Imaging hadrons and nuclei through generalized parton distributions

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The proton is a rich and complicated system

- It’s more than just three quarks
  - The masses don’t even add up
    
    \[ 2m_u + m_d \approx 9.4 \text{ MeV} \]
    
    \[ m_p \approx 940 \text{ MeV} \]

- Quark-gluon interactions somehow generate mass
- How does this happen?
- ...and where is the mass inside the proton?
  - This is what imaging is all about

On right: artist’s impression of the proton, CERN
1. The light front
   ▶ How do spatial densities work for quantum relativistic systems?

2. The energy momentum tensor
   ▶ Where is the mass inside the proton?

3. Generalized parton distributions
   ▶ How do we get 3D images of quark and gluon distributions?
I. The Light Front
Quantum objects have two kinds of spatial extent:

1. Distance between constituents
2. Wave packet size

We want this!

Reduce to overall wave packet and **internal structure**

\[ \psi_{\text{total}}(r_1, r_2, t) = \Psi(R, t)\psi(r, t) \]

Internal structure is the interesting part
We measure structure via scattering

This gives momentum info

Get position info with **Fourier transform**:

\[
\psi(r, t) = \int \frac{d^3p}{(2\pi)^3} \tilde{\psi}(p) e^{i(p \cdot r - Et)}
\]

Only works when wave packets factorize:

\[
\psi_{\text{total}}(r_1, r_2, t) = \Psi(R, t)\psi(r, t)
\]

**Relativity** makes this break down

Image credit: Jefferson Lab
Relativity of simultaneity

- Momentum-space wave packet contains boosted versions of composite system
  - Boosts mix up planes of simultaneity
  - Internal constituents get boosted to **different times**

![Diagram showing momentum-space wave packet and boosts mixing planes of simultaneity]

- Cannot decompose structure to overall wave packet $\otimes$ internal structure at fixed $t$
- **Light front coordinates** fix this by defining a **new, boost-invariant time variable**!
Partonic images are described by **light front coordinates**.

Light front coordinates are a different foliation of spacetime.
- Involves redefining *equal-time surfaces*.

\[
x^\pm = t \pm z \quad \mathbf{x}_\perp = (x, y) \quad x^+ = t + z = \text{time}
\]
Synchronization vs. seeing

- Fixed $t$ surfaces require coordination
  - Distant clocks must be synchronized
  - Must wait for light to arrive
  - Reconstruct after the fact
  - Ruined by boosts

- Fixed $x^+$ surfaces require looking
  - Look in the $+z$ direction
  - That’s a fixed $x^+$ surface
  - Invariant under boosts!
  - Allow relativistic densities
Lorentz-boosted objects appear rotated.

- Terrell rotation (PR116, 1959)
- Optical effect: contraction + delay

Light front transverse boost undoes Terrell rotation:

\[ B_x^{(LF)} = K_x - J_y \]

- Standard boost + counter-rotation
- Leaves \( x^+ \) (time) invariant
- Part of the Galilean subgroup

Dice images by Ute Kraus, https://www.spacetime.travel.org/
Poincaré group has a $(2 + 1)$D **Galilean subgroup**.

- $x^+$ is time and $x_\perp$ is space under this subgroup.
- $P^+ = E_p + p_z$ is the central charge.
- $x^+$ and $P^+$ are invariant under this subgroup!

Light front time gives **fully relativistic** 2D picture that looks a lot like non-relativistic physics.

