PION NUCLEUS DRELL–YAN PROCESS

AND

PARTON TRANSVERSE MOMENTUM IN THE PION

JLab Theory Seminar
9/5/18

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Universidad Nacional Autónoma de México
(IFUNAM)
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OUTLINE

- Drell-Yan in πN scattering
- Drell-Yan with transverse momentum
  - Pion dynamics
  - Effects on DY cross section
- Outlook
OUTLINE

- Drell-Yan in $\pi N$ scattering
- Drell-Yan with transverse momentum
  - Pion dynamics
  - Effects on DY cross section
- Outlook

Based on
with F.A. Ceccopieri, S. Noguera & S. Scopetta
FOCUS ON THE PION
\[ Q^2 = M^2 \]
\[ s = 2P_1 \cdot P_2 \]
\[ \tau = \frac{Q^2}{s} \equiv \text{finite as } Q^2, s \to \infty \]
Pion-proton Drell-Yan differential cross-section

\[ d\sigma = \sum_{ab} \int dx_a \int dx_b f_a(x_a, \mu) f_b(x_b, \mu) \, d\hat{\sigma}_{ab}(x_a P_a, x_b P_b, Q, \alpha_s(\mu), \mu) \]

\[ x_a x_b = \tau \quad Q^2 = M^2 \]

\[ s = 2P_1 \cdot P_2 \]

\[ \tau = \frac{Q^2}{s} \equiv \text{finite as } Q^2, s \to \infty \]
Pion-proton Drell-Yan differential cross-section

\[ d\sigma = \sum_{ab} \int dx_a \int dx_b f_a^{\pi}(x_a, \mu) f_b^{\pi}(x_b, \mu) d\hat{\sigma}_{ab}(x_a P_a, x_b P_b, Q, \alpha_s(\mu), \mu) \]

\[ x_a x_b = \tau \]
\[ Q^2 = M^2 \]
\[ s = 2P_1 \cdot P_2 \]
\[ \tau = \frac{Q^2}{s} \equiv \text{finite as } Q^2, s \to \infty \]
Pion-proton Drell-Yan:
main source of information on pion structure

E615 extraction
(joint proton and pion PDF)

Momentum fraction carried by valence quarks → allows $Q_0$ fixing

Nambu - Jona-Lasinio (NJL)

$Q_0 = 0.29\text{GeV}$, for the LO evolution;
$Q_0 = 0.43\text{GeV}$, for the NLO evolution.

$\Lambda_{\text{LO}} = 0.174 \text{ GeV}$
$\Lambda_{\text{NLO}} = 0.246 \text{ GeV}$
THE PION IN NJL

Distribution Functions as Functions of $x$

Calculation in QCD not possible $\rightarrow$ Nonperturbative Objects

Approaching QCD by Models, Effective theories, ... $\rightarrow$ not an exact calculation

Pion $\rightarrow$ Chiral Low-Energy Models

Why NJL?

- Quarks degrees of freedom
- Relation current $\leftrightarrow$ constituent quarks
- Pion as a Goldstone mode
- Pion as a Bound-State in the sense of Bethe-Salpeter
- Choice of a covariant regularization scheme

Why are we confident about the use of NJL?

- Calculations of PDFs in NJL $\rightarrow$ OK
- Non perturbative QCD $\rightarrow$ Evolution of initial PDF
- Comparison with data $\rightarrow$ OK

Long story of successful results and predictions
THE PION IN NJL: DISTRIBUTION AMPLITUDE

Mind the scale of y-axis!

$Q_0^2$ = $2 \text{GeV}^2$

$Q_0 = 0.29 \text{GeV}$, for the LO evolution; $Q_0 = 0.43 \text{GeV}$, for the NLO evolution.

$\Lambda_{\text{LO}} = 0.174 \text{ GeV}$

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Pion-proton Drell-Yan: main source of information on pion structure

E615 extraction (joint proton and pion PDF)

Momentum fraction carried by valence quarks → allows $Q_0$ fixing

Nambu - Jona-Lasinio (NJL)

What if instead we map the pion PDF onto the DY X-section?

$Q_0 = 0.29\text{GeV}$, for the LO evolution;
$Q_0 = 0.43\text{GeV}$, for the NLO evolution.

