Spectral analysis of the gauge invariant quark propagator



Caroline S. R. Costa

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Based on: arXiv 2307.10152 In collaboration with: Alberto Accardi (Jlab) Andrea Signori (Università degli Studi di Torino)



Outline

Introduction

Gauge invariant quark propagator

Quark propagator spectral representation

Conclusions

Confinement: Quarks

and gluons are not

asymptotic states of

QCD: are confined

inside hadrons

Confinement: Quarks

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QCD: are confined

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DCSB: Mass

generation

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These QCD features are intimately related to *hadronization*

How color neutral and massive hadron emerge out of colored and massless quarks and gluons?



- These QCD features are intimately related to hadronization
- How color neutral and massive hadron emerge out of colored and massless quarks and gluons?





 e^+

 e^{-}

Ι



AA, Bacchetta (2017)



AA, Bacchetta (2017)

 $\Xi_{ij}(k;w) = \operatorname{Disc} \int \frac{d^4\xi}{(2\pi)^4} e^{ik\cdot\xi} \frac{\operatorname{Tr}_c}{N_c} \langle \Omega | \left[\mathcal{T} W_1(\infty,\xi;w)\psi_i(\xi) \right] \left[\overline{\mathcal{T}} \overline{\psi}_j(0) W_2(0,\infty;w) \right] | \Omega \rangle$

$$\Xi_{ij}(k;w) = \operatorname{Disc} \int \frac{d^4\xi}{(2\pi)^4} e^{\mathrm{i}k\cdot\xi} \frac{\operatorname{Tr}_c}{N_c} \langle \Omega | \left[\mathcal{T} W_1(\infty,\xi;w)\psi_i(\xi) \right] \left[\overline{\mathcal{T}} \overline{\psi}_j(0) W_2(0,\infty;w) \right] | \Omega \rangle$$

 Hadronization of a quark into an unobserved jet of particles (fully inclusive)



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 Hadronization of a quark into an unobserved jet of particles (fully inclusive)



$$\Xi_{ij}(k;n_+) = \text{Disc} \int \frac{d^4\xi}{(2\pi)^4} e^{ik\cdot\xi} \frac{\text{Tr}_c}{N_c} \langle \Omega | \psi_i(\xi) \overline{\psi}_j(0) W(0,\xi;n_+) | \Omega \rangle$$

Gauge invariant generalization of the fully dressed quark propagator



Can be given a convolution representation

$$\Xi_{ij}(k;w) = \operatorname{Disc} \int d^4 p \frac{\operatorname{Tr}_c}{N_c} \langle \Omega | i \widetilde{S}_{ij}(p;v) \widetilde{W}(k-p;w,v) | \Omega \rangle$$

where

$$i\widetilde{S}_{ij}(p,v) = \int \frac{d^4\xi}{(2\pi)^4} e^{i\xi \cdot p} \mathcal{T} \psi_i(\xi) \overline{\psi}_j(0)$$
$$\widetilde{W}(k-p;w,v) = \int \frac{d^4\xi}{(2\pi)^4} e^{i\xi \cdot (k-p)} W(0,\xi;w,v)$$

Can be given a convolution representation

$$\Xi_{ij}(k;w) = \operatorname{Disc} \int d^4 p \frac{\operatorname{Tr}_c}{N_c} \langle \Omega | i \widetilde{S}_{ij}(p;v) \widetilde{W}(k-p;w,v) | \Omega \rangle$$

Decomposition of the quark bilinear operator

(axial gauges)

$$i\tilde{S}_{ij}(p,v) = \hat{s}_3(p^2, p \cdot v) \not\!\!\!p_{ij} + \sqrt{p^2} \hat{s}_1(p^2, p \cdot v) \,\mathbb{I}_{ij} + \hat{s}_0(p^2, p \cdot v) \not\!\!\!p_{ij}$$

Can be given a convolution representation

$$\Xi_{ij}(k;w) = \text{Disc} \int d^4 p \frac{\text{Tr}_c}{N_c} \langle \Omega | i \widetilde{S}_{ij}(p;v) \widetilde{W}(k-p;w,v) | \Omega \rangle$$

