On the gauge invariant quark propagator

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Jefferson Lab
Exploring the Nature of Matter

Outline

- QCD basics
- Gauge invariant quark propagator
- Quark propagator spectral representation
- Conclusions

QCD Basics

$$\mathcal{L}_{\mathrm{QCD}} = \sum_q \bar{\psi}_q^i (i \not{D}^{ij} - \delta^{ij} m_q) \psi_q^j - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}$$

QCD Basics

$$\mathcal{L}_{\text{QCD}} = \sum_q \bar{\psi}_q^i (i\not{D}^{ij} - \delta^{ij} m_q)\psi_q^j - \frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu}$$

$$D_\mu = \partial_\mu + igA_\mu$$

$$G_{\mu\nu} = t^a G^a_{\mu\nu}$$

$$G_{\mu\nu} = t^a \left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f_{abc} A^b_\mu A^c_\nu\right)$$

QCD Basics

- Local gauge transformation:

$$U(x) = \exp\left[i\theta_a(x)t_a\right]$$

- Fields transform as:

$$A_\mu(x) \to U(x)A_\mu(x)U^\dagger(x) - \frac{i}{g}\left((\partial_\mu U^\dagger(x))\right)U(x)$$

- Abelian case:

$$A_\mu(x) \to A_\mu(x) - \frac{1}{g}\partial_\mu\theta(x)$$

QCD Basics

$$S_{\mathrm{YM}} = -\frac{1}{4}\int \mathrm{d}^4x\, G^a_{\mu\nu} G^{\mu\nu}_a$$

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$$S_{\mathrm{YM}} = \int \mathrm{d}^4x \left[\frac{1}{2} A^a_\mu \left(\partial^2 g^{\mu\nu} - \partial^\mu \partial^\nu\right) A^a_\nu + \frac{g}{2} f^{abc} \left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\right) A^{\mu b} A^{\nu c} - \frac{1}{4} f^{abc} f^{cde} A^a_\mu A^b_\nu A^{\mu c} A^{\nu d}\right]$$

QCD Basics

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Not invertible

QCD Basics

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$$S_{\rm YM} = \int \mathrm{d}^4x \left[\frac{1}{2} A^a_\mu \left(\partial^2 g^{\mu\nu} - \partial^\mu \partial^\nu\right) A^a_\nu + \frac{g}{2} f^{abc} \left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\right) A^{\mu b} A^{\nu c} - \frac{1}{4} f^{abc} f^{cde} A^a_\mu A^b_\nu A^{\mu c} A^{\nu d}\right]$$

Not invertible

- Requires gauge fixing in order to quantize the theory

QCD Basics

$$\partial \cdot A^a = 0 \qquad D^{ab}_{\mu\nu}(p) = \left[-g_{\mu\nu} + (1-\xi)\frac{p_\mu p_\nu}{p^2} \right] \delta^{ab} D(p^2) \qquad \text{(covariant gauges)}$$

$$n \cdot A^a = 0 \quad D^{ab}_{\mu\nu}(p) = \left[-g_{\mu\nu} + \frac{n_\mu p_\nu + n_\nu p_\mu}{n \cdot p} - (n^2 + \xi p^2)\frac{p_\mu p_\nu}{(n \cdot p)^2} \right] \delta^{ab} D(p^2) \qquad \text{(axial gauges)}$$

$$\xi \to 0 \qquad n^2 = 0$$

$$D^{ab}_{\mu\nu}(p) = \left[-g_{\mu\nu} + \frac{n_\mu p_\nu + n_\nu p_\mu}{n \cdot p} \right] \delta^{ab} D(p^2) \qquad \text{(light cone gauge)}$$

⟶ Instead, can build a gauge invariant quantity from the beginning

QCD Basics

$\partial \cdot A^a = 0$ $$D^{ab}_{\mu\nu}(p) = \left[-g_{\mu\nu} + (1-\xi)\,\frac{p_\mu p_\nu}{p^2}\right]\delta^{ab} D(p^2)$$ (covariant gauges)

$n \cdot A^a = 0$ $$D^{ab}_{\mu\nu}(p) = \left[-g_{\mu\nu} + \frac{n_\mu p_\nu + n_\nu p_\mu}{n \cdot p} - (n^2 + \xi p^2)\,\frac{p_\mu p_\nu}{(n \cdot p)^2}\right]\delta^{ab} D(p^2)$$ (axial gauges)

$$\xi \to 0 \qquad n^2 = 0$$

$$D^{ab}_{\mu\nu}(p) = \left[-g_{\mu\nu} + \frac{n_\mu p_\nu + n_\nu p_\mu}{n \cdot p}\right]\delta^{ab} D(p^2)$$ (light cone gauge)

