Distillation in Lattice QCD Calculations of Nucleon Isovector Charges g_S, g_A, g_T

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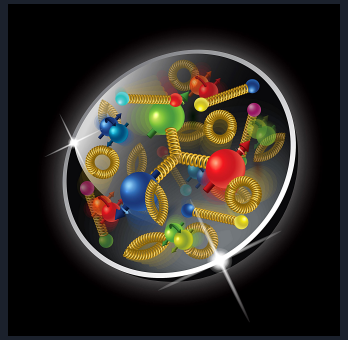
arXiv:1810.09991 [hep-lat]

Spin/Flavor Structure of the Nucleon



- Hadron properties from quarks/gluons

$$\frac{1}{2} = \frac{1}{2}\sum_q \Delta q + \sum_q L_q^z + J_q^z$$

$$\langle r_E^2 \rangle = -\frac{6}{G_E(0)} \frac{\mathrm{d}G_E(Q^2)}{\mathrm{d}Q^2}\Big|_{Q^2=0}$$

- Axial coupling

$$pp \to de^+\nu_e$$
$$n \to pe^-\overline{\nu}_e$$

$$g_A^{u-d}\left(Q^2\right) = \int_0^1 \mathrm{d}x \left[\Delta u^+\left(x, Q^2\right) - \Delta d^+\left(x, Q^2\right)\right]$$

- Transversity measured at SoLID (Hall A) and CLAS (Hall B)

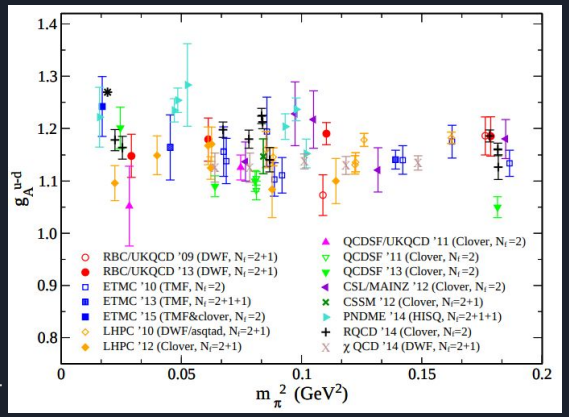
$$\delta q\left(Q^2\right) = \int_0^1 \mathrm{d}x \left[h_1^q\left(x, Q^2\right) - h_1^{\bar{q}}\left(x, Q^2\right)\right] \longrightarrow g_T^{u-d}, g_S^{u-d}$$ & novel BSM interactions

A Benchmark

➔ Flavor non-singlet axial-vector current $A^3_\mu = \overline{\psi}\gamma_\mu\gamma_5\frac{\tau^3}{2}\psi$

$$\langle N(p',s) \mid A^3_\mu \mid N(p,s)\rangle = \overline{u}_N(p',s)\left[\gamma_\mu\gamma_5 G_A^{u-d}(q^2) + \frac{q_\mu}{2M_N}\gamma_5 G_P^{u-d}(q^2)\right]u_N(p,s)$$

$$g_A^{u-d} \equiv G_A^{u-d}(0)$$



M. Constantinou, PoS CD15 (2015) 009, arXiv:1511.00215 [hep-lat]

modified actions & chiral symmetry

H.-W. Lin et al, LA-UR-18-25337, MSUHEP-18-010, arXiv:1806.10604 [hep-lat]
S. Ohta (RBC, UKQCD, LHP), PoS LATTICE2015, 124 (2016), arXiv:1511.05126 [hep-lat]
N. Yamanaka et al. (JLQCD), Phys. Rev. D98, 054516 (2018), arXiv: 1805.10507 [hep-lat]

$\mathcal{O}(a)$ improved currents

J. Liang, et al., Phys. Rev. D96, 034519 (2017), arXiv:1612.04388 [hep-lat]
Y.-B. Yang et al., Phys. Rev. D93, 034503 (2016), arXiv:1509.04616 [hep-lat]

tmQCD - $\mathcal{O}(a)$ improvement at maximal twist

C. Alexandrou et al., Phys. Rev. D95, 114514 (2017), arXiv:1703.08788 [hep-lat]
C. Alexandrou et al., Phys. Rev. D96, 054507 (2017), arXiv:1705.03399 [hep-lat]

2+1 & 2+1+1 flavor-QCD

R. Gupta et al., Phys. Rev. D98, 034503 (2018), arXiv:1806.09006 [hep-lat]
R. Gupta et al., EPJ Web Conf. 175, 06029 (2018), arXiv:1801.03130 [hep-lat]

Recent Reconciliation

C. C. Chang et al., Nature 558, 91 (2018), arXiv:1805.12130 [hep-lat]

$$g_A^{u-d}/g_V^{u-d} = 1.2724(23)$$

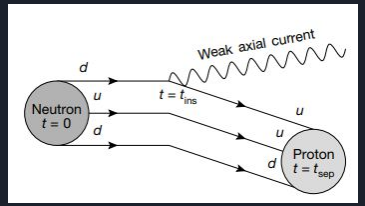
M. Tanabashi et al. (Particle Data Group), Phys. Rev. D98, 030001 (2018).

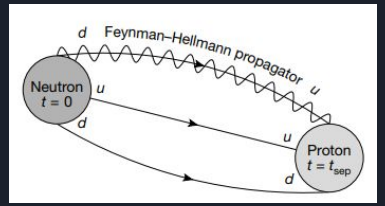
$$C^{2\text{pt}}(t) = \langle \mathcal{O}(t)\,\mathcal{O}(0)^\dagger \rangle = \mathcal{Z}^{-1} \int \mathcal{D}[\psi, \overline{\psi}, A_\mu] e^{-S[\psi,\overline{\psi},A_\mu]}\,\mathcal{O}(t)\,\mathcal{O}(0)^\dagger$$

$$\mathcal{Z}_\lambda^{-1} \int \mathcal{D}[\psi, \overline{\psi}, A_\mu] e^{-(S+S_\lambda)[\psi,\overline{\psi},A_\mu]}\,\mathcal{O}(t)\,\mathcal{O}(0)^\dagger$$

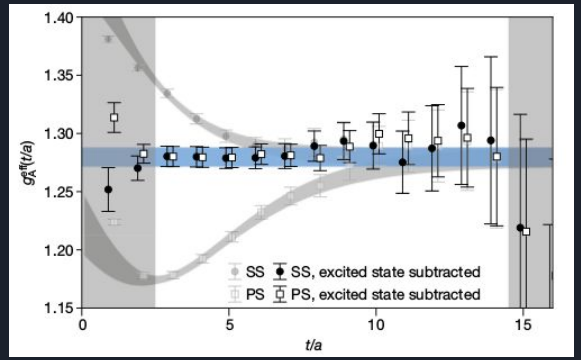
➔ Modify action via $S_\lambda = \lambda \int \mathrm{d}^4x\, j(x)$

$$\frac{\partial m_\lambda^{\text{eff}}(t)}{\partial \lambda}\Big|_{\lambda=0} = \left[\frac{\partial_\lambda C(t)}{C(t)} - \frac{\partial_\lambda C(t+1)}{C(t+1)}\right]\Big|_{\lambda=0} \to g_{00}$$