Decomposition of the quark bilinear operator

(axial gauges)

$$i\widetilde{S}_{ij}(p,v) = \hat{s}_3(p^2, p \cdot v) \not p_{ij} + \sqrt{p^2} \hat{s}_1(p^2, p \cdot v) \mathbb{I}_{ij} + \hat{s}_0(p^2, p \cdot v) \not \psi_{ij}$$
(lightlike axial gauges) $\hat{s}_3(p^2)$ $\hat{s}_1(p^2)$ $\hat{s}_1(p^2)$

 $\hat{s}_3(p^2) \,\, \hat{s}_1(p^2) \,\, \hat{s}_0(p^2)\,$: spectral operators







$$\frac{\operatorname{Tr}_{c}}{N_{c}} \langle \Omega | i \tilde{S}(p) | \Omega \rangle = \frac{1}{(2\pi)^{4}} \int_{-\infty}^{\infty} d\sigma^{2} \rho(\sigma^{2}) \frac{i}{p^{2} - \sigma^{2} + i\epsilon} \theta(\sigma^{2})$$

$$\rho(p^{2}) = \rho_{3}(p^{2}) \not p + \sqrt{p^{2}} \rho_{1}(p^{2}) + \frac{p^{2}}{p \cdot v} \rho_{0}(p^{2}) \not p$$

$$\operatorname{Disc} \frac{\operatorname{Tr}_{c}}{N_{c}} \langle \Omega | i \tilde{S}(p) | \Omega \rangle = \frac{1}{(2\pi)^{3}} \rho(p^{2}) \theta(p^{2}) \theta(p^{-})$$

$$\operatorname{Disc} \frac{\operatorname{Tr}_{c}}{N_{c}} \langle \Omega | \hat{s}_{3,1,0}(p,v) | \Omega \rangle = \frac{1}{(2\pi)^{3}} \rho_{3,1,0}(p^{2}) \theta(p^{2}) \theta(p^{-})$$

$$\frac{i}{i \operatorname{solated}} \frac{i}{i \operatorname{solate}} \frac{i}{i \operatorname{solate}} \frac{i}{i \operatorname{sol$$

Boost quark at large light-cone momentum:

Integrate out the suppressed component of the quark momentum:

$$k^{-} \gg |\mathbf{k}_{\perp}| \gg k^{+}$$

$$w = n^{+}$$

$$J_{ij}(k^{-}, \vec{k}_{\perp}; n_{+}) \equiv \frac{1}{2} \int dk^{+} \Xi_{ij}(k; n_{+})$$

Generalizes the perturbative quark propagator that appears in inclusive and semi-inclusive DIS



 $W_{\rm TMD}(\xi^+, \xi_{\perp}) = \mathcal{U}_{n_+}[0^-, 0^+, \mathbf{0}_{\perp}; 0^-, \infty^+, \mathbf{0}_{\perp}] \mathcal{U}_{n_{\perp}}[0^-, \infty^+, \mathbf{0}_{\perp}; 0^-, \infty^+, \xi_{\perp}] \mathcal{U}_{n_+}[0^-, \infty^+, \xi_{\perp}; 0^-, \xi^+, \xi_{\perp}]$

$$W_{\text{coll}}(\xi^+) = \mathcal{U}_{n_+}[0^-, 0^+, \mathbf{0}_{\perp}; 0^-, \xi^+, \mathbf{0}_{\perp}]$$

$$J(k^{-}, \mathbf{k}_{\perp}; n_{+}) = \frac{1}{2} \alpha(k^{-}) \gamma^{+} + \frac{\Lambda}{k^{-}} \left[\zeta(k^{-}) \mathbb{I} + \alpha(k^{-}) \frac{\mathbf{k}_{\perp}}{\Lambda} \right] + \frac{\Lambda^{2}}{2(k^{-})^{2}} \omega(k^{-}, \mathbf{k}_{\perp}^{2}) \gamma^{-}$$

$$J(k^{-}, \boldsymbol{k}_{\perp}; n_{+}) = \frac{1}{2} \alpha(k^{-}) \gamma^{+} + \frac{\Lambda}{k^{-}} \left[\zeta(k^{-}) \mathbb{I} + \alpha(k^{-}) \frac{\boldsymbol{k}_{\perp}}{\Lambda} \right] + \frac{\Lambda^{2}}{2(k^{-})^{2}} \omega(k^{-}, \boldsymbol{k}_{\perp}^{2}) \gamma^{-}$$
$$\alpha(k^{-}) = J^{[\gamma^{-}]}$$
$$\zeta(k^{-}) = \frac{k^{-}}{\Lambda} J^{[\mathbb{I}]}$$
$$\omega(k^{-}, \boldsymbol{k}_{\perp}^{2}) = \left(\frac{k^{-}}{\Lambda}\right)^{2} J^{[\gamma^{+}]}$$

