Unraveling Pion and Kaon Structure with the DSEs

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Jefferson Lab Theory Seminar

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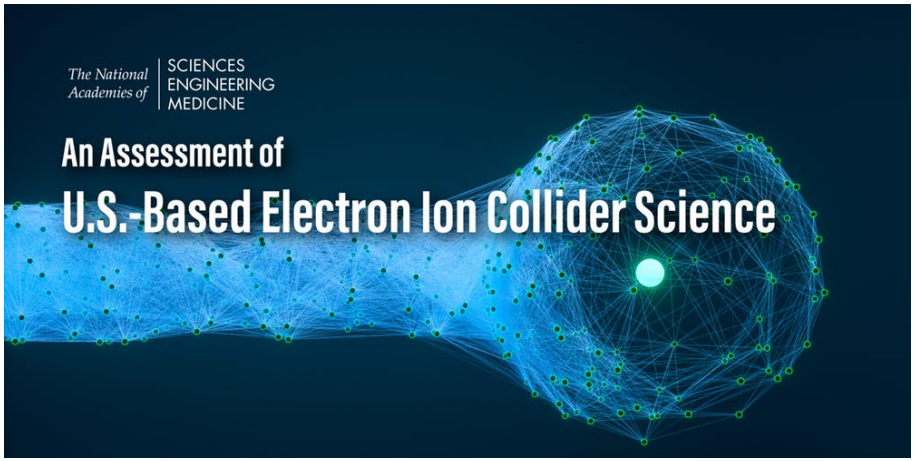
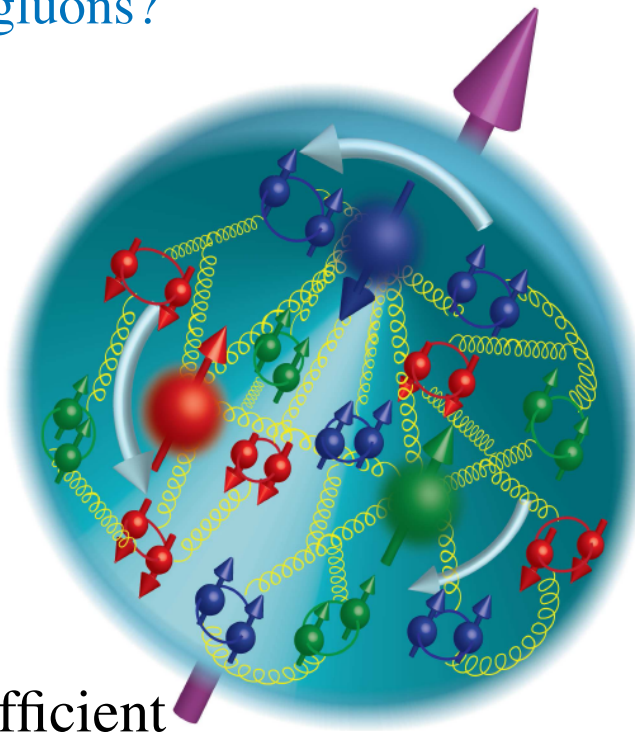
U.S. DEPARTMENT OF ENERGY | Office of Science

Argonne NATIONAL LABORATORY

High-priority science questions



- The NAS *Assessment of a U.S. based Electron Ion Collider* identified three high-priority science questions
  - How does the mass of the nucleon arise?
  - How does the spin of the nucleon arise?
  - What are the emergent properties of dense systems of gluons?
- What do we know about hadron masses?
  - The three current quarks needed to define the nucleon quantum numbers contribute only $\sim 1\%$ to its mass
  - In chiral limit nucleon mass $\sim 900\,\mathrm{MeV}$; *Higgs mechanism is largely irrelevant for visible mass*
  - The chiral symmetry of $\mathcal{L}_{\mathrm{QCD}}$ is dynamically broken $\Longrightarrow \sim 500\,\mathrm{MeV}$ mass splittings in hadron spectrum & massless Goldstone bosons in chiral limit (π, K, η)



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  - must also understand the mass of the pion ($u\bar{d}, \ldots$) and kaon ($u\bar{s}, \ldots$)

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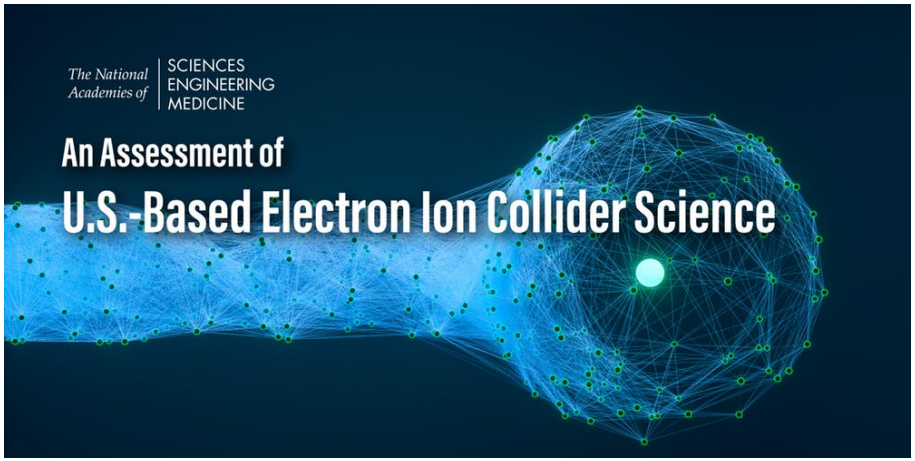

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N^* $J^\pi = \frac{1}{2}^-$ $\sim 1500\,\mathrm{MeV}$

a_1 $J^\pi = 1^+$ $\sim 1245\,\mathrm{MeV}$

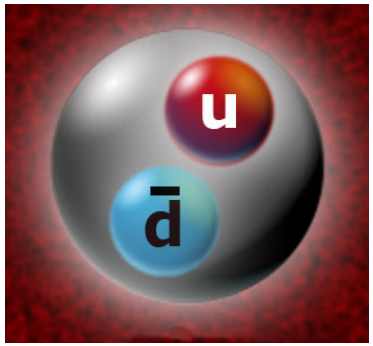
N $J^\pi = \frac{1}{2}^+$ $939\,\mathrm{MeV}$

ρ $J^\pi = 1^-$ $775\,\mathrm{MeV}$

σ $J^\pi = 0^+$ $\sim 500\,\mathrm{MeV}$

π $J^\pi = 0^-$ $139\,\mathrm{MeV}$

Story of the Pion and Kaon

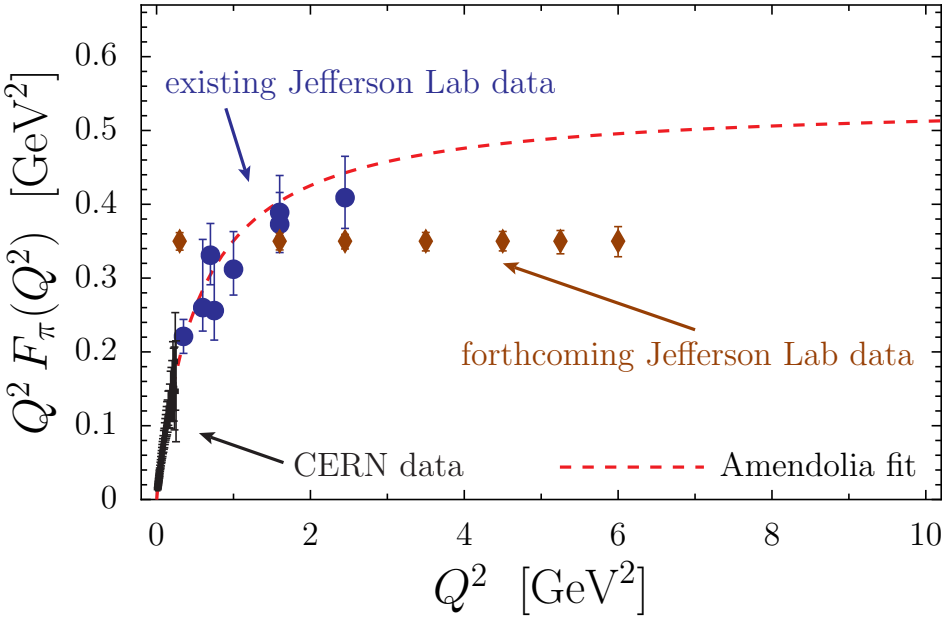


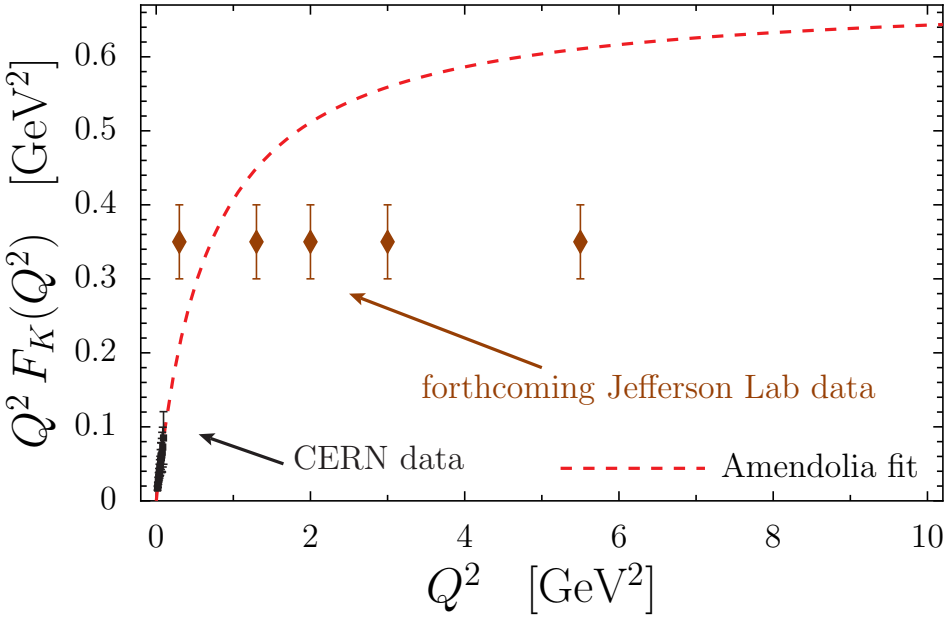
- In 1935 Yukawa postulated a strongly interacting particle ["(π-) meson"] as a mediator for the strong nuclear force
  - in 1947 both the π and shortly afterwards the K were discovered from cosmic ray tracks in a photographic emulsion
- Today, pion still regarded as the mediator of the strong force in *ab inito* approaches to nuclear structure; the kaon has played an important role in establishing quark model, and understanding flavor breaking & CP violation
- Formally the pion and kaon are now understood as both a bound state of a *dressed-quark* and a *dressed-antiquark* in QFT and the Goldstone mode associated with DCSB in QCD
- This dichotomous nature has numerous ramifications near chiral limit *e.g.*:

$$f_\pi^2\, m_\pi^2 \simeq \frac{1}{2}\,(m_u + m_d)\,\langle \psi\bar{\psi}\rangle\,,\;\; H_\pi^{u-d}(x,\xi\to 1,0) = \varphi_\pi\left(\frac{1+x}{2}\right),\;\; D_\pi(0) = -1$$

- Perturbative QCD can also make predictions for pion and kaon structure – therefore π and K provide an ideal laboratory to test and understand QCD

What we know about the Pion and Kaon





- Pion and kaon structure is slowly being revealed using: π^-/K^- beams at CERN; Sullivan type experiments at Jefferson Lab; π^- beam at Fermilab; and $e^+e^- \to \pi^+\pi^-,\ K^+K^-$ in the time-like region
- 40 years of experiments has revealed, e.g.
  - $r_{\pi^+} = 0.672 \pm 0.008, \qquad r_{K^+} = 0.560 \pm 0.031, \qquad r_{K^0} = -0.277 \pm 0.018$
- Still a lot more to learn about pion and kaon structure:
  - quark and gluon PDFs; TMDs including Boer-Mulders function; $q, g \to \pi/K$ fragmentation functions, quark and gluon GPDs; gravitational form factors

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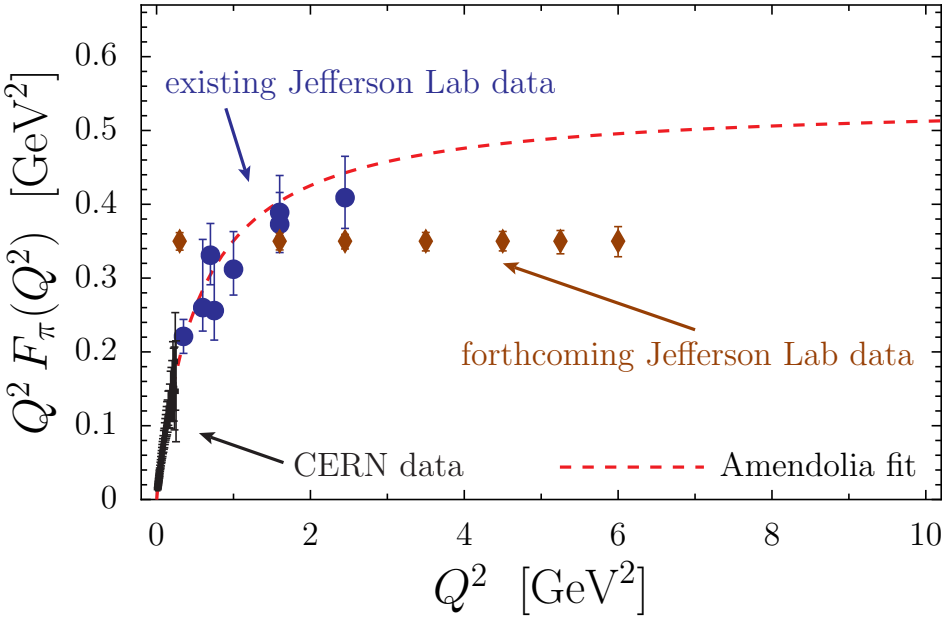

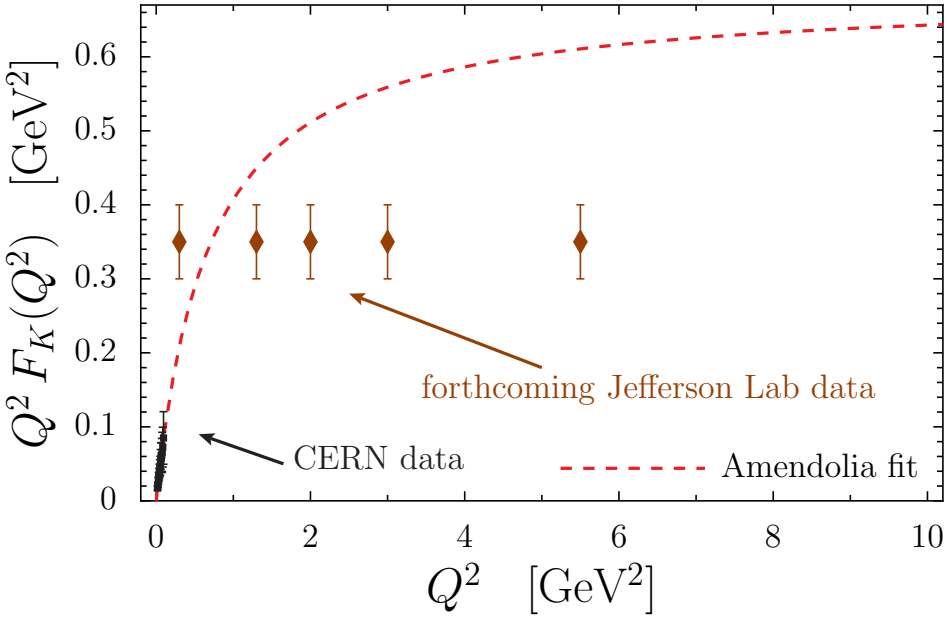

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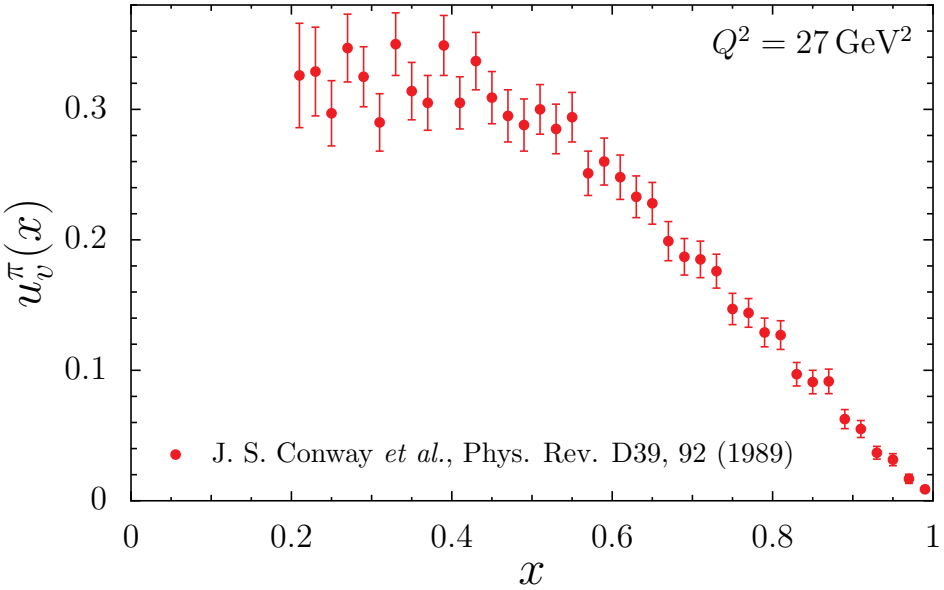

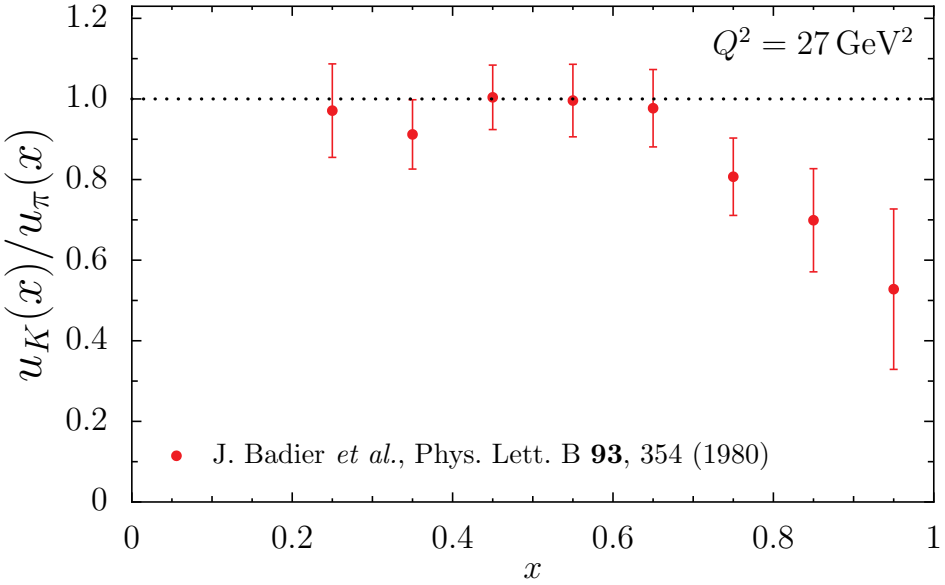

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Hadron Masses in QCD

- Quark/gluon contributions to masses (& angular momentum) are accessed via matrix elements of QCD's (symmetric) energy-momentum tensor

$$T^{\mu\nu} = T^{\nu\mu}, \qquad \partial_\mu T^{\mu\nu} = \partial_\mu T_q^{\mu\nu} + \partial_\mu T_g^{\mu\nu} = 0, \qquad T^{\mu\nu} = \underset{\text{[traceless]}}{\overline{T}^{\mu\nu}} + \underset{\text{[trace]}}{\widehat{T}^{\mu\nu}}$$

- Renormalized (perturbatively) trace piece of $T^{\mu\nu}$ takes the form

$$T^\mu_\mu = \sum_{q=u,d,s} \underbrace{m_q\,(1+\gamma_m)\,\overline{\psi}_q\psi_q}_{\text{quark mass term}} + \underbrace{\frac{\tilde{\beta}(g)}{2\,g}\,F^{\mu\nu,a}F^a_{\mu\nu}}_{\text{trace anomaly}}$$

- At zero momentum transfer

$$\langle p\,|T^{\mu\nu}|\,p\rangle = 2\,p^\mu p^\nu \quad \Longrightarrow \quad \langle p\,|T^\mu_\mu|\,p\rangle = 2\,m^2$$

  - in chiral limit entire hadron mass from gluons!
  - Dmitri Kharzeev – Proton Mass workshops at Temple University and ECT*

- Understanding difference in pion and proton is key to hadron masses:

$$\langle \pi\,|T^\mu_\mu|\,\pi\rangle = 2\,m_\pi^2 \overset{\text{chiral limit}}{\to} 0, \qquad \langle N\,|T^\mu_\mu|\,N\rangle = 2\,m_N^2$$