$\Lambda_{LO} = 0.174\text{ GeV}$
$\Lambda_{NLO} = 0.246\text{ GeV}$
Comparison of integrated X-section with theory at NLO:

- pion from NJL
- proton from CTEQ06M

We find

\[ Q_0^2 = 0.21 \text{ GeV}^2 / Q_0 = 0.46 \text{ GeV} \]

with \( \chi^2/\text{dof} = 2 \)
DRELL-YAN WITH TRANSVERSE MOMENTUM

With measured $Q_T$ of order $Q$

$$\frac{d\sigma}{dQ^2 \, dy \, dQ_T^2} = \frac{4\pi^2 \alpha^2}{9Q^2 s} \sum_{a,b} \int_{x_{\pi}}^{1} \frac{d\xi_{\pi}}{\xi_{\pi}} \int_{x_{P}}^{1} \frac{d\xi_{P}}{\xi_{P}} T_{ab}(\cdots) f_{a/\pi}(\xi_{\pi}, \mu) f_{b/P}(\xi_{P}, \mu)$$
DRELL-YAN WITH TRANSVERSE MOMENTUM

\[
\frac{d\sigma}{dQ^2 dy dQ_T^2} = \frac{4\pi^2 \alpha^2}{9Q^2_s} \sum_{a,b} \int_{\pi x}^1 \frac{d\xi_{\pi}}{\xi_{\pi}} \int_{\pi P}^1 \frac{d\xi_{P}}{\xi_{P}} T_{ab}(\cdots) f_{a/\pi}(\xi_{\pi}, \mu) f_{b/P}(\xi_{P}, \mu)
\]

With measured \( Q_T \) of order \( Q \)
DRELL-YAN WITH TRANSVERSE MOMENTUM

\[ \frac{d\sigma}{dQ^2 dy dQ^2_T} = \frac{4\pi^2 \alpha^2}{9Q^2s} \sum_{a,b} \int_{x_{\pi}}^{1} \frac{d\xi_{\pi}}{\xi_{\pi}} \int_{x_P}^{1} \frac{d\xi_{P}}{\xi_{P}} T_{ab}(\cdots) f_{a/\pi}(\xi_{\pi}, \mu) f_{b/P}(\xi_{P}, \mu) \]

With measured \( Q_T \) of order \( Q \)

\[ q^2 = (k + k')^2 \]
\[ x_{\pi} = \frac{Q^2}{2P_{\pi} \cdot q}, \quad x_P = \frac{Q^2}{2P_P \cdot q} \]
\[ \tau = \frac{Q^2}{s} \text{ fixed and finite as } Q^2, s \to \infty \]

\[ q^\mu = (x_{\pi} P_{\pi}^+, x_P P_P^-, \vec{q}_T) \]

\[ y = \frac{1}{2} \ln \frac{x_{\pi}}{x_P} \]
REGION OF TRANSVERSE MOMENTUM

$q_T \sim \Lambda$

$q_T \sim Q$

$Q^2 \gg \Lambda^2$

REGION OF TRANSVERSE MOMENTUM

$q_T \sim \Lambda$

$q_T \sim Q$

Transverse momentum totally from hard subprocesses

$Q^2 \gg \Lambda^2$

REGION OF TRANSVERSE MOMENTUM

$Q^2 \gg \Lambda^2$

Transverse momentum from non-perturbative dynamics:
Transverse Momentum PDFs

Transverse momentum totally from hard subprocesses

Transverse momentum from non-perturbative dynamics: Transverse Momentum PDFs

Transverse momentum totally from hard subprocesses

COURTESY OF T. ROGERS

**REGION OF TRANSVERSE MOMENTUM**

Transverse momentum from non-perturbative dynamics:
Transverse Momentum PDFs

Transverse momentum totally from hard subprocesses

$q_T \sim \Lambda$

$q_T \sim Q$

$Q^2 >> \Lambda^2$

\[ \frac{d\sigma}{dQ^2 dy dQ_T^2} = \frac{4\pi^2 \alpha^2}{9Q^2 s} \int \frac{d^2b}{2\pi^2} e^{i\bar{Q}_T \cdot \vec{b}} \sum_{a,b} \int_{x_\pi}^1 \frac{d\xi_\pi}{\xi_\pi} \int_{x_\pi}^1 \frac{d\xi_P}{\xi_P} f_{a/\pi}(\xi_\pi, \mu) f_{b/P}(\xi_P, \mu) \]

\[ \times \sum_j e_j^2 C_{ja}(x_\pi/\xi_\pi, b; Qb; \mu) C_{jb}(x_P/\xi_P, b; Qb; \mu) \]

\[ C_{ja}(x_\pi/\xi_\pi, b; Qb; \mu) = \delta_{ja} \delta(x_\pi/\xi_\pi - 1) + \frac{\alpha_s(\mu)}{\pi} C_{ja}^{(1)} + \cdots \]