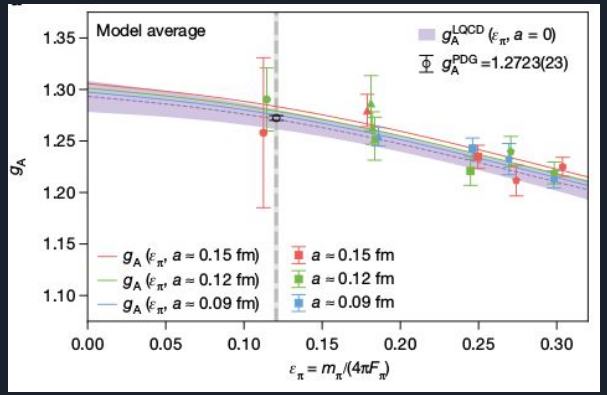




$$S^\Gamma(y,x) = \sum_{(\vec{z},t_z)} S(y,z)\,\Gamma\,S(z,x)$$



Demonstration of summation method



Physical point extrapolation - infinite volume, continuum extrapolation

Outline

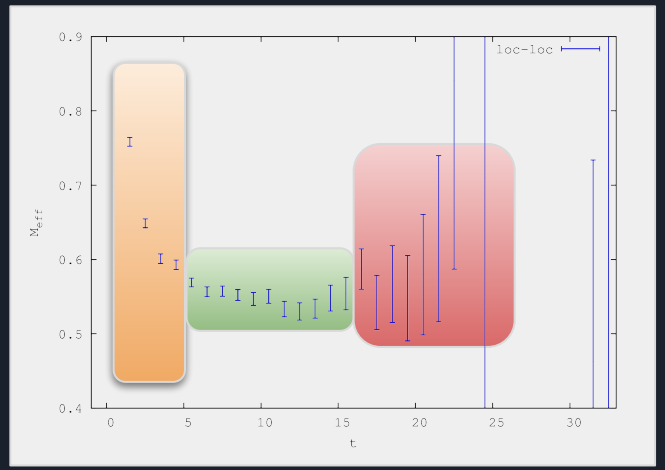
- Baryon correlation functions
- Excited-state contamination
  - Spatial smearing
  - Distillation
- Variational Method
- Interpolator constructions
- Methodology for Extraction
- Results/discussion for $g_S^{u-d}, g_A^{u-d}, g_T^{u-d}$
- Form factors and assorted future directions

A Judicial Choice

- Baryon correlation functions and a decaying signal

$$C(t) = \langle \mathcal{O}(t)\, \mathcal{O}^\dagger(0) \rangle \sim e^{-m_N t}$$
$$\sigma^2_{C(t)} = \langle C(t)\, C^\dagger(t) \rangle - \langle C(t) \rangle^2 \sim e^{-3 m_\pi t}$$

$$C(t)/\sigma^2_{C(t)} \sim e^{-\left(m_N - \frac{3}{2} m_\pi\right) t}$$



- Large Euclidean times insufficient

- Construct better interpolators

$$\langle 0 \mid \mathcal{O} \mid n \rangle \,|_{n>0} \ll \langle 0 \mid \mathcal{O} \mid n = 0 \rangle$$

Smearing

- Point-like interpolators couple to states at all energy scales
  - Confinement-scale physics

One remedy → Spatial “smearing”

$$\tilde{\psi}(\vec{x},t) = \sum_{\vec{y}} S[U](\vec{x},\vec{y})\,\psi(\vec{y},t)$$

$$J_{\sigma,n_\sigma}(t) = \left(1 + \frac{\sigma\nabla^2(t)}{n_\sigma}\right)^{n_\sigma}$$

C. R. Allton et al. (UKQCD), Phys. Rev. D47, 5128 (1993), arXiv 9303009 [hep-lat]

$$\tilde{\psi}(\vec{x},t) = \lim_{n_\sigma\to\infty} J_{\sigma,n_\sigma}(t)\,\psi(\vec{x},t) \to e^{\sigma\nabla^2(t)}\psi(\vec{x},t)$$

➔ Low modes contribute appreciably to $\tilde{\psi}(\vec{x},t)$

Distillation

$$J_{\sigma,n_\sigma} = e^{\sigma\nabla^2} = \sum_\lambda e^{-\sigma\lambda} |\lambda\rangle\langle\lambda|$$

- ➔ Low-mode approximation to some gauge-covariant smearing kernel (e.g. $\mathcal{D}ist = \sum_{i=1}^{N} |\lambda\rangle\langle\lambda|$)
- ➔ Considering the Jacobi-smearing kernel

$$-\nabla^2_{ab}(\vec{x},\vec{y};t) = 6\delta_{xy}\delta_{ab} - \sum_{j=1}^{3}\left[\tilde{U}_j(\vec{x},t)_{ab}\,\delta_{x+\hat{j},y} + \tilde{U}_j^\dagger(\vec{x}-\hat{j},t)_{ab}\,\delta_{x-\hat{j},y}\right]$$

$$-\nabla^2\xi^{(k)} = \lambda^{(k)}\xi^{(k)}$$

- ➔ Define Distillation of $\mathrm{rk}(\mathcal{D}ist) = N \ll N_c \times V_3$

$$\Box(\vec{x},\vec{y};t)_{ab} = \sum_{k=1}^{N}\xi_a^{(k)}(\vec{x},t)\,\xi_b^{(k)\dagger}(\vec{y},t)$$

M. Peardon et al., Phys. Rev. D80, 054506 (2009), arXiv:0905.2160 [hep-lat]

- ➔ Generic interpolator smeared with Distillation

$$\mathcal{O}_i(t) \propto \epsilon^{abc} S_i^{\alpha\beta\gamma}(\mathcal{D}_1\Box u)_a^\alpha(\mathcal{D}_2\Box d)_b^\beta(\mathcal{D}_3\Box u)_c^\gamma(t)$$

When the Dust Settles

$$C_{ij}^{2\mathrm{pt}}(t) = \langle \mathcal{O}_i(t)\, \overline{\mathcal{O}}_j(0) \rangle$$

$$C_{ij,\Gamma}^{3\mathrm{pt}}(t,\tau) = \langle \mathcal{O}_i(t)\, \mathcal{J}(\tau)\, \overline{\mathcal{O}}_j(0) \rangle$$

$$\mathcal{O}_i(t) \propto \epsilon^{abc} S_i^{\alpha\beta\gamma} (\mathcal{D}_1 \Box u)_a^\alpha (\mathcal{D}_2 \Box d)_b^\beta (\mathcal{D}_3 \Box u)_c^\gamma (t)$$

- ➔ Factorization of correlation functions

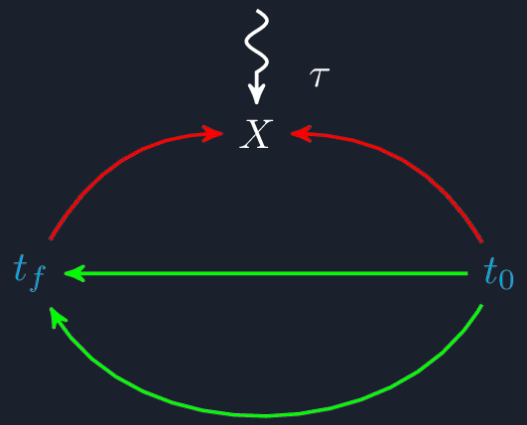
$$C_{ij}^{2\mathrm{pt}}(t) = \Phi_i^{(l,m,n)}(t)\, \Phi_j^{(\bar{l},\overline{m},\overline{n})\dagger}(0) \left[\tau_d^{l\bar{l}}(t,0)\, \tau_u^{m\overline{m}}(t,0)\, \tau_u^{n\overline{n}}(t,0) - \tau_d^{l\bar{l}}(t,0)\, \tau_u^{m\overline{n}}(t,0)\, \tau_u^{n\overline{m}}(t,0) \right]$$

$$C_{ij}^{3\mathrm{pt}}(t,\tau)_d = C_{ij}^{2\mathrm{pt}}(t) \Big|_{\tau_d^{l\bar{l}}(t,0) \mapsto S^l(t,\tau)^{-1} \Gamma(\tau) S^{\bar{l}}(\tau,0)}$$

- ➔ Quark propagation separated from interpolator construction
  - ◆ angular/radial structure of states w/o recalculating $M^{-1}(t',t)$
- ➔ Momentum projection at source/sink

