# Spectral analysis of the gauge invariant quark propagator 



Caroline S. R. Costa

2023, Sept 14th

Based on: arXiv 2307.10152
In collaboration with:
Alberto Accardi (Jlab)
Andrea Signori (Università degli Studi di Torino)

## Introduction

## Gauge invariant quark propagator

Outline
Quark propagator spectral representation

Conclusions
$\square$ Confinement: Quarks
and gluons are not
asymptotic states of
QCD: are confined
inside hadrons
$\square$ Confinement: Quarks
and gluons are not
asymptotic states of
QCD: are confined
inside hadrons
$\square$ DCSB: Mass
generation
$\square$ Confinement: Quarks
and gluons are not
asymptotic states of
QCD: are confined
inside hadrons
$\square$ DCSB: Mass
generation
$\square$ These QCD features are intimately related to hadronization
$\square$ How color neutral and massive hadron emerge out of colored and massless quarks and gluons?
$\square$ Confinement: Quarks and gluons are not asymptotic states of QCD: are confined inside hadrons
$\square$ DCSB: Mass generation

Nonperturbative: Gauge invariant quark propagator/jet correlator

$\square$ These QCD features are intimately related to hadronization
$\square$ How color neutral and massive hadron emerge out of colored and massless quarks and gluons?





## Gauge invariant quark propagator

$$
\Xi_{i j}(k ; w)=\operatorname{Disc} \int \frac{d^{4} \xi}{(2 \pi)^{4}} e^{\mathrm{i} k \cdot \xi} \frac{\operatorname{Tr}_{c}}{N_{c}}\langle\Omega|\left[\mathcal{T} W_{1}(\infty, \xi ; w) \psi_{i}(\xi)\right]\left[\overline{\mathcal{T}} \bar{\psi}_{j}(0) W_{2}(0, \infty ; w)\right]|\Omega\rangle
$$

## Gauge invariant quark propagator

$$
\Xi_{i j}(k ; w)=\operatorname{Disc} \int \frac{d^{4} \xi}{(2 \pi)^{4}} e^{\mathrm{i} k \cdot \xi} \frac{\operatorname{Tr}_{c}}{N_{c}}\langle\Omega|\left[\mathcal{T} W_{1}(\infty, \xi ; w) \psi_{i}(\xi)\right]\left[\overline{\mathcal{T}} \bar{\psi}_{j}(0) W_{2}(0, \infty ; w)\right]|\Omega\rangle
$$

$\square$ Hadronization of a quark into an unobserved jet of particles (fully inclusive)

$q \rightarrow X$ amplitude

## Gauge invariant quark propagator

$$
\Xi_{i j}(k ; w)=\operatorname{Disc} \int \frac{d^{4} \xi}{(2 \pi)^{4}} e^{\mathrm{i} k \cdot \xi} \frac{\operatorname{Tr}_{c}}{N_{c}}\langle\Omega|\left[\mathcal{T} W_{1}(\infty, \xi ; w) \psi_{i}(\xi)\right]\left[\overline{\mathcal{T}} \bar{\psi}_{j}(0) W_{2}(0, \infty ; w)\right]|\Omega\rangle
$$

$\square$ Hadronization of a quark into an unobserved jet of particles
 (fully inclusive)

$$
\Xi_{i j}\left(k ; n_{+}\right)=\operatorname{Disc} \int \frac{d^{4} \xi}{(2 \pi)^{4}} e^{\mathrm{i} k \cdot \xi} \frac{\operatorname{Tr}_{c}}{N_{c}}\langle\Omega| \psi_{i}(\xi) \bar{\psi}_{j}(0) W\left(0, \xi ; n_{+}\right)|\Omega\rangle
$$