Towards Wilson lines in an Abelian theory

- Quark fields transform as:

$$\psi(x) \to e^{i\theta(x)}\psi(x)$$

$$\overline{\psi}(x) \to \overline{\psi}(x)e^{-i\theta(x)}$$

$$\Rightarrow \quad \overline{\psi}(x)\psi(x) \to \overline{\psi}(x)\psi(x)$$

$$\Rightarrow \quad \overline{\psi}(y)\psi(x) \to \overline{\psi}(y)\psi(x)e^{-i(\theta(y)-\theta(x))}$$

$$\overline{\psi}(y)\psi(x) \quad \to \quad \overline{\psi}(y)\psi(x)e^{-i(\theta(y)-\theta(x))}$$

- In PDFs:

$$\overline{\psi}(0,\omega^-,\mathbf{0_T})\psi(0) \quad \to \quad \overline{\psi}(0,\omega^-,\mathbf{0_T})\psi(0)e^{-i\theta(\omega^-)}$$

- In TMDs:

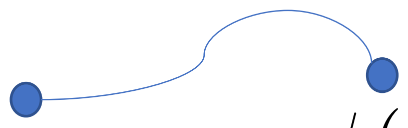
$$\overline{\psi}(0,\omega^-,\omega_\mathbf{T})\psi(0) \quad \to \quad \overline{\psi}(0,\omega^-,\omega_\mathbf{T})\psi(0)e^{-i\theta(\omega^-,\omega_\mathbf{T})}$$

What to do?

Based on T. Rogers notes

We want to compare gauge transformations that we would perform in these two fields at two different points, but with Δx arbitrarily small

- Consider the limit



$$\Delta x^{\mu} = \Delta x \to 0$$

How to make $\overline{\psi}(x+\Delta x)\psi(x)$ gauge invariant?

$$\theta(x+\Delta x) = \theta(x) + \Delta x^{\mu}\partial_{\mu}\theta(x)$$

Now we define

$$A_{\mu}(x) \to A_{\mu}(x) - \frac{1}{g}\partial_{\mu}\theta(x)$$

$$L_1 = e^{-i\Delta x_{\mu}A^{\mu}(x)g}$$

- Can verify that

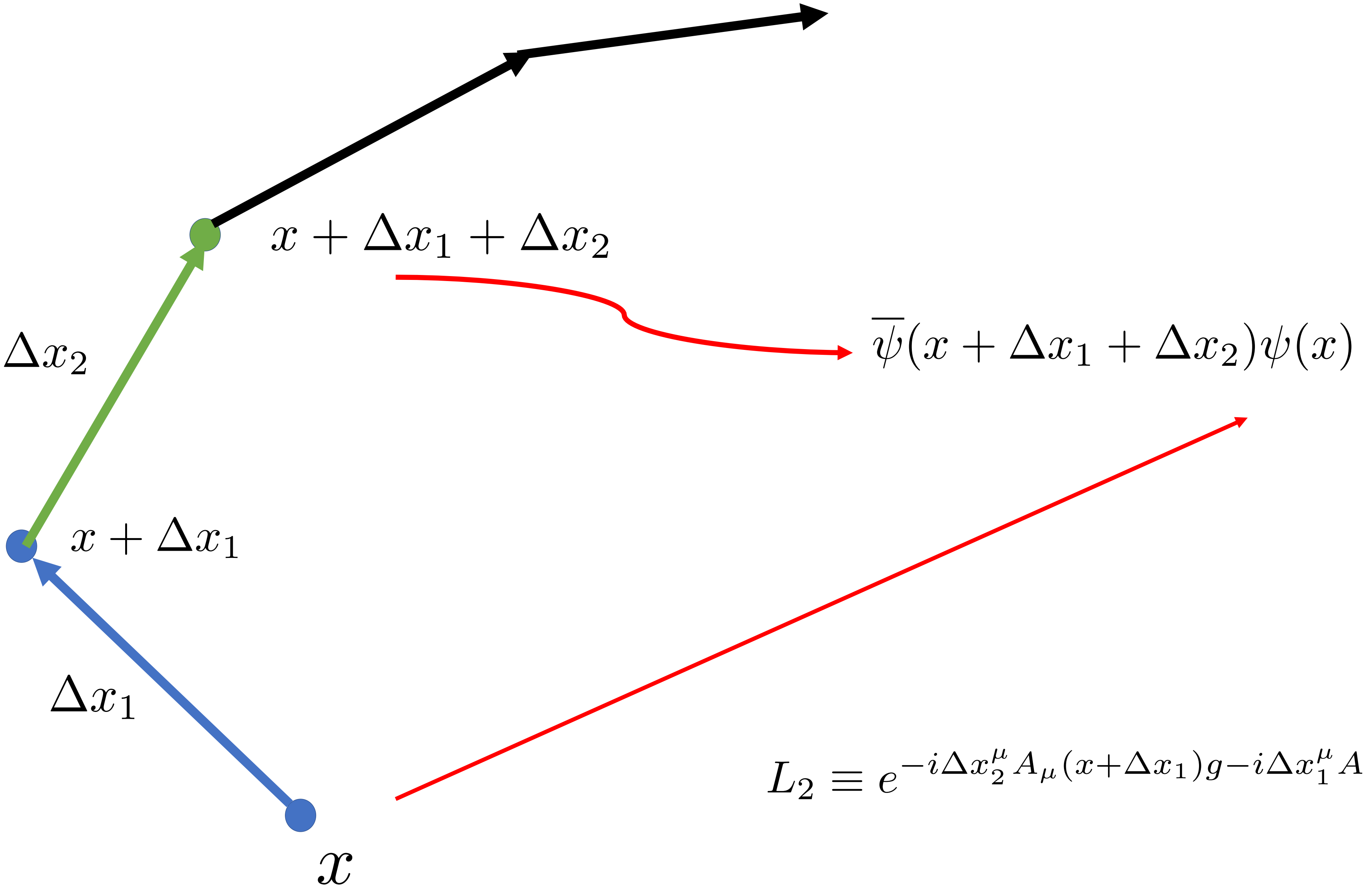
$$L_1 \quad \rightarrow \quad e^{-i\Delta x_\mu A^\mu(x) g + i\Delta x_\mu \partial^\mu \theta(x)}$$