\( Q_T \) appears at NLO
\[ \frac{d\sigma}{dQ^2\,dy\,dQ_T^2} = \frac{4\pi^2\alpha^2}{9Q^2s} \int \frac{d^2b}{2\pi^2} e^{i\vec{Q}_T\cdot\vec{b}} \sum_{a,b} \int_{x_\pi}^1 \frac{d\xi_\pi}{\xi_\pi} \int_{x_P}^1 \frac{d\xi_P}{\xi_P} f_{a/\pi}(\xi_\pi, \mu_b) f_{b/P}(\xi_P, \mu_b) \]

\[ \times \exp \left( -C_F \frac{\alpha_s(q)}{2\pi} \int b_0^2/b^2 \frac{dq^2}{q^2} \left[ 2 \ln \frac{Q^2}{\mu^2} - 3 \right] + \text{H.O.} \right) \]

\[ \times \sum_j e_j^2 C_{ja}(x_\pi/\xi_\pi, b; 2e^{-\gamma}; \mu_b) C_{jb}(x_P/\xi_P, b; 2e^{-\gamma}; \mu_b) \]

\[ \times e^{S_{NP}(b)} \, e^{S_{NP}(b)} \]

Taming $b \sim 1/\Lambda$:

- **b-prescription**
  \[ \mu_b = 2e^{-\gamma}/b^* \]
  \[ b^* = b/\sqrt{1 + b^2/b_{\text{max}}^2} \]


\[
\frac{d\sigma}{dQ^2 dy dQ_T^2} = \frac{4\pi^2 \alpha^2}{9Q^2 s} \int \frac{d^2 b}{2\pi^2} e^{i\vec{Q}_T \cdot \vec{b}} \sum_{a,b} \int_{x_\pi}^{1} \frac{d\xi_\pi}{\xi_\pi} \int_{x_P}^{1} \frac{d\xi_P}{\xi_P} f_{a/\pi}(\xi_\pi, \mu_b) f_{b/P}(\xi_P, \mu_b)
\]

× \exp \left( -C_F \frac{\alpha_s(q)}{2\pi} \int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2} \left[ \frac{2\ln\frac{Q^2}{\mu^2}}{2} - 3 \right] + \text{H.O.} \right)

× \sum_j e_{j}^2 C_{ja}(x_\pi/\xi_\pi, b; 2e^{-\gamma}; \mu_b) C_{jb}(x_P/\xi_P, b; 2e^{-\gamma}; \mu_b)

\times e^{S_{NP}^{\piP}(b)} e^{S_{NP}^{P}(b)}

\text{Taming } b \sim 1/\Lambda:

