BChPT x 1/Nc

Jose L. Goity

Hampton University and Jefferson Lab

Cake Seminar JLab, October 10, 2018

works with Alvaro Calle-Cordon, Jose Alarcon, Ishara Fernando, Ruben Flores-Mendieta

BChPT: a long saga...

Numerous versions:

- Ordinary (only spin 1/2 baryons) BChPT: relativistic, non-relativistic
- BChPT including spin 3/2 baryons: relativistic, non-relativistic
- Different regularization schemes

Key issue: limited convergence range

- GB ChPT: expansion in powers of p^2
- BChPT: expansion in powers of p

An even bigger issue: what happens at large N_c?

- GB ChPT:loops suppressed by factors of $1/N_c$

meson theory becomes tree level at large N_c

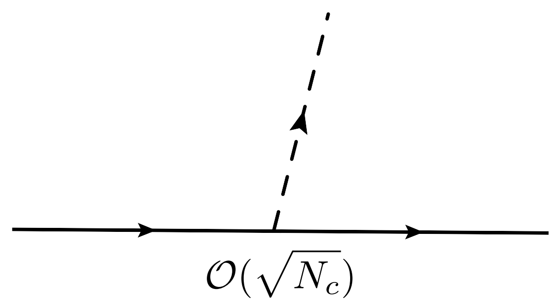
- BChPT: loops enhanced by factors of N_c

baryon theory needs a formulation consistent with N_c power constraints

N_c power of different hadronic quantities can be determined using various arguments: QMs, Feynman diagrams, etc

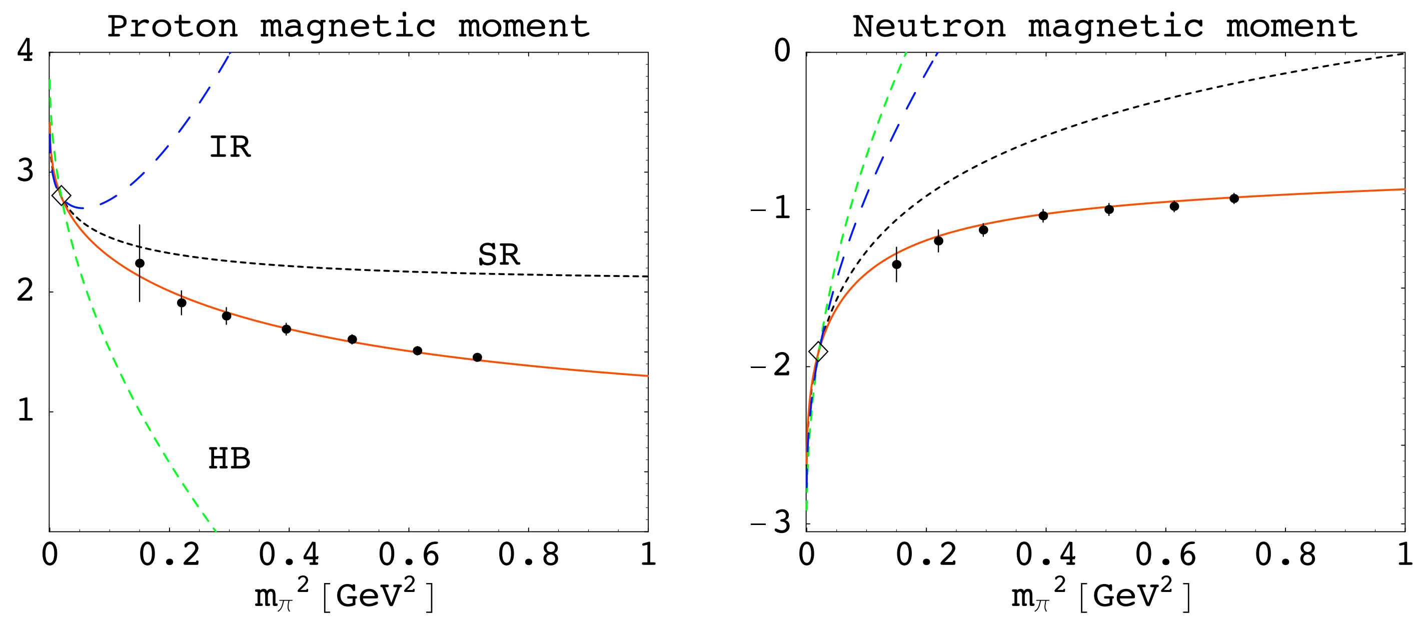
'tHooft expansion: N_f fixed, m_q fixed, m_ρ fixed

$M_\pi = \mathcal{O}({N_c}^0)$
$F_\pi = \mathcal{O}(\sqrt{N_c})$
$m_B = \mathcal{O}(N_c)$
$g_A = \mathcal{O}(N_c) \Rightarrow$



Well defined large N_c limit imposes constraints!

One illustration of the problem: magnetic moment vs m_q



ordinary BChPT

[Holstein et al (2005)]

OUTLINE

- BChPT x 1/Nc: brief basics
- Masses, sigma terms
- Vector charges in SU(3)
- Axial couplings in SU(3)
- Summary, comments

• The need for combining BChPT and 1/Nc

- Ordinary BChPT (only S=1/2 baryons) has poor convergence

- $g_{\pi N}$ is large: need for large CTs

- Inclusion of S=3/2 baryons gives significant improvement in convergence: [Jenkins & Manohar; many others]

- Consistency with $1/N_c$ expansion of QCD necessary

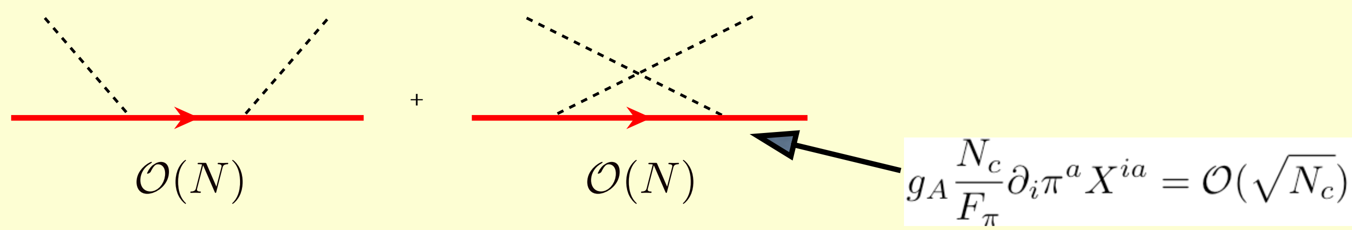
Baryons vs Nc

- Emergent dynamical spin-flavor symmetry

[Gervais & Sakita; Dashen & Manohar] last millenium

π couples to axial currents

$$\mathcal{L}_{\pi B_{\rm int}} = i\frac{g_A}{F_\pi}\partial_\mu \pi^a A^{a\mu} \qquad g_A = \mathcal{O}(N_c)$$



$$\sim \frac{k^i k'^j}{k_0}\frac{\mathring{g}_A^2 N_c^2}{F_\pi^2}\langle B' \mid [X^{ia}, X^{jb}] \mid B\rangle$$

must be order N_c^0

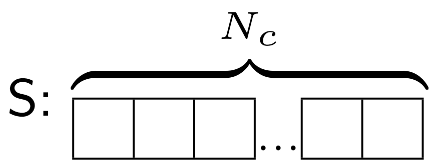
X^{ia} axial current

$$[X^{ia}, X^{jb}] = \mathcal{O}(1/N_c)$$ **key requirement at large Nc**

$\{T^a, S^i, X^{ia}\}$ generate contracted $SU(2N_f)$ dynamical symmetry

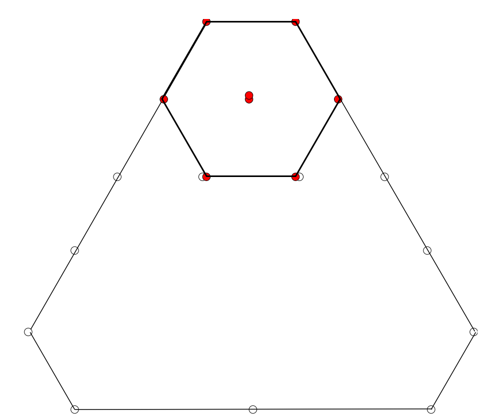
[Gervais & Sakita; Dashen & Manohar (last millenium)] classify baryons in multiplets of $SU(2N_f)$ with generators $\{T^a, S^i, G^{ia}\}$

$$G^{ia} = N_c X^{ia}$$



ground state baryons: tower with $S = \frac{1}{2} \cdots \frac{N_c}{2}$

$N_f = 3$ states in $SU(2) \times SU(3)$: $[S, R] = [S, (2S, \frac{1}{2}(N - 2S))]$



Spin-flavor Symmetry

- symmetry of spectrum at large N_c
- dynamical symmetry: not a Noether symmetry!
- imposes constraints in effective Lagrangians: relations between LECs

spin flavor symmetry as starting point for the 1/Nc expansion

$\frac{1}{N_c}$ expansion as spin-flavor operator product expansion

$$\langle B' \mid \hat{O}_{QCD} \mid B\rangle = \sum_n C_n \frac{1}{N_c^{\nu_n -1}} \langle B' \mid \hat{O}_n \mid B\rangle$$