$\square$ Gauge invariant generalization of the fully dressed quark propagator


## Gauge invariant quark propagator

$\square$ Can be given a convolution representation

$$
\Xi_{i j}(k ; w)=\operatorname{Disc} \int d^{4} p \frac{\operatorname{Tr}_{c}}{N_{c}}\langle\Omega| i \widetilde{S}_{i j}(p ; v) \widetilde{W}(k-p ; w, v)|\Omega\rangle
$$

where

$$
\begin{aligned}
i \widetilde{S}_{i j}(p, v) & =\int \frac{d^{4} \xi}{(2 \pi)^{4}} e^{i \xi \cdot p} \mathcal{T} \psi_{i}(\xi) \bar{\psi}_{j}(0) \\
\widetilde{W}(k-p ; w, v) & =\int \frac{d^{4} \xi}{(2 \pi)^{4}} e^{i \xi \cdot(k-p)} W(0, \xi ; w, v)
\end{aligned}
$$

## Gauge invariant quark propagator

$\square$ Can be given a convolution representation

$$
\Xi_{i j}(k ; w)=\operatorname{Disc} \int d^{4} p \frac{\operatorname{Tr}_{c}}{N_{c}}\langle\Omega| i \widetilde{S}_{i j}(p ; v) \widetilde{W}(k-p ; w, v)|\Omega\rangle
$$

$\square$ Decomposition of the quark bilinear operator

$$
i \widetilde{S}_{i j}(p, v)=\hat{s}_{3}\left(p^{2}, p \cdot v\right) \not p_{i j}+\sqrt{p^{2}} \hat{s}_{1}\left(p^{2}, p \cdot v\right) \mathbb{I}_{i j}+\hat{s}_{0}\left(p^{2}, p \cdot v\right) \psi_{i j}
$$

## Gauge invariant quark propagator

$\square$ Can be given a convolution representation

$$
\Xi_{i j}(k ; w)=\operatorname{Disc} \int d^{4} p \frac{\operatorname{Tr}_{c}}{N_{c}}\langle\Omega| i \widetilde{S}_{i j}(p ; v) \widetilde{W}(k-p ; w, v)|\Omega\rangle
$$

$\square$ Decomposition of the quark bilinear operator

$$
i \widetilde{S}_{i j}(p, v)=\underbrace{\hat{s}_{3}\left(p^{2}, p \cdot v\right) \not p}_{\hat{s}_{3}\left(p^{2}\right)}+\underbrace{}_{i j}+\underbrace{p_{1}^{2}}_{\hat{s}_{1}\left(p^{2}\right)} \hat{s}_{1}\left(p^{2}, p \cdot v\right) \mathbb{I}_{i j}+\underbrace{\hat{s}_{0}\left(p^{2}, p \cdot v\right)}_{\frac{p^{2}}{p \cdot v} \hat{s}_{0}\left(p^{2}\right)} \psi_{i j}
$$

$$
\hat{s}_{3}\left(p^{2}\right) \hat{s}_{1}\left(p^{2}\right) \hat{s}_{0}\left(p^{2}\right): \text { spectral operators }
$$

## Spectral representation of the quark propagator in the Icg

$$
\frac{\operatorname{Tr}_{\mathrm{c}}}{N_{c}}\langle\Omega| i \tilde{S}(p)|\Omega\rangle=\frac{1}{(2 \pi)^{4}} \int_{-\infty}^{\infty} d \sigma^{2} \rho\left(\sigma^{2}\right) \frac{i}{p^{2}-\sigma^{2}+i \epsilon} \theta\left(\sigma^{2}\right)
$$



## Spectral representation of the quark propagator in the lcg

$$
\frac{\operatorname{Tr}_{\mathrm{c}}}{N_{c}}\langle\Omega| i \tilde{S}(p)|\Omega\rangle=\frac{1}{(2 \pi)^{4}} \int_{-\infty}^{\infty} d \sigma^{2} \rho\left(\sigma^{2}\right) \frac{i}{p^{2}-\sigma^{2}+i \epsilon} \theta\left(\sigma^{2}\right)
$$

