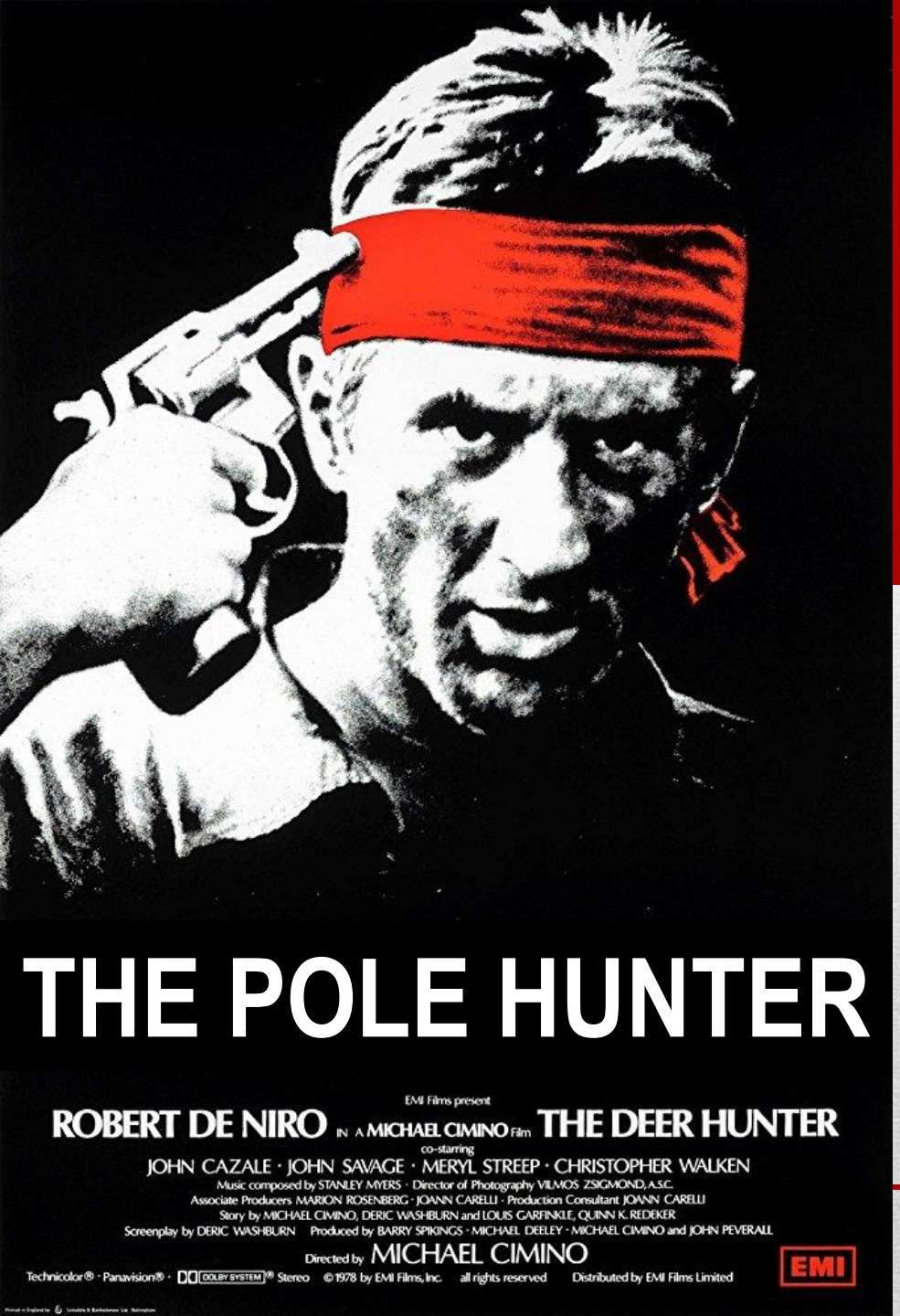
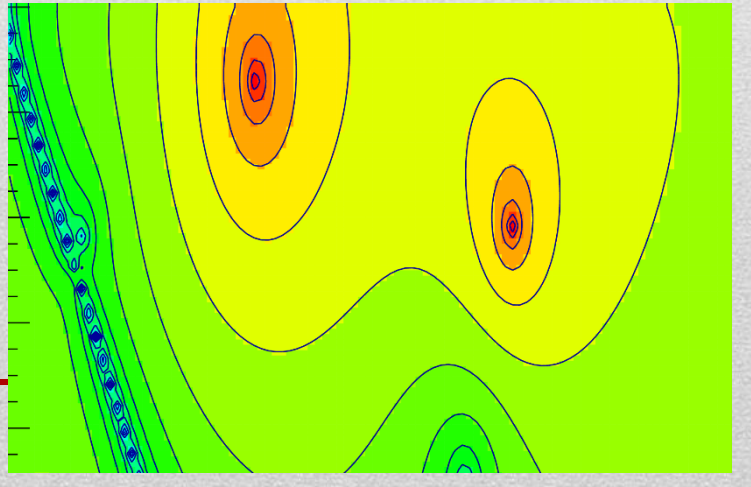


Alessandro Pilloni

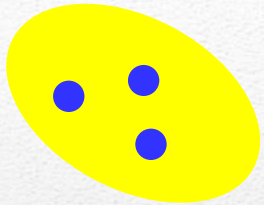
Cake Seminar, October 24th 2018



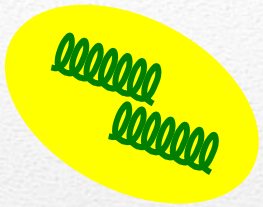
Hadron Spectroscopy

Meson

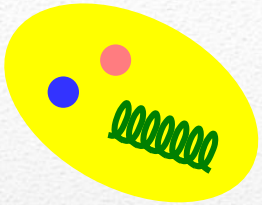
Baryon



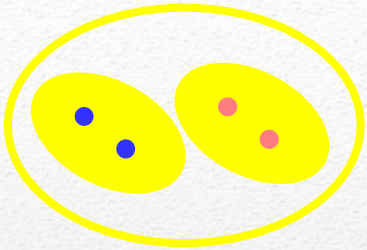
Glueball

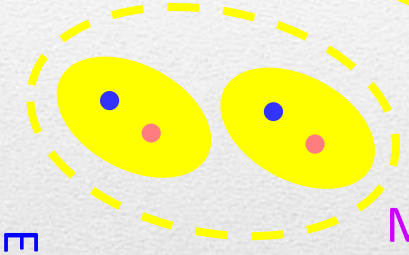


Hybrids

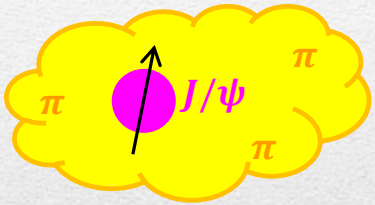


Tetraquark



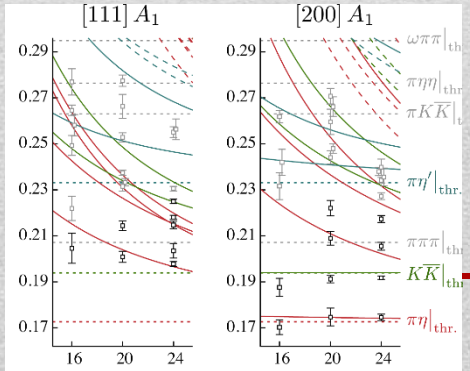


Molecule



Hadroquarkonium

Experiment

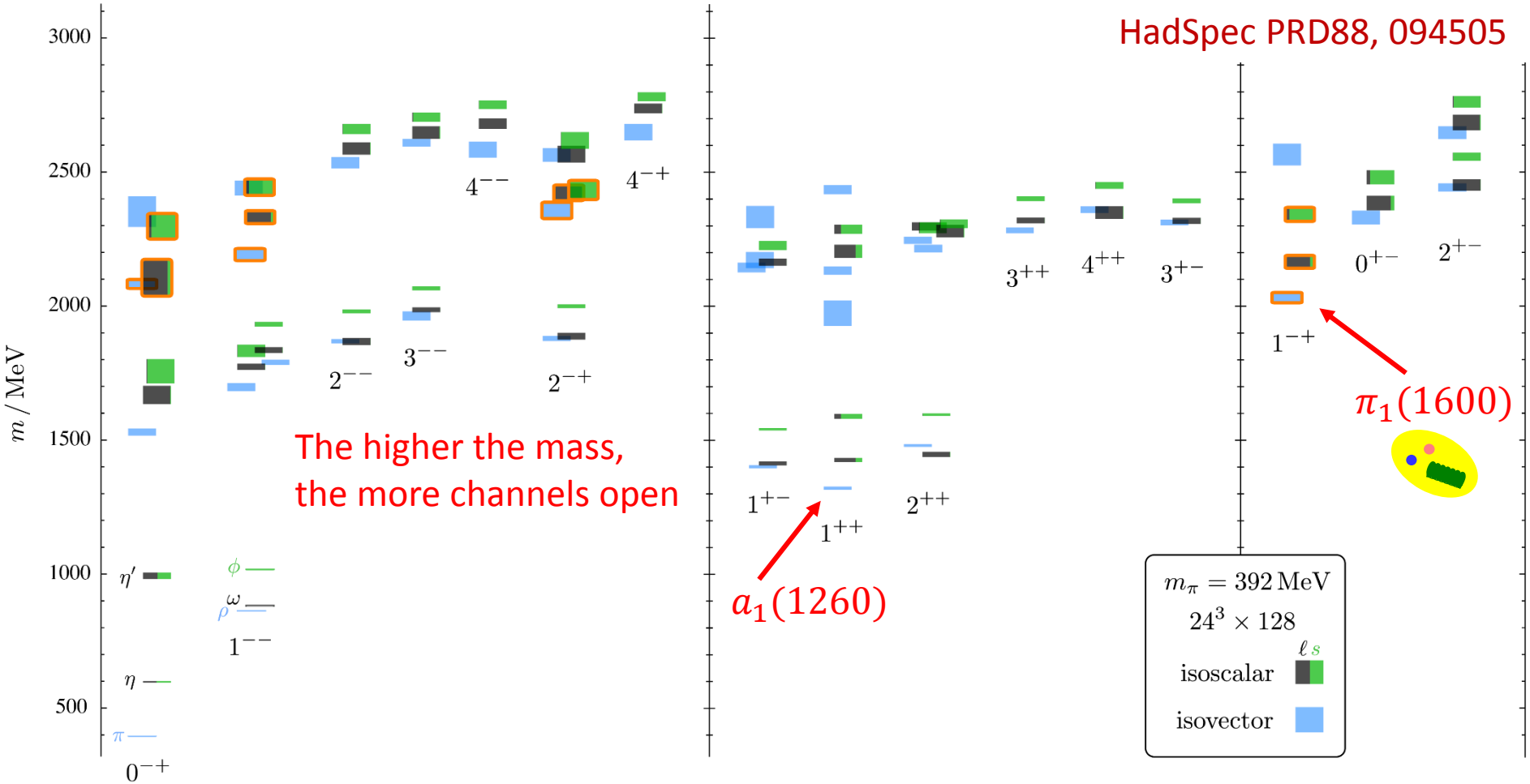


Lattice QCD

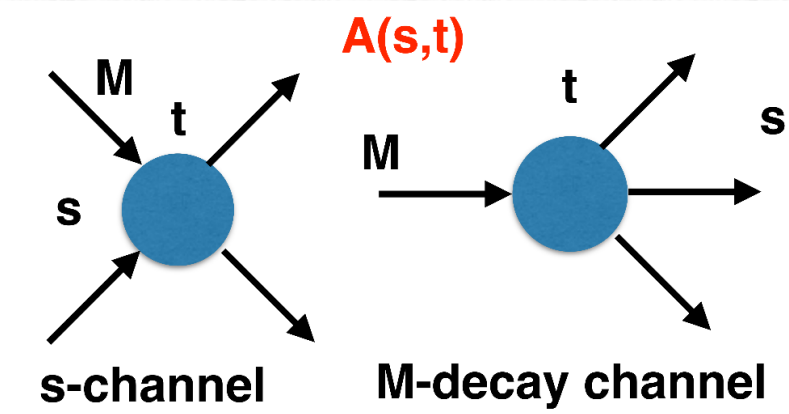
Data → Amplitude analysis → Properties, Model building

Interpretations on the spectrum leads to understanding fundamental laws of nature

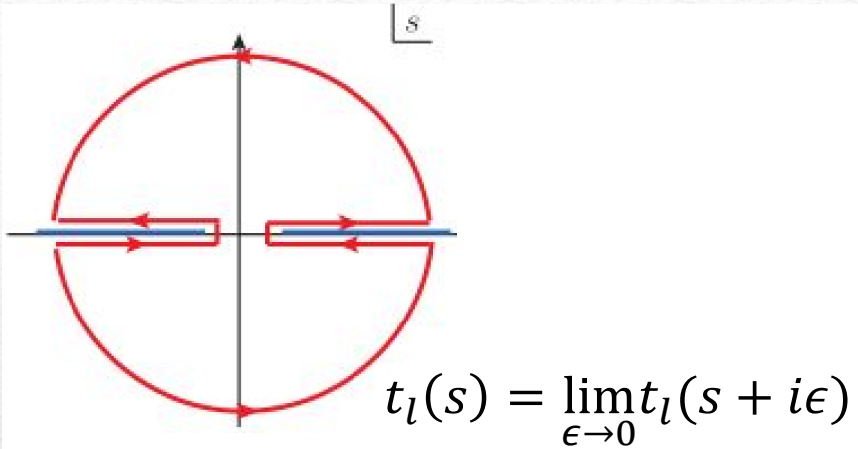
Light spectrum (1-particle correlators)



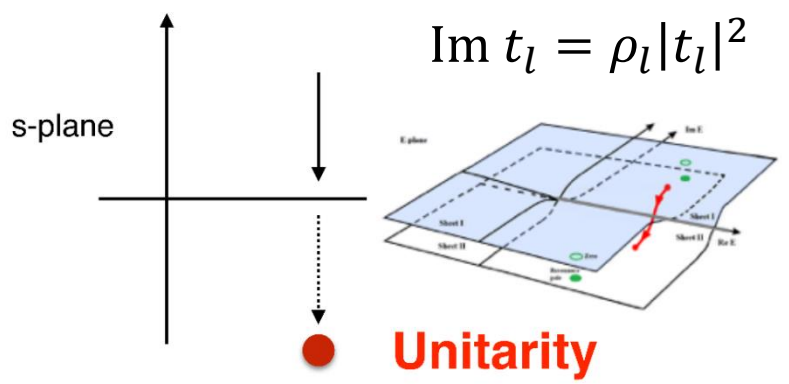
S-Matrix principles



Crossing



Analyticity



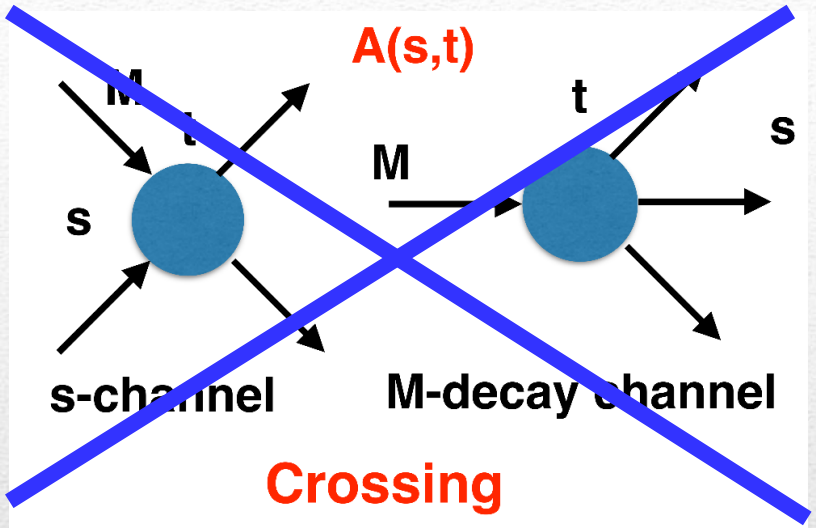
Unitarity

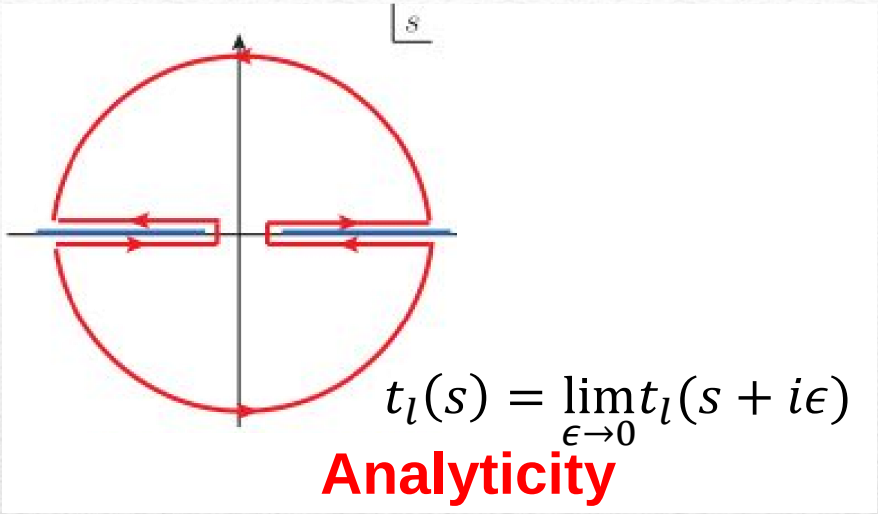
These are constraints the amplitudes have to satisfy, but do not fix the dynamics

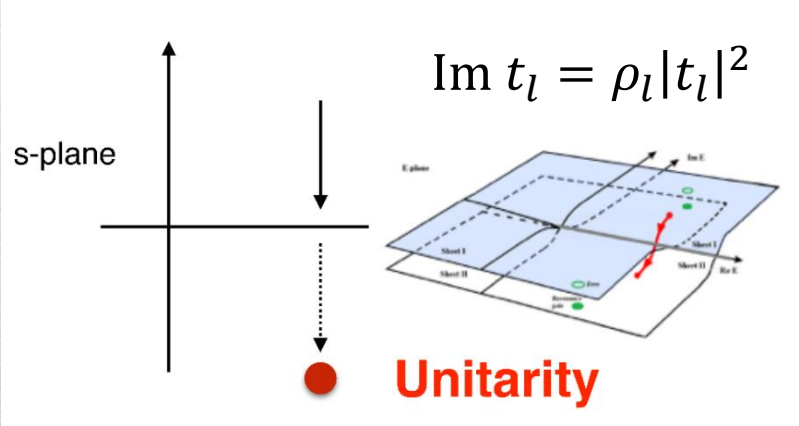
Impose the constraints, parameterize the ignorance, extract the physics

+ Lorentz, discrete & global symmetries

S-Matrix principles







Unitarity opens a cut on the real axis

Resonances (QCD states) are poles in the unphysical Riemann sheets

Unitarity controls the interference pattern between resonances and background

+ Lorentz, discrete & global symmetries

The hybrid π_1

Hybrid hunting

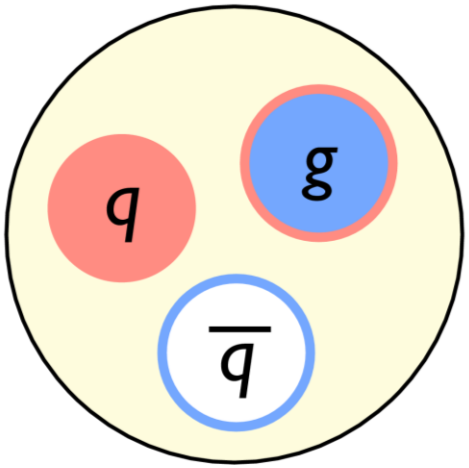
$\bar{\psi}\,\gamma^{\mu}F_{\mu\nu}\psi$ smells of glue

"gluonic field"

$(J^{PC})_g = 1^{+-}$

mass ≈ 1.0-1.5 GeV

color-octet $q\bar{q}$ pair

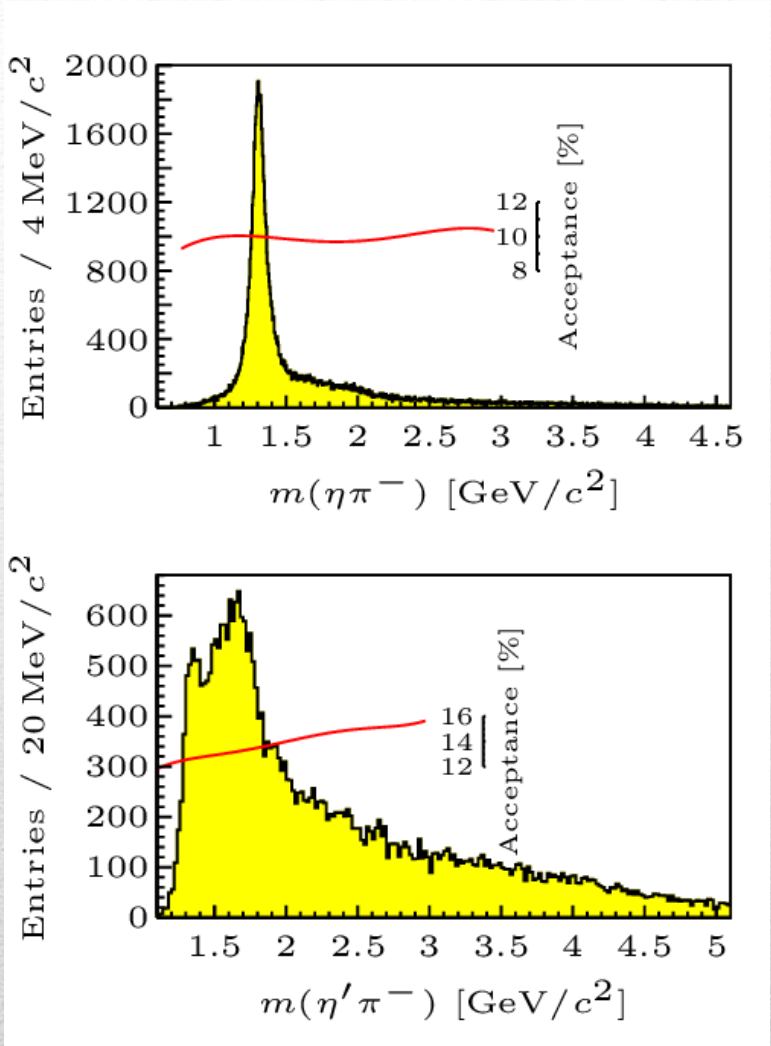


Lightest Hybrids

	$S_{q\bar{q}} = 1$	$S_{q\bar{q}} = 0$
J^{PC}:	0^{-+}, 1^{-+}, 2^{-+}	1^{--}

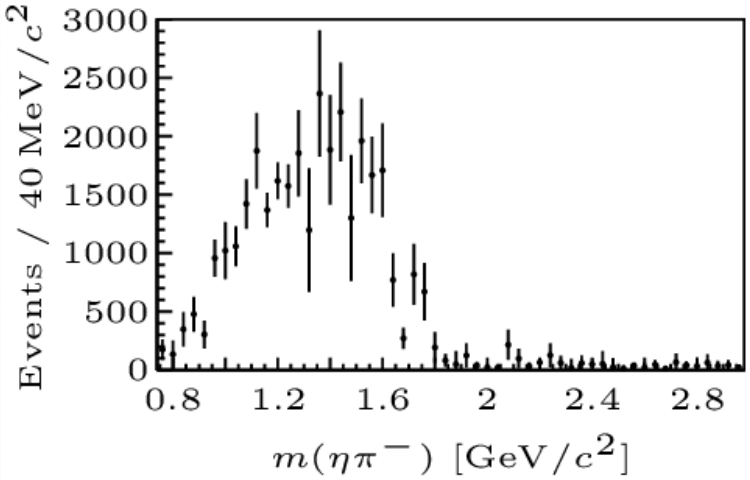
"exotic hybrid"

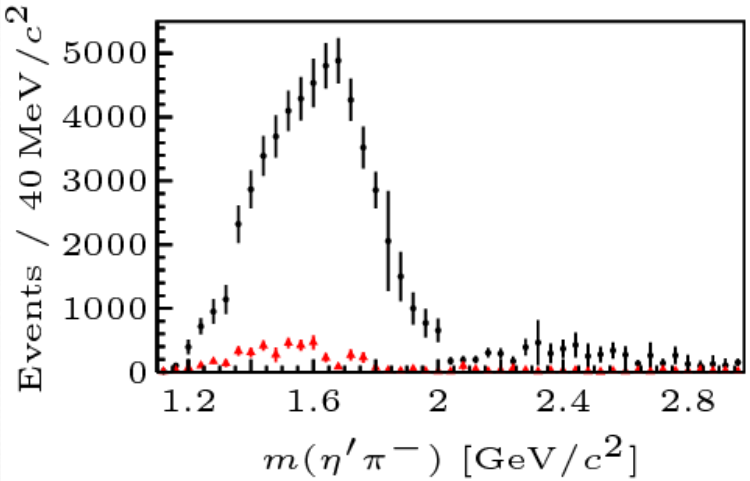
M. R. Shepherd
Hadron 2017, Salamanca
September 28, 2017



Small signal in data, requires refined PWA

Two hybrid states?





$\pi_1(1400)$ $I^G(J^{PC}) = 1^-(1^{-+})$

See also the mini-review under non-$q\bar{q}$ candidates in PDG 2006 , Journal of Physics G33 1 (2006).

$\pi_1(1400)$ MASS	1354 ± 25 MeV (S = 1.8)
$\pi_1(1400)$ WIDTH	330 ± 35 MeV

Decay Modes

Mode		Fraction (Γ_i/Γ)	Scale Factor/ Conf. Level
Γ_1	$\eta\pi^0$	seen	
Γ_2	$\eta\pi^-$	seen	
Γ_3	$\eta'\pi$		

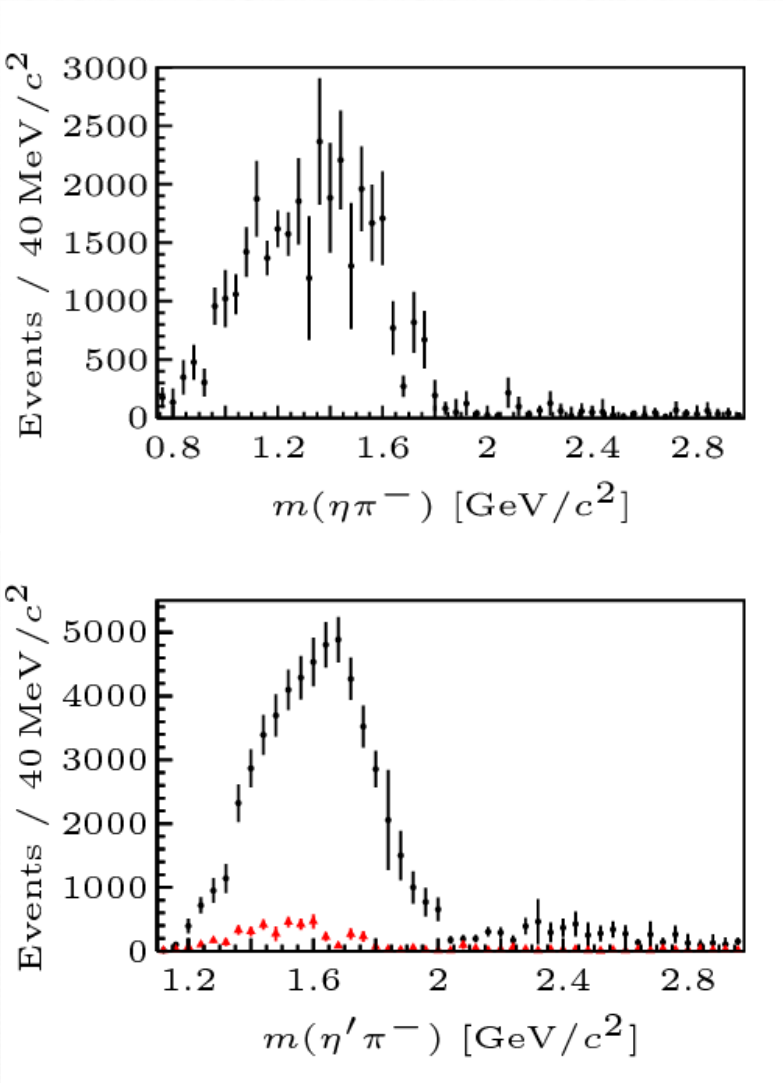
$\pi_1(1600)$ $I^G(J^{PC}) = 1^-(1^{-+})$

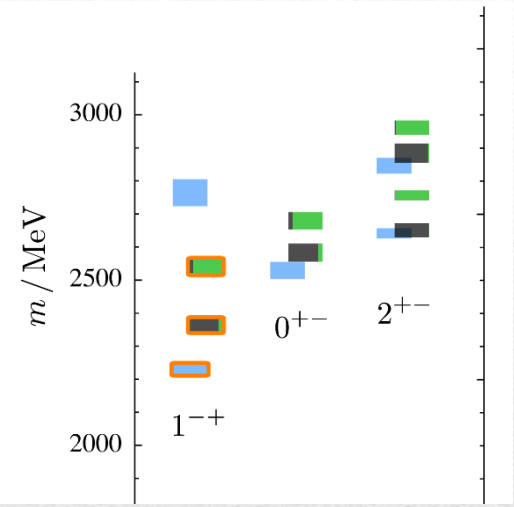
$\pi_1(1600)$ MASS	1662^{+8}_{-9} MeV
$\pi_1(1600)$ WIDTH	241 ± 40 MeV (S = 1.4)

Decay Modes

Mode		Fraction (Γ_i/Γ)	Scale Factor/ Conf. Level
Γ_1	$\pi\pi\pi$	seen	
Γ_2	$\rho^0\pi^-$	seen	
Γ_3	$f_2(1270)\pi^-$	not seen	
Γ_4	$b_1(1235)\pi$	seen	
Γ_5	$\eta'(958)\pi^-$	seen	
Γ_6	$f_1(1285)\pi$	seen	

Two hybrid states?