$$J(k^-, \boldsymbol{k}_\perp; n_+) = \frac{1}{2}\alpha(k^-)\gamma^+ + \frac{\Lambda}{k^-} \left[\zeta(k^-)\mathbb{I} + \alpha(k^-)\frac{\boldsymbol{k}_\perp}{\Lambda}\right] + \frac{\Lambda^2}{2(k^-)^2}\omega(k^-, \boldsymbol{k}_\perp^2)\gamma^-$$

$$J(k^{-}, \boldsymbol{k}_{T}; n_{+}) = \frac{\theta(k^{-})}{4(2\pi)^{3} k^{-}} \left\{ k^{-} \gamma^{+} + \boldsymbol{k}_{T} + M_{j} \mathbb{I} + \frac{K_{j}^{2} + \boldsymbol{k}_{T}^{2}}{2k^{-}} \gamma^{-} \right\}$$

$$J(k^{-}, \mathbf{k}_{\perp}; n_{+}) = \frac{1}{2}\alpha(k^{-})\gamma^{+} + \frac{\Lambda}{k^{-}} \left[\zeta(k^{-})\mathbb{I} + \alpha(k^{-})\frac{\mathbf{k}_{\perp}}{\Lambda}\right] + \frac{\Lambda^{2}}{2(k^{-})^{2}}\omega(k^{-}, \mathbf{k}_{\perp}^{2})\gamma^{-}$$
$$\mathbf{k}_{\perp} + m\mathbf{k}_{\perp} + m\mathbf{k}_{\perp} + m\mathbf{k}_{\perp} + m\mathbf{k}_{\perp} + \frac{m^{2} + \mathbf{k}_{\perp}^{2}}{2k^{-}}\gamma^{-}$$

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$$J(k^{-}, \mathbf{k}_{\perp}; n_{+}) = \frac{1}{2}\alpha(k^{-})\gamma^{+} + \frac{\Lambda}{k^{-}} \left[\zeta(k^{-})\mathbb{I} + \alpha(k^{-})\frac{\mathbf{k}_{\perp}}{\Lambda} \right] + \frac{\Lambda^{2}}{2(k^{-})^{2}}\omega(k^{-}, \mathbf{k}_{\perp}^{2})\gamma^{-} \\ \mathbf{k} + m = k^{-}\gamma^{+} + \mathbf{k}_{\perp} + m\mathbb{I} + \frac{m^{2} + \mathbf{k}_{\perp}^{2}}{2k^{-}}\gamma^{-} \\ J(k^{-}, \mathbf{k}_{T}; n_{+}) = \frac{\theta(k^{-})}{4(2\pi)^{3}k^{-}} \left\{ k^{-}\gamma^{+} + \mathbf{k}_{T} + M_{j}\mathbb{I} + \frac{K_{j}^{2} + k_{T}^{2}}{2k^{-}}\gamma^{-} \right\} \\ \text{Average mass of all the} \\ \text{hadronization products} \\ \text{produced during the} \\ \text{fragmentation of a quark} \end{cases}$$



$$(k) = J^{[]} = \frac{\theta(k)}{2(2\pi)^3} \int_0^\infty dp^2 \rho_3(p^2)$$

☐ In *any gauge*:

$$(k) = J^{[]} = \frac{\theta(k)}{2(2\pi)^3} \int_0^\infty dp^2 \rho_3(p^2)$$
$$\zeta(k) = \frac{k}{\Lambda} J^{[]} = \frac{\theta(k)}{2\Lambda(2\pi)^3} \int dp^2 \sqrt{p^2} \rho_1(p^2)$$

