Relativistic mass densities: from the light front to generalized parton distributions

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The proton is a rich and complicated system

It’s more than just three quarks

The masses don’t even add up

\[2m_u + m_d \approx 9.4 \text{ MeV}\]
\[m_p \approx 940 \text{ MeV}\]

Quark-gluon interactions somehow generate mass

How does this happen?

…and where is the mass inside the proton?

This is what imaging is all about

On right: artist’s impression of the proton, CERN
1. The light front  
   - How do spatial densities work for quantum relativistic systems?

2. The energy momentum tensor  
   - Where is the mass inside the proton?

3. Generalized parton distributions  
   - How do we get 3D images of quark and gluon distributions?
I. The Light Front
Quantum objects have two kinds of spatial extent:
1. Distance between constituents
2. Wave packet size

Reduce to overall wave packet and **internal structure**

\[ \psi_{\text{total}}(r_1, r_2, t) = \Psi(R, t)\psi(r, t) \]

Internal structure is the interesting part
We measure structure via scattering
This gives momentum info
Get position info with Fourier transform:

\[
\psi(r, t) = \int \frac{d^3p}{(2\pi)^3} \tilde{\psi}(p) e^{i(p \cdot r - Et)}
\]

Only works when wave packets factorize:

\[
\psi_{\text{total}}(r_1, r_2, t) = \Psi(R, t)\psi(r, t)
\]

Relativity makes this break down

Image credit: Jefferson Lab
Relativity of simultaneity

- Momentum-space wave packet contains boosted versions of composite system
  - Boosts mix up planes of simultaneity
  - Internal constituents get boosted to different times

- Cannot decompose structure to overall wave packet $\otimes$ internal structure at fixed $t$
- **Light front coordinates** fix this by defining a new, boost-invariant time variable!
Partonic images are described by **light front coordinates**.

**Light front coordinates** are a different foliation of spacetime.  
- Involves redefining *equal-time surfaces*.

\[
x^\pm = t \pm z \quad \mathbf{x}_\perp = (x, y) \quad x^+ = t + z = \text{time}
\]

**Minkowski coordinates**

**Light front coordinates**
Synchronization vs. seeing

- Fixed $t$ surfaces require coordination
  - Distant clocks must be synchronized
  - Must wait for light to arrive
  - Reconstruct after the fact
  - Ruined by boosts

- Fixed $x^+$ surfaces require looking
  - Look in the $+z$ direction
  - That’s a fixed $x^+$ surface
  - Invariant under boosts!
  - Allow relativistic densities
Lorentz-boosted objects appear rotated.

- **Terrell rotation** (PR116, 1959)
- Optical effect: contraction + delay

**Light front transverse boost** undoes Terrell rotation:

\[ B_{x}^{(LF)} = K_x - J_y \]

- Standard boost + counter-rotation
- Leaves \( x^+ \) (time) invariant
- Part of the **Galilean subgroup**

Dice images by Ute Kraus, https://www.spacetime-travel.org/
Poincaré group has a \((2 + 1)D\) Galilean subgroup.

- \(x^+\) is time and \(x_\perp\) is space under this subgroup.
- \(P^+ = E_p + p_z\) is the central charge.
- \(x^+\) and \(P^+\) are invariant under this subgroup!

Light front time gives fully relativistic 2D picture that looks a lot like non-relativistic physics.

- But with \(P^+\) in place of \(m\).

\[
\frac{dP_\perp}{dx^+} = P^+ \frac{d^2 x_\perp}{dx^{+2}}
\]

\[
H = H_{\text{rest}} + \frac{P_\perp^2}{2P^+}
\]

\[
v_\perp = \frac{P_\perp}{P^+}
\]
Wave packet separation works for *transverse* spatial coordinates

Works thanks to the Galilean subgroup

Stuck with 2D spatial densities

**Generalized parton distributions** give back a third dimension

But the third dimension *must be* a momentum

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See AF & Miller PRD108 (2023) 034008 for formal info
II. The Energy-Momentum Tensor
The energy-momentum tensor describes **density** and **flow** of energy & momentum.

