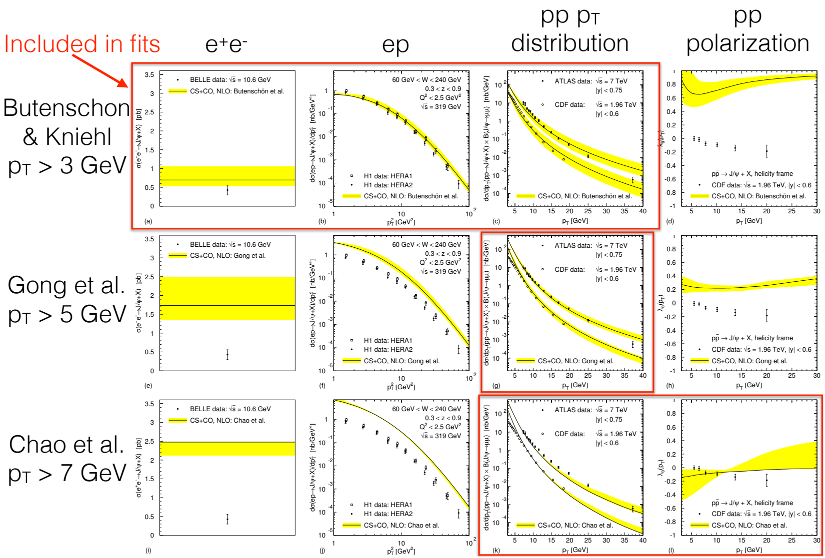
[13]



[14]

[13] R. Aaij *et al.* (LHCb Collaboration), Eur. Phys. J. C **71**, 1645 (2011).
[14] G. Aad *et al.* (ATLAS Collaboration), Nucl. Phys. B **850**, 387 (2011).

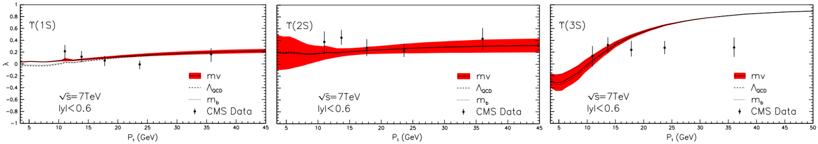
Polarization Puzzle[15]



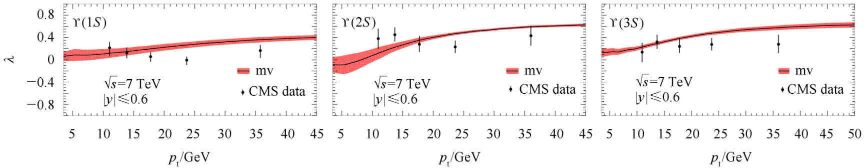
[15]N. Brambilla *et al.*, Eur. Phys. J. C **74**, 2981 (2014).

$\Upsilon(nS)$ Polarization in NRQCD

B. Gong, L. P. Wan, J. X. Wang and H. F. Zhang, Phys. Rev. Lett. **112**, 032001 (2014).



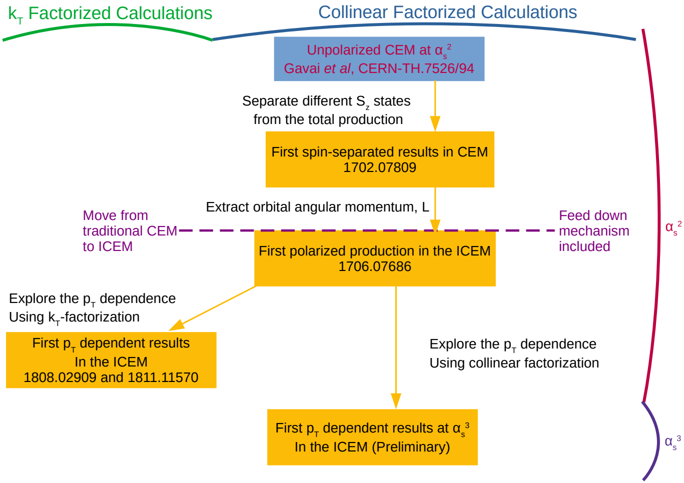
Update: Y. Feng, B. Gong, L. P. Wan, and J. X. Wang, Chin. Phys. **C39**, 123102 (2015).