O_n : tensor operator product of spin-flavor generators and momenta
ν_n : spin-flavor n-bodyness of O_n

Example: mass operator in chiral limit:
$$H_{QCD} \Rightarrow N_c m_0 + C_{HF} \frac{1}{N_c} \hat{S}^2 + \mathcal{O}(\frac{1}{N_c^3}) \hat{S}^4 + \cdots$$
expansion is in $1/N_c^2$, $m_\Delta - m_N = \mathcal{O}(\frac{1}{N_c})$

A test: $g_A s$

$\frac{g_A^{N\Delta}}{g_A^N} = 1 + \mathcal{O}(\frac{1}{N_c^2})$ [Dashen & Manohar]

$g_A^N = -1.2724 \pm 0.0023 \qquad g_A^{N\Delta} = -1.235 \pm 0.011$

• BChPT x 1/Nc: brief basics

- $m_B = \mathcal{O}(N_c) \Rightarrow$ HB expansion is a $1/N_c$ expansion
- Lagrangians built with chiral and spin-flavor tensor operators:

$$\mathbf{B}^\dagger \; T_\chi \otimes T_{SF} \; \mathbf{B}$$

$$\mathbf{B} = \begin{pmatrix} B_{S=1/2} \\ B_{S=3/2} \\ \vdots \\ B_{S=N_c/2} \end{pmatrix} \qquad \text{GS tower of baryon fields}$$

T_χ chiral tensor $\quad T_{SF}$ spin-flavor tensor product of SU(6) generators

chiral and $1/N_c$ power counting determined by operators

LECs: chosen to be $\mathcal{O}(N_c^0)$, have a $1/N_c$ expansion themselves

each Lagrangian term has a well defined *leading* chiral and $1/N_c$ power

need to link chiral and $1/N_c$ expansions: small mass scale $\Delta_{HF} = m_{3/2} - m_{1/2}$

ξ expansion: $\xi = \mathcal{O}(1/N_c) = \mathcal{O}(p)$

Lagrangians to order ξ^3

$$\mathcal{L}_B^{(1)} = \mathbf{B}^\dagger(iD_0 - \mathring{g}_A u^{ia} G^{ia} - \frac{C_{\mathrm{HF}}}{N_c}\hat{S}^2 + \frac{c_1}{2\Lambda}\,\hat{\chi}_+)\mathbf{B}$$

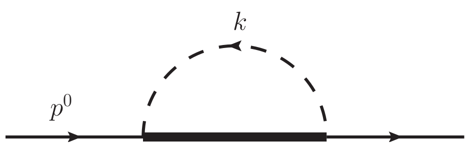
$$\mathring{g}_A = \tfrac{6}{5} g_A^N$$

$$\mathcal{L}_{\mathbf{B}}^{(2)} = \mathbf{B}^\dagger\Big(\tfrac{c_2}{\Lambda}\,\chi_+^0 + \tfrac{C_1^A}{N_c} u^{ia} S^i T^a + \tfrac{\tau_1}{N_c}(u_0^a G^{ia} D_i + \overleftarrow{D}_i u_0^a G^{ia})$$
$$+\tfrac{1}{m}(\vec{B}_+^0 + \vec{B}_+^a T^a)\cdot\vec{S} + \tfrac{1}{2m}(2(\kappa_0\,\vec{B}_+^0 + \kappa_1\,\vec{B}^a T^a)\cdot\vec{S} + \tfrac{6}{5}\kappa_2\,B_+^{ia} G^{ia}) + \cdots\Big)\mathbf{B}$$

$$\mathcal{L}_{\mathbf{B}}^{(3)} = \mathbf{B}^\dagger\Big(\tfrac{1}{2m} D^\mu D_\mu + \tfrac{c_3}{N_c\,\Lambda^3}\,\hat{\chi}_+^2 + \tfrac{h_1}{N_c^3}\hat{S}^4 + \tfrac{h_2}{N_c^2\Lambda}\hat{\chi}_+\hat{S}^2$$
$$+\tfrac{h_3}{N_c\Lambda}\chi_+^0\hat{S}^2 + \tfrac{h_4}{N_c\,\Lambda}\,\chi_+^a\{S^i, G^{ia}\} + \tfrac{C_2^A}{N_c^2} u^{ia}\{\hat{S}^2, G^{ia}\}$$
$$+\tfrac{C_4^A}{N_c^2} u^{ia} S^i S^j G^{ja} + \tfrac{D_1^A}{\Lambda^2}\chi_+^0 u^{ia} G^{ia} + \tfrac{D_2^A}{\Lambda^2}\chi_+^a u^{ia} S^i$$
$$+\tfrac{D_3^A(d)}{\Lambda^2} d^{abc}\chi_+^a u^{ib} G^{ic} + \tfrac{D_3^A(f)}{\Lambda^2} f^{abc}\chi_+^a u^{ib} G^{ic}$$
$$+(\tfrac{1}{8m^2} + \tfrac{g_0}{\Lambda^2})\,\partial_i E_{+i}^0 + (\tfrac{1}{8m^2} + \tfrac{g_1}{\Lambda^2})\,(D_i E_{+i})^a T^a + \cdots\Big)\mathbf{B}$$

[E. Jenkins; R. Flores-Mendieta et al; A. Calle-Cordon & JLG; I. Fernando & JLG]

Chiral loops



$$= \int \frac{d^d k}{(2\pi)^d} \, \frac{i}{k^2 - M_\pi^2} \, \frac{i}{p^0 + k^0 - \underbrace{(m_{B'} - m_B)}_{\mathcal{O}(1/N_c)}} \times \text{vertex factors}$$

contains non-analytic terms:

$$(M_\pi^2 - (m_\Delta - m_N)^2)^{\frac{3}{2}}, \; tanh^{-1}\left(\frac{(m_\Delta - m_N)}{\sqrt{1/(-M_\pi^2 + (m_\Delta - m_N)^2}}\right)$$

link $1/N_c$ and chiral expansions:

ξ expansion: $\xi = \mathcal{O}(1/N_c) = \mathcal{O}(p)$

equivalent to not expanding non-analytic terms

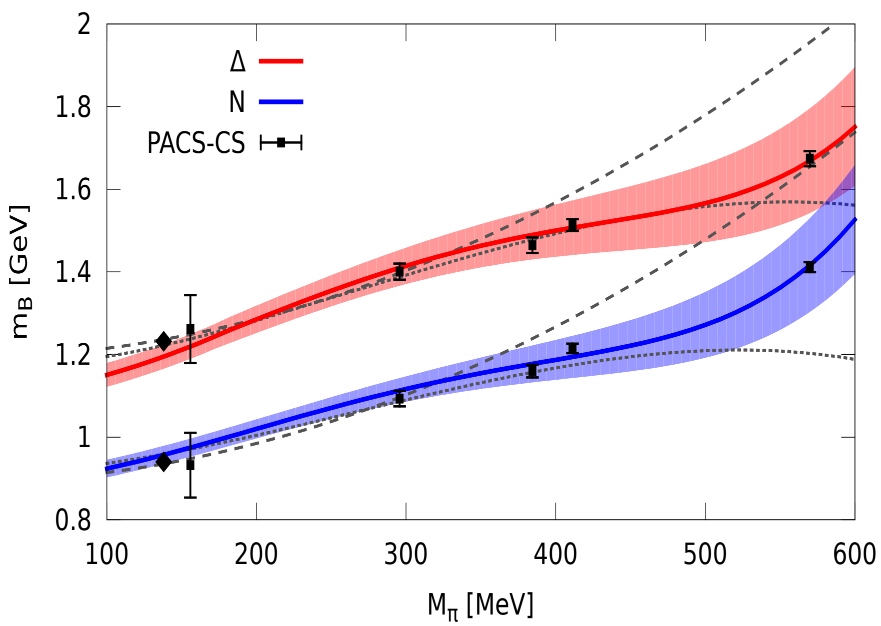
$$\nu_\xi = 1 + 3L + \frac{n_\pi}{2} + \sum_i n_i \left(\nu_{O_i} + \nu_{p_i} - 1\right)$$

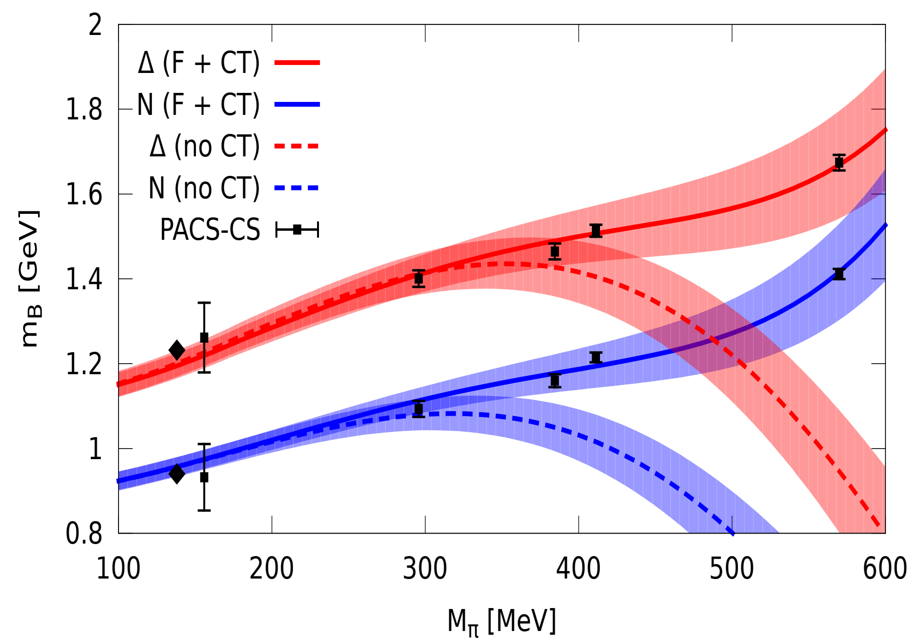
• Masses, sigma terms: SU(3)