The energy-momentum tensor describes density and flow of energy & momentum.

\[ T^{\hat{\mu}\nu}(x) = \begin{bmatrix} T^{+0}(x) & T^{+1}(x) & T^{+2}(x) & T^{+3}(x) \\ T^{10}(x) & T^{11}(x) & T^{12}(x) & T^{13}(x) \\ T^{20}(x) & T^{21}(x) & T^{22}(x) & T^{23}(x) \\ T^{30}(x) & T^{31}(x) & T^{32}(x) & T^{33}(x) \end{bmatrix} \]

**Energy density**

**Momentum densities**

**Energy fluxes**

**Stress tensor**

**Static densities:** \( n_\mu T^{\mu\nu}(x) \)
Noether’s theorems and spacetime distortions

- Conserved current from *local* spacetime translations (**Noether’s second theorem**):

\[ x \mapsto x + \xi(x) \]

- **Noether’s theorems**: symmetries imply conservation laws
- *Local* translation: move spacetime differently everywhere

- The **energy-momentum tensor** is a response to these deformations

\[ \Delta S_{QCD} = \int d^4 x \ T_{QCD}^{\mu \nu}(x) \partial_\mu \xi_\nu(x) \]

- Conserved if the action is invariant

AF, Phys. Rev. D 106 (2022) 125012
The energy-momentum tensor is parametrized using \textit{gravitational form factors}.

- It’s just a name.
- The energy-momentum tensor is the source of gravitation.
- But we don’t really use gravitation to measure them.

Spin-zero example:

\[
\langle p' | \hat{T}^{\mu\nu}(0) | p \rangle = 2 P^\mu P^\nu A(\Delta^2) + \frac{1}{2}(\Delta^\mu \Delta^\nu - \Delta^2 g^{\mu\nu}) D(\Delta^2)
\]

- \( A(\Delta^2) \) encodes momentum density.
- \( D(\Delta^2) \) encodes stress distributions (anisotropic pressures).
- Mix of both encodes energy density.
\[ \mathcal{E}(b_{\perp}) = m \int \frac{d^2 \Delta_{\perp}}{(2\pi)^2} \left( A(-\Delta_{\perp}^2) + \frac{\Delta_{\perp}^2}{4m^2} D(-\Delta_{\perp}^2) \right) e^{-i\Delta_{\perp} \cdot b_{\perp}} \]
Proton transverse momentum density

\[ \mathcal{P}(b_\perp) = m(\hat{s} \cdot \hat{e}_z) \int \frac{d^2 \Delta_\perp}{(2\pi)^2} \hat{e}_z \times i \Delta_\perp J(-\Delta_\perp^2) e^{-i\Delta_\perp \cdot b_\perp} \]

\[ T^{\mu\nu}(x) = \begin{bmatrix} T^{+0}(x) & T^{+1}(x) & T^{+2}(x) & T^{+3}(x) \\ T^{10}(x) & T^{11}(x) & T^{12}(x) & T^{13}(x) \\ T^{20}(x) & T^{21}(x) & T^{22}(x) & T^{23}(x) \\ T^{30}(x) & T^{31}(x) & T^{32}(x) & T^{33}(x) \end{bmatrix} \]

Energy density

Momentum densities

Energy fluxes

Stress tensor

Formalism in AF & Miller, PRD108 (2023) 094026
**Density of** $p^+ = E + p_z$ **in deuterium nucleus**

AF & Cosyn, PRD106 (2022) 114013
Transversely-polarized deuteron

**Transverse, $m_s = +1$**

Transversely polarized ($m_s = +1$)

**Transverse, $m_s = 0$**

Transversely polarized ($m_s = 0$)

▶ Distortions from optical effects—*relativistic wheel*

AF & Cosyn, PRD106 (2022) 114013
The relativistic wheel

Static wheel

- Static wheel has regularly-placed spokes

Spinning wheel

- Spinning wheel has distortions
  - Spokes moving away are redshifted.  
    - Appear to move slower, pile up
  - Spokes moving towards are blueshifted.  
    - Appear to move faster, become sparse

- These same distortions are present in the deuteron!  
  - The deuteron is a relativistic wheel!