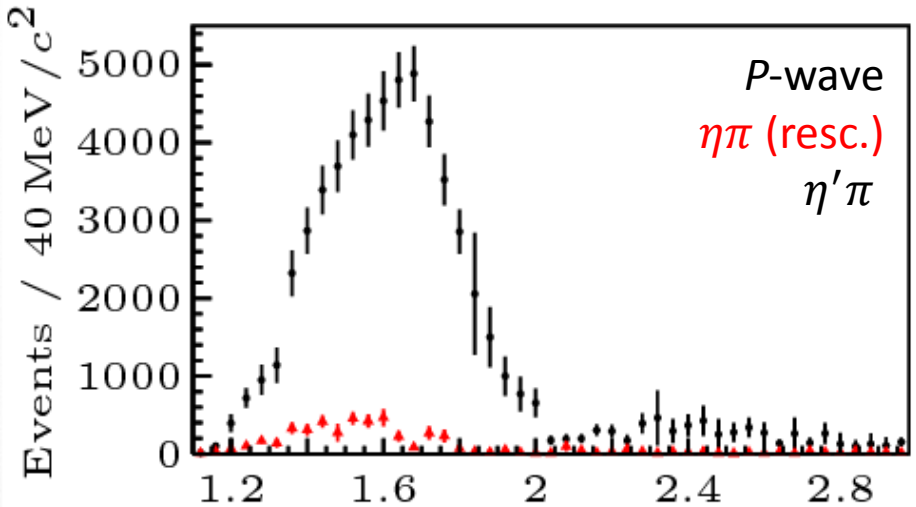
Neither lattice nor models predict two isovector 1^{-+} states in that region

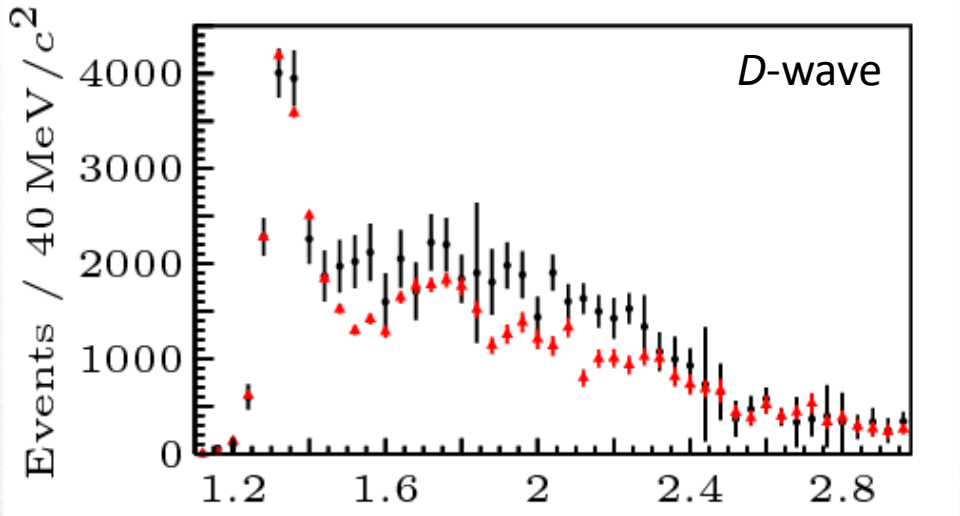
A hybrid meson ($\mathbf{8} \otimes \mathbf{8}$) cannot decay into $\eta\pi$ in the chiral limit

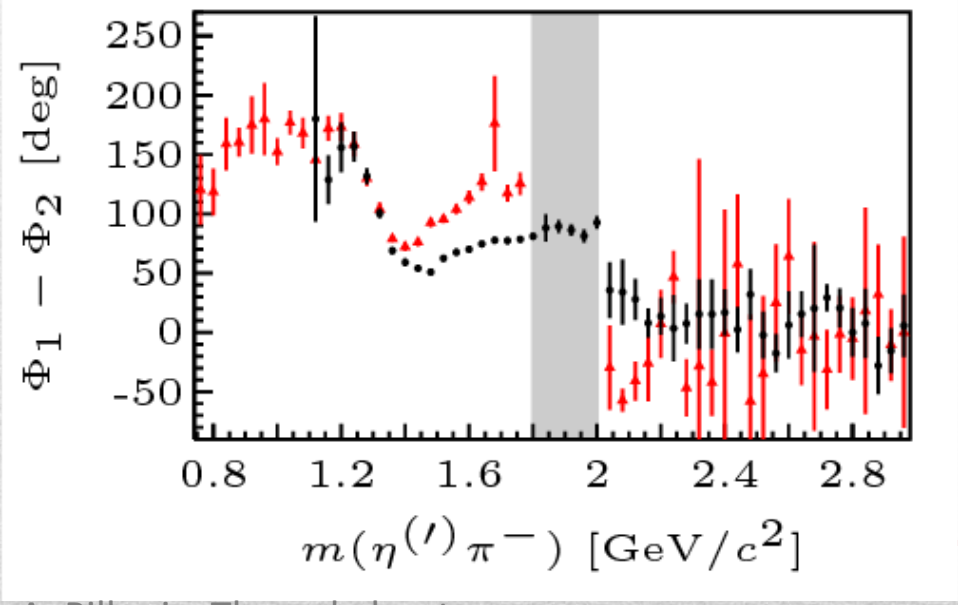
Tetraquark ($\mathbf{10} \oplus \overline{\mathbf{10}}$)? Requires doubly charged

Data

COMPASS, PLB740, 303-311







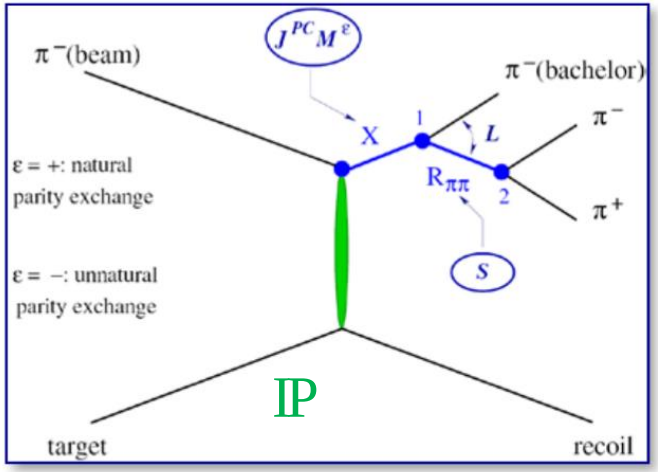
A sharp drop appears at 2 GeV in *P*-wave intensity and phase

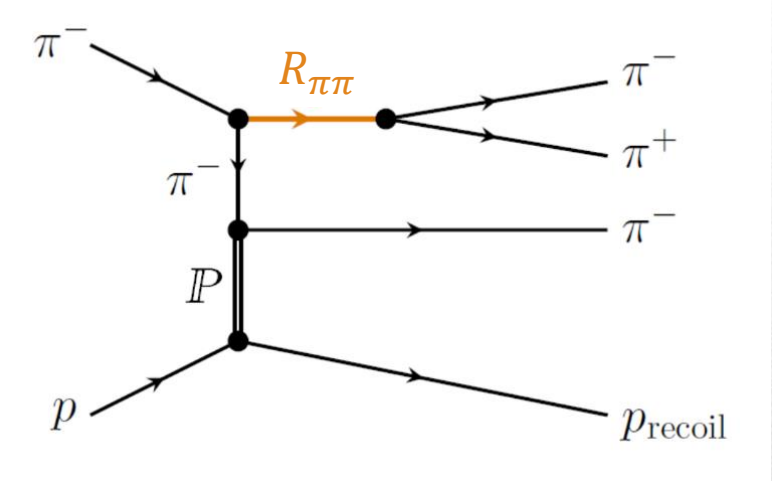
No convincing physical motivation for it

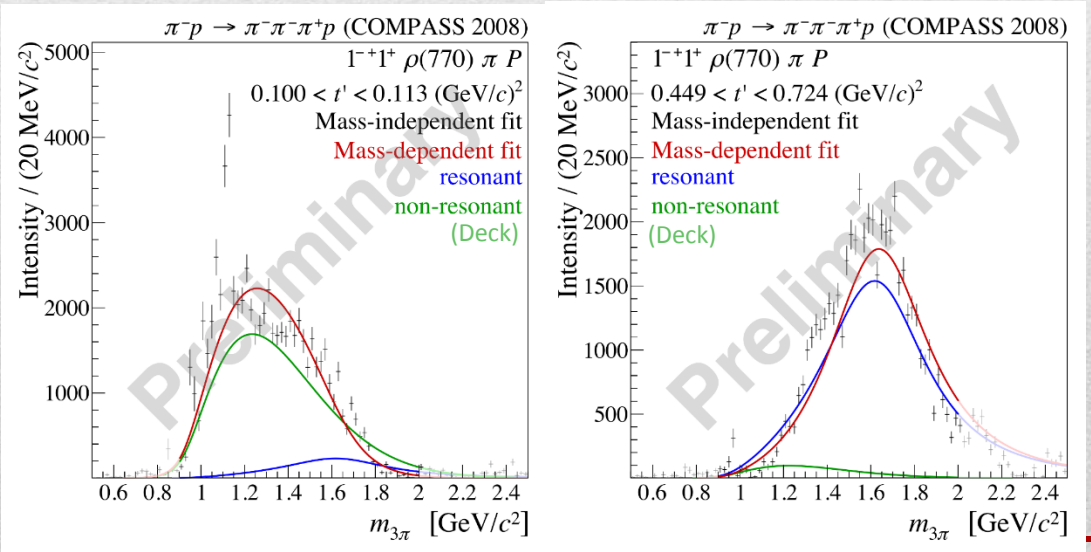
It affects the position of the $a_2'(1700)$

We decided to fit up to 2 GeV only

$\rho\pi$ channel and Deck amplitude





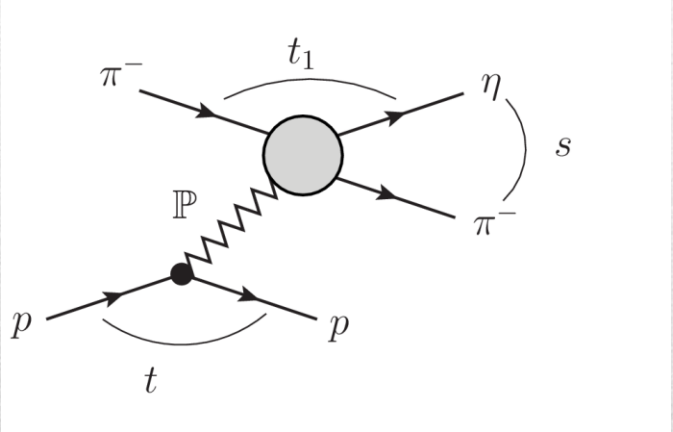


The production allows for a nonresonant component (Deck effect)
The singularity is close to the physical region, peaking background

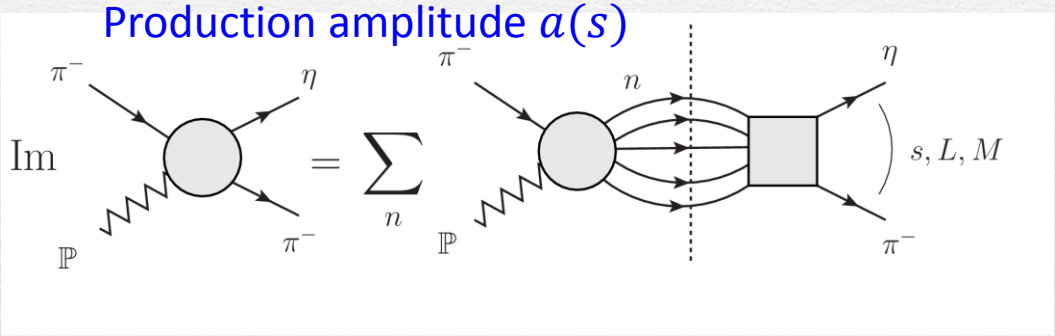
We do not include this channel

Amplitudes for $\eta^{(\prime)}\pi$

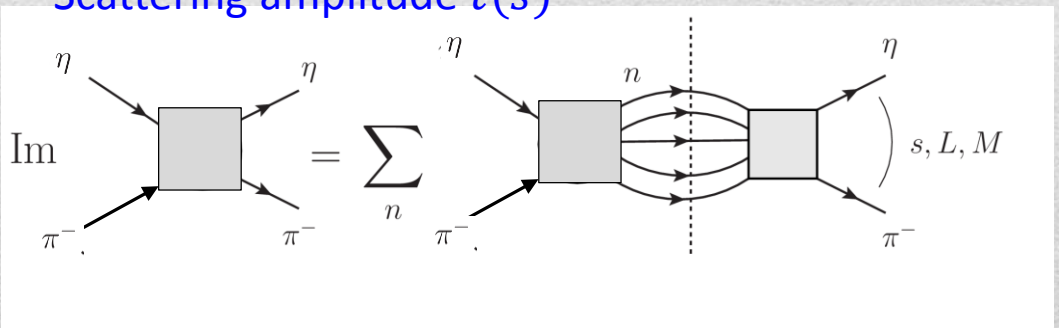
We build the partial wave amplitudes according to the N/D method



Production amplitude $a(s)$



Scattering amplitude $t(s)$



$$\mathrm{Im}\, a(s) = \rho\, a(s)\, t^*(s)$$

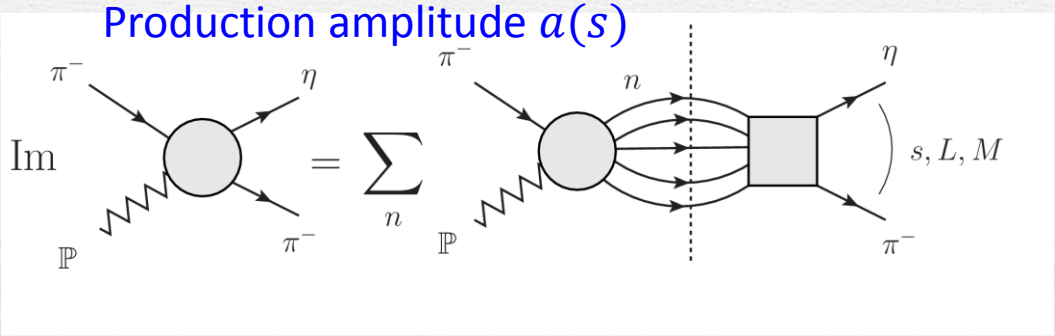
$$\frac{d\sigma}{d\sqrt{s}} \propto \frac{\rho}{\sqrt{s}} |p^L q^{L-1}\, a(s)|^2$$

Amplitudes for $\eta^{(\prime)}\pi$

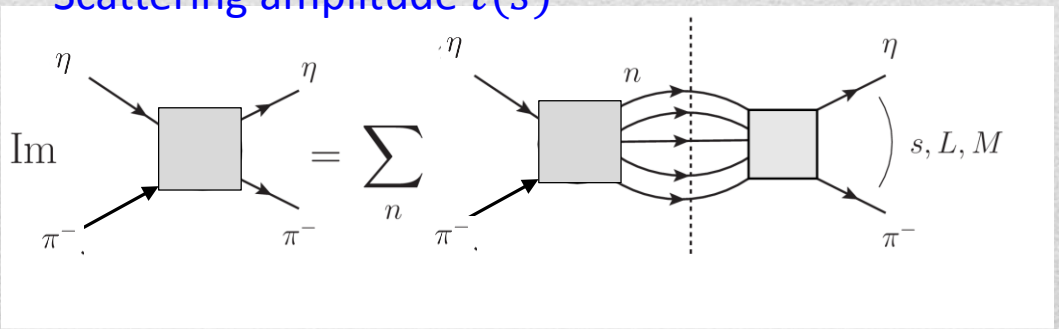
We build the partial wave amplitudes according to the N/D method

$a(s)$ is an effective $2 \to 2$ process, where the Pomeron is treated as a vector quasi-particle with virtuality $t_{\text{eff}} = -0.1\ \text{GeV}^2$

Production amplitude $a(s)$



Scattering amplitude $t(s)$



Amplitudes for $\eta^{(\prime)}\pi$

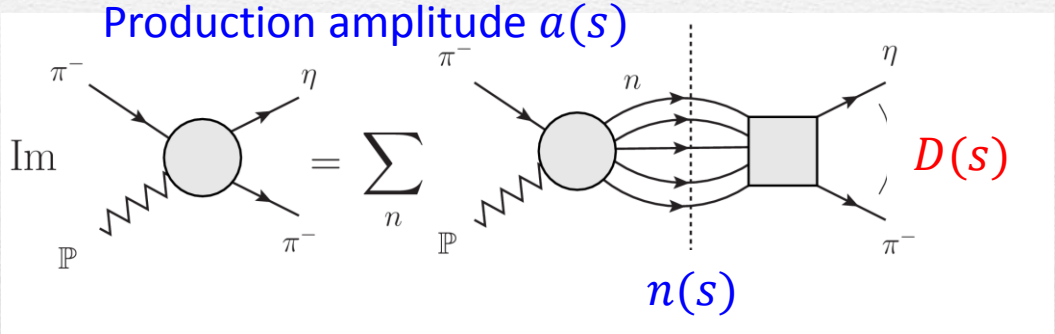
We build the partial wave amplitudes according to the N/D method

$$t(s) = \frac{N(s)}{D(s)}, a(s) = \frac{n(s)}{D(s)}$$

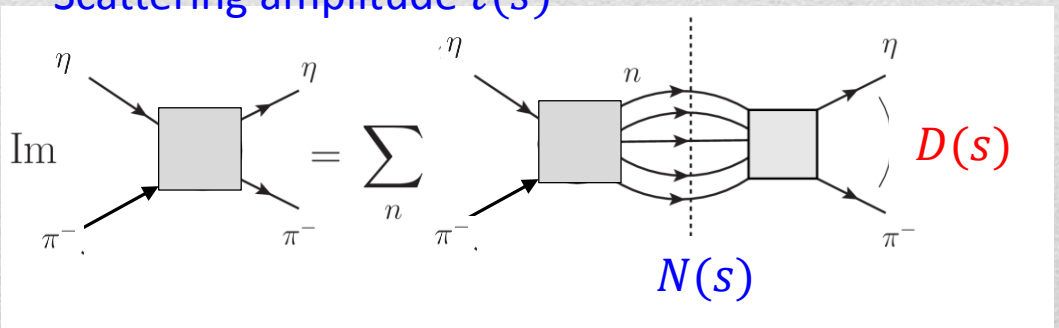
The $D(s)$ has only right hand cuts;
it contains all the Final State Interactions
constrained by unitarity → universal

$$\mathrm{Im}\, D(s) = -\rho\, N(s)$$

Production amplitude $a(s)$



Scattering amplitude $t(s)$



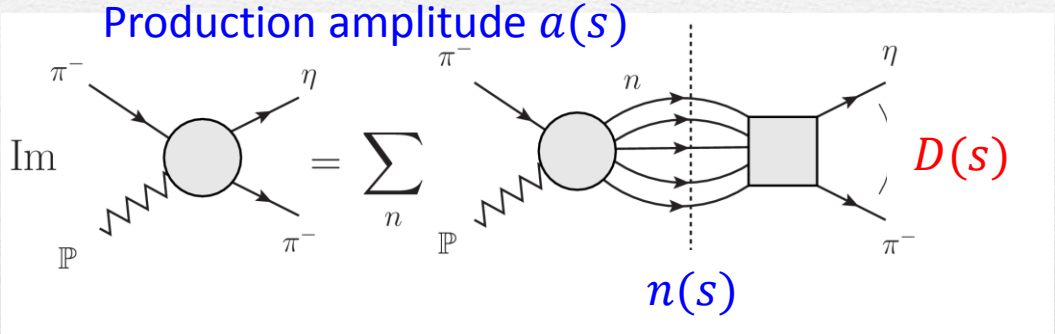
Amplitudes for $\eta^{(\prime)}\pi$

We build the partial wave amplitudes according to the N/D method

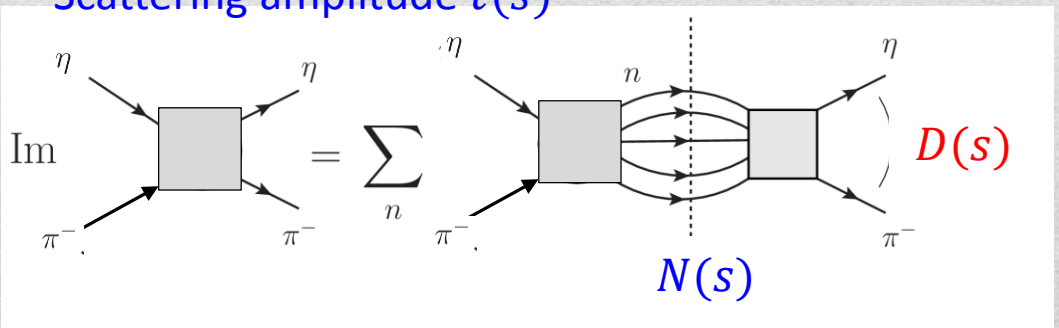
$$t(s) = \frac{N(s)}{D(s)}, a(s) = \frac{n(s)}{D(s)}$$

The $n(s), N(s)$ have left hand cuts only, they depend on the exchanges → process-dependent, smooth

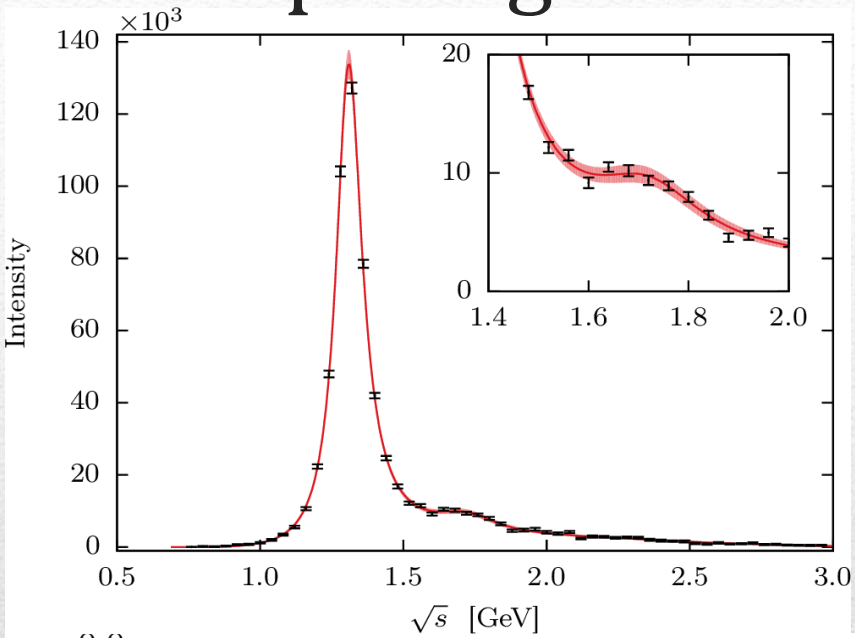
Production amplitude $a(s)$



Scattering amplitude $t(s)$

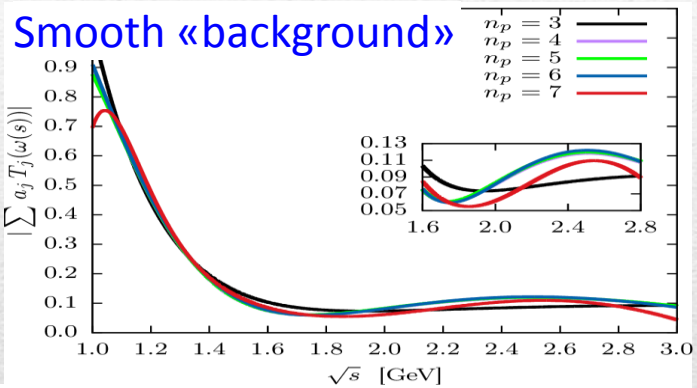


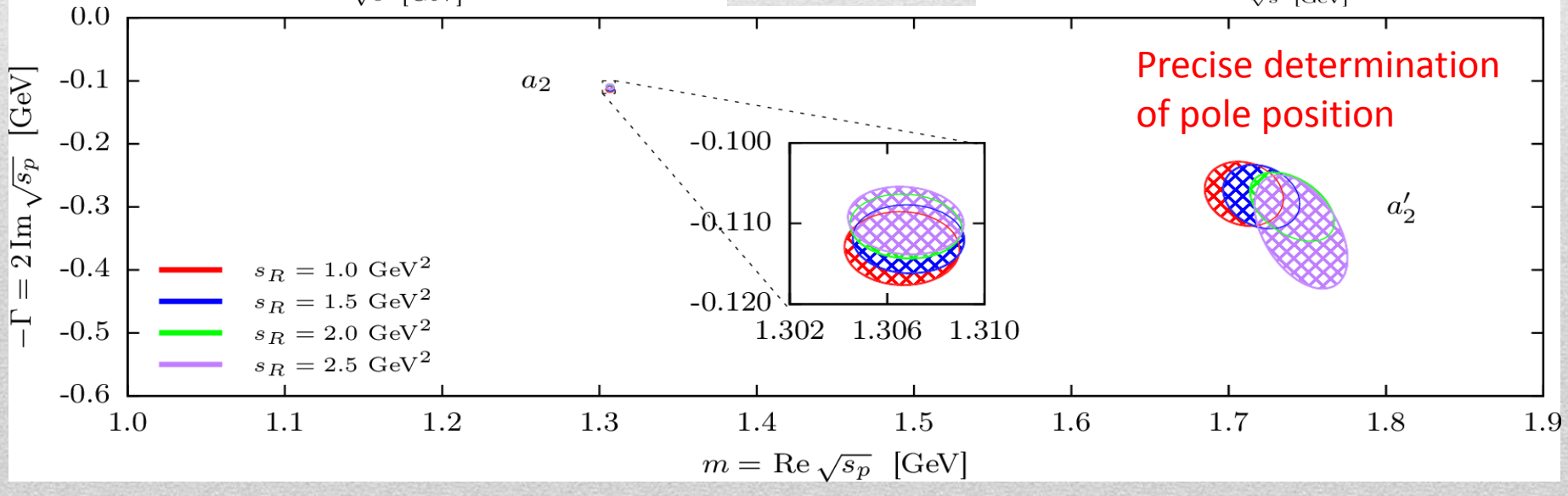
Recap: single channel $\eta\pi$



Test against the D-wave $\eta\pi$ data, where the a_2 and the a_2' show up

A. Jackura, M. Mikhasenko, AP *et al.* (JPAC & COMPASS), PLB779, 464-472





Coupled channel: the model

A. Rodas, AP *et al.* 1810.04171

Two channels, $i, k = \eta\pi, \eta'\pi$ Two waves, $J = P, D$ 37 fit parameters

$$D^J_{ki}(s) = \left[K^J(s)^{-1}\right]_{ki} - \frac{s}{\pi}\int_{s_k}^{\infty} ds' \frac{\rho N^J_{ki}(s')}{s'(s'-s-i\epsilon)}$$

$$K^J_{ki}(s) = \sum_R \frac{g_k^{(R)} g_i^{(R)}}{m_R^2 - s} + c^J_{ki} + d^J_{ki}\, s$$

$$\rho N^J_{ki}(s') = \delta_{ki} \frac{\lambda^{J+1/2}\left(s', m^2_{\eta^{(\prime)}}, m^2_\pi\right)}{(s'+s_R)^{2J+1+\alpha}}$$

$$n^J_k(s) = \sum_{n=0}^{3} a^{J,k}_n T_n\left(\frac{s}{s+s_0}\right)$$

Coupled channel: the model

A. Rodas, AP *et al.* 1810.04171

Two channels, $i, k = \eta\pi, \eta'\pi$ Two waves, $J = P, D$ 37 fit parameters

$$D^J_{ki}(s) = \left[K^J(s)^{-1}\right]_{ki} - \frac{s}{\pi}\int_{s_k}^{\infty} ds' \frac{\rho N^J_{ki}(s')}{s'(s'-s-i\epsilon)}$$

$$K^J_{ki}(s) = \sum_R \frac{g_k^{(R)} g_i^{(R)}}{m_R^2 - s} + c^J_{ki} + d^J_{ki}\, s$$

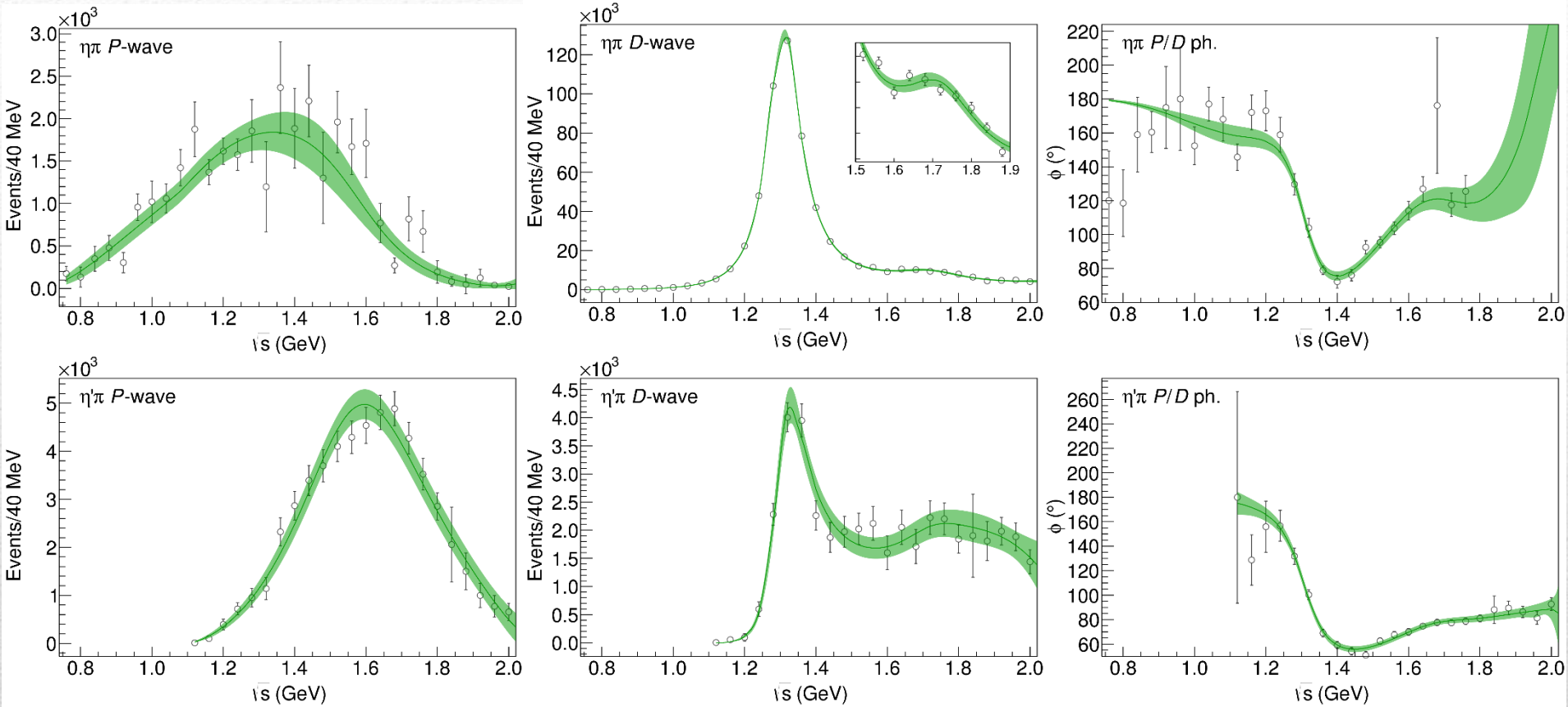
1 *K*-matrix pole for the P-wave
2 *K*-matrix poles for the D-wave

$$\rho N^J_{ki}(s') = \delta_{ki} \frac{\lambda^{J+1/2}\left(s', m^2_{\eta^{(\prime)}}, m^2_\pi\right)}{(s'+s_R)^{2J+1+\alpha}}$$

$$n^J_k(s) = \sum_{n=0}^{3} a^{J,k}_n T_n\left(\frac{s}{s+s_0}\right)$$

Left-hand scale (Blatt-Weisskopf radius) $s_R = s_0 = 1\ \mathrm{GeV}^2$
$\alpha = 2$ as in the single channel, 3rd order polynomial for $n^J_k(s)$