$$\begin{aligned}
L_1 \quad &\rightarrow \quad e^{-i\Delta x_\mu A^\mu(x) g + i\Delta x_\mu \partial^\mu \theta(x) + i\theta(x) - i\theta(x)} \\
&\rightarrow \quad e^{-i\Delta x_\mu A^\mu(x) g - i\theta(x) + i\Delta x_\mu \partial^\mu \theta(x) + i\theta(x)} \\
&\rightarrow \quad e^{-i\Delta x_\mu A^\mu(x) g - i\theta(x) + \overbrace{i\Delta x_\mu \partial^\mu \theta(x) + i\theta(x)}^{= i\theta(x + \Delta(x))}} \\
&\rightarrow \quad e^{i\theta(x+\Delta(x))} \, L_1 \, e^{-i\theta(x)}
\end{aligned}$$

$$\overline{\psi}(x + \Delta x) L_1 \psi(x) \quad \rightarrow \quad \overline{\psi}(x + \Delta x) e^{-i\theta(x+\Delta x)} e^{i\theta(x+\Delta(x))} L_1 e^{-i\theta(x)} e^{i\theta(x)} \psi(x)$$

$$\overline{\psi}(x + \Delta x) L_1 \psi(x) \quad \rightarrow \quad \overline{\psi}(x + \Delta x) \, \overbrace{e^{-i\theta(x+\Delta x)} e^{i\theta(x+\Delta(x))}}^{=1} \, L_1 \, \overbrace{e^{-i\theta(x)} e^{i\theta(x)}}^{=1} \, \psi(x)$$

- Build up a path of infinitesimal steps:



$$L_2 \equiv e^{-i\Delta x_2^\mu A_\mu(x+\Delta x_1)g - i\Delta x_1^\mu A_\mu(x)g}$$

- Can verify that

$$L_2 \quad \rightarrow \quad e^{-i\Delta x_2^\mu A_\mu(x+\Delta x_1)g+i\Delta x_2^\mu \partial_\mu \theta(x+\Delta x_1)-i\Delta x_1^\mu A_\mu(x)+i\Delta x_1^\mu \partial^\mu \theta(x)}$$

$$\rightarrow \quad e^{-i\Delta x_2^\mu A_\mu(x+\Delta x_1)g+i\Delta x_2^\mu \partial_\mu \theta(x+\Delta x_1)g-i\Delta x_1^\mu A_\mu(x)g+i\Delta x_1^\mu \partial^\mu \theta(x)+i\theta(x)-i\theta(x)}$$

$$\rightarrow \quad e^{-i\Delta x_2^\mu A_\mu(x+\Delta x_1)g-i\Delta x_1^\mu A_\mu(x)g-i\theta(x)+\underbrace{i\Delta x_2^\mu \partial_\mu \theta(x+\Delta x_1)g+\overbrace{i\Delta x_1^\mu \partial_\mu \theta(x)+i\theta(x)}^{=i\theta(x+\Delta x_1)}}_{i\theta(x+\Delta x_1+\Delta x_2)}}$$

$$\rightarrow \quad e^{i\theta(x+\Delta x_1+\Delta x_2)}\, L_2\, e^{-i\theta(x)}$$

such that

$$\overline{\psi}(x+\Delta x_1+\Delta x_2)L_2\psi(x) \quad \rightarrow \quad \overline{\psi}(x+\Delta x_1+\Delta x_2)\, L_2\, \psi(x)$$

is gauge invariant.

