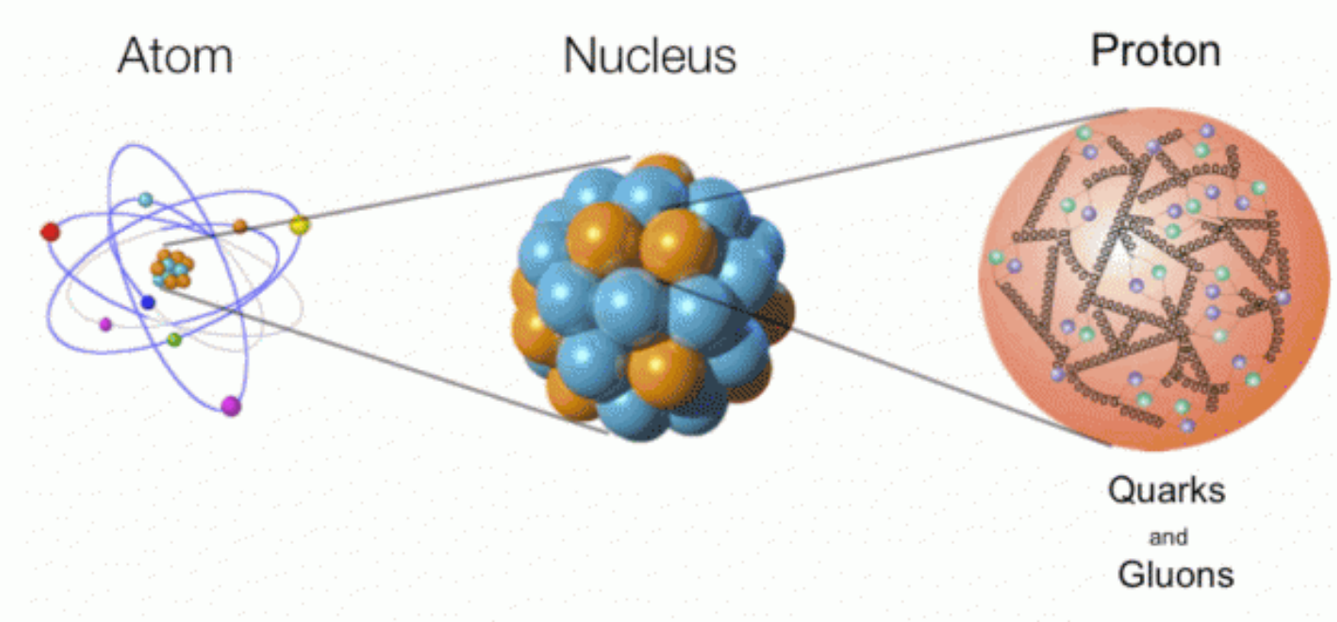
Multiple Particles Interaction in finite volume

Peng Guo

California State University-Bakersfield
&
JPAC

JLab Seminar, Feb.11rd, 2010

Hadron Spectroscopy and QCD: Exploring the nature of matter



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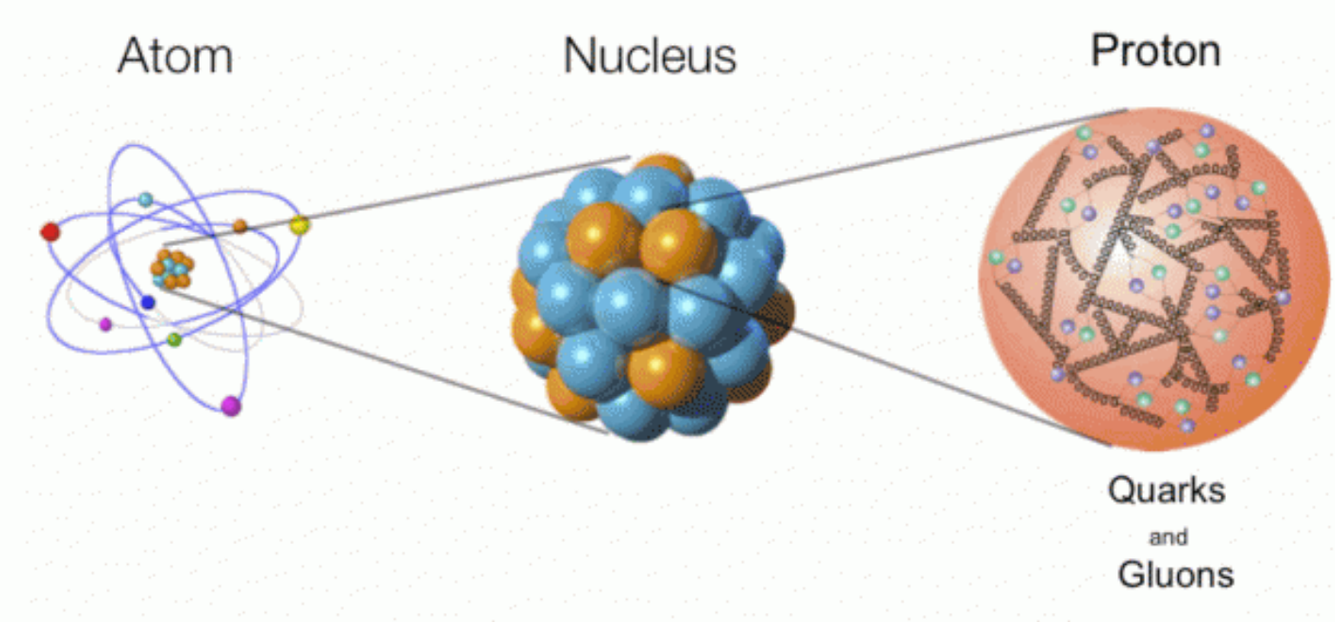