WF renormalization factor is $\mathcal{O}(N_c)$!
plays key role in N_c power counting consistency in loops

- mass corrections are $\mathcal{O}(N_c)$ (terms proportional to M_{GB}^3)
- SU(3) mass splitting of course $\mathcal{O}(N_c^0)$

$\mathrm{SU}(2)$



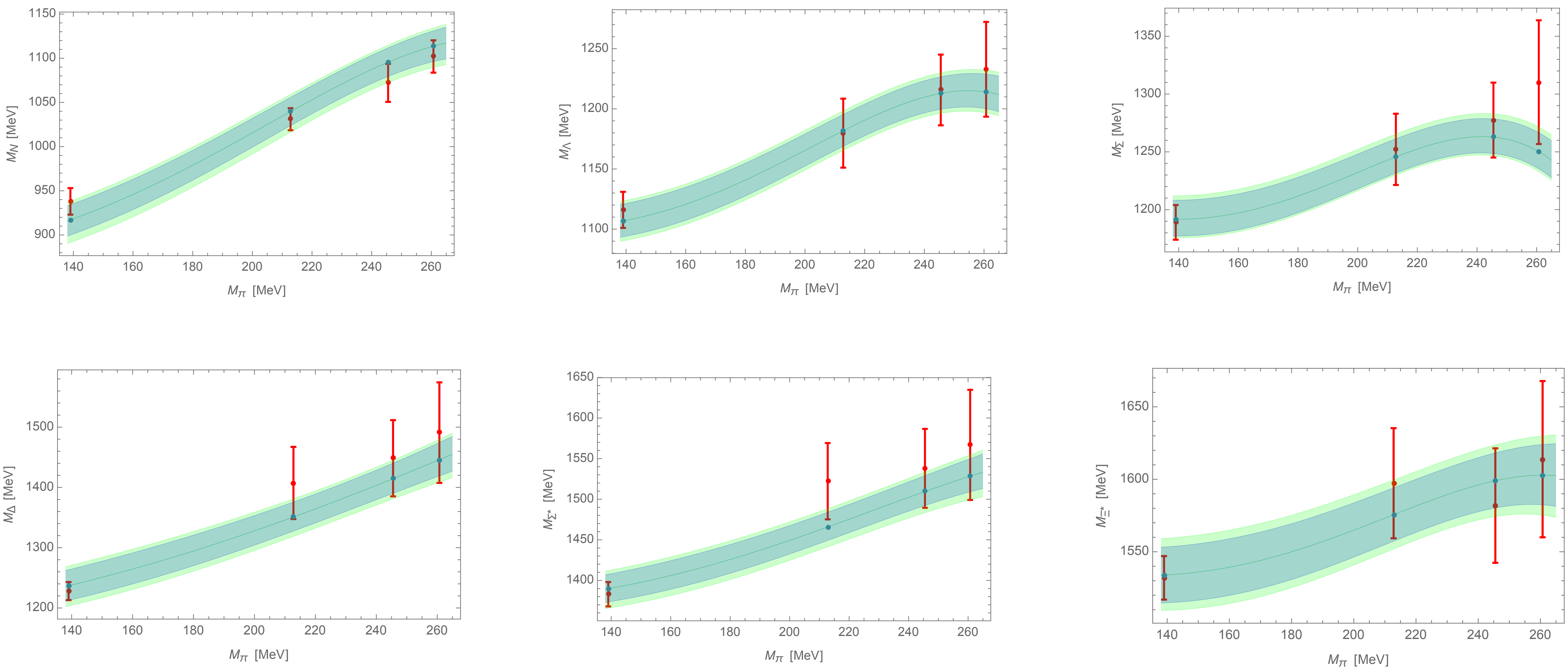


[A. Calle-Cordon & JLG]

$SU(3)$

[I. Fernando & JLG]

M_π dependency from LQCD ($M_K \sim 500$ MeV):
poor convergence above $M_\pi \sim 250$ MeV



[Alexandrou et al (2014), ETMC LQCD Coll.
octet and decuplet baryon masses

Mass relations

GMO

$$\Delta_{GMO} = \text{ Th: } \left(\frac{g_A^N(LO)}{g_A^N}\right)^2 44 \pm 5 \text{ MeV vs Exp: } 25.6 \pm 1.5 \text{ MeV}$$

$$\Delta_{GMO} = -\left(\frac{\mathring{g}_A}{4\pi F_\pi}\right)^2 \left(\frac{2\pi}{3}(M_K^3 - \frac{1}{4}M_\pi^3 - \frac{2}{\sqrt{3}}(M_K^2 - \frac{1}{4}M_\pi^2)^{\frac{3}{2}})\right.$$
$$\left. + \frac{2C_{HF}}{N_c}\left(-M_K^2 \log M_K^2 + \frac{1}{4}M_\pi^2 \log M_\pi^2 + (M_K^2 - \frac{1}{4}M_\pi^2)\log(\frac{4}{3}M_K^2 - \frac{1}{3}M_\pi^2)\right)\right) + \mathcal{O}(1/N_c^3)$$
$$= 37 \text{ MeV} + \mathcal{O}(1/N_c^3)$$

in large N_c, Δ_{GMO} is $\mathcal{O}(1/N_c)$

ES

$$\Delta_{ES} = m_{\Xi^*} - 2m_{\Sigma^*} + m_\Delta =$$
$$\text{Th: } -\left(\frac{g_A^N(LO)}{g_A^N}\right)^2 6.5\text{MeV vs Exp: } -4 \pm 7\text{MeV} = \mathcal{O}(1/N_c)$$

GR

$$\Delta_{GR} = m_{\Xi^*} - m_{\Sigma^*} - (m_\Xi - m_\Sigma) = 0, \quad \text{Exp: } 21 \pm 7 \text{ MeV},$$

$$\Delta_{GR} = \frac{h_2}{\Lambda}\frac{12}{N_c}(M_K^2 - M_\pi^2) + \underbrace{\text{O}(1/N_c) \text{ UV finite no-analytic terms})}_{\sim 68 \text{ MeV} \times \left(\frac{g_A^N(LO)}{g_A^N}\right)^2}$$

πN σ-term

$$\hat{\sigma} = \frac{\hat{m}}{2m_N}\langle N|\bar{u}u + \bar{d}d - 2\bar{s}s|N\rangle \qquad \sigma_{\pi N} = \hat{\sigma} + \frac{2\hat{m}}{m_s}\sigma_s.$$

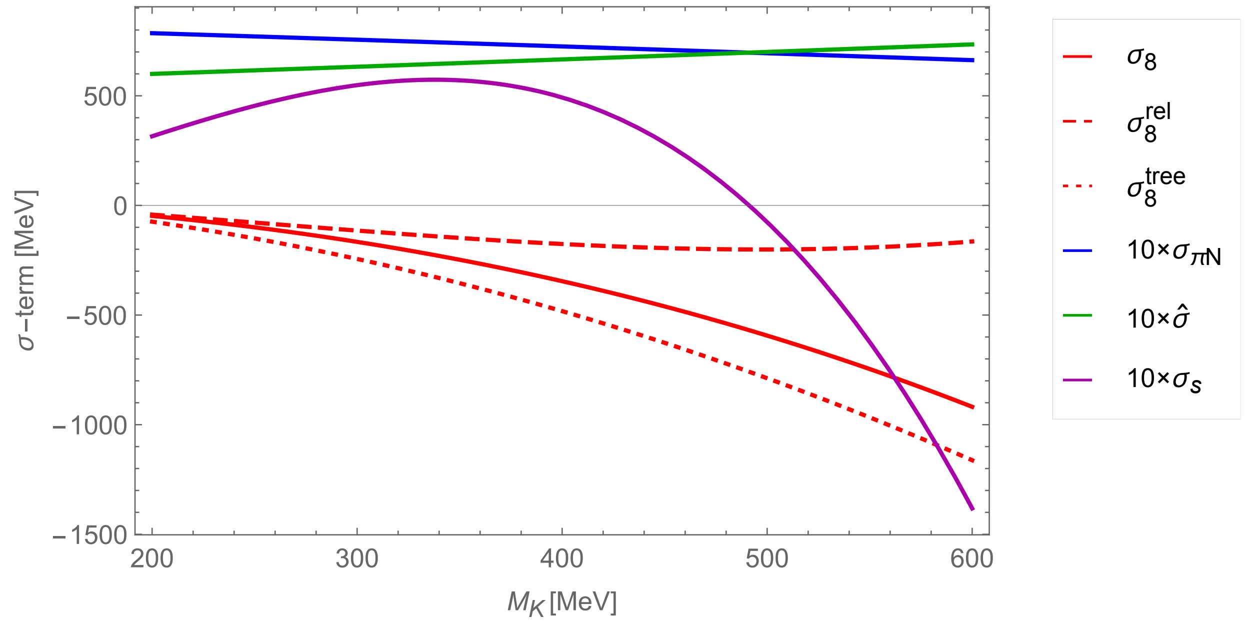
$$\hat{\sigma} = \underbrace{\frac{\hat{m}}{m_s - \hat{m}}\left(\frac{N_c+3}{6}m_\Xi + \frac{2N_c-3}{3}m_\Sigma - \frac{5N_c-3}{6}m_N\right)}_{@N_c=3:\ \sim 23\mathrm{MeV}} + \Delta\hat{\sigma} \leftarrow \mathcal{O}(N_c)$$