Wilson lines

- Need to take the continuum limit of infinitely many infinitesimals Δx's

$$x_j = x_{j-1} + \Delta x_j$$

- Can finally write down the Wilson line

$$W(y, x; C) = \exp\left(\lim_{\substack{\Delta x \to 0 \\ N \to \infty}} \sum_{j=1}^{N} i \Delta x_j^\mu A_\mu(x_{j\ \ 1}) g \right) = \mathcal{P} \exp\left(ig \int_C dz^\mu A_\mu(z) \right)$$

Order of the fields matter: first fields on the path are written leftmost

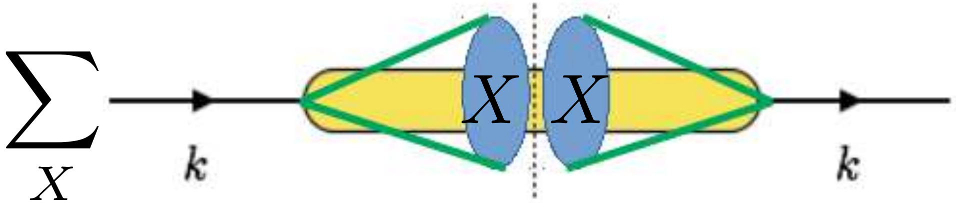
$$\Rightarrow \quad \boxed{\overline{\psi}(y) W(y, x; C) \psi(x)} \quad \Rightarrow$$

Gauge invariant

Gauge invariant quark propagator

$$\Xi_{ij}(k;w) = \text{Disc} \int \frac{d^4\xi}{(2\pi)^4} e^{\mathrm{i}k\cdot\xi} \, \frac{\mathrm{Tr}_c}{N_c} \, \langle\Omega|\big[\,\mathcal{T}\, W_1(\infty,\xi;w)\psi_i(\xi)\big]\,\big[\,\overline{\mathcal{T}}\,\overline{\psi}_j(0) W_2(0,\infty;w)\big]|\Omega\rangle$$

- Hadronization of a quark into an unobserved jet of particles (fully inclusive)



$$\Xi_{ij}(k;n_+) = \text{Disc} \int \frac{d^4\xi}{(2\pi)^4} e^{\mathrm{i}k\cdot\xi} \, \frac{\mathrm{Tr}_c}{N_c} \, \langle\Omega|\psi_i(\xi)\overline{\psi}_j(0) W(0,\xi;n_+)|\Omega\rangle$$

- Gauge invariant generalization of the fully dressed quark propagator

Gauge invariant quark propagator

- Can be given a convolution representation

$$\Xi_{ij}(k;w) = \mathrm{Disc}\int d^4p\,\frac{\mathrm{Tr}_c}{N_c}\,\langle\Omega|i\widetilde{S}_{ij}(p;v)\widetilde{W}(k-p;w,v)|\Omega\rangle$$

where

$$i\widetilde{S}_{ij}(p,v) = \int \frac{d^4\xi}{(2\pi)^4}\,e^{i\xi\cdot p}\,\mathcal{T}\,\psi_i(\xi)\overline{\psi}_j(0)$$

$$\widetilde{W}(k-p;\,w,v) = \int \frac{d^4\xi}{(2\pi)^4}\,e^{i\xi\cdot(k-p)}\,W(0,\xi;w,v)$$

Gauge invariant quark propagator

- Can be given a convolution representation

$$\Xi_{ij}(k;w) = \mathrm{Disc} \int d^4p \frac{\mathrm{Tr}_c}{N_c} \langle\Omega| i\widetilde{S}_{ij}(p;v)\widetilde{W}(k-p;w,v)|\Omega\rangle$$

- Decomposition of the quark bilinear operator

(axial gauges)

$$i\widetilde{S}_{ij}(p,v) = \hat{s}_3(p^2, p\cdot v)\not{p}_{ij} + \sqrt{p^2}\,\hat{s}_1(p^2, p\cdot v)\,\mathbb{I}_{ij} + \hat{s}_0(p^2, p\cdot v)\not{v}_{ij}$$

Gauge invariant quark propagator

- Can be given a convolution representation

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- Decomposition of the quark bilinear operator

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(lightlike axial gauges) $\hat{s}_3(p^2)$ $\hat{s}_1(p^2)$ $\dfrac{p^2}{p\cdot v}\hat{s}_0(p^2)$