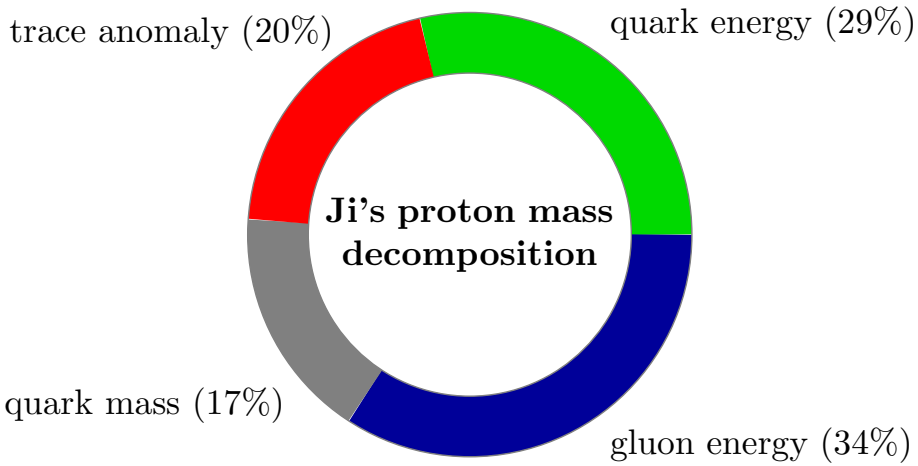
Rest Frame Hadron Mass Decompositions

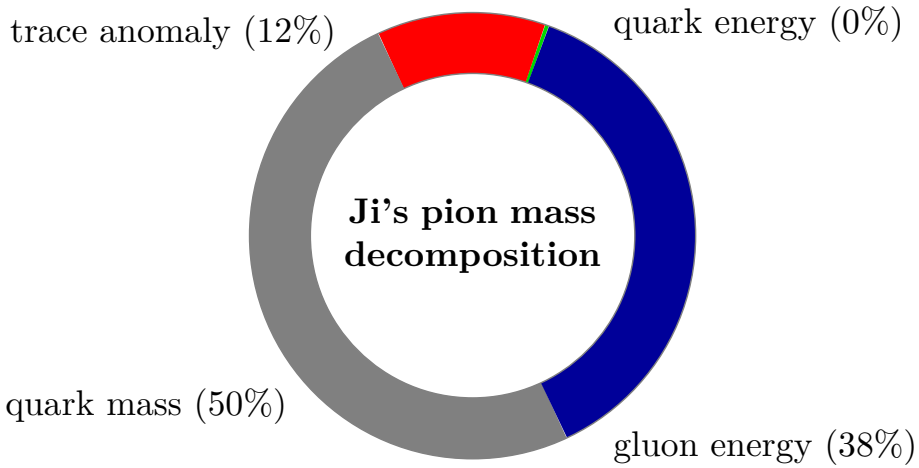
- Xiangdong Ji proposed hadron mass decomposition [PRL **74**, 1071 (1995); PRD **52**, 271 (1995)]

$$m_p = \left.\frac{\langle p \,|\int d^3x\, T^{00}(0,\vec{x})|\, p\rangle}{\langle p|p\rangle}\right|_{\text{at rest}} = \underbrace{M_q + M_g}_{\text{quark and gluon energies}} + \underbrace{M_m}_{\text{quark mass}} + \underbrace{M_a}_{\text{trace anomaly}}$$

$$M_q = \tfrac{3}{4}\,(a-b)\,m_p, \quad M_g = \tfrac{3}{4}\,(1-a)\,m_p, \quad M_m = b\,m_p, \quad M_a = \tfrac{1}{4}\,(1-b)\,m_p,$$

- a = quark momentum fraction, b related to sigma-term or anomaly contribution
- [See Cédric Lorcé, EPJC **78**, (2018) for decomposition with pressure effects]





- In chiral limit ($m_q \to 0$) pion has no rest frame ($m_\pi = 0$) – how to interpret Ji's pion mass decomposition?

Hadron Mass Decomposition – Experiment

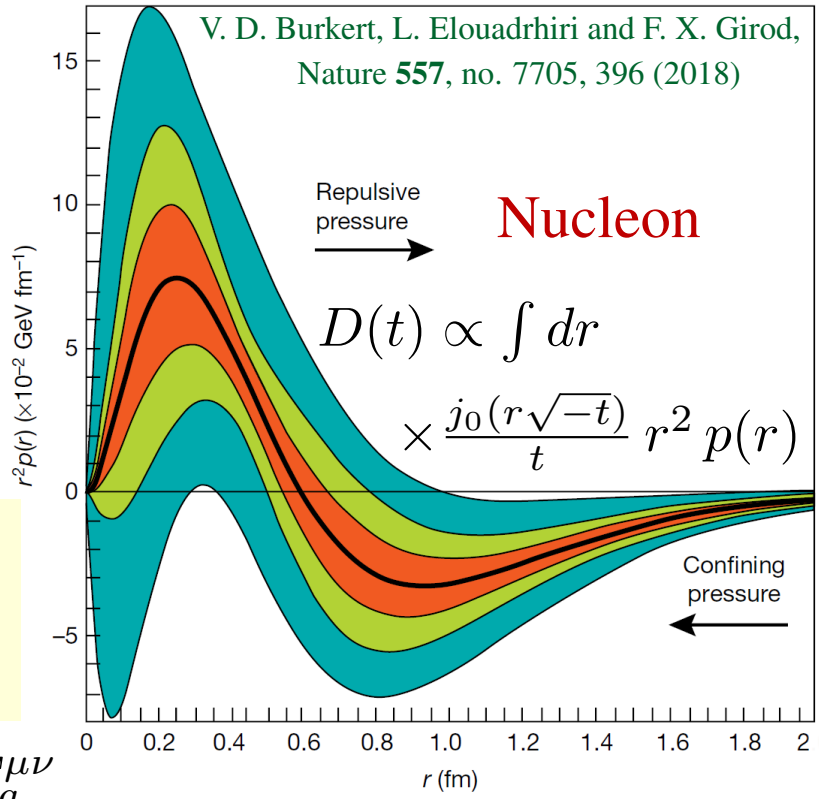
- Gravitational form factors of the pion:

$$\langle p' | T^{\mu\nu}_{q,g} | p \rangle = A^{q,g}_{\pi}(t)\, P^{\mu}\, P^{\nu} + D^{q,g}_{\pi}(t)\left(q^{\mu}q^{\nu} - q^{2} g^{\mu\nu}\right) + \bar{c}^{q,g}_{\pi}(t)\, g^{\mu\nu}$$

- A graviton probe can only measure $T^{\mu\nu} = T^{\mu\nu}_{q} + T^{\mu\nu}_{g}$, where

$$A^{q}(0) + A^{g}(0) = 1, \quad D^{q}_{\pi}(0) + D^{g}_{\pi}(0) \overset{m_q \to 0}{=} -1$$

$$\partial_{\mu} T^{\mu\nu} = 0 \implies \bar{c}^{q}_{\pi}(t) = -\bar{c}^{g}_{\pi}(t)$$

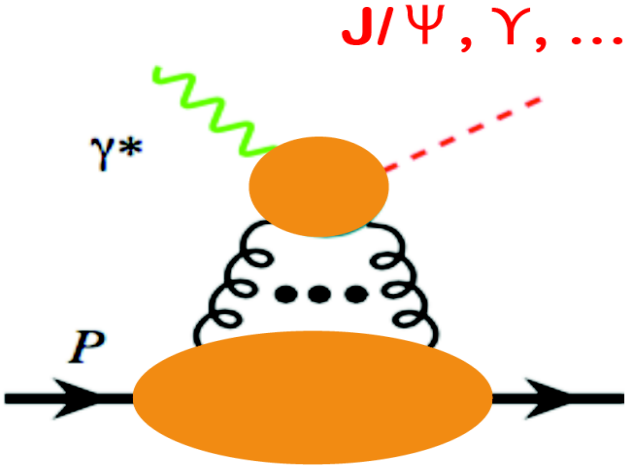


- However, GPDs can access both $T^{\mu\nu}_{q}$ and $T^{\mu\nu}_{g}$

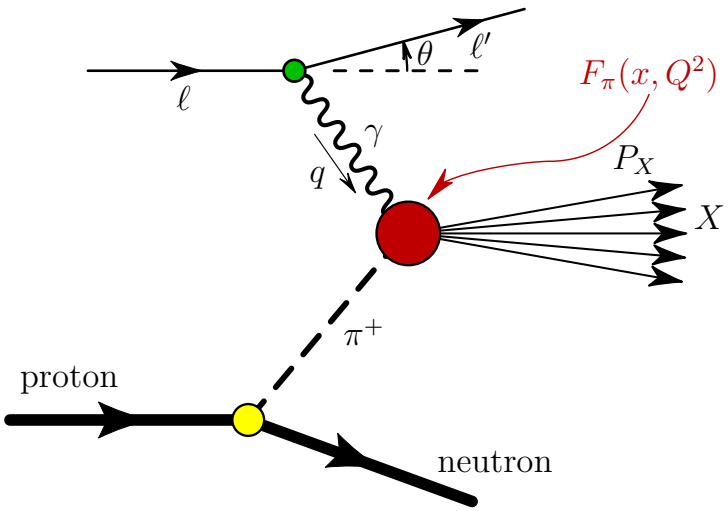
$$\int dx\, x\, H^{q,g}_{\pi}(x,\xi,t) = A^{q,g}_{\pi}(t) + \xi^{2}\, D^{q,g}_{\pi}(t)$$

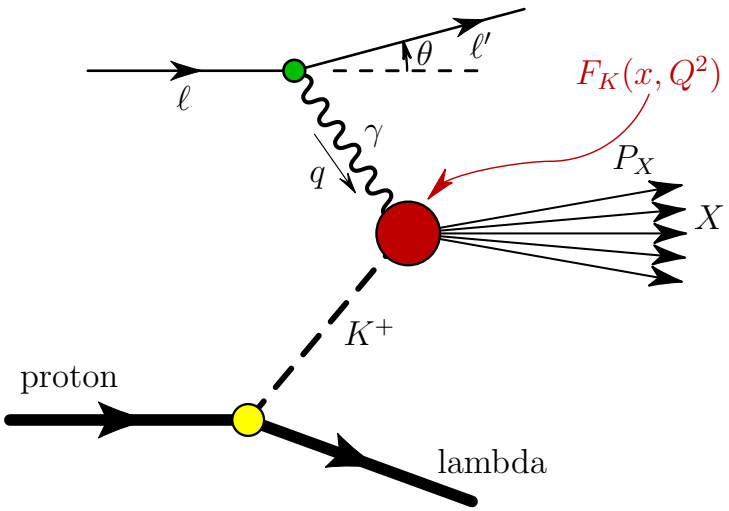
  - measuring/calculating pion and kaon GPDs would shed-light on mass and confinement

- Trace anomaly contribution can likely be accessed through $J/\psi, \Upsilon$ production at threshold



Pion & Kaon Structure at JLab and an EIC

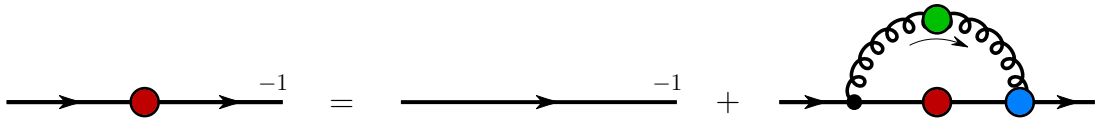




- At Jefferson Lab and an EIC pion and kaon structure can be accessed via the so-called *Sullivan* processes
  - initial pion/kaon is off mass-shell – need extrapolation to pole
  - proven results for form factors – what about quark and gluon PDFs, TMDs, GPDs, *etc*, at an EIC?
- Explored this ideal at a series of workshops on "*Pion and Kaon Structure at an Electron–Ion Collider*" (PIEIC)
  - 1−2 June 2017, Argonne National Laboratory www.phy.anl.gov/theory/pieic2017/
  - 24−25 May 2018, The Catholic University of America www.jlab.org/conferences/pieic18/

QCD's Dyson-Schwinger Equations

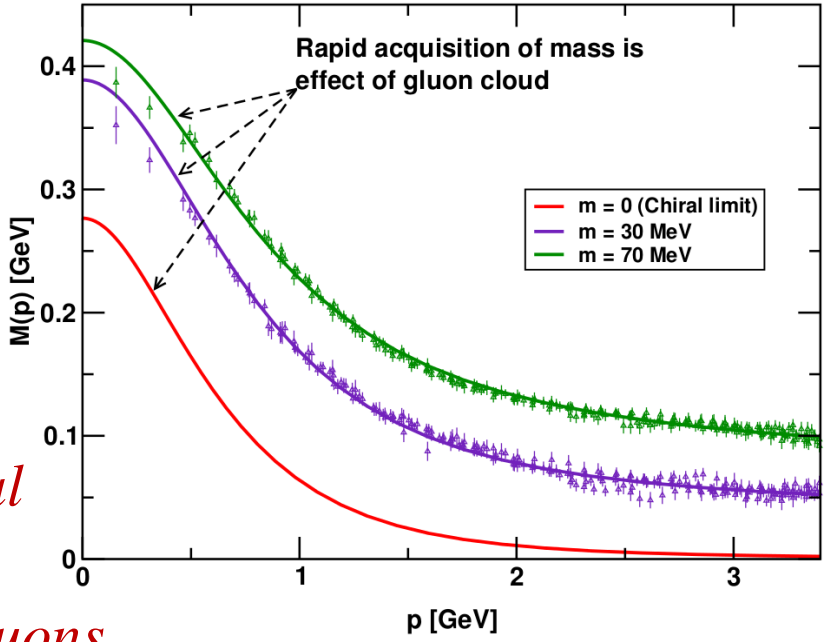
- The equations of motion of QCD $\Longleftrightarrow$ QCD's Dyson–Schwinger equations
  - an infinite tower of coupled integral equations
  - tractability $\Longrightarrow$ must implement a symmetry preserving truncation
- The most important DSE is QCD's gap equation $\Longrightarrow$ quark propagator



  - ingredients – dressed gluon propagator & dressed quark-gluon vertex

$$S(p) = \frac{Z(p^2)}{i\not{p} + M(p^2)}$$

[M. S. Bhagwat *et al.*, Phys. Rev. **C 68**, 015203 (2003)]



- Mass function, $M(p^2)$, exhibits dynamical mass generation, even in chiral limit
  - *mass function is gauge dependent and therefore NOT an observable!*
- *Hadron masses are generated by dynamical chiral symmetry breaking – caused by a cloud of gluons dressing the quarks and gluons*

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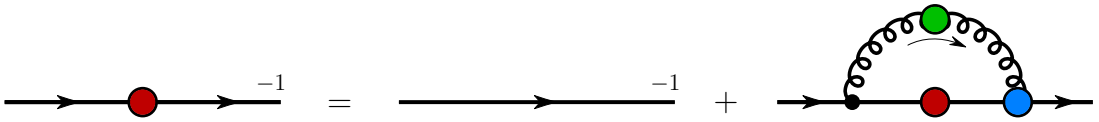

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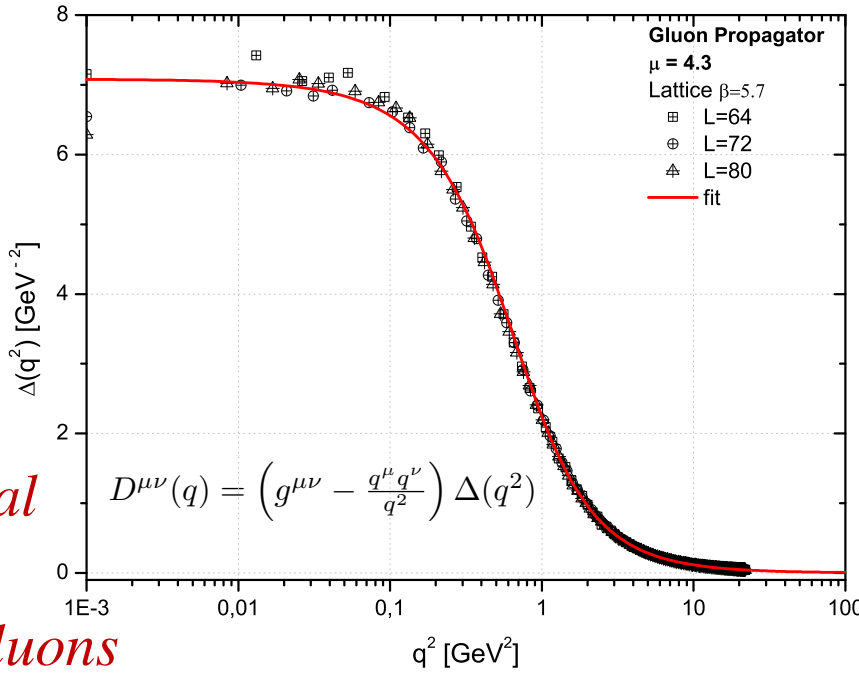
[A. C. Aguilar *et al*, Phys. Rev. **D81**, 034003 (2010)]



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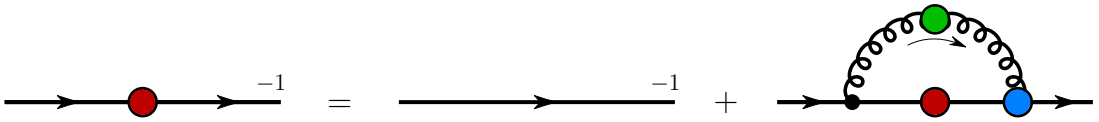

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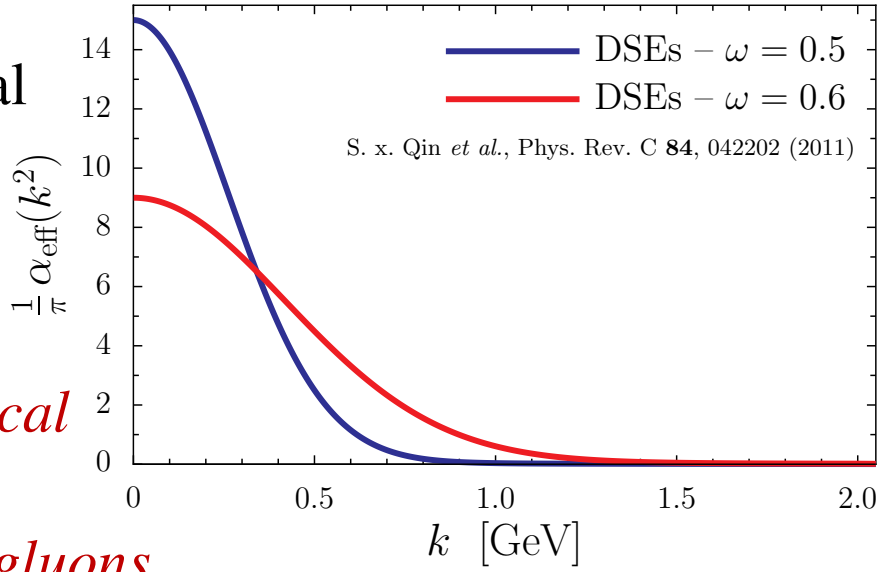
[S. x. Qin *et al.*, Phys. Rev. C 84, 042202 (2011)]



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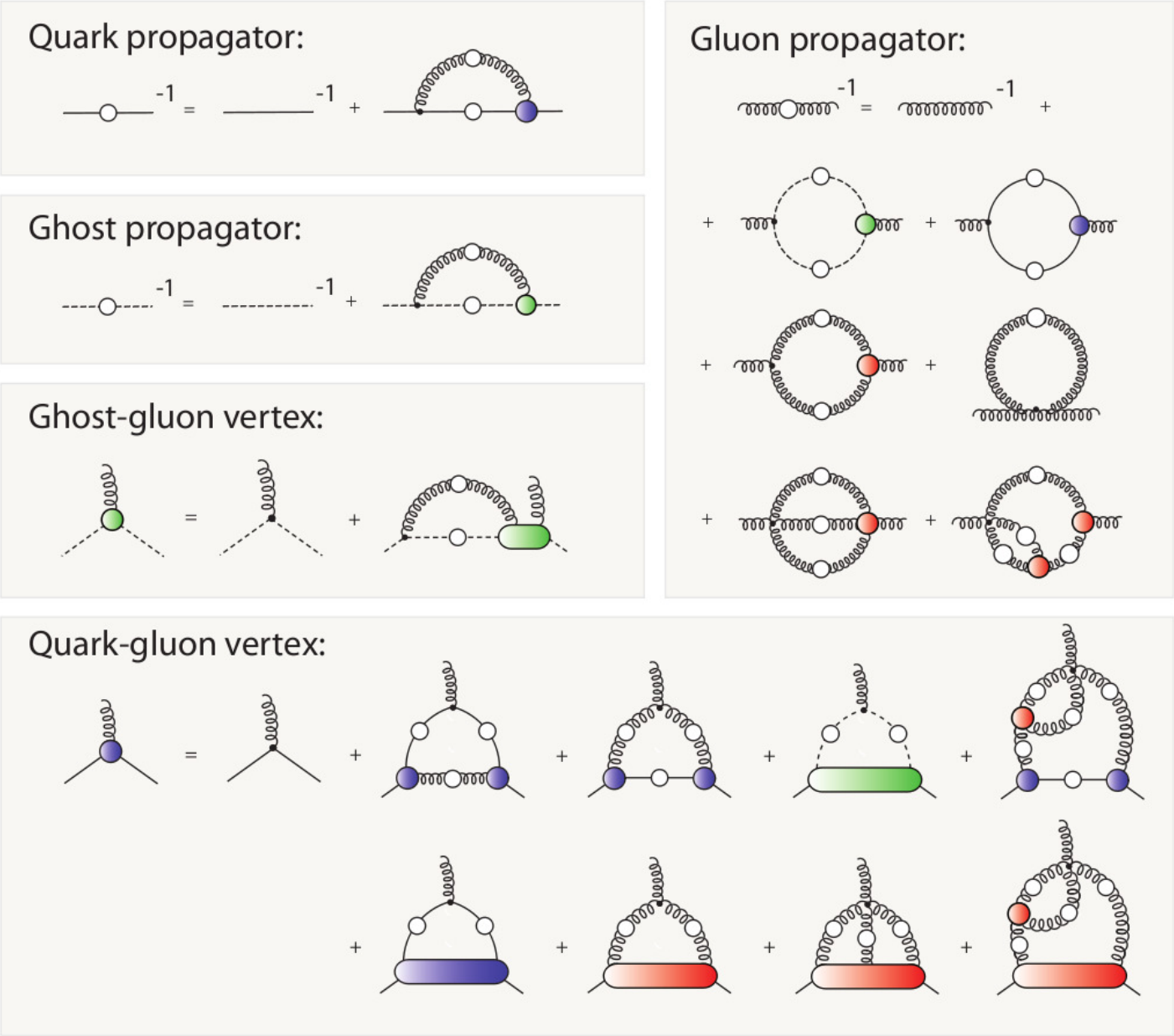
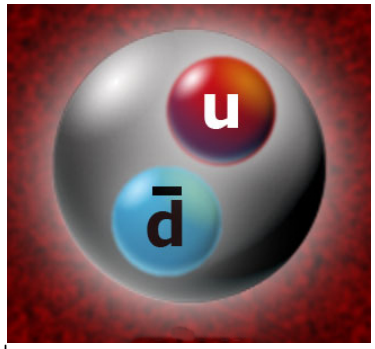


Image courtesy of Gernot Eichmann

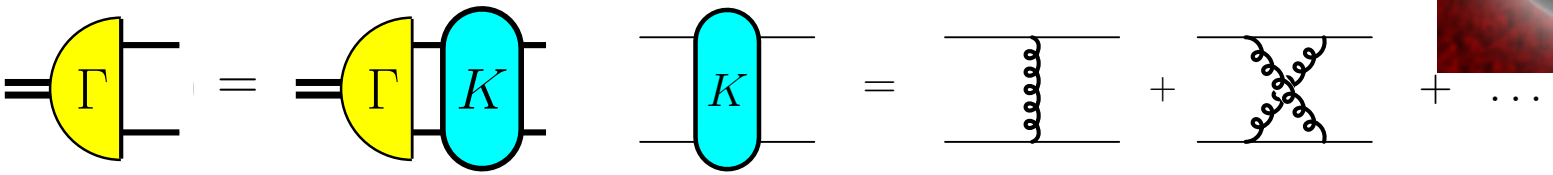
ETC!