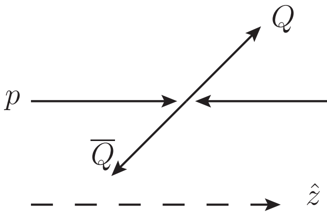
- polarization of $\Upsilon(nS)$ is better described than for J/ψ
- polarization prediction in NRQCD is improved by including the feed down decays from χ_b states (bottom row)

Polarized Production in the CEM and ICEM

- No polarization calculations made in the CEM family before 2017.
- It is worth revisiting back the CEM to calculate polarized results
- VC and Ramona Vogt made a few calculations using the (I)CEM.



Polarization in collinearly factorized calculation at $\mathcal{O}(\alpha_s^2)$



Polarization in HX/CS/GJ frames

- At $\mathcal{O}(\alpha_s^2)$, $Q\bar{Q}$ can only be produced with $p_T = 0$.
- There is no difference in the three commonly used axes.
- We started by projecting the spin of the heavy quarks onto beam axis.

Scattering amplitudes used at $\mathcal{O}(\alpha_s^2)$

In terms of the Dirac spinors u and v, the individual amplitudes at leading order are

$$
\begin{aligned}
\mathcal{A}_{qq} &= \frac{g_s^2}{\hat{s}}[\overline{u}(p')\gamma_\mu v(p)][\overline{v}(k)\gamma^\mu u(k')] \,, \\
\mathcal{A}_{gg,s} &= -\frac{g_s^2}{\hat{s}}\Big\{ -2k'\cdot\epsilon(k)[\overline{u}(p')\not{\epsilon}(k')v(p)] \\
&+ 2k\cdot\epsilon(k')[\overline{u}(p')\not{\epsilon}(k)v(p)] \\
&+ \epsilon(k)\cdot\epsilon(k')[\overline{u}(p')(\not{k}'-\not{k})v(p)]\Big\} \,, \\
\mathcal{A}_{gg,t} &= -\frac{g_s^2}{\hat{t}-M^2}\overline{u}(p')\not{\epsilon}(k')(\not{k}-\not{p}+M)\not{\epsilon}(k)v(p) \,, \\
\mathcal{A}_{gg,u} &= -\frac{g_s^2}{\hat{u}-M^2}\overline{u}(p')\not{\epsilon}(k)(\not{k}'-\not{p}+M)\not{\epsilon}(k')v(p) \,,
\end{aligned}
$$

- $\mathcal{A}$'s are separated according to the S_z of the final state
- Orbital Angular Momentum is extracted before squaring the amplitudes

Orbital Angular Momentum

At leading order, the final state $Q\overline{Q}$ is produced with no dependence on the azimuthal angle and thus $L_z = 0$. To extract the projection on a state with orbital-angular-momentum quantum number L, we determine the corresponding Legendre component $\mathcal{A}_L$ in the amplitudes by

$$\begin{aligned}
\mathcal{A}_{L=0} &= \frac{1}{2}\int_{-1}^{1} dx \mathcal{A}(x = \cos\theta) \,, \\
\mathcal{A}_{L=1} &= \frac{3}{2}\int_{-1}^{1} dx \; x\mathcal{A}(x = \cos\theta) \,.
\end{aligned}$$

$L = 2$ amplitudes are not needed for S and χ states production.