A 3pt Calculation with Distillation

$$\square\,(\vec{x},\vec{y};t)_{ab} = \sum_{k=1}^{N} \xi_a^{(k)}\,(\vec{x},t)\,\xi_b^{(k)\dagger}\,(\vec{y},t)$$



- Solution Vectors

$$S_{\alpha\beta}^{(k)}\,(\vec{x},t';t) = M_{\alpha\beta}^{-1}\,(t',t)\,\xi^{(k)}\,(t)$$

- Perambulators

$$\tau_{\alpha\beta}^{kl}\,(t',t) = \xi^{(k)\dagger}\,(t')\,M_{\alpha\beta}^{-1}\,(t',t)\,\xi^{(l)}\,(t)$$

- Elementals

$$\Phi_{\mu\nu\sigma}^{(i,j,k)}\,(t) = \epsilon^{abc}\left(\mathcal{D}_1\xi^{(i)}\right)^a\left(\mathcal{D}_2\xi^{(j)}\right)^b\left(\mathcal{D}_3\xi^{(k)}\right)^c(t)\,S_{\mu\nu\sigma}$$

Variational Method

- Exploit redundancy of interpolators in a symmetry channel
- Optimal linear combination to project onto $|\mathbf{n}\rangle$

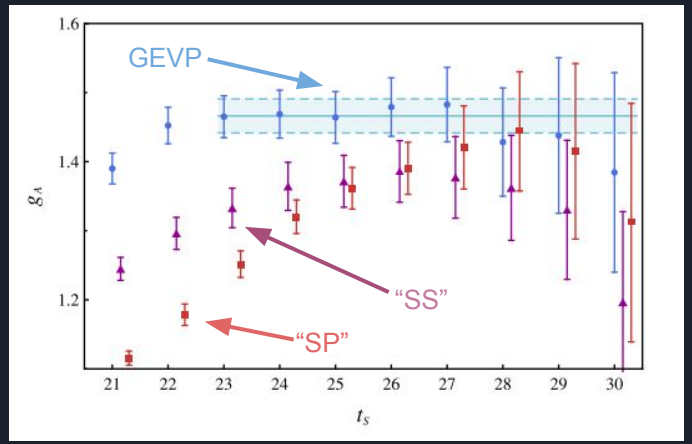
$$C(t)\, v_{\mathbf{n}}(t,t_0) = \lambda_{\mathbf{n}}(t,t_0)\, C(t_0)\, v_{\mathbf{n}}(t,t_0)$$

$$C_{ij}(t') = \langle \mathcal{O}_i(t')\, \mathcal{O}_j^\dagger(0)\rangle$$

$$v_{\mathbf{n}'}^\dagger C(t_0)\, v_{\mathbf{n}} = \delta_{\mathbf{n}',\mathbf{n}}$$

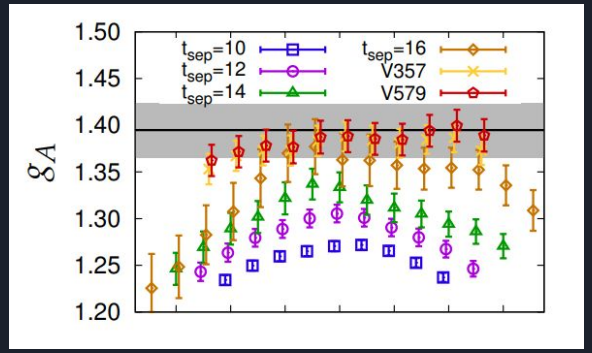
- Fixed t_0 and solved for $t > t_0$
- Solutions yield (organized by $|\lambda_n(t,t_0)|$)
  - "Principal correlator" $\lambda_{\mathbf{n}}(t,t_0) \sim e^{-E_{\mathbf{n}}(t-t_0)}$
  - Interpolator weights $\mathcal{O}_{\mathbf{n}}^{\mathrm{opt}\,\dagger} = \sum_i v_{\mathbf{n}}^i(t,t_0)\, \mathcal{O}_i^\dagger$

Variational Method in Structure Calculations



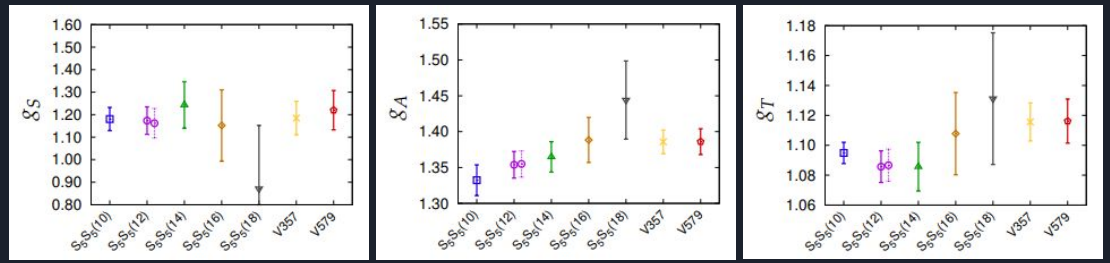
B. J. Owen et al., Phys. Lett. B723, 217 (2013), arXiv:1212.4668 [hep-lat]

$$v_{\mathbf{n}}^{\top} C\left(t_{\text{sep}}, \tau\right) v_{\mathbf{n}}$$



2-state fits vs. variational method

➔ Variational method and better determined matrix elements



B. Yoon et al., Phys. Rev. D93, 114506 (2016), arXiv:1602.07737 [hep-lat]

Approach and Aims

- Investigate inability to isolate ground-state from excitations
- Explore efficacy of four types of operators
  - Jacobi-smeared
  - Distillation
- Employ variational method on two distilled bases
- Forward-limit of z-polarized Nucleons st. $\vec{p} = 0$
- Reliable extrapolation of matrix element for earlier source-sink separations
- Abstain from calculating renormalized charges, and from performing infinite volume, chiral, and continuum limits

Interpolator Construction

- Simplest interpolator consistent with nucleon

$$\mathcal{N}_\alpha(x) = \epsilon^{abc}\left[u^{a\top}(x)\, C\frac{(1\pm\gamma_4)}{2}\gamma_5 d^b(x)\right] u^c_\alpha(x)$$

"Jacobi-SS"

$\sigma = 5.0, n_\sigma = 60$

- Projectors

$$\mathcal{P}^{2\text{pt}} = (1+\gamma_4)/2$$

$$\mathcal{P}^{3\text{pt}} = \mathcal{P}^{2\text{pt}}(1+i\gamma_5\gamma_3)$$

- Analyze projected correlation functions

$$C^{2\text{pt}}(t) = \sum_{\vec{x}} \langle \mathcal{P}^{2\text{pt}}_{\beta\alpha} \mathcal{N}_\alpha(\vec{x},t)\, \overline{\mathcal{N}}_\beta(0)\rangle$$

$$C^{3\text{pt}}(t,\tau) = \sum_{\vec{x},\vec{z}} \langle \mathcal{P}^{3\text{pt}}_{\beta\alpha} \mathcal{N}_\alpha(\vec{x},t)\, \mathcal{O}^{u-d}_\Gamma(\vec{x},\tau)\, \overline{\mathcal{N}}_\beta(0)\rangle$$

Distilled Interpolator Construction

$$\mathcal{O}_i\,(t) \propto \epsilon^{abc}\, S_i^{\alpha\beta\gamma}\,(\mathcal{D}_1\Box u)_a^{\alpha}\,(\mathcal{D}_2\Box d)_b^{\beta}\,(\mathcal{D}_3\Box u)_c^{\gamma}\,(t)$$

$$\left(N_M \otimes \left(\tfrac{1}{2}^+\right)^1_M \otimes D^{[0]}_{L=0,S}\right)^{J^P=\frac{1}{2}^+}$$

$$\left(N_M \otimes \left(\tfrac{3}{2}^+\right)^1_S \otimes D^{[2]}_{L=1,M}\right)^{J^P=\frac{1}{2}^+}$$

$$\left(N_M \otimes \left(\tfrac{3}{2}^+\right)^1_S \otimes D^{[2]}_{L=2,M}\right)^{J^P=\frac{1}{2}^+}$$

$$\left(N_M \otimes \left(\tfrac{1}{2}^+\right)^1_M \otimes D^{[2]}_{L=0,M}\right)^{J^P=\frac{1}{2}^+}$$

$$\left(N_M \otimes \left(\tfrac{1}{2}^+\right)^1_M \otimes D^{[2]}_{L=0,S}\right)^{J^P=\frac{1}{2}^+}$$