$\uparrow$ $\mathcal{O}(N_c)$

$2.3 \times 10^5 \mathrm{MeV}^3 \times \frac{g_A^2}{F_\pi^2} \sim 40\ \mathrm{MeV}$

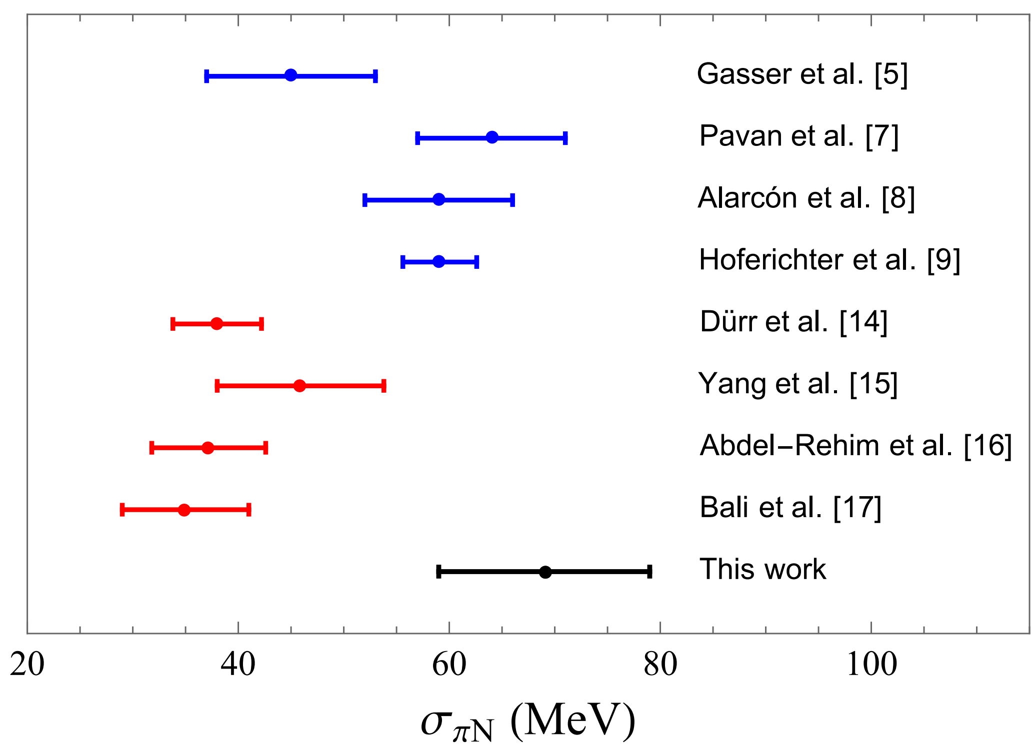
40 % from 8 in loop and 60 % from 10

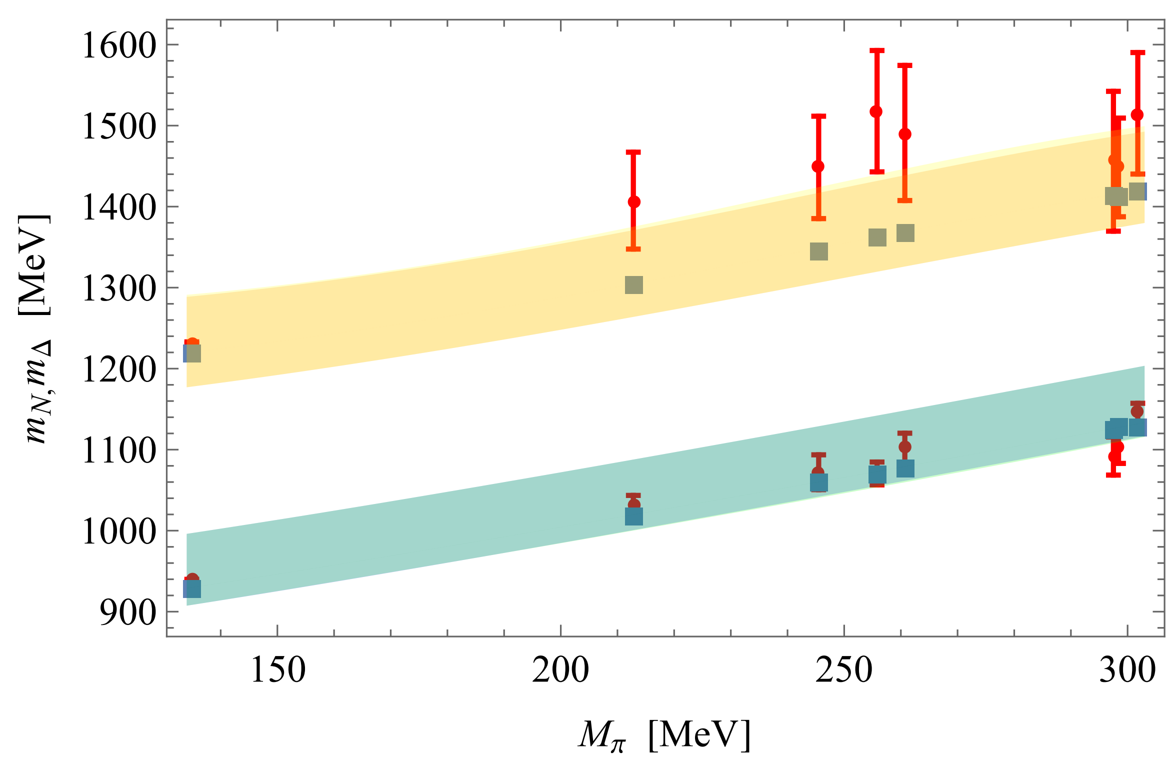
- $\frac{\Delta\sigma_8}{\Delta_{GMO}} \sim -13$: independent of g_A/F_π,
  virtually independent of C_{HF},
  mild dependence on M_K, M_π !
- $\frac{\Delta\sigma_8}{\Delta_{GMO}}$ changes little if one turns off decuplet!
  but g_A from Δ_{GMO} too large, clashes with axial couplings



$$\hat{\sigma} = 70 \pm 9 \text{ MeV} \quad \oplus \quad LQCD \quad \sigma_{\pi N} = 69 \pm 10 \text{ MeV}$$

[LQCD: Alexandrou et al (2016)]





[J.M.Alarcon, I.Fernando & JLG (2018)]