$|J, J_z\rangle$ States

Two helicity combinations that result in $S_z = 0$ are added and normalized to give contribution to the spin triplet state ($S = 1$). We calculate the amplitudes for $J = 0, 1, 2$:

$$
\begin{aligned}
\mathcal{A}_{J=1,J_z=\pm1} &= \mathcal{A}_{L=0,L_z=0;S=1,S_z=\pm1}\ , (\text{S States}) \\
\mathcal{A}_{J=1,J_z=0} &= \mathcal{A}_{L=0,L_z=0;S=1,S_z=0}\ , (\text{S States}) \\
\mathcal{A}_{J=0,J_z=0} &= -\sqrt{\frac{1}{3}}\mathcal{A}_{L=1,L_z=0;S=1,S_z=0}\ , (\chi_0 \text{ States}) \\
\mathcal{A}_{J=1,J_z=\pm1} &= \mp\frac{1}{\sqrt{2}}\mathcal{A}_{L=1,L_z=0;S=1,S_z=\pm1}\ , (\chi_1 \text{ States}) \\
\mathcal{A}_{J=1,J_z=0} &= 0\ , (\chi_1 \text{ States}) \\
\mathcal{A}_{J=2,J_z=\pm2} &= 0\ , (\chi_2 \text{ States}) \\
\mathcal{A}_{J=2,J_z=\pm1} &= \frac{1}{\sqrt{2}}\mathcal{A}_{L=1,L_z=0;S=1,S_z=\pm1}\ , (\chi_2 \text{ States}) \\
\mathcal{A}_{J=2,J_z=0} &= \sqrt{\frac{2}{3}}\mathcal{A}_{L=1,L_z=0;S=1,S_z=0}\ . (\chi_2 \text{ States})
\end{aligned}
$$

Production formula in collinear CEM at $\mathcal{O}(\alpha_s^2)$

CEM using collinear factorization approach

$$\sigma = F_Q \sum_{i,j} \int_{M_{\mathcal{Q}}^2}^{4m_H^2} d\hat{s} \int dx_1 dx_2 f_{i/p}(x_1, \mu^2) f_{j/p}(x_2, \mu^2) \hat{\sigma}_{ij}(\hat{s}) \delta(\hat{s} - x_1 x_2 s) \,,$$

- Convoluted with the CTEQ6L1 parton distribution functions (PDFs)
- α_s is calculated at one-loop level
- We took the factorization and renormalization scales to be $\mu^2 = \hat{s}$
- $1.27 < m_c < 1.50$ GeV, $4.5 < m_b < 5.0$ GeV
- Assumed that the polarization is unchanged by the transition from the parton level to the hadron level

Feed Down Production[16]

CEM polarization calculations assume two pions are emitted from an S state feed down and a photon is emitted from a P state feed down.

$$R_{J/\psi}^{J_z=0} = \sum_{\psi,J_z} c_\psi S_\psi^{J_z} R_\psi^{J_z} \, , R_{\Upsilon(1\mathrm{S})}^{J_z=0} = \sum_{\Upsilon,J_z} c_\Upsilon S_\Upsilon^{J_z} R_\Upsilon^{J_z} \, ,$$

Q	M_Q (GeV)	c_Q	$S_Q^{J_z=0}$	$S_Q^{J_z=\pm 1}$
J/ψ	3.10	0.62	1	0
ψ(2S)	3.69	0.08	1	0
χ_{c1}(1P)	3.51	0.16	0	1/2
χ_{c2}(1P)	3.56	0.14	2/3	1/2
Υ(1S)	9.46	0.52	1	0
Υ(2S)	10.0	0.1	1	0
Υ(3S)	10.4	0.02	1	0
χ_{b1}(1P)	9.89	0.13	0	1/2
χ_{b2}(1P)	9.91	0.13	2/3	1/2
χ_{b1}(2P)	10.3	0.05	0	1/2
χ_{b2}(2P)	10.3	0.05	2/3	1/2

[16] S. Digal, P. Petreczky, and H. Satz, Phys. Rev. D **64**, 094015 (2001).