- But with $P^+$ in place of $m$.

\[
\frac{dP_\perp}{dx^+} = P^+ \frac{d^2x_\perp}{dx^+^2}
\]

\[
H = H_{\text{rest}} + \frac{P_\perp^2}{2P^+}
\]

\[
v_\perp = \frac{P_\perp}{P^+}
\]
Wave packet separation works for *transverse* spatial coordinates

Works thanks to the Galilean subgroup

Stuck with 2D spatial densities

*Generalized parton distributions* give back a third dimension

But the third dimension *must be* a momentum

See AF & Miller PRD108 (2023) 034008 for formal info
II. The Energy-Momentum Tensor
The energy-momentum tensor describes **density** and **flow** of energy & momentum.

\[
T^\mu_\nu(x) = \begin{bmatrix}
T^{+0}(x) & T^{+1}(x) & T^{+2}(x) & T^{+3}(x) \\
T^{10}(x) & T^{11}(x) & T^{12}(x) & T^{13}(x) \\
T^{20}(x) & T^{21}(x) & T^{22}(x) & T^{23}(x) \\
T^{30}(x) & T^{31}(x) & T^{32}(x) & T^{33}(x)
\end{bmatrix}
\]

**Energy density**

**Momentum densities**

**Energy fluxes**

**Stress tensor**

Static densities: \(n_\mu T^{\mu\nu}(x)\)
Noether’s theorems and spacetime distortions

- Conserved current from *local* spacetime translations (**Noether’s second theorem**):

\[ x \mapsto x + \xi(x) \]

- **Noether’s theorems**: symmetries imply conservation laws
- *Local* translation: move spacetime differently everywhere

- The **energy-momentum tensor** is a response to these deformations

\[ \Delta S_{\text{QCD}} = \int d^4x \, T_{\text{QCD}}^{\mu\nu}(x) \partial_\mu \xi_\nu(x) \]

- Conserved if the action is invariant

AF, Phys. Rev. D 106 (2022) 125012
The energy-momentum tensor is parametrized using **gravitational form factors**

- It’s just a name.
- The energy-momentum tensor is the source of gravitation
- But we don’t really use gravitation to measure them

**Spin-zero example:**

\[
\langle p'|\hat{T}^{\mu\nu}(0)|p\rangle = 2P^\mu P^\nu A(\Delta^2) + \frac{1}{2}(\Delta^\mu \Delta^\nu - \Delta^2 g^{\mu\nu}) D(\Delta^2)
\]

- \(A(\Delta^2)\) encodes momentum density
- \(D(\Delta^2)\) encodes stress distributions (anisotropic pressures)
- Mix of both encodes energy density
$T^\mu_\nu(x) = \begin{bmatrix}
T^+0(x) \\
T^{10}(x) \\
T^{20}(x) \\
T^{30}(x) \\
\end{bmatrix} = \begin{bmatrix}
T^+1(x) & T^+2(x) & T^+3(x) \\
T^{11}(x) & T^{12}(x) & T^{13}(x) \\
T^{21}(x) & T^{22}(x) & T^{23}(x) \\
T^{31}(x) & T^{32}(x) & T^{33}(x) \\
\end{bmatrix}$

Energy density

Momentum densities

Energy fluxes

Stress tensor

Formalism in AF & Miller, PRD108 (2023) 094026
Proton transverse momentum density

\[ T^{\mu\nu}(x) = \begin{bmatrix}
  T^{+0}(x) & T^{+1}(x) & T^{+2}(x) & T^{+3}(x) \\
  T^{10}(x) & T^{11}(x) & T^{12}(x) & T^{13}(x) \\
  T^{20}(x) & T^{21}(x) & T^{22}(x) & T^{23}(x) \\
  T^{30}(x) & T^{31}(x) & T^{32}(x) & T^{33}(x)
\end{bmatrix} \]
Helicity ±1

Helicity state ($\lambda = \pm 1$)

Density of $p^+ = E + p_z$ in deuterium nucleus

Helicity 0

Helicity state ($\lambda = 0$)

AF & Cosyn, PRD106 (2022) 114013
Transversely-polarized deuteron

Transverse, $m_s = +1$

Transversely polarized ($m_s = +1$)

Distortions from optical effects—relativistic wheel

Transverse, $m_s = 0$

Transversely polarized ($m_s = 0$)

AF & Cosyn, PRD106 (2022) 114013
The relativistic wheel

Static wheel

- Static wheel has regularly-placed spokes

Spinning wheel

- Spinning wheel has distortions
  - Spokes moving away are redshifted.
    - Appear to move slower, pile up
  - Spokes moving towards are blueshifted.
    - Appear to move faster, become sparse

- These same distortions are present in the deuteron!
  - The deuteron is a relativistic wheel!