\bullet \text{ b-prescription}

\mu_b = 2e^{-\gamma} / b^*

b^* = b / \sqrt{1 + b^2 / b_{\text{max}}^2}

\[
\frac{d\sigma}{dQ^2 dy dq_T^2} = \frac{4\pi^2 \alpha^2}{9Q^2 s} \int \frac{d^2b}{2\pi^2} e^{i\vec{q}_T \cdot \vec{b}} \sum_{a,b} \sum_j e_j^2 \\
\times \int_{x_\pi}^{1} \frac{d\xi}{\xi} \int_{x_P}^{1} \frac{d\xi_P}{\xi_P} f_{a/\pi}(\xi \pi, \bar{b}; \zeta, \mu_b) f_{b/P}(\xi_P, \bar{b}; \zeta_P, \mu_b) \\
\times \exp \left( -C_F \frac{\alpha_s(q)}{2\pi} \int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2} \left[ 2 \ln \frac{Q^2}{\mu^2} - 3 \right] + \text{H.O} \right) \\
\times C_{ja} \left( x_\pi/\xi; \bar{b}; 2e^{-\gamma}; \mu_b \right) C_{jb} \left( x_P/\xi_P; \bar{b}; 2e^{-\gamma}; \mu_b \right)
\]
\[ \frac{d\sigma}{dQ^2 dy dq_T^2} = \frac{4\pi^2 \alpha^2}{9Q^2 s} \int \frac{d^2 b}{2\pi^2} e^{i\vec{q}_T \cdot \vec{b}} \sum_{a,b} \sum_j e_j^2 \times \int_{x_\pi}^{1} \frac{d\xi_\pi}{\xi_\pi} \int_{x_P}^{1} \frac{d\xi_P}{\xi_P} \left( f_{a/\pi}(\xi_\pi, \vec{b}; \zeta_\pi, \mu_b) f_{b/P}(\xi_P, \vec{b}; \zeta_P, \mu_b) \right) \times \exp \left( -C_F \frac{\alpha_s(q)}{2\pi} \int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2} \left[ 2 \ln \frac{Q^2}{\mu^2} - 3 \right] + \text{H.O} \right) \times C_{ja} \left( x_\pi/\xi_\pi, \vec{b}; \gamma e^{-\gamma}; \mu_b \right) C_{jb} \left( x_P/\xi_P, \vec{b}; \gamma e^{-\gamma}; \mu_b \right) \]
\[ \frac{d\sigma}{dQ^2 dy dq_T^2} = \frac{4\pi^2 \alpha^2}{9Q^2 s} \int \frac{d^2b}{2\pi^2} e^{i\vec{q}_T \cdot \vec{b}} \sum_{a,b} \sum_{j} e_j^2 \]

\[ \times \left( \int_{x_{\pi}}^{1} \frac{d\xi_{\pi}}{\xi_{\pi}} \right) \left( \int_{x_{P}}^{1} \frac{d\xi_{P}}{\xi_{P}} \right) \left( f_{a/\pi}(\xi_{\pi}, \vec{b}; \zeta_{\pi}, \mu_{b}) f_{b/P}(\xi_{P}, \vec{b}; \zeta_{P}, \mu_{b}) \right) \]

\[ \times \exp \left( -C_F \frac{\alpha_s(q)}{2\pi} \int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2} \left[ 2 \ln \frac{Q^2}{\mu^2} - 3 \right] + \text{H.O} \right) \]

\[ \times C_{ja} \left( x_{\pi}/\xi_{\pi}, \vec{b}; 2e^{-\gamma}; \mu_{b} \right) C_{jb} \left( x_{P}/\xi_{P}, \vec{b}; 2e^{-\gamma}; \mu_{b} \right) \]

Ideally, we’d use

- full TMD for both hadrons
- either from pheno. or similar models

But,

- no pheno proton TMD available (when we started this…)
- no model similar to NJL for the proton
STRATEGY

- use a phenomenologically estimated \( f_{b/P}(\xi_P; \mu_b) \times e^{S_{NP}(b)} \)
- PDF from CTEQ6M
- use the pion TMD from the NJL model \( f_{a/\pi}(\xi_\pi, \vec{b}; \xi_\pi, \mu_b) \)
- [Noguera, S. Scopetta, JHEP 1511, 102 (2015)]
- redefine the hadronic scale of PDF from DY integrated data
- interpret the \( k_T \)-dependence of the model onto the (unintegrated) DY data
STRATEGY

- use a phenomenologically estimated $f_{b/P}(\xi_P; \mu_b) \times e^{S_{NP}^P(b)}$
  - PDF from CTEQ6M
- use the pion TMD from the NJL model $f_{a/\pi}(\xi_\pi, \vec{b}, \zeta_\pi; \mu_b)$
  - [Noguera, S. Scopetta, JHEP 1511, 102 (2015)]
- redefine the hadronic scale of PDF from DY integrated data
- interpret the $k_T$-dependence of the model onto the (unintegrated) DY data

from now on: $e^{S_{NP}^P(b)} \rightarrow S_{NP}^P(b)$
THE NON-PERTURBATIVE PART

\[
\frac{d\sigma}{dQ^2 dy dQ'^2} \sim \frac{4\pi^2\alpha^2}{9Q'^2s} \left(\frac{2\pi}{2}\right)^{-2} \int d^2b e^{iQ_T \cdot b} \sum_j e_j^2 \tilde{W}_j(b_*, Q, x_A, x_B)_{\text{pert}}
\]

\[
\times \exp\left[-\ln\left(\frac{Q^2}{Q_0^2}\right) g_1(b) - g_{j/A}(x_A, b) - g_{j/B}(x_B, b)\right]
\]