Pion & Kaon Wave Functions

Calculating and Predicting Pion Structure



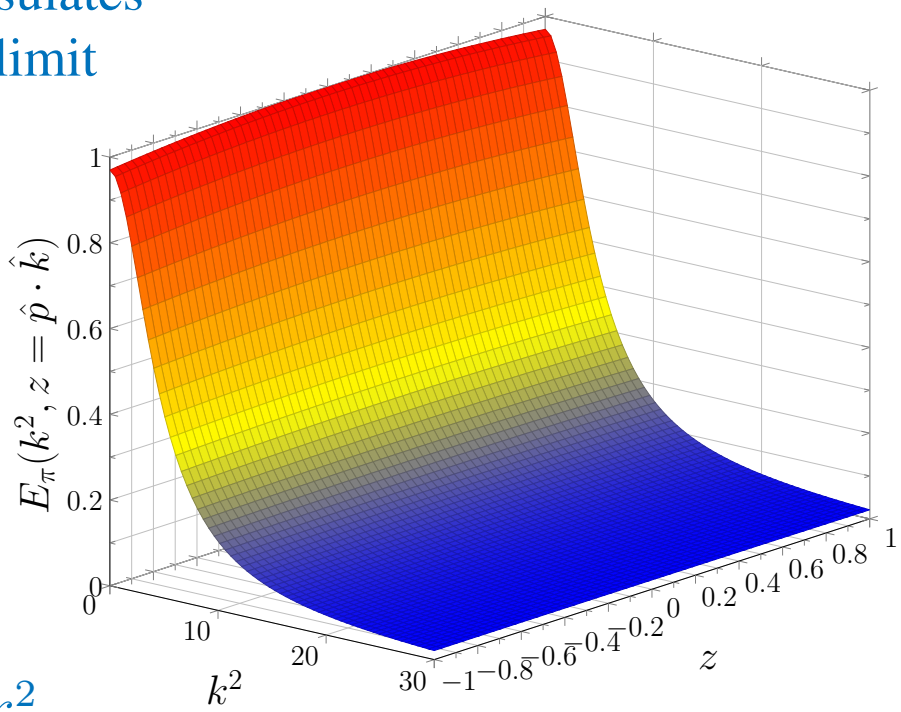
- In QFT a two-body bound state (*e.g.* a pion, kaon, *etc*) is described by the Bethe-Salpeter equation (BSE):



  - the kernel must yield a solution that encapsulates the consequences of DCSB, *e.g.*, in chiral limit $m_\pi = 0$ & $m_\pi^2 \propto m_u + m_d$

- Pion Bethe-Salpeter vertex

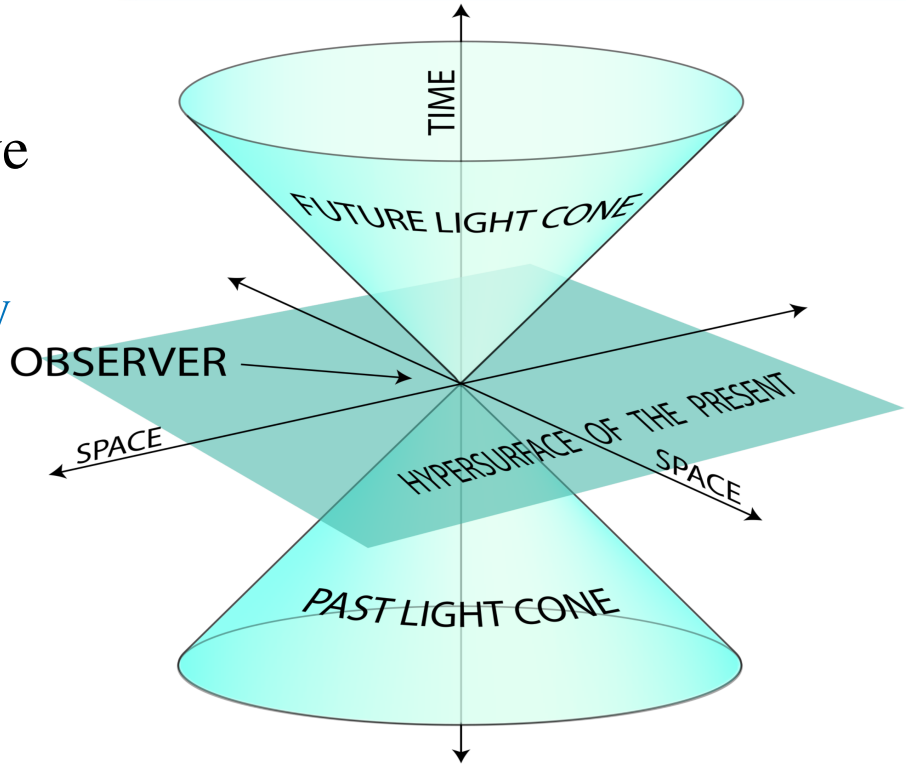
$$\Gamma_\pi(p,k) = \gamma_5 \Big[E_\pi(p,k) + \not{p}\, F_\pi(p,k) + \not{k}\, k\cdot p\, G_\pi(p,k) + i\sigma^{\mu\nu} k_\mu p_\nu\, H_\pi(p,k) \Big]$$



  - $\chi_{\rm BSE} = S(k+\frac{1}{2}p)\,\Gamma_\pi(p,k)\,S(k-\frac{1}{2}p)$
  - large relative momentum: $E_\pi \sim F_\pi \sim 1/k^2$

- Challenging to go beyond rainbow-ladder trunction and maintain symmetries

Light-Front Wave Functions



- In equal-time quantization a hadron wave function is a frame-dependent concept
  - boost operators are dynamical, that is, they are interaction dependent
- In high energy scattering experiments particles move at near speed of light
  - natural to quantize a theory at equal light-front time: $\tau = (t + z)/\sqrt{2}$
- Light-front quantization $\Longrightarrow$ light-front wave functions, which have some interesting properties
  - frame dependence is trivial, and yield a probability interpretation
  - boosts are kinematical – *not dynamical*
- BSE wave function $\Longrightarrow$ light-front wave functions (LFWFs) $[\psi(x, \boldsymbol{k}_T)]$
  $\Longrightarrow$ parton distribution amplitudes (PDAs) $[\varphi(x)]$

$$\psi(x, \boldsymbol{k}_T) = \int dk^- \; \chi_{\rm BSE}(p, k), \qquad \varphi(x) = \int d^2\boldsymbol{k}_T \; \psi(x, \boldsymbol{k}_T)$$

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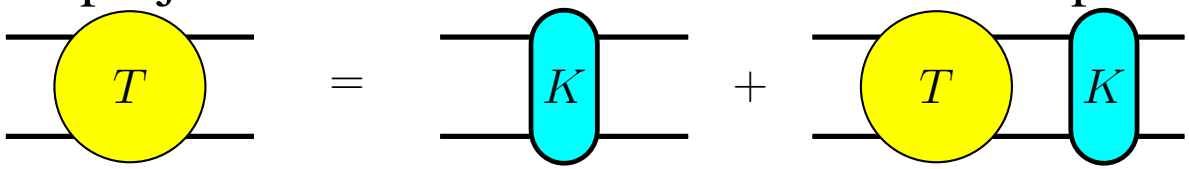
DSEs + Light-Front Wave Functions

- On light-front hadronic states can be represented by a Fock-state expansion

$$\left|\pi^+\right\rangle = \left|u\bar{d}\,\right\rangle + \left|u\bar{d}\,g\right\rangle + \left|u\bar{d}\,gg\right\rangle + \ldots + \left|u\bar{d}\,q\bar{q}\right\rangle + \left|u\bar{d}\,q\bar{q}\,g\right\rangle + \ldots$$

- Associated with each Fock-state is a number of LFWFs
  - diagonalizing the light-cone QCD Hamiltonian operator $\Longrightarrow$ LFWFs
  - *methods include*: discretized lightcone quantization, basis light-front quantization, and holographic QCD

- LFWFs can be projected from solutions to the Bethe-Salpeter equation



  - BSE self-consistently sums an infinite number of Fock states
  - in rainbow-ladder, e.g, $\left|\pi^+\right\rangle = \left|u\bar{d}\,\right\rangle + \left|u\bar{d}\,g\right\rangle + \left|u\bar{d}\,gg\right\rangle + \ldots$

- Obtaining LFWFs from DSE solutions of the BSE has several key features
  - in the DSEs emergent pheonmena, such as confinement and DCSB, arise through the infinite sum of diagrams
  - these effects are encoded in DSE dressed propagators and BS amplitudes, and therefore the projected LFWFs

DSEs + Light-Front Wave Functions

- On light-front hadronic states can be represented by a Fock-state expansion

$$|\pi^+\rangle = |u\bar{d}\,\rangle + |u\bar{d}\,g\rangle + |u\bar{d}\,gg\rangle + \ldots + |u\bar{d}\,q\bar{q}\rangle + |u\bar{d}\,q\bar{q}\,g\rangle + \ldots$$

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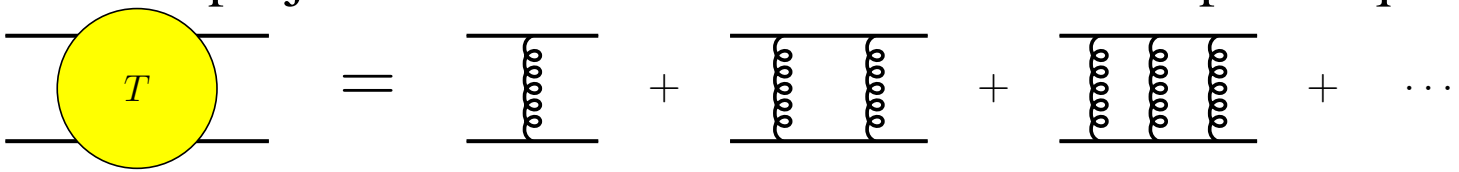

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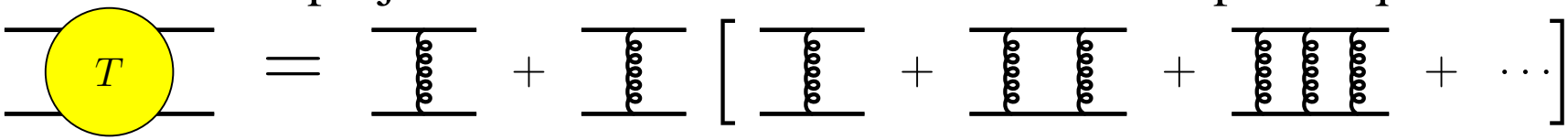

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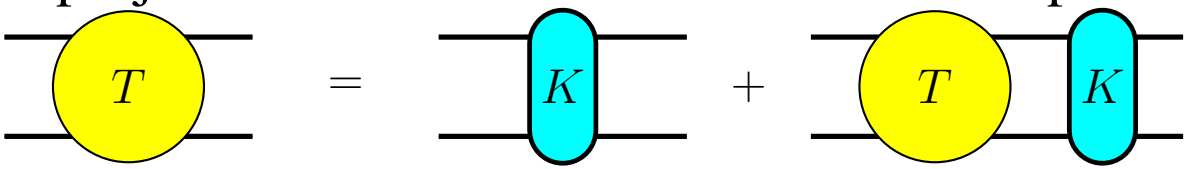

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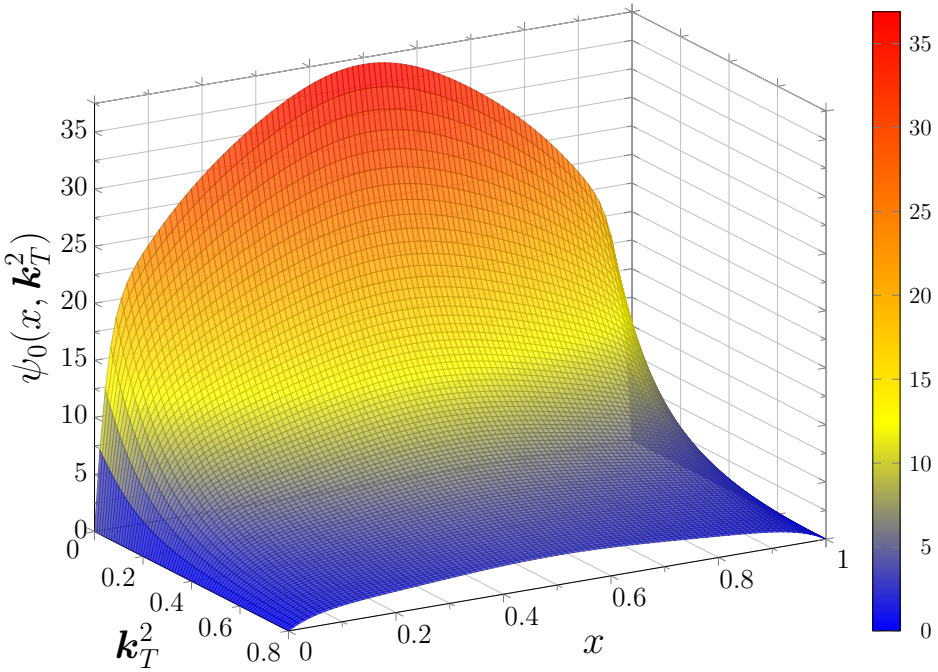
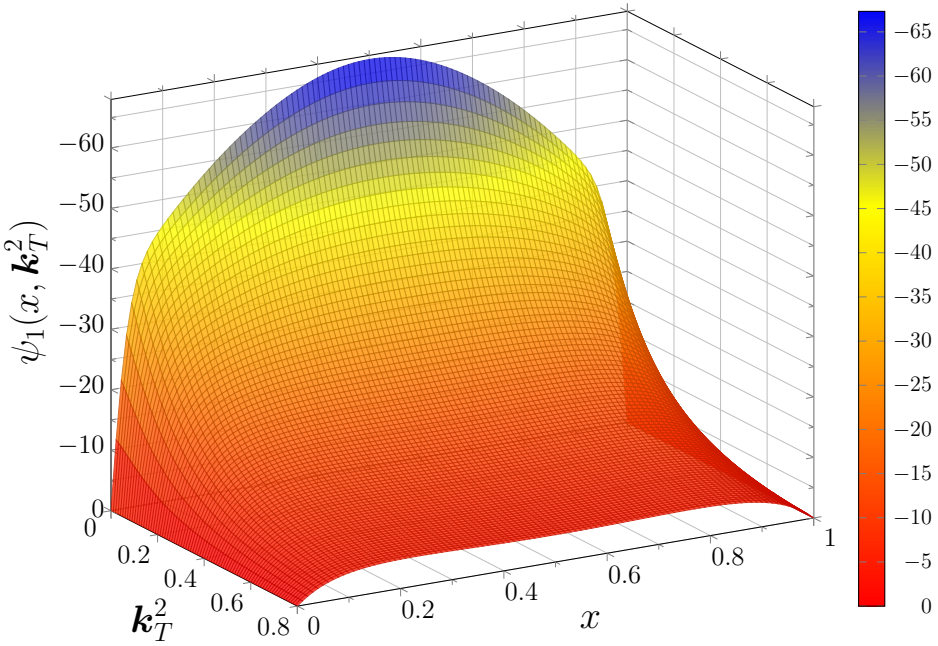
Pion & Kaon Tomography

Pion and Kaon LFWFs

[Chao Shi and ICC, PRL (to appear) arXiv:1806.04799 [nucl-th]]





- Pion has two leading Fock-state LFWFs: $\psi_{\uparrow\downarrow}(x, \boldsymbol{k}_T^2)$ & $\psi_{\uparrow\uparrow}(x, \boldsymbol{k}_T^2)$

$$\psi_0(x, \boldsymbol{k}_T^2) = \sqrt{3}\, i \int \frac{dk^+ dk^-}{2\,\pi} \mathrm{Tr}_D \left[\gamma^+ \gamma_5\, \chi(k,p)\right] \delta\left(k^+ - x\, p^+\right);\ \psi_1(x, \boldsymbol{k}_T^2) = \ldots$$

- DSE result finds broad (almost) concave functions at hadronic scales, with features at small $\boldsymbol{k}_T^2$ driven by DCSB
  - at large $\boldsymbol{k}_T^2$ find same power-law behavior as predicted by perturbative QCD
  - in this domain: $\psi_0(x, \boldsymbol{k}_T^2) \propto x(1-x)/\boldsymbol{k}_T^2$ & $\psi_1(x, \boldsymbol{k}_T^2) \propto x(1-x)/\boldsymbol{k}_T^4$

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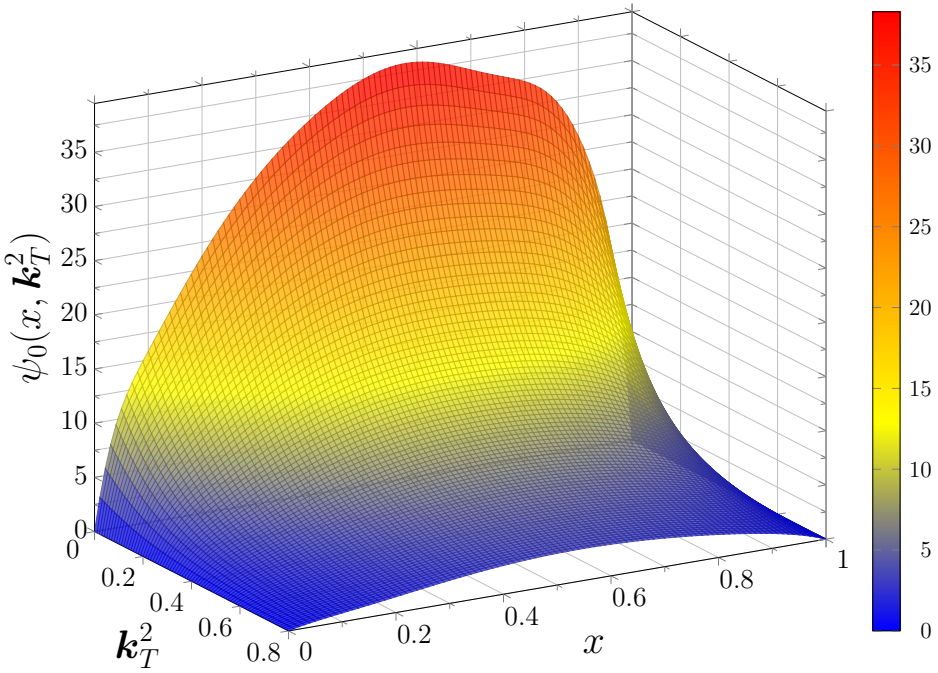