- Also see videos at:
  https://www.spacetimetravel.org/rad
  (green wheel is relevant case)
Stress tensor

\[ T^{\mu\nu}(x) = \begin{bmatrix} T^{+0}(x) & T^{+1}(x) & T^{+2}(x) & T^{+3}(x) \\ T^{10}(x) & T^{11}(x) & T^{12}(x) & T^{13}(x) \\ T^{20}(x) & T^{21}(x) & T^{22}(x) & T^{23}(x) \\ T^{30}(x) & T^{31}(x) & T^{32}(x) & T^{33}(x) \end{bmatrix} \]

- **Energy density**
- **Momentum densities**
- **Energy fluxes**
- **Stress tensor**

- Stresses are **gross** forces
  - There even if net force is zero!
  - e.g., water pressure, stress in a bridge

- Stresses are also momentum flows
Radial eigenpressures in the deuteron

**Helicity ±1**

Helicity state ($\lambda = +1$)

**Helicity 0**

Helicity state ($\lambda = 0$)

AF & Cosyn, PRD106 (2022)
Radial pressure *negative* at center.

- Partons feel like they’re being pulled apart?
- Cause of deuteron’s famous donut shape?
- Recent paper by Garcia Martin-Caro *et al.* also has negative radial pressures!
  - Skyrme model calculation
  - arxiv:2312.12984

**Future work:** flows/fluxes in deuteron
- Formalism in AF & Miller, PRD108 (2023)

AF & Cosyn, PRD106 (2022)
What about the third dimension?

- $z$ or $x^-$ doesn’t transform in a Galilean manner.
  - Absorbs relativistic wonkiness.

- $p^+$—its conjugate momentum—does.
  
  $$p^+ = E + p_z$$
  
  - Transforms like time ($x^+$); is invariant.

- **Momentum fractions** are additionally invariant under longitudinal boosts.
  
  $$x \equiv \frac{p_{\text{parton}}^+}{p_{\text{hadron}}^+}$$

- We’re stuck with a *partially* spatial picture.
  - 2 spatial dimensions $b_\perp + 1$ momentum fraction $x$.

NJL model results for proton, AF & Cloët

- PRC101 (2020) 035203
- PRC103 (2021) 045204
III. Generalized Parton Distributions
Generalized parton distributions

- GPDs are formally defined using light cone correlators.
  - Amplitude for a quark being at two spacetime locations.
- For quarks (in the light cone gauge):
  \[
  \mathcal{M}^q[\mathcal{O}] = \frac{1}{2} \int \frac{dz}{2\pi} e^{-i(P \cdot n)z} x \langle p' \vert \bar{q} \left( \frac{n z}{2} \right) \mathcal{O} q \left( -\frac{n z}{2} \right) \vert p \rangle
  \]
- Analogous definitions exist for gluons.
- \( \mathcal{O} \) chosen by which GPD we want; e.g.,
  - \( \gamma^+ \) for helicity-independent, leading twist GPDs.
- Each correlator is decomposed into Lorentz structures; for the proton:
  \[
  \mathcal{M}^q[\gamma^+] = \bar{u}' \left[ \gamma^+ H^q(x, \xi, t; Q^2) + \frac{i\sigma^\gamma n}{2m_p} E^q(x, \xi, t; Q^2) \right] u
  \]
- The Lorentz-invariant functions of \( x, \xi, t, \) and \( Q^2 \) are the GPDs.
The GPD variables

\[
\begin{align*}
x &= \frac{(k + k') \cdot n}{(p + p') \cdot n} \\
\xi &= \frac{(p - p') \cdot n}{(p + p') \cdot n} \\
t &= \Delta^2 = (p' - p)^2 \\
n &\text{defines the light front, i.e., } n \cdot V \equiv V^+
\end{align*}
\]