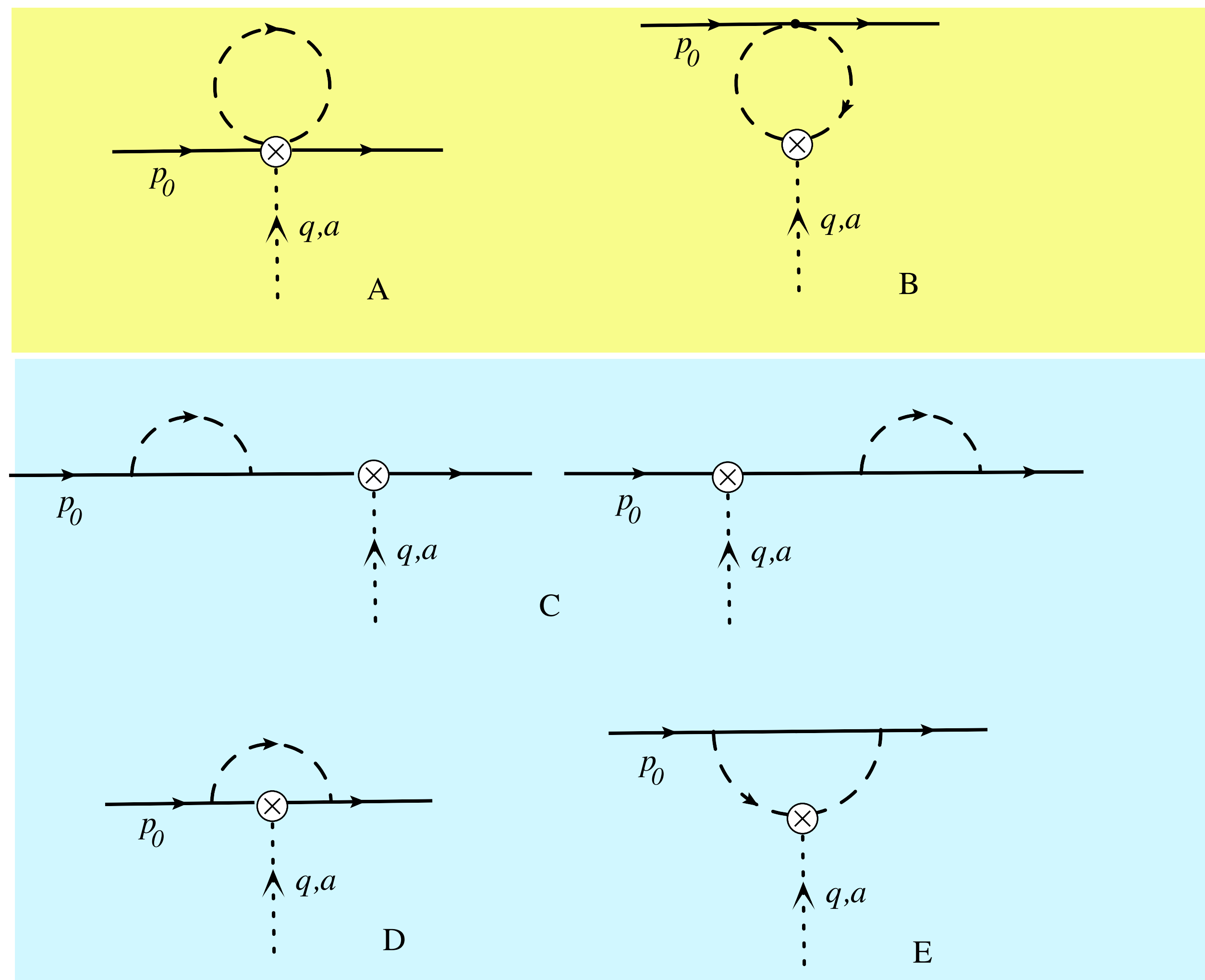
Fit	$\frac{\mathring{g}_A}{F_\pi}$ MeV^{-1}	$\frac{M_0}{N_c}$ MeV	C_{HF} MeV	c_1	c_2	h_2	h_3	h_4	α MeV	β MeV
1	0.0126(2)	364(1)	166(23)	−1.48(4)	0	0	0.67(9)	0.56(2)	−1.63(24)	2.16(22)
2	0.0126(3)	213(1)	179(20)	−1.49(4)	−1.02(5)	−0.018(20)	0.69(7)	0.56(2)	−1.62(24)	2.14(22)
3	0.0126*	262(30)	147(52)	−1.55(3)	−0.67(8)	0	0.64(3)	0.63(3)	−1.63*	2.14*

Fit	$\Delta^{\text{phys}}_{GMO}$ MeV	σ_8 MeV	$\Delta\sigma_8$ MeV	$\hat{\sigma}$ MeV	$\sigma_{\pi N}$ MeV	σ_s MeV	σ_3 MeV	$\sigma_{u+d}(p-n)$ MeV
1	25.6(1.1)	−583(24)	−382(13)	70(3)(6)	–	–	−1.0(3)	−1.6(6)
2	25.5(1.5)	−582(55)	−381(20)	70(7)(6)	69(8)(6)	−3(32)	−1.0(4)	−1.6(8)
3	25.8*	−615(80)	−384(2)	74(1)(6)	65(15)(6)	−121(15)	–	–

Relations between σ terms for octet and decuplet
deviations are finite (calculable) at order ξ^3
need more accurate baryon masses from LQCD to test

$$\sigma_{Nm_s} = \frac{m_s}{8\hat{m}}\left(-4(N_c-1)\sigma_{N\hat{m}} + (N_c+3)\sigma_{\Lambda\hat{m}} + 3(N_c-1)\sigma_{\Sigma\hat{m}}\right)$$
$$\sigma_{\Lambda m_s} = \frac{m_s}{8\hat{m}}\left(-4(N_c-3)\sigma_{N\hat{m}} + (N_c-5)\sigma_{\Lambda\hat{m}} + 3(N_c-1)\sigma_{\Sigma\hat{m}}\right)$$
$$\sigma_{\Sigma m_s} = \frac{m_s}{8\hat{m}}\left(-4(N_c-3)\sigma_{N\hat{m}} + (N_c+3)\sigma_{\Lambda\hat{m}} + (3N_c-11)\sigma_{\Sigma\hat{m}}\right)$$
$$\sigma_{\Delta m_s} = \frac{m_s}{8\hat{m}}\left(-4(N_c-1)\sigma_{\Delta\hat{m}} - 5(N_c-3)(\sigma_{\Lambda\hat{m}} - \sigma_{\Sigma\hat{m}}) + 4N_c\sigma_{\Sigma^*\hat{m}}\right)$$
$$\sigma_{\Sigma^* m_s} = \frac{m_s}{8\hat{m}}\left(-(N_c-3)(4\sigma_{\Delta\hat{m}} + 5\sigma_{\Lambda\hat{m}} - 5\sigma_{\Sigma\hat{m}}) + 4(N_c-2)\sigma_{\Sigma^*\hat{m}}\right).$$

Vector charges



SU(3) breaking corrections to the vector currents:

Ademollo-Gatto theorem at $\mathcal{O}(\xi^2)$
non-analytic calculable corrections to AGTh $\mathcal{O}(N_c^0)$,

different spin baryons in loop give $\mathcal{O}(N_c)$ terms!
key cancellations give N_c consistency

SU(3) breaking to vector charges

[R.Flores-Mendieta & JLG; I.P.Fernando & JLG]

Charge	$\frac{f_1}{f_1^{SU(3)}}$	$\frac{f_1}{f_1^{SU(3)}} - 1$ [Flores-Mendieta & JLG:2014] HBChPT×1/N_c	[Villadoro:2006] HBChPT with 8 and 10	[Lacour et al:2007] HBChPT only 8	[Geng et al:2009] RBChPT with 8 and 10
Λp	0.952	−0.048	−0.080	−0.097	−0.031
$\Sigma^- n$	0.966	−0.034	−0.024	0.008	−0.022
$\Xi^-\Lambda$	0.953	−0.047	−0.063	−0.063	−0.029
$\Xi^-\Sigma^0$	0.962	−0.038	−0.076	−0.094	−0.030

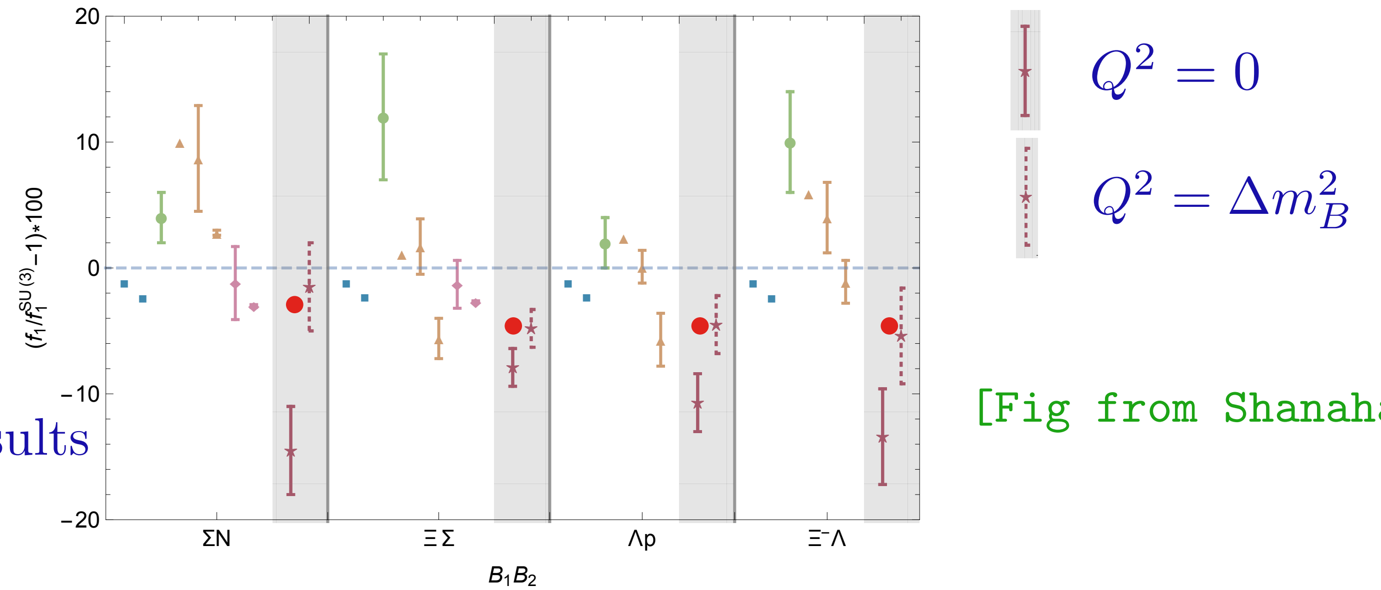
LQCD

$$f_1^{\Sigma\to N}(0) = -0.9662(43), \quad f_1^{\Xi\to\Sigma}(0) = +0.9742(28)$$

[S. Sasaki, (2017)]

- parameter free result

tension with the $Q^2 = 0$ LQCD results



[Fig from Shanahan et al (2015)]