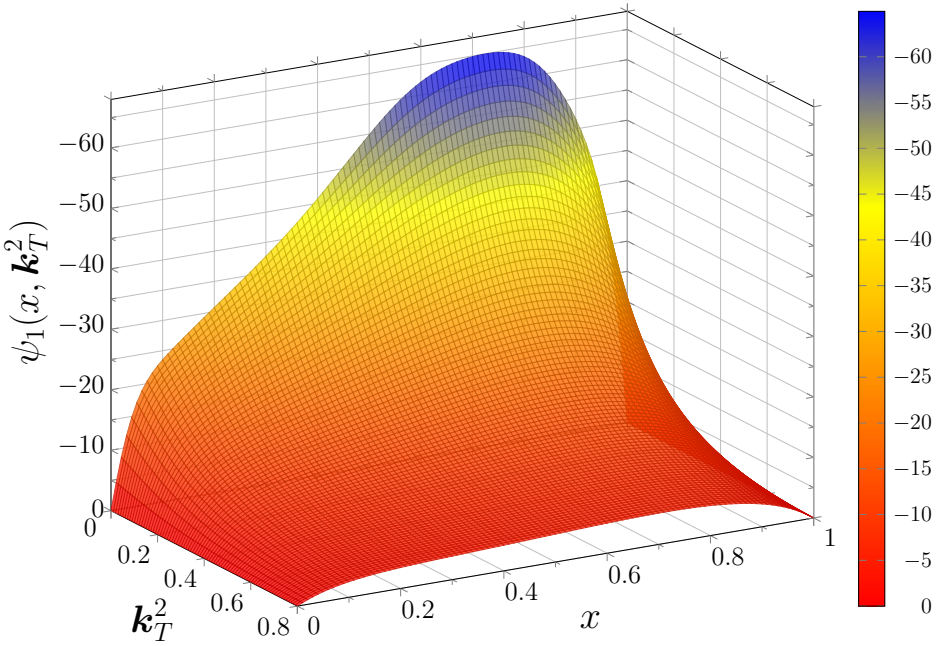

- Kaon has two leading Fock-state LFWFs: $\psi_{\uparrow\downarrow}(x, \boldsymbol{k}_T^2)$ & $\psi_{\uparrow\uparrow}(x, \boldsymbol{k}_T^2)$

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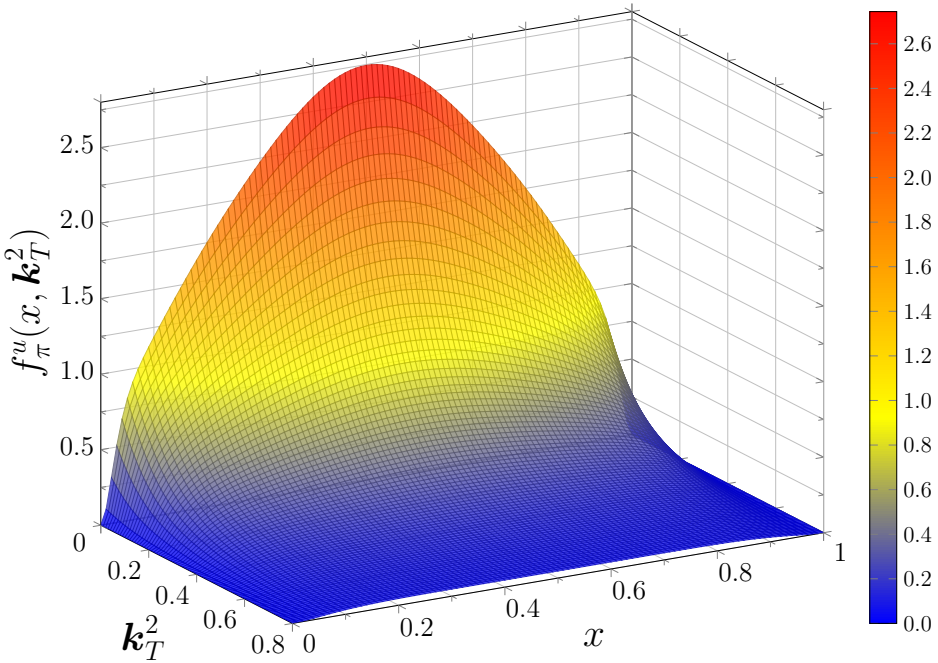
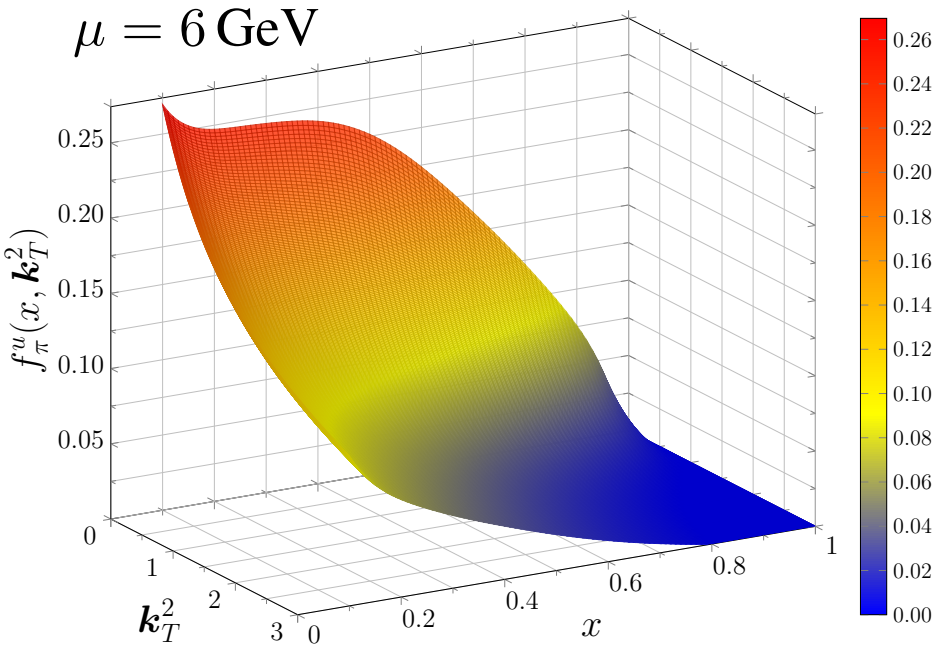
- Significant flavor breaking effects in kaon's LFWFs, particularly $\psi_{\uparrow\uparrow}(x, \boldsymbol{k}_T^2)$
  - however at large $\boldsymbol{k}_T^2$ pion and kaon LFWFs have same scaling behavior
- In general both pion and kaon LFWFs do not factorize in x and $\boldsymbol{k}_T^2$

Pion's T-even TMD

[Chao Shi and ICC, PRL (to appear) arXiv:1806.04799 [nucl-th]]



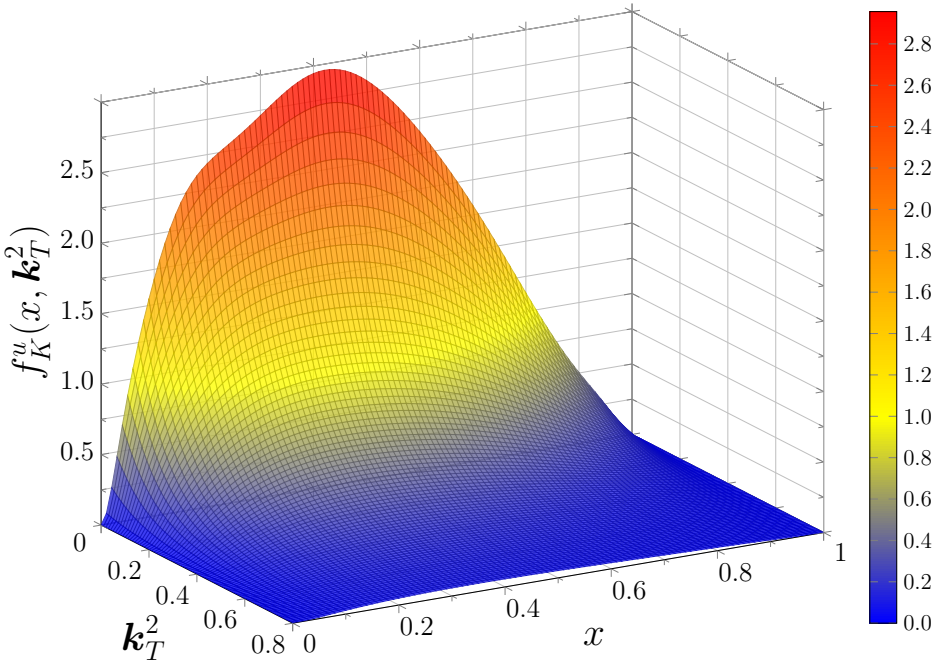


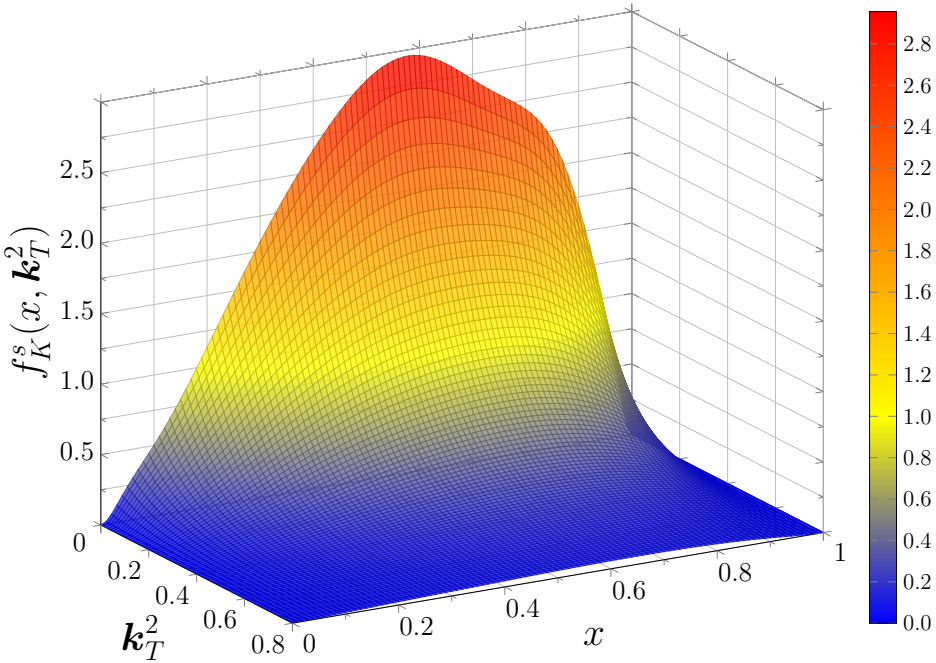
- Using pion's LFWFs straightforward to make predictions for pion TMDs

$$f(x, \boldsymbol{k}_T^2) \propto \left|\psi_{\uparrow\downarrow}(x, \boldsymbol{k}_T^2)\right|^2 + \boldsymbol{k}_T^2 \left|\psi_{\uparrow\uparrow}(x, \boldsymbol{k}_T^2)\right|^2$$

  - numerous features inherited from LFWFs: TMDs are broad functions as a result of DCSB and peak at zero relative momentum ($x = 1/2$)
  - evolution from model scale ($\mu = 0.52\,\mathrm{GeV}$) to $\mu = 6\,\mathrm{GeV}$ results in significant broadening in $\langle \boldsymbol{k}_T^2 \rangle$, from $0.16\,\mathrm{GeV}^2$ to $0.69\,\mathrm{GeV}^2$
- Need careful treatment of gauge link to study pion Boer-Mulders function

Kaon's T-even TMD





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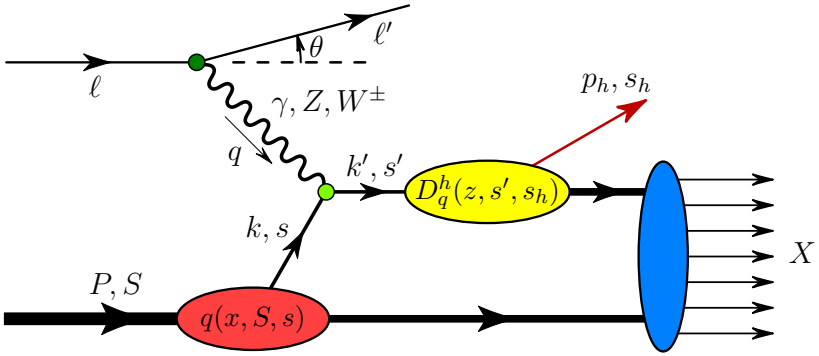
  - numerous features inherited from LFWFs
  - TMDs are broad functions as a result of DCSB and with significant flavor breaking effects
- TMDs satisfy: $f_K^s(x, \boldsymbol{k}_T^2) = f_K^u(1-x, \boldsymbol{k}_T^2)$; $f(x, \boldsymbol{k}_T^2) \to x^2(1-x)^2/\boldsymbol{k}_T^4$
- In general both pion and kaon LFWFs do not factorize in x and $\boldsymbol{k}_T^2$

Probing Transverse Momentum

leading twist		quark polarization		
		unpolarized [U]	longitudinal [L]	transverse [T]
nucleon polarization	U	$f_1 =$ unpolarized		$h_1^\perp =$ Boer-Mulders
	L		$g_1 =$ helicity	$h_{1L}^\perp =$ worm gear 1
	T	$f_{1T}^\perp =$ Sivers	$g_{1T}^\perp =$ worm gear 2	$h_1 =$ transversity $h_{1T}^\perp =$ pretzelosity



- Measuring the pion/kaon TMDs will be a challenge, however progress can be made now by studing the $q \to \pi/K$ TMD fragmentation functions
- Fragmentation functions are particularly important and interesting
  - *potentially fragmentation functions can shed the most light on confinement and DCSB – because they describe how a fast moving (massless) quark or gluon becomes a tower of hadrons*
- Also interesting tool with which to probe color entanglement at an EIC
  - over what length scales can colored correlations be observed?

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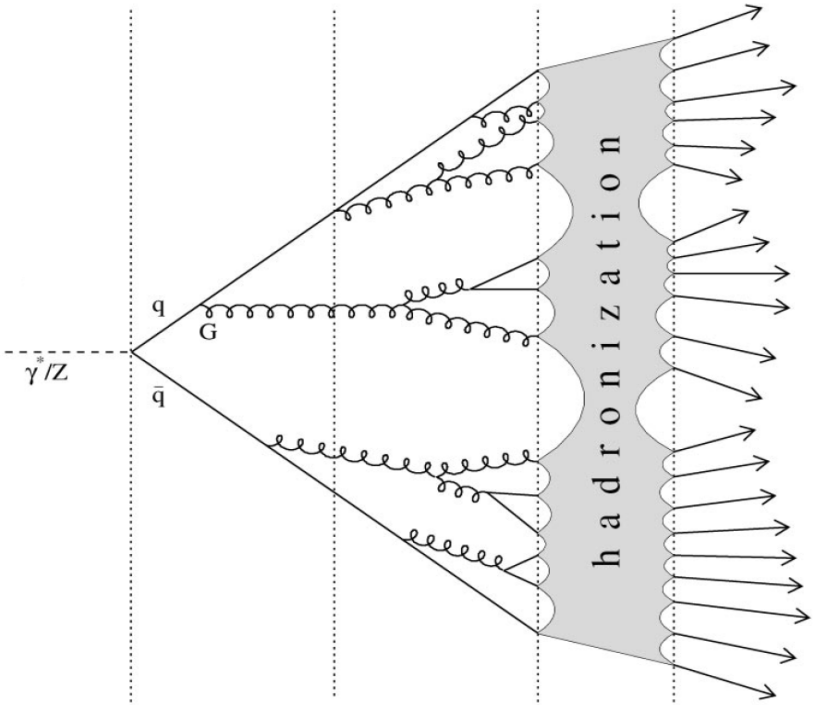

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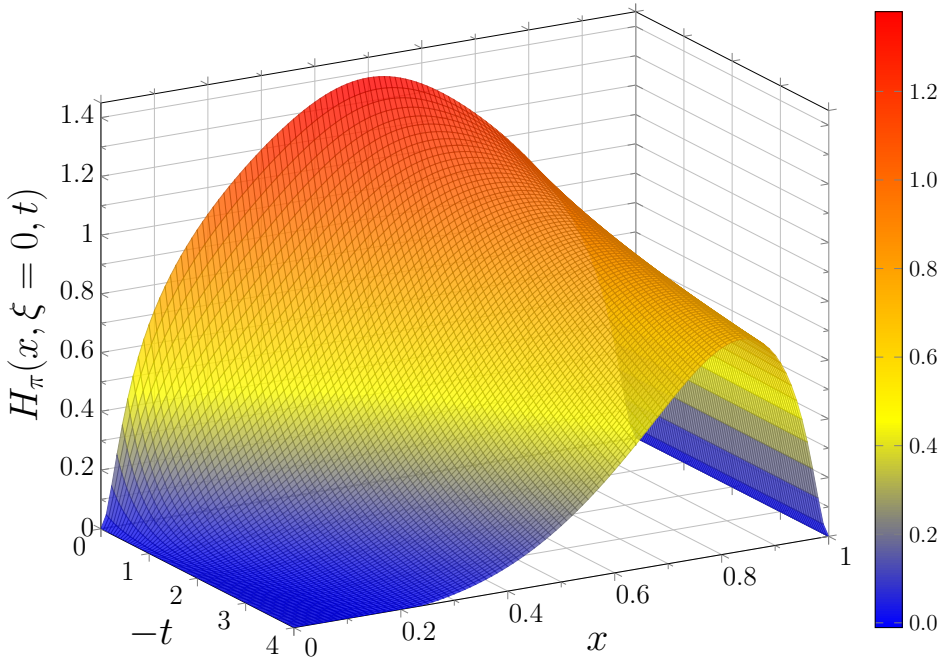
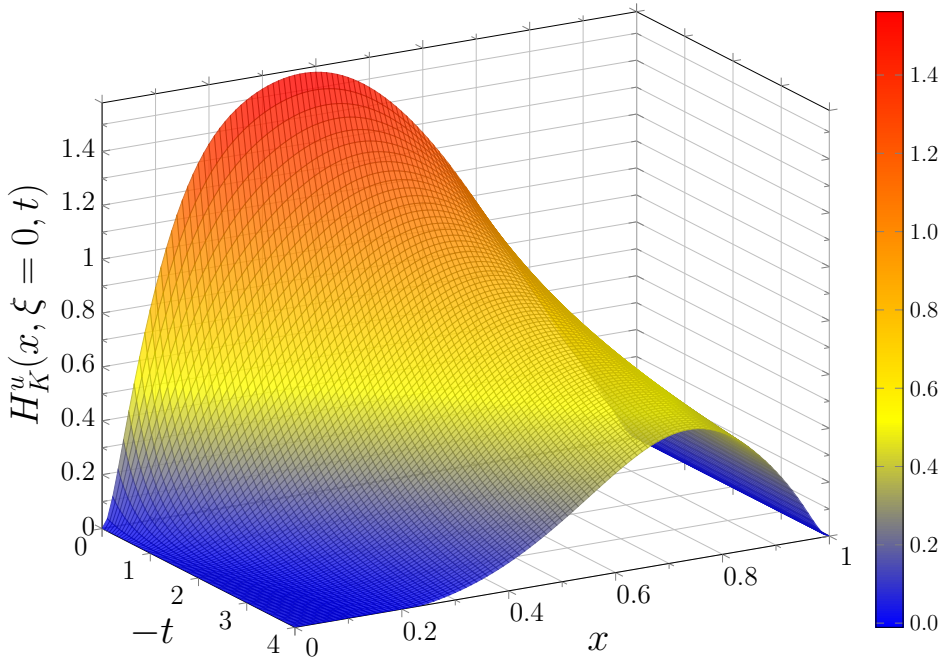
Pion and Kaon GPDs

[Chao Shi and ICC, forthcoming publication]





- Straightforward to make predictions for pion and kaon GPDs from overlaps of LFWFs – only one type of GPD at leading twist

$$H_\pi(x, 0, t) = \int d\boldsymbol{k}_T \left[\psi_0(x, \hat{\boldsymbol{k}}_T)\psi_0(x, \boldsymbol{k}_T) + (\hat{k}_1 + i\hat{k}_2)(k_1 - ik_2)\,\psi_1(x, \hat{\boldsymbol{k}}_T)\psi_1(x, \boldsymbol{k}_T)\right]$$

  - access to DGLAP region $[x > \xi]$ only with leading Fock state
- Our Fock-state expansion is in terms of dressed quarks and gluons
  - as the momentum transfer t increases the dressing of the quarks and gluons is stripped away