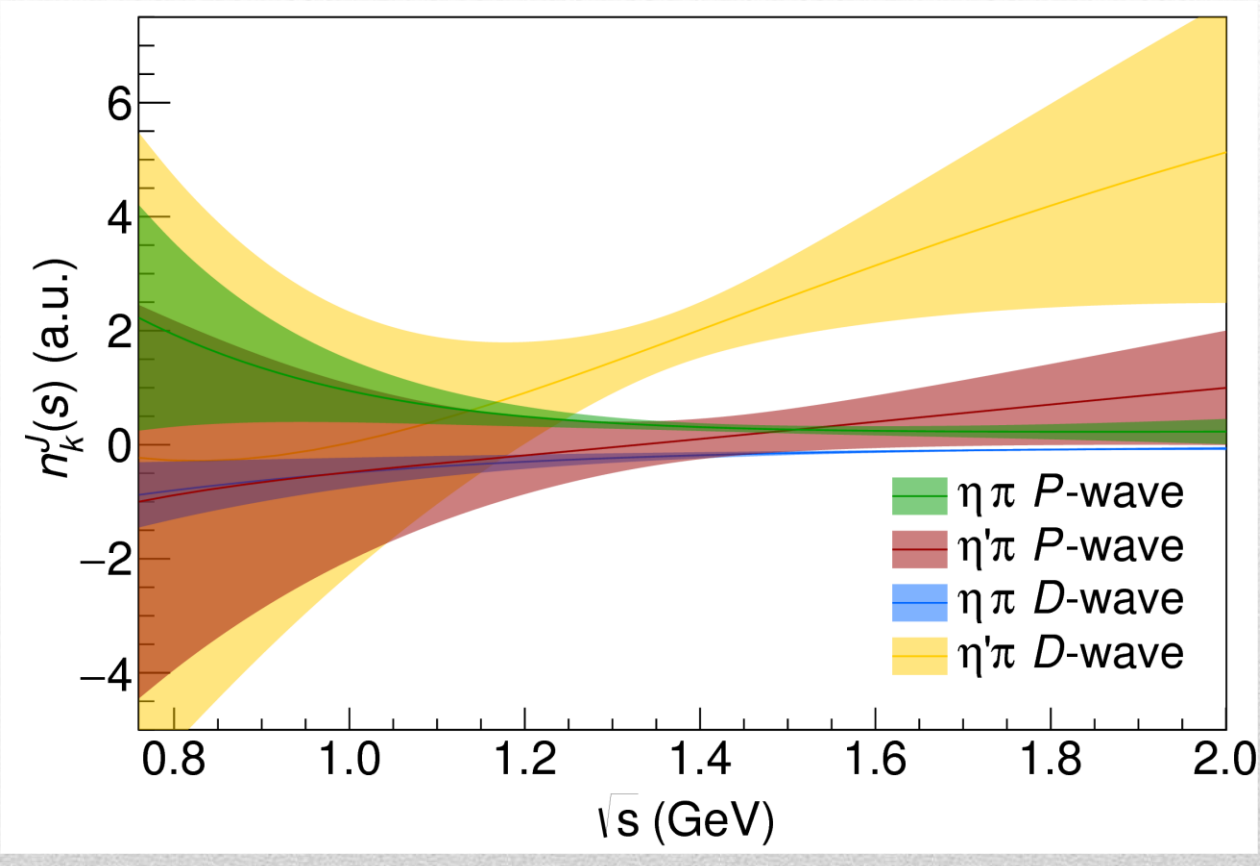
Fit



$\chi^2/\mathrm{dof} = 162/122 \sim 1.3$, statistical error estimated via 50k bootstraps

Bands show the 2σ error

Polynomial in the numerator



The numerator should be smooth and have variation milder that the typical resonance width

This happens indeed

Correlations

Denominator parameters uncorrelated with the numerator ones ✓

Denominator parameters uncorrelated between *P*- and *D*-wave ✓

Production (numerator) parameters

K-matrix «pole» parameters

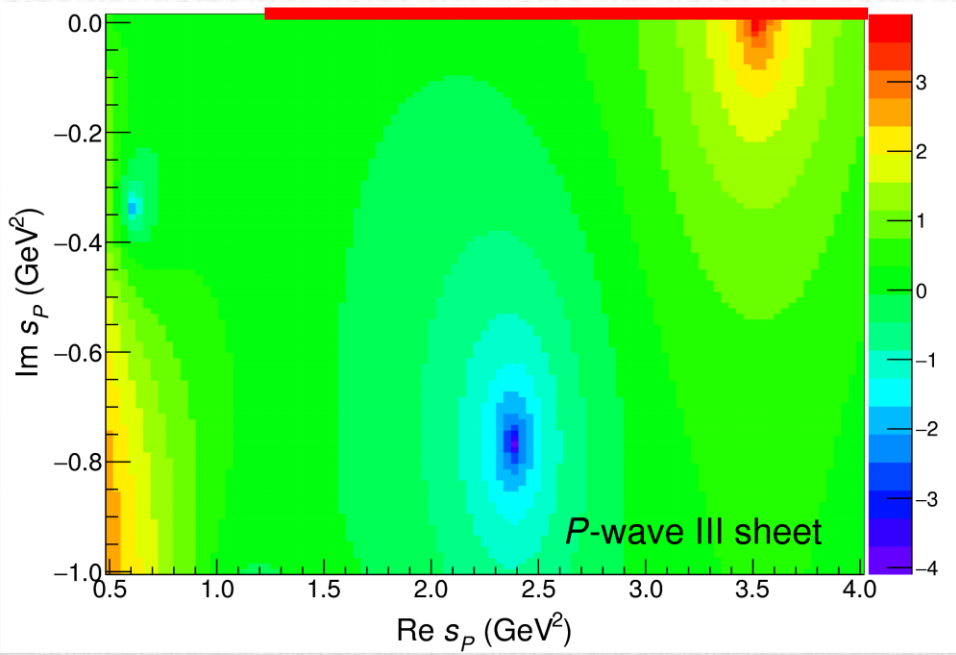
	$a_0^{P,\eta\pi}$	$a_1^{P,\eta\pi}$	$a_2^{P,\eta\pi}$	$a_3^{P,\eta\pi}$	$a_0^{D,\eta\pi}$	$a_1^{D,\eta\pi}$	$a_2^{D,\eta\pi}$	$a_3^{D,\eta\pi}$	$a_0^{P,\eta'\pi}$	$a_1^{P,\eta'\pi}$	$a_2^{P,\eta'\pi}$	$a_3^{P,\eta'\pi}$	$a_0^{D,\eta'\pi}$	$a_1^{D,\eta'\pi}$	$a_2^{D,\eta'\pi}$	$a_3^{D,\eta'\pi}$
$m^2_{D,2}$	16	-16	16	-15	-20	16	-19	1	-5	5	-5	6	15	-19	6	-7
$g^{D,2}_{\eta'\pi}$	45	-45	44	-43	-8	-3	-5	-8	-40	41	-41	41	-4	-2	-4	-6
$g^{D,2}_{\eta\pi}$	13	-13	13	-13	-3	-8	-2	-8	-1	2	-3	7	-10	5	-8	-4
$m^2_{D,1}$	24	-23	21	-15	-4	5	-15	1	-25	20	-9	-12	5	-4	-13	2
$g^{D,1}_{\eta'\pi}$	9	-9	10	-12	18	4	-27	32	-5	7	-11	19	9	10	-24	35
$g^{D,1}_{\eta\pi}$	23	-22	20	-15	-0	1	-13	1	-24	20	-9	-12	1	-0	-16	4
$m^2_{P,1}$	25	-24	24	-23	-21	12	-9	-6	-26	28	-31	36	2	-10	7	-12
$g^{P,1}_{\eta'\pi}$	-12	11	-11	9	8	-7	7	-0	14	-13	12	-9	5	-5	7	-2
$g^{P,1}_{\eta\pi}$	-6	6	-7	10	-6	10	-3	3	5	-5	8	-11	11	-11	11	-4
Γ_{π_1}	22	-23	23	-25	-4	5	-9	3	-3	2	1	-5	6	-6	0	-1
m_{π_1}	-10	9	-8	4	12	-8	3	3	6	-6	7	-7	-6	11	-9	8
$\Gamma_{a'_2}$	-17	17	-16	14	-21	25	-8	6	17	-15	8	4	26	-27	28	-10
$m_{a'_2}$	8	-9	9	-11	-17	21	-13	8	-3	4	-8	16	19	-21	17	-7
Γ_{a_2}	-3	3	-4	4	-6	4	1	-4	2	-3	4	-7	5	-7	6	-4
m_{a_2}	-6	6	-5	5	-12	14	-5	3	7	-6	4	-0	13	-15	15	-7

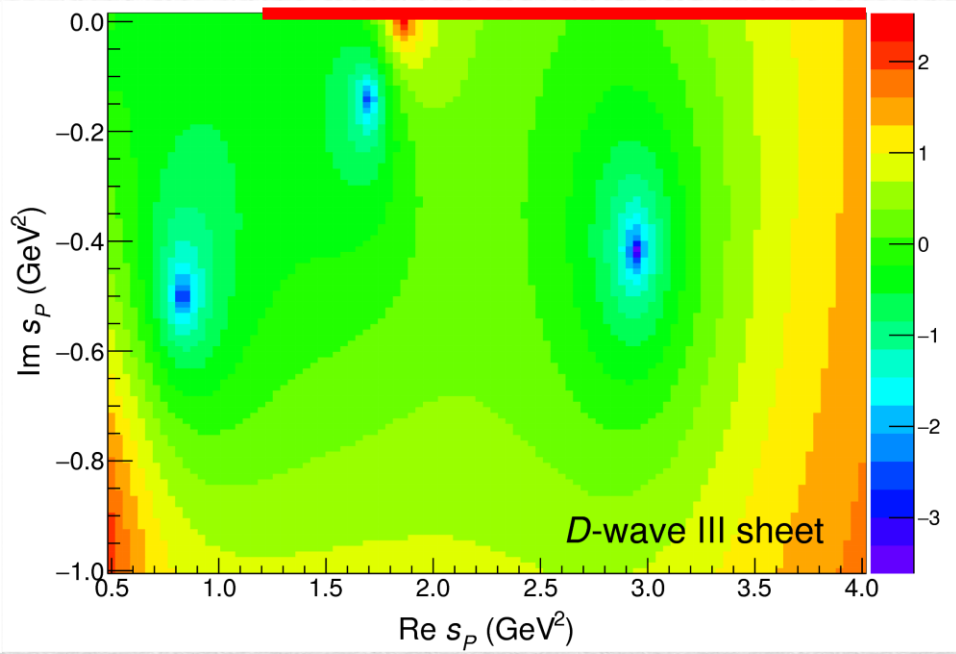
K-matrix «bkg» parameters

	$c^P_{\eta\pi,\eta\pi}$	$c^P_{\eta\pi,\eta'\pi}$	$c^P_{\eta'\pi,\eta'\pi}$	$c^D_{\eta\pi,\eta\pi}$	$c^D_{\eta\pi,\eta'\pi}$	$c^D_{\eta'\pi,\eta'\pi}$	$d^P_{\eta\pi,\eta\pi}$	$d^P_{\eta\pi,\eta'\pi}$	$d^P_{\eta'\pi,\eta'\pi}$	$d^D_{\eta\pi,\eta\pi}$	$d^D_{\eta\pi,\eta'\pi}$	$d^D_{\eta'\pi,\eta'\pi}$
$d^D_{\eta'\pi,\eta'\pi}$	-27	0	-22	-57	-53	84	22	-3	20	-20	-91	100
$d^D_{\eta\pi,\eta'\pi}$	32	-4	22	32	63	-76	-29	4	-18	-18	100	-91
$d^D_{\eta\pi,\eta\pi}$	-22	11	1	67	-15	-17	24	-4	-4	100	-18	-20
$d^P_{\eta'\pi,\eta'\pi}$	-28	-74	-90	-8	-17	20	23	72	100	-4	-18	20
$d^P_{\eta\pi,\eta'\pi}$	-45	-94	-60	2	-5	2	45	100	72	-4	4	-3
$d^P_{\eta\pi,\eta\pi}$	-92	-30	-24	-8	-9	13	100	45	23	24	-29	22
$c^D_{\eta'\pi,\eta'\pi}$	-18	-7	-19	-26	-84	100	13	2	20	-17	-76	84
$c^D_{\eta\pi,\eta'\pi}$	10	6	16	-19	100	-84	-9	-5	-17	-15	63	-53
$c^D_{\eta\pi,\eta\pi}$	14	3	10	100	-19	-26	-8	2	-8	67	32	-57
$c^P_{\eta'\pi,\eta'\pi}$	31	67	100	10	16	-19	-24	-60	-90	1	22	-22
$c^P_{\eta\pi,\eta'\pi}$	41	100	67	3	6	-7	-30	-94	-74	11	-4	0
$c^P_{\eta\pi,\eta\pi}$	100	41	31	14	10	-18	-92	-45	-28	-22	32	-27

K-matrix «bkg» parameters

Complex plane



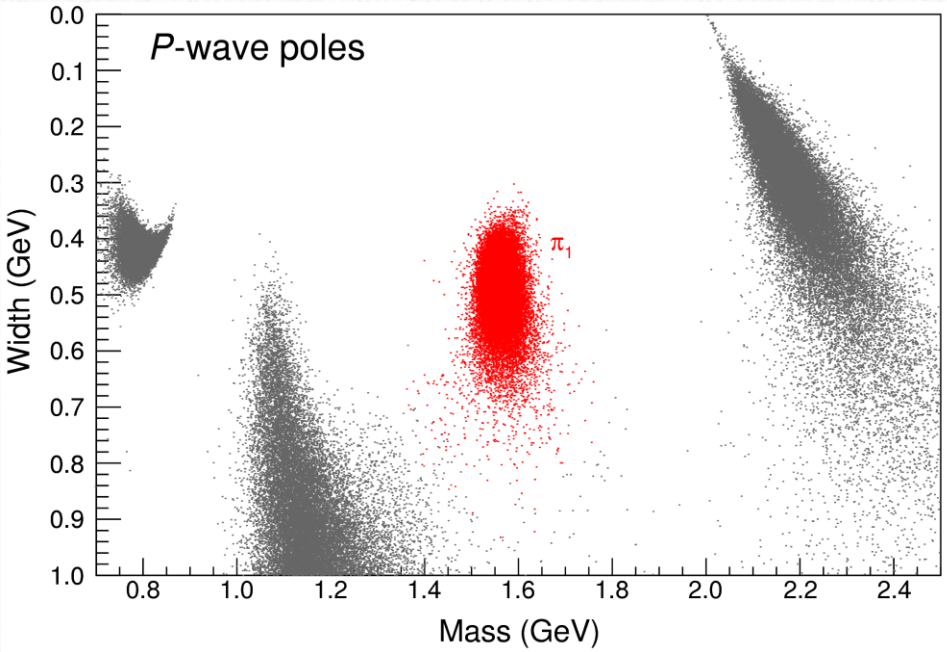


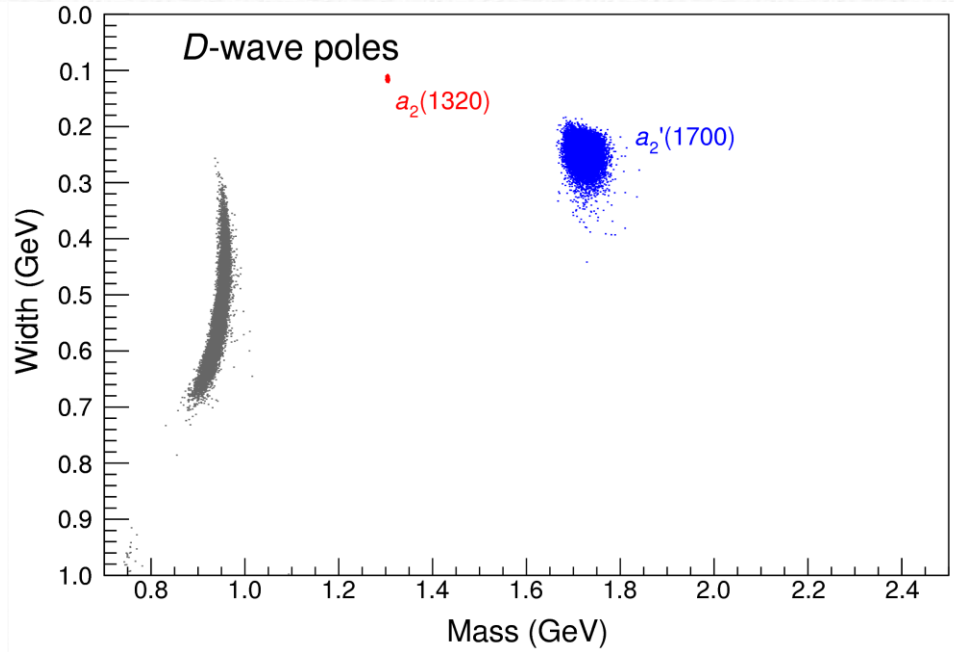
For the best fit solution, we look at the closest Riemann sheet in the complex plane
We see a limited amount of poles in the relevant region

We also see some close-to-threshold poles, which are artifacts of the model for $N(s)$

How to distinguish the two?

Bootstrap

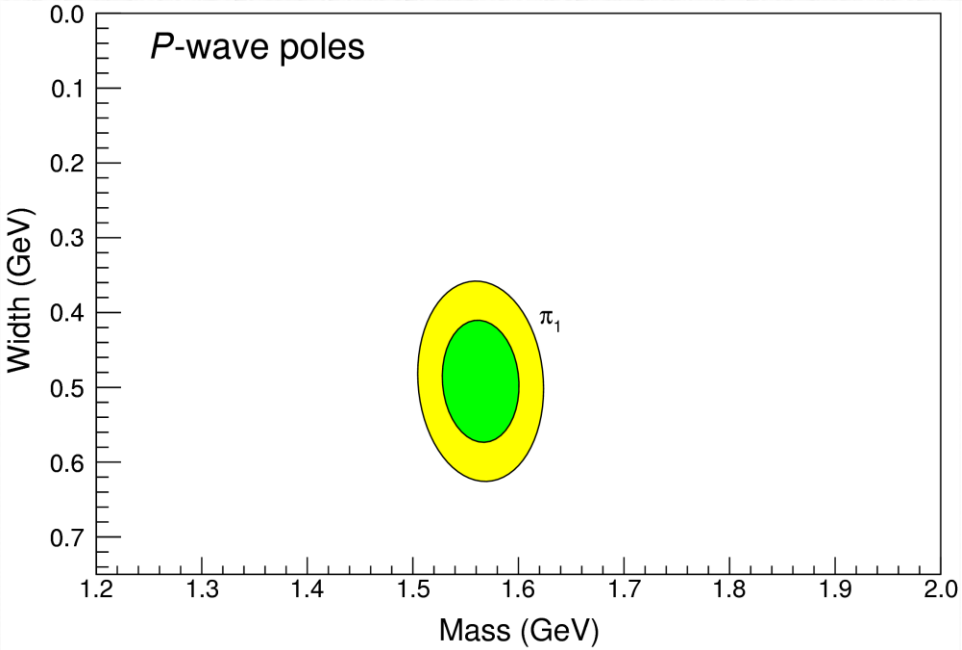


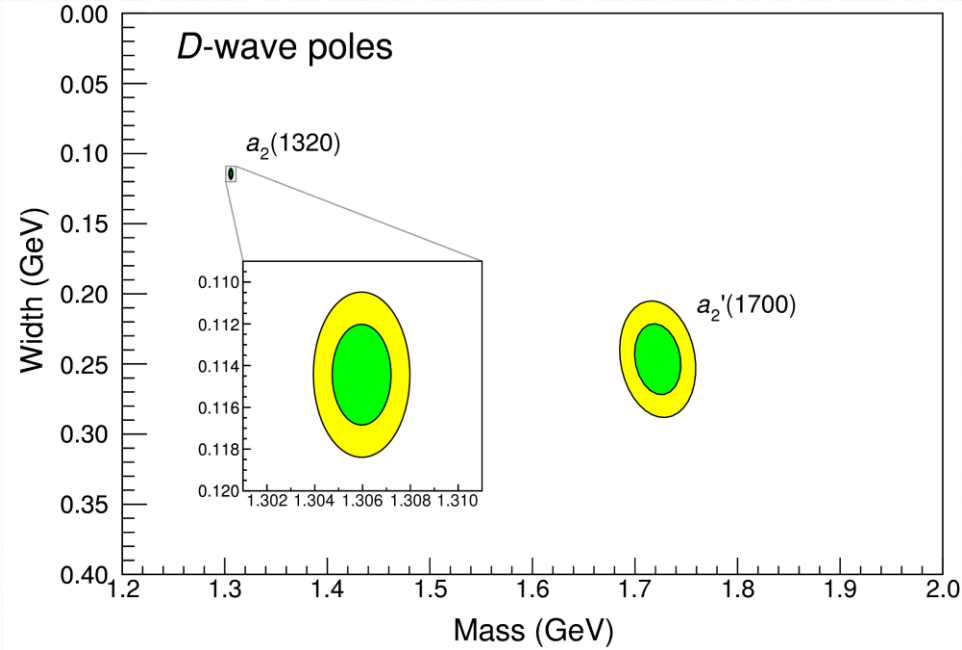


We can identify the poles in the region $m \in [1.2, 2]$ GeV, $\Gamma \in [0, 1]$ GeV

Two stable isolated poles are indentifiable in the *D*-wave
Only one is stable in the *P*-wave

Result (stat. error only)

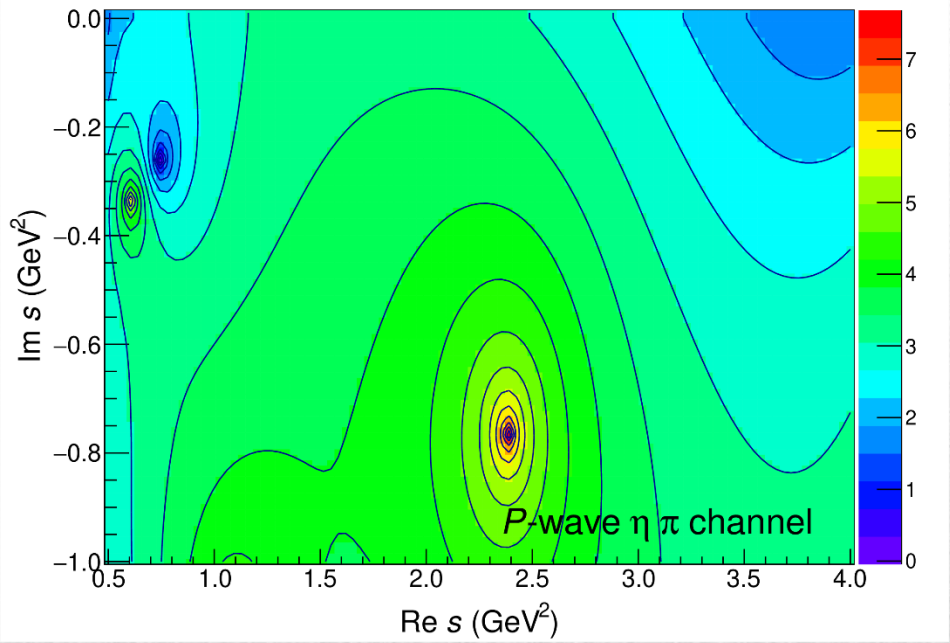


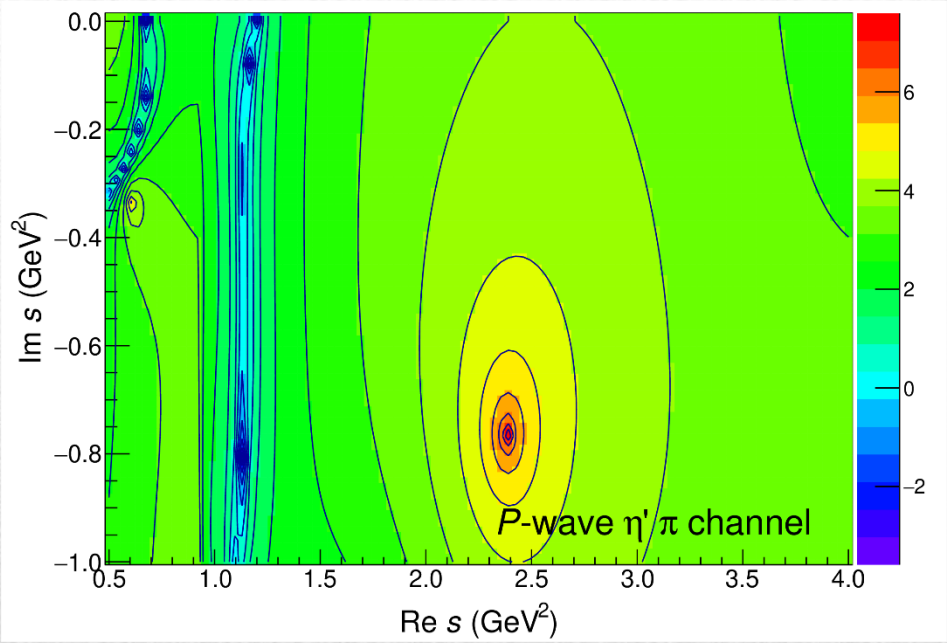


The variance of the bootstrapped poles gives the statistical error

Poles	Mass (MeV)	Width (MeV)
$a_2(1320)$	1306.0 ± 0.8	114.4 ± 1.6
$a_2'(1700)$	1722 ± 15	247 ± 17
π_1	1564 ± 24	492 ± 54

Again into the complex plane





The strength of the pole propagates differently in the two channels

In $\eta\pi$ the strength move to lighter values

Systematic studies

- Change of functional form and parameters in the denominator

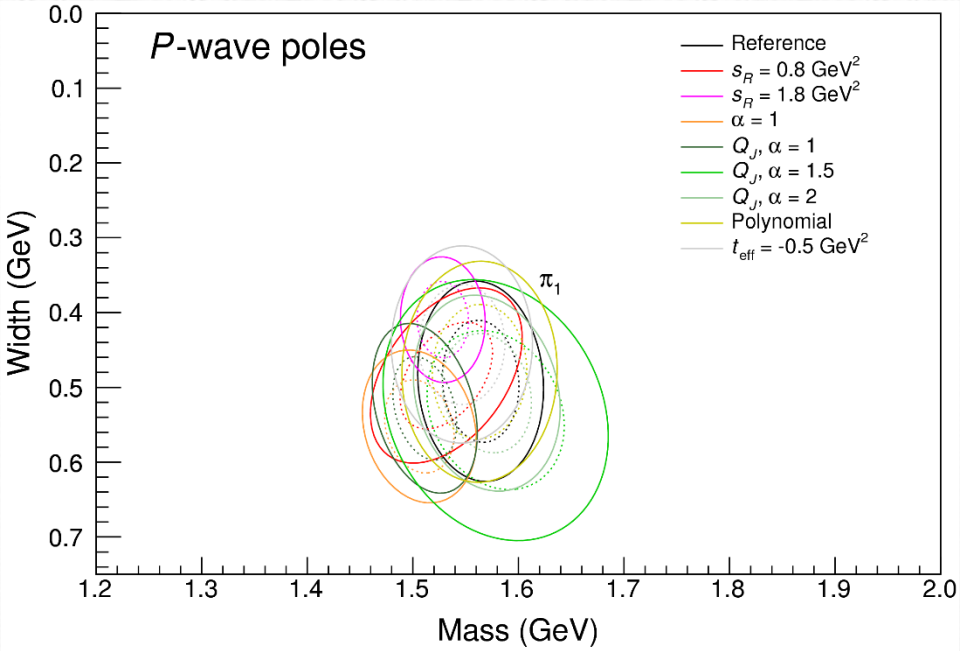
$$\rho N_{ki}^{J}(s') = g\,\delta_{ki}\,\frac{\lambda^{J+1/2}\left(s', m_{\eta^{(\prime)}}^2, m_\pi^2\right)}{\left(s'+s_R\right)^{2J+1+\alpha}}$$