Presenting Polarization

- The tendency for quarkonium states of spin J to be in a particular $|J, J_z\rangle$ state is known as polarization
- For S state ($J = 1$) quarkonium, if $J_z = 0$, then it is longitudinally polarized
- If $J_z = \pm 1$, then it is transversely polarized
- It is typical to represent the polarization in terms of the polarization parameter, λ_ϑ, which ranges from -1 to $+1$
- For the S states, $\lambda_\vartheta = -1$ refers to pure longitudinal production while $\lambda_\vartheta = +1$ refers to pure transverse production

$J^P = 1^-$ (S states)[17]

$$\lambda_\vartheta = \frac{\sigma^{J_z=+1} + \sigma^{J_z=-1} - 2\sigma^{J_z=0}}{\sigma^{J_z=+1} + \sigma^{J_z=-1} + 2\sigma^{J_z=+0}}$$

[17] P. Faccioli, C. Lourenco, J. Seixas, and H. K. Wohri, Eur. Phys. J. C **69**, 657 (2010).

Presenting Polarization

- For the χ_1 ($J = 1$) and χ_2 ($J = 2$) states, the polarization parameter is defined as the polarization parameter of the product J/ψ or Υ(nS) if production comes purely from χ state feed down
- $\chi_c \to J/\psi + \gamma$, $\chi_b \to \Upsilon(n\mathrm{S}) + \gamma$

$J^P = 1^+$ (χ_1 P states)[18]

$$\lambda_\vartheta = \frac{2\sigma^{J_z=0} - \sigma^{J_z=+1} - \sigma^{J_z=-1}}{2\sigma^{J_z=0} + 3\sigma^{J_z=+1} + 3\sigma^{J_z=-1}}$$

$J^P = 2^+$ (χ_2 P states)[18]

$$\lambda_\vartheta = \frac{-6\sigma^{J_z=0} - 3\sigma^{J_z=+1} + 6\sigma^{J_z=+2} - 3\sigma^{J_z=-1} + 6\sigma^{J_z=-2}}{10\sigma^{J_z=0} + 9\sigma^{J_z=+1} + 6\sigma^{J_z=+2} + 9\sigma^{J_z=-1} + 6\sigma^{J_z=-2}}$$

[18] P. Faccioli *et al.*, Phys. Lett. B **773**, 476 (2017).

Polarization Parameters

In our calculation, we have $\sigma^{J_z=\pm 2} = 0$ and $\sigma^{J_z=+1} = \sigma^{J_z=-1}$, so the polarization parameters can be written as:

$J^P = 1^-$ (S states)

$$\lambda_\vartheta = \frac{1 - 3R^{J_z=0}}{1 + R^{J_z=0}}$$

$J^P = 1^+$ (χ_1 P states)

$$\lambda_\vartheta = \frac{-1 + 3R^{J_z=0}}{3 - R^{J_z=0}}$$

$J^P = 2^+$ (χ_2 P states)

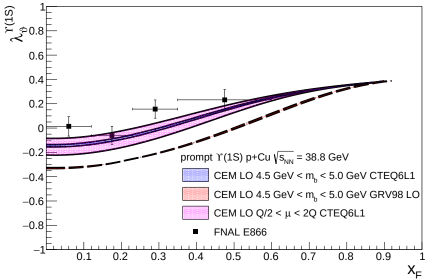
$$\lambda_\vartheta = \frac{-3 - 3R^{J_z=0}}{9 + R^{J_z=0}}$$

Comparing x_F Dependence with Fixed-Target Data[19]

CEM polarization calculation using collinear factorization:

$J^P = 1^-$ (S states)

$$\lambda_\vartheta = \frac{1 - 3R^{J_z=0}}{1 + R^{J_z=0}}$$



x_F $(x_1 - x_2)$ Dependence (EPS09 for Cu PDFs)

- longitudinally polarized at small $|x_F|$ and transversely polarized at large $|x_F|$
- prediction is consistent with the ~ 0 polarization for $\Upsilon(1S)$

[19] C. N. Brown *et al.* (NuSea Collaboration), Phys. Rev. Lett. **86**, 2529 (2001).