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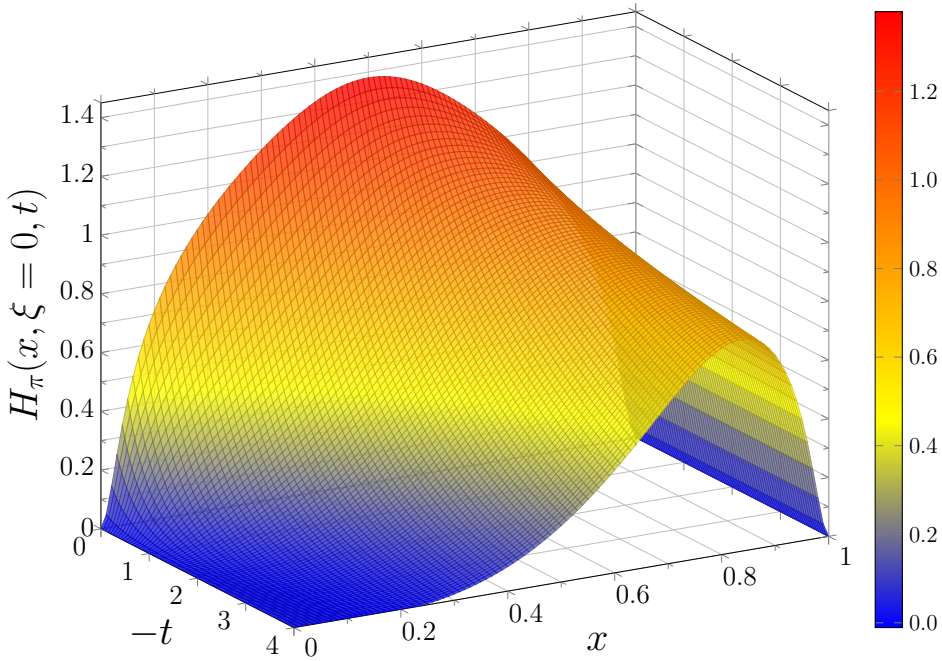

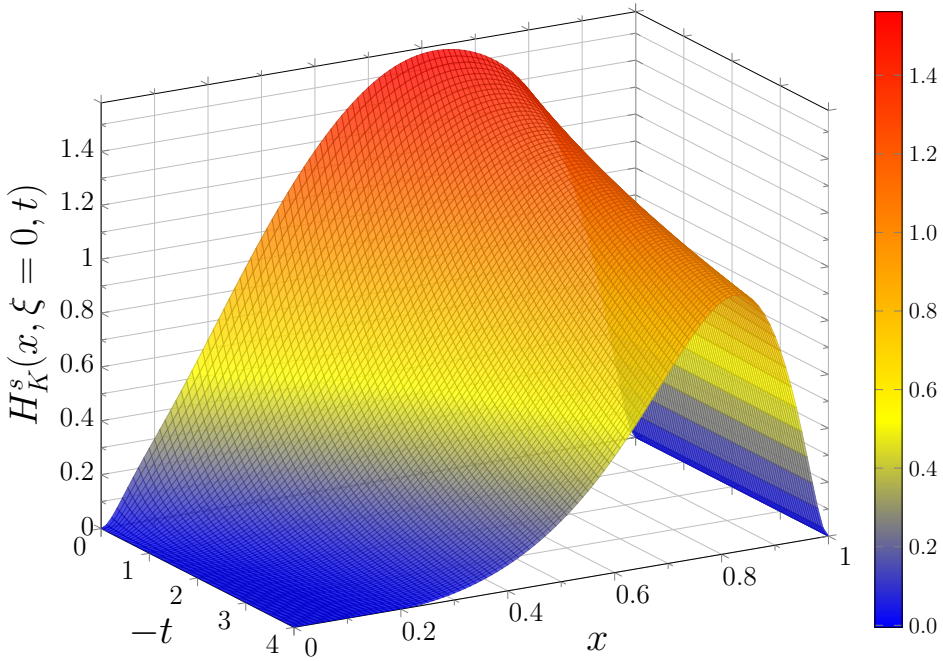

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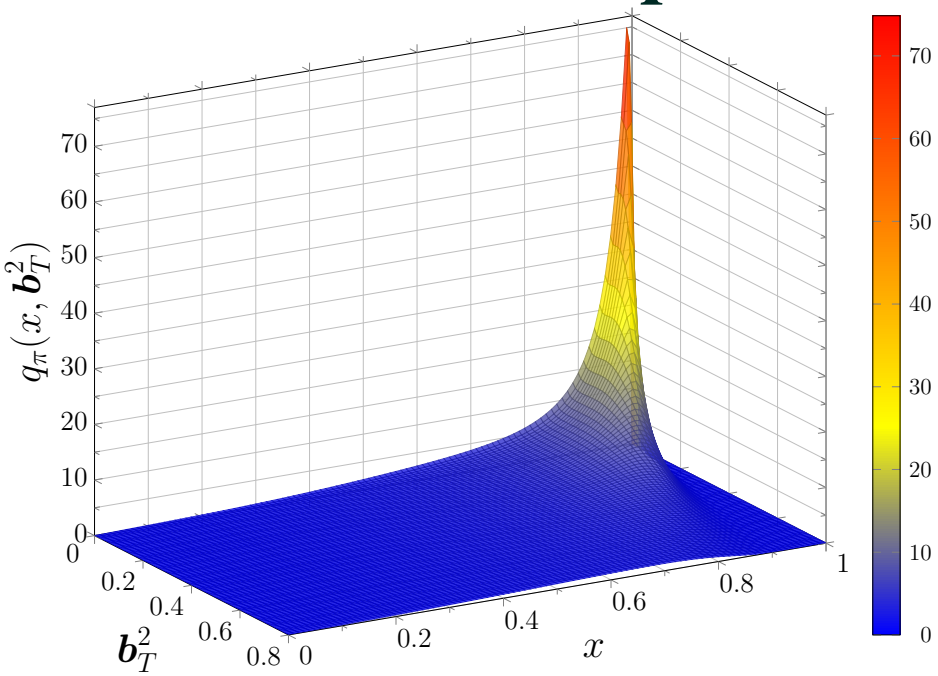
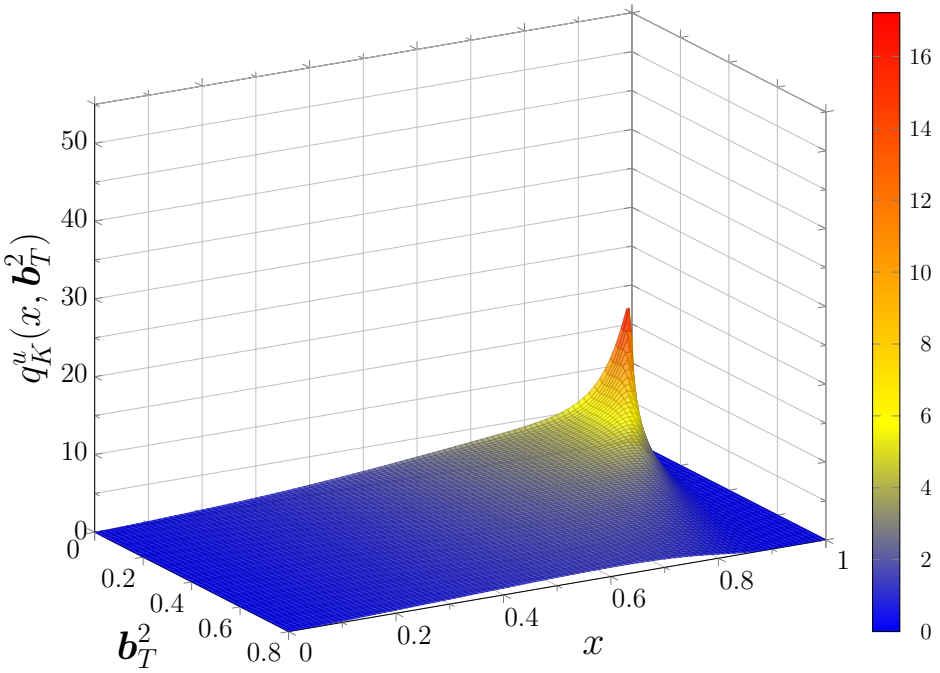
Pion and Kaon Impact-Parameter PDFs





- Impact parameter dependent parton distributions defined by

$$q(x, \boldsymbol{b}_T) = \int \frac{d^2\,\boldsymbol{\Delta}_T}{(2\pi)^2}\, e^{-i\,\boldsymbol{\Delta}_T\cdot\boldsymbol{b}_T}\, H(x, 0, -\boldsymbol{\Delta}_T^2)$$

- IPD PDFs have a probability interpretation
  - probability for a parton to have light-cone momentum fraction x and impact parameter $\boldsymbol{b}_T$; as $x \to 1$ must have $\boldsymbol{b}_T \to 0$
- $q(x, \boldsymbol{b}_T^2)$ peaks near $x \simeq 1$ and $\boldsymbol{b}_T^2 \simeq 0$ because phase space is reduced here, however this region contributes very little to the PDF

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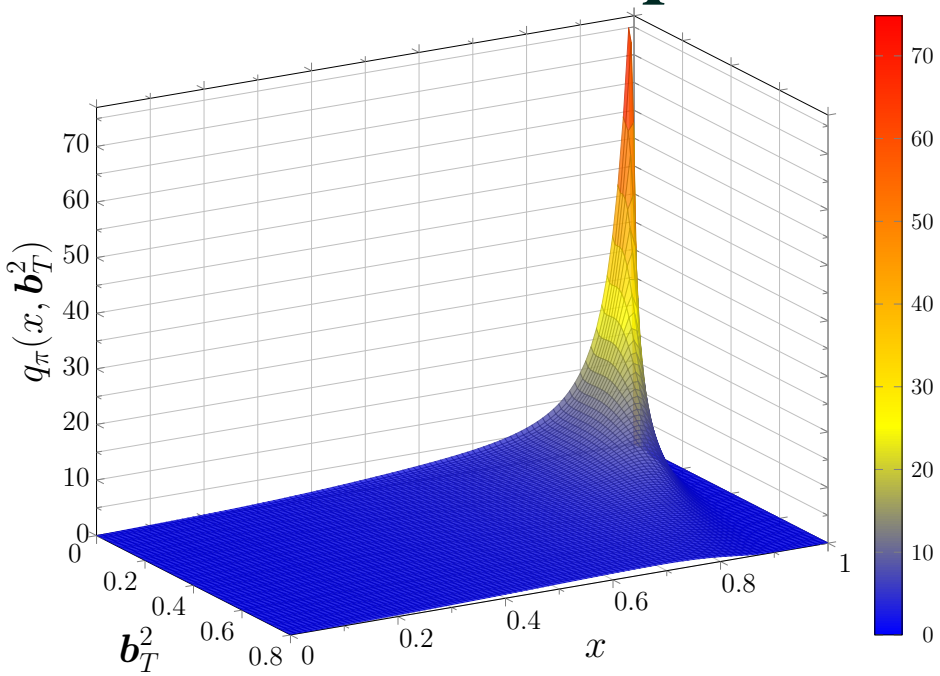

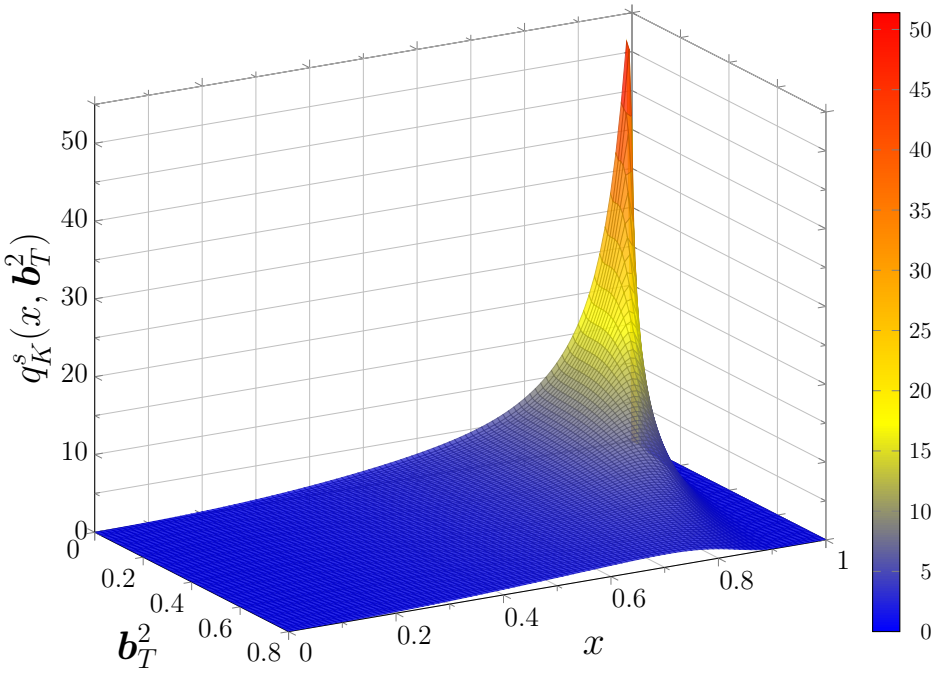

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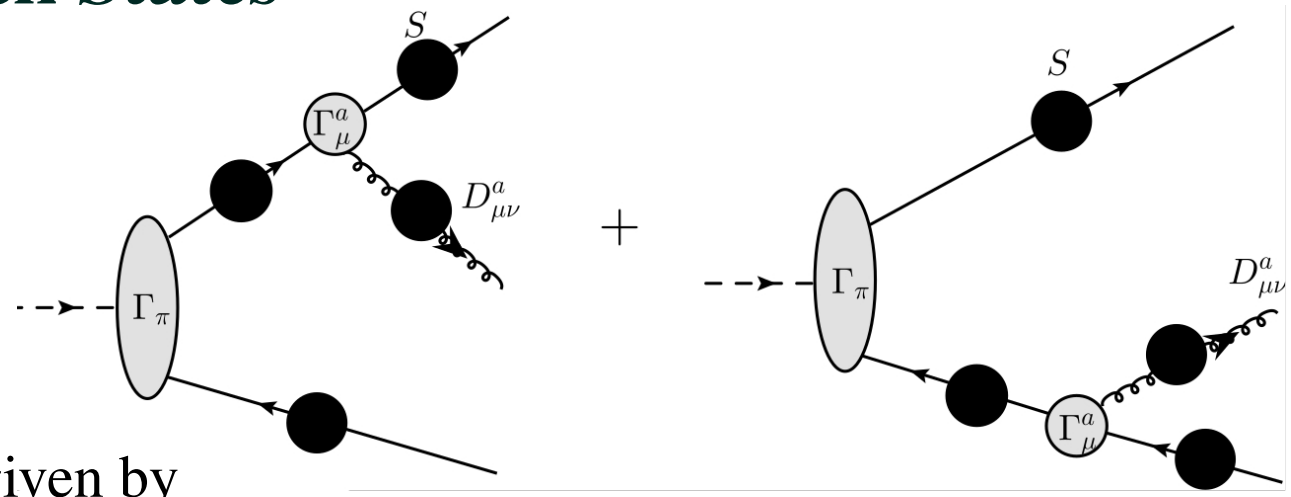
DSEs + Higher Fock States

- From existing DSE ingredients can project out higher Fock states



- For example, the $|q\bar{q}\,g\rangle$ Fock state is given by

$$\psi_{\lambda_1\lambda_2\lambda_3}(x_1,x_2,\boldsymbol{k}_{1T},\boldsymbol{k}_{2T}) \sim \int \frac{dk_1^- dk_2^-}{(2\pi)^2}\; \bar{u}(x_1P^+,\boldsymbol{k}_{1T},\lambda_1)\gamma^+\chi^\mu(k_1,k_2;P)\gamma^+ v(x_2P^+,\boldsymbol{k}_{2T},\lambda_2)\,\varepsilon^*_\mu(\lambda_3)$$

  - for a pion there are nine 5-dimensional LFWFs associated with $|q\bar{q}\,g\rangle$ Fock state

- Key question: *When is a leading Fock-state approximation reliable?*
  - leading Fock state dominates at (very) large x and/or (very) large Q^2
  - can generate numerous higher Fock states using e.g. DGLAP evolution – however non-perturbative content is missing

- Increasing difficult to calculate these higher Fock-state LFWFs and their impact on observables – *need to use full BSE solutions*

Pion PDAs & Form Factors

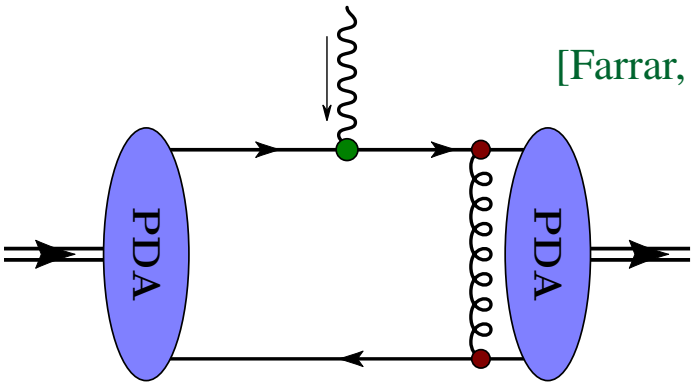
Pion's Parton Distribution Amplitude

- pion's PDA – $\varphi_\pi(x)$: *is a probability amplitude that describes the momentum distribution of a quark and antiquark in the bound-state's valence Fock state*

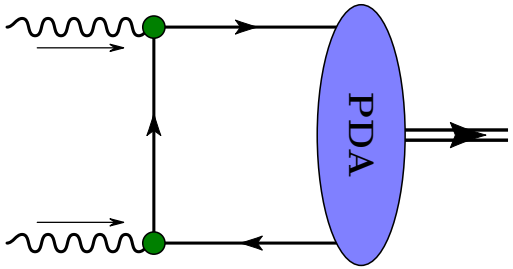
$$f_\pi\,\varphi_\pi(x) = Z_2 \int \frac{d^4k}{(2\pi)^2}\,\delta\left(k^+ - x\,p^+\right)\,\mathrm{Tr}\left[\gamma^+\gamma_5\;S(k)\,\Gamma_\pi(k,p)\,S(k-p)\right]$$

  - it's a function of the light-cone momentum fraction $x = \frac{k^+}{p^+}$ and the scale Q^2
  - asymptotic result is: $\varphi_\pi^{\text{asy}}(x) = 6\,x\,(1-x)$; $\quad \varphi(x) = \int d\boldsymbol{k}_T\,\psi(x,\boldsymbol{k}_T)$

[Farrar, Jackson; Lepage, Brodsky; Radyushkin, Efremov]



$$Q^2\,F_\pi(Q^2) \to 16\pi\,f_\pi^2\,\alpha_s(Q^2)$$

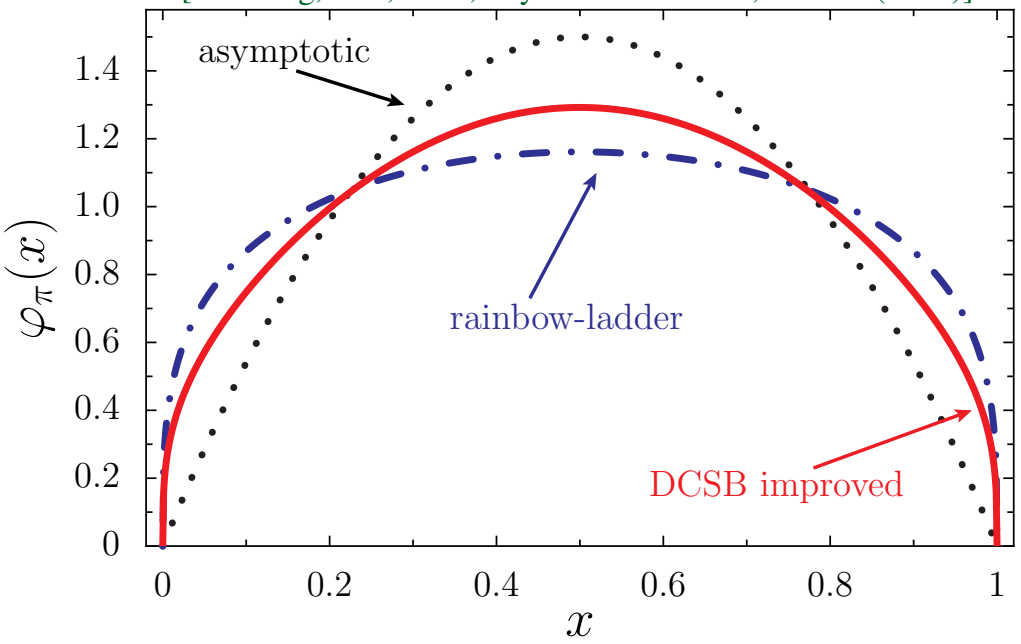


$$Q^2\,F_{\gamma^*\gamma\pi}(Q^2) \to 2\,f_\pi$$

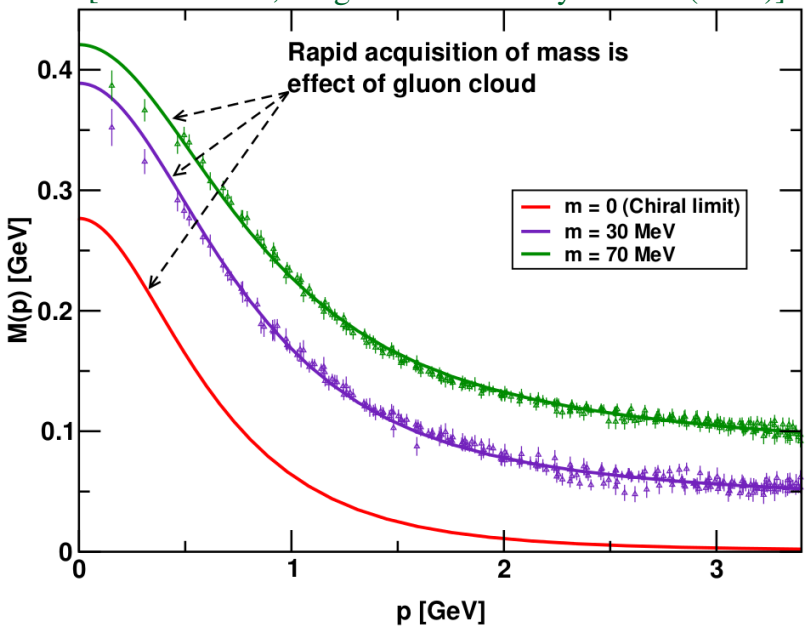
- PDAs enter numerous hard exclusive scattering processes
- Domain of validity of such results can be explored via comparison with data and non-perturbative calculations that maintain a connection to QCD
  - thereby revealing the inner workings of QCD and the strong interaction