- \( x \) is average momentum fraction of struck parton.
- \( 2\xi \) is the skewness: momentum fraction lost by struck parton.
- \( t \) is the invariant momentum transfer.
- GPDs also depend on renormalization scale \( Q^2 \).
Negative momentum fraction means antiparticle

- $x$ and $\xi$ give qualitative picture of reaction
- $x = \pm \xi$ means one quark line has zero (light front) momentum
  - Instantaneous propagation (at fixed light front time)
Several GPDs become PDFs when $p' = p$, i.e., $t = 0$ and $\xi = 0$.

Definition of light cone correlator:

\[
\mathcal{M}^q[\mathcal{O}] = \frac{1}{2} \int \frac{dz}{2\pi} \ e^{-i(P \cdot n)zx} \langle p' | \bar{q} \left(\frac{n \cdot z}{2}\right) \mathcal{O} q \left(-\frac{n \cdot z}{2}\right) | p \rangle
\]

This is how PDFs are formally defined, provided $p' = p$.

See e.g., Collins's *Foundations of Perturbative QCD*.

For the proton:

\[
H^q(x, 0, 0; Q^2) = q(x; Q^2)
\]

\[
\tilde{H}^q(x, 0, 0; Q^2) = \Delta q(x; Q^2)
\]
Non-skewed GPDs

Non-skewed GPD ($\xi = 0$)

\[ H^q(x, \xi = 0, t) \]

2D Fourier transform

Impact parameter PDF

- Partially spatial structure recovered when $\xi = 0$
- 2D Fourier transform gives 2D spatial structure at fixed light front time
- Third dimension is momentum fraction $x$

Calculations in figures: AF & Cloët, PRC101 (2020) 035203
Why consider skewness?

Why consider skewness if non-skewed GPDs give spatial densities?

One reason is **polynomiality**:

\[
\int_{-1}^{1} dx \, x^{s-1} H(x, \xi, t; Q^2) = \sum_{l=0}^{s} A_{s,l}(t; Q^2)(-2\xi)^l
\]

- Mellin moment gives polynomial in $\xi$
- **This is a consequence of Lorentz covariance**
- Time and parity make it either even or odd (usually even)
- Maximum power of $s$

$A_{s,l}(t : Q^2)$ are **generalized form factors**

- $s = 1$: electromagnetic and axial form factors.
- $s = 2$: gravitational form factors—**only empirical means of accessing these**

...also, $\xi \neq 0$ in empirically-accessible processes.
Hard exclusive reactions are used to measure GPDs.

- **Deeply virtual Compton scattering** (DVCS) to probe quark structure.
- Deeply virtual meson production (DVMP), e.g., $J/\psi$ or $\Upsilon$ to probe gluon structure.
- Single-diffractive hard exclusive reactions (SDHEPs) to refine $x$ dependence.
- …and more!

Measured at Jefferson Lab and the upcoming Electron Ion Collider.
Deeply virtual Compton scattering (DVCS) is one method to probe GPDs.

Loop in diagram: $x$ is integrated out

Integrated quantities seen in experiment: Compton form factors

$$\mathcal{H}(\xi, t; Q^2) = \int_{-1}^{1} dx C'(x, \xi) H(x, \xi, t; Q^2) \overset{\text{LO}}{=} \int_{-1}^{1} dx \left[ \frac{1}{\xi - x - i0} + \frac{1}{\xi + x - i0} \right] H(x, \xi, t; Q^2)$$

Other processes have different coefficients—more $x$ sensitivity
Need to invert the relationship:

\[ \mathcal{H}(\xi, t; Q^2) = \int_{-1}^{1} dx \, C(x, \xi) H(x, \xi, t; Q^2) \]

Shadow GPDs impose a mighty obstacle:

\[ \int_{-1}^{1} dx \, C'(x, \xi) H_s(x, \xi, t; Q_0^2) = 0 \]

\[ H(x, \xi, t, Q_0^2) + H_s(x, \xi, t, Q_0^2) \] gives the same DVCS amplitude at \( Q^2 = Q_0^2 \).