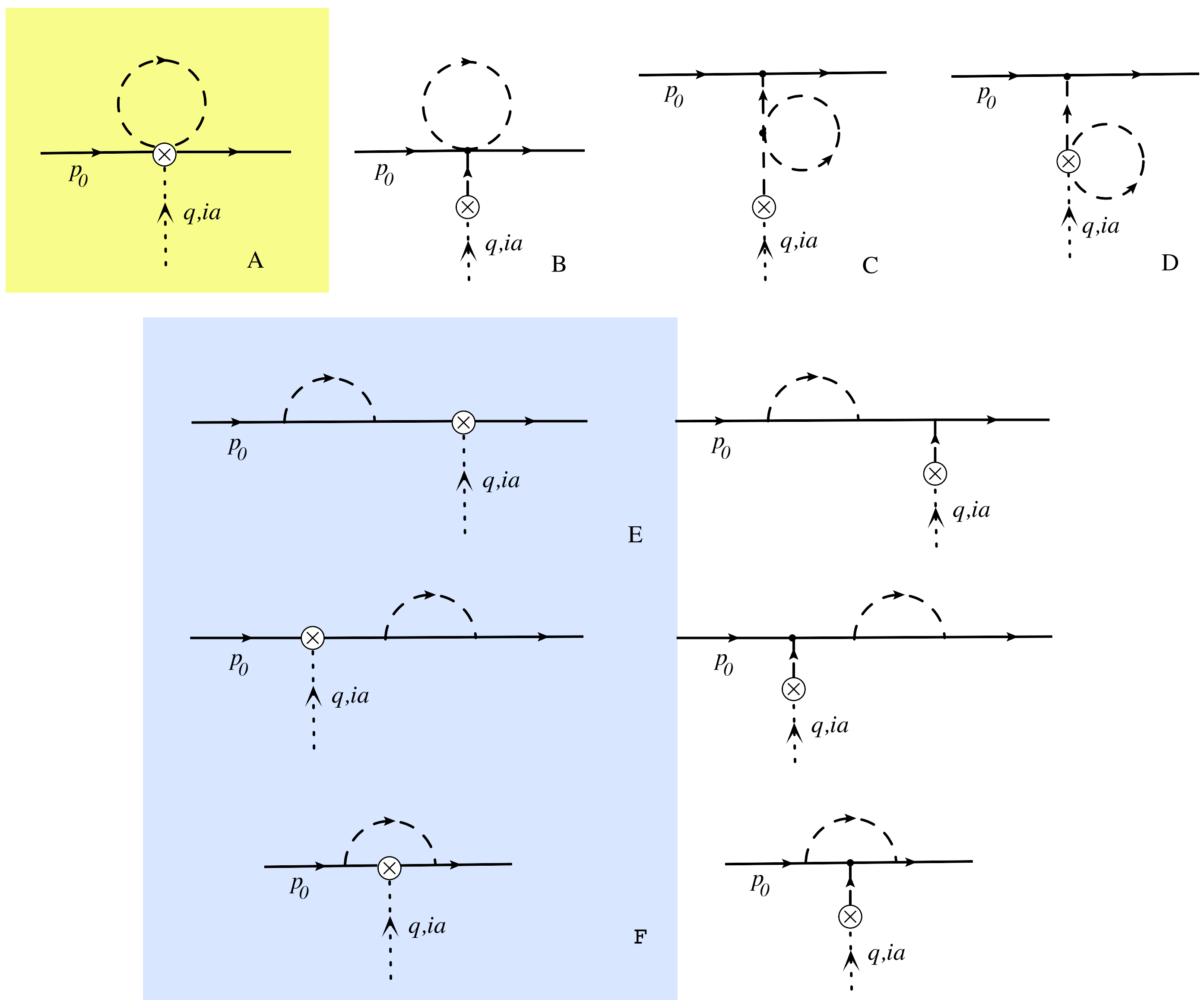
Vector current form factors:

- EM and SU(3): work in progress (I. Fernando and JLG)
- Charge FF: peripheral component (J. M. Alarcon and C. Weiss)

Axial-vector currents

[Flores-Mendieta, Hernandez & Hofmann; Fernando & JLG] [SU(2): A. Calle-Cordon & JLG]

Definition of axial couplings $\langle B' \mid A^{ia} \mid B\rangle = \frac{6}{5} g_A^{aBB'} \langle B' \mid G^{ia} \mid B\rangle$



E, F violate N_c power counting:
cancellation of such terms in E+F

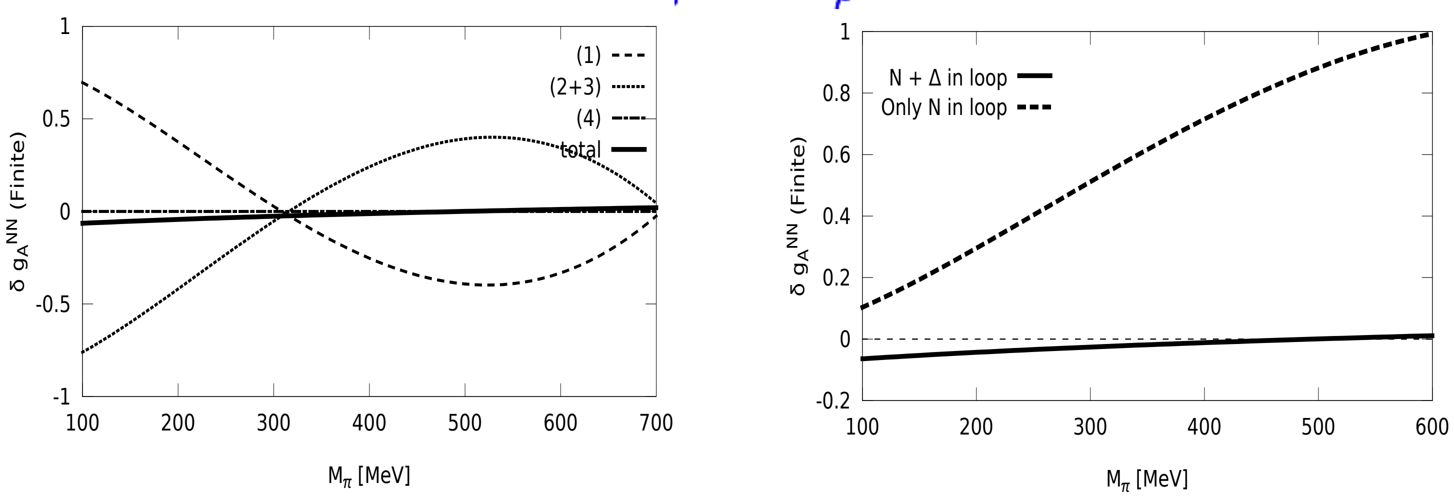
Cancellation of N_c power violating terms between diagrams

Polynomial pieces

$$((1)+\tfrac{1}{2}((2)+(3)))^{Poly} = (\lambda_\epsilon+1)\ \tfrac{1}{2}\ M^2_{ab}\ [[\Gamma, G^{ia}], G^{ib}]$$
$$+(\lambda_\epsilon+2)\ \tfrac{1}{3}\ \Big([[\Gamma,[\delta\hat{m},G^{ia}]],[\delta\hat{m},G^{ia}]] + 2[[G^{ia},\Gamma],[\delta\hat{m},[\delta\hat{m},G^{ia}]]]\Big)$$

No-analytic pieces: SU(2)