## Spectral representation of the quark propagator in the Icg

$$
\begin{gathered}
\frac{\operatorname{Tr}_{\mathrm{c}}}{N_{c}}\langle\Omega| i \tilde{S}(p)|\Omega\rangle=\frac{1}{(2 \pi)^{4}} \int_{-\infty}^{\infty} d \sigma^{2} \rho\left(\sigma^{2}\right) \frac{i}{p^{2}-\sigma^{2}+i \epsilon} \theta\left(\sigma^{2}\right) \\
\rho\left(p^{2}\right)=\rho_{3}\left(p^{2}\right) \not p+\sqrt{p^{2}} \rho_{1}\left(p^{2}\right)+\frac{p^{2}}{p \cdot v} \rho_{0}\left(p^{2}\right) \psi{ }_{\rho\left(m^{2}\right)}
\end{gathered}
$$

## Spectral representation of the quark propagator in the Icg

$$
\begin{gathered}
\frac{\operatorname{Tr}_{\mathrm{c}}}{N_{c}}\langle\Omega| i \tilde{S}(p)|\Omega\rangle=\frac{1}{(2 \pi)^{4}} \int_{-\infty}^{\infty} d \sigma^{2} \rho\left(\sigma^{2}\right) \frac{i}{p^{2}-\sigma^{2}+i \epsilon} \theta\left(\sigma^{2}\right) \\
\rho\left(p^{2}\right)=\rho_{3}\left(p^{2}\right) \not p+\sqrt{p^{2}} \rho_{1}\left(p^{2}\right)+\frac{p^{2}}{p \cdot v} \rho_{0}\left(p^{2}\right) \psi \\
\operatorname{Disc} \frac{\operatorname{Tr}_{c}}{N_{c}}\langle\Omega| i \tilde{S}(p)|\Omega\rangle=\frac{1}{(2 \pi)^{3}} \rho\left(p^{2}\right) \theta\left(p^{2}\right) \theta\left(p^{-}\right) \\
\operatorname{Disc} \frac{\operatorname{Tr}_{c}}{N_{c}}\langle\Omega| \hat{s}_{3,1,0}(p, v)|\Omega\rangle=\frac{1}{(2 \pi)^{3}} \rho_{3,1,0}\left(p^{2}\right) \theta\left(p^{2}\right) \theta(p)
\end{gathered}
$$

## Integrated g.i. quark propagator

Boost quark at large light-cone momentum:

$$
k^{-} \gg\left|\mathbf{k}_{\perp}\right| \gg k^{+}
$$

$$
w=n^{+}
$$

Integrate out the suppressed component of the quark momentum:

$$
J_{i j}\left(k^{-}, \vec{k}_{\perp} ; n_{+}\right) \equiv \frac{1}{2} \int d k^{+} \Xi_{i j}\left(k ; n_{+}\right)
$$

$\square$ Generalizes the perturbative quark propagator that appears in inclusive and semi-inclusive DIS

$$
\begin{gathered}
W_{\mathrm{TMD}}\left(\xi^{+}, \boldsymbol{\xi}_{\perp}\right)=\mathcal{U}_{n_{+}}\left[0^{-}, 0^{+}, \mathbf{0}_{\perp} ; 0^{-}, \infty^{+}, \mathbf{0}_{\perp}\right] \mathcal{U}_{\boldsymbol{n}_{\perp}}\left[0^{-}, \infty^{+}, \mathbf{0}_{\perp} ; 0^{-}, \infty^{+}, \boldsymbol{\xi}_{\perp}\right] \mathcal{U}_{n_{+}}\left[0^{-}, \infty^{+}, \boldsymbol{\xi}_{\perp} ; 0^{-}, \xi^{+}, \boldsymbol{\xi}_{\perp}\right] \\
W_{\mathrm{coll}}\left(\xi^{+}\right)=\mathcal{U}_{n_{+}}\left[0^{-}, 0^{+}, \mathbf{0}_{\perp} ; 0^{-}, \xi^{+}, \mathbf{0}_{\perp}\right]
\end{gathered}
$$