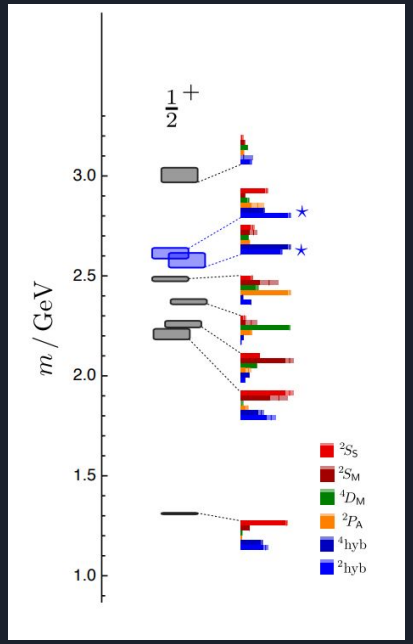
$$\left(N_M \otimes \left(\tfrac{1}{2}^+\right)^1_M \otimes D^{[2]}_{L=1,A}\right)^{J^P=\frac{1}{2}^+}$$

$$\left(N_M \otimes \left(\tfrac{1}{2}^+\right)^1_M \otimes D^{[2]}_{L=1,M}\right)^{J^P=\frac{1}{2}^+}$$

$^2S_S\,\frac{1}{2}^+$

$\hat{\mathcal{P}}_3$

$\hat{\mathcal{P}}_7$



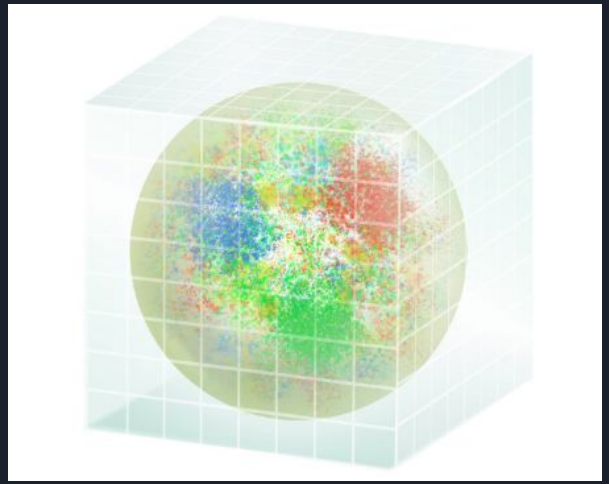
Extracted $\frac{1}{2}^+$ Nucleon states - $m_\pi = 524$ MeV

R. G. Edwards et al., Phys. Rev. D84, 074508 (2011), arXiv:1104.5152 [hep-ph]

J. Dudek and R. Edwards, Phys. Rev. D85, 054016 (2012), arXiv:1201.2349 [hep-ph]

Lattice Parameters

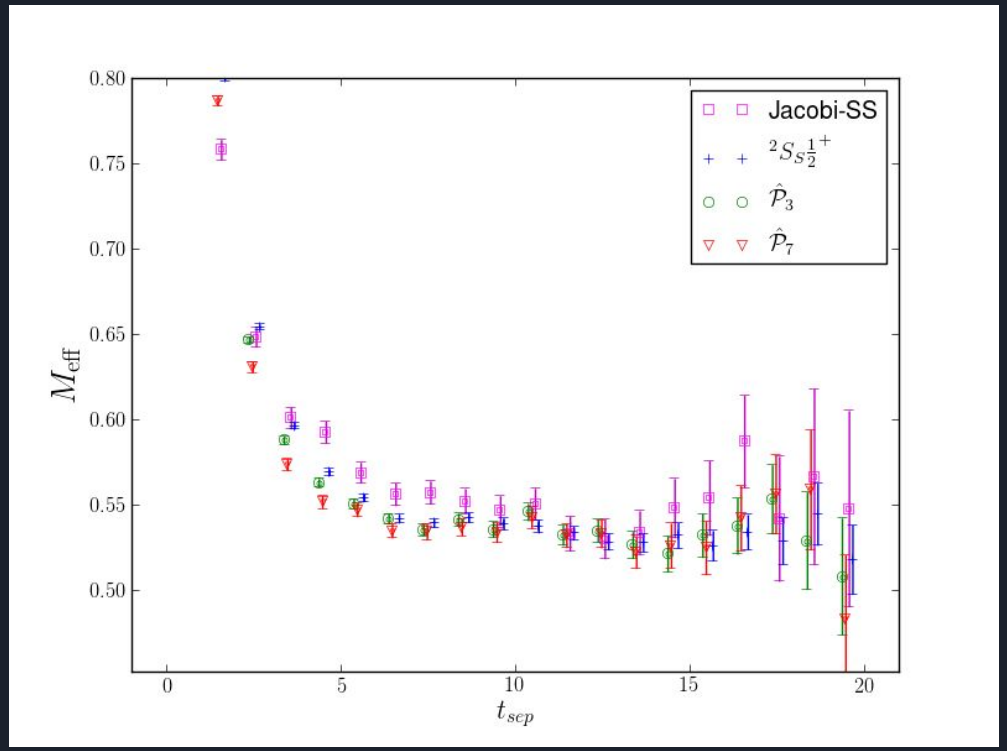
- $32^3 \times 64$ Isotropic lattices
- 2+1 flavor QCD
  - Clover Wilson action - $\mathcal{O}(a)$ improved
- 350 configurations - 10 steps in generating HMC algorithm
- $a = 0.09840(4)\ \mathrm{fm}$ & $\beta = 6.3$
  - $m_\pi \simeq 356\ \mathrm{MeV}$ & $m_\pi L \simeq 5.7$
- $\mathrm{rk}\,(\mathcal{D}ist) = 64$
  - 10 iterations Stout smearing
  - $\rho_{ij} = 0.08 \quad \rho_{\mu 4} = \rho_{4\mu} = 0$



credit: Joanna Griffin (JLab)

Effective Mass

$$M_{\text{eff}}\left(t_{\text{sep}}+0.5\right)=\frac{1}{a}\ln\frac{C^{2\text{pt}}\left(t_{\text{sep}}\right)}{C^{2\text{pt}}\left(t_{\text{sep}}+1\right)}$$



*Averaged over 3 source positions

Effective Mass Fits

$$C^{2\text{pt}}(t) = |\mathbf{a}|^2 e^{-M_0 t} + |\mathbf{b}|^2 e^{-M_1 t}$$

$\hat{\mathcal{O}}$	t_{fit}	$\lvert\mathbf{a}\rvert^2$	M_0	$\lvert\mathbf{b}\rvert^2$	M_1	χ^2_r
Jacobi-SS	[2,16] [3,16] [4,16]	4.12(25)e-08 3.81(42)e-08 4.14(48)e-08	0.534(6) 0.536(9) 0.54(01)	3.70(25)e-08 3.12(35)e-08 5.3(5.9)e-08	1.04(08) 0.91(11) 1.13(42)	0.842 0.753 0.667
${}^2S_S\,\frac{1}{2}^+$	[2,16] [3,16] [4,16]	1.45(02)e-02 1.43(03)e-02 1.42(04)e-02	0.536(2) 0.534(2) 0.534(3)	1.69(06)e-02 1.35(14)e-02 1.31(52)e-02	1.25(03) 1.14(06) 1.13(15)	1.535 1.268 1.407
$\hat{\mathcal{P}}_3$	[2,16] [3,16] [4,16]	1.07(1)e+00 1.05(2)e+00 1.05(3)e+00	0.536(2) 0.535(2) 0.535(2)	1.21(7)e+00 0.935(157) 1.00(63)e+00	1.32(04) 1.21(08) 1.23(21)	1.159 1.069 1.185
$\hat{\mathcal{P}}_7$	[2,16] [3,16] [4,16]	1.00(1)e+00 0.98(2)e+00 0.98(4)e+00	0.535(2) 0.533(2) 0.533(3)	1.08(13)e+00 0.68(24)e+00 0.48(53)e+00	1.43(08) 1.23(17) 1.13(39)	0.737 0.668 0.729