Pion PDA from the DSEs

[L. Chang, ICC, *et al.*, Phys. Rev. Lett. **110**, 132001 (2013)]



[C.D. Roberts, Prog. Part. Nucl. Phys. 61 50 (2008)]



- Both DSE results – each using a different Bethe-Salpeter kernel – exhibit a pronounced broadening compared with the asymptotic pion PDA
  - scale of calculation is given by renormalization point $\xi = 2\,\mathrm{GeV}$
- A realization of DCSB on the light-front
- *ERBL evolution demonstrates that the pion's PDA remains broad & concave for all accessible scales in current and conceivable experiments*
- Broading of PDA influences the Q^2 evolution of the pion's EM form factor

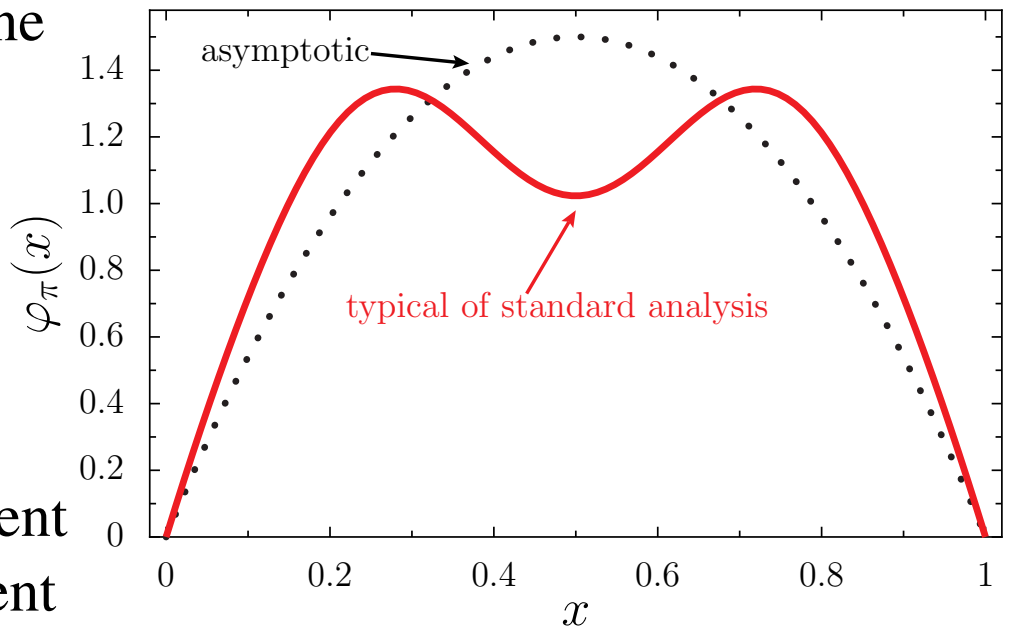
Pion PDA from Lattice QCD

- Currently, lattice QCD can determine only one non-trivial moment *e.g.*

$$\int dx\,(2\,x-1)^2\varphi_\pi(x) = 0.2361\,(41)\,(39)$$

[V. M. Braun, *et al.*, Phys. Rev. D **92**, no. 1, 014504 (2015)]

  - scale is $Q^2 = 4\,\mathrm{GeV}^2$



- Standard practice to fit first coefficient of "*asymptotic expansion*" to moment

$$\varphi_\pi(x,Q^2) = 6\,x\,(1-x)\left[1+\sum\nolimits_{n=2,\,4,\ldots} a_n^{3/2}(Q^2)\,C_n^{3/2}(2x-1)\right]$$

  - however this expansion is guaranteed to converge rapidly only when $Q^2 \to \infty$
  - method results in a *double-humped* pion PDA – not supported by BSE WFs

- Advocate using a *generalized expansion*

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- Find good agreement with DSE result

Pion PDA from Lattice QCD

- Currently, lattice QCD can determine only one non-trivial moment *e.g.*

$$\int dx\,(2\,x-1)^2 \varphi_\pi(x) = 0.2361\,(41)\,(39)$$

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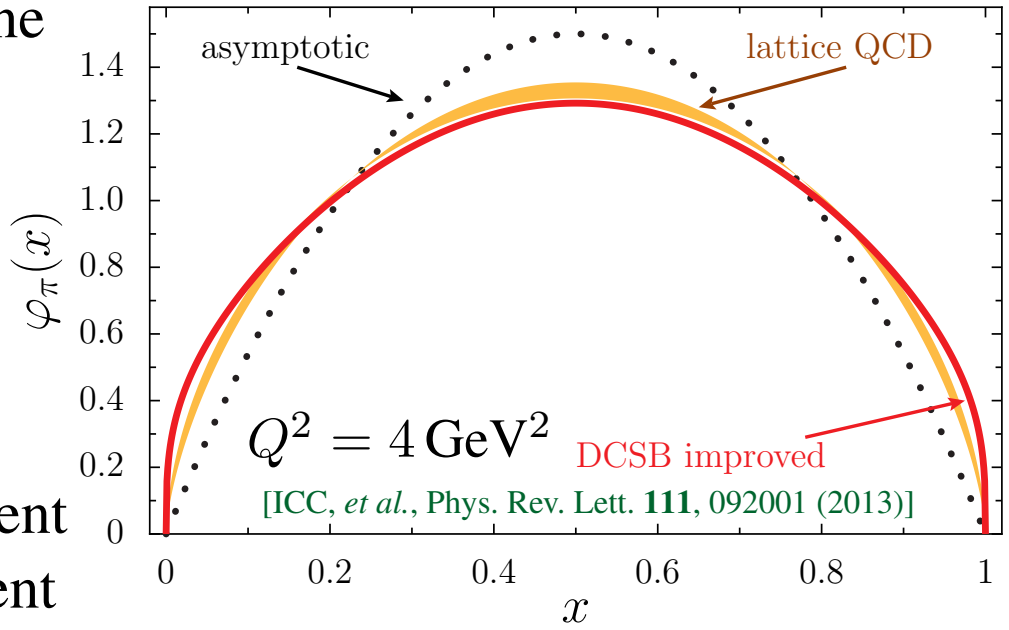

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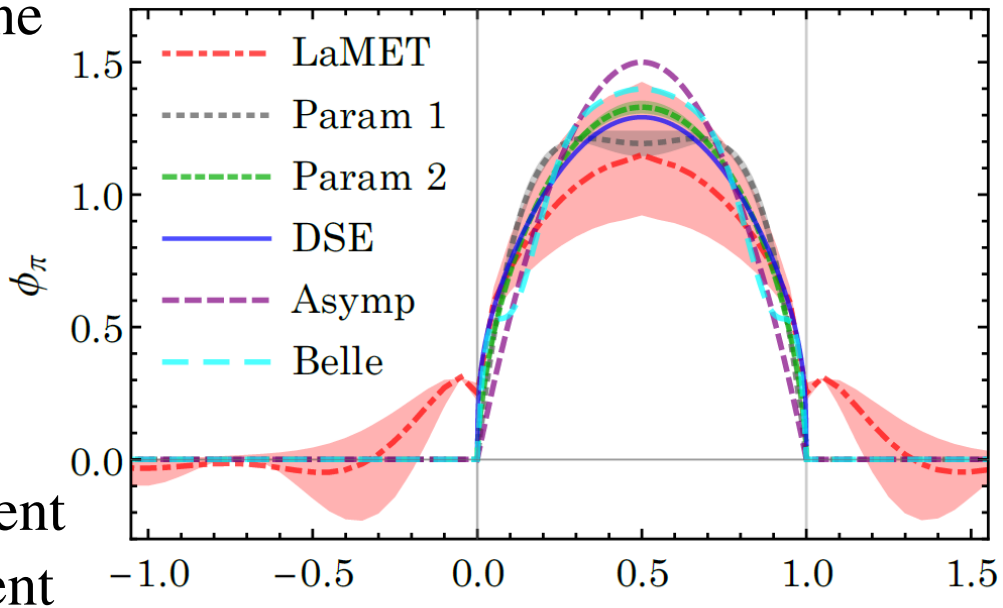
[J. H. Zhang, J. W. Chen, *et al.*, Phys. Rev. D **95**, no. 9, 094514 (2017)]



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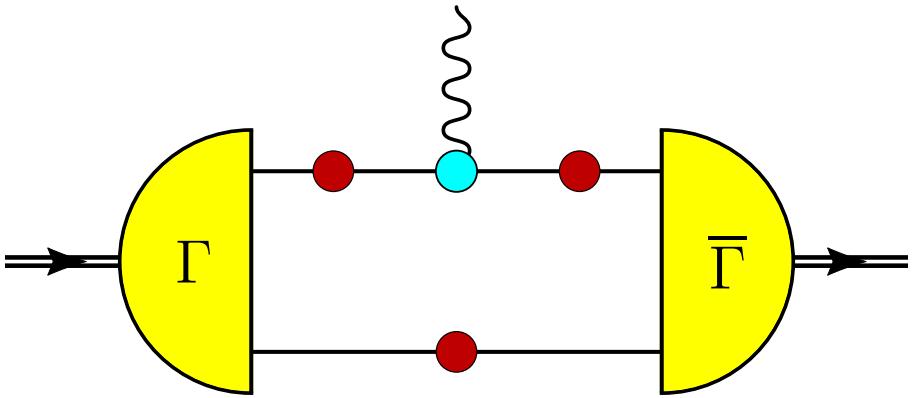
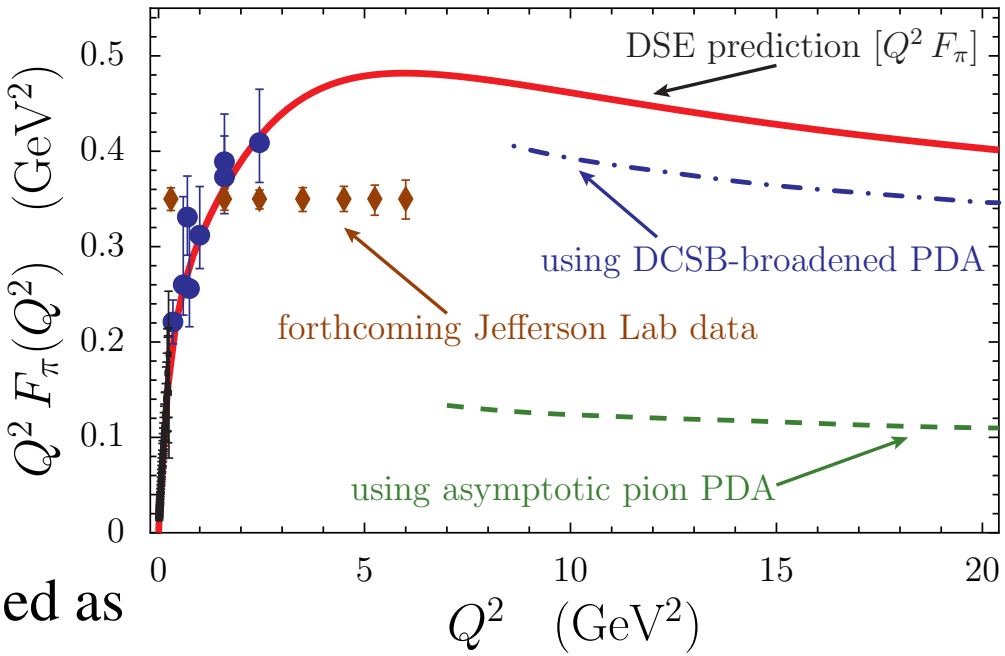
Pion Elastic Form Factor



[L. Chang, ICC, *et al.*, Phys. Rev. Lett. **111**, 141802 (2013)]



- The QCD prediction can be expressed as

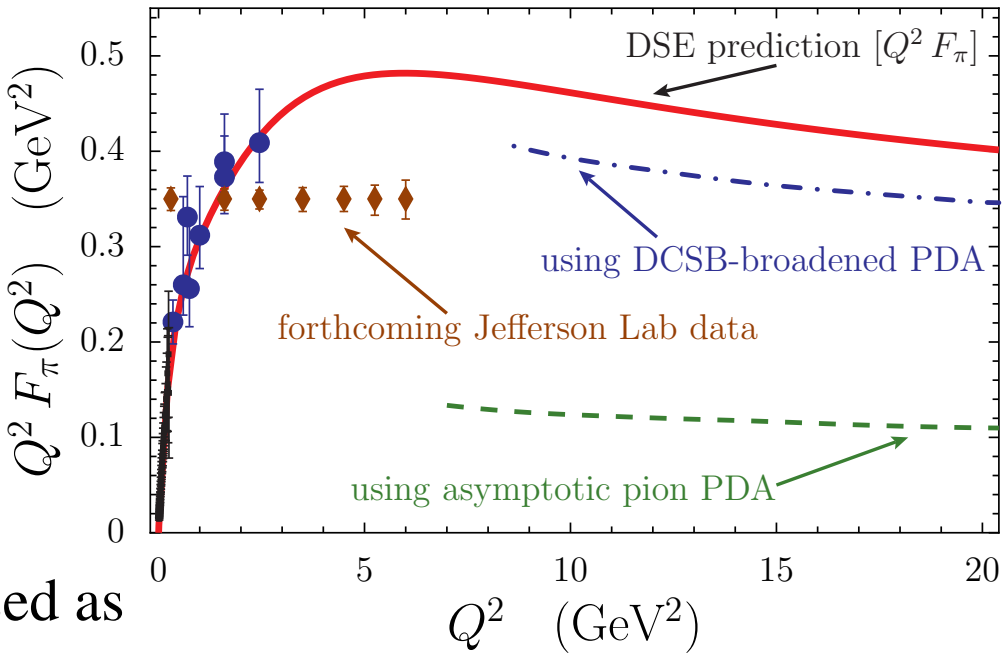
$$Q^2 F_\pi(Q^2) \overset{Q^2 \gg \Lambda_{\rm QCD}^2}{\sim} 16\,\pi\, f_\pi^2\, \alpha_s(Q^2)\, w_\pi^2; \qquad w_\pi = \frac{1}{3}\int_0^1 dx\, \frac{1}{x}\, \varphi_\pi(x)$$

- Find consistency between the *direct pion form factor calculation* and the QCD hard-scattering formula – if DSE pion PDA is used
  - 15% disagreement may be explained by higher-order corrections
- At an EIC preliminary studies [Garth Huber – PIEIC 2018] suggest pion form factor can be measured to $Q^2 \gtrsim 30\,\mathrm{GeV}^2$

Pion Elastic Form Factor

- Direct, symmetry-preserving computation of pion form factor predicts maximum in $Q^2\,F_\pi(Q^2)$ at $Q^2 \approx 6\,\mathrm{GeV}^2$
  - magnitude of this product is determined by strength of DCSB at all accessible scales

[L. Chang, ICC, *et al.*, Phys. Rev. Lett. **111**, 141802 (2013)]



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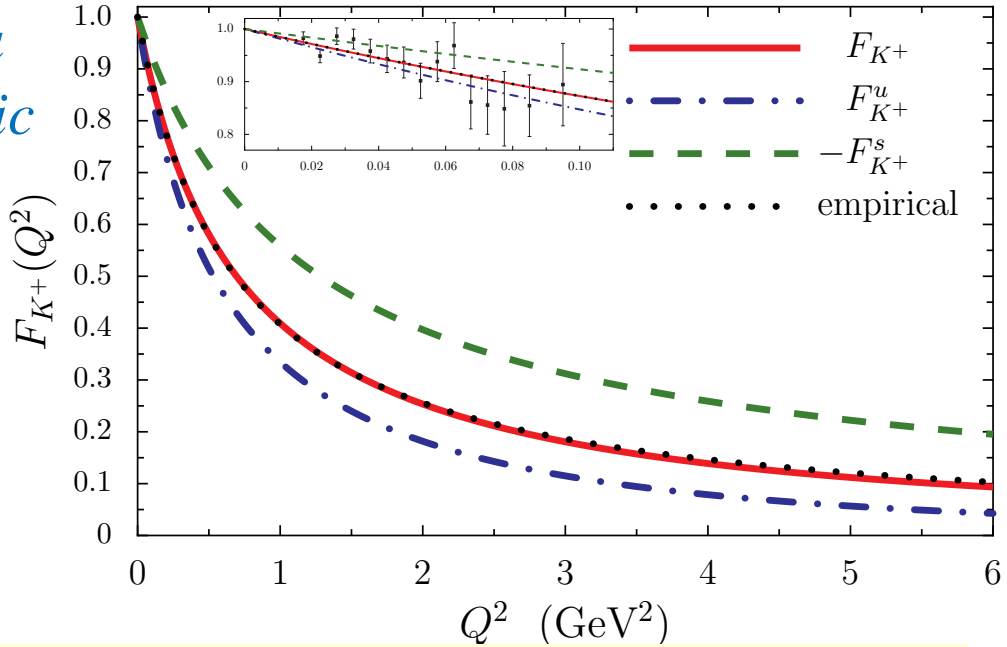
Form Factors and Confinement

[P. T. P. Hutauruk, ICC and A. W. Thomas, PRC **94**, (2016)]

- Space-like form factors: *ability of a hadron to absorb an electromagnetic current and remain intact*
- Form factors must be a sensitive measure of confinement in QCD
  - but what are they telling us?
- Consider quark-sector kaon form factors (with charge symmetry)



$$F_{K^+}(Q^2) = e_u\, F^u_K(Q^2) + e_s\, F^s_K(Q^2), \qquad F_{K^0}(Q^2) = e_d\, F^u_K(Q^2) + e_s\, F^s_K(Q^2)$$

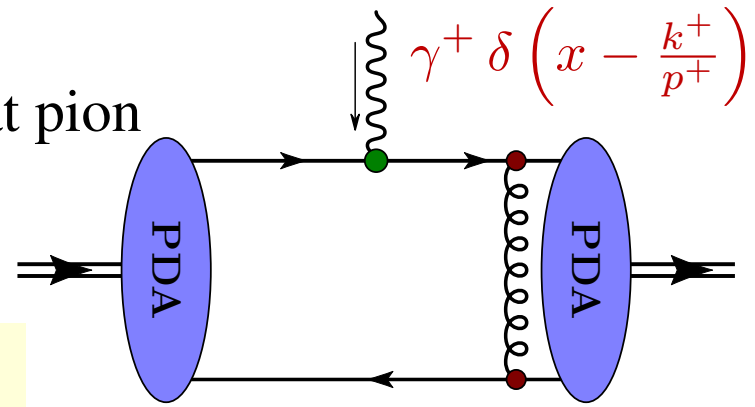
- Empirical radius: $r_{K^0} = -0.277 \pm 0.018 \implies |r^u_K| > |r^s_K|$; DSE results:

$$r_{K^0} = -0.272; \qquad r^u_K = 0.646; \qquad r^s_K = -0.441$$

- For kaon's quark-sector form factors remarkable flavor dependence remains
  - s quark much harder than the u quark
  - confinement? If probe strikes a light u quark it is much harder for the hadron to remain intact – compared to when an s quark is struck

Pion PDFs

Perturbative QCD and Pion PDFs



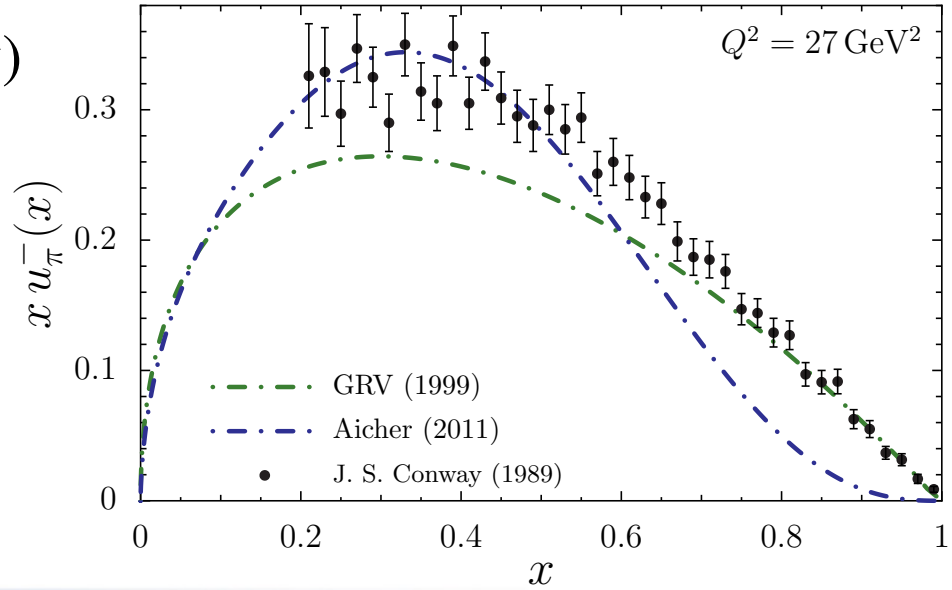
- Longstanding perturbative QCD prediction that pion PDF near $x = 1$ behaves as: $q_\pi(x) \simeq (1-x)^2$
- From LFWF perspective expect:

$$q_\pi(x) \sim a_1(\mu^2)\,(1-x)^2 + a_2(\mu^2)\,(1-x)^4 + \ldots$$

  - even power because of Drell-Yan-Levy relation or Gribov-Lipatov reciprocity
- Since large-x quarks are source of large-x gluons expect

$$g_\pi(x) \sim b_1(\mu^2)\,(1-x)^3 + b_2(\mu^2)\,(1-x)^5 + \ldots; \quad g_\pi(x) \sim (1-x)^{3+\gamma(\mu^2)}$$