Calculation at $\mathcal{O}(\alpha_s^2)$ using k_T-factorization

In our calculations using k_T-factorization, we compute the scattering amplitdues $\mathcal{A}(\mathcal{R}\mathcal{R} \to Q\overline{Q})$:

$$\begin{aligned} \mathcal{A}(\mathcal{R}\mathcal{R} \to Q\overline{Q}) &= \epsilon(k)^\mu \epsilon(k')^\nu \mathcal{A}_{\mu\nu}(gg \to Q\overline{Q}) \,, \\ \epsilon(k)^\mu &= (0, \frac{\vec{k}_T}{|k_T|}, 0) \,, \end{aligned}$$

$\mathcal{A}$'s are separated according to the S_z of the final state. We then determine the corresponding Legendre component $\mathcal{A}_L$ in the amplitudes by

$$\begin{aligned} \mathcal{A}_{L=0} &= \frac{1}{2}\int_{-1}^{1} dx \mathcal{A}(x = \cos\theta) \,, \\ \mathcal{A}_{L=1} &= \frac{3}{2}\int_{-1}^{1} dx \, x\mathcal{A}(x = \cos\theta) \,. \end{aligned}$$

$L = 2$ amplitudes are not needed for S and χ states production. Only $\mathcal{A}_{gg}$'s are used in the k_T-factorization approach

Production in k_T-factorized ICEM

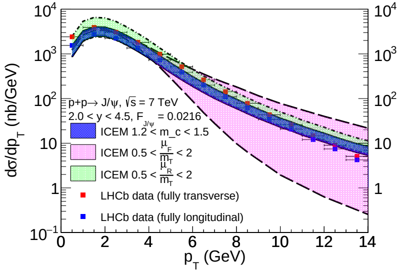
Production cross section

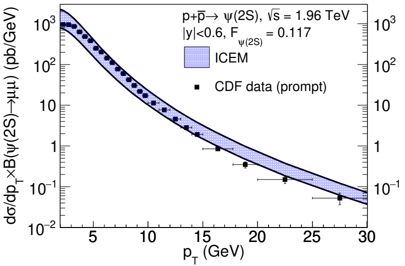
$$\begin{aligned}\sigma &= F_Q \int_{M_{\mathcal{Q}}^2}^{4m_H^2} d\hat{s} \int dx_1 \int dx_2 \int d{k_{1T}}^2 \int d{k_{2T}}^2 \int \frac{d\phi_1}{2\pi} \int \frac{d\phi_2}{2\pi} \\ &\times \Phi_1(x_1, k_{1T}, Q_1)\Phi_2(x_2, k_{2T}, Q_2)\hat{\sigma}(\mathcal{R} + \mathcal{R} \to Q\overline{Q}) \\ &\times \delta(\hat{s} - x_1 x_2 s + |\vec{k}_{1T} + \vec{k}_{2T}|^2)\end{aligned}$$

Parameters used

- We used JH-2013[5] unintegrated (transverse-momentum-dependent) PDF set for $\Phi(x, k_T, Q)$
- factorization scale set at $Q = m_T$
- $1.27 < m_c < 1.50$ GeV, $4.5 < m_b < 5.0$ GeV
- $\frac{1}{2} < \frac{\mu_r}{m_T} < 2$

Charmonium production in k_T-factorized ICEM[20]