☐ In *any gauge*:

$$(k_{-}) = J^{[-]} = \frac{\theta(k_{-})}{2(2\pi)^{3}} \int_{0}^{\infty} dp^{2} \rho_{3}(p^{2})$$
$$\zeta(k_{-}) = \frac{k_{-}}{\Lambda} J^{[]} = \frac{\theta(k_{-})}{2\Lambda(2\pi)^{3}} \int dp^{2} \sqrt{p^{2}} \rho_{1}(p^{2})$$
$$\omega(k_{-}, \mathbf{k_{T}}) = \left(\frac{k_{-}}{\Lambda}\right)^{2} J^{[-]} = \frac{\theta(k_{-})}{(2\Lambda)^{2}(2\pi)^{3}} \left[\mu_{j}^{2} + \tau_{j}^{2} + \mathbf{k_{T}}^{2}\right]$$

In *any gauge*:

$$(k_{-}) = J^{[-]} = \frac{\theta(k_{-})}{2(2\pi)^{3}} \int_{0}^{\infty} dp^{2} \rho_{3}(p^{2})$$

$$\zeta(k_{-}) = \frac{k_{-}}{\Lambda} J^{[]} = \frac{\theta(k_{-})}{2\Lambda(2\pi)^{3}} \int dp^{2} \sqrt{p^{2}} \rho_{1}(p^{2})$$

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Sum rules

In *any gauge*:

$$1 = \int_{0}^{\infty} dp^{2} \rho_{3}(p^{2})$$
$$M_{j} = \int_{0}^{\infty} dp^{2} \sqrt{p^{2}} \rho_{1}(p^{2})$$
$$0 = \int_{0}^{\infty} dp^{2} p^{2} \rho_{0}(p^{2})$$

Can be used to verify actual calculations of the quark propagator!

Sum rules

🖵 In <u>any gauge:</u>



Can be used to verify actual calculations of the quark propagator!

In <u>any gauge:</u>

 $M_j = \int dp^2 \sqrt{p^2} \rho_1(p^2)$

leading twist		quark operator			
		unpolarized [U]	longitudinal [L]	transverse [T]	
target polarization	U	$f_1 = \bigcirc$ unpolarized		$h_1^\perp = \bigotimes_{\text{Boer-Mulders}} - \bigotimes_{\text{Boer-Mulders}}$	
	L		$g_1 = \longrightarrow - \longleftrightarrow$ helicity	$h_{1L}^{\perp} = \underbrace{\swarrow}_{\text{worm gear 1}} _{\text{worm gear 1}} _{\text{worm gear 1}}$	
	т	$f_{1T}^{\perp} = \underbrace{\bullet}_{\text{Sivers}} - \underbrace{\bullet}_{\text{V}}$	$g_{1T} = \underbrace{\bigstar}_{\text{worm gear } 2} - \underbrace{\bigstar}_{\text{gear } 2}$	$h_{1} = \underbrace{\begin{pmatrix} \bullet \\ \bullet \\ transversity \end{pmatrix}}_{transversity}$ $h_{1T}^{\perp} = \underbrace{\begin{pmatrix} \bullet \\ \bullet \\ pretzelosity \end{pmatrix}}_{pretzelosity}$	
	TENSOR	$egin{aligned} & f_{1LL}(x,m{k}_T^2) \ & f_{1LT}(x,m{k}_T^2) \ & f_{1TT}(x,m{k}_T^2) \ \end{aligned}$	$egin{array}{l} g_{1TT}(x,oldsymbol{k}_T^2)\ g_{1LT}(x,oldsymbol{k}_T^2) \end{array}$	$egin{array}{lll} h_{1LL}^{\perp}(x,m{k}_{T}^{2})\ h_{1TT}, & h_{1TT}^{\perp}\ h_{1LT}, & h_{1LT}^{\perp} \end{array}$	

Gauge invariant generalization of the gauge dependent dressed quark mass

Experimentally accessible in double spin assymetry measurements!

(table from Satvir Kaur's talk yesterday)

In *light-cone gauge*:

$$K_{j}^{2} = \mu_{j}^{2} + \sum_{j}^{2} = \int_{0}^{\infty} dp^{2} p^{2} \rho_{3}^{\log}(p^{2})$$

Final state interactions "vanish"

In *light-cone gauge*:

$$K_{j}^{2} = \mu_{j}^{2} + \sum_{j=1}^{2} = \int_{0}^{\infty} dp^{2} \ p^{2} \ \rho_{3}^{\log}(p^{2})$$