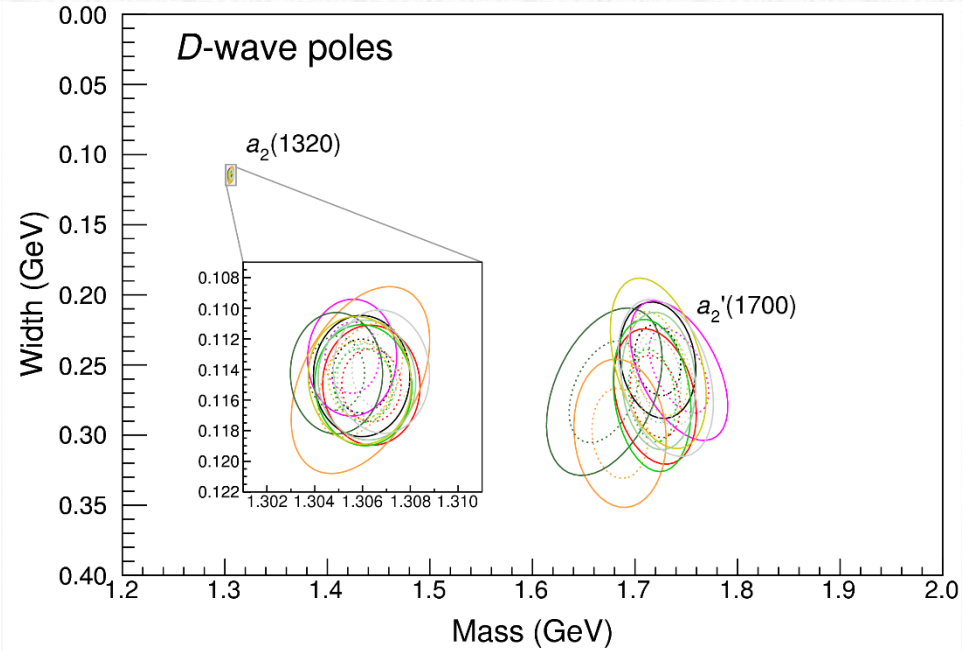
  - Default: $s_R = 1\ \mathrm{GeV}^2$. We try $s_R = 0.8, 1.8\ \mathrm{GeV}^2$
  - Default: $\alpha = 2$. We try $\alpha = 1$
  - We also try a different function: $\rho N_{ki}^{J}(s') = g\,\delta_{ki}\,\frac{Q_J(z_{s'})}{s'^{\alpha}\lambda^{1/2}(s', m_{\eta^{(\prime)}}, m_\pi)}$ with $\alpha = 2, 1.5, 1$

- Change of parameters in the numerator

  - Default: $t_{\mathrm{eff}} = -0.1\ \mathrm{GeV}^2$. We try $t_{\mathrm{eff}} = -0.5\ \mathrm{GeV}^2$
  - Default: 3rd order polynomial. We try 4th

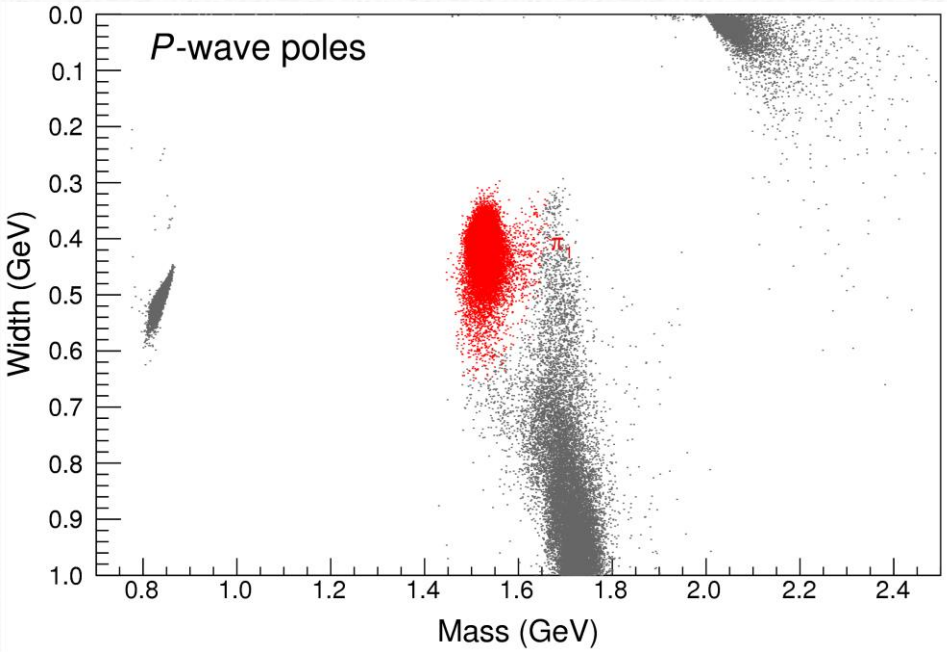
Systematic studies

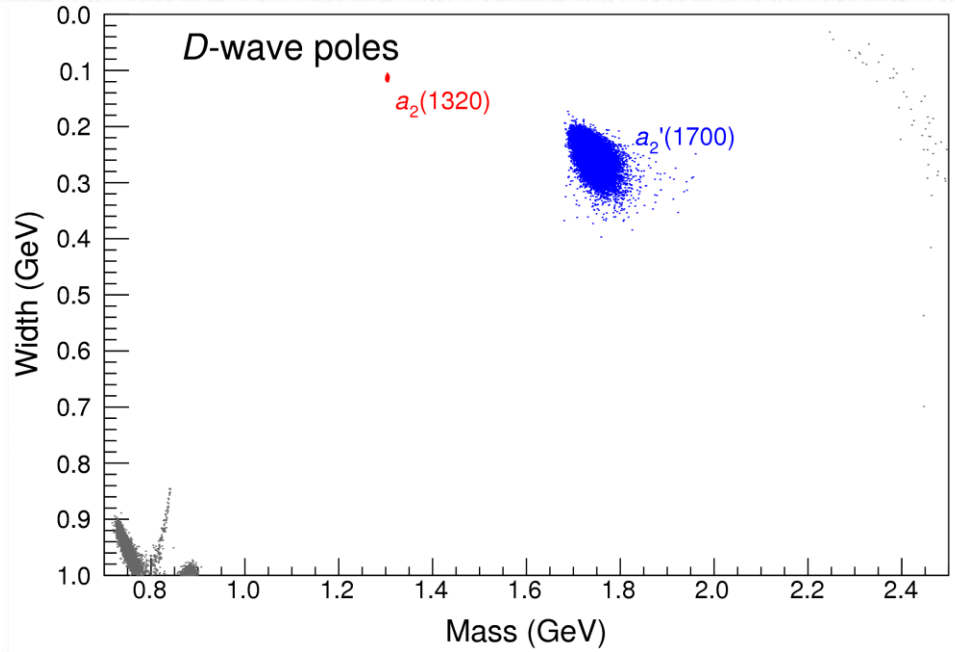




For each class, the maximum deviation of mass and width is taken as a systematic error
Deviation smaller than the statistical error are neglected
Systematic of different classes are summed in quadrature

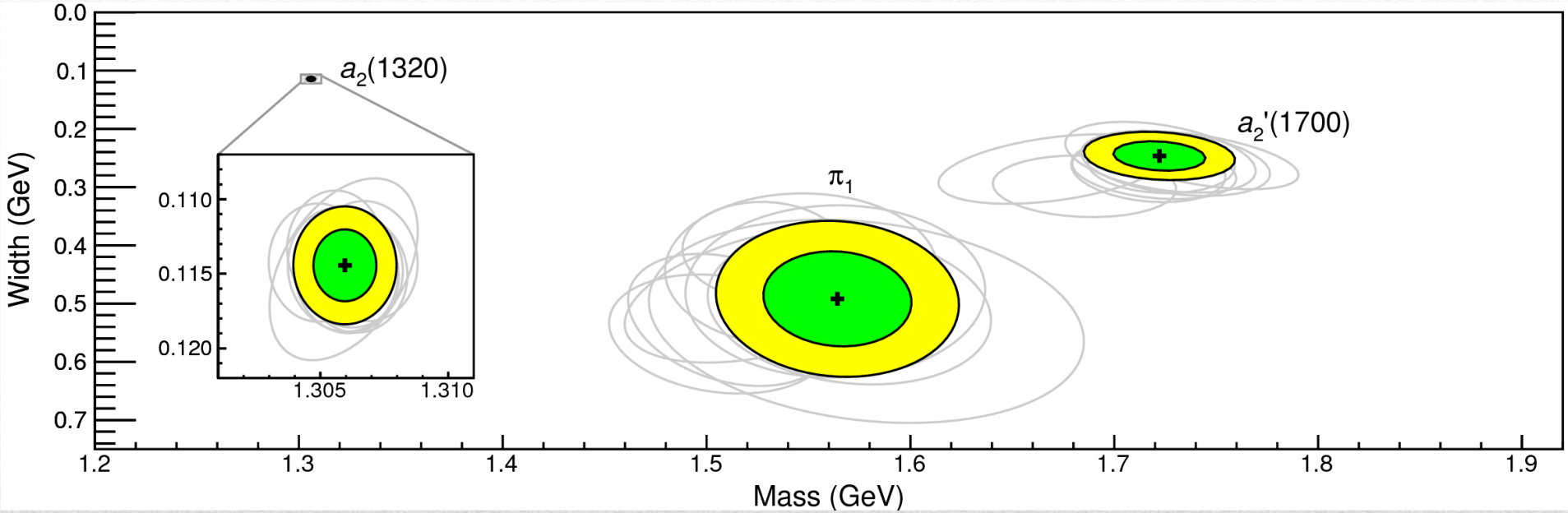
Bootstrap for $s_R = 1.8\ \mathrm{GeV}^2$





Our skepticism about a second pole in the relevant region is confirmed:
It is unstable and not trustable

Final results



Poles	Mass (MeV)	Width (MeV)
$a_2(1320)$	$1306.0 \pm 0.8 \pm 1.3$	$114.4 \pm 1.6 \pm 0.0$
$a_2'(1700)$	$1722 \pm 15 \pm 67$	$247 \pm 17 \pm 63$
π_1	$1564 \pm 24 \pm 86$	$492 \pm 54 \pm 102$

The a_1

The $a_1(1260)$

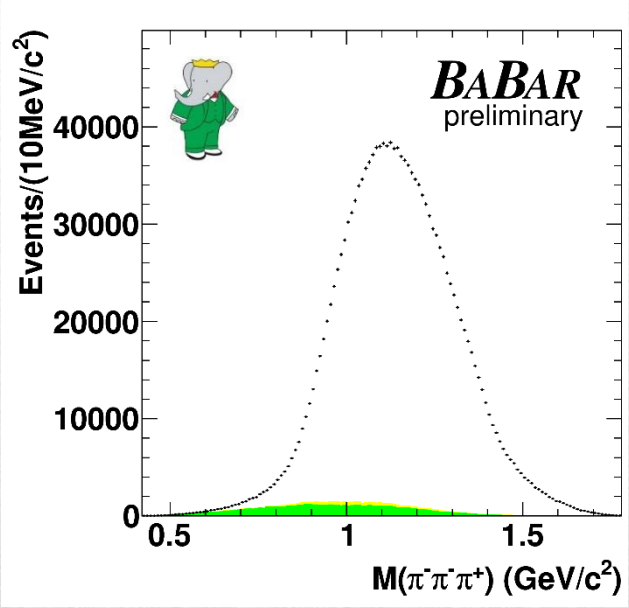
$a_1(1260)$ [k]

$I^G(J^{PC}) = 1^-(1^{++})$

Mass $m = 1230 \pm 40$ MeV [l]
Full width $\Gamma = 250$ to 600 MeV

$a_1(1260)$ DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$(\rho\pi)_{S-\text{wave}}$	seen	353
$(\rho\pi)_{D-\text{wave}}$	seen	353
$(\rho(1450)\pi)_{S-\text{wave}}$	seen	†
$(\rho(1450)\pi)_{D-\text{wave}}$	seen	†
$\sigma\pi$	seen	–
$f_0(980)\pi$	not seen	179
$f_0(1370)\pi$	seen	†
$f_2(1270)\pi$	seen	†
$K\overline{K}^*(892)+$ c.c.	seen	†
$\pi\gamma$	seen	608

3π final states



Despite it has been known since forever, the resonance parameters of the $a_1(1260)$ are poorly determined
The production (and model) dependence is affecting their extraction

The $a_1(1260)$

$a_1(1260)$ WIDTH

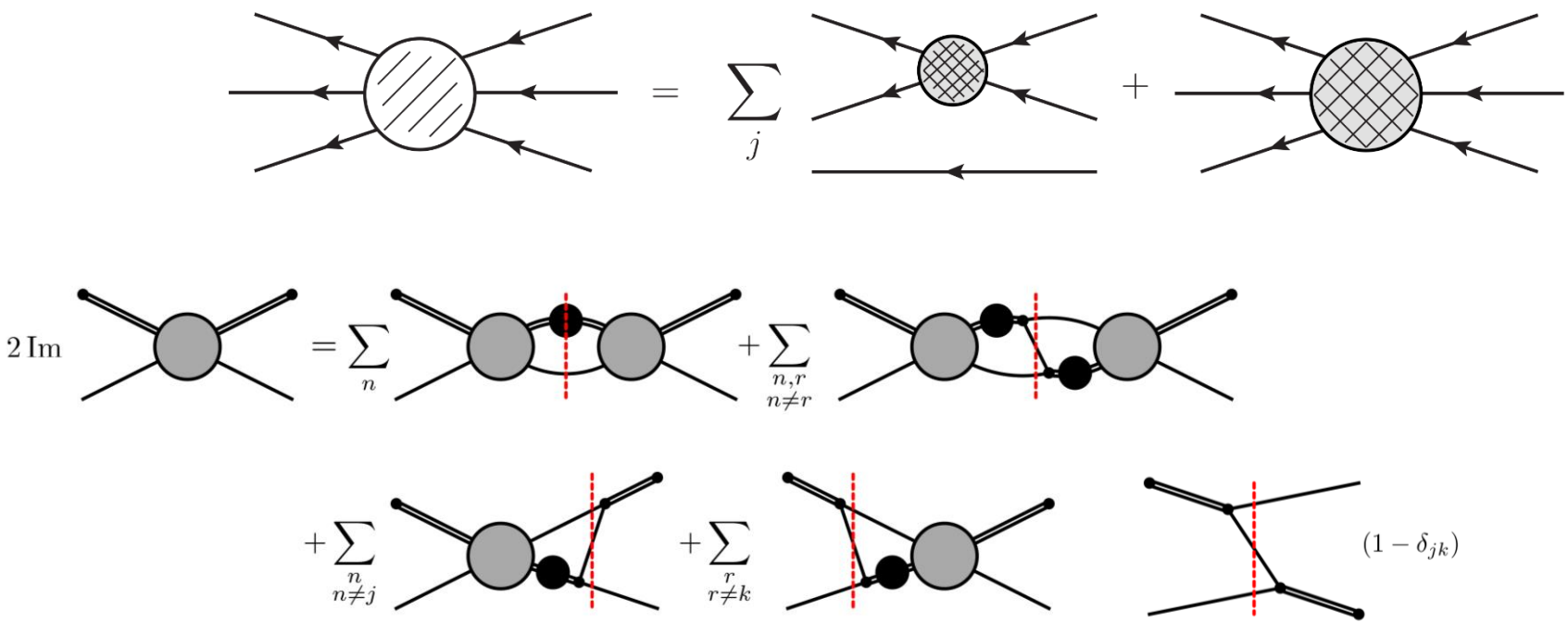
INSPIRE search

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMMENT
250 to 600	**OUR ESTIMATE**				
$367 \pm 9 \,^{+28}_{-25}$	420k	ALEKSEEV	2010	COMP	190 $\pi^- \to \pi^-\pi^-\pi^+ Pb'$
• • • We do not use the following data for averages, fits, limits, etc. • • •					
$410 \pm 31 \pm 30$		[1] AUBERT	2007AU	BABR	10.6 $e^+ e^- \to \rho^0 \rho^\pm \pi^\mp \gamma$
520 - 680	6360	[2] LINK	2007A	FOCS	$D^0 \to \pi^-\pi^+\pi^-\pi^+$
480 ± 20		[3] GOMEZ-DUMM	2004	RVUE	$\tau^+ \to \pi^+\pi^+\pi^-\nu_\tau$
580 ± 41	90k	SALVINI	2004	OBLX	$\bar{p}\, p \to 2\,\pi^+ 2\,\pi^-$
460 ± 85	205	[4] DRUTSKOY	2002	BELL	$B^{(*)}\, K^-\, K^{*0}$
$814 \pm 36 \pm 13$	37k	[5] ASNER	2000	CLE2	10.6 $e^+ e^- \to \tau^+\tau^-$, $\tau^- \to \pi^-\pi^0\pi^0\nu_\tau$

The extraction of the resonance in the τ decay should be the cleanest, but the determination of the pole is still unstable

3-body stuff

Having a 3π final state requires implementing 3-body unitarity

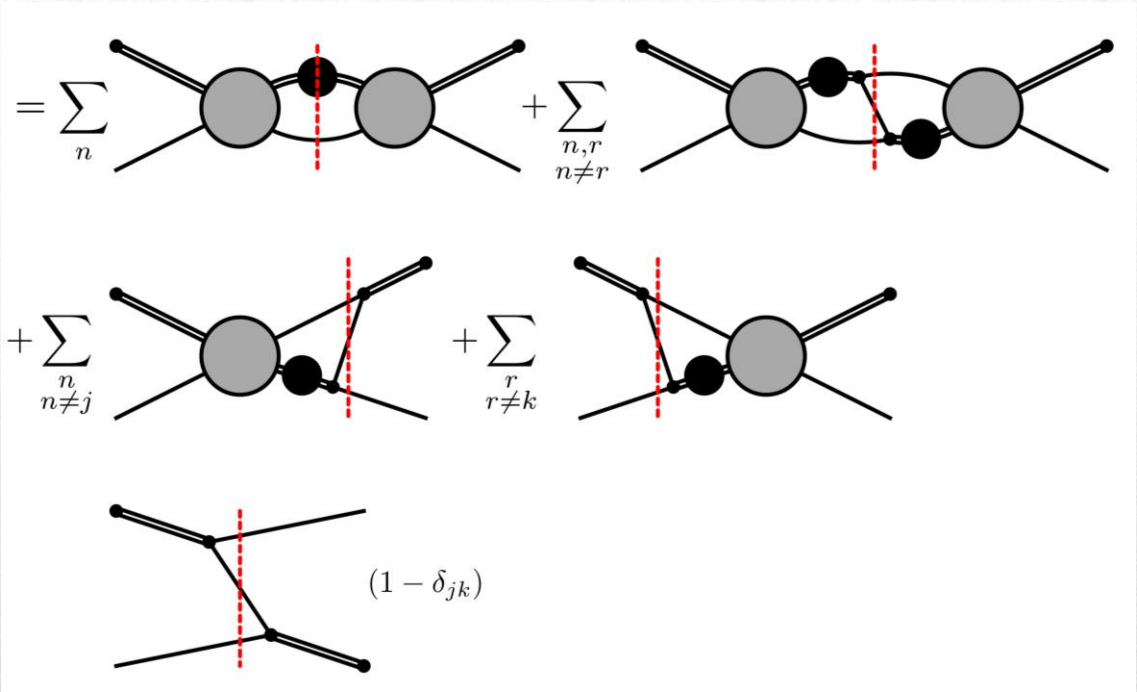


A. Jackura

M. Mai, B. Hu, M. Doring, AP, A. Szczepaniak EPJA53, 9, 177
A. Jackura, *et al.*, 1809.10523

→ See Andrew's talk on Monday

Factorizable model

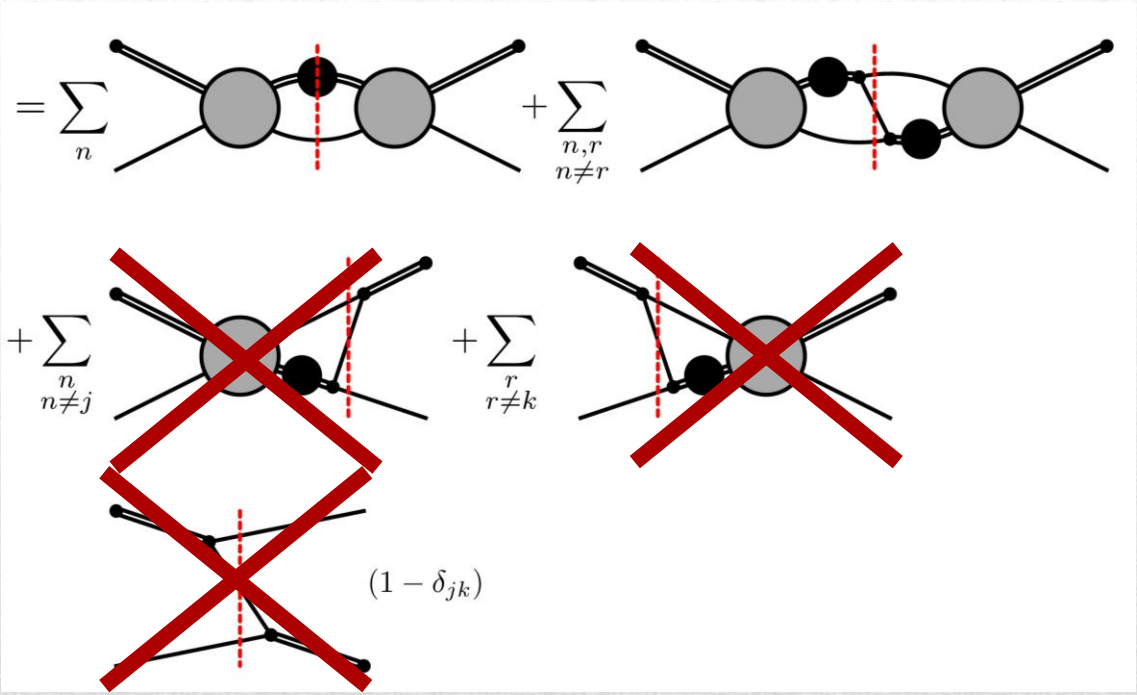


M. Mikhasenko, AP, *et al.*, 1810.00016

One can amputate the 2-body amplitude from the inital and final state

$$\mathcal{A}\left(\sigma_j', s, \sigma_k\right) = f(\sigma_j')\hat{\mathcal{A}}\left(\sigma_j', s, \sigma_k\right) f(\sigma_k)$$

Factorizable model



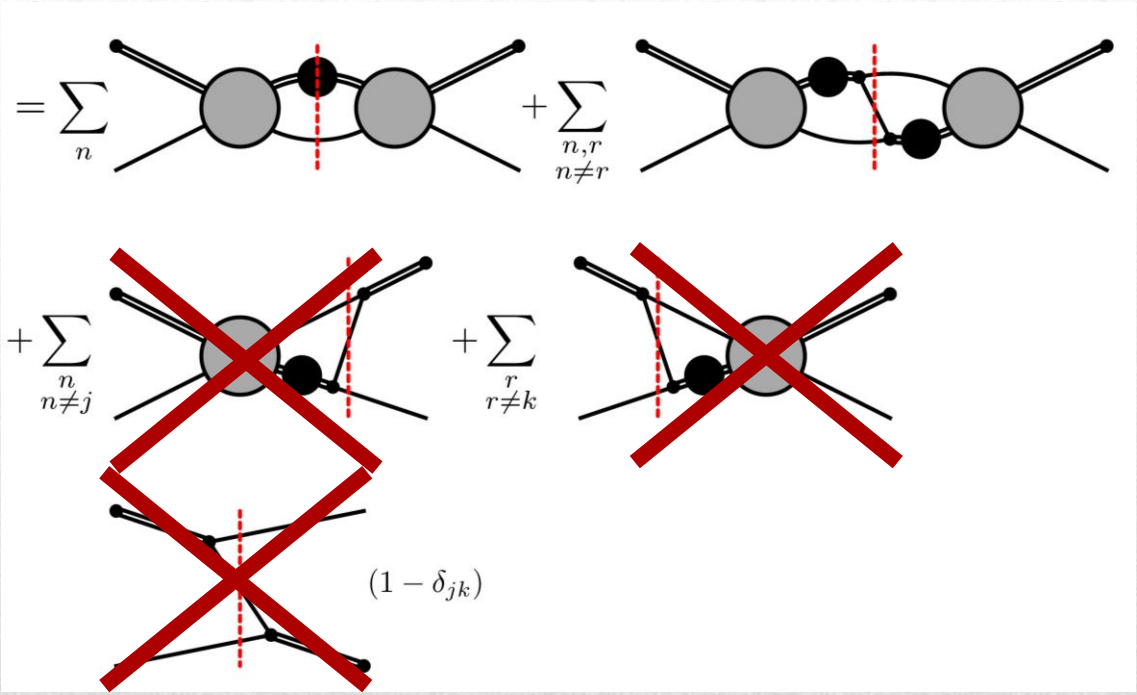
M. Mikhasenko, AP, *et al.*, 1810.00016

One can amputate the 2-body amplitude from the inital and final state

If one neglects the effect of the disconnected diagrams to unitarity, is it possible to suppress the dependence of the reduced amplitude on the 2-body invariant masses

$$\mathcal{A}\left(\sigma'_j, s, \sigma_k\right) = f(\sigma'_j)\hat{\mathcal{A}}\left(\cancel{\sigma'_j}, s, \cancel{\sigma_k}\right) f(\sigma_k)$$

Factorizable model



M. Mikhasenko, AP, *et al.*, 1810.00016

The unitarity equation is now algebraic and easier to handle

$$\mathrm{Im}\,\hat{\mathcal{A}}(s) = \hat{\mathcal{A}}(s)\,\hat{\mathcal{A}}(s)^{\dagger} \int d\Phi_3 \left| \sum_j f(\sigma_j) \right|^2$$

Integral over the Dalitz plot (*aka* quasi 2-body)

More about the model

We consider ALEPH data of $\tau^- \to \pi^-\pi^+\pi^-\nu_\tau$

CLEO estimated the dominant decay mode to be $a_1(1260) \to \rho\,\pi$ in *S*-wave

This statement is model dependent, and would be desiderable to perform a combined fit of the subchannels. However, no data are available → we consider $\rho^0\,\pi^-$ *S*-wave only
The faible $\pi^-\pi^-$ interaction is neglected

$$f(\sigma) = \mathcal{N}\frac{p(\sigma)R}{\sqrt{1+(p(\sigma)R)^2}}\frac{1}{m_\rho^2 - \sigma - i m_\rho \Gamma_\rho(\sigma)}$$

$$\Gamma(\sigma) = \Gamma_\rho \times \frac{p^3(\sigma)}{\sqrt{\sigma}\sqrt{1+(p(\sigma)R)^2}} \Bigg/ \frac{p(m_\rho^2)}{m_\rho\sqrt{1+(p(m_\rho^2)R)^2}}$$

Standard *P*-wave Breit-Wigner with Blatt-Weisskopf barrier factors

$$\hat{\mathcal{A}}(s) = \frac{c}{m^2 - s - i g^2 C(s)/2}$$

Standard *S*-wave Breit-Wigner with modified phase space

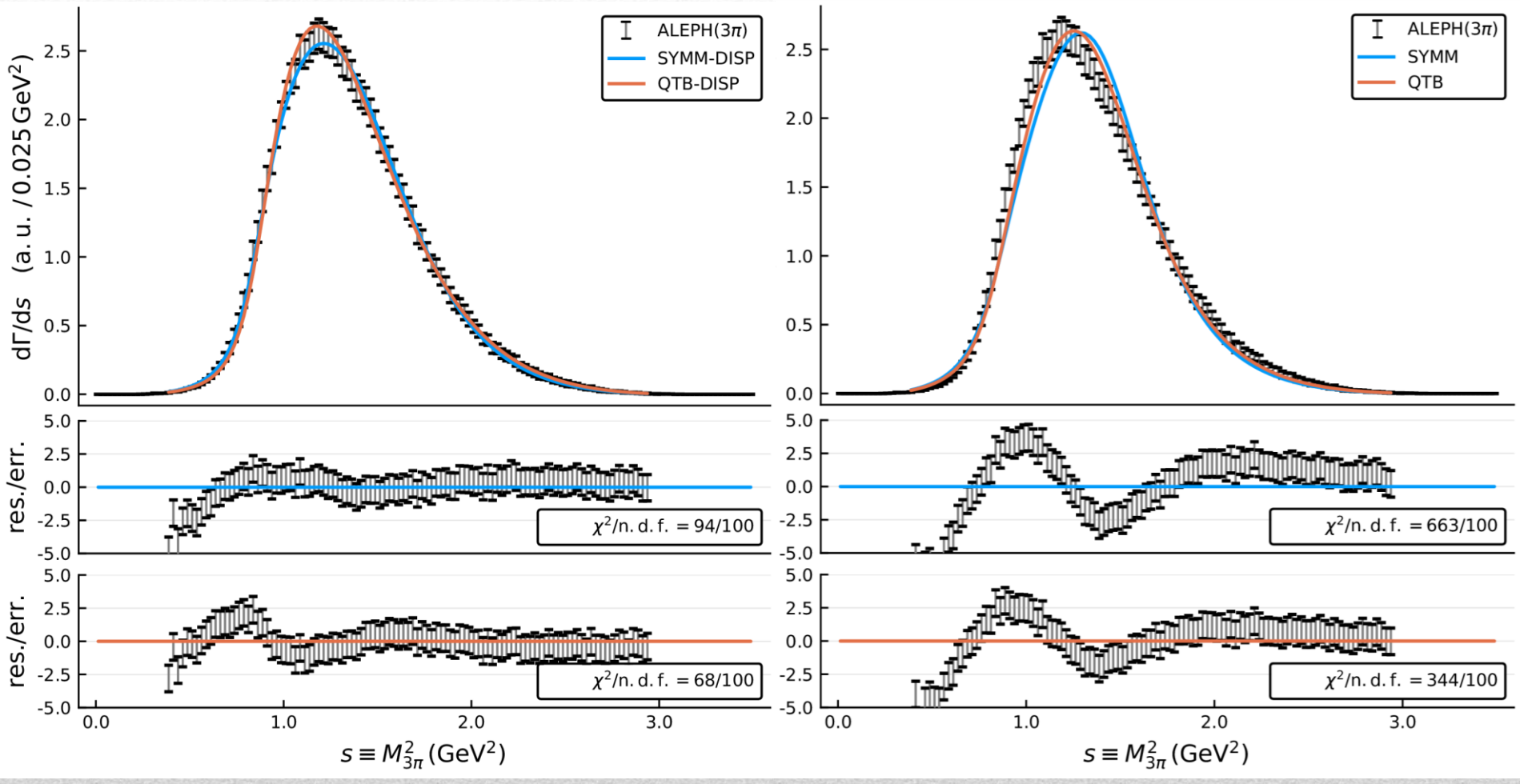
More about the model

We consider ALEPH data of $\tau^- \to \pi^-\pi^+\pi^-\nu_\tau$

$$\hat{\mathcal{A}}(s) = \frac{c}{m^2 - s - ig^2 C(s)/2}$$

$$C(s) = \begin{cases} \frac{1}{2}\int d\Phi_3 \left| \sum_\lambda f_\rho(\sigma_1) D^1_{0\lambda}(\Omega_1) D^1_{\lambda 0}(\Omega_{23}) - f_\rho(\sigma_3) D^1_{0\lambda}(\Omega_3) D^1_{\lambda 0}(\Omega_{12}) \right|^2 \equiv \rho_{\mathrm{SYMM}}(s) \\ \int d\Phi_3 \left| \sum_\lambda f_\rho(\sigma_1) D^1_{0\lambda}(\Omega_1) D^1_{\lambda 0}(\Omega_{23}) \right|^2 \equiv \rho_{\mathrm{QTB}}(s) \\ l_0 + \frac{s}{\pi}\int ds' \frac{\rho_{\mathrm{SYMM}}(s')}{s'(s'-s-i\epsilon)} \equiv \rho_{\text{SYMM-DISP}}(s) \\ l_0 + \frac{s}{\pi}\int ds' \frac{\rho_{\mathrm{QTB}}(s')}{s'(s'-s-i\epsilon)} \equiv \rho_{\text{QTB-DISP}}(s) \end{cases}$$