- Also see videos at:  
  https://www.spacetime-travel.org/rad  
  (green wheel is relevant case)
Stress tensor

\[ T^{\mu\nu}(x) = \begin{bmatrix}
T^{+0}(x) & T^{+1}(x) & T^{+2}(x) & T^{+3}(x) \\
T^{10}(x) & T^{11}(x) & T^{12}(x) & T^{13}(x) \\
T^{20}(x) & T^{21}(x) & T^{22}(x) & T^{23}(x) \\
T^{30}(x) & T^{31}(x) & T^{32}(x) & T^{33}(x)
\end{bmatrix} \]

- Energy density
- Momentum densities
- Energy fluxes
- Stress tensor

▶ Stresses are **gross** forces
  ▶ There even if net force is zero!
  ▶ e.g., water pressure, stress in a bridge

▶ Stresses are also momentum flows

▶ Excellent review on stresses in Polyakov & Schweitzer, IJMP A33 (2018)
Radial eigenpressures in the deuteron

**Helicity ±1**

Helicity state ($\lambda = +1$)

**Helicity 0**

Helicity state ($\lambda = 0$)

AF & Cosyn, PRD106 (2022)
Radial eigenpressures in the deuteron

- Radial pressure *negative* at center.
- Partons feel like they’re being pulled apart?
- Cause of deuteron’s famous donut shape?
- Recent paper by Garcia Martin-Caro *et al.* also has negative radial pressures!
  - Skyrme model calculation
  - arxiv:2312.12984
- **Future work:** flows/fluxes in deuteron
  - Formalism in AF & Miller, PRD108 (2023)

AF & Cosyn, PRD106 (2022)
What about the third dimension?

- $z$ or $x^-$ doesn’t transform in a Galilean manner.
  - Absorbs relativistic wonkiness.

- $p^+$—its conjugate momentum—does.

$$p^+ = E + p_z$$

- Transforms like time ($x^+$); is invariant.

- **Momentum fractions** are additionally invariant under longitudinal boosts.

$$x \equiv \frac{p^+_{\text{parton}}}{p^+_{\text{hadron}}}$$

- We’re stuck with a partially spatial picture.
  - 2 spatial dimensions $b_\perp + 1$ momentum fraction $x$.

NJL model results for proton, AF & Cloët

- PRC101 (2020) 035203
- PRC103 (2021) 045204
III. Generalized Parton Distributions
Generalized parton distributions

GPDs are defined through **light cone correlators**.
- Amplitude for parton to be at two spacetime locations.
- Both locations along the light cone.

Correlator evaluated inside hadron.

\[
\mathcal{M}_q[\mathcal{O}] = \frac{1}{2} \int \frac{dz}{2\pi} e^{-i(P\cdot n)z} \langle p' | \bar{q} \left( \frac{n z}{2} \right) \mathcal{O}_q \left( -\frac{n z}{2} \right) | p \rangle
\]

Breakdown of correlator defines the **generalized parton distributions**:

\[
\mathcal{M}_q[\gamma^+] = \bar{u}' \left[ \gamma^q H^q(x, \xi; Q^2) + \frac{i \sigma^{n \Delta}}{2m_p} E^q(x, \xi; Q^2) \right] u
\]

Example of helicity-independent GPDs for spin-half target.
The GPD variables

\[ x = \frac{(k + k') \cdot n}{(p + p') \cdot n} \]
\[ \xi = \frac{(p - p') \cdot n}{(p + p') \cdot n} \]
\[ t = \Delta^2 = (p' - p)^2 \]
\[ n \text{ defines the light front, i.e., } n \cdot V \equiv V^+ \]

- **x** is average momentum fraction of struck parton.
- **2\(\xi\)** is the **skewness**: momentum fraction lost by struck parton.
- **t** is the invariant momentum transfer.
- GPDs also depend on renormalization scale \(Q^2\).
DGLAP and ERBL regions

\[ x + \xi \quad x - \xi \quad x > \xi \]
\[ x + \xi \quad \xi - x \quad -\xi < x < \xi \]
\[ \xi - x \quad -x - \xi \quad x < -\xi \]