One parameterization of the non-perturbative contribution

Here:

\[
S_{NP}^{\pi W}(b) = S_{NP}^{\pi}(b) \sqrt{S_{NP}^{pp}(b)}
\]

purely comes from the dynamics of the model

\[
b_*(b, b_{\text{max}}) = \frac{b}{\sqrt{1 + \left(\frac{b}{b_{\text{max}}}\right)^2}} \quad \text{with} \quad b_{\text{max}} = 1.5 \text{GeV}^{-1}
\]

Pion DY
FULL TRANSVERSE MOMENTUM DEPENDENCE FOR THE PION

\[ f(x; \mu) \times \exp\left(\frac{g_{j/P}(b)}{P}\right) = f(x, b; \mu) \]

TMD PDFs

\[ f_{1,\pi}(x, k_T^2) = \frac{3}{4 \pi^3} g_{\pi qq}^2 \theta(x) \theta(1-x) \sum_{i=0}^{2} c_i \times \left\{ \frac{1}{k_T^2 + M_i^2 - m_\pi^2 x (1-x)} + \frac{m_\pi^2 x (1-x)}{[k_T^2 + M_i^2 - m_\pi^2 x (1-x)]^2} \right\} \]

[Noguera & Scopetta, JHEP11, 102]
THE PION IN A CHIRAL MODEL

\[ f_\pi(x, b; \mu) \xrightarrow{\text{chiral lim}} f'_\pi(x; \mu) f''_\pi(b) \]

**Our interpretation:**

\[ \exp\left(\frac{g_j}{\pi}(b)\right) = f''_\pi(b) \]

\[ \rightarrow \text{no "}g_1(b)\text{" is this model picture} \]

\[
\begin{align*}
  f''_\pi(b) &= \frac{3}{2\pi^2} \left(\frac{m}{f_\pi}\right)^2 \sum_{i=0,2} \int dk_T k_T J_0(bk_T) \frac{a_i}{k_T^2 + m_i^2} \\
  &= \frac{3}{2\pi^2} \left(\frac{m}{f_\pi}\right)^2 \sum_{i=0,2} a_i K_0(m_i b)
\end{align*}
\]
The pion in a chiral model

\[
f_\pi(x, b; \mu) \xrightarrow{\text{chiral lim}} f'_\pi(x; \mu) f''_\pi(b)
\]

\[\exp(g_{j/\pi}(b)) = f''_\pi(b)\]

Our interpretation: no “\(g_1(b)\)” is this model picture

\[
f''_\pi(b) = \frac{3}{2\pi^2} \left(\frac{m}{f_\pi}\right)^2 \sum_{i=0,2} \int dk_T k_T J_0(bk_T) \frac{a_i}{k_T^2 + m_i^2}
\]

\[= \frac{3}{2\pi^2} \left(\frac{m}{f_\pi}\right)^2 \sum_{i=0,2} a_i K_0(m_i b)\]

We assumed that factorization of the transverse momentum occurs at \(Q_0\) only.

[Noguera & Scopetta, JHEP11, 102]
THE PION IN A CHIRAL MODEL

[Noguera & Scopetta, JHEP11, 102]
THE PION IN A CHIRAL MODEL

\[ \exp \left( -\frac{q_T^2}{\langle q_T^2 \rangle} \right) \]

\[ \frac{\pi \langle q_T^2 \rangle}{\langle q_T^2 \rangle} \]

Pion DY

[Noguera & Scopetta, JHEP11, 102]
THE PION IN A CHIRAL MODEL

Pion DY

[Noguera & Scopetta, JHEP11, 102]
THE PION IN A CHIRAL MODEL

[Noguera & Scopetta, JHEP11, 102]
THE PION IN A CHIRAL MODEL

Pion dynamics \(\to\) differs from a gaussian

\[ f_{\pi T}(0, s, k_T^2) [\text{GeV}^{-2}] \]

\[ k_T [\text{GeV}] \]

\[ 0.0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1.0 \]

\[ 0.0 \quad 0.5 \quad 1.0 \]

\[ \exp\left(-\frac{q_T^2}{\langle q_T^2 \rangle}\right) \]

\[ \frac{\pi \langle q_T^2 \rangle}{\pi \langle q_T^2 \rangle} \]

\[ 0.0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1.0 \quad 1.2 \quad 1.4 \]