$\mu = m_\rho$



cancellations to accuracy $1/N_c^2$

in large N_c persist at $N_c = 3$

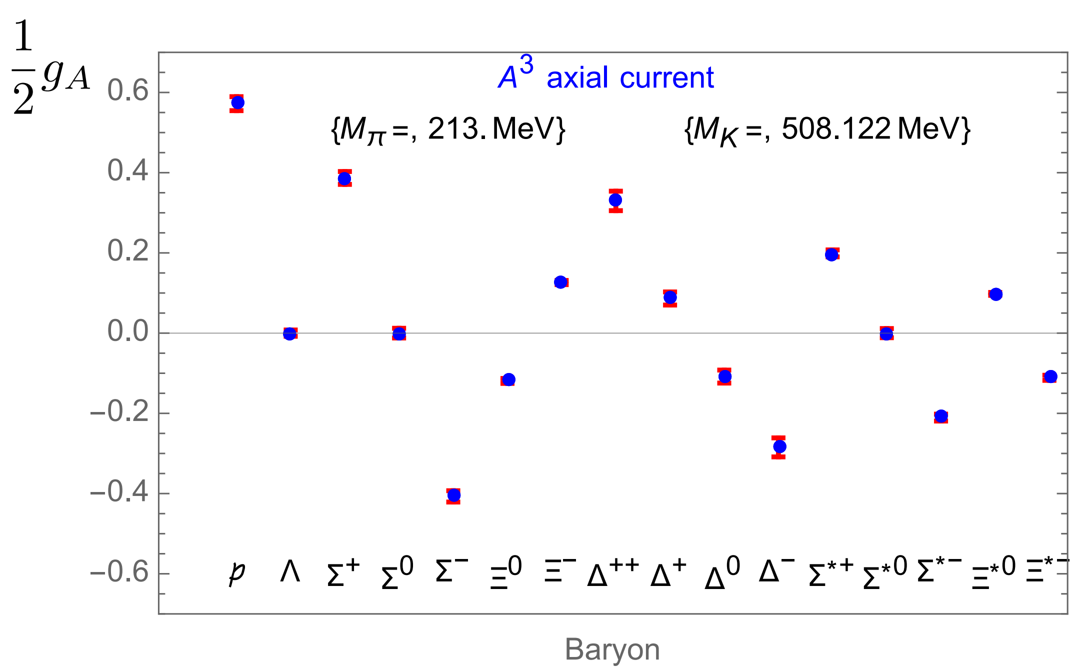
Fit to SU(3) LQCD g_A's

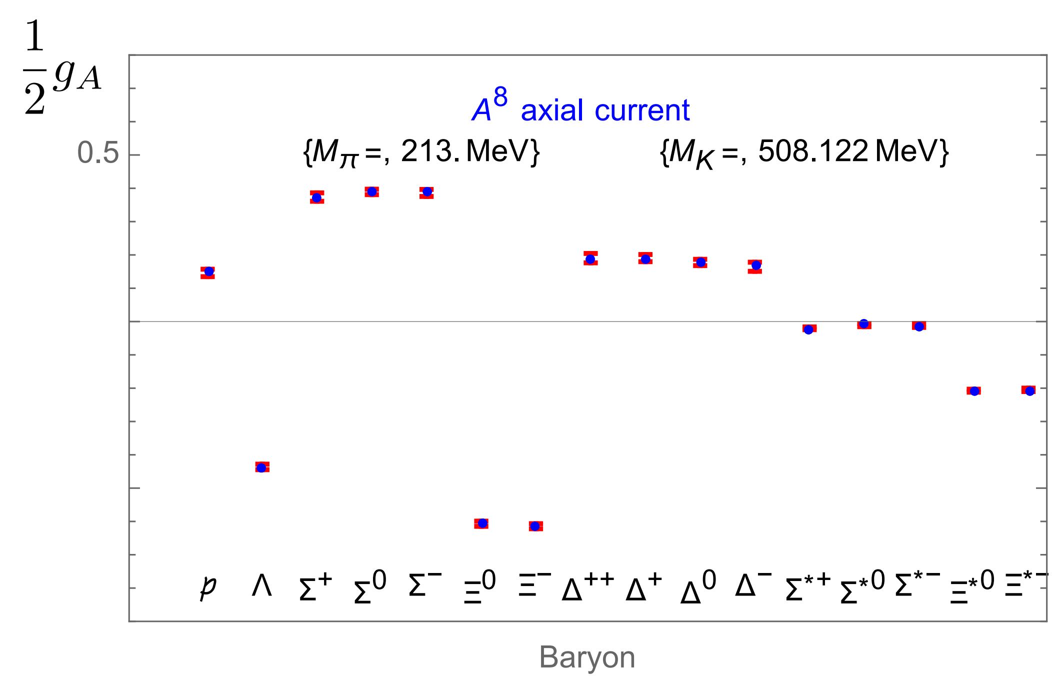
Key observed feature:@ fixed M_K, g_A's have little dependence on M_π

SU(3) calculation by Cyprus Group [Alexandrou et al, (2016)]
g_A^{3BB} and g_A^{8BB}

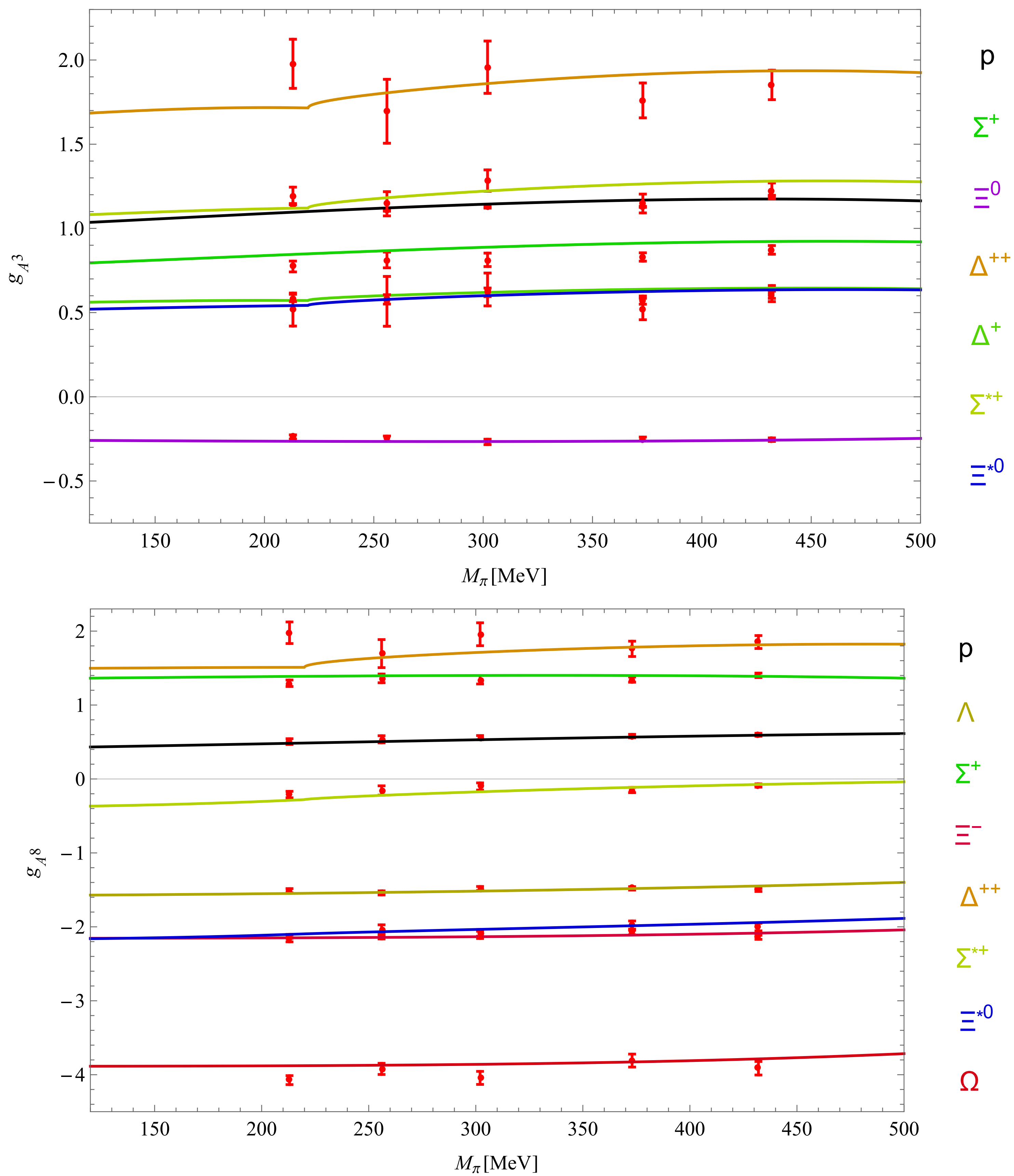
Fit	$\chi^2_{\rm dof}$	$\overset{\circ}{g}_A$	$\delta\overset{\circ}{g}_A$	C_1^A	C_2^A	C_3^A	C_4^A	D_1^A	D_2^A	D_3^A	D_4^A
LO	3.9	1.35	...	...	...	...	...	...	...	...	...
NLO Tree	0.91	1.42	...	−0.18	...	...	...	...	0.009	...	...
NLO Full	1.08	1.02	0.15	−1.11	0.	1.08	0.	−0.56	−0.02	−0.08	0.
	1.13	1.04	0.08	−1.17	0.	1.15	0.	−0.59	−0.02	−0.09	0.
	1.19	1.06	0.	−1.23	0.	1.21	0.	−0.62	−0.03	−0.09	0.

[I. Fernando & JLG (2018)]





Mild M_π dependence of axial couplings **cannot** be described without the cancellations of N_c violating terms



Observations on axial couplings

- show most prominently the need for theory consistent with $1/N_c$ expansion
- natural fit at one-loop of the axial couplings from SU(3) LQCD
- impossible to fit g_As of octet when turning off decuplet
- in SU(3): $g_A^N(LO)/g_A^N \sim 0.8$ from fit to axial couplings and from Δ_{GMO}
- numerous relations among axial couplings with calculable corrections

Summary and comments

- Based on general arguments of Nc scaling, we know how to implement the 1/Nc expansion at hadron level

- In baryons it requires implementing a dynamical spin-flavor symmetry, broken at subleading orders in 1/Nc: use to implement BChPT x 1/Nc