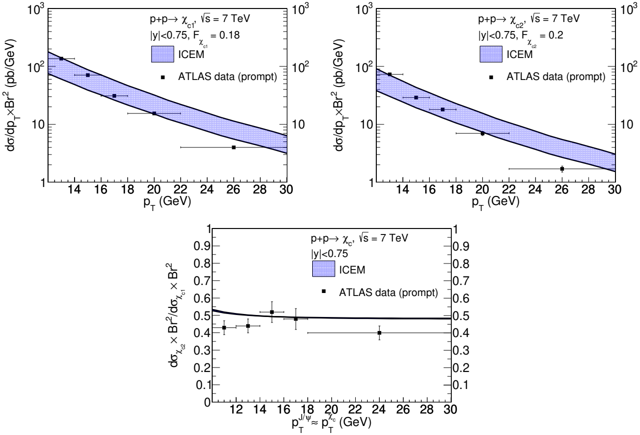




- We obtained $F_{J/\psi}$ while assumming a constant direct-to-inclusive ratio of 0.62 for J/ψ.
- We also compare our directly produced $\psi(2S)$ to the prompt production of $\psi(2S)$ to obtain $F_{\psi(2S)}$.
- The ICEM with k_T-factorization is able to describe the yield, but having a strong dependence on factorization scale at high p_T.

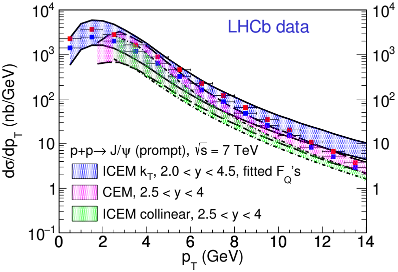
[20] V. Cheung and R. Vogt, Phys. Rev. D **98**, 114029 (2018) and **99**, 034007 (2019).

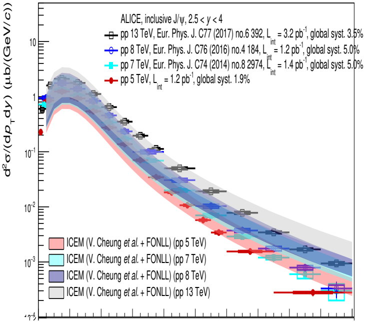
χ_c production in k_T-factorized ICEM[20]



- We also compare our results to χ_c production at ATLAS to obtain the $F_{\mathcal{Q}}$'s as well.
- We found the relative production is stable at high p_T. This is consistent with the data.

Prompt and inclusive J/ψ in k_T-factorized ICEM[20]

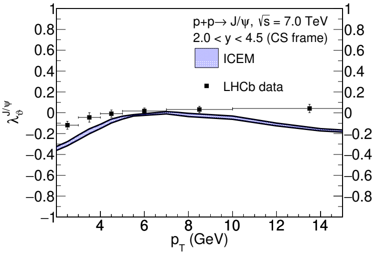


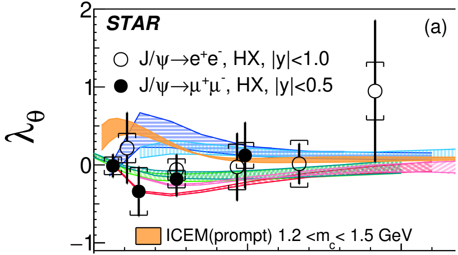


- With all the $F_{\mathcal{Q}}$'s fitted for all S states and P states, the prompt J/ψ yield can be calculated.
- The k_T-factorized ICEM agrees with previous collinear (I)CEM calculations.
- When B feed-down is also added using FONLL, we found agreement with inclusive J/ψ production in a large range of beam energies.

J/ψ polarization in k_T-factorized ICEM[20]

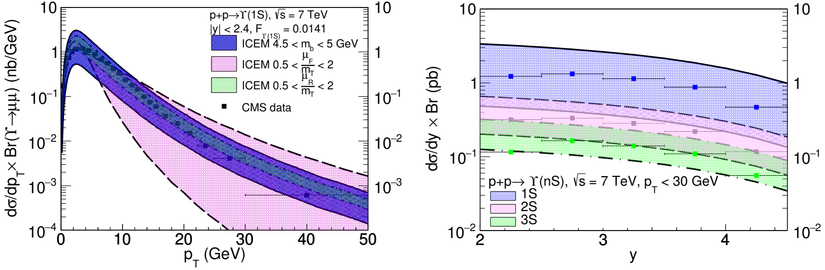
Polarization is independent of $F_{\mathcal{Q}}$ and scales, mass is the only uncertainty





- We found the prompt production of J/ψ is slightly longitudinally polarized in the CS frame.
- Slightly transversely polarized in the HX frame.
- Agreement with polarization data is frame-dependent at low p_T.