[0.0 \quad 0.5 \quad 1.0 \quad 1.5 \quad 2.0 \quad 2.5 \quad 3.0]

[Noguera & Scopetta, JHEP11, 102]
THE PION IN A CHIRAL MODEL

Pion dynamics → differs from a gaussian

Transverse profile → no dpdce on x or M

[Noguera & Scopetta, JHEP11, 102]
DRELL-YAN WITH PION DYNAMICS

Next-to-Leading Log

\[ \sigma_{DY\pi N} \equiv \frac{d\sigma}{d\tau dy dp_T^2} = \sum_q \frac{\sigma_{q\bar{q}}^0}{2} \int_0^\infty db \ J_0(b p_T) \ e^{S(b, b_{max}, Q, C_1)} \ e^{S_{NP}^\pi (b)} \ e^{S_{NP}^N (b)} \cdot \left[ \left( f_{qa}^\pi (x_a, \mu_b) \otimes C_{aa'} \right) \left( F_{\bar{q}b}^N (x_b, \mu_b) \otimes C_{bb'} \right) + q \leftrightarrow \bar{q} \right], \]

Wilson coeff. at order \( \alpha_s \)
CTEQ6M PDFs evolved at NLO

Proton \( b_{max} = 0.86 \ \text{GeV}^{-1} \)
Pion \( b_{max} = \text{educated guess/adjusted to data} \)
\[ = b_0/Q_0 = 2.44 \ \text{GeV}^{-1} \]
☑️ stability upon variation of regulator
Cross section in Q-bins

- overall magnitude
- small qT
- stability upon b-prescription

- higher qT
- Gaussian profile ~indistinguishable

No free parameters
Only Q₀ is fixed beforehand

with KN param.
Cross section in x-bins

- overall magnitude
- small qT
- stability upon b-prescription

- higher qT
- Gaussian profile ~indistinguishable

No free parameters
Only $Q_0$ is fixed beforehand

with KN param.
theoretical calculation underestimates the experimental data in that region, particularly
turns out that, using the two parameters of the Sudakov form factor for the pion meson, the
Figure 1
proton structure. The pion is treated in the Nambu–Jona–
ture. In particular we have focused on the study of differential
scattering has been presented. The main goal of our work
4Conclusions
has been the test of model predictions, obtained within the
scattering calculation. Without the inclusion of the finite, fixed order, contributions
appears instead more questionable. In this complicated sce-
which populate the

All interpretations of this effect, however, are not conclusive

Fit
[Pasquini & et al, Phys.Rev.D90]

Only gaussian

[Polossak & et al, JHEP08-137]
CSS evolution affects the transverse mmt distribution.

[Pasquini & et al, Phys.Rev.D90]
PION STRUCTURE FROM DY?

Degrees of freedom change governed by the chiral symmetry.
CONCLUSIONS

- Pion-proton collision to $\mu^+\mu^-$
- We have included pion nonperturbative dynamics in DY cross section
- Slight change in shape w.r.t. pure gaussians
- Need to understand another function: $g_K(b)$

Importance of nonperturbative inserts in perturbative evolution!

Exciting physics ahead!
OUTLOOK

- Predictions for COMPASS-II
- Go to polarized case
  - T-odd TMDs and universality
- Go to the modern TMD description of the factorized form

\[
\frac{d\sigma}{d^2q \, d\Omega} = \frac{2}{s} \sum_j \frac{d\sigma_{jj}(Q, \mu Q, \alpha_s(\mu Q))}{d\Omega} \int d^2b_T \, e^{iq_T \cdot b_T} \, \tilde{f}_{j/A}(z_A, b_T; Q_0^2, \mu Q_0) \, \tilde{f}_{j/B}(z_B, b_T; Q_0^2, \mu Q_0) \\
\times \exp \left\{ -g_K(b_T; b_{\text{max}}) + K(b_s; \mu_b) - \int_{\mu_b}^{\mu_{Q_0}} \frac{d\mu'}{\mu'} \, \gamma_K(\alpha_s(\mu')) \ln \frac{Q^2}{Q_0^2} \right\} \\
\times \exp \left\{ \int_{\mu_{Q_0}}^{\mu_{Q_0}'} \frac{d\mu'}{\mu'} \left[ 2\gamma_j(\alpha_s(\mu'); 1) - \ln \frac{Q^2}{(\mu')^2} \gamma_K(\alpha_s(\mu')) \right] \right\}
\]