- Understanding the role of gluons and confinement: exotic hadrons and gluonic excitations

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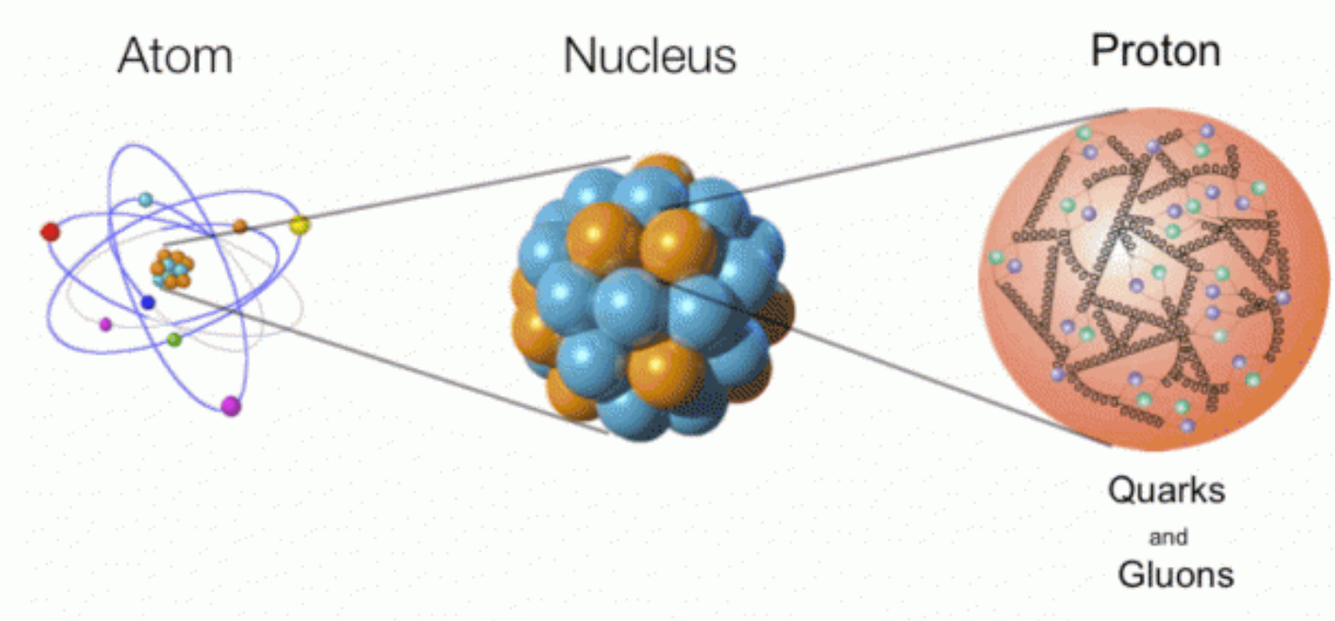