- Pion-induced Drell-Yan data (Conway) and a resent analysis (Sato), also including leading-neutron data, find $q(x) \sim (1-x)^1$ near $x = 1$
- Potential resolution to this puzzle provided by Aicher *et al.* $\Longrightarrow$ soft-gluon resummation



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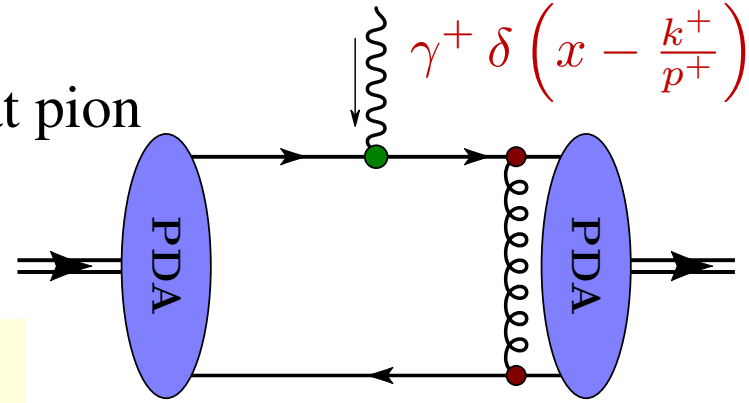

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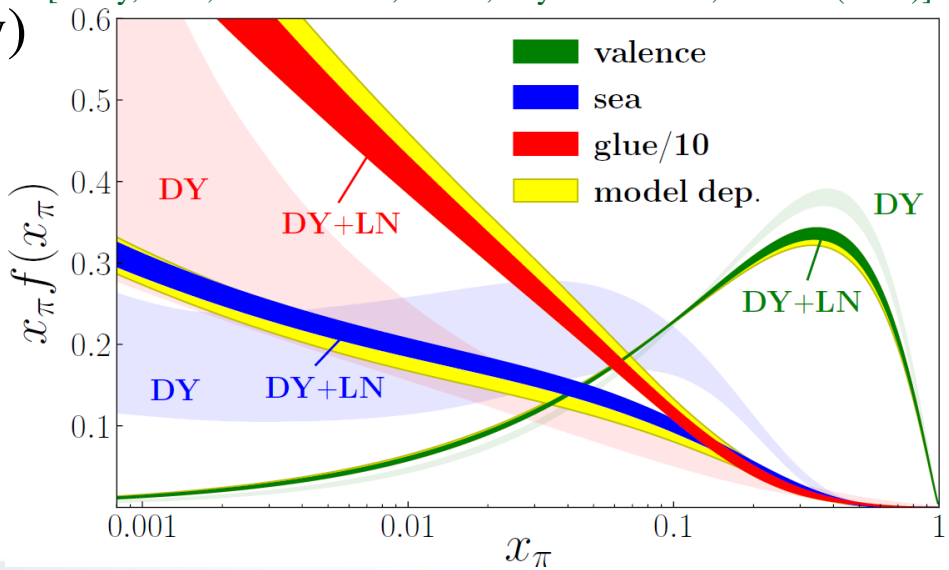
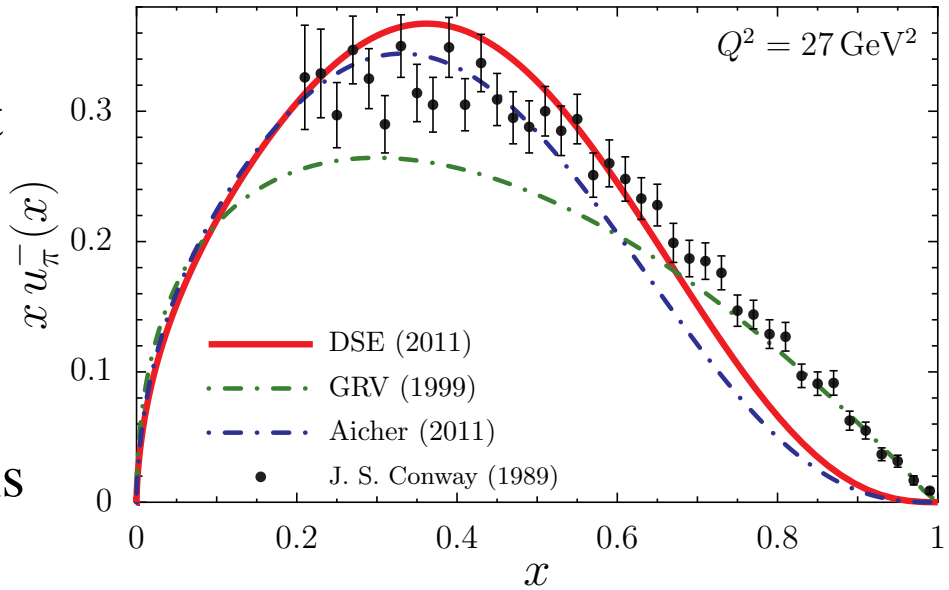
[Barry, Sato, Melnitchouk, and Ji, Phys. Rev. Lett., 152001 (2018)]



DSEs + Pion PDFs

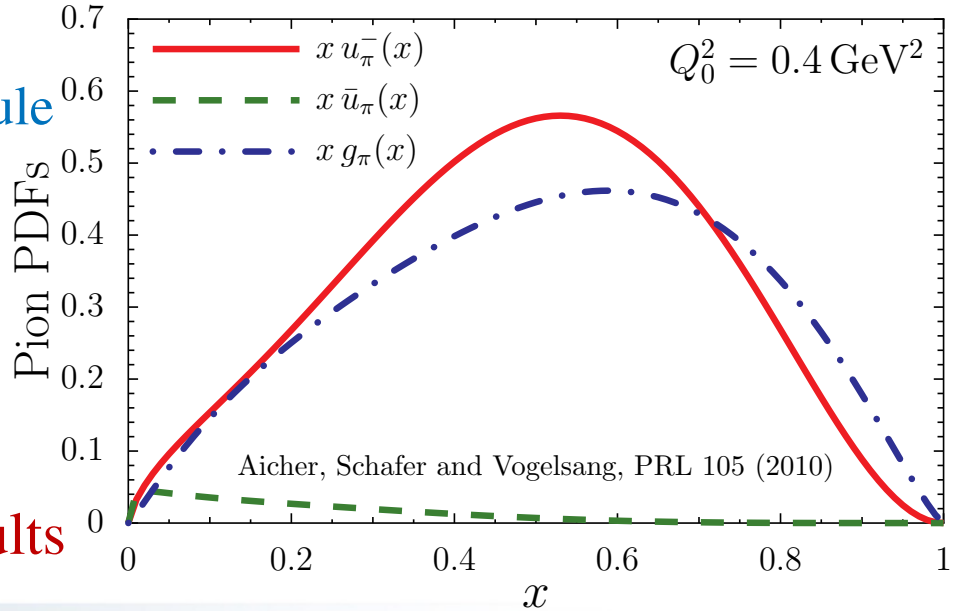
- DSE prediction [Hecht (2001); etc] that $q_\pi(x) \simeq (1-x)^2$ as $x \to 1$
  - related to $1/k^2$ dependence of BSE kernel at large relative momentum



- Until recently DSEs PDFs calculations used Ward-Identity ansatz (WIA)

$$\Lambda_q(z,p,n) \to \Lambda_q^{\mathrm{WIA}}(z,p,n) = \delta\,(1-z)\;\; n^\mu \frac{\partial}{\partial p^\mu} S_q^{-1}(p)$$

  - WIA respects baryon sum rule but not higher moments e.g. momentum sum rule
  - momentum is not distributed correctly between quarks and gluons

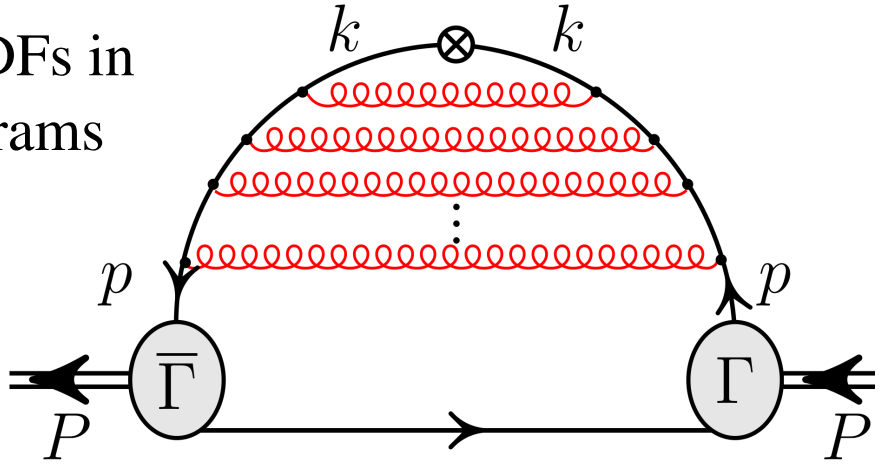


- Aicher: $g_\pi(x) \sim (1-x)^{1.3}$ as $x \to 1$
  - pQCD predicts: $g_\pi(x) \sim (1-x)^3$
- Inconsistencies in Aicher & DSE results

Pion PDFs – Self-Consistent DSE Calculations

- To self-consistently determine hadron PDFs in rainbow-ladder must sum all planar diagrams

$$q(x) \propto \mathrm{Tr} \int \frac{d^4p}{(2\pi)^4}\, \overline{\Gamma}_M(p,P)\, S(p)$$
$$\times\, \Gamma_q(x,p,n)\, S(p)\, \Gamma_M(p,P)\, S(p-P)$$



- DSEs are formulated in Euclidean space – evaluate $q(x)$ by taking moments
- The *hadron dependent* vertex $\Gamma_q(x,p,n)$ satisfies an inhomogeneous BSE
- However can define a *hadron independent* vertex $\Lambda_q(x,p,n)$

$$\Gamma_q(x,p,n) = \iint dy\, dz\, \delta(x-yz)\, \delta\!\left(y - \frac{p\cdot n}{P\cdot n}\right) \Lambda_q(z,p,n)$$

- $\Lambda_q(x,p,n)$ satisfies the inhomogeneous BSE

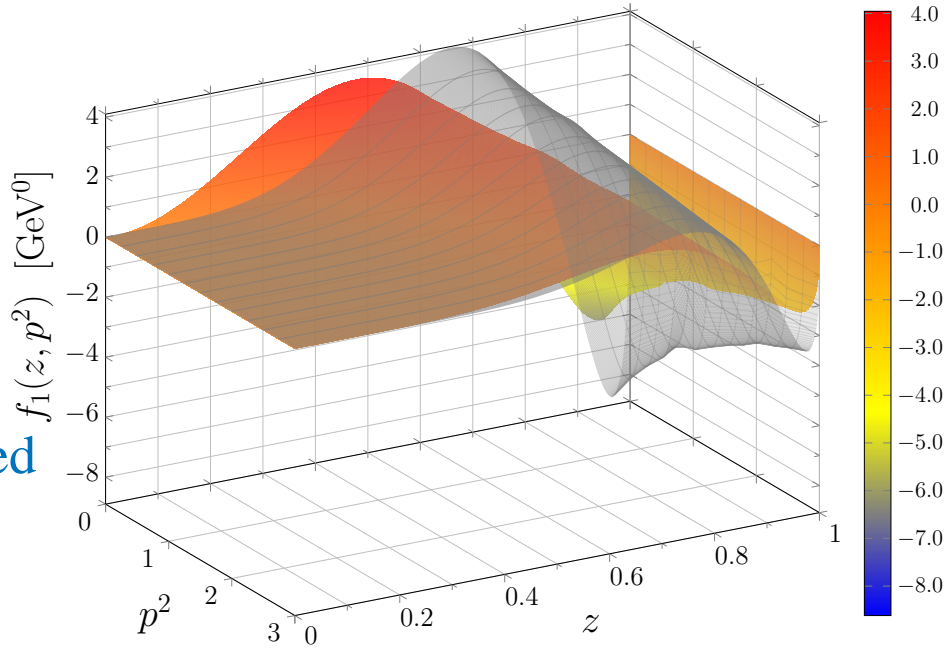
$$\Lambda_q(z,p,n) = iZ_2\, \not{n}\, \delta(1-z) - \iint du\, dw\, \delta(z-uw) \int \frac{d^4\ell}{(2\pi)^4} \delta\!\left(w - \frac{\ell\cdot n}{p\cdot n}\right)$$
$$\times\, \gamma_\mu\, \mathcal{K}_{\mu\nu}(p-\ell)\, S(\ell)\, \Lambda_q(u,\ell,n)\, S(\ell)\, \gamma_\nu$$

PDFs of a Dressed Quark

- *Hadron independent* vertex has form

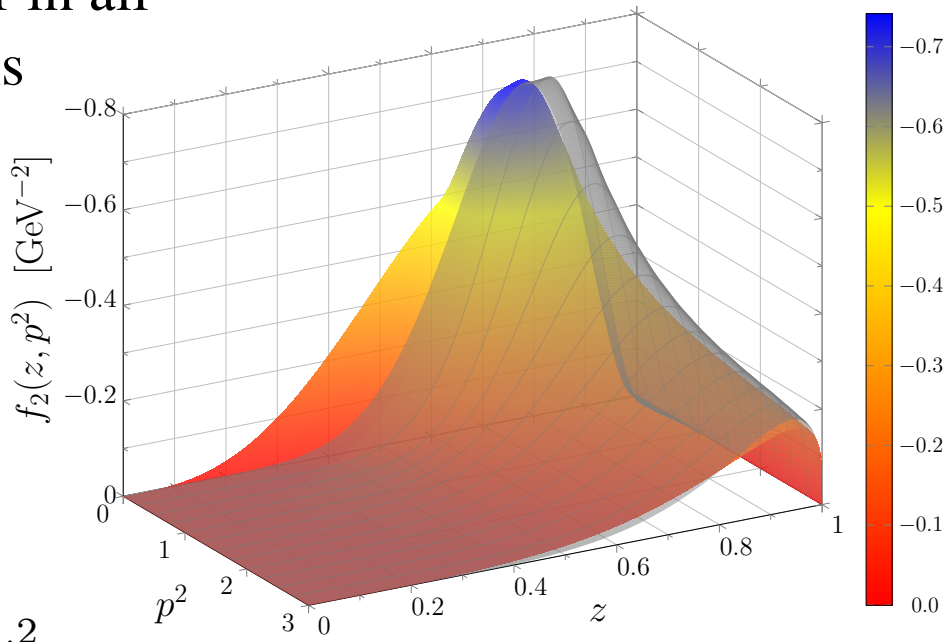
$$\Lambda_q(z,p,n) = i\not{p}\,\delta(1-z) + i\not{p}\,f_1^q(z,p^2) + n\cdot p\left[i\not{p}\,f_2^q(z,p^2) + f_3^q(z,p^2)\right]$$

  - the functions $f_i^q(z,p^2)$ can be interpreted as unpolarized PDFs in a dressed quark of virtuality p^2



- These functions are universal – appear in all RL-DSE unpolarzied PDF calculations

- Distributed support in z is immediate indication gluons carry significant momentum
  - heavier s quark support nearer $z=1$
  - WIA $\implies \Lambda_q(z,p,n) \propto \delta(1-z)$



- Renormalization condition means dressing functions vanish when $p^2=\mu^2$

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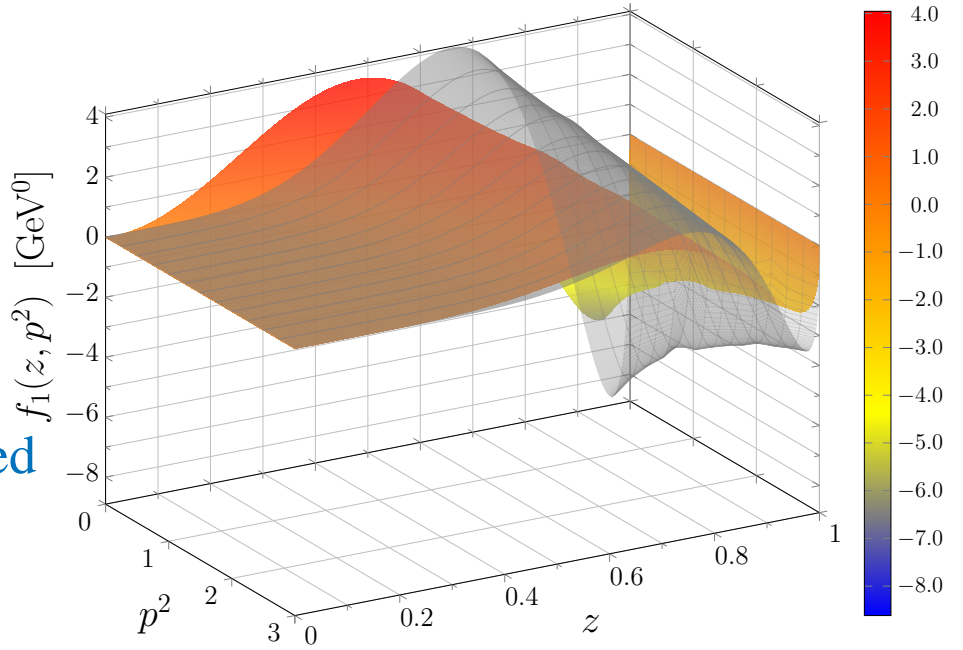

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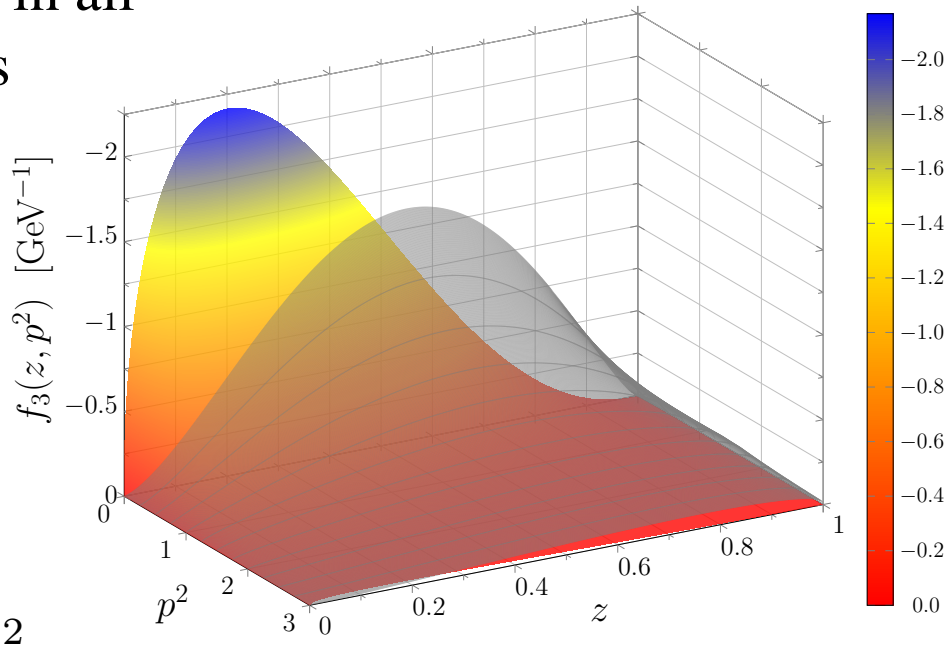

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Pion PDFs: Self-Consistent DSE Results