Bertone, et al., PRD103 (2021) 114019

Evolution partially mitigates this.
Need to invert the relationship:

\[ H(\xi, t; Q^2) = \int_{-1}^{1} dx \, C(x, \xi) H(x, \xi, t; Q^2) \]

- Shadow GPDs entail large uncertainties
- Large kinematic lever-arm can reduce uncertainty near \( \xi \approx 0 \)
- Formal constraints (positivity) help reduce uncertainty too
- Focus of topical collaboration on **quark-gluon tomography (QGT)**
  - First analysis (Eric Moffat, AF et al.) in PRD108 (2023)
  - Will need to train AI on mock data—requires good **model calculations**
Stick to observables when extracting
  - e.g., Compton form factors
Use equivalence classes of GPDs
  - \( H \sim H + H_{\text{shadow}} \)
Other reactions
  - Single diffractive hard exclusive processes (SDHEP), Qiu & Yu PRD107 (2023)
  - Has different shadows than DVCS
Use lattice results to further constrain GPDs

Model-testing paradigm
  - The traditional scientific method
  - Make predictions via models
  - Use experiment to test predictions
  - DVCS etc. serve to rule out models
  - Better/more measurements cull survivors
▶ SciDAC on femtoscale imaging.
▶ Collaboration between theory, experiment, computer science & applied math.
▶ **Event-level analysis** with AI/ML.
▶ Designed for exascale platforms.
  ▶ Will utilize Aurora at Argonne!

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**Event-level QCF inference framework**

- **Module 1:** Noise → Parameter Generator → Parameters
- **Module 2:** Trial QCF model → EIC → hermes → Jefferson Lab → Experimental Events
  - MCMC
  - Trial PMD
  - Idealized Theory Events
- **Module 3:** Detector model → Simulated Events → Event level Discriminator
- **Module 4:** Experimental Events → Optimize QCF parameters
GPD phenomenology
- Framework for connecting theory to experiment
- Pipeline to use AI/ML for fitting GPDs to data
- Modular code packages the community can use

Model calculations for nuclear GPDs
- “Conventional” models for baseline
- Models of extra-nucleonic QCD effects
- Identify expected experimental signatures

Considerable overlap with Quark-Gluon Tomography collaboration
Nuclear GPDs are highly desirable:
- Understand QCD dynamics in the nuclear environment
- Distribution of mass & forces
- Lorentz covariance needed for relation to form factors

Deuteron is ideal target:
- Simplest nucleus
- Proton + neutron + \( NN \) interaction
- Spin-one: gluon transversity PDF encodes extra-nucleonic but intrinsic glue

Ongoing work using a non-local separable kernel for \( NN \) interactions:
- This can be and is done covariantly.
- Matched to precision wave functions in NR limit (e.g., AV18).
- Covariance required to relate GPDs to the energy-momentum tensor
So far: good agreement with electromagnetic observables

Manuscript with these and PDFs in preparation.

Near future: energy-momentum tensor and GPD calculations.
  Relevant formalism for spin-one targets in:
  Cosyn, AF, Pire: PRD99 (2019); Cosyn, Cotogno, AF, Lorcé: EPJC 79 (2019); AF & Cosyn: PRD106 (2022)

Longer term plans:
  Medium modifications & intrinsic glue
  Heavier nuclei
  Revisit the proton with lessons learned (more sophisticated proton GPDs!)
- The proton is a rich and complicated system
- Nuclei (like the deuteron) are too
- The energy-momentum tensor gives us access to this richness
  - Tied to the origin of proton mass
  - Generalized parton distributions give empirical access
- There’s a lot more yet to learn
  - There are more exciting calculations to do
  - The Electron-Ion Collider will tell us more!

Thank you for your time!