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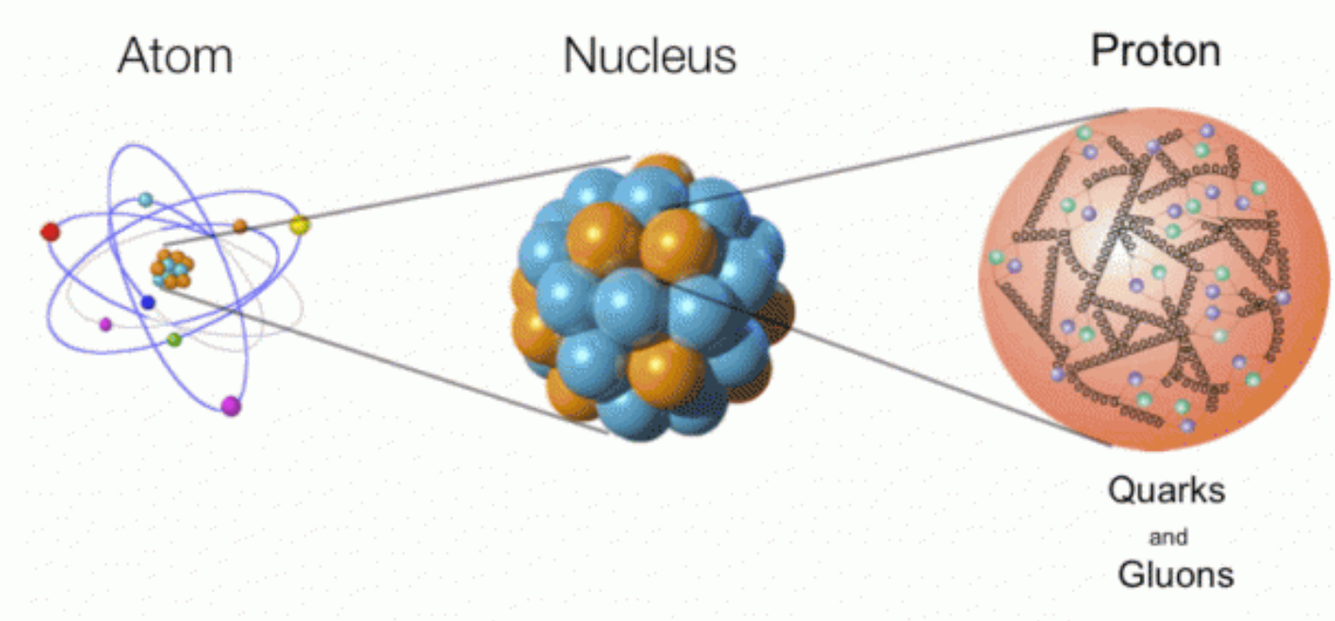

- Understanding the role of gluons and confinement: exotic hadrons and gluonic excitations

- Understanding the dynamics of physical states and formation of resonances and bound states

Hadron Spectroscopy and QCD: Exploring the nature of matter

- Establishing connections between experimental observables and theoretical predictions at heart of matter

long distance behavior:
decays, scattering

Experiments

short distance nature:
quarks and gluons

Theories

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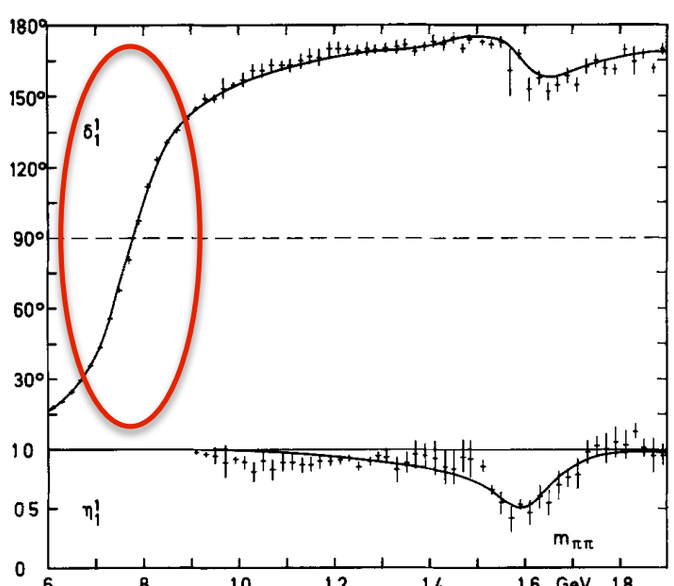
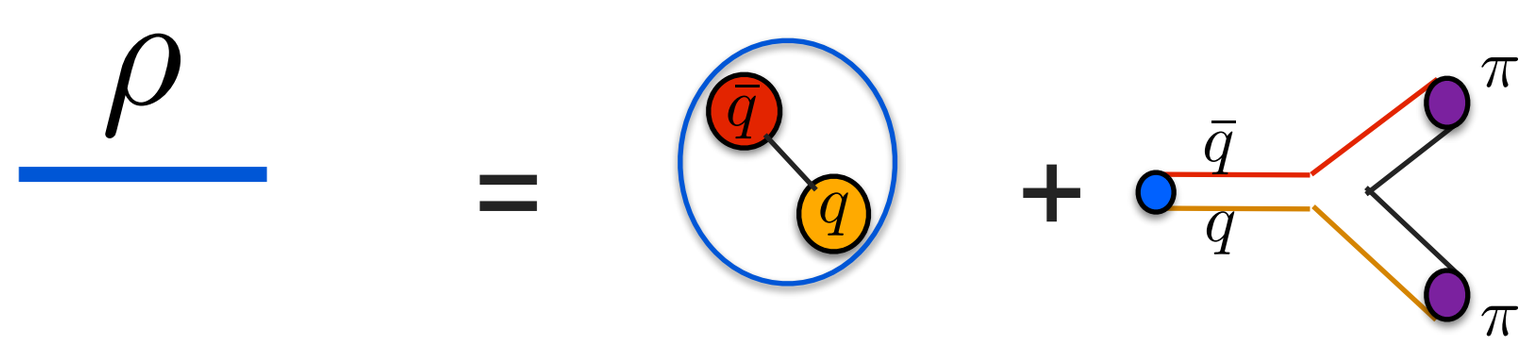