Matrix Element Extraction

- Degree of excited-state contamination

$$g_{\text{eff}}^{\Gamma}\left(t_{\text{sep}}, \tau\right) = \frac{C_{\Gamma}^{3\text{pt}}\left(t_{\text{sep}}, \tau\right)}{C_{\text{fit}}^{2\text{pt}}\left(t_{\text{sep}}\right)}$$

"Effective" matrix element $\mathcal{M}_{\text{eff}}$

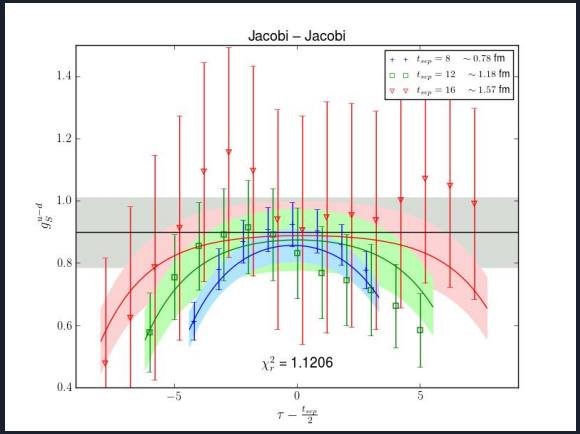
- Simultaneous fits

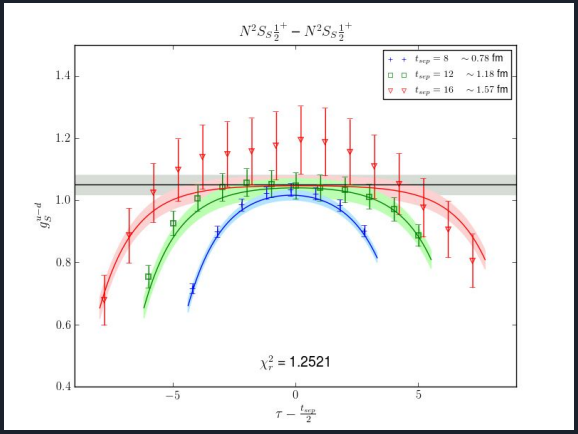
$$C_{\text{fit}}^{2\text{pt}}(t) = e^{-M_0 t}\left[|\mathbf{a}|^2 + |\mathbf{b}|^2 e^{-(M_1 - M_0)t}\right]$$

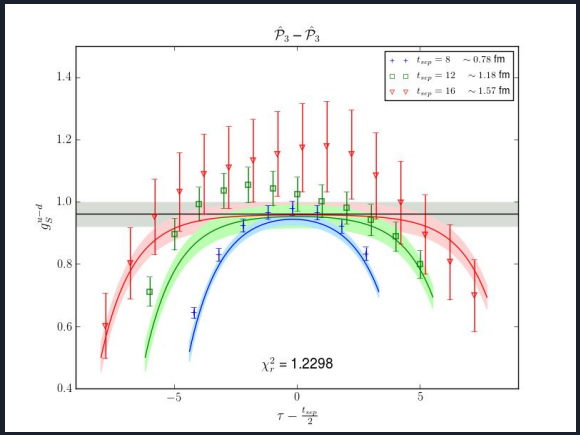
$$C_{\text{fit}}^{3\text{pt}}\left(t_{\text{sep}}, \tau\right) = e^{-M_0 t_{\text{sep}}}\left(\mathcal{A} + \mathcal{B} e^{-\Delta m t_{\text{sep}}} + \mathcal{C} e^{-\Delta m t_{\text{sep}}/2} \cosh\left[\Delta m\left(\tau - t_{\text{sep}}/2\right)\right]\right)$$

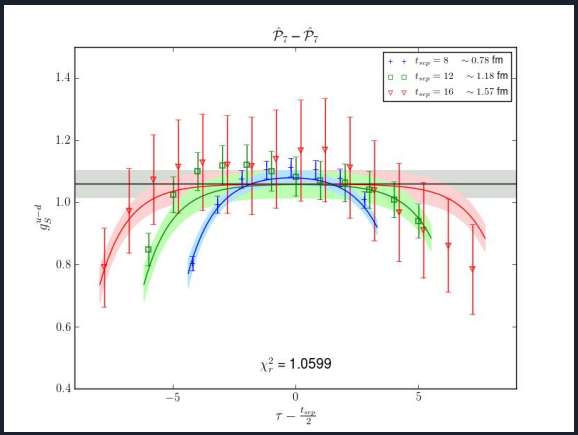
- Simultaneous fit windows $t_{\text{sep}} \in \{8, 12, 16\}$
  - $t_{2\text{pt}}^{\text{fit}}/a \in [2, 16]$
  - $\tau_{\text{fit}} \in \left[0 + \tau_{\text{buff}}, t_{\text{sep}} - \tau_{\text{buff}}\right]$