- Treating gluon contributions correctly has important impact on PDFs
  - mantains result: $q_\pi(x) \overset{x\to 1}{\sim} (1-x)^2$
  - but pushes support of $q_\pi(x)$ to larger x
- *Immediate consequence of gluon dressing is that gluons carry 35% of pion's and 30% of kaon's momentum*
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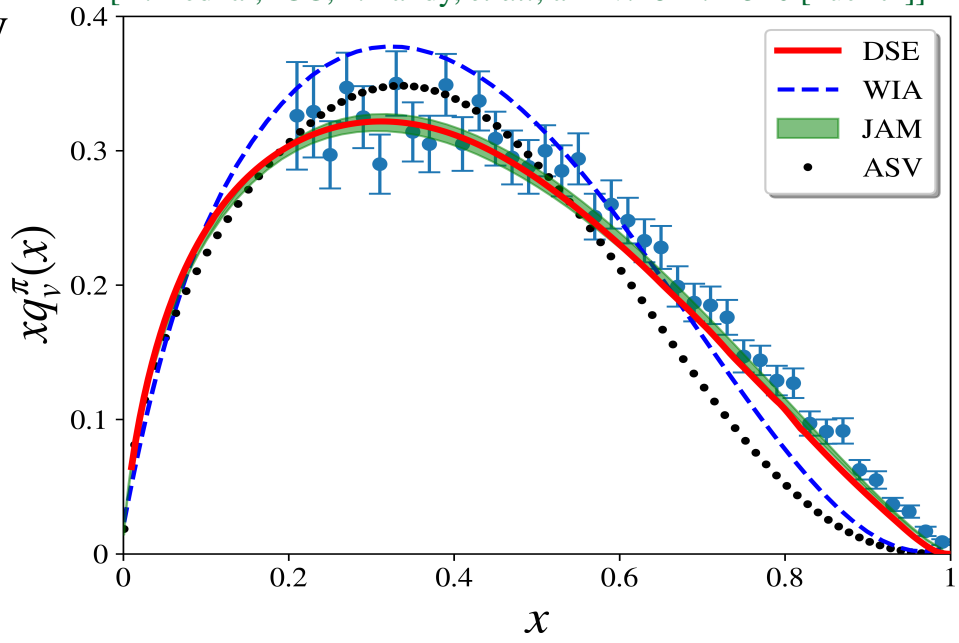
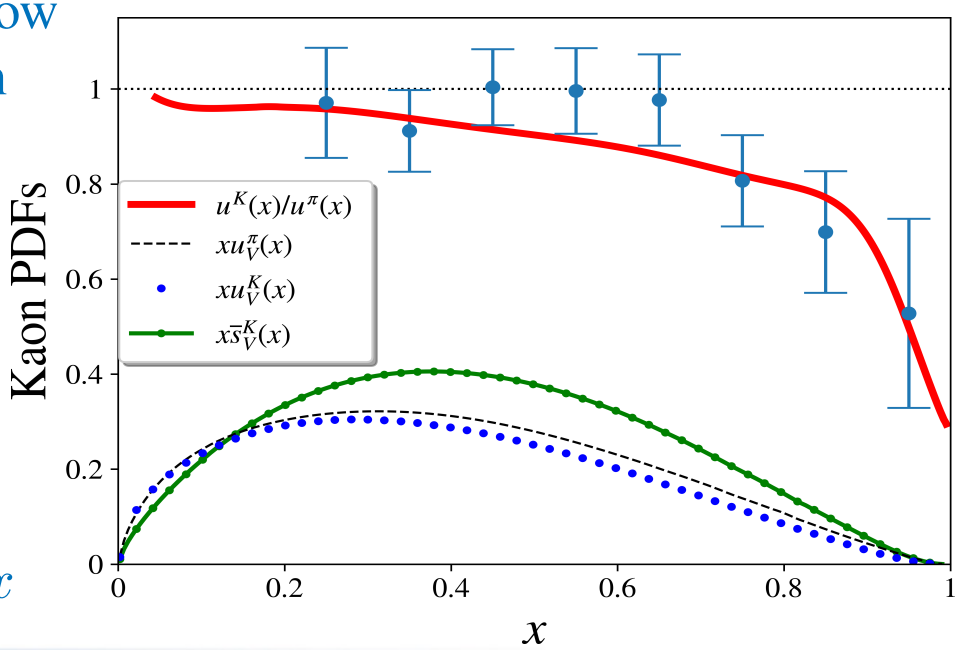
  - cannot be replicated by DGLAP – DSE splitting functions are dressed
- Consistent result for $u_K(x)/u_\pi(x)$
  - but need new data on pion and kaon PDFs to solve puzzle of PDFs at large x

[K. Bednar, ICC, P. Tandy, *et al.*, arXiv:1811.12310 [nucl-th]]





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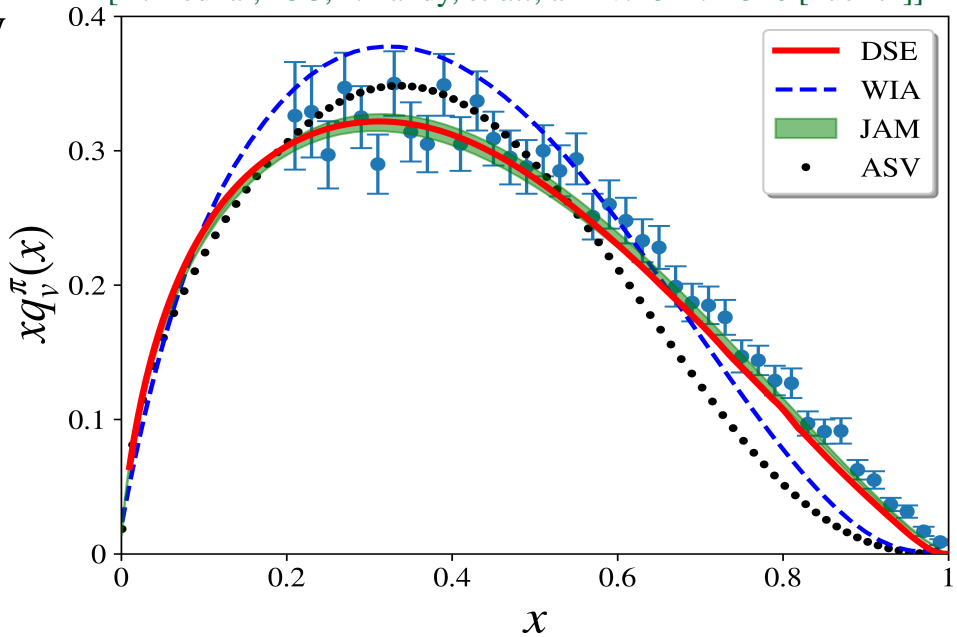
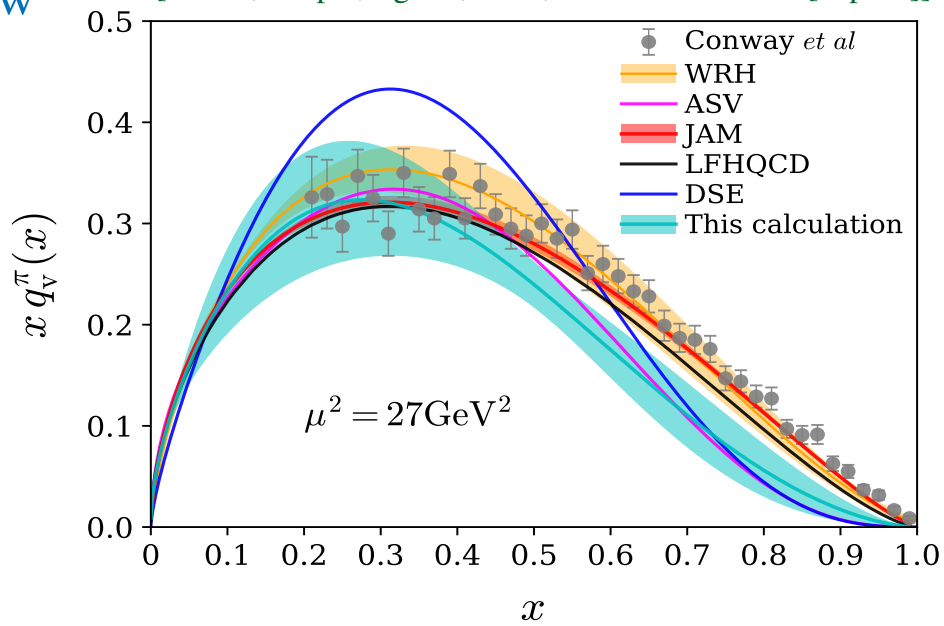
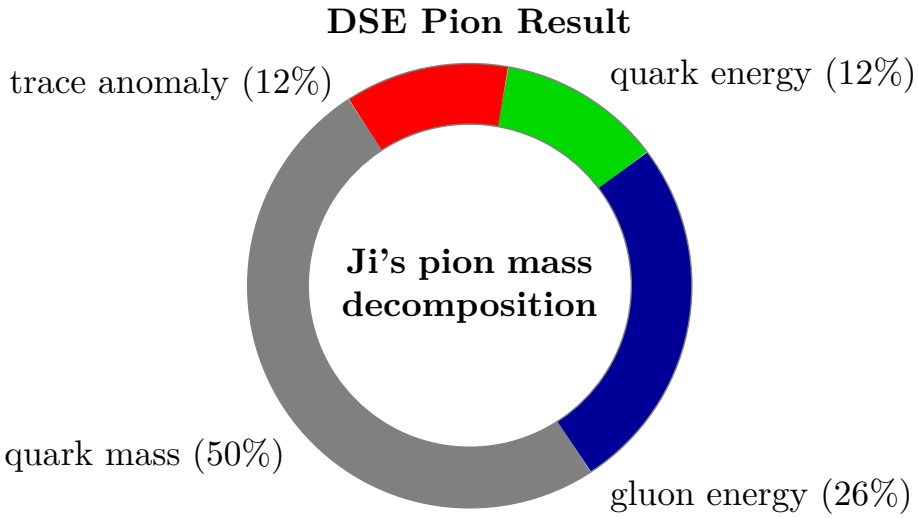
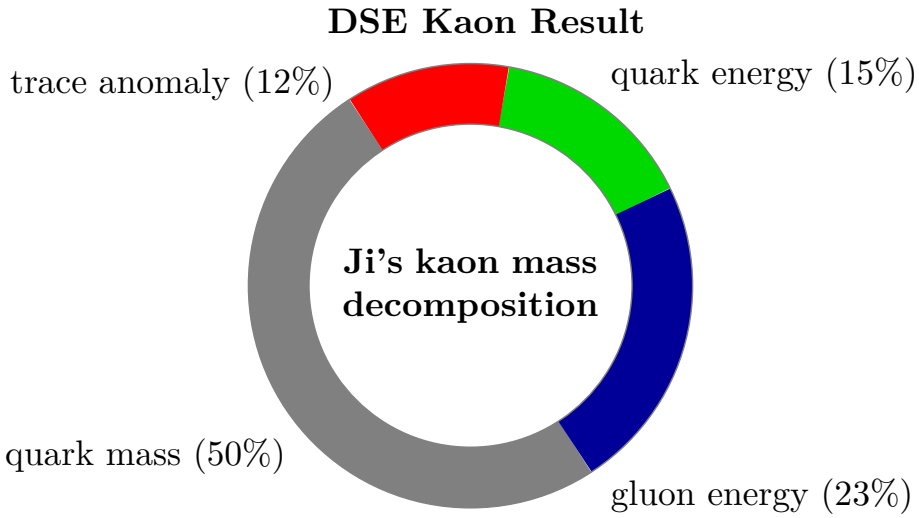


[Sufian, Karpie, Egerer, *et al.*, arXiv:1901.03921 [hep-lat]]



Pion & Kaon Mass Decomposition





- Use the calculated DSE momentum fractions and Ji's mass decomposition to determine a mass budget for the pion & kaon

$$M_q = \tfrac{3}{4}\,(a-b)\,m_\pi, \quad M_g = \tfrac{3}{4}\,(1-a)\,m_\pi, \quad M_m = b\,m_\pi, \quad M_a = \tfrac{1}{4}\,(1-b)\,m_\pi$$

  - a = quark momentum fraction [assumes $\gamma_m = 0$]
  - b related to sigma-term or anomaly contribution: LO χPT $\implies b = \frac{1}{2}$

- In the covariant decomposition and interpretation

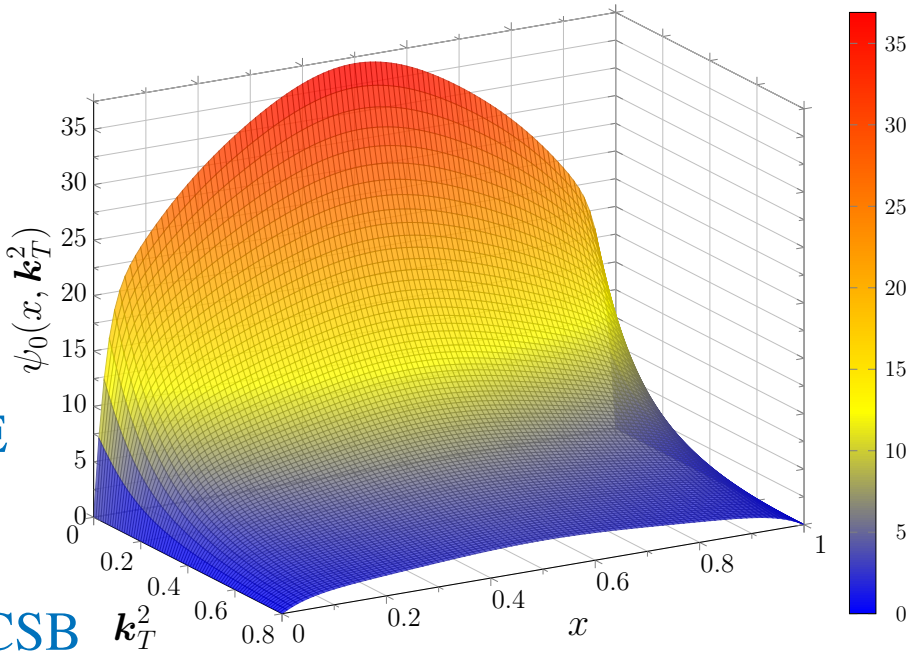
$$\langle \pi \,|T^\mu{}_{\mu;q,g}|\, \pi \rangle = A_\pi^{q,g}(0)\, m_\pi^2 + 4\,\bar{c}_\pi^{q,g}(0)$$

  - in chiral limit expect $\bar{c}_\pi^{q,g}(0) = 0$; therefore $\langle m_\pi^2 \rangle_q = 65\%$, $\langle m_\pi^2 \rangle_g = 35\%$

Conclusion

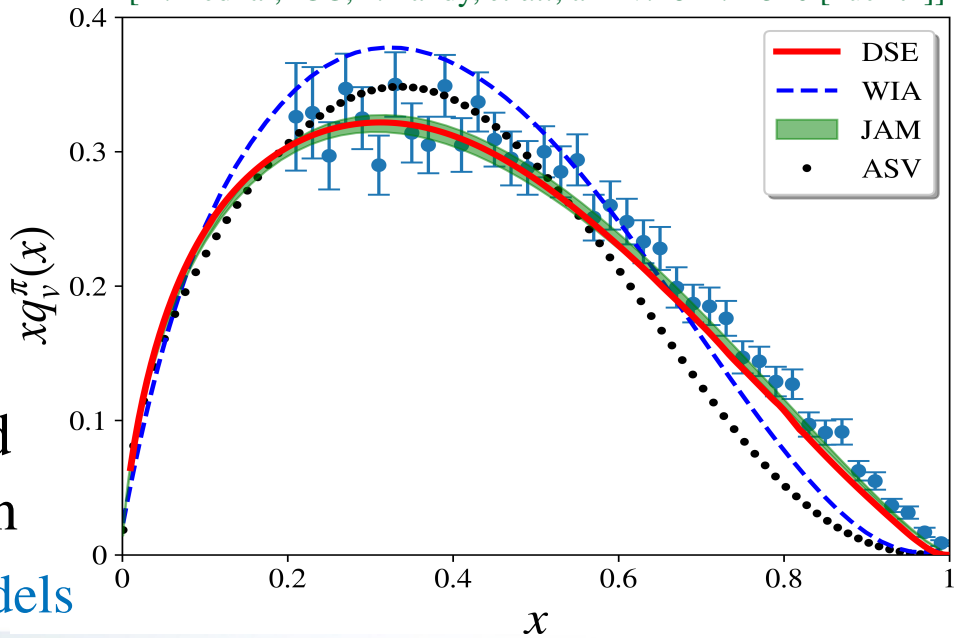
[Chao Shi and ICC, PRL (to appear) arXiv:1806.04799 [nucl-th]]

- Using DSE solutions to the BSE we determined the leading Fock-state LFWFs for the pion and kaon
  - using these LFWFs straightforward to determine FFs, PDFs, TMDs, GPDs, etc
  - key advantage of DSE method is the BSE sums an infinite number of Fock states – therefore LFWFs encapslate effect from emergent phenomena: confinement & DCSB



[K. Bednar, ICC, P. Tandy, *et al.*, arXiv:1811.12310 [nucl-th]]

- For pion and kaon PDFs included for first time gluons self-consistently
  - correct RL-DSE pion PDFs in excellent argeement with Conway *et al.* data and recent JAM analysis
  - agrees with $x \to 1$ pQCD prediction



- Much work remains in experiment and theory to understand the pion and kaon
  - need effort from lattice, pQCD, and models

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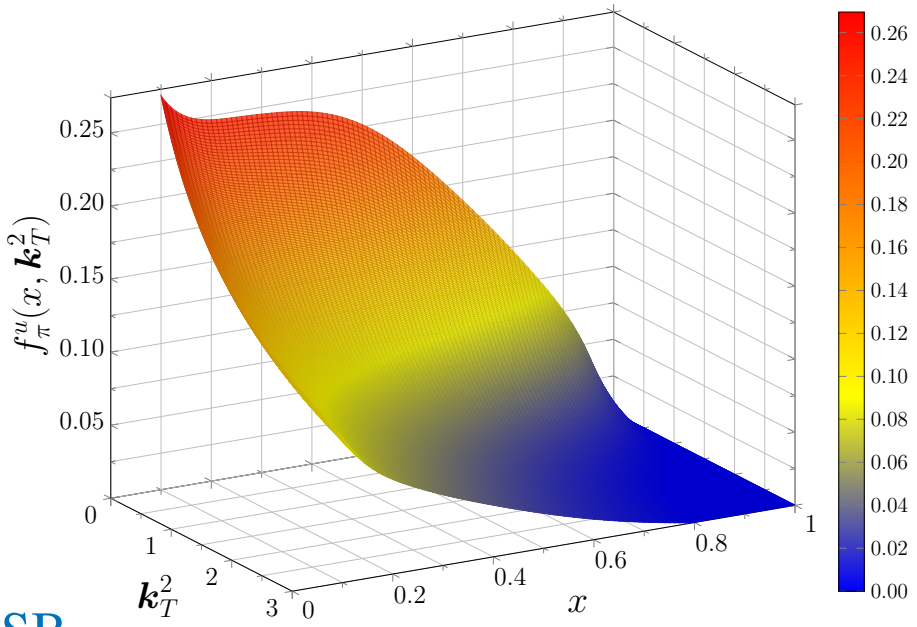
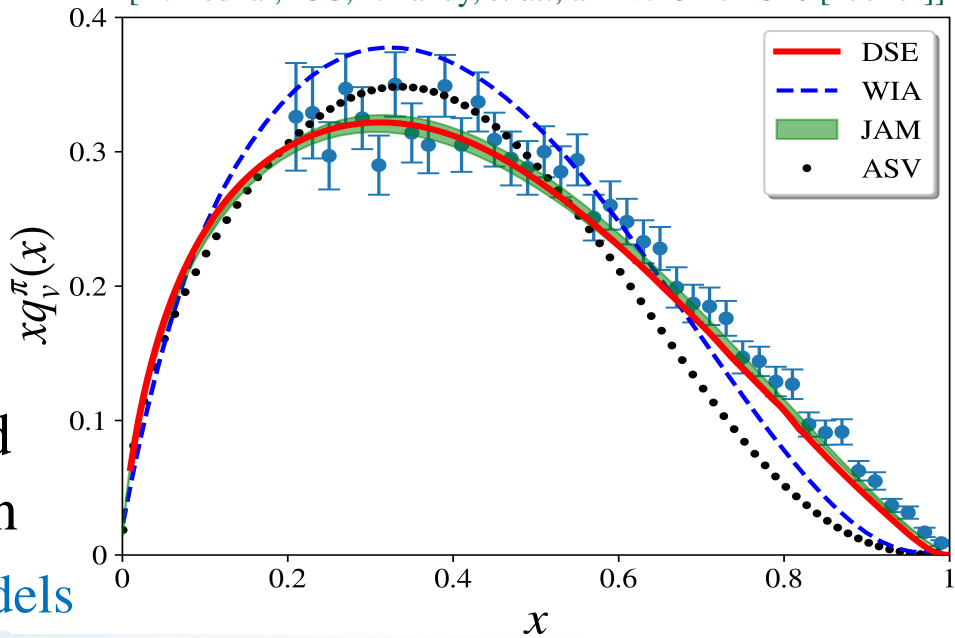
  - using these LFWFs straightforward to determine FFs, PDFs, TMDs, GPDs, etc
  - key advantage of DSE method is the BSE sums an infinite number of Fock states – therefore LFWFs encapslate effect from emergent phenomena: confinement & DCSB



- For pion and kaon PDFs included for first time gluons self-consistently
  - correct RL-DSE pion PDFs in excellent argeement with Conway *et al.* data and recent JAM analysis
  - agrees with $x \to 1$ pQCD prediction

[K. Bednar, ICC, P. Tandy, *et al.*, arXiv:1811.12310 [nucl-th]]



- Much work remains in experiment and theory to understand the pion and kaon
  - need effort from lattice, pQCD, and models