## Integrated g.i. quark propagator

$\square$ Expand in Dirac structures, in powers of $1 / k^{-}$

$$
J\left(k^{-}, \boldsymbol{k}_{\perp} ; n_{+}\right)=\frac{1}{2} \alpha\left(k^{-}\right) \gamma^{+}+\frac{\Lambda}{k^{-}}\left[\zeta\left(k^{-}\right) \mathbb{I}+\alpha\left(k^{-}\right) \frac{k_{\perp}}{\Lambda}\right]+\frac{\Lambda^{2}}{2\left(k^{-}\right)^{2}} \omega\left(k^{-}, \boldsymbol{k}_{\perp}^{2}\right) \gamma^{-}
$$

## Integrated g.i. quark propagator

$\square$ Expand in Dirac structures, in powers of $1 / k^{-}$

$$
J\left(k^{-}, \boldsymbol{k}_{\perp} ; n_{+}\right)=\frac{1}{2} \alpha\left(k^{-}\right) \gamma^{+}+\frac{\Lambda}{k^{-}}\left[\zeta\left(k^{-}\right) \mathbb{\Psi}+\alpha\left(k^{-}\right) \frac{k_{\perp}}{\Lambda}\right]+\frac{\Lambda^{2}}{2\left(k^{-}\right)^{2}} \omega\left(k^{-}, \boldsymbol{k}_{\perp}^{2}\right) \gamma^{-}
$$

$$
\alpha\left(k^{-}\right)=J^{\left[\gamma^{-}\right]}
$$

$$
\zeta\left(k^{-}\right)=\frac{k^{-}}{\Lambda} J^{[\mathbb{]}]}
$$

$$
\omega\left(k^{-}, \boldsymbol{k}_{\perp}^{2}\right)=\left(\frac{k^{-}}{\Lambda}\right)^{2} J^{\left[\gamma^{+}\right]}
$$

## Integrated g.i. quark propagator

$\square$ Expand in Dirac structures, in powers of $1 / k^{-}$

$$
J\left(k^{-}, \boldsymbol{k}_{\perp} ; n_{+}\right)=\frac{1}{2} \alpha\left(k^{-}\right) \gamma^{+}+\frac{\Lambda}{k^{-}}\left[\zeta\left(k^{-}\right) \mathbb{I}+\alpha\left(k^{-}\right) \frac{\not k_{\perp}}{\Lambda}\right]+\frac{\Lambda^{2}}{2\left(k^{-}\right)^{2}} \omega\left(k^{-}, \boldsymbol{k}_{\perp}^{2}\right) \gamma^{-}
$$

$$
J\left(k^{-}, \boldsymbol{k}_{T} ; n_{+}\right)=\frac{\theta\left(k^{-}\right)}{4(2 \pi)^{3} k^{-}}\left\{k^{-} \gamma^{+}+\not k_{T}+M_{j} \mathbb{I}+\frac{K_{j}^{2}+\boldsymbol{k}_{T}^{2}}{2 k^{-}} \gamma^{-}\right\}
$$

## Integrated g.i. quark propagator

$\square$ Expand in Dirac structures, in powers of $1 / k^{-}$

$$
\begin{aligned}
& J\left(k^{-}, \boldsymbol{k}_{\perp} ; n_{+}\right)= \frac{1}{2} \alpha\left(k^{-}\right) \gamma^{+}+\frac{\Lambda}{k^{-}}\left[\zeta\left(k^{-}\right) \mathbb{I}+\alpha\left(k^{-}\right) \frac{\not k_{\perp}}{\Lambda}\right]+\frac{\Lambda^{2}}{2\left(k^{-}\right)^{2}} \omega\left(k^{-}, \boldsymbol{k}_{\perp}^{2}\right) \gamma^{-} \\
& \not k+m=k^{-} \gamma^{+}+\not k_{\perp}+m \mathbb{I}+\frac{m^{2}+\boldsymbol{k}_{\perp}^{2}}{2 k^{-}} \gamma^{-} \\
& J\left(k^{-}, \boldsymbol{k}_{T} ; n_{+}\right)= \frac{\theta\left(k^{-}\right)}{4(2 \pi)^{3} k^{-}}\left\{k^{-} \gamma^{+}+\not k_{T}+M_{j} \mathbb{I}+\frac{K_{j}^{2}+\boldsymbol{k}_{T}^{2}}{2 k^{-}} \gamma^{-}\right\}
\end{aligned}
$$