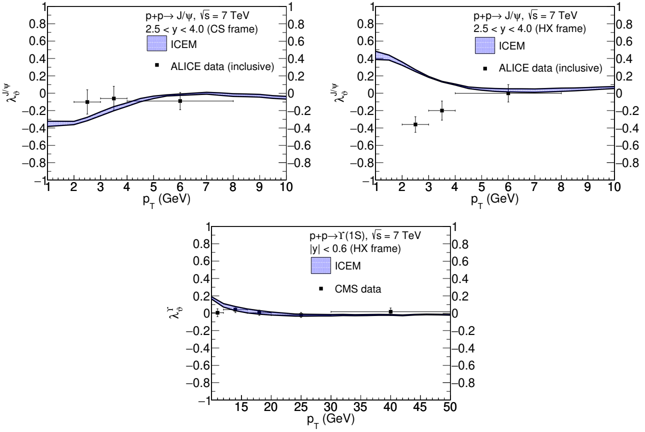
Υ production in k_T-factorized ICEM[20]



- The p_T-distributions for Υ production also have a strong dependence on factorization scale at high p_T.
- When the factorization scale is set at m_T, both p_T and y distributions are described.

Υ polarization in k_T-factorized ICEM[20]

Polarization is independent of $F_{\mathcal{Q}}$ and scales, mass is the only uncertainty



- Agreement with polarization data is also frame-dependent at low p_T.
- At high p_T, the polarization becomes unpolarized.

Collinear ICEM at $\mathcal{O}(\alpha_s^3)$

- We consider all 16 diagrams from gg$\to$ c$\bar{c}$g, 5(+5) from gq($\bar{q}$)$\to$ c$\bar{c}$ q($\bar{q}$), and 5 from q$\bar{q}$$\to$ c$\bar{c}$g with the projection operator applied at the diagram level.
- The c$\bar{c}$ produced are the proto-charmonium before hardonization. The mass of the charmonium will then fix the relative momentum of the heavy quark, k.
- $p_\psi = p_c + p_{\bar{c}}$, $k = \frac{1}{2}(p_c - p_{\bar{c}})$
- The polarized cross sections are then computed using the appropriate polarization vector for the charmonium:
- $\epsilon_{\psi,0} = \frac{1}{m_\psi}(p, 0, 0, E)$, $\epsilon_{\psi,\pm} = \mp\frac{1}{\sqrt{2}}(0, 1, \pm i, 0)$
- All final state momenta are integrated while restricting $p_\psi \cdot k = 0$
- We used the CT14 PDFs in our calculations.
- k_T-smearing is applied to the initial state partons to provide better description at low p_T

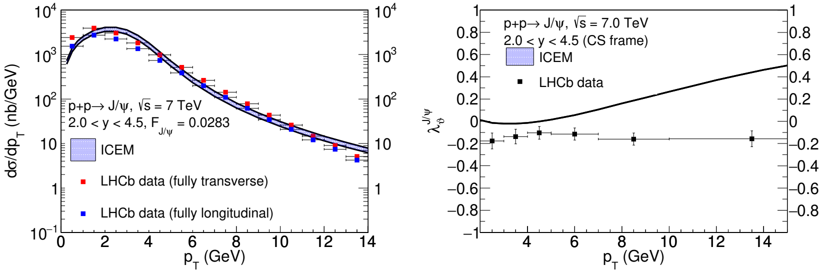
Collinear polarized ICEM at $\mathcal{O}(\alpha_s^3)$