[Collins & Rogers, PRD91]

- Use knowledge on pion to fix NP parameters
- Redefine/evaluate the hadronic scale from TMD pheno.
BACKUP SLIDES
THE THEORY OF THE STRONG INTERACTIONS

QM: \[ \hat{H} = \hat{H}_0 + \lambda \hat{H}_1 + \cdots, \quad \lambda << \]

QCD: \[
A = A_0 + \frac{\alpha_s(Q)}{4\pi} A_1 + \left(\frac{\alpha_s(Q)}{4\pi}\right)^2 A_2 + \cdots
\]

Notwithstanding these open issues, a rather stable and well-defined world average value emerges from the compilation of current determinations of \( \alpha_s \):

\[
\alpha_s(M_Z^2) = 0.1184 \pm 0.0007.
\]

The results also provide a clear signature and proof of the energy dependence of \( \alpha_s \), in full agreement with the QCD prediction of Asymptotic Freedom. This is demonstrated in Fig. 9.4, where results of \( \alpha_s(Q^2) \) obtained at discrete energy scales \( Q \), now including those based just on NLO QCD, are summarized and plotted.

\[
\begin{align*}
\alpha_s(Q) & = \alpha_s(M_Z^2) + A_1 \left(\frac{\alpha_s(Q)}{\alpha_s(M_Z^2)}\right) + A_2 \left(\frac{\alpha_s(Q)}{\alpha_s(M_Z^2)}\right)^2 + \cdots, \\
A & = A_0 + \frac{\alpha_s(Q)}{4\pi} A_1 + \left(\frac{\alpha_s(Q)}{4\pi}\right)^2 A_2 + \cdots
\end{align*}
\]

April 2012

QM: \[ \hat{H} = \hat{H}_0 + \lambda \hat{H}_1 + \cdots, \quad \lambda << \]

Confinement

Asymptotic freedom

Pion DY
Resolution
Notwithstanding these open issues, a rather stable and well-defined world average value emerges from the compilation of current determinations of $\alpha_s$: $\alpha_s(M_Z^2) = 0.1184 \pm 0.0007$.

The results also provide a clear signature and proof of the energy dependence of $\alpha_s$, in full agreement with the QCD prediction of Asymptotic Freedom. This is demonstrated in Fig. 9.4, where results of $\alpha_s(Q^2)$ obtained at discrete energy scales $Q$, now including those based just on NLO QCD, are summarized and plotted.

Figure 9.4: Summary of measurements of $\alpha_s$ as a function of the respective energy scale $Q$. The respective degree of QCD perturbation theory used in the extraction of $\alpha_s$ is indicated in brackets (NLO: next-to-leading order; NNLO: next-to-next-to-leading order; res. NNLO: NNLO matched with resummed next-to-leading logs; N$^3$LO: next-to-NNLO).
DRELL-YAN PROCESSES
Pion-proton collision to a lepton pair

Process called Drell-Yan

That's a pion

That's a proton
bottom panel, to that of the KN05 proton, pared, in the top panel, to the profile of the WLS pion.

remark that the NJL pion transverse distribution in Eq. (12) and the distributions presented in Fig. 3

We now turn to the discussion of the perturbative part of the pion non-perturbative form factor depends on both the hard scale and the parton momenta. A fit is then performed up to different values of the scale $Q^2$. The latter, at variance with the Sudakov form factor, Eq. (19), has a structure similar to the pion's non-perturbative part.

As a matter of fact, in that paper the proton non-perturbative form factor, Eq. (3), has a structure similar to the four-momentum of the proton. Therefore we plot in each panels, as a representative case, the curves corresponding to both form factors.

One may notice that, for this model, the width of the distribution of Ref. [20] is smaller with respect to the NJL one, implying a larger average transverse momentum. In the bottom panel of Fig. 3 we compare the NJL transverse distribution to the pion parametrisation is therefore mainly illustrative and quantitative conclusions can be hardly reached. All the fits of the same cross section data used in the present paper.