Fit to data

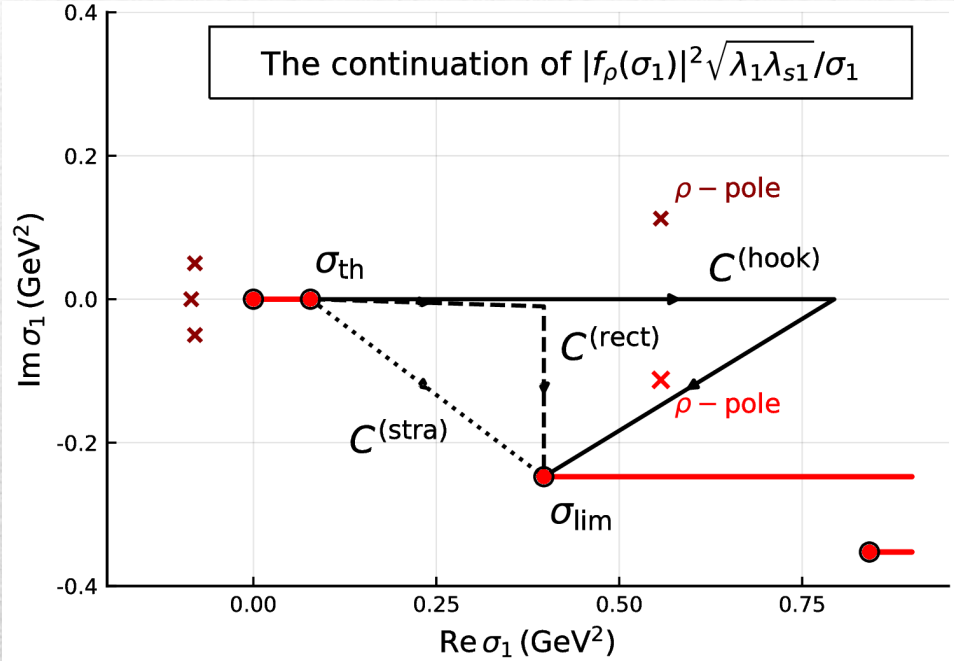


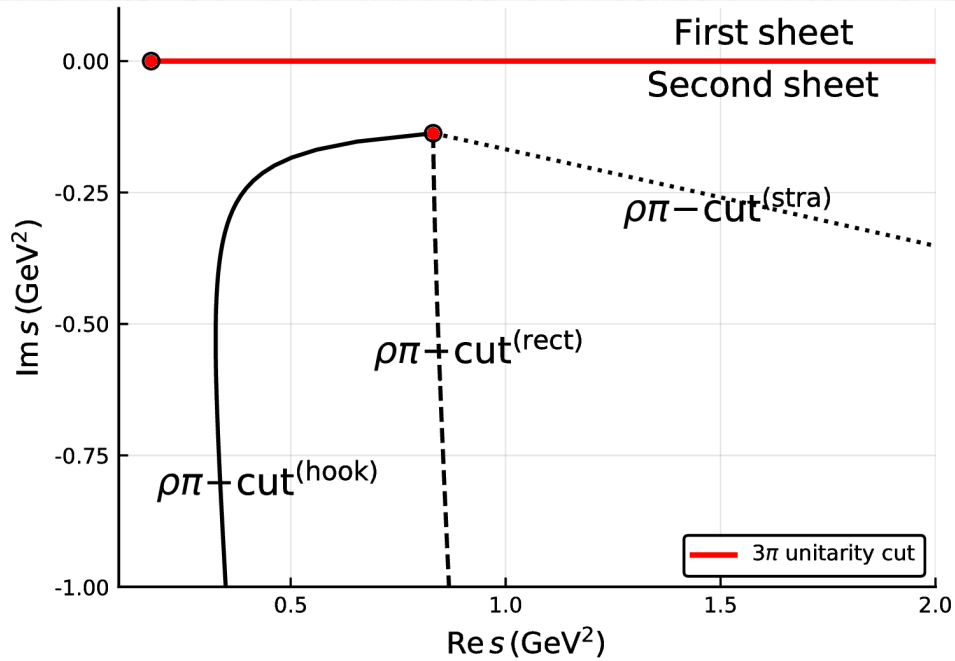
Dispersive models look better

Analytic continuation

$$\rho_{\mathrm{QTB}}(s) \propto \frac{1}{s}\int_{4m_\pi^2}^{(\sqrt{s}-m_\pi)^2} d\sigma_1 f^{(II)}(\sigma_1) f^{(I)}(\sigma_1)\frac{\sqrt{\lambda_1\lambda_{s1}}}{\sigma_1}$$

Going to complex s means making the integration path complex



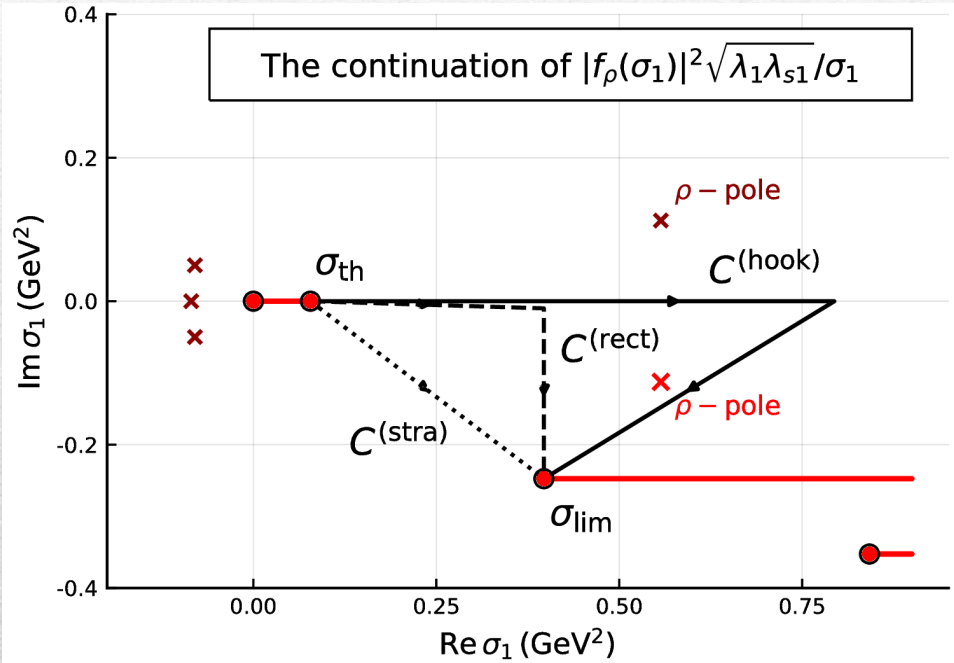


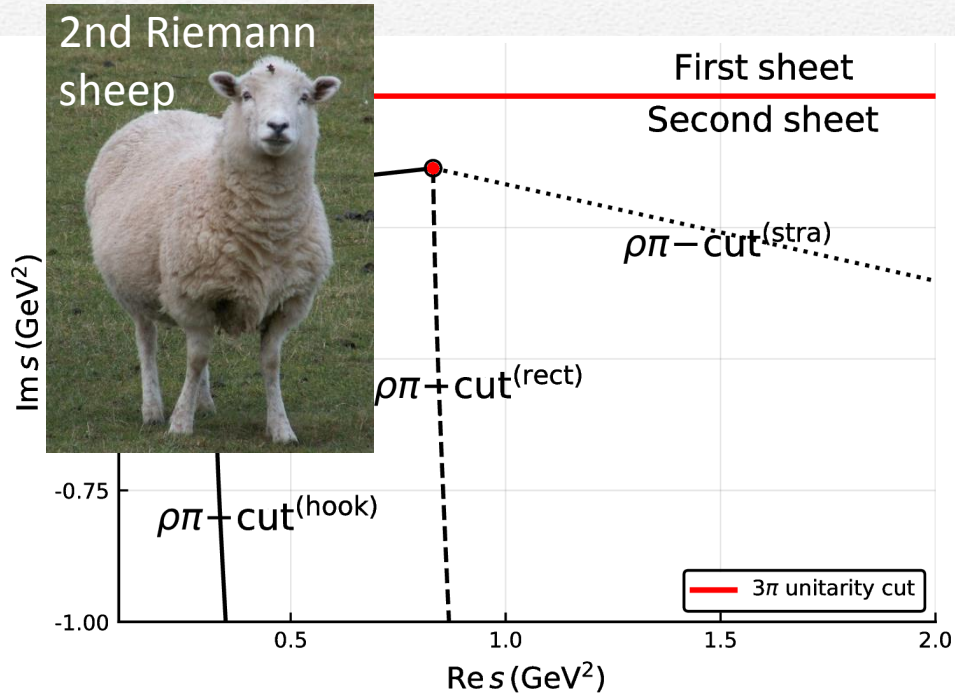
When the integration boundary hits the ρ pole in $f(\sigma_1)$, the woolly cut opens

Analytic continuation

$$\rho_{\mathrm{QTB}}(s) \propto \frac{1}{s}\int_{4m_\pi^2}^{(\sqrt{s}-m_\pi)^2} d\sigma_1 f^{(II)}(\sigma_1) f^{(I)}(\sigma_1)\frac{\sqrt{\lambda_1\lambda_{s1}}}{\sigma_1}$$

Going to complex s means making the integration path complex





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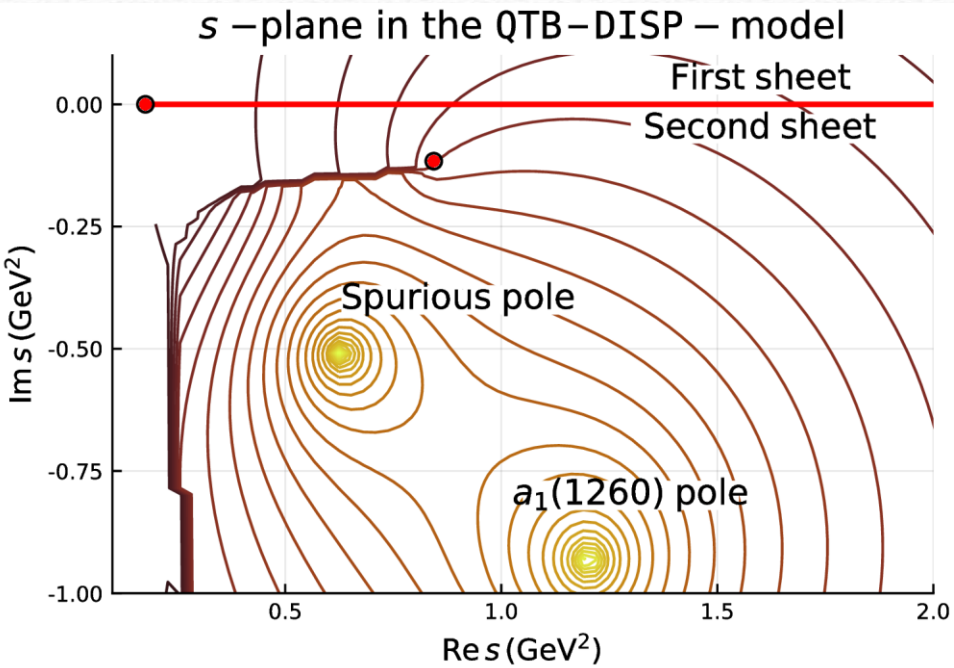
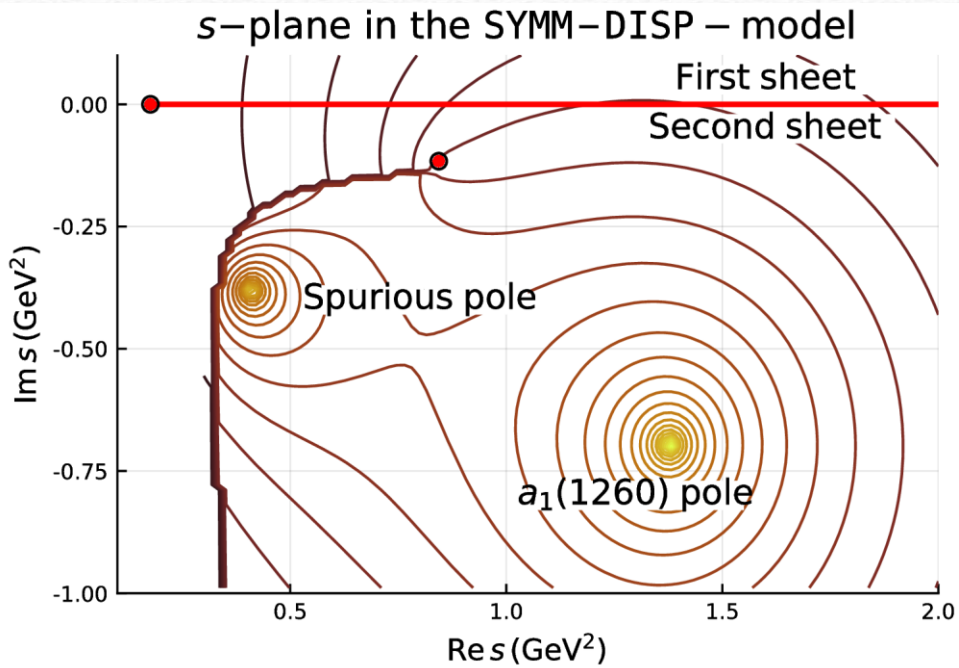
Analytic continuation

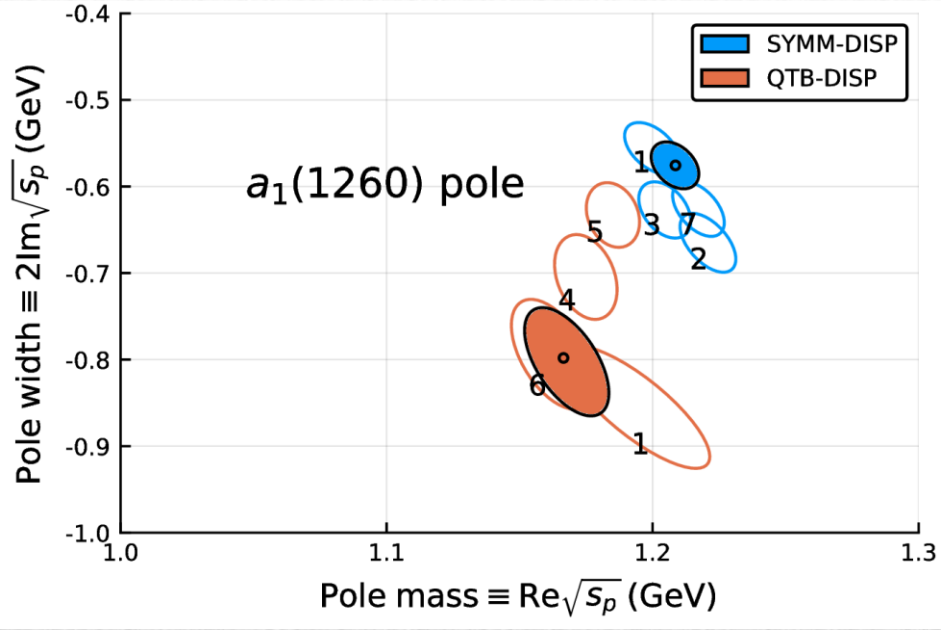




An additional pole appear, almost hidden under the woolly cut
It can be traced back to the $1/s$ factor of the phase space

Systematics

#	Fit studies	QTB-DISP χ^2/n.d.f.	SYMM-DISP χ^2/n.d.f.
1	$s < 2\ \mathrm{GeV}^2$	53/62	81/62
2	$R' = 3\ \mathrm{GeV}^{-1}$		18/100
3	$m'_\rho = m_\rho + 10\ \mathrm{MeV}$		83/100
4	$m'_\rho = m_\rho - 10\ \mathrm{MeV}$	37/100	
5	$m'_\rho = m_\rho - 20\ \mathrm{MeV}$	30/100	
6	$\Gamma'_\rho = \Gamma_\rho + 5\ \mathrm{MeV}$	66/100	
7	$\Gamma'_\rho = \Gamma_\rho - 30\ \mathrm{MeV}$		36/100



$$m_p^{(a_1(1260))} = (1209 \pm 4^{+12}_{-9})\,\mathrm{MeV}, \quad \Gamma_p^{(a_1(1260))} = (576 \pm 11^{+80}_{-20})\,\mathrm{MeV}$$

Conclusions

We perform a coupled-channel analysis to the $\eta^{(\prime)}\pi$ COMPASS data

We can describe data with a model which generates a single stable pole in the relevant region of the *P*-wave

The pole position is sufficiently stable upon changes of the model

We also extract the resonant parameters of $a_2^{(\prime)}$

Poles	Mass (MeV)	Width (MeV)
$a_2(1320)$	$1306.0 \pm 0.8 \pm 1.3$	$114.4 \pm 1.6 \pm 0.0$
$a_2'(1700)$	$1722 \pm 15 \pm 67$	$247 \pm 17 \pm 63$
π_1	$1564 \pm 24 \pm 86$	$492 \pm 54 \pm 102$

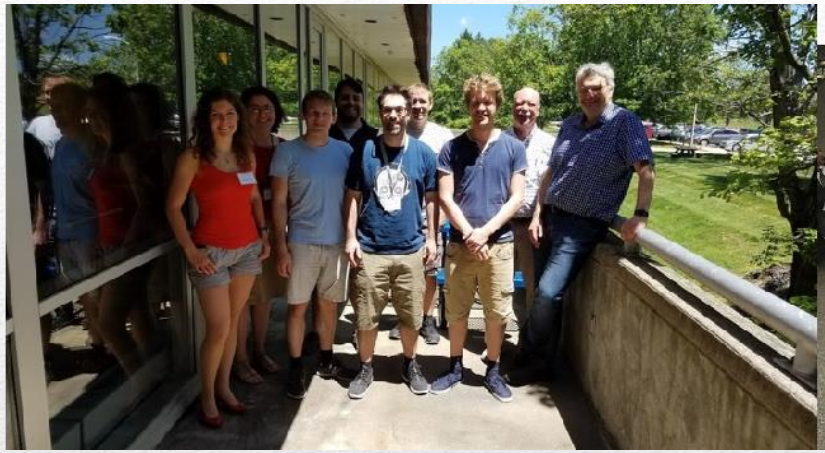
We perform the analysis of $\tau \to 3\pi\nu$ ALEPH data

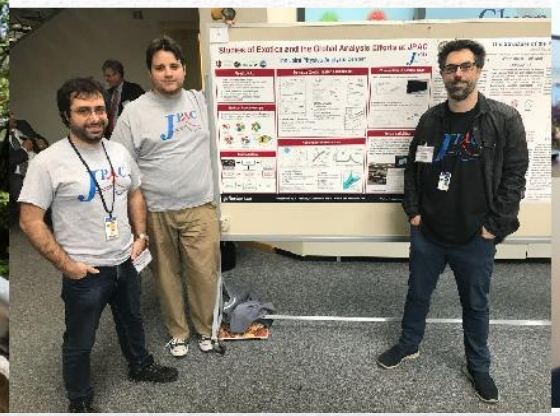
We consider a simplified quasi 2-body model, with a reduced unitarity equation easier to handle

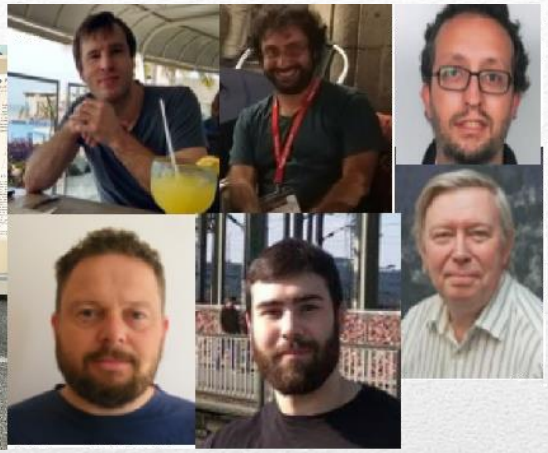
The $a_1(1260)$ pole position is determined

$$m_p^{(a_1(1260))} = (1209 \pm 4^{+12}_{-9})\,\mathrm{MeV}$$
$$\Gamma_p^{(a_1(1260))} = (576 \pm 11^{+80}_{-20})\,\mathrm{MeV}$$

Joint Physics Analysis Center



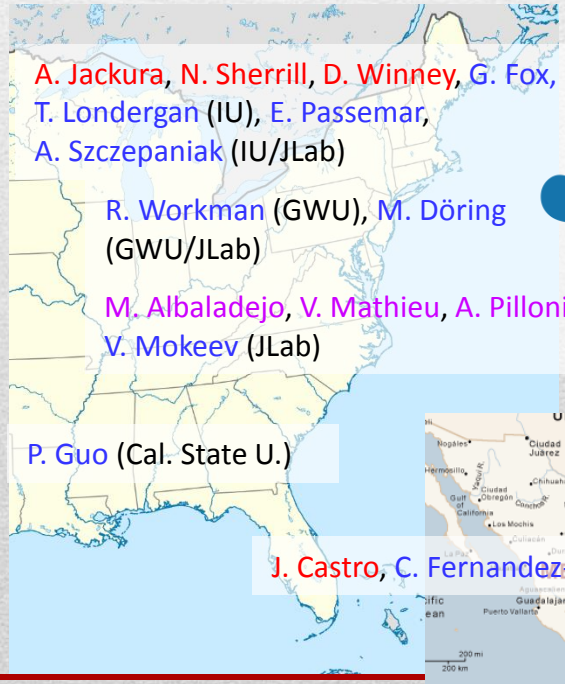




BACKUP

Joint Physics Analysis Center

- We aim at developing new theoretical tools, to get insight on QCD using first principles of QFT (unitarity, analyticity, crossing symmetry, low and high energy constraints,…) to extract the physics out of the data
- Many other ongoing projects (both for meson and baryon spectroscopy, and for high energy observables), with a particular attention to producing complete reaction models for the golden channels in exotic meson searches



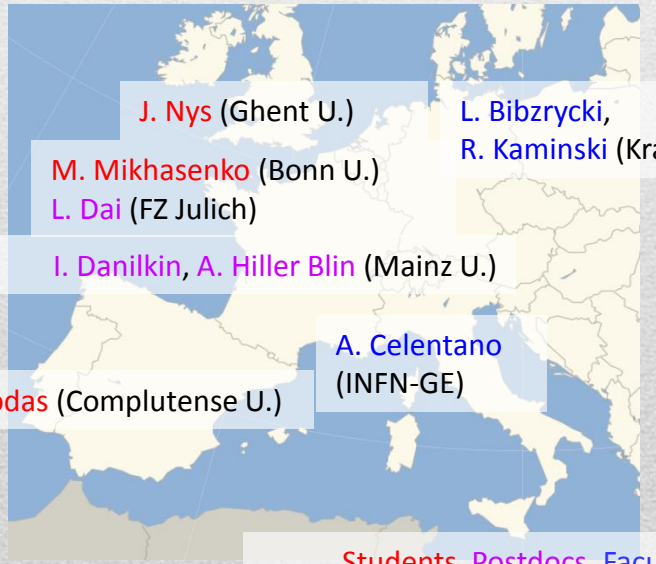
A. Jackura, N. Sherrill, D. Winney, G. Fox, T. Londergan (IU), E. Passemar, A. Szczepaniak (IU/JLab)

R. Workman (GWU), M. Döring (GWU/JLab)

M. Albaladejo, V. Mathieu, A. Pilloni, V. Mokeev (JLab)

P. Guo (Cal. State U.)

J. Castro, C. Fernandez-Ramirez (UNAM)



J. Nys (Ghent U.)

L. Bibzrycki, R. Kaminski (Krakow)

M. Mikhasenko (Bonn U.)

L. Dai (FZ Julich)

I. Danilkin, A. Hiller Blin (Mainz U.)

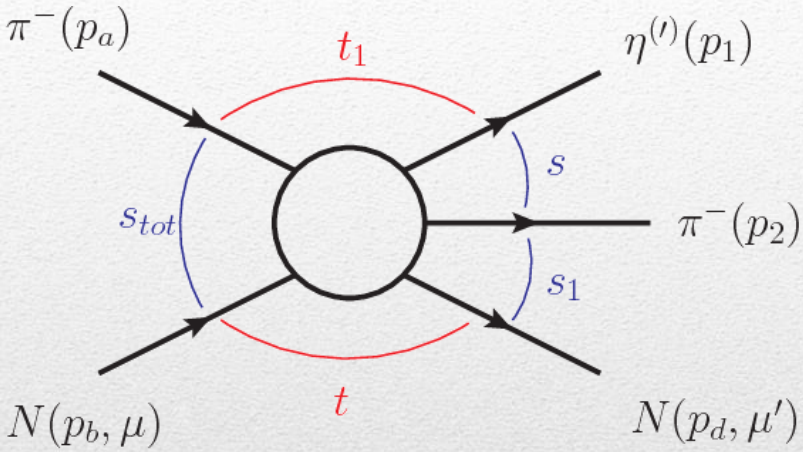
A. Celentano (INFN-GE)

A. Rodas (Complutense U.)

Students, Postdocs, Faculties

Formalism

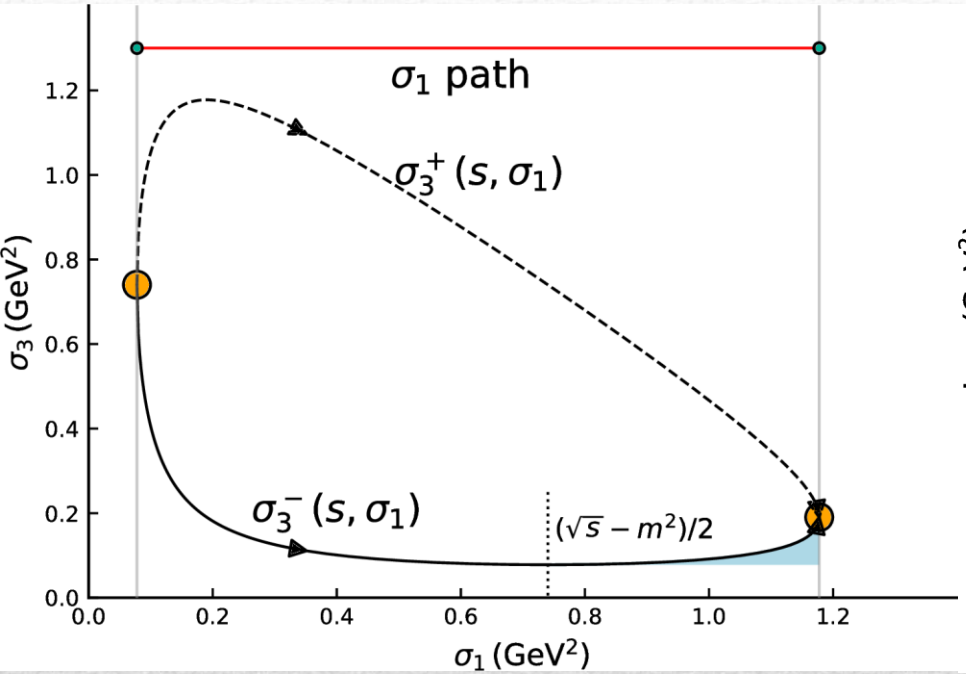
- Process is at fixed s_{tot}, and integrated t. Interested in resonances in s
- Recoil proton kinematically decouples from final state $\eta\pi$

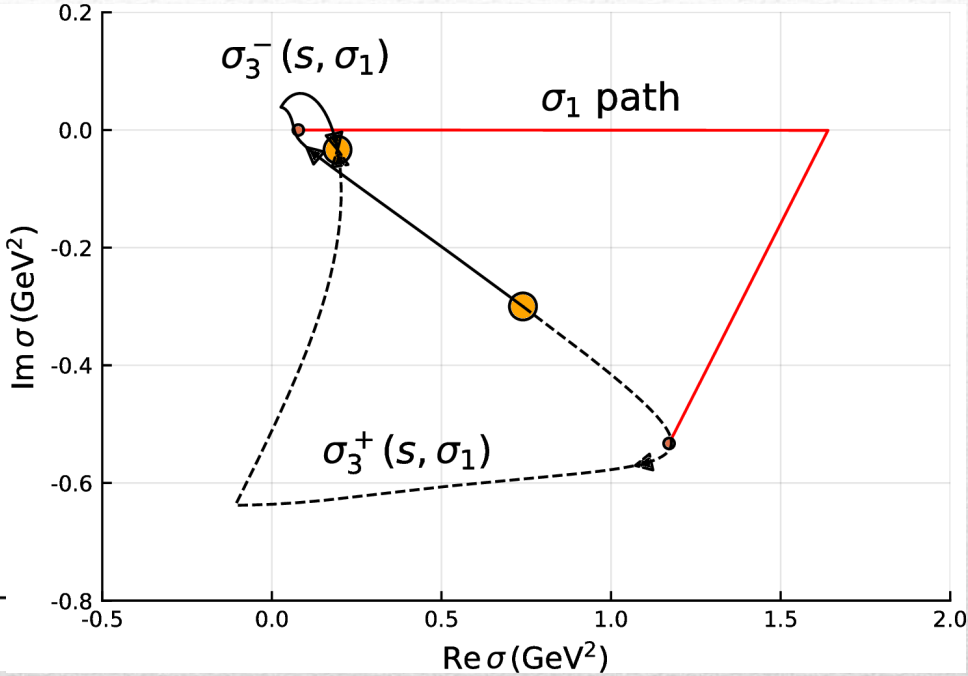


- Expand amplitude into partial waves

$$A_{\mu'\mu}(s_{tot}, s, t, s_1, t_1) = \sum_{LM\epsilon} a^{\epsilon}_{LM,\mu'\mu}(s_{tot}, t, s) Y^{\epsilon}_{LM}(\theta, \phi)$$