B.Hyams et al.
Nucl.Phys.B64(1973) 134

- Current stand

Hadron Spectroscopy and QCD: Exploring the nature of matter

Current stand

Experiments

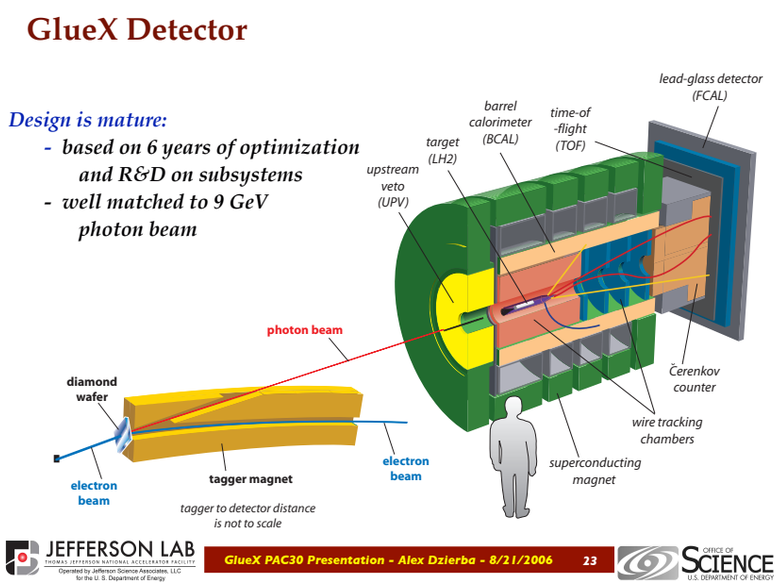
QCD

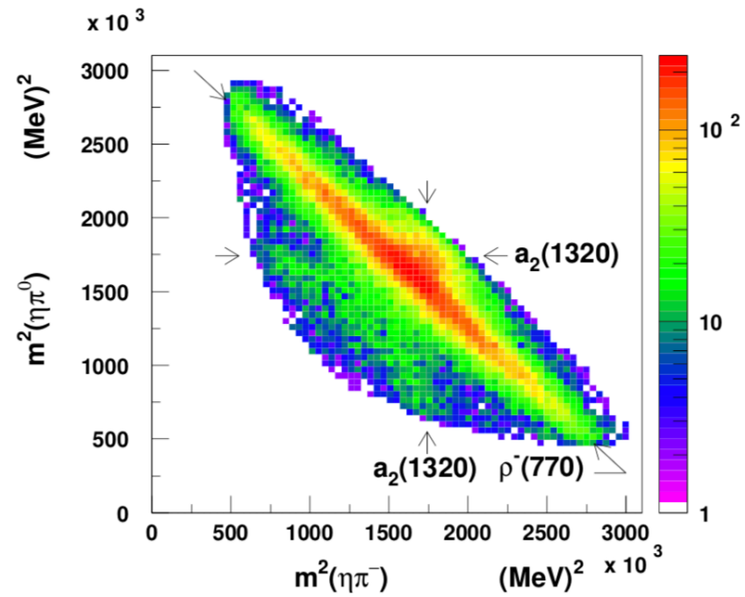
- Compromise

- Compromise

Hadron Dynamics

Hadron Spectroscopy and QCD: Exploring the nature of matter





Compromise

Physics analysis (JPAC)