But in other gauges

Final state interactions "vanish"

$$K_j^2 = \mu_j^2 + \tau_j^2$$

$$\tau_j^2 = (2\pi)^3 \int_0^\infty dp^2 \operatorname{Disc} \frac{\operatorname{Tr}_{\mathbf{c}}}{\operatorname{N}_{\mathbf{c}}} \langle \Omega | \hat{\sigma}_3(p^2) ig \, \boldsymbol{D}_\perp \left(\boldsymbol{A}^\perp(\boldsymbol{\xi}_\perp) + \mathcal{Z}^\perp(\boldsymbol{\xi}_\perp) \right)_{\boldsymbol{\xi}_\perp = 0} | \Omega \rangle$$

$$\mathcal{Z}^{\perp}(\boldsymbol{\xi}_{\perp}) = \int_{0}^{\infty^{+}} ds^{+} \boldsymbol{D}_{\perp} \left(U_{n_{+}}[0^{-}, 0^{+}, \boldsymbol{\xi}_{\perp}; 0^{-}, s^{+}, \boldsymbol{\xi}_{\perp}] G^{\perp -}(0^{-}, s^{+}, \boldsymbol{\xi}_{\perp}) U_{n^{+}}[0^{-}, s^{+}, \boldsymbol{\xi}_{\perp}; 0^{-}, \infty^{+}, \boldsymbol{\xi}_{\perp}] \right) |\Omega\rangle$$

In *light-cone gauge*:

$$K_{j}^{2} = \mu_{j}^{2} + \sum_{j=1}^{2} = \int_{0}^{\infty} dp^{2} \ p^{2} \ \rho_{3}^{\log}(p^{2})$$

But in other gauges

Final state interactions "vanish"

 $|\Omega\rangle$

$$K_j^2 = \mu_j^2 + \tau_j^2$$

$$\tau_{j}^{2} = (2\pi)^{3} \int_{0}^{\infty} dp^{2} \operatorname{Disc} \frac{\operatorname{Tr}_{c}}{\operatorname{N}_{c}} \langle \Omega | \hat{\sigma}_{3}(p^{2}) ig \, \boldsymbol{D}_{\perp} \left(\boldsymbol{A}^{\perp}(\boldsymbol{\xi}_{\perp}) + \boldsymbol{\mathcal{Z}}^{\perp}(\boldsymbol{\xi}_{\perp}) \right)_{\boldsymbol{\xi}_{\perp}=0} | \Omega \rangle$$
$$\boldsymbol{\mathcal{Z}}^{\perp}(\boldsymbol{\xi}_{\perp}) = \int_{0}^{\infty^{+}} ds^{+} \boldsymbol{D}_{\perp} \left(U_{n_{+}}[0^{-}, 0^{+}, \boldsymbol{\xi}_{\perp}; 0^{-}, s^{+}, \boldsymbol{\xi}_{\perp}] G^{\perp-}(0^{-}, s^{+}, \boldsymbol{\xi}_{\perp}) U_{n^{+}}[0^{-}, s^{+}, \boldsymbol{\xi}_{\perp}; 0^{-}, \infty^{+}, \boldsymbol{\xi}_{\perp}] \right)$$

Completed the analysis of the gauge invariant quark propagator

Full calculation of the twist-4 coefficient

□ Formal demonstration of the gauge invariance of the twist-2, twist-3 and twist-4

coefficients of the g.i. quark propagator/jet correlator

New sum rules (needed: numerical checks)

Completed the analysis of the gauge invariant quark propagator

Full calculation of the twist-4 coefficient

Formal demonstration of the gauge invariance of the twist-2, twist-3 and twist-4 coefficients of the g.i. quark propagator/jet correlator

> New sum rules for the quark spectral functions (needed: numerical checks)

In particular:

> Second moment of ρ_0 vanishes

 \succ First moment of the chiral odd quark spectral function gives a mass M_i that

is a gauge invariant generalization of the gauge dependent quark mass

 \square M_i color screened gauge invariant mass

- > Non-vanishing even in the chiral limit
- Provides a direct way to probe dynamical chiral symmetry breaking

 \square M_j color screened gauge invariant mass

- > Non-vanishing even in the chiral limit
- Provides a direct way to probe dynamical chiral symmetry breaking (In progress)



 \square M_i color screened gauge invariant mass

> Non-vanishing even in the chiral limit

Provides a direct way to probe dynamical chiral symmetry breaking

It's calculable, but moreover.. It can be measured!

(In progress)