## Integrated g.i. quark propagator

$\square$ Expand in Dirac structures, in powers of $1 / k^{-}$

$$
\begin{aligned}
& J\left(k^{-}, \boldsymbol{k}_{\perp} ; n_{+}\right)= \frac{1}{2} \alpha\left(k^{-}\right) \gamma^{+}+\frac{\Lambda}{k^{-}}\left[\zeta\left(k^{-}\right) \mathbb{I}+\alpha\left(k^{-}\right) \frac{\not k_{\perp}}{\Lambda}\right]+\frac{\Lambda^{2}}{2\left(k^{-}\right)^{2}} \omega\left(k^{-}, \boldsymbol{k}_{\perp}^{2}\right) \gamma^{-} \\
& \not \not k+m=k^{-} \gamma^{+}+\not k_{\perp}+m \mathbb{I}+\frac{m^{2}+\boldsymbol{k}_{\perp}^{2}}{2 k^{-}} \gamma^{-} \\
& J\left(k^{-}, \boldsymbol{k}_{T} ; n_{+}\right)= \frac{\theta\left(k^{-}\right)}{4(2 \pi)^{3} k^{-}}\left\{k^{-} \gamma^{+}+\not k_{T}+M_{j} \mathbb{I}+\frac{K_{j}^{2}+\boldsymbol{k}_{T}^{2}}{2 k^{-}} \gamma^{-}\right\}
\end{aligned}
$$

## Integrated g.i. quark propagator

$\square$ Expand in Dirac structures, in powers of $1 / k^{-}$

$$
\begin{gathered}
J\left(k^{-}, \boldsymbol{k}_{\perp} ; n_{+}\right)=\frac{1}{2} \alpha\left(k^{-}\right) \gamma^{+}+\frac{\Lambda}{k^{-}}\left[\zeta\left(k^{-}\right) \mathbb{I}+\alpha\left(k^{-}\right) \frac{\not k_{\perp}}{\Lambda}\right]+\frac{\Lambda^{2}}{2\left(k^{-}\right)^{2}} \omega\left(k^{-}, \boldsymbol{k}_{\perp}^{2}\right) \gamma^{-} \\
J\left(k^{-}, \boldsymbol{k}_{T} ; n_{+}\right)=\frac{\theta\left(k^{-}\right)}{4(2 \pi)^{3} k^{-}}\left\{k^{-} \gamma^{+}+\not k_{T}+\not k_{\perp}+m \mathbb{I}+\frac{m^{2}+\boldsymbol{k}_{\perp}^{2}}{2 k^{-}} \gamma^{-}\right. \\
\begin{array}{l}
\text { Average mass of all the } \\
\text { hadronization products } \\
\text { produced during the } \\
\text { fragmentation of a quark }
\end{array}
\end{gathered}
$$