$g_S^{u-d} \quad - \quad \tau_{\text{buff}} = 2$

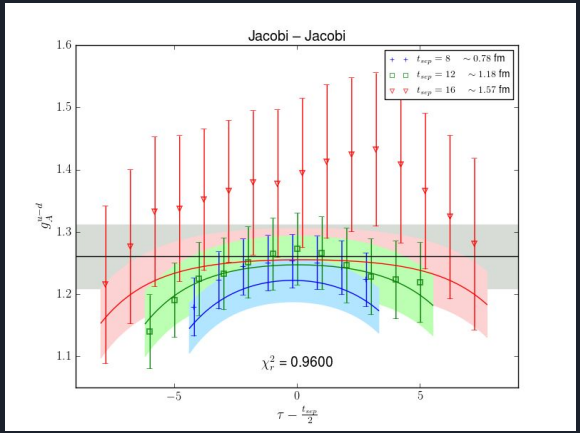


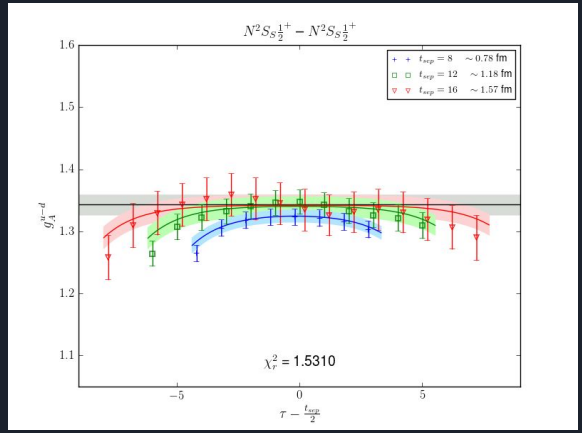


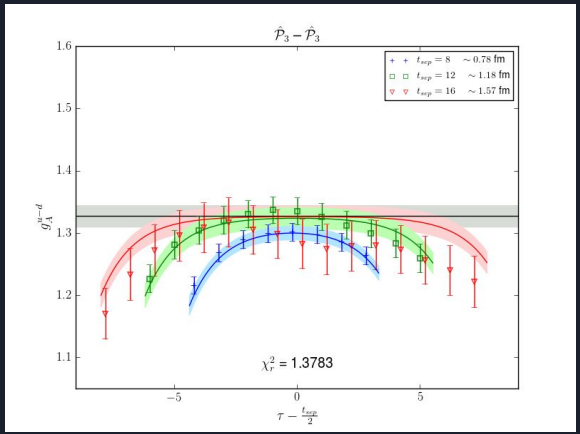


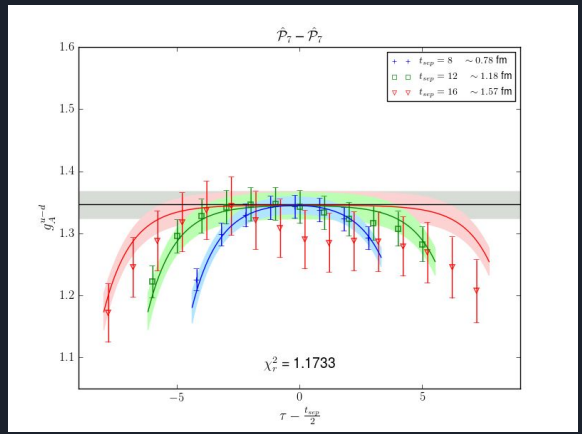


$g_A^{u-d} \quad - \quad \tau_{\text{buff}} = 2$

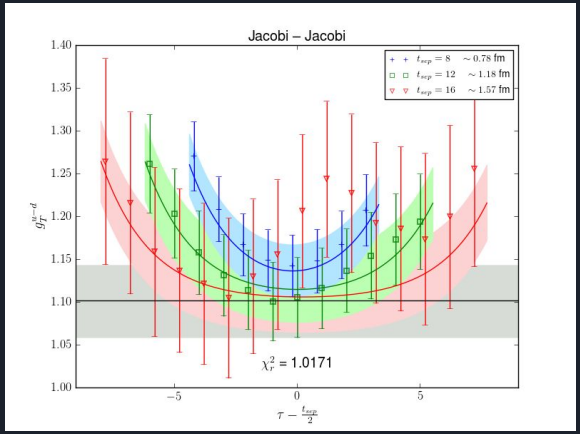


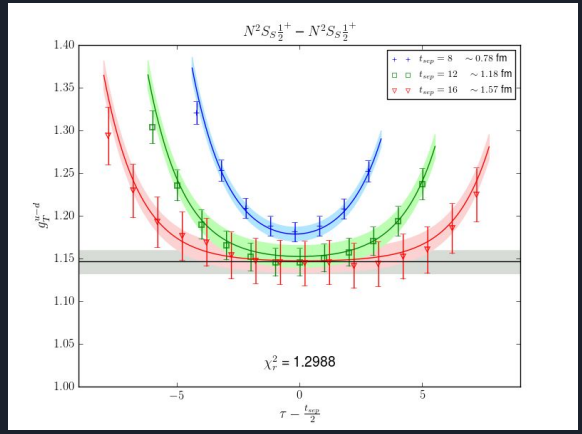


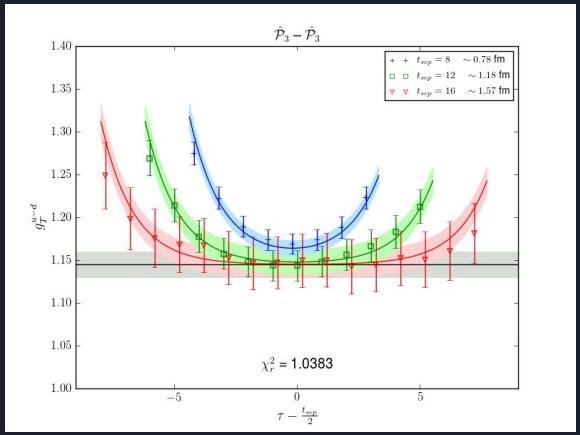


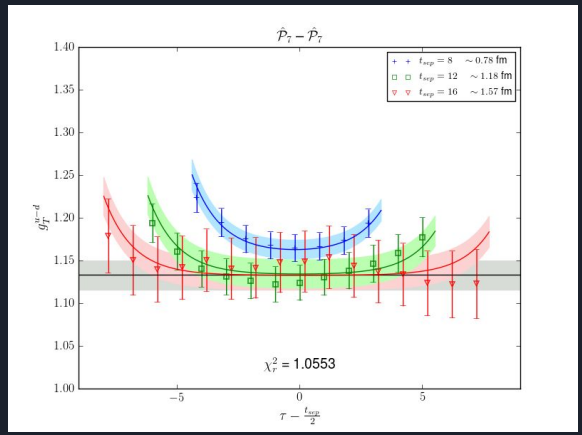


$g_T^{u-d} \quad - \quad \tau_{\text{buff}} = 2$

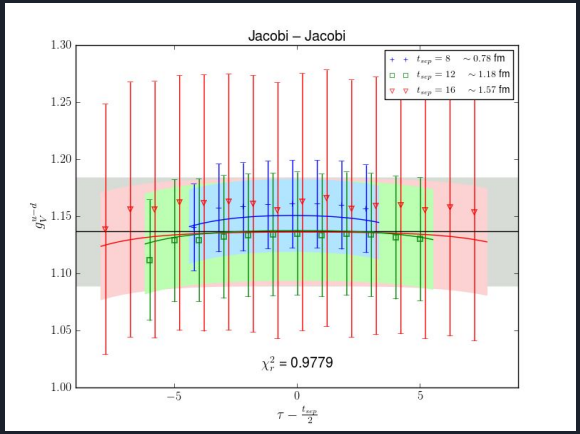


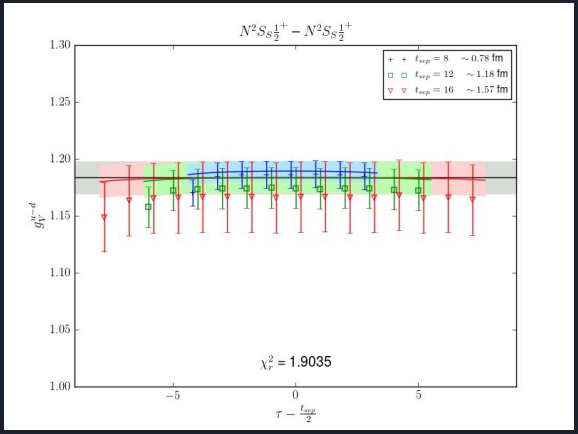


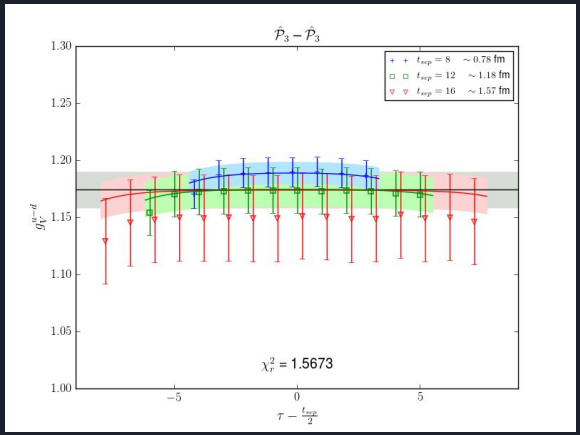


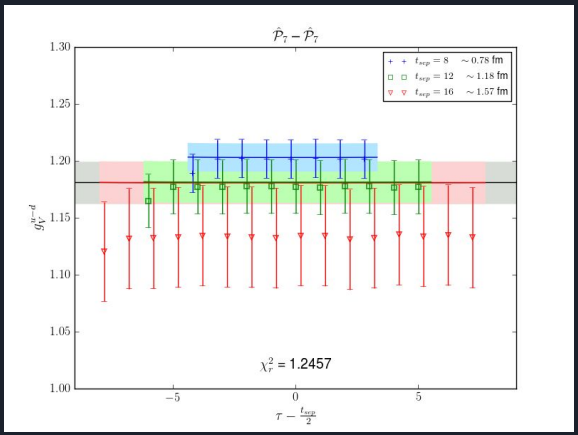


$g_V^{u-d} \quad - \quad \tau_{\text{buff}} = 2$









Effects of Distillation

- Substantial reduction in statistical uncertainty
  - momentum projections $\{t_{\rm src}, t_{\rm snk}, \tau\}$
- Control of excited-states
  - suppression of excited-states *viz* g_T^{u-d}
- Variationally-optimized interpolators
  - less dramatic over ${}^2S_S\,\frac{1}{2}^+$
  - $\hat{\mathcal{P}}_3$ some taming of excited-states
  - $\hat{\mathcal{P}}_7$ Fidelity and consistency in $\mathcal{M}_{\rm eff}$
    - extraction of charges possible for $t_{\rm sep} \sim 0.78\ {\rm fm}$
  - no additional Dirac inversions...