Hadron Dynamics

Hadron Spectroscopy and QCD: Exploring the nature of matter

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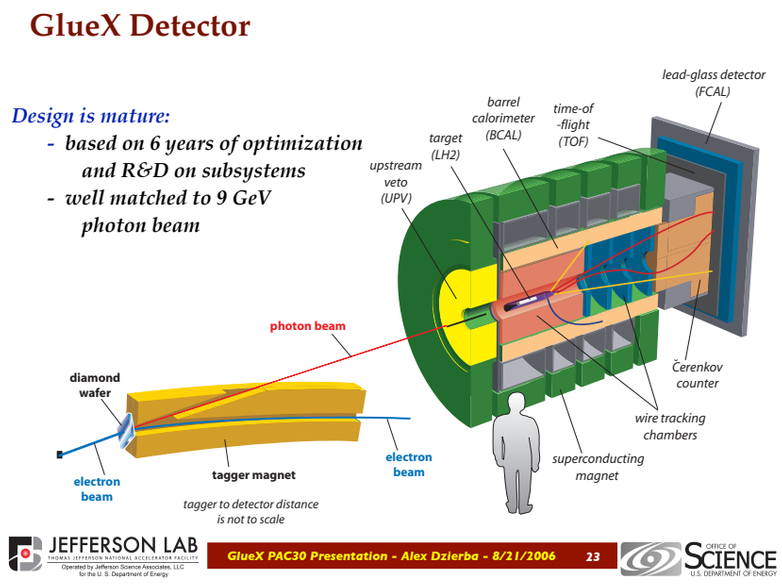

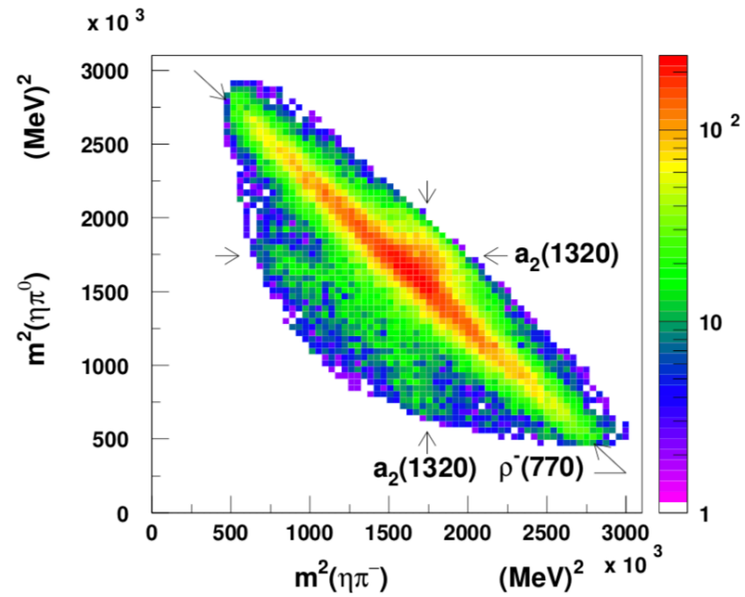

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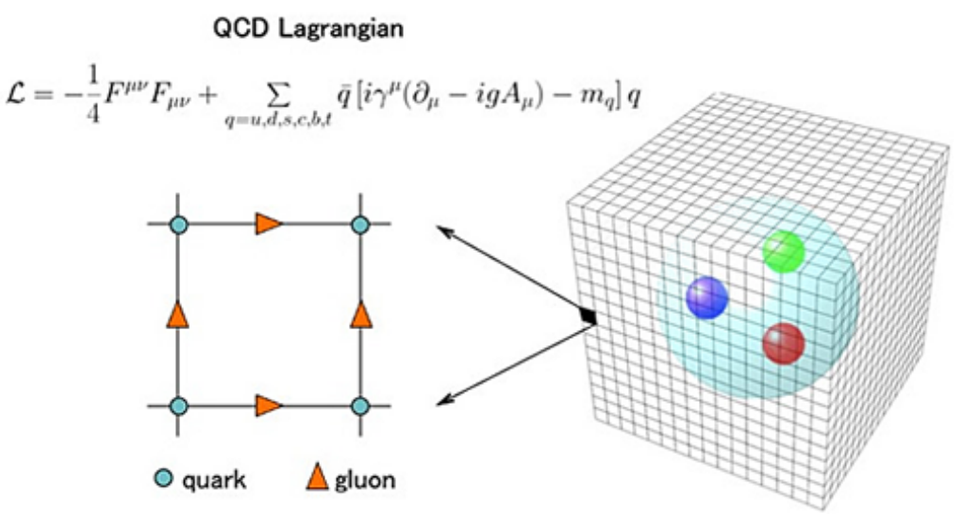