## Integrated g.i. quark propagator

$\square$ Expand in Dirac structures, in powers of $1 / k^{-}$

$$
\begin{aligned}
& J\left(k^{-}, \boldsymbol{k}_{\perp} ; n_{+}\right)= \frac{1}{2} \alpha\left(k^{-}\right) \gamma^{+}+\frac{\Lambda}{k^{-}}\left[\zeta\left(k^{-}\right) \mathbb{I}+\alpha\left(k^{-}\right) \frac{\not k_{\perp}}{\Lambda}\right]+\frac{\Lambda^{2}}{2\left(k^{-}\right)^{2}} \omega\left(k^{-}, \boldsymbol{k}_{\perp}^{2}\right) \gamma^{-} \\
& \not \not k+m=k^{-} \gamma^{+}+\not k_{\perp}+m \mathbb{I}+\frac{m^{2}+\boldsymbol{k}_{\perp}^{2}}{2 k^{-}} \gamma^{-} \\
& J\left(k^{-}, \boldsymbol{k}_{T} ; n_{+}\right)= \frac{\theta\left(k^{-}\right)}{4(2 \pi)^{3} k^{-}}\left\{k^{-} \gamma^{+}+\not k_{T}+M_{j} \mathbb{I}+\frac{K_{j}^{2}+\boldsymbol{k}_{T}^{2}}{2 k^{-}} \gamma^{-}\right\}
\end{aligned}
$$

Average mass of all the hadronization products produced during the
fragmentation of a quark
$\square$ In any gauge:

$$
(k)=J^{[\quad]}=\frac{\theta(k)}{2(2 \pi)^{3}} \int_{0}^{\infty} d p^{2} \rho_{3}\left(p^{2}\right)
$$

$\square$ In any gauge:

$$
\begin{aligned}
(k) & =J^{[\quad]}=\frac{\theta(k)}{2(2 \pi)^{3}} \int_{0}^{\infty} d p^{2} \rho_{3}\left(p^{2}\right) \\
\zeta(k) & =\frac{k}{\Lambda} J^{[]}=\frac{\theta(k)}{2 \Lambda(2 \pi)^{3}} \int d p^{2} \sqrt{p^{2}} \rho_{1}\left(p^{2}\right)
\end{aligned}
$$

$\square$ In any gauge:

$$
\begin{aligned}
(k) & =J^{[]}=\frac{\theta(k)}{2(2 \pi)^{3}} \int_{0}^{\infty} d p^{2} \rho_{3}\left(p^{2}\right) \\
\zeta(k) & =\frac{k}{\Lambda} J^{[]}=\frac{\theta(k)}{2 \Lambda(2 \pi)^{3}} \int d p^{2} \sqrt{p^{2}} \rho_{1}\left(p^{2}\right) \\
\omega\left(k, \boldsymbol{k}_{\boldsymbol{T}}\right) & \left.=\left(\frac{k}{\Lambda}\right)^{2} J^{[\quad]}=\frac{\theta(k)}{(2 \Lambda)^{2}(2 \pi)^{3}} \mu_{j}^{2}+\tau_{j}^{2}+\boldsymbol{k}_{T}^{2}\right)
\end{aligned}
$$

$\square$ In any gauge:

$$
\begin{aligned}
&(k)=J^{[]}=\frac{\theta(k)}{2(2 \pi)^{3}} \int_{0}^{\infty} d p^{2} \rho_{3}\left(p^{2}\right) \\
& \zeta(k)=\frac{k}{\Lambda} J^{[]}=\frac{\theta(k)}{2 \Lambda(2 \pi)^{3}} \int d p^{2} \sqrt{p^{2}} \rho_{1}\left(p^{2}\right) \\
& \omega\left(k, \boldsymbol{k}_{\boldsymbol{T}}\right)\left.\left.=\left(\frac{k}{\Lambda}\right)^{2} J^{[ }\right]=\frac{\theta(k)}{(2 \Lambda)^{2}(2 \pi)^{3}} \mu_{j}^{2}+\tau_{j}^{2}+\boldsymbol{k}_{\boldsymbol{T}}^{2}\right) \\
& K_{j}^{2}
\end{aligned}
$$

## Sum rules

$\square$ In any gauge:

$$
\begin{aligned}
1 & =\int_{0}^{\infty} d p^{2} \rho_{3}\left(p^{2}\right) \\
M_{j} & =\int_{0}^{\infty} d p^{2} \sqrt{p^{2}} \rho_{1}\left(p^{2}\right) \\
0 & =\int_{0}^{\infty} d p^{2} p^{2} \rho_{0}\left(p^{2}\right)
\end{aligned}
$$

$\square$ Can be used to verify actual calculations of the quark propagator!