Numerical Costs

- Point-to-all propagators $S(\vec{x},\vec{z})_{ba}^{\beta\alpha} = \sum_{\vec{y},\gamma,c} D^{-1}(\vec{x},\vec{y})_{bc}^{\beta\gamma}\,\delta(\vec{y}-\vec{z})\,\delta_{\gamma\alpha}\delta_{ca}$
  - 12 per source
  - sequential props $N_{\mathrm{src}}\left[12+24N_{\mathrm{seps}}\right]N_{\mathrm{cfg}}$
  - 2pt GEVP $N_{\mathrm{src}}\left[12\left(1+N_{\mathrm{ops}}\right)+24N_{\mathrm{seps}}\right]N_{\mathrm{cfg}}$
  - 3pt GEVP $N_{\mathrm{src}}\,N_{\mathrm{ops}}\left[12+24N_{\mathrm{seps}}\,N_{\mathrm{ops}}\right]N_{\mathrm{cfg}}$ $\mathcal{O}\left(10^4\right)$
- Solution vectors

$$S_{\alpha\beta}^{(k)}(\vec{x},t';t) = D_{\alpha\beta}^{-1}(t',t)\,\xi^{(k)}(t)$$

$3N_{\mathrm{src}}\,N_{\mathrm{eigs}}\left(1+N_{\mathrm{seps}}\right)N_{\mathrm{cfg}}$

$\mathcal{O}\left(2000\right)$

- Construction/Contractions

$\mathcal{O}\left(V_3^2\right)$ perambulators

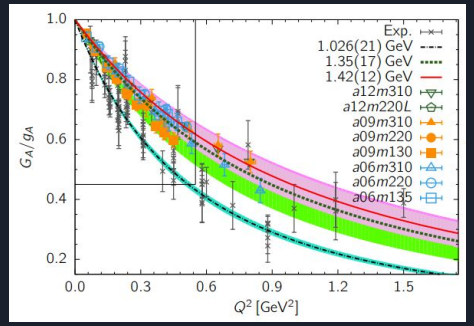
$\mathcal{O}\left(V_3^3\right)$ elementals

Closing Thoughts

- ➔ Forward-scattering of $\vec{p} = \vec{0}$ nucleons
  - ◆ $\vec{p} \neq \vec{0}$ states with $q^2 = 0$
- ➔ Form Factors (JLab at high Q^2)
  - ◆ $\nu/\bar{\nu}$ oscillation parameters & $\sigma_{\nu N}$
  - ◆ $\mathcal{M}_{\nu N} \rightarrow$ nuclear models
- ➔ Flavor-diagonal charges
  - ◆ e.g. qEDMs → nEDM
- ➔ A new era in Lattice calculations
  - ◆ e.g. pseudo-/quasi-PDFs & GLCS
  - ◆ Need for control over all systematics is paramount

$g_{A_i^3} = g_{A_4^3}$?

J. Liang et al., Phys. Rev. D96, 034519 (2017), arXiv:1612.04388 [hep-lat]



R. Gupta et al., Phys. Rev. D96, 114503, arXiv:1705.06834 [hep-lat]

$$\mathcal{P}\left(x, -z^2\right) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathrm{d}\nu \, e^{-ix\nu} \mathcal{M}\left(\nu, -z^2\right)$$

$$\sigma_n\left(\omega, \xi^2, P^2\right) = \sum_a \int_{-1}^{1} \frac{\mathrm{d}x}{x} f_a\left(x, \mu^2\right) K_n^a\left(x\omega, \xi^2, x^2 P^2, \mu^2\right) + \mathcal{O}\left(\xi^2 \Lambda_{\mathrm{QCD}}^2\right)$$

Correlator Behavior

$$C^{2\text{pt}}(t) = 2\sum_n |Z_n|^2 e^{-M_n t}$$

$$C^{2\text{pt}}_{\text{fit}}(t) = e^{-M_0 t}\left[|\mathbf{a}|^2 + |\mathbf{b}|^2 e^{-(M_1 - M_0)t}\right]$$

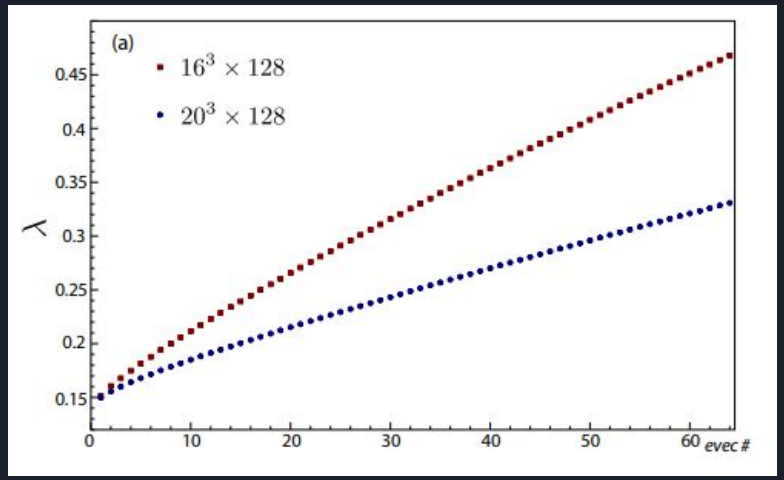
$$C^{3\text{pt}}(t,\tau) = \sum_{\vec{x},\vec{z}} \langle \mathcal{P}^{3\text{pt}}_{\beta\alpha} \mathcal{N}_\alpha(\vec{x},t)\, \mathcal{O}^{u-d}_\Gamma(\vec{x},\tau)\, \overline{\mathcal{N}}_\beta(0)\rangle$$

$$C^{3\text{pt}}(t_{\text{sep}},\tau) = \left(\frac{|Z_0|^2}{4M_0^2}\mathcal{J}_{00} e^{-M_0 t_{\text{sep}}} + \frac{|Z_1|^2}{4M_1^2}\mathcal{J}_{11} e^{-M_1 t_{\text{sep}}}\right) + \frac{Z_0 Z_1}{2M_0 M_1}\mathcal{J}_{01} e^{-\frac{(M_1+M_0)}{2} t_{\text{sep}}} \cosh\left[(M_1 - M_0)(\tau - t_{\text{sep}}/2)\right]$$

$$C^{3\text{pt}}_{\text{fit}}(t_{\text{sep}},\tau) = e^{-M_0 t_{\text{sep}}}\left(\mathcal{A} + \mathcal{B} e^{-\Delta m t_{\text{sep}}} + \mathcal{C} e^{-\Delta m t_{\text{sep}}/2}\cosh\left[\Delta m(\tau - t_{\text{sep}}/2)\right]\right)$$