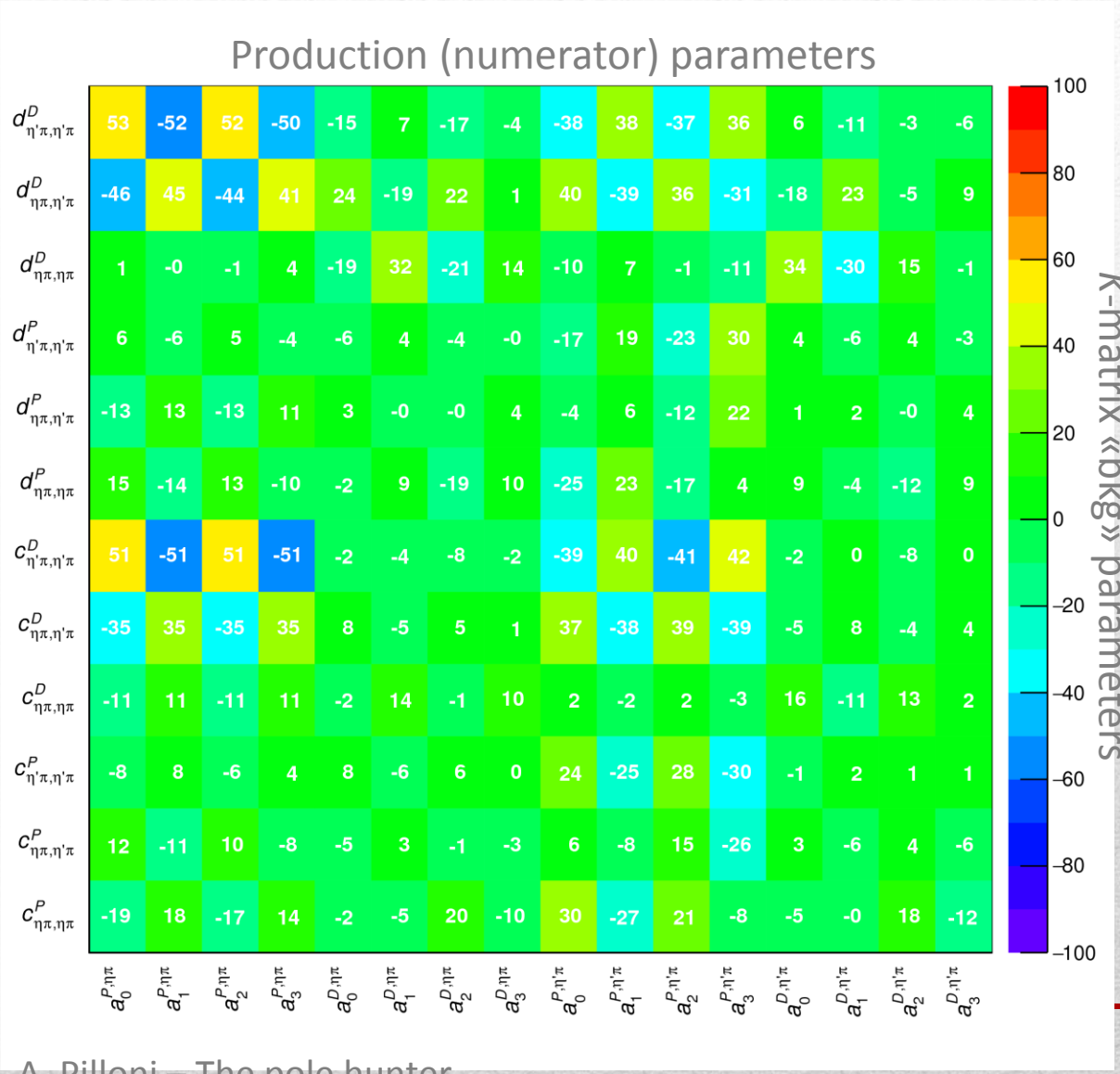
Analytic continuation





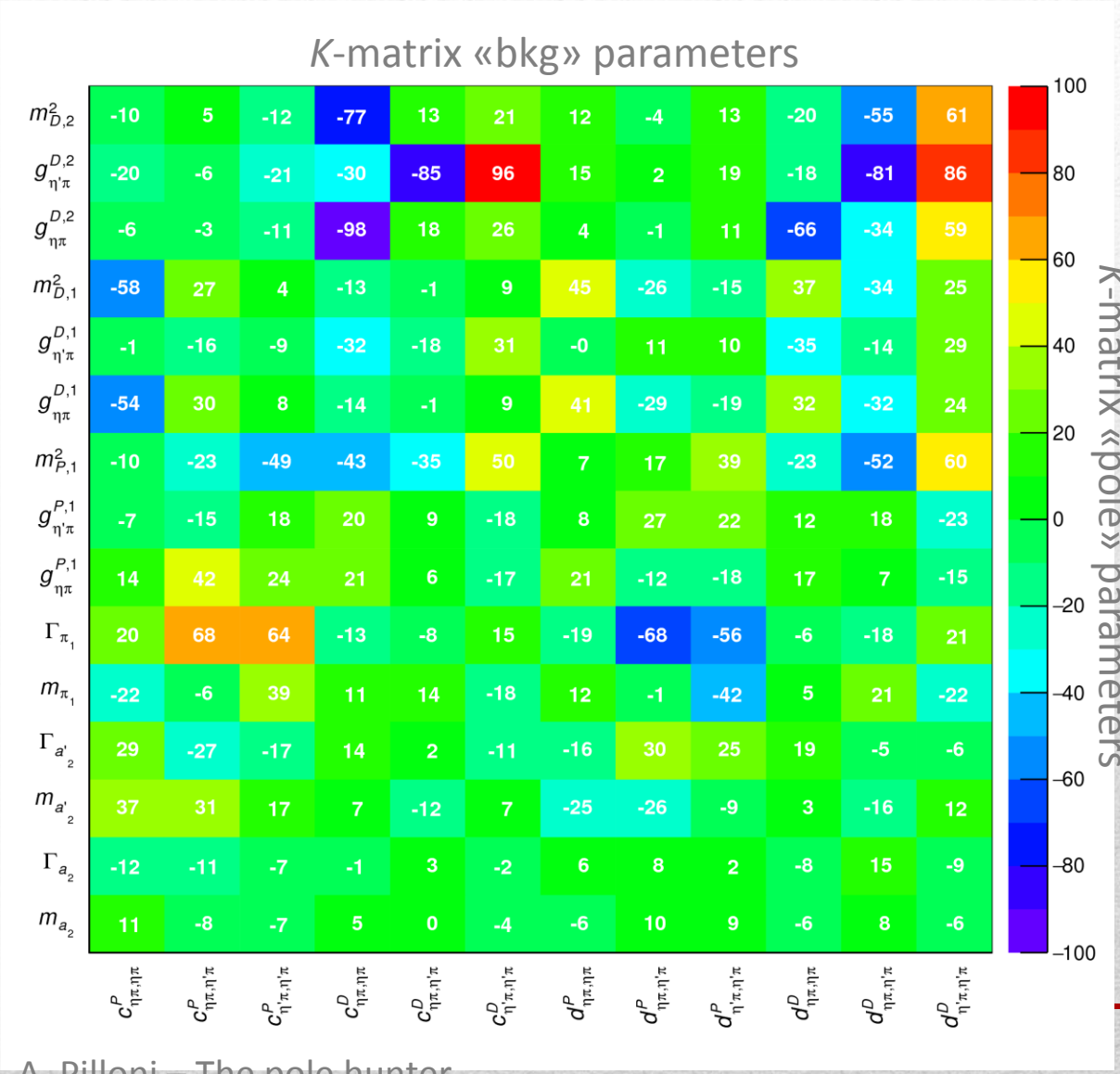
$$\rho_{\rm INT}(s) = \frac{1}{2\pi(8\pi)^2 s}\int_{4m_\pi^2}^{\sigma_{\rm lim}} {\rm d}\sigma_1 \int_{\sigma_3^-(\sigma_1,s)}^{\sigma_3^+(\sigma_1,s)} {\rm d}\sigma_3 \frac{f_\rho^*(\sigma_1)}{\sqrt{\sigma_1-4m_\pi^2}}\frac{f_\rho(\sigma_3)}{\sqrt{\sigma_3-4m_\pi^2}}$$
$$\times \frac{W(\sqrt{s},\sqrt{\sigma_1},\sqrt{\sigma_3})}{((\sqrt{s}+\sqrt{\sigma_1})^2-m_\pi^2)((\sqrt{s}+\sqrt{\sigma_3})^2-m_\pi^2)}.$$

Correlations



Denominator parameters not very correlated with the numerator ones ✓

Correlations



Denominator parameters uncorrelated between *P*- and *D*-wave ✓

Formalism

- The differential cross section is

$$\frac{d\sigma}{ds} = \frac{1}{2(4\pi)^4\sqrt{s}}\left(\frac{\hbar c}{m_N P_{lab}}\right)^2 \frac{1}{2}\sum_{LM\epsilon}\int_{t_-}^{t_+} dt\,|\mathbf{p}|\sum_{\mu\mu'}|a^{\epsilon}_{LM,\mu'\mu}(s_{tot},t,s)|^2$$

$$\equiv \frac{\mathcal{N}}{\sqrt{s}}\sum_{LM\epsilon}\mathcal{I}^{\epsilon}_{LM}(s_{tot},s)$$

where the intensity distribution is defined

$$\mathcal{I}^{\epsilon}_{LM}(s_{tot},s) = \int_{t_-}^{t_+} dt\,|\mathbf{p}|\sum_{\mu\mu'}|a^{\epsilon}_{LM,\mu'\mu}(s_{tot},t,s)|^2$$

- Model will be compared to intensity distributions given by COMPASS

Systematic studies

Systematic	Poles	Mass (MeV)	Deviation (MeV)	Width (MeV)	Deviation (MeV)
		Variation of the function $\rho N(s')$			
$s_R = 0.8\,\mathrm{GeV}^2$	$a_2(1320)$	1306.4	0.4	115.0	0.6
	$a_2'(1700)$	1720	−3	272	26
	π_1	1532	−33	484	−8
$s_R = 1.8\,\mathrm{GeV}^2$	$a_2(1320)$	1305.6	−0.4	113.2	−1.2
	$a_2'(1700)$	1743	21	254	7
	π_1	1528	−36	410	−82
Systematic assigned	$a_2(1320)$		0.0		0.0
	$a_2'(1700)$		21		26
	π_1		36		82
$\alpha = 1$	$a_2(1320)$	1305.9	−0.1	114.7	0.3
	$a_2'(1700)$	1685	−37	299	52
	π_1	1506	−58	552	60
Systematic assigned	$a_2(1320)$		0.0		0.0
	$a_2'(1700)$		37		52
	π_1		58		60
$Q_J, \alpha = 1$	$a_2(1320)$	1304.9	−1.1	114.2	−0.2
	$a_2'(1700)$	1670	−52	269	22
	π_1	1511	−53	528	36
$Q_J, \alpha = 1.5$	$a_2(1320)$	1306.0	0.1	115.0	0.6
	$a_2'(1700)$	1717	−5	272	25
	π_1	1578	14	530	39
$Q_J, \alpha = 2$	$a_2(1320)$	1306.2	0.2	114.7	0.3
	$a_2'(1700)$	1723	1	261	15
	π_1	1570	6	508	16
Systematic assigned	$a_2(1320)$		1.1		0.0
	$a_2'(1700)$		52		25
	π_1		53		0

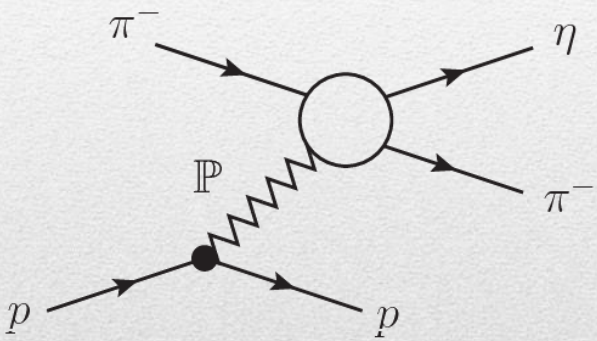
Systematic studies

Variation of the numerator function $n(s)$

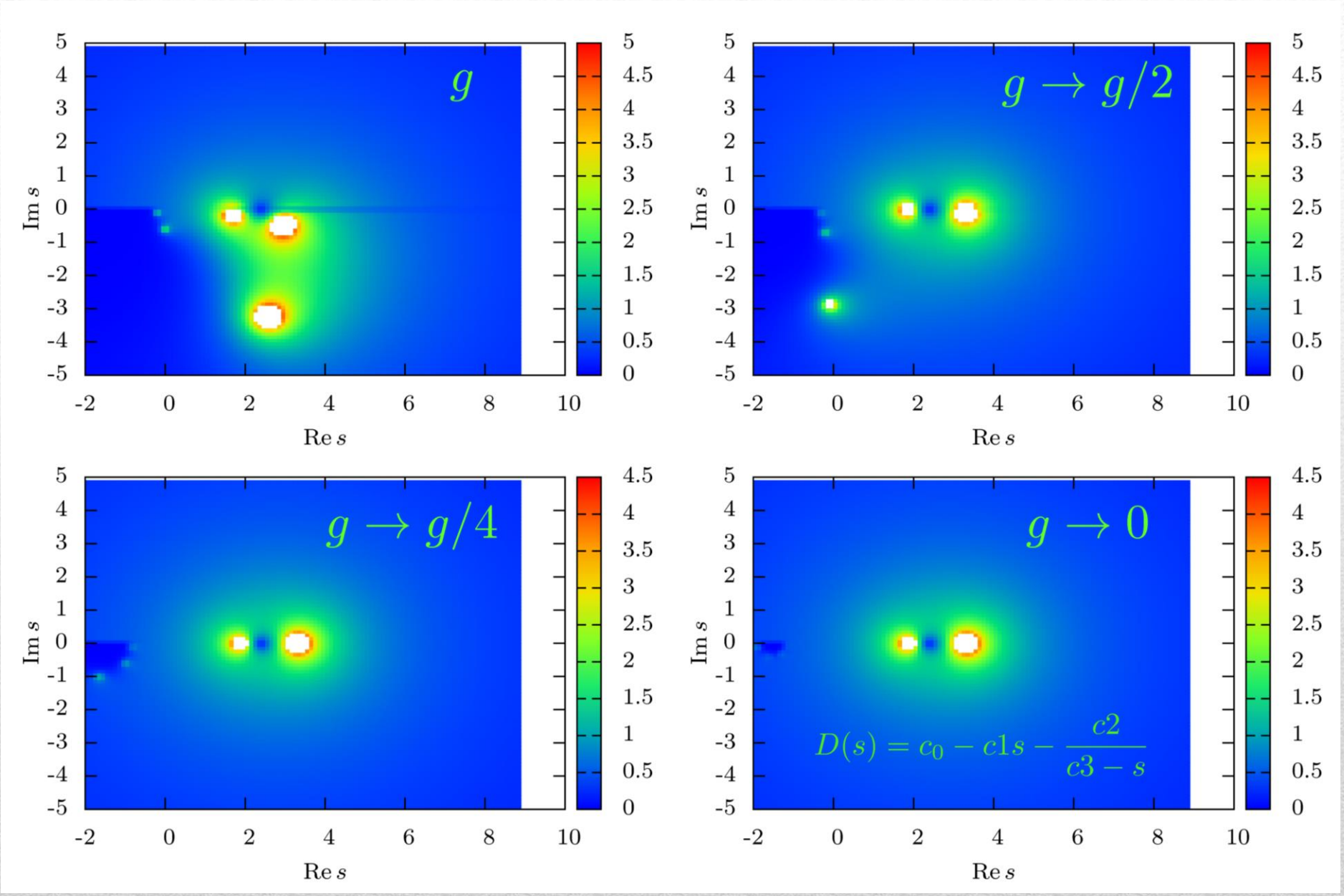
Polynomial expansion	$a_2(1320)$	1305.9	-0.1	114.7	0.3
	$a_2'(1700)$	1723	1	249	2
	π_1	1563	-1	479	-13
Systematic assigned	$a_2(1320)$		0.0		0.0
	$a_2'(1700)$		0		0
	π_1		0		0
$t_{\rm eff} = -0.5\,{\rm GeV}^2$	$a_2(1320)$	1306.8	0.8	114.1	-0.3
	$a_2'(1700)$	1730	8	259	13
	π_1	1546	-18	443	-49
Systematic assigned	$a_2(1320)$		0.8		0.0
	$a_2'(1700)$		0		0
	π_1		0		0

Formalism

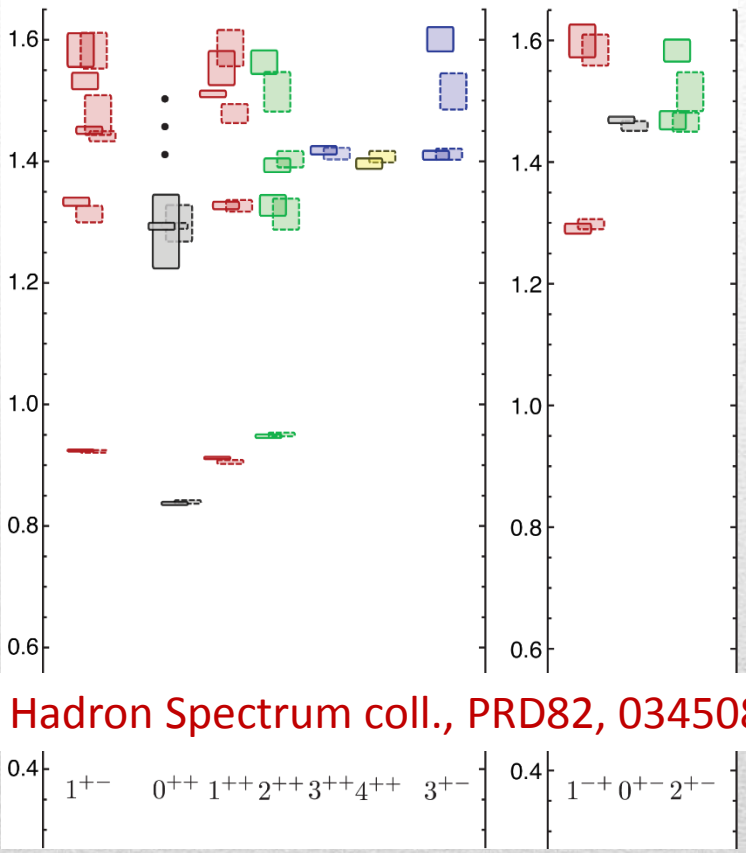
- $\pi p \to \eta\pi p$ is high-energy peripheral process $\Longrightarrow$ pomeron dominated exchange
- Factorize pomeron-nuclear vertex
- Pomeron has effective mass $\sqrt{-t}$



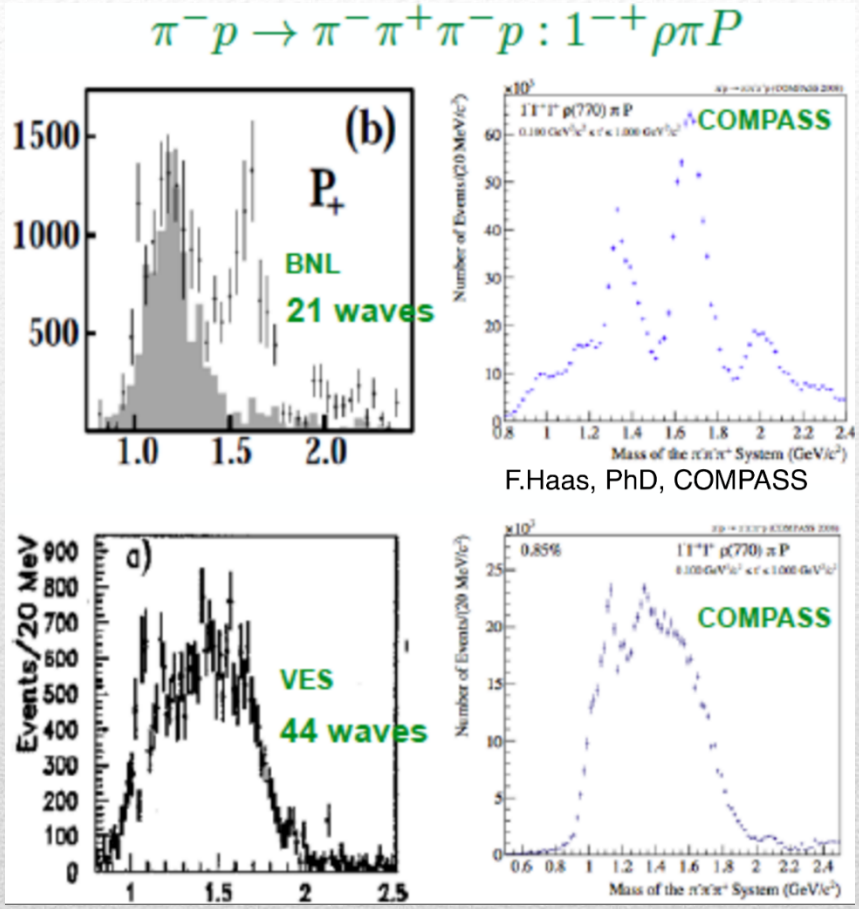
- Denote $p = |\mathbf{p}|$ the momentum of the $\eta\pi$ system, and $q = |\mathbf{q}|$ the momentum of the $\pi\mathbb{P}$ system


g
$g \to g/2$
$g \to g/4$
$g \to 0$
$D(s) = c_0 - c1s - \frac{c2}{c3 - s}$
Im s
Re s

Hybrids



Hadron Spectrum coll., PRD82, 034508



Signatures as $J^{PC} = 1^{-+}$ are not allowed in the quark model, Coulomb gauge QCD and flux tube predict glue excitation to be a quasi-particle with $J^{PC} = 1^{+-}, q\bar{q}g$ states expected

Need some constraint to draw robust conclusions about the existence of exotic states

Recap: single channel $\eta\pi$

The denominator $D(s)$ contains all the FSI constrained by unitarity → universal

$$D(s) = (K^{-1})(s) - \frac{s}{\pi}\int_{s_i}^{\infty} \frac{\rho(s')N(s')}{s'\,(s'-s)}ds'$$

$$K(s) = \sum_R \frac{g_R^2}{M_R^2 - s} \quad \textbf{OR} \quad K^{-1}(s) = c_0 - c_1 s + \sum_i \frac{c_i}{M_i^2 - s}$$

$$\rho(s)N(s) = g\frac{\lambda^{(2l+1)/2}\left(s, m_\pi^2, m_\eta^2\right)}{\left(s+s_R\right)^7} \qquad n(s) = \sum_n a_n T_n\left(\frac{s}{s+s_0}\right)$$

Recap: single channel $\eta\pi$

The denominator $D(s)$ contains all the FSI constrained by unitarity → universal

$$D(s) = (K^{-1})(s) - \frac{s}{\pi}\int_{s_i}^{\infty} \frac{\rho(s')N(s')}{s'\,(s'-s)}ds'$$

$$K(s) = \sum_R \frac{g_R^2}{M_R^2 - s}$$

K-matrix
more QFT motivated
poles on the 1st sheet unlikely

OR

$$K^{-1}(s) = c_0 - c_1 s + \sum_i \frac{c_i}{M_i^2 - s}$$

CDD parameterization
more *S*-matrix motivated
poles on the 1st sheet impossible

Recap: single channel $\eta\pi$

The denominator $D(s)$ contains all the FSI constrained by unitarity → universal

$$D(s) = (K^{-1})(s) - \frac{s}{\pi}\int_{s_i}^{\infty} \frac{\rho(s')N(s')}{s'\,(s'-s)}ds'$$

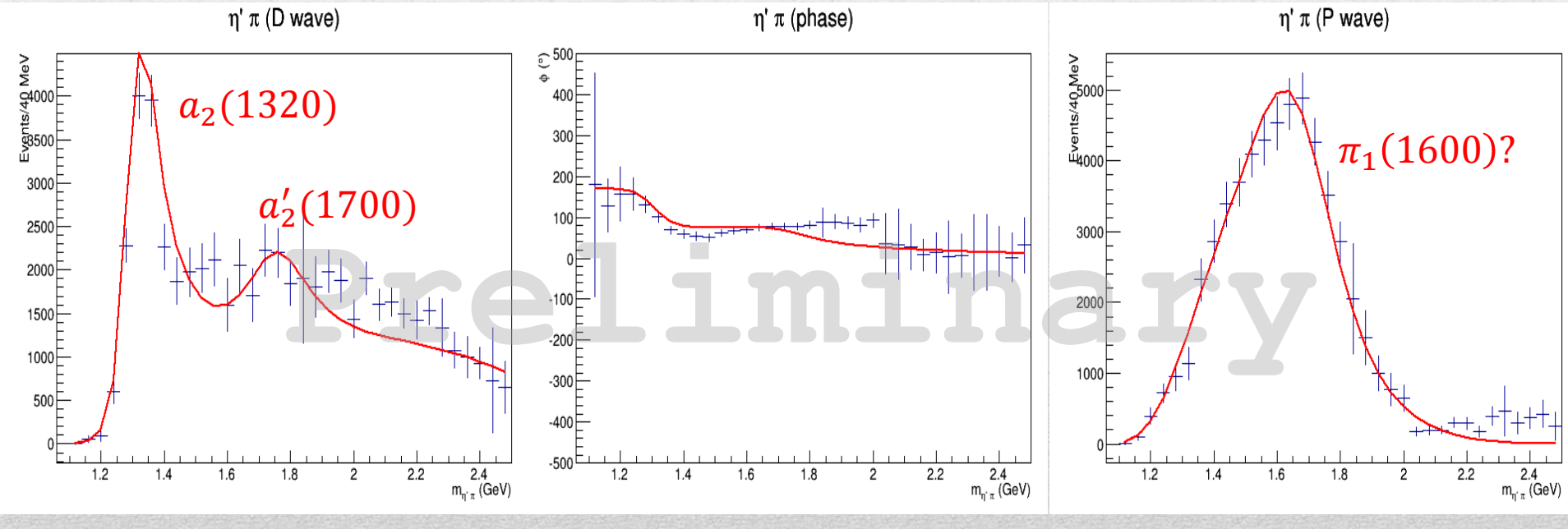
Numerator functions
know about crossed channel dynamics
unconstrained, we use a smooth model

$$\rho(s)N(s) = g\frac{\lambda^{(2l+1)/2}\left(s, m_\pi^2, m_\eta^2\right)}{\left(s+s_R\right)^7} \qquad n(s) = \sum_n a_n T_n\left(\frac{s}{s+s_0}\right)$$

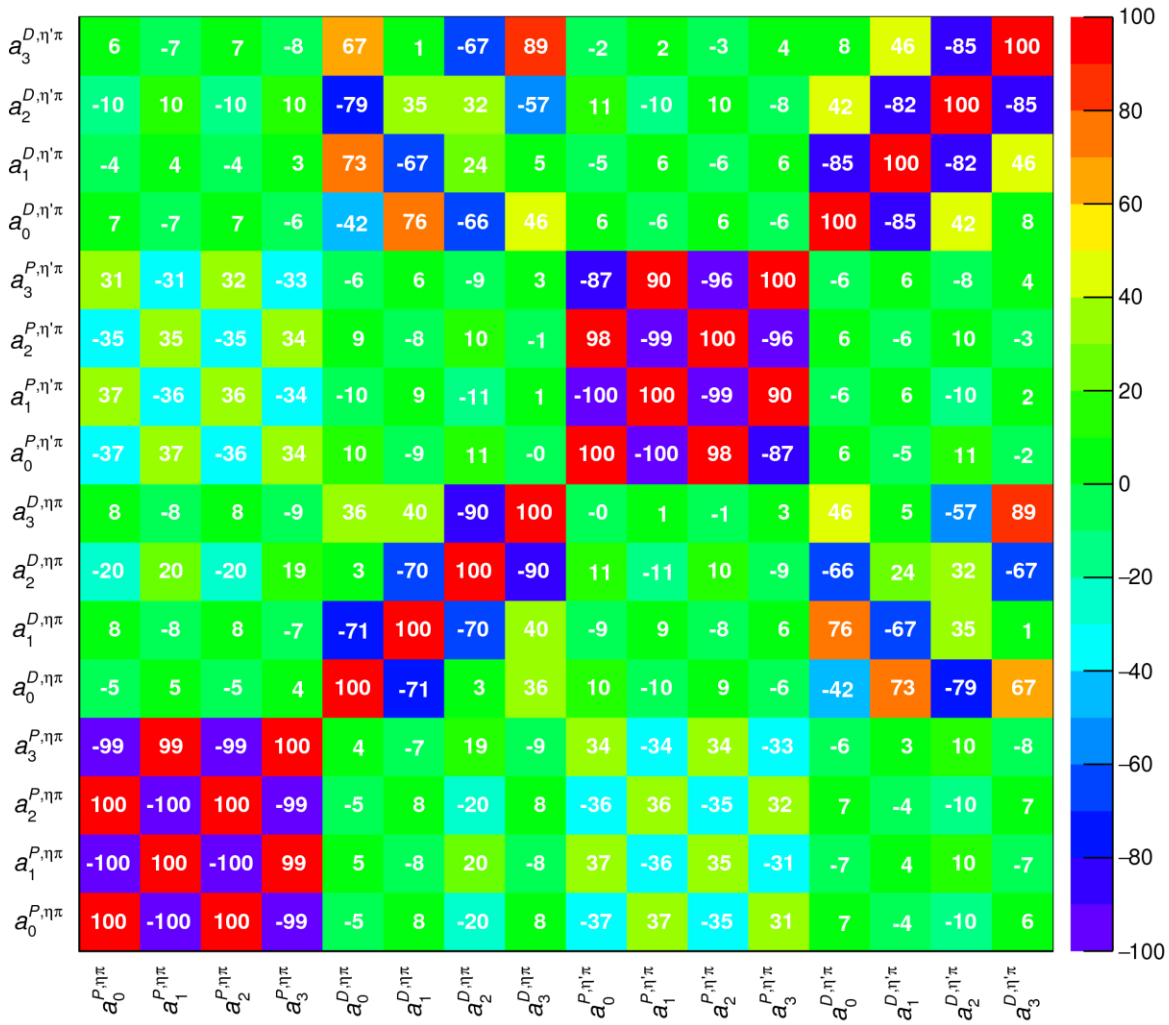
Searching for resonances in $\eta\pi$

$$m(a_2) = (1307 \pm 1 \pm 6)\ \text{MeV} \qquad m(a_2') = (1720 \pm 10 \pm 60)\ \text{MeV}$$
$$\Gamma(a_2) = (112 \pm 1 \pm 8)\ \text{MeV} \qquad \Gamma(a_2') = (280 \pm 10 \pm 70)\ \text{MeV}$$