$\hat{s}_3(p^2)$ $\hat{s}_1(p^2)$ $\hat{s}_0(p^2)$: spectral operators

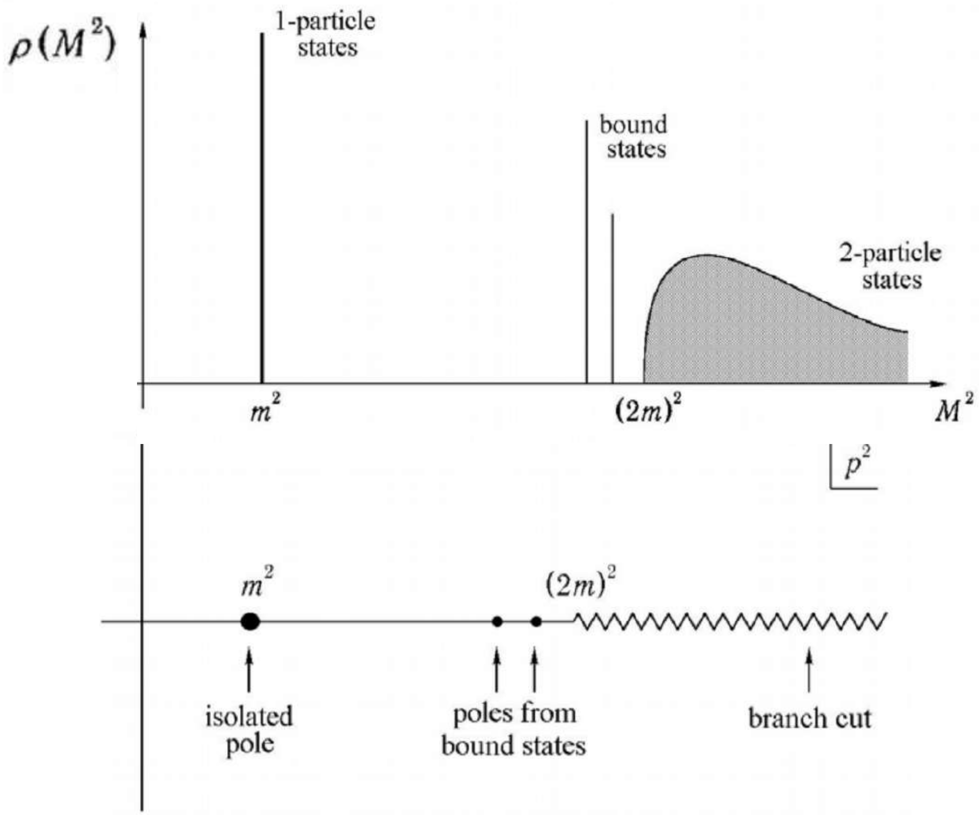
Spectral representation of the quark propagator in the lcg

$$\frac{\mathrm{Tr}_c}{N_c}\langle\Omega|i\tilde{S}(p)|\Omega\rangle = \frac{1}{(2\pi)^4}\int_{-\infty}^{\infty} d\sigma^2 \rho(\sigma^2)\frac{i}{p^2-\sigma^2+i\epsilon}\theta(\sigma^2)$$

$$\rho(p^2) = \rho_3(p^2)\not{p} + \sqrt{p^2}\rho_1(p^2) + \frac{p^2}{p\cdot v}\rho_0(p^2)\not{v}$$

$$\mathrm{Disc}\frac{\mathrm{Tr}_c}{N_c}\langle\Omega|i\tilde{S}(p)|\Omega\rangle = \frac{1}{(2\pi)^3}\rho(p^2)\theta(p^2)\theta(p^-)$$

$$\mathrm{Disc}\frac{\mathrm{Tr}_c}{N_c}\langle\Omega|\hat{s}_{3,1,0}(p,v)|\Omega\rangle = \frac{1}{(2\pi)^3}\rho_{3,1,0}(p^2)\theta(p^2)\theta(p\quad)$$



Integrated g.i. quark propagator

- Boost quark at large light-cone momentum:

$k^- \sim Q$

$$k^- \gg |\mathbf{k}_\perp| \gg k^+$$

$w = n^+$

Integrate out the supressed component of the quark momentum:

$$J_{ij}(k^-, \vec{k}_\perp; n_+) \equiv \frac{1}{2} \int dk^+ \Xi_{ij}(k; n_+)$$

- Generalizes the perturbative quark propagator that appears in in Inclusive and semi-inclusive DIS

$$W_{\text{TMD}}(\xi^+, \boldsymbol{\xi}_\perp) = \mathcal{U}_{n_+}[0^-, 0^+, \mathbf{0}_\perp; 0^-, \infty^+, \mathbf{0}_\perp]\, \mathcal{U}_{\boldsymbol{n}_\perp}[0^-, \infty^+, \mathbf{0}_\perp; 0^-, \infty^+, \boldsymbol{\xi}_\perp]\, \mathcal{U}_{n_+}[0^-, \infty^+, \boldsymbol{\xi}_\perp; 0^-, \xi^+, \boldsymbol{\xi}_\perp]$$

$$W_{\text{coll}}(\xi^+) = \mathcal{U}_{n_+}[0^-, 0^+, \mathbf{0}_\perp; 0^-, \xi^+, \mathbf{0}_\perp]$$