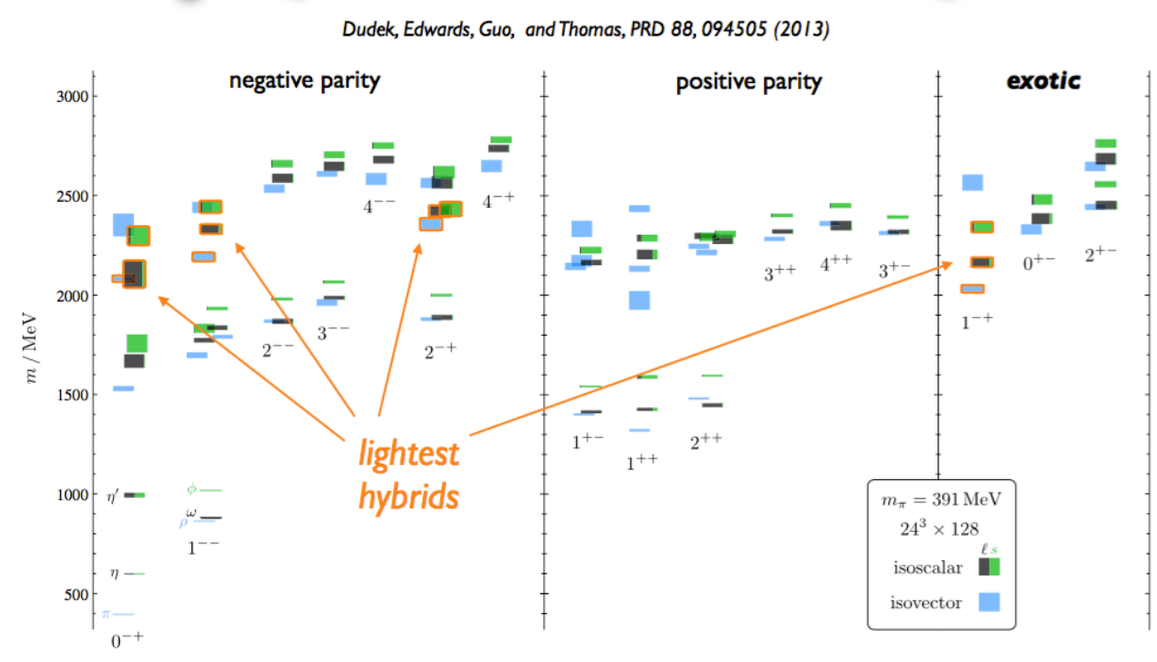

One of Challenges: few-body dynamics and strongly coupled systems

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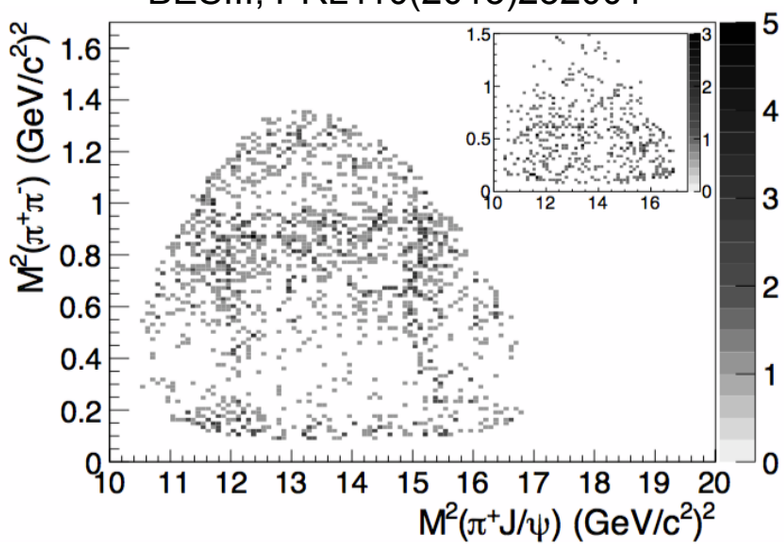