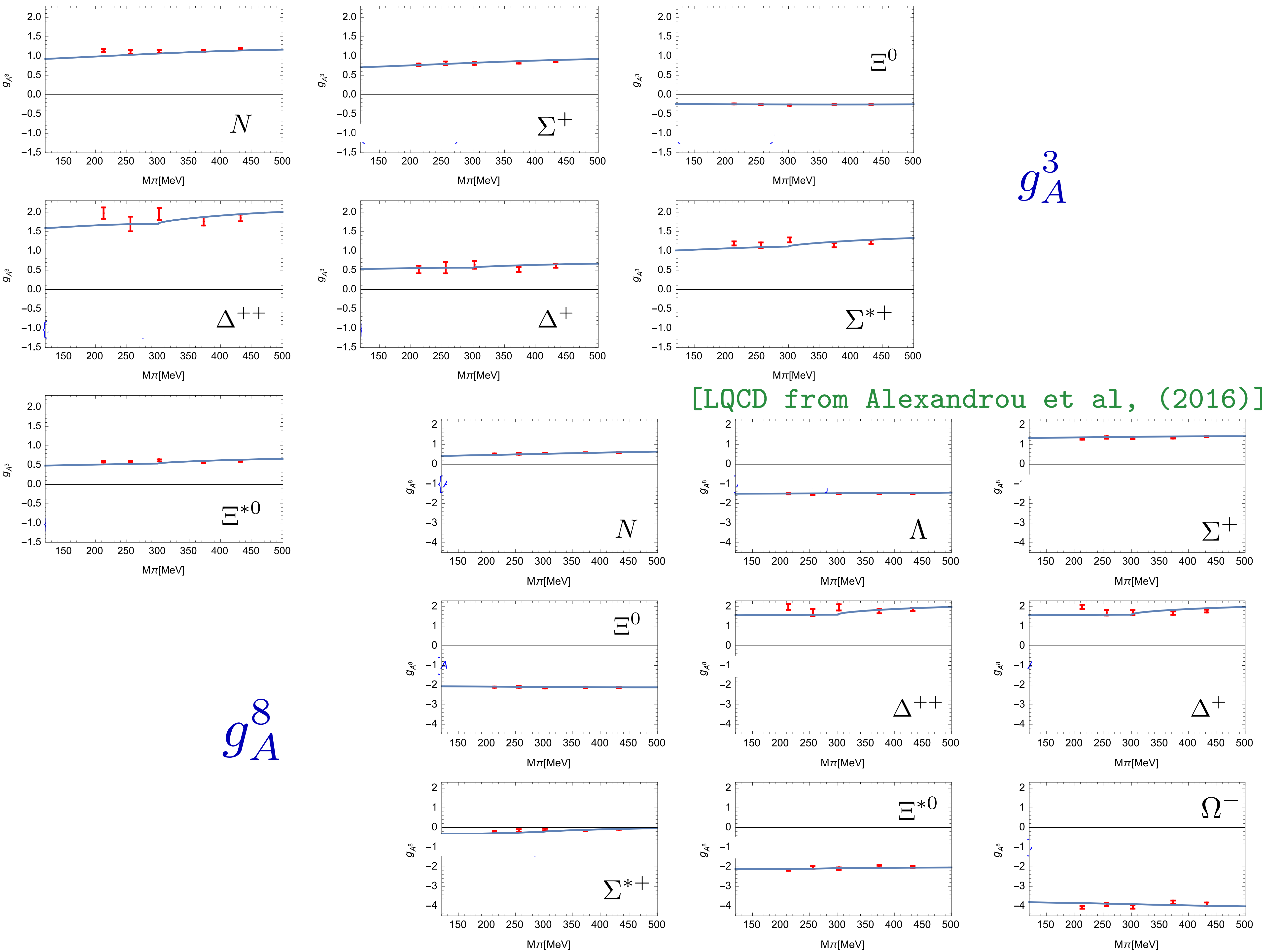
- BChPT x 1/Nc expansion improves convergence as it eliminates consistently large Nc power violating terms from loop corrections.

- Convergence improvement is especially important in SU(3) BChPT

- Axial couplings are good testing ground thanks to inputs from LQCD

- Predictions at order ξ^3 : calculable corrections to mass relations, SU(3) vector charges and almost parameter free prediction for $\hat{\sigma}$

- 1/Nc requirements impact broadly on BChPT, so... much more to be done !!!



g_A^3
N
Σ^+
Ξ^0
Δ^{++}
Δ^+
Σ^{*+}
Ξ^{*0}
[LQCD from Alexandrou et al, (2016)]
g_A^8
N
Λ
Σ^+
Ξ^0
Δ^{++}
Δ^+
Σ^{*+}
Ξ^{*0}
Ω^-
Mπ[MeV]

Why we need to combine ChPT and 1/Nc

QCD expansion parameters: $m_q \quad (q = u, d, s); \qquad 1/N_c$

m_q and low energy/momenta $\rightarrow$ ChPT

$1/N_c \rightarrow \; N_c$ scalings of hadron masses and couplings

$1/N_c$ expansion
Pheno: OZI; VMD
LQCD @ varying N_c: string tension ; F_π; baryon masses

Need for combining ChPT and $1/N_c$ expansion

[Herrera-Siklody, Latorre, Pascual & Taron; Kaiser & Leutwyler]

Effective theories need to agree with
chiral dynamics and $1/N_c$ power counting

Large Nc baryons and chiral symmetry

$1/N_c$ × heavy baryon expansion is a natural combination [Jenkins]

LO chiral Lagrangian

$$\mathcal{L}_{\mathbf{B}}^{(1)} = \mathbf{B}^\dagger \left(iD_0 + \mathring{g}_A u^{ia} G^{ia} - \frac{C_{HF}}{N_c} \hat{\vec{S}}^2 - \frac{c_1}{2\Lambda} \hat{\chi}_+ \right) \mathbf{B} \qquad g_A = \frac{5}{6}\mathring{g}_A$$

$\mathbf{B}$ is the baryon spin-flavor multiplet field

$N_f = 3$ states in $SU(2) \times SU(3)$: $[S, R] = [S, (2S, \frac{1}{2}(N - 2S))]$

LO all GB-baryon couplings given in terms of g_A

from Δ width: $g_A^{\Delta N} = 1.235 \pm 0.011$ vs $g_A^{NN} = 1.267 \pm 0.004$

Small scales: $p,\ M_{GB},\ m_\Delta - m_N = \mathcal{O}(1/N_c)$

Chiral and $1/N_c$ expansions do not commute!:
need to link power countings

ξ or small scale expansion:
$\mathcal{O}(p) = \mathcal{O}(1/N_c) = \mathcal{O}(\xi)$

NLO Lagrangians

$$\mathcal{L}_{\mathbf{B}}^{(2)} = \mathbf{B}^\dagger \Big(\big(\tfrac{z_1}{N_c} + \tfrac{z_2}{N_c}\hat{S}^2 + \tfrac{z_3}{\Lambda^2}\, N_c\, \chi_+^0\big)\, i\tilde{D}_0$$
$$+\big(-\frac{1}{2N_c m_0} + \tfrac{w_1}{\Lambda}\big)\vec{D}^2 + \big(\frac{1}{2N_c m_0} - \tfrac{w_2}{\Lambda}\big)\tilde{D}^2 + \tfrac{c_2}{\Lambda}\,\chi_+^0$$
$$+\tfrac{C_1^A}{N_c} u^{ia} S^i T^a + \tfrac{C_2^A}{N_c}\epsilon^{ijk} u^{ia}\{S^j, G^{ka}\}$$
$$+\kappa_0\, \epsilon^{ijk} F^0_{+ij} S^k + \kappa_1\, \epsilon^{ijk} F^a_{+ij} G^{ka} + \rho_0 F^0_{-0i} S^i + \rho_1 F^a_{-0i} G^{ia}$$
$$+\tfrac{\tau_1}{N_c} u_0^a G^{ia} D_i + \tfrac{\tau_2}{N_c^2} u_0^a S^i T^a D_i + \tfrac{\tau_3}{N_c}\nabla_i u_0^a S^i T^a + \tau_4 \nabla_i u_0^a G^{ia} + \cdots \Big)\mathbf{B}$$

$$\mathcal{L}_{\mathbf{B}}^{(3)} = \mathbf{B}^\dagger \Big(\tfrac{z_4}{\Lambda^2}\, \tilde{\chi}_+\, i\tilde{D}_0 + \tfrac{z_5}{\Lambda^2}\, [i\tilde{D}_0,\, \tilde{\chi}_+] + \tfrac{c_3}{N_c\,\Lambda^3}\, \hat{\chi}_+^2$$
$$+\tfrac{h_1}{N_c^3}\hat{S}^4 + \tfrac{h_2}{N_c^2\Lambda}\hat{\chi}_+\hat{S}^2 + \tfrac{h_3}{N_c\Lambda}\chi_+^0\hat{S}^2 + \tfrac{h_4}{N_c\,\Lambda}\,\chi_+^a\{S^i, G^{ia}\}$$
$$+\tfrac{C_3^A}{N_c^2} u^{ia}\{\hat{S}^2, G^{ia}\} + \tfrac{C_4^A}{N_c^2} u^{ia} S^i S^j G^{ja}$$
$$+\tfrac{D_1^A}{\Lambda^2}\chi_+^0 u^{ia} G^{ia} + \tfrac{D_2^A}{\Lambda^2}\chi_+^a u^{ia} S^i + \tfrac{D_3^A(d)}{\Lambda^2} d^{abc}\chi_+^a u^{ib} G^{ic} + \tfrac{D_3^A(f)}{\Lambda^2} f^{abc}\chi_+^a u^{ib} G^{ic}$$
$$+g_E\, [D_i, F_{+i0}] + \alpha_1\, \tfrac{i}{N_c}\epsilon^{ijk} F^a_{+0i} G^{ia} D_k + \beta_1\, \tfrac{i}{N_c} F^a_{-ij} G^{ia} D_j + \cdots \Big)\mathbf{B}$$