- The coupled channel analysis involving the exotic P-wave is ongoing, as well as the extention to the GlueX production mechanism and kinematics

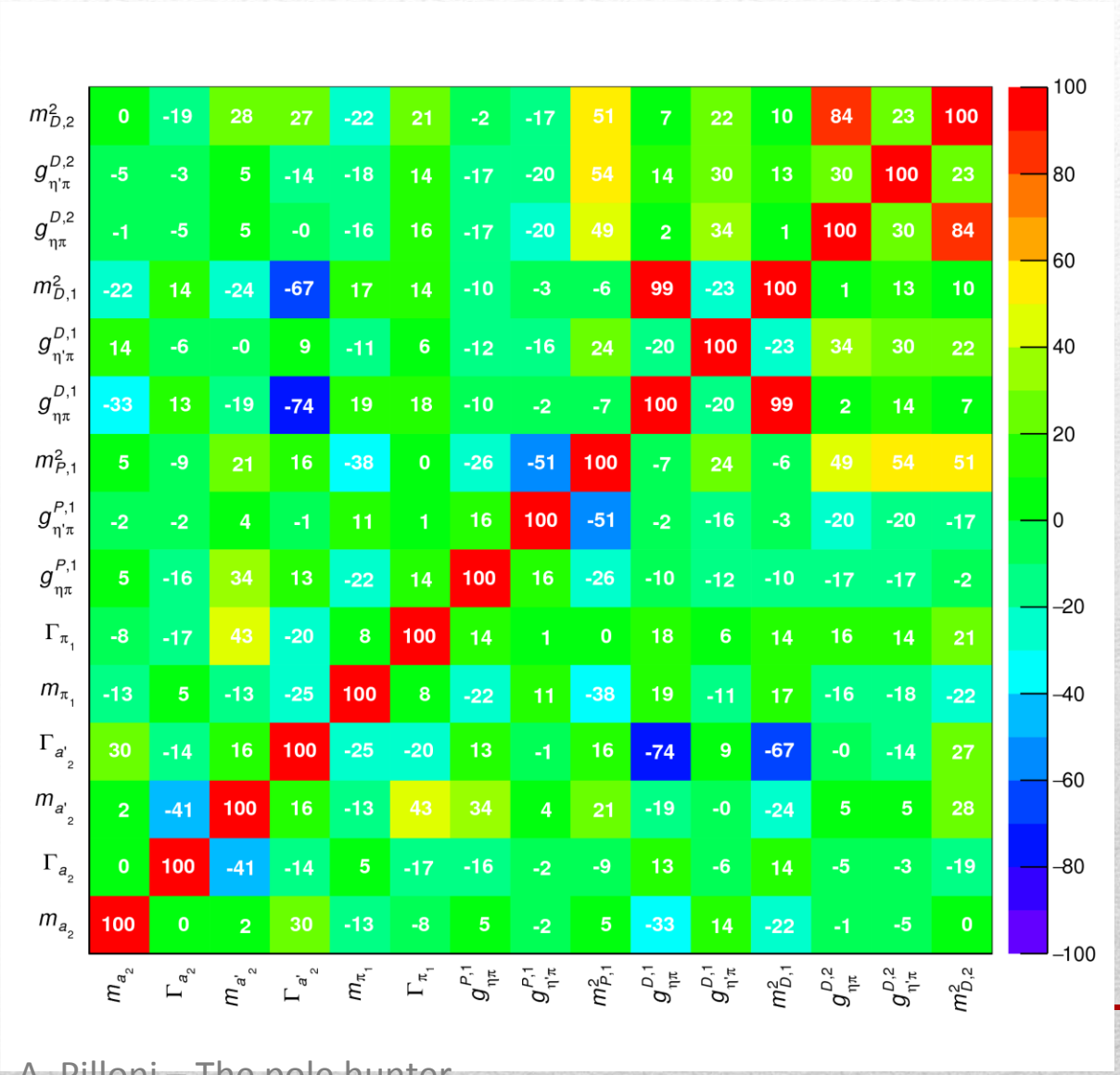


Correlations

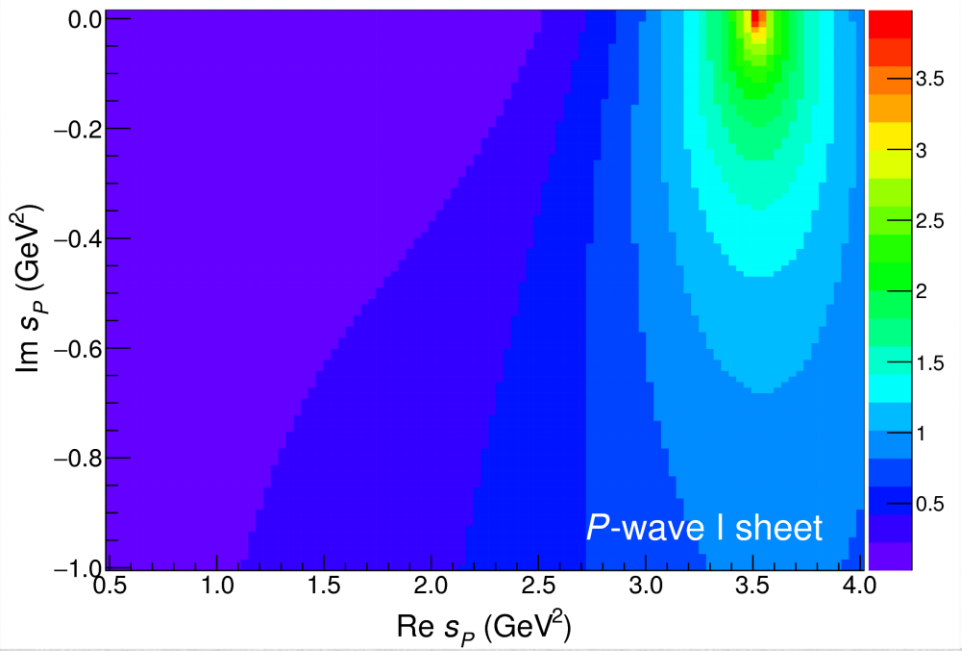


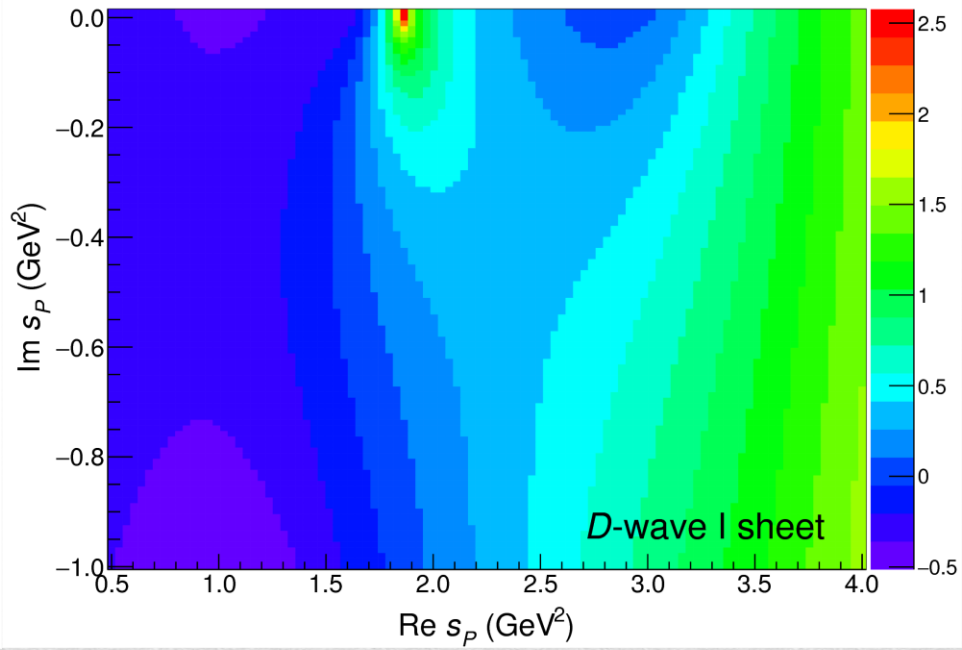
Polynomial parameters uncorrelated between *P*- and *D*-wave ✓

Correlations



Complex plane



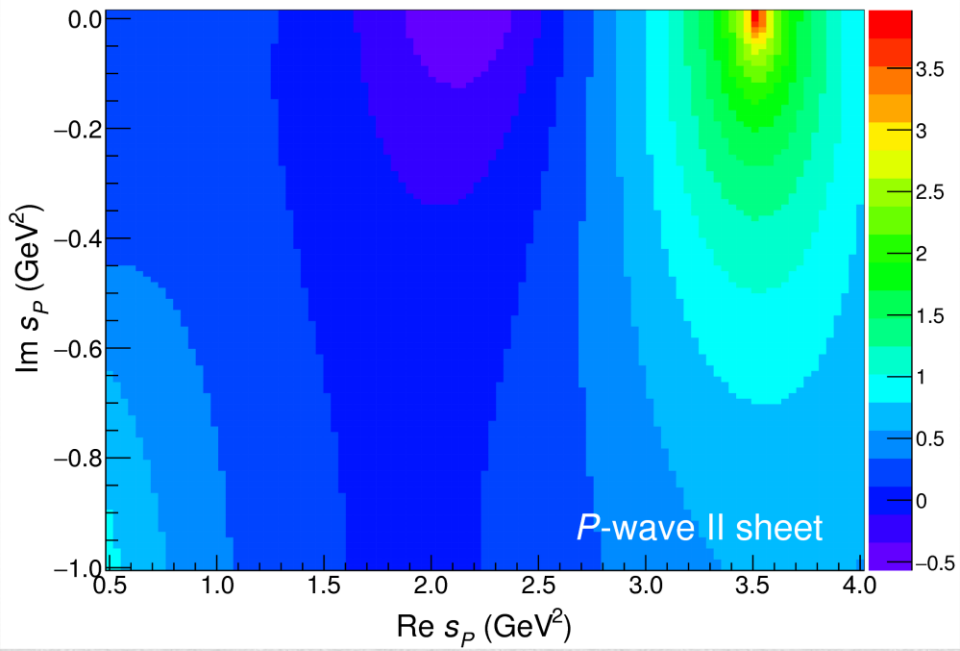


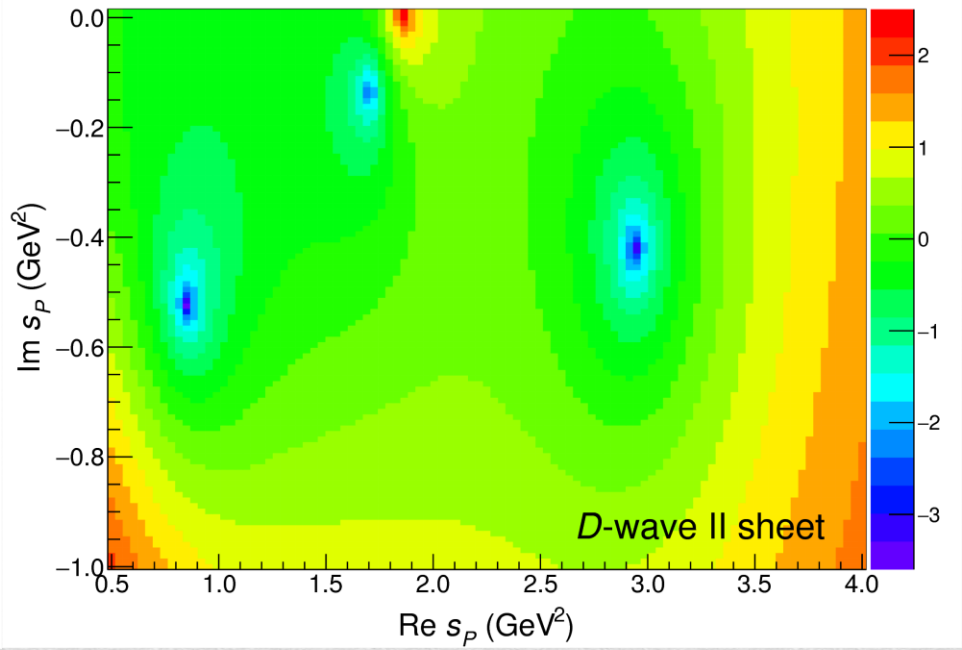
For the best fit solution, we look at the closest Riemann sheet in the complex plane
We see a limited amount of poles in the relevant region

We also see some close-to-threshold poles, which are artifacts of the model for $N(s)$

How to distinguish the two?

Complex plane





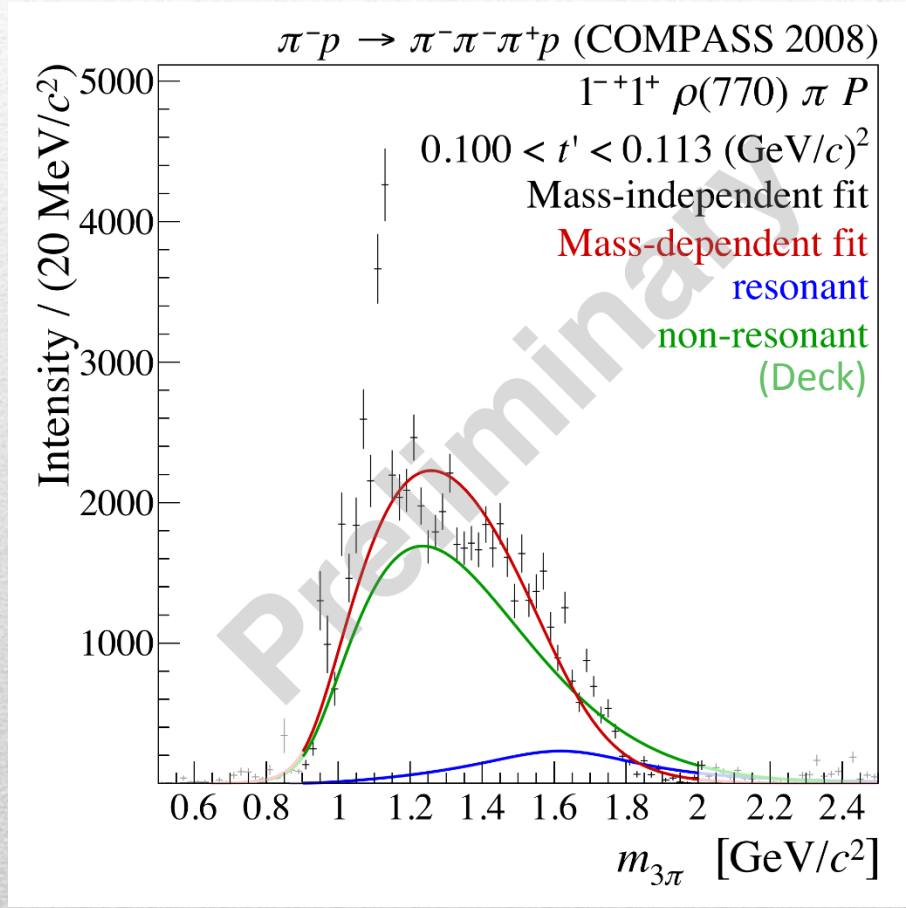
For the best fit solution, we look at the closest Riemann sheet in the complex plane
We see a limited amount of poles in the relevant region

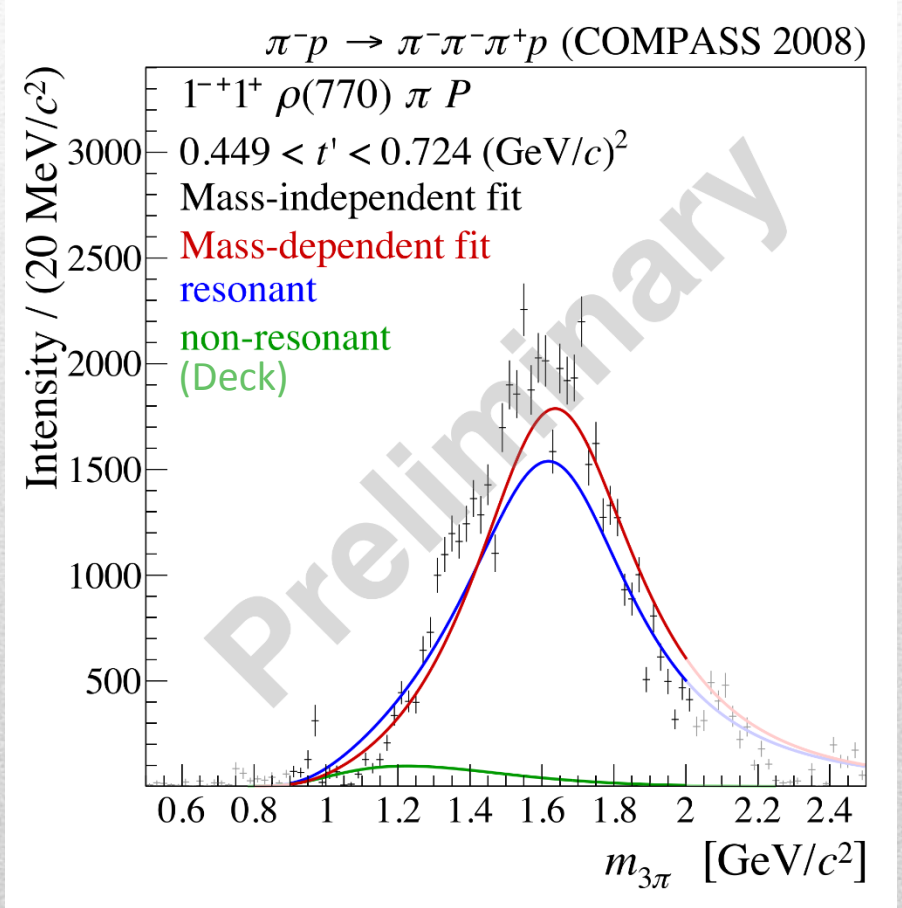
We also see some close-to-threshold poles, which are artifacts of the model for $N(s)$

How to distinguish the two?

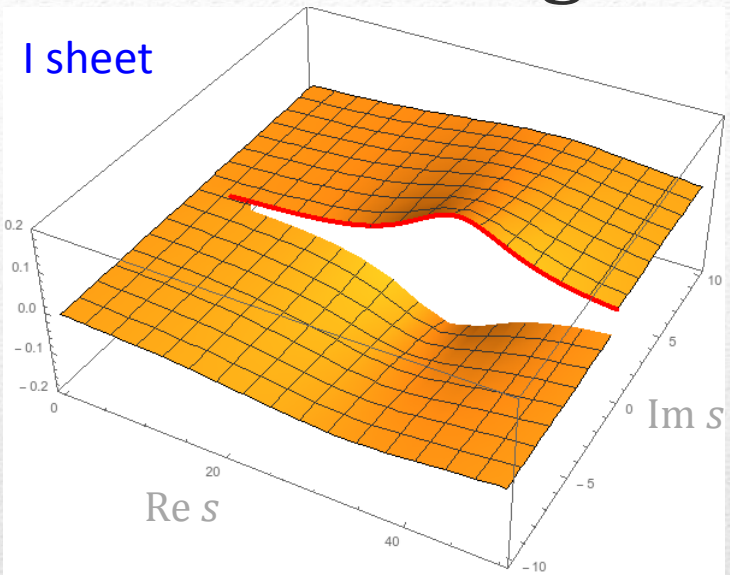
$\pi_1(1600) \to \rho\pi \to \pi\pi\pi$

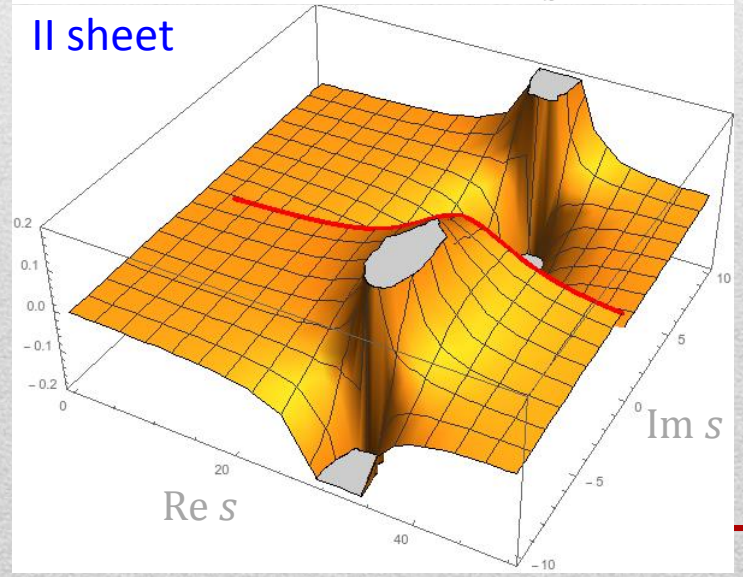
The strength of the Deck effect depends on the momentum transferred t, but the precise estimates rely on the model for the Deck amplitude





Pole hunting

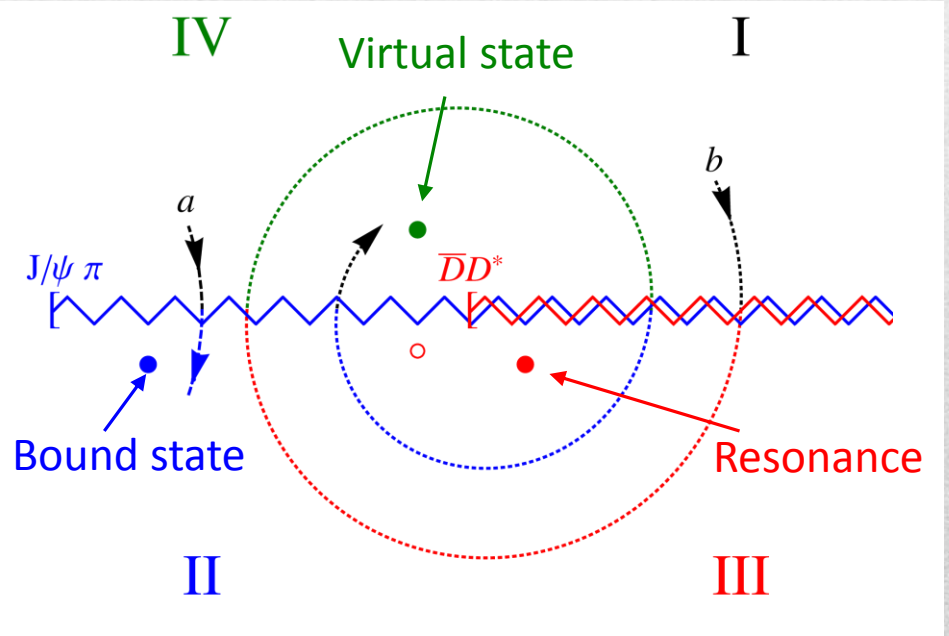




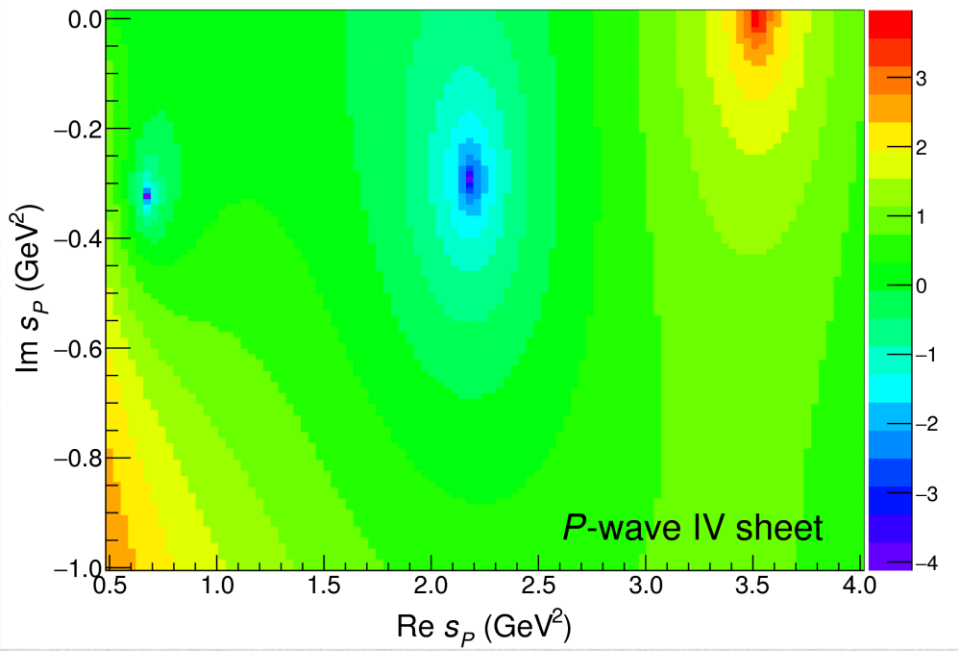
More complicated structure when more thresholds arise: two sheets for each new threshold

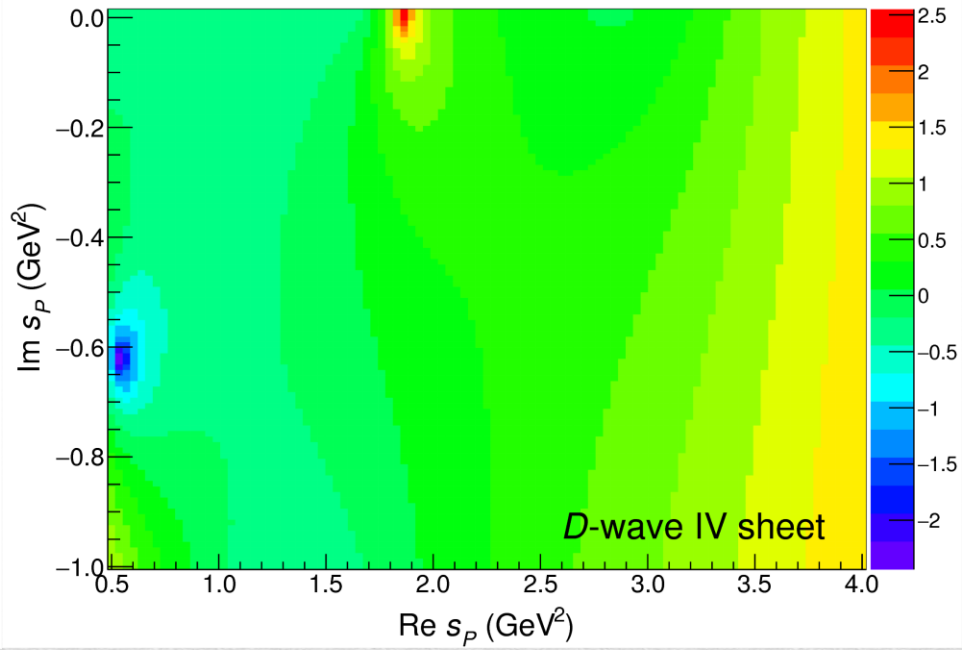
III sheet: usual resonances

IV sheet: cusps (virtual states)



Complex plane





For the best fit solution, we look at the closest Riemann sheet in the complex plane
We see a limited amount of poles in the relevant region

We also see some close-to-threshold poles, which are artifacts of the model for $N(s)$

How to distinguish the two?