- DGLAP region
  Quark out, quark in

- ERBL region
  Quark and antiquark out/in

- DGLAP region
  Antiquark out, antiquark in

- Negative momentum fraction means antiparticle
- \( x \) and \( \xi \) give qualitative picture of reaction
- \( x = \pm \xi \) means one quark line has zero (light front) momentum
  - Instantaneous propagation (at fixed light front time)
Several GPDs become PDFs when $p' = p$, i.e., $t = 0$ and $\xi = 0$.

Definition of light cone correlator:

$$\mathcal{M}^q[\mathcal{O}] = \frac{1}{2} \int \frac{dz}{2\pi} e^{-i(P\cdot n)z x} \langle p' | \bar{q} \left( \frac{n z}{2} \right) \mathcal{O} q \left( -\frac{n z}{2} \right) | p \rangle$$

This is how PDFs are formally defined, provided $p' = p$.

See e.g., Collins’s *Foundations of Perturbative QCD*.

For the proton:

$$H^q(x, 0, 0; Q^2) = q(x; Q^2)$$

$$\tilde{H}^q(x, 0, 0; Q^2) = \Delta q(x; Q^2)$$
Non-skewed GPDs

Non-skewed GPD ($\xi = 0$)

$H_q(x, \xi = 0, t)$

$-1 \leq t \leq 0$ (GeV$^2$)

$0 \leq x \leq 1$

$2D$ Fourier transform

Impact parameter PDF

$\rightarrow$

- Partially spatial structure recovered when $\xi = 0$
- $2D$ Fourier transform gives $2D$ spatial structure at fixed light front time
- Third dimension is momentum fraction $x$

Calculations in figures: AF & Cloët, PRC101 (2020) 035203
Why consider skewness?

- Why consider skewness if non-skewed GPDs give spatial densities?
- One reason is **polynomiality**:

\[
\int_{-1}^{1} dx \ x^{s-1} H(x, \xi, t; Q^2) = \sum_{l=0}^{s} A_{s,l}(t; Q^2) (-2\xi)^l
\]

- Mellin moment gives polynomial in $\xi$
- **This is a consequence of Lorentz covariance**
- Time and parity make it either even or odd (usually even)
- Maximum power of $s$

- $A_{s,l}(t : Q^2)$ are **generalized form factors**
  - $s = 1$: electromagnetic and axial form factors.
  - $s = 2$: gravitational form factors—**most promising empirical means of accessing these**
- …also, $\xi \neq 0$ in empirically-accessible processes.
Hard exclusive reactions are used to measure GPDs.

- Deeply virtual Compton scattering (DVCS) to probe quark structure.
- Deeply virtual meson production (DVMP), e.g., $J/\psi$ or $\Upsilon$ to probe gluon structure.
- Single-diffractive hard exclusive reactions (SDHEPs) to refine $x$ dependence.
- …and more!

Measured at Jefferson Lab and the upcoming Electron Ion Collider.
Deeply virtual Compton scattering (DVCS) is one method to probe GPDs.

Loop in diagram: \( x \) is integrated out

Integrated quantities seen in experiment: Compton form factors

\[
\mathcal{H}(\xi, t; Q^2) = \int_{-1}^{1} dx C(x, \xi) H(x, \xi, t; Q^2) \overset{\text{LO}}{=} \int_{-1}^{1} dx \left[ \frac{1}{\xi - x - i0} \mp \frac{1}{\xi + x - i0} \right] H(x, \xi, t; Q^2)
\]

Other processes have different coefficients—more \( x \) sensitivity
Need to invert the relationship:

\[
\mathcal{H}(\xi, t; Q^2) = \int_{-1}^{1} dx \ C(x, \xi) H(x, \xi, t; Q^2)
\]

Shadow GPDs impose a mighty obstacle:

\[
\int_{-1}^{1} dx \ C(x, \xi) H_s(x, \xi, t; Q_0^2) = 0
\]