## Sum rules

$\square$ In any gauge:

$$
\begin{aligned}
1 & =\int_{0}^{\infty} d p^{2} \rho_{3}\left(p^{2}\right) \\
M_{j} & =\int_{0}^{\infty} d p^{2} \sqrt{p^{2}} \rho_{1}\left(p^{2}\right) \\
0 & =\int_{0}^{\infty} d p^{2} p^{2} \rho_{0}\left(p^{2}\right)
\end{aligned}
$$

$\square$ Can be used to verify actual calculations of the quark propagator!

$$
M_{j}=\int d p^{2} \sqrt{p^{2}} \rho_{1}\left(p^{2}\right)
$$

| leading |  | quark operator |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | unpolarized [U] | longitudinal [L] | transverse [T] |
|  | U | $\begin{aligned} & f_{1}=\bigodot \\ & \text { unpolarized } \end{aligned}$ |  | $h_{1}^{\frac{1}{1}}=\underset{\text { Boer-Mulders }}{(i)}-\left(\frac{1}{2}\right.$ |
|  | L |  | $g_{1}=\bigodot \rightarrow-\bigodot \rightarrow$ | $h_{1 L}^{1}=\underset{\text { worm gear } 1}{\rightarrow-\longrightarrow}$ |
|  | T | $f_{1 T}^{1}=\bigodot_{\text {Sivers }}^{\uparrow}-\bigodot$ |  |  |
|  | T <br> E <br> N <br> N <br> S <br> R | $\begin{array}{r} \hline f_{1 L L}\left(x, \boldsymbol{k}_{T}^{2}\right) \\ f_{1 L T}\left(x, \boldsymbol{k}_{T}^{2}\right) \\ f_{1 T T}\left(x, \boldsymbol{k}_{T}^{2}\right) \end{array}$ | $\begin{array}{r} g_{1 T T}\left(x, \boldsymbol{k}_{T}^{2}\right) \\ g_{1 U T}\left(x, \boldsymbol{k}_{T}^{2}\right) \\ \hline \end{array}$ | $h_{1 L L}^{\perp}\left(x, \boldsymbol{k}_{T}^{2}\right)$ <br> $h_{1 T T}, h_{1 T T}^{\perp}$ <br> $h_{1 L T}, h_{1 L T}^{\perp}$ |

Gauge invariant generalization of the gauge dependent dressed quark mass
Experimentally accessible in double spin assymetry measurements!
(table from Satvir Kaur's talk yesterday)
$\square$ In light-cone gauge:

$$
K_{j}^{2}=\mu_{j}^{2}+2=\int_{0}^{\infty} d p^{2} p^{2} \rho_{3}^{\operatorname{lcg}}\left(p^{2}\right)
$$

$\square \ln$ light-cone gauge:

$$
K_{j}^{2}=\mu_{j}^{2}+\sum^{2}=\int_{0}^{\infty} d p^{2} p^{2} \rho_{3}^{\operatorname{lcg}}\left(p^{2}\right)
$$

$\square$ But in other gauges

$$
K_{j}^{2}=\mu_{j}^{2}+\tau_{j}^{2}
$$

$$
\tau_{j}^{2}=(2 \pi)^{3} \int_{0}^{\infty} d p^{2} \operatorname{Disc} \frac{\mathrm{Tr}_{\mathrm{c}}}{\mathrm{~N}_{\mathrm{c}}}\langle\Omega| \hat{\sigma}_{3}\left(p^{2}\right) i g D_{\perp}\left(\boldsymbol{A}^{\perp}\left(\boldsymbol{\xi}_{\perp}\right)+\mathcal{Z}^{\perp}\left(\boldsymbol{\xi}_{\perp}\right)\right)_{\boldsymbol{\xi}_{\perp}=0}|\Omega\rangle
$$