Integrated g.i. quark propagator

- Expand in Dirac structures, in powers of $1/k^-$

$$J(k^-, \boldsymbol{k}_\perp; n_+) = \frac{1}{2}\alpha(k^-)\gamma^+ + \frac{\Lambda}{k^-}\left[\zeta(k^-)\mathbb{I} + \alpha(k^-)\frac{\not{\boldsymbol{k}}_\perp}{\Lambda}\right] + \frac{\Lambda^2}{2(k^-)^2}\omega(k^-, \boldsymbol{k}_\perp^2)\gamma^-$$

$$\alpha(k^-) = J^{[\gamma^-]}$$

$$\zeta(k^-) = \frac{k^-}{\Lambda}J^{[\mathbb{I}]}$$

$$\omega(k^-, \boldsymbol{k}_\perp^2) = \left(\frac{k^-}{\Lambda}\right)^2 J^{[\gamma^+]}$$

Integrated g.i. quark propagator

$$J(k^-, \boldsymbol{k}_\perp; n_+) = \frac{1}{2}\alpha(k^-)\gamma^+ + \frac{\Lambda}{k^-}\left[\zeta(k^-)\mathbb{I} + \alpha(k^-)\frac{\not{\boldsymbol{k}}_\perp}{\Lambda}\right] + \frac{\Lambda^2}{2(k^-)^2}\omega(k^-, \boldsymbol{k}_\perp^2)\gamma^-$$

$$J(k^-, \boldsymbol{k}_T; n_+) = \frac{\theta(k^-)}{4(2\pi)^3\, k^-}\left\{k^-\,\gamma^+ + \not{k}_T + M_j\mathbb{I} + \frac{K_j^2 + \boldsymbol{k}_T^2}{2k^-}\gamma^-\right\}$$

Integrated g.i. quark propagator

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$$\not{k} + m = k^-\,\gamma^+ + \not{k}_\perp + m\mathbb{I} + \frac{m^2 + \boldsymbol{k}_\perp^2}{2k^-}\,\gamma^-$$

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Integrated g.i. quark propagator

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Integrated g.i. quark propagator

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$$J(k^-, \boldsymbol{k}_T; n_+) = \frac{\theta(k^-)}{4(2\pi)^3\, k^-}\left\{k^-\gamma^+ + \not{k}_T + M_j\mathbb{I} + \frac{K_j^2 + \boldsymbol{k}_T^2}{2k^-}\gamma^-\right\}$$

Average mass of all the hadronization products produced during the fragmentation of a quark

- In *__any gauge__*

$$(k\ ) = J^{[\ \ ]} = \frac{\theta(k\ )}{2(2\pi)^3}\int_0^\infty dp^2\,\rho_3(p^2)$$

$$\zeta(k\ ) = \frac{k}{\Lambda}J^{[\,]} = \frac{\theta(k\ )}{2\Lambda(2\pi)^3}\int dp^2\,\sqrt{p^2}\,\rho_1(p^2)$$

$$\omega(k\ ,\boldsymbol{k_T}) = \left(\frac{k}{\Lambda}\right)^2 J^{[\ \ ]} = \frac{\theta(k\ )}{(2\Lambda)^2(2\pi)^3}\ \ \mu_j^2+\tau_j^2+\boldsymbol{k_T^2})$$

$$K_j^2 = \mu_j^2+\tau_j^2$$

Sum rules

- In ***<u>any gauge</u>***

$$1 = \int_0^\infty dp^2 \rho_3(p^2)$$

$$M_j = \int_0^\infty dp^2 \ \sqrt{p^2} \ \rho_1(p^2)$$

$$0 = \int_0^\infty dp^2 \, p^2 \, \rho_0(p^2)$$

- In any gauge,

$$M_j = \int dp^2 \, \sqrt{p^2} \, \rho_1(p^2)$$

Gauge invariant generalization of the gauge dependent dressed quark mass

(l.c.g)

$$K_j^2 = \mu_j^2 + \cancel{\tau_j^2} = \int_0^\infty dp^2 \; p^2 \; \rho_3^{\mathrm{lcg}}(p^2)$$