Production distribution

$$\frac{d^2\sigma}{dp_T dy} = F_{\mathcal{Q}} \sum_{i,j=\{q,\bar{q},g\}} \int_{M_{\mathcal{Q}}}^{2m_H} dm_\psi \int d\hat{s} dx_1 dx_2 f_{i/p}(x_1,\mu^2) f_{j/p}(x_2,\mu^2) d\hat{\sigma}_{ij\to c\bar{c}+X} \ ,$$

- First p_T-dependent polarization results using collinear factorization
- Should not have strong dependence on factorization scale as in k_T-factorized approach.
- $1.18 < m_c < 1.36$ GeV
- $\mu_F/m = 2.1^{+2.55}_{-0.85}$
- $\mu_R/m = 1.6^{+0.11}_{-0.12}$
- same set of variations used in MV (2016) and NVF (2013)

ICEM polarized cross sections using collinear factorization



- k_T-smearing gives a small kick $< k_T^2 > \sim 1$ GeV^2 to the inital state parton. This is not needed in k_T-factorization approach as the uPDFs are k_T-dependent.
- These are preliminary results[21] with uncertainty bands constructed by varying the charm quark mass. Uncertainties from factorization and renormalization scales are not included yet.
- We find some agreement with the polarization data.

[21] V. Cheung and R. Vogt, in progress.

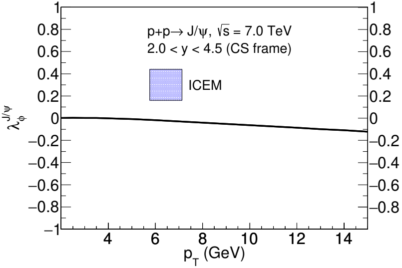
Other polarization parameters in collinear ICEM[21]

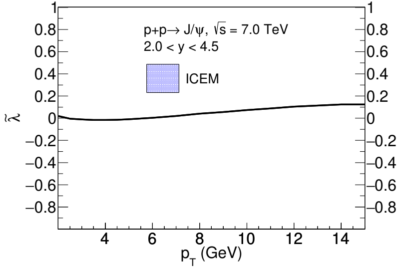
Azimuthal anisotropy[17]

$$\lambda_\phi = \frac{2Re[a_{+1}a^*_{-1}]}{\mathcal{N} + a_0^2}$$

Frame invariant parameter[17]

$$\tilde{\lambda} = \frac{\lambda_\theta - 3\lambda_\phi}{1 - \lambda_\phi}$$





- We found that the invariant polarization parameter is close to zero.

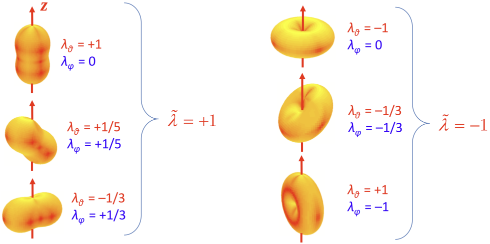
Other polarization parameters in collinear ICEM

Azimuthal anisotropy[17]

$$\lambda_\phi = \frac{2Re[a_{+1}a^*_{-1}]}{\mathcal{N} + a_0^2}$$

Frame invariant parameter[17]

$$\tilde{\lambda} = \frac{\lambda_\theta - 3\lambda_\phi}{1 - \lambda_\phi}$$



- Calculating invariant $\tilde{\lambda}$ removes frame-induced kinematic dependencies
- Our preliminary result is near-unpolarized

Conclusion and Future

In this talk, I

- reviewed different models developed to desribe the quarkonium yield in high-energy collisions
- reviewed recent attempts to solve the polarization puzzle
- suggested the (I)CEM is worth exploring in polarized production

Questions to be answered in the future:

- NRQCD is rigorous, but still can't describe the η_c production and J/ψ polarization simutaneously using the same LDMEs and HQSS.
- CEM is less rigorous. It describes the yield and perhaps polarization in hadroproduction. How about other collision systems? How about η_c for ICEM?
- How do $c\bar{c}$'s end up in J/ψ? Can we describe the mechanism beyond $F_{\mathcal{Q}}$'s and LDME's?