$$
\mathcal{Z}^{\perp}\left(\boldsymbol{\xi}_{\perp}\right)=\int_{0}^{\infty^{+}} d s^{+} \boldsymbol{D}_{\perp}\left(U_{n_{+}}\left[0^{-}, 0^{+}, \boldsymbol{\xi}_{\perp} ; 0^{-}, s^{+}, \boldsymbol{\xi}_{\perp}\right] G^{\perp-}\left(0^{-}, s^{+}, \boldsymbol{\xi}_{\perp}\right) U_{n^{+}}\left[0^{-}, s^{+}, \boldsymbol{\xi}_{\perp} ; 0^{-}, \infty^{+}, \boldsymbol{\xi}_{\perp}\right]\right)|\Omega\rangle
$$

$\square \ln$ light-cone gauge:

$$
K_{j}^{2}=\mu_{j}^{2}+\sum^{2}=\int_{0}^{\infty} d p^{2} p^{2} \rho_{3}^{\operatorname{lcg}}\left(p^{2}\right)
$$

$\square$ But in other gauges
Final state interactions "vanish"

$$
K_{j}^{2}=\mu_{j}^{2}+\tau_{j}^{2}
$$

$$
\tau_{j}^{2}=(2 \pi)^{3} \int_{0}^{\infty} d p^{2} \operatorname{Disc} \frac{\mathrm{Tr}_{\mathrm{c}}}{\mathrm{~N}_{\mathrm{c}}}\langle\Omega| \hat{\sigma}_{3}\left(p^{2}\right) i g D_{\perp}\left(\boldsymbol{A}^{\perp}\left(\boldsymbol{\xi}_{\perp}\right)+\mathcal{Z}^{\perp}\left(\xi_{\perp}\right)\right)_{\boldsymbol{\xi}_{\perp}=0}|\Omega\rangle
$$

$$
\mathcal{Z}^{\perp}\left(\boldsymbol{\xi}_{\perp}\right)=\int_{0}^{\infty^{+}} d s^{+} \boldsymbol{D}_{\perp}\left(U_{n_{+}}\left[0^{-}, 0^{+}, \boldsymbol{\xi}_{\perp} ; 0^{-}, s^{+}, \boldsymbol{\xi}_{\perp}\right] G^{\perp-}\left(0^{-}, s^{+}, \boldsymbol{\xi}_{\perp}\right) U_{n^{+}}\left[0^{-}, s^{+}, \boldsymbol{\xi}_{\perp} ; 0^{-}, \infty^{+}, \boldsymbol{\xi}_{\perp}\right]\right)|\Omega\rangle
$$

## Summary

$\square$ Completed the analysis of the gauge invariant quark propagator
$\square$ Full calculation of the twist-4 coefficient
$\square$ Formal demonstration of the gauge invariance of the twist-2, twist-3 and twist-4 coefficients of the g.i. quark propagator/jet correlator
> New sum rules (needed: numerical checks)

## Summary

$\square$ Completed the analysis of the gauge invariant quark propagator
$\square$ Full calculation of the twist-4 coefficient
$\square$ Formal demonstration of the gauge invariance of the twist-2, twist-3 and twist-4 coefficients of the g.i. quark propagator/jet correlator
> New sum rules for the quark spectral functions (needed: numerical checks)
$\square$ In particular:
$\Rightarrow$ Second moment of $\rho_{0}$ vanishes
$>$ First moment of the chiral odd quark spectral function gives a mass $M_{j}$ that
is a gauge invariant generalization of the gauge dependent quark mass

## Summary

$\square M_{j}$ color screened gauge invariant mass
> Non-vanishing even in the chiral limit
> Provides a direct way to probe dynamical chiral symmetry breaking

## Summary

$\square M_{j}$ color screened gauge invariant mass
> Non-vanishing even in the chiral limit
> Provides a direct way to probe dynamical chiral symmetry breaking (In progress)


## Summary

$\square M_{j}$ color screened gauge invariant mass
> Non-vanishing even in the chiral limit
> Provides a direct way to probe dynamical chiral symmetry breaking
> It's calculable, but moreover.. It can be measured!