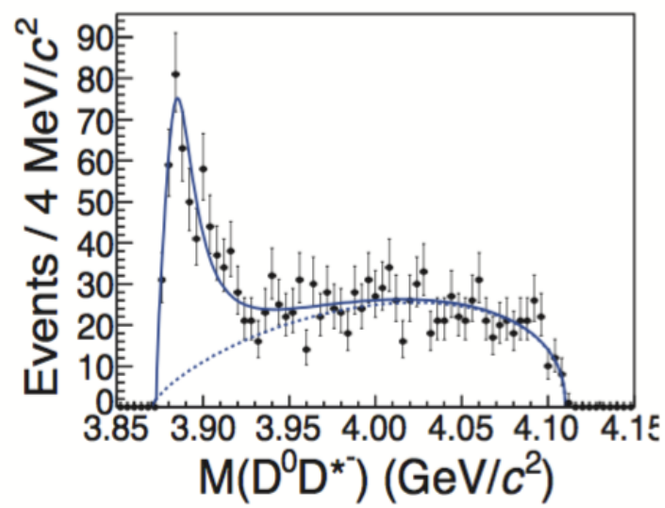

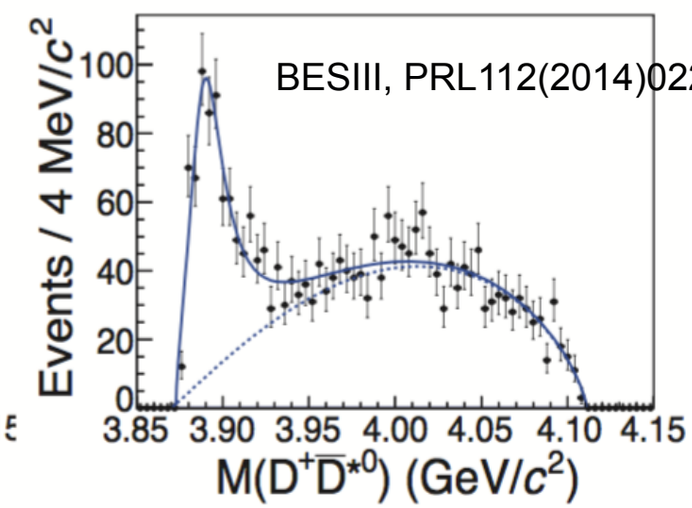

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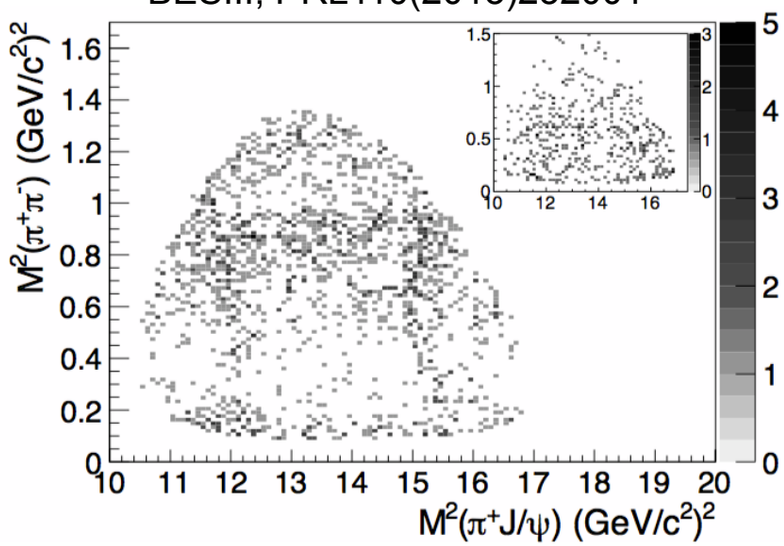