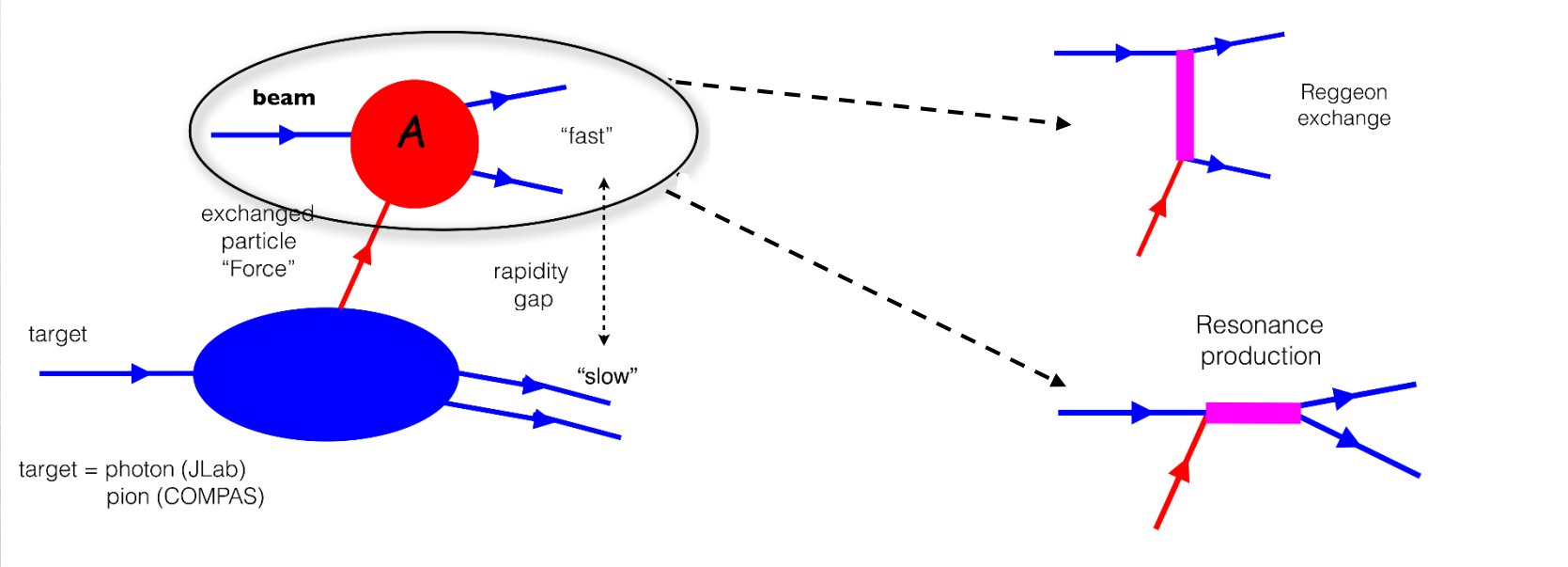
Regge exchange

Resonances are poles in s for fixed l
dominate low energy region

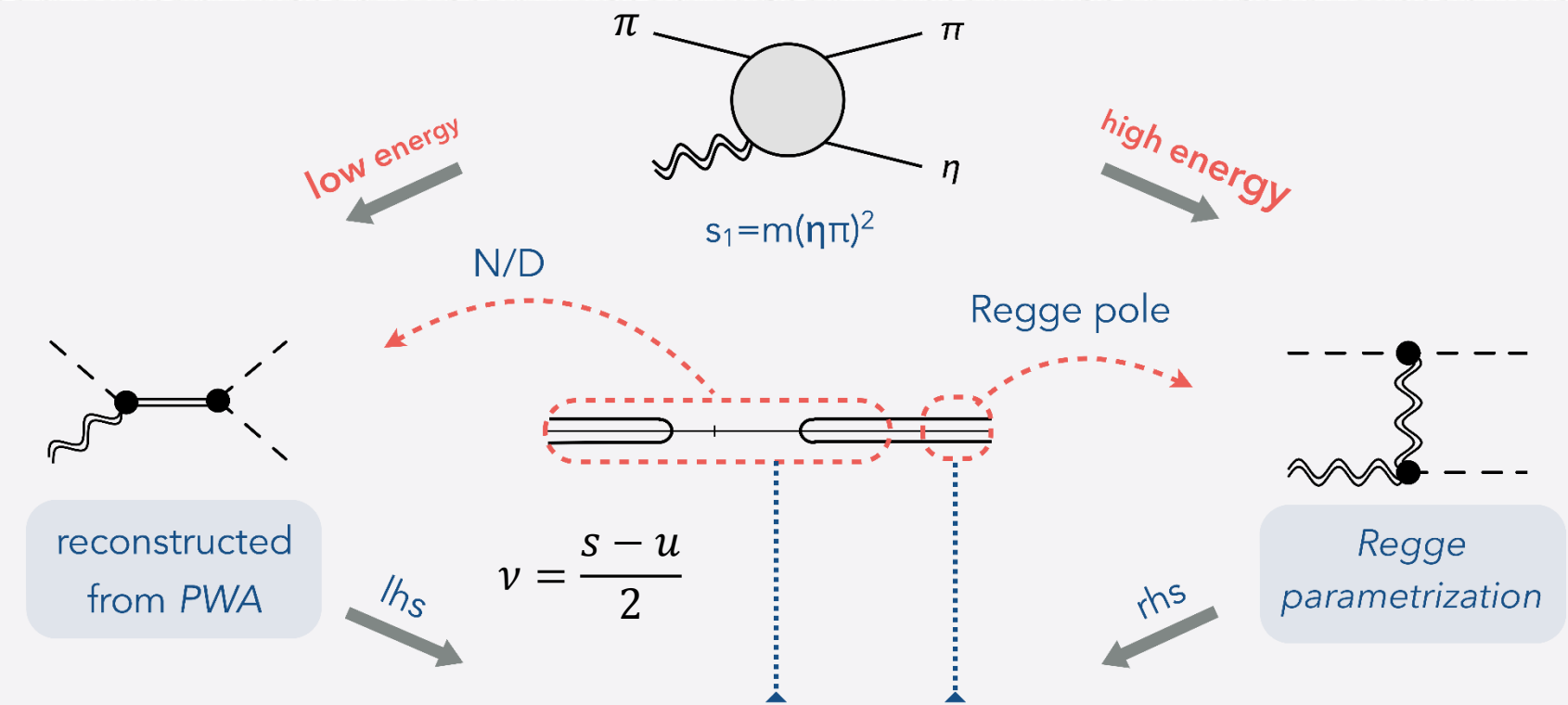
$$A_l \sim \frac{g_1 g_2}{s_p - s}$$

Reggeons are poles in l for fixed s
dominate high energy region

$$A \sim \sum s^l \sim g_1(t) g_2(t) s^{\alpha_\pm(t)} \left(-\frac{e^{i\pi\alpha^\pm(t)} \pm 1}{\sin \pi\, \alpha^\pm(t)} \right)$$



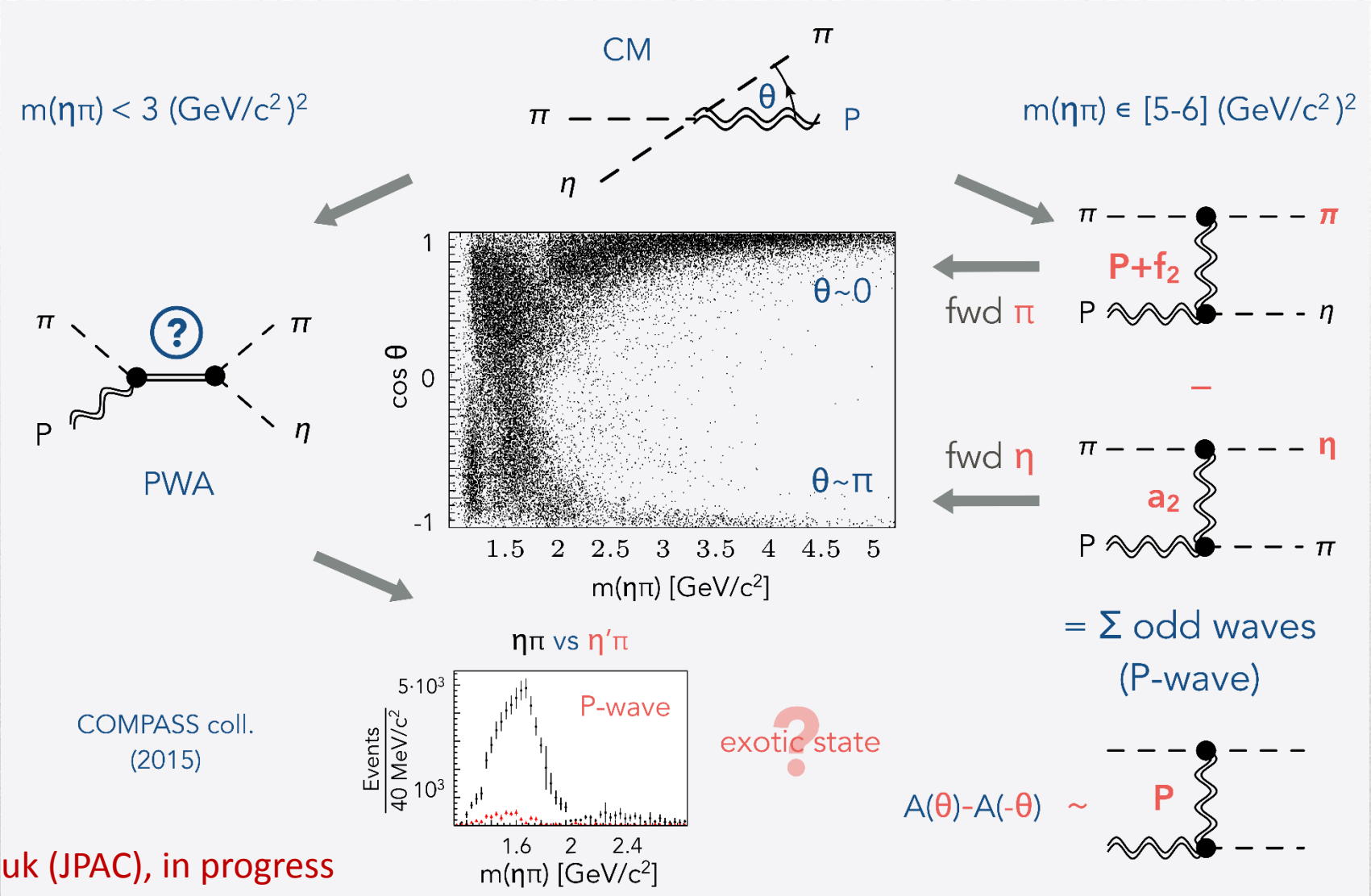
Finite energy sum rules



$$\frac{1}{\Lambda^n}\int_{\nu_{th}}^{\Lambda} Im\, A(\nu,t)\, \nu^n d\nu = \frac{\beta(t)\Lambda^{\alpha(t)+1}}{\alpha(t)+n+1}$$

aim: *first systematic analysis of* ***peripheral production*** *using* ***FESR***

$\eta\pi$ production

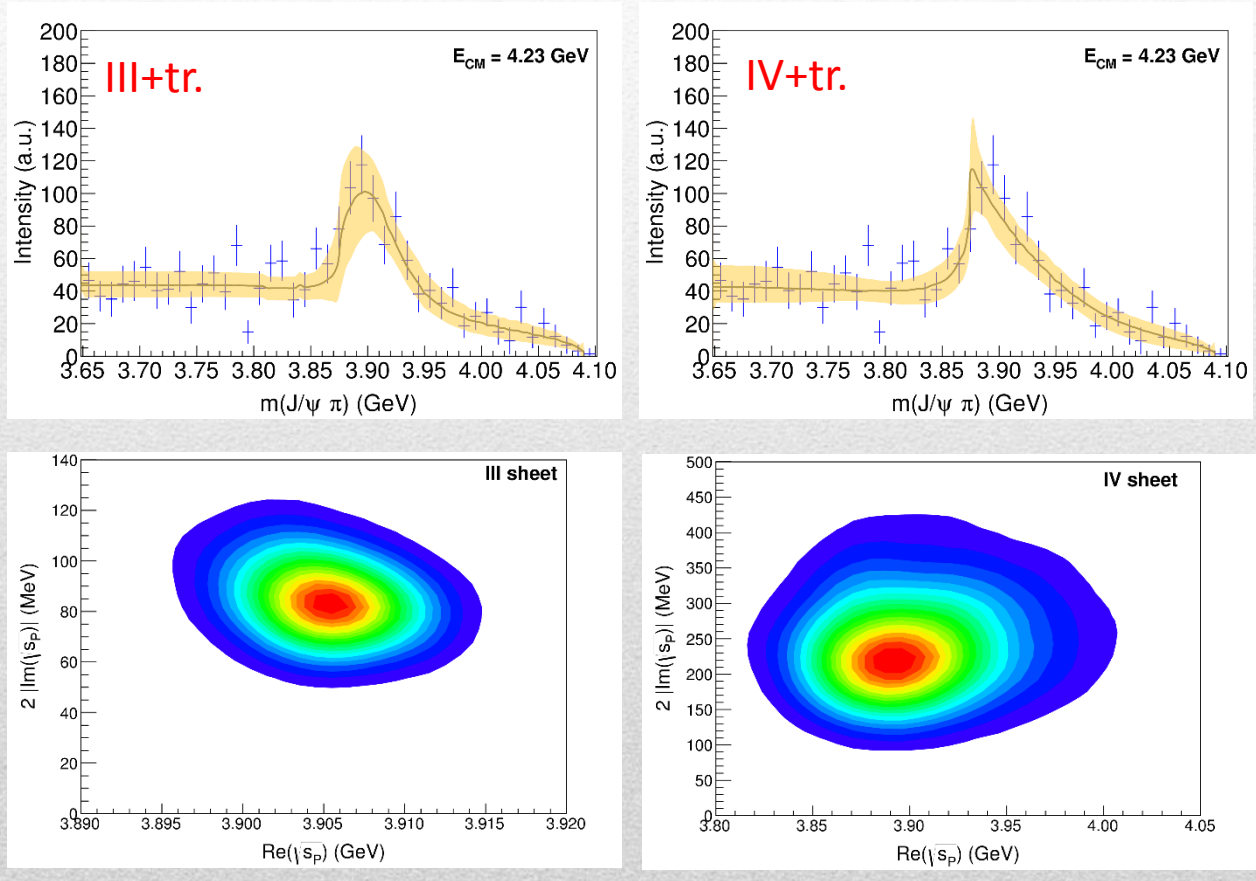


Model dependence and physics

Understanding the model dependence is mandatory: models with similar fit qualities can lead to dramatically different physical interpretations

E.g. $e^+e^- \to J/\psi\, \pi\pi$ and the $Z_c(3900)$

AP *et al.* (JPAC), PLB772, 200



Recipes to build an amplitude

M. Mikhasenko, AP, J. Nys *et al.* (JPAC), EPJC78, 3, 229

The literature abounds with discussions on the optimal approach to construct the amplitudes for the hadronic reactions

- Helicity formalism

  Jacob, Wick, Annals Phys. 7, 404 (1959)

- Covariant tensor formalisms

  Chung, PRD48, 1225 (1993)
  Chung, Friedrich, PRD78, 074027 (2008)
  Filippini, Fontana, Rotondi, PRD51, 2247 (1995)
  Anisovich, Sarantsev, EPJA30, 427 (2006)

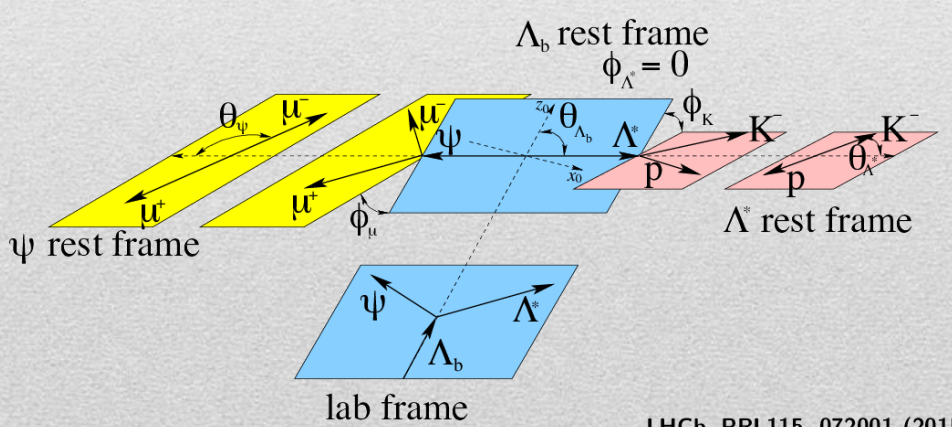
The common lore is that the former one is nonrelativistic, especially when expressed in terms of LS couplings
and the latter takes into account the proper relativistic corrections

How helicity formalism works

- Helicity formalism enforces the constraints about rotational invariance
- It allows us to fix the angular dependence of the amplitude
- What about energy dependence?

Example: $B \to \psi K^* \to \pi K$

$$\mathcal{M}^{K^*}_{\Delta\lambda_\mu} \equiv \sum_n \sum_{\lambda_{K^*}} \sum_{\lambda_\psi} \mathcal{H}^{B\to K^*_n \psi}_{\lambda_{K^*},\,\lambda_\psi} \delta_{\lambda_{K^*},\lambda\psi}$$



LHCb. PRL115. 072001 (2015)

$$\mathcal{H}^{K^*_n \to K\pi} D^{J_{K^*_n}}_{\lambda_{K^*},\,0}(\phi_K, \theta_{K^*}, 0)^* \; R_{K^*{}_n}(m_{K\pi}) D^1_{\lambda_\psi,\,\Delta\lambda_\mu}(\phi_\mu, \theta_\psi, 0)^*,$$

Each set of angles is defined in a different reference frame

How tensor formalism works

The method is based on the construction of explicitly covariant expressions.

- To describe the decay $a \to bc$, we first consider the polarization tensor of each particle, $\varepsilon^{i}_{\mu_1 \ldots \mu_{j_i}}(p_i)$
- We combine the polarizations of b and c into a "total spin" tensor $S_{\mu_1 \ldots \mu_S}(\varepsilon_b, \varepsilon_c)$
- Using the decay momentum, we build a tensor $L_{\mu_1 \ldots \mu_L}(p_{bc})$ to represent the orbital angular momentum of the bc system, orthogonal to the total momentum of p_a
- We contract S and L with the polarization of a

Tensor $\times R_X(m)$ which contain resonances and form factors

What do we know?

- Energy dependence is not constrained by symmetry
- Still, there are some known properties one can enforce

$$R_X(m) = B'_{L^X_{\Lambda_b^0}}(p, p_0, d)\left(\frac{p}{M_{\Lambda_b^0}}\right)^{L^X_{\Lambda_b^0}}$$

$$\mathrm{BW}(m|M_{0X}, \Gamma_{0X})\, B'_{L_X}(q, q_0, d)\left(\frac{q}{M_{0X}}\right)^{L_X}$$

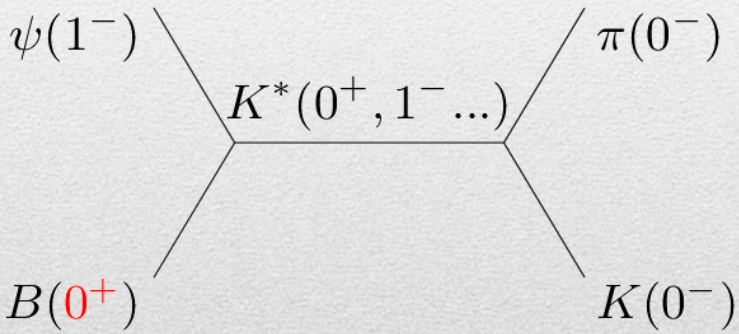
- Kinematical singularities: e.g. barrier factors (known)
- Left hand singularities (need model, e.g. Blatt-Weisskopf)
- Right hand singularities = resonant content (Breit Wigner, K-matrix...)

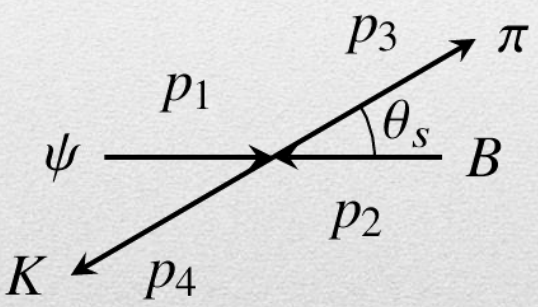
Kinematics

- Kinematical singularities appear because of the spin of the external particle involved
- We can write the most general covariant parametrization of the amplitude as
  tensor of external polarizations $\otimes$ scalar amplitudes
- Scalar amplitudes must be kinematical singularities free
- They can be matched to the helicity amplitudes
- We can get the minimal energy dependent factor
- Any other additional energy factor would be model-dependent

$B \to \psi\, \pi K$

To consider the effect of spin, let's consider $B \to \psi \pi K$
We focus on the parity violating amplitude for the K^* isobars, scattering kinematics





$$p = \text{incoming 3-momentum in the COM} = \frac{\lambda_{12}^{1/2}}{2\sqrt{s}}$$

$$= \frac{\sqrt{[s-(m_1+m_2)^2]\,[s-(m_1-m_2)^2]}}{2\sqrt{s}}$$

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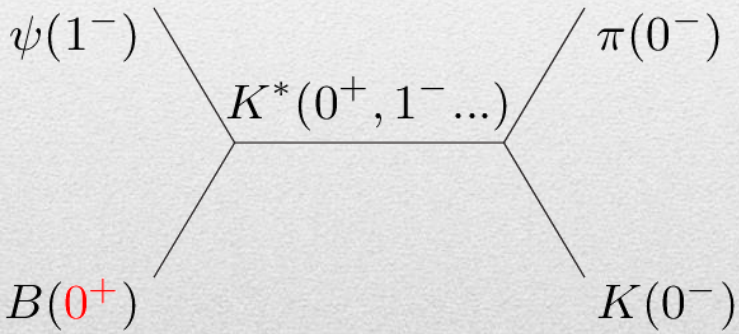
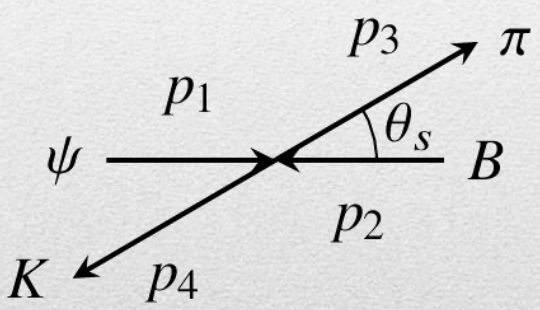


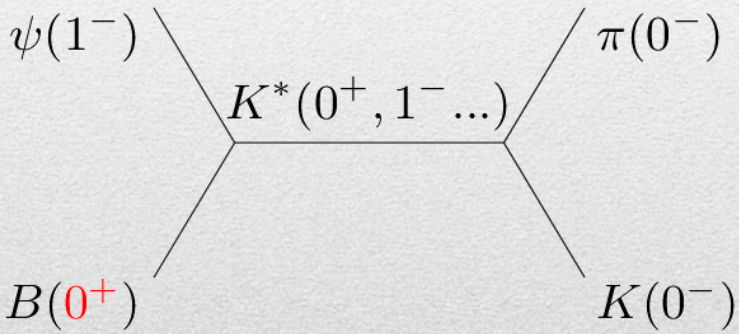


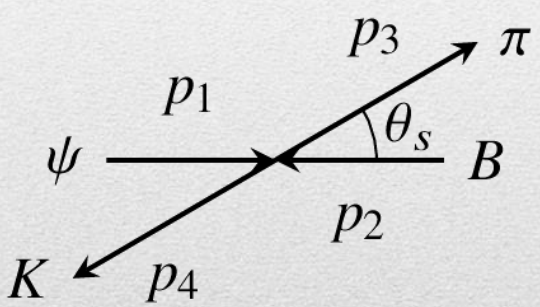
$$q = \text{outgoing 3-momentum in the COM} = \frac{\lambda_{34}^{1/2}}{2\sqrt{s}}$$

$$= \frac{\sqrt{[s-(m_3+m_4)^2]\,[s-(m_3-m_4)^2]}}{2\sqrt{s}}$$

$B \to \psi\, \pi K$

To consider the effect of spin, let's consider $B \to \psi \pi K$
We focus on the parity violating amplitude for the K^* isobars, scattering kinematics





z_s = cosine of the scatt. angle in the COM

$$= \frac{s(t-u) + (m_1^2 - m_2^2)(m_3^2 - m_4^2)}{\lambda_{12}^{1/2}\lambda_{34}^{1/2}} = \frac{\text{polynomial}}{\lambda_{12}^{1/2}\lambda_{34}^{1/2}}$$

Helicity amplitudes

$$A_\lambda = \frac{1}{4\pi} \sum_{j=|\lambda|}^{\infty} (2j+1) A^j_\lambda(s)\, d^j_{\lambda 0}(z_s)$$

$$d^j_{\lambda 0}(z_s) = \hat{d}^j_{\lambda 0}(z_s) \xi_{\lambda 0}(z_s), \qquad \xi_{\lambda 0}(z_s) = \left(\sqrt{1-z_s^2}\right)^\lambda$$

$\hat{d}^j_{\lambda 0}(z_s)$ is a polynomial of order $j - |\lambda|$ in z_s,
The kinematical singularities of $A^j_\lambda(s)$ can be isolated by writing

$$A^j_0 = \frac{m_1}{p\sqrt{s}}\, (pq)^j\ \hat{A}^j_0 \qquad \text{for } j \geq 1,$$

$$A^j_\pm = q\ (pq)^{j-1}\ \hat{A}^j_\pm \qquad \text{for } j \geq 1,$$

$$A^0_0 = \frac{p\sqrt{s}}{m_1}\, \hat{A}^0_0 \qquad \text{for } j = 0,$$

Identify covariants

Two helicity couplings $\rightarrow$ two independent covariant structures

Important: we are not imposing any intermediate isobar

$$A_\lambda(s,t) = \varepsilon_\mu(\lambda, p_1)\left[(p_3 - p_4)^\mu - \frac{m_3^2 - m_4^2}{s}(p_3 + p_4)^\mu\right] C(s,t)$$
$$+ \varepsilon_\mu(\lambda, p_1)(p_3 + p_4)^\mu B(s,t)$$

$$C(s,t) = \frac{1}{4\pi\sqrt{2}} \sum_{j>0} (2j+1)(pq)^{j-1} \hat{A}_+^j(s)\, \hat{d}_{10}^j(z_s)$$

$$B(s,t) = \frac{1}{4\pi}\hat{A}_0^0 + \frac{1}{4\pi}\frac{4m_1^2}{\lambda_{12}} \sum_{j>0} (2j+1)(pq)^j \left[\hat{A}_0^j(s)\hat{d}_{00}^j(z_s) + \frac{s + m_1^2 - m_2^2}{\sqrt{2}m_1^2}\hat{A}_+^j(s)\, z_s \hat{d}_{10}^j(z_s)\right]$$

Everything looks fine but the λ_{12} in the denominator

The brackets must vanish at $\lambda_{12} = 0 \Rightarrow s = s_\pm = (m_1 \pm m_2)^2$,

$\hat{A}_+^j$ and $\hat{A}_0^j$ cannot be independent

General expression and comparison

$$\hat{A}^j_+ = \langle j-1,0;1,1|j,1\rangle g_j(s) + f_j(s)$$

$$\hat{A}^j_0 = \langle j-1,0;1,0|j,0\rangle \frac{s+m_1^2-m_2^2}{2m_1^2} g_j'(s) + f_j'(s)$$

$g_j(s_\pm) = g_j'(s_\pm)$, and $f_j(s), f_j'(s) \sim O(s-s_\pm)$
All these four functions are free of kinematic singularity.

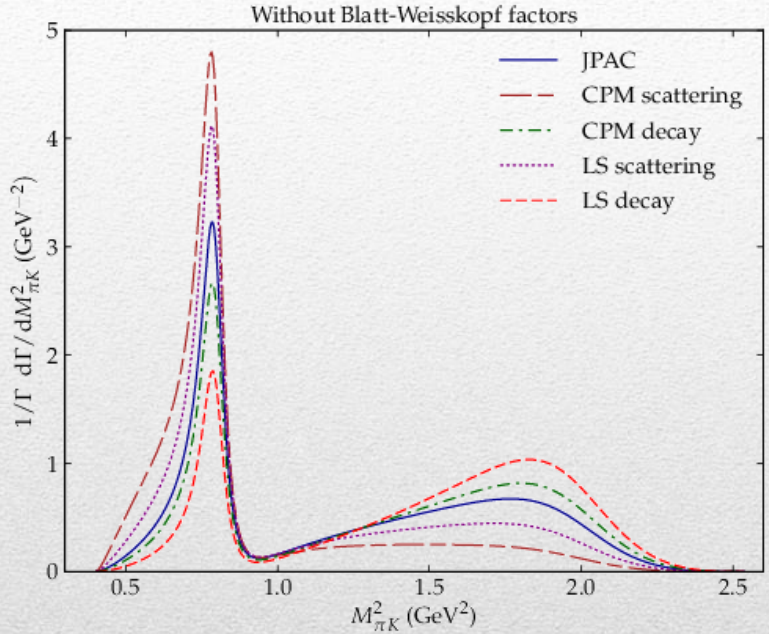
Comparison with tensor formalisms ($j=1$)

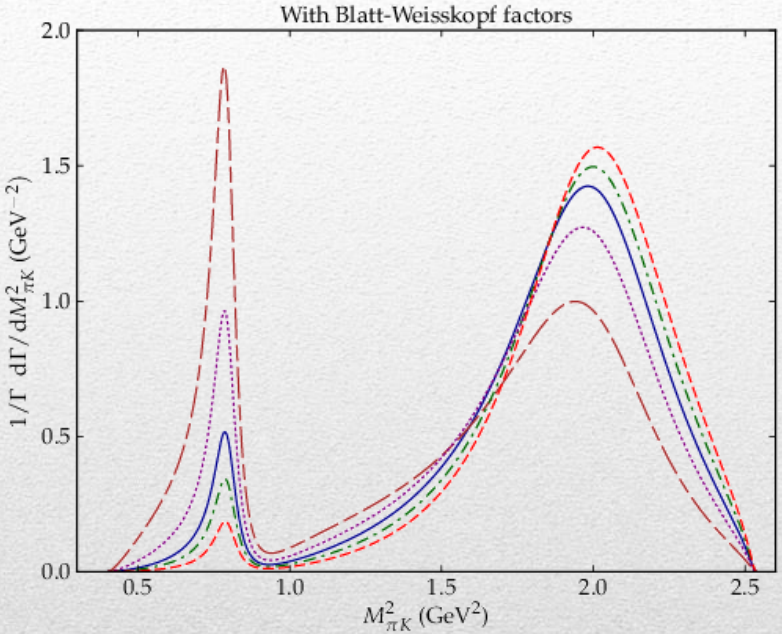
$$g_1 = g_1' = \frac{4\pi}{3} g_S, \quad f_1 = \frac{2\pi\lambda_{12}}{3s} g_D, \quad f_1' = -\frac{4\pi\lambda_{12}}{3s}\frac{s+m_1^2-m_2^2}{m_1^2} g_D.$$

If the g_S, g_D are the usual Breit-Wigner, the g', f' are fine

There is no unique recipe to build the right amplitude, but one can ensure the right singularities to be respected

General expression and comparison





We consider the example of two intermediate $K^*(892)$ and $K^*(1410)$

We set $g_S(s) = 0$ and $g_D(s) =$ sum of Breit-Wigner

For the plot on the right we multiply the amplitudes by the Blatt-Weisskopf barrier factors

Interactive tools

- Completed projects are fully documented on interactive portals
- These include description on physics, conventions, formalism, etc.
- The web pages contain source codes with detailed explanation how to use them. Users can run codes online, change parameters, display results.

http://www.indiana.edu/~jpac/