➔ Use of distillation introduces overall factors of lattice spatial volume V_3

Scaling of Distillation Space



Laplacian eigenvalues on differing spatial extents

M. Peardon et al., Phys. Rev. D80, 054506 (2009), arXiv:0905.2160 [hep-lat]

- ➔ Resolution of distillation space scales with V_3

$$\xi_a^{(k)}(\vec{x},t) \to N_D \times (N_c \times V_3)$$

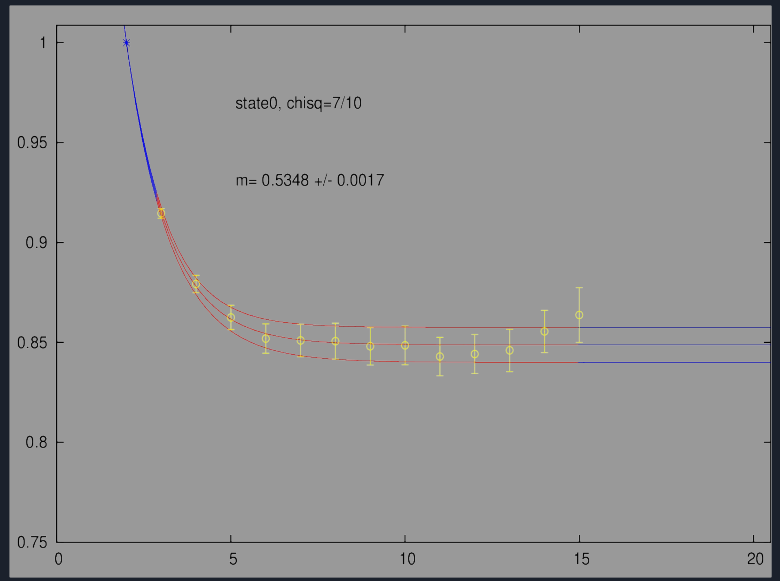
- ➔ Elemental construction

$$\Phi_{\alpha\beta\gamma}^{(i,j,k)}(t) = \epsilon^{abc} S_{\alpha\beta\gamma} \left(\mathcal{D}_1 \xi^{(i)}\right)_\alpha^a \left(\mathcal{D}_2 \xi^{(j)}\right)_\beta^b \left(\mathcal{D}_3 \xi^{(k)}\right)_\gamma^c \sim \mathcal{O}\left(V_3^3\right)$$

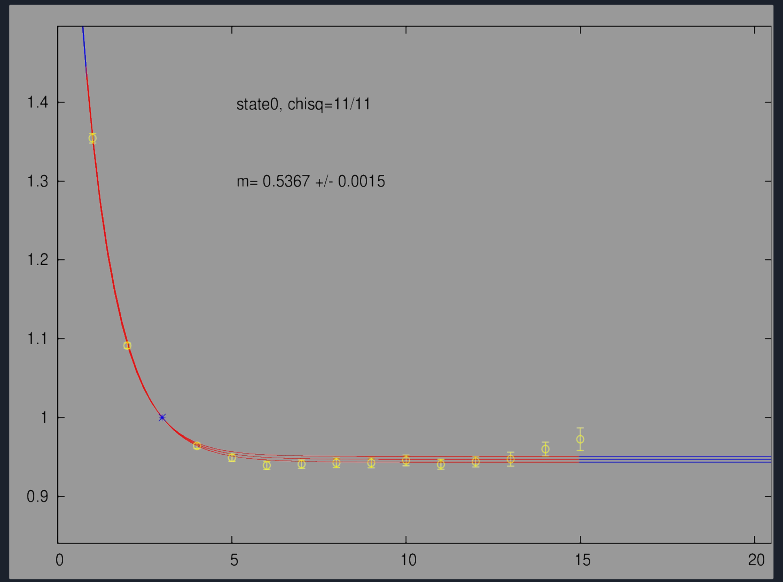
- ➔ Perambulator construction

$$\tau_{\alpha\beta}^{kl}(t',t) = \xi^{(k)\dagger}(t') M_{\alpha\beta}^{-1}(t',t) \xi^{(l)}(t) \sim \mathcal{O}\left(V_3^2\right)$$

Principal Correlators



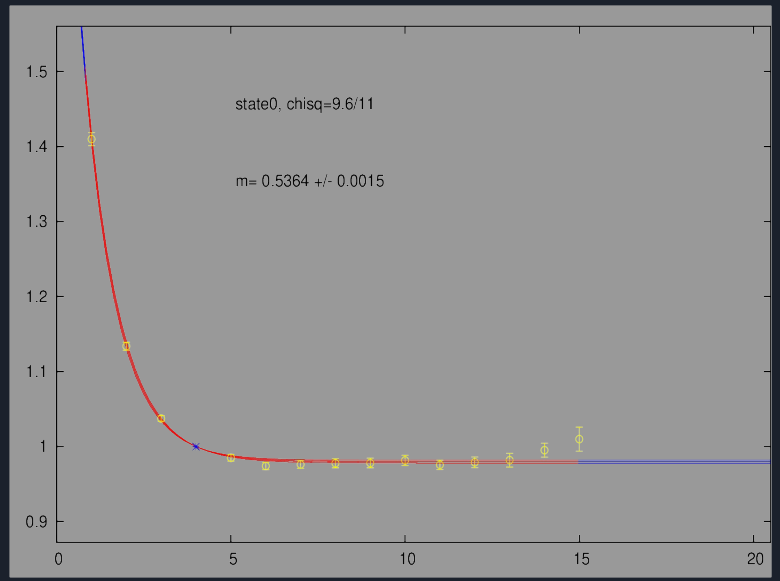
$\lambda_{n=0} \qquad t_0 = 2, t_Z = 5$



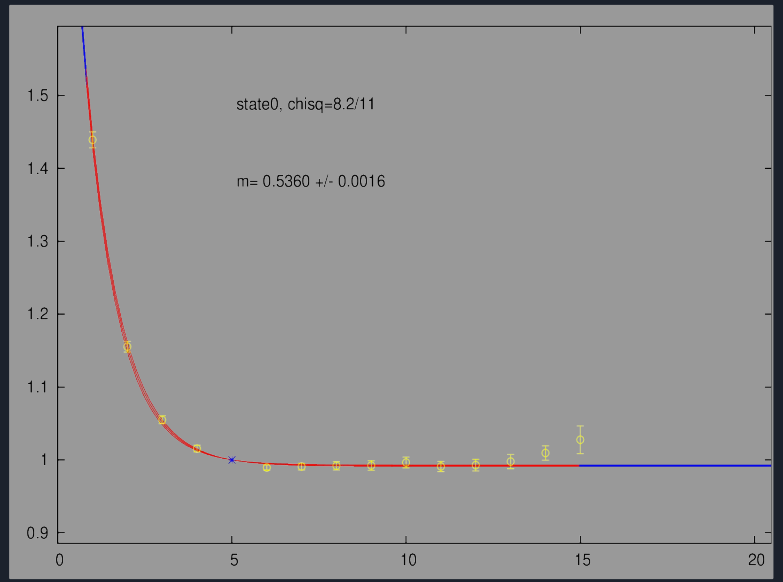
$\lambda_{n=0} \qquad t_0 = 3, t_Z = 5$

$$\lambda_{\mathbf{n}}(t) = (1 - A_{\mathbf{n}})\, e^{-m_{\mathbf{n}}(t-t_0)} + A_{\mathbf{n}} e^{-m_{\mathbf{n}'}(t-t_0)}$$

Principal Correlators



$\lambda_{n=0}$ $t_0 = 4, t_Z = 5$



$\lambda_{n=0}$ $t_0 = 5, t_Z = 7$

$$\lambda_{\mathbf{n}}(t) = (1 - A_{\mathbf{n}})\, e^{-m_{\mathbf{n}}(t-t_0)} + A_{\mathbf{n}} e^{-m_{\mathbf{n}'}(t-t_0)}$$