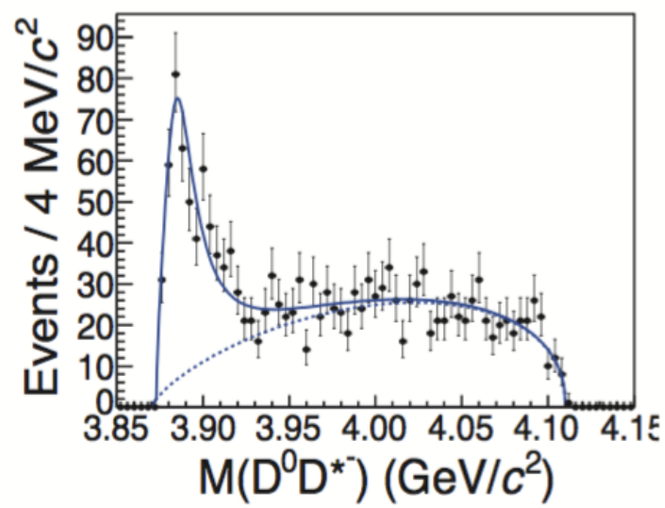

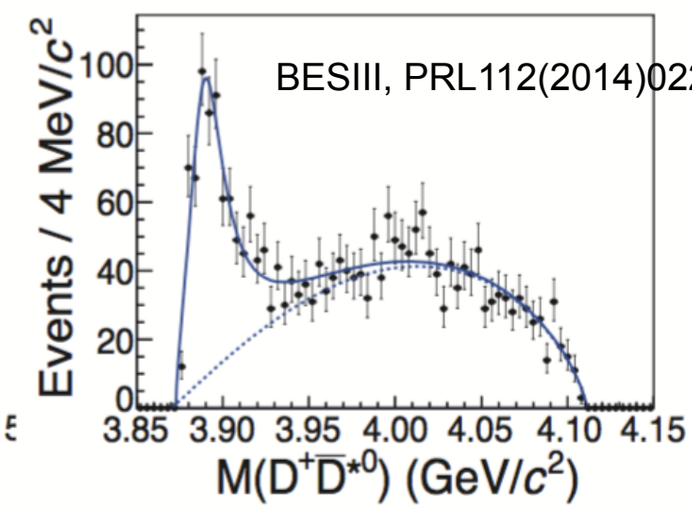

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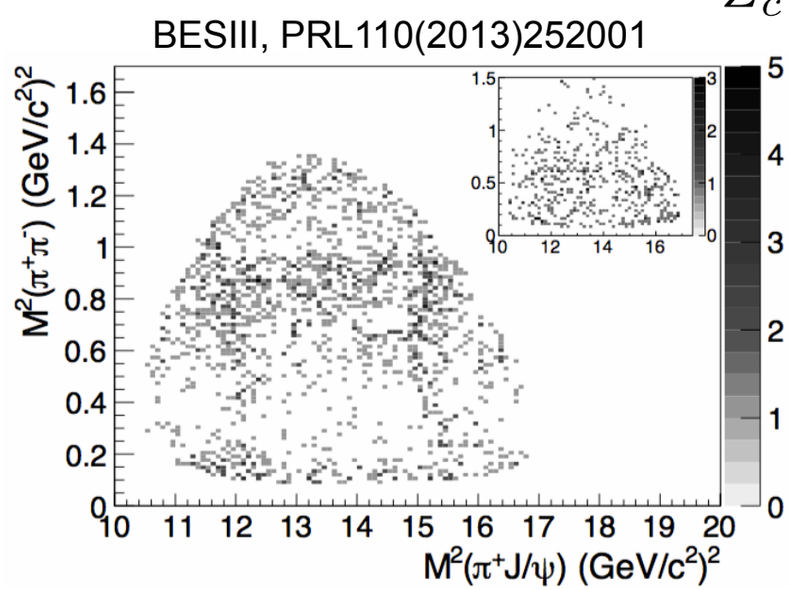

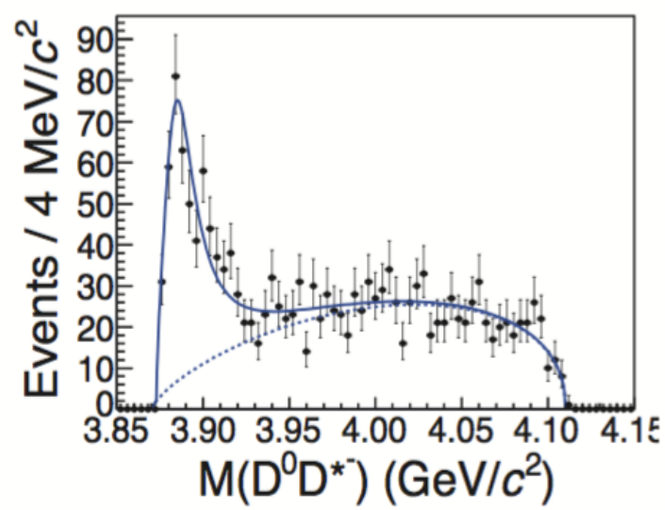

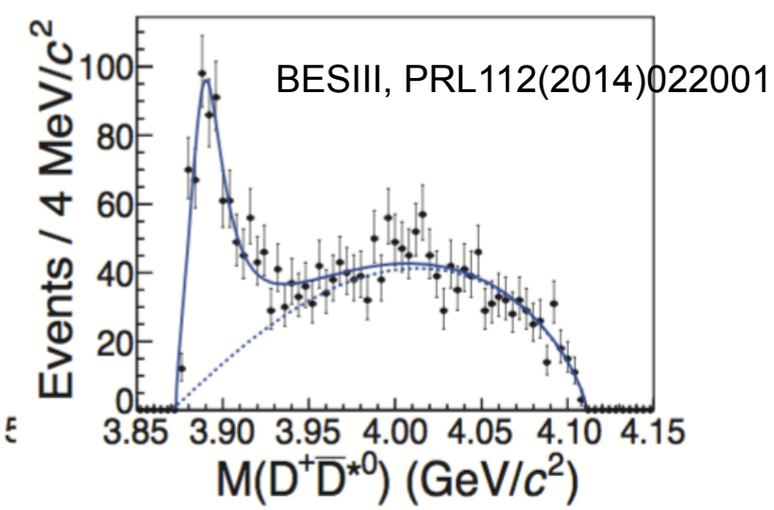

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