- \( H(x, \xi, t, Q_0^2) + H_s(x, \xi, t, Q_0^2) \) gives the same DVCS amplitude at \( Q^2 = Q_0^2 \).
- Bertone, et al., PRD103 (2021) 114019

Evolution partially mitigates this.
Need to invert the relationship:

\[
\mathcal{H}(\xi, t; Q^2) = \int_{-1}^{1} dx \ C(x, \xi) H(x, \xi, t; Q^2)
\]

- Shadow GPDs entail large uncertainties
- Large kinematic lever-arm can reduce uncertainty near \(\xi \approx 0\)
- Formal constraints (positivity) help reduce uncertainty too

First analysis (Eric Moffat, AF et al.) in PRD108 (2023)
Stick to observables when extracting
e.g., Compton form factors
Use equivalence classes of GPDs
\[ H \sim H + H_{\text{shadow}} \]
Other reactions
Single diffractive hard exclusive processes (SDHEP), Qiu & Yu PRD107 (2023) 1
Has different shadows than DVCS
Use lattice results to further constrain GPDs
Model-testing paradigm
The traditional scientific method
Make predictions via models
Use experiment to test predictions
DVCS etc. serve to rule out models
Better/more measurements cull survivors
SciDAC: Femtoscale Imaging of Nuclei using Exascale Platforms

- SciDAC on femtoscale imaging.
- Collaboration between theory, experiment, computer science & applied math.
- **Event-level analysis** with AI/ML.
- Designed for exascale platforms.
  - Will utilize Aurora at Argonne!

Event-level QCF inference framework

- **Module 1**: Noise
  - Parameter Generator
  - Parameters

- **Module 2**: Trial QCF model
  - EIC
  - hermes
  - Compass
  - Jefferson Lab
  - MCMC
  - Trial PMD
  - Idealized Theory Events

- **Module 3**: Detector model
  - Optimize QCF parameters
  - Simulated Events

- **Module 4**: Experimental Events
  - Event level Discriminator
  - Trial PMD
  - MCMC
  - Simulated Events
Ongoing and future research

- GPD phenomenology
  - Framework for connecting theory to experiment
  - Pipeline to use AI/ML for fitting GPDs to data
  - Modular code packages the community can use

- Model calculations for nuclear GPDs
  - “Conventional” models for baseline
  - Models of extra-nucleonic QCD effects
  - Identify expected experimental signatures

Deuteron GPD, AF (unpublished)
GPDs and the EMT of nuclei

**Nuclear GPDs** are highly desirable:
- Understand QCD dynamics in the nuclear environment
- Distribution of mass & forces
- **Lorentz covariance** needed for relation to form factors

**Deuteron** is ideal target:
- Simplest nucleus
- Proton + neutron + $NN$ interaction
- Spin-one: gluon transversity PDF encodes extra-nucleonic but intrinsic glue

\[ k_1 - k_2; k_3 - k_4 = K(k_1 - k_2; k_3 - k_4) \]

**Ongoing work** using a non-local separable kernel for $NN$ interactions:
- This can be and is done covariantly.
- Covariance required to relate GPDs to the energy-momentum tensor
Deuteron structure

So far: decent agreement with empirical electromagnetic form factors

Model parameters determined just by $t = 0$ values!

Manuscript with these and PDFs in preparation.

Near future: energy-momentum tensor and GPD calculations.

Relevant formalism for spin-one targets in:

Cosyn, AF, Pire: PRD99 (2019); Cosyn, Cotogno, AF, Lorcé: EPJC 79 (2019); AF & Cosyn: PRD106 (2022)

Longer term plans:

Medium modifications & intrinsic glue

Heavier nuclei

Revisit the proton with lessons learned (more sophisticated proton GPDs!)
The proton is a rich and complicated system

- Nuclei (like the deuteron) are too
- The energy-momentum tensor gives us access to this richness
  - Tied to the origin of proton mass
  - Generalized parton distributions give empirical access

There’s a lot more yet to learn
- There are more exciting calculations to do
- The Electron-Ion Collider will tell us more!

Thank you for your time!