Invariant mass of the particles produced in the quark's fragmentation process

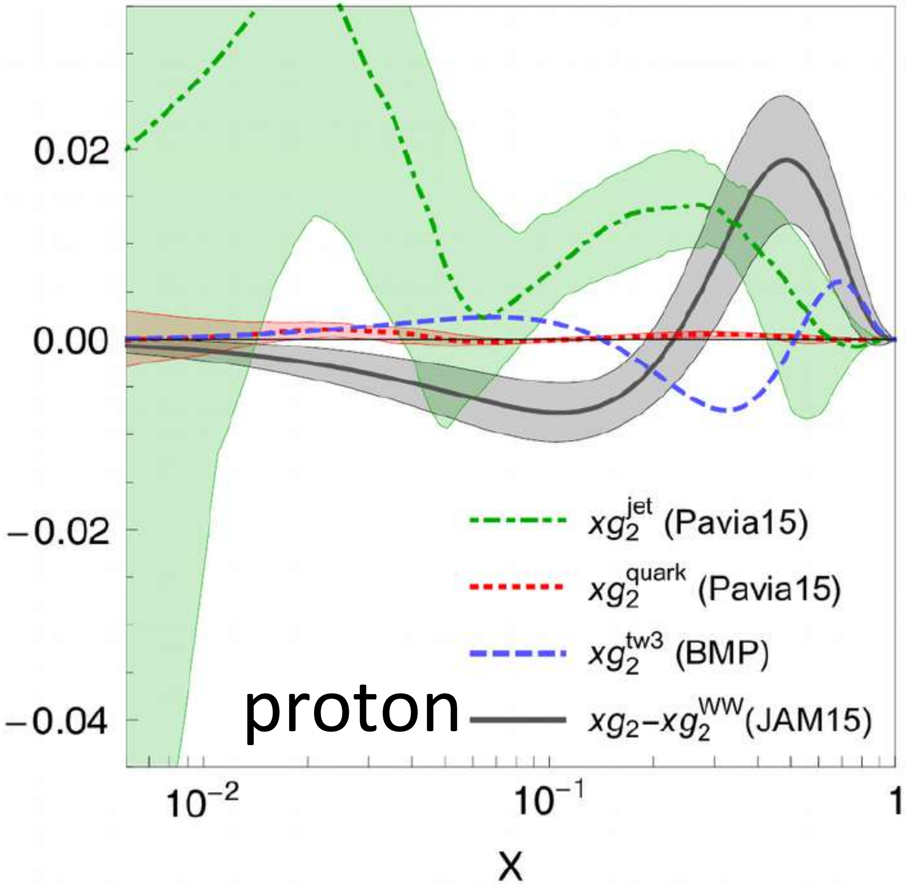
- In other gauges,

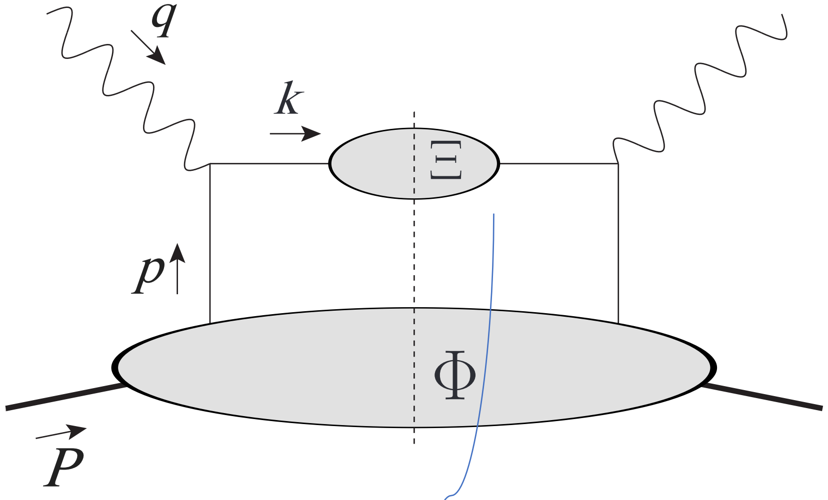
$$K_j^2 = \mu_j^2 + \tau_j^2$$

Final state interactions

$$\tau_j^2 = (2\pi)^3 \int_0^\infty dp^2 \, \mathrm{Disc} \frac{\mathrm{Tr_c}}{\mathrm{N_c}} \langle\Omega|\hat{\sigma}_3(p^2) ig \, \boldsymbol{D}_\perp \left(\boldsymbol{A}^\perp(\boldsymbol{\xi}_\perp) + \mathcal{Z}^\perp(\boldsymbol{\xi}_\perp)\right)_{\boldsymbol{\xi}_\perp = 0} |\Omega\rangle$$

$$\mathcal{Z}^\perp(\boldsymbol{\xi}_\perp) = \int_0^{\infty^+} ds^+ \boldsymbol{D}_\perp \Big(U_{n_+}[0^-, 0^+, \boldsymbol{\xi}_\perp; 0^-, s^+, \boldsymbol{\xi}_\perp] G^{\perp -}(0^-, s^+, \boldsymbol{\xi}_\perp) U_{n^+}[0^-, s^+, \boldsymbol{\xi}_\perp; 0^-, \infty^+, \boldsymbol{\xi}_\perp] \Big) |\Omega\rangle$$





dynamically generated mass: nonvanishing even when m_q =0

g_2^{quark}

$$g_2(x_B) - g_2^{WW}(x_B) = \frac{1}{2}\sum_a e_a^2 \left(g_2^{tw-3}(x_B) + \frac{m_q}{M}\left(\frac{h_1^q}{x}\right)^{\star}(x_B) + \frac{M_q - m_q}{M}\frac{h_1^q(x_B)}{x_B} \right)$$

AA, Bacchetta, AA (2017)

- M_j provides a gauge invariant generalization of the gauge dependent dressed quark mass
- Non-vanishing even in the chiral limit
- Provides a direct way to probe dynamical mass generation
- Not only of theoretical interest..
- It’s calculable, but moreover.. It can be measured!
- Needed: Numerical checks of the sum rules

Snowmass 2021 White Paper
Upgrading SuperKEKB with a Polarized Electron Beam: Discovery Potential and Proposed Implementation

April 13, 2022

US Belle II Group [1]
and
Belle II/SuperKEKB e- Polarization Upgrade Working Group [2]

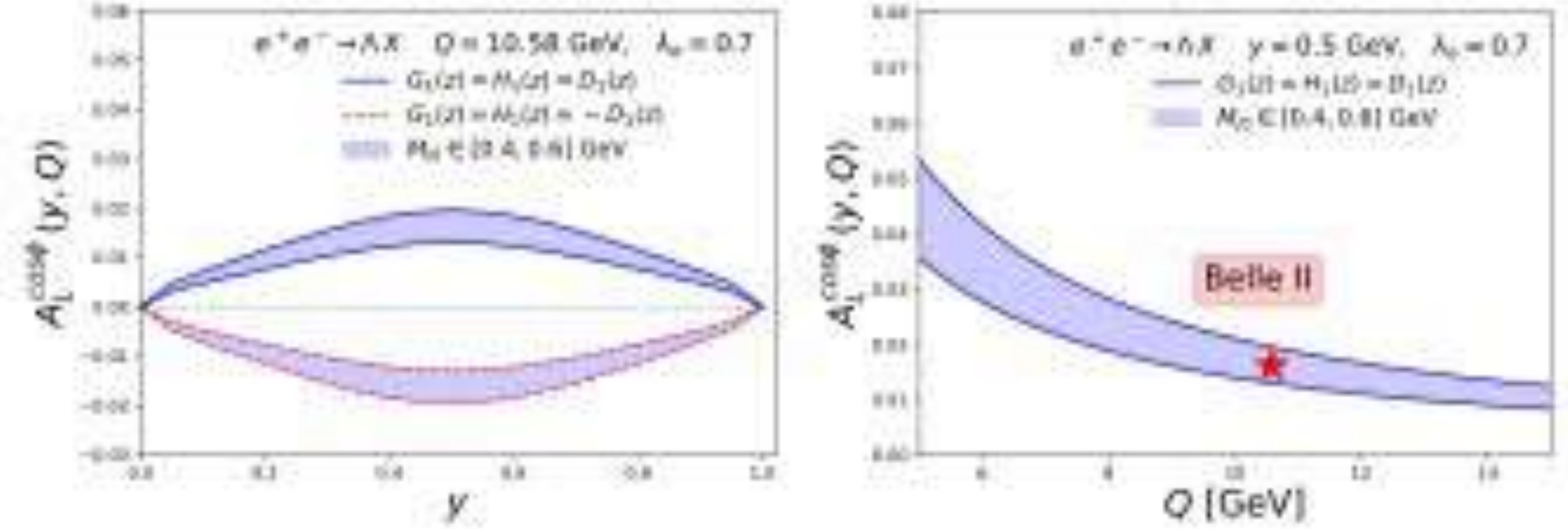


Figure 13: The Fourier components $A_L^1(y)$ and $A_L^{\cos\phi}(y, Q)$ of the longitudinal electron spin asymmetry as a function of y at the SuperKEKB nominal energy $Q = 10.58$ GeV. The band in the $\cos\phi$ modulation indicates the sensitivity of the measurement to $\pm 20\%$ variation in the jet mass at the initial scale. The rightmost panel shows the $A_L^{\cos\phi}$ modulation as a function of Q at fixed $y = 0.5$, along with its 20% sensitivity to M_j, which also slightly increases at lower energies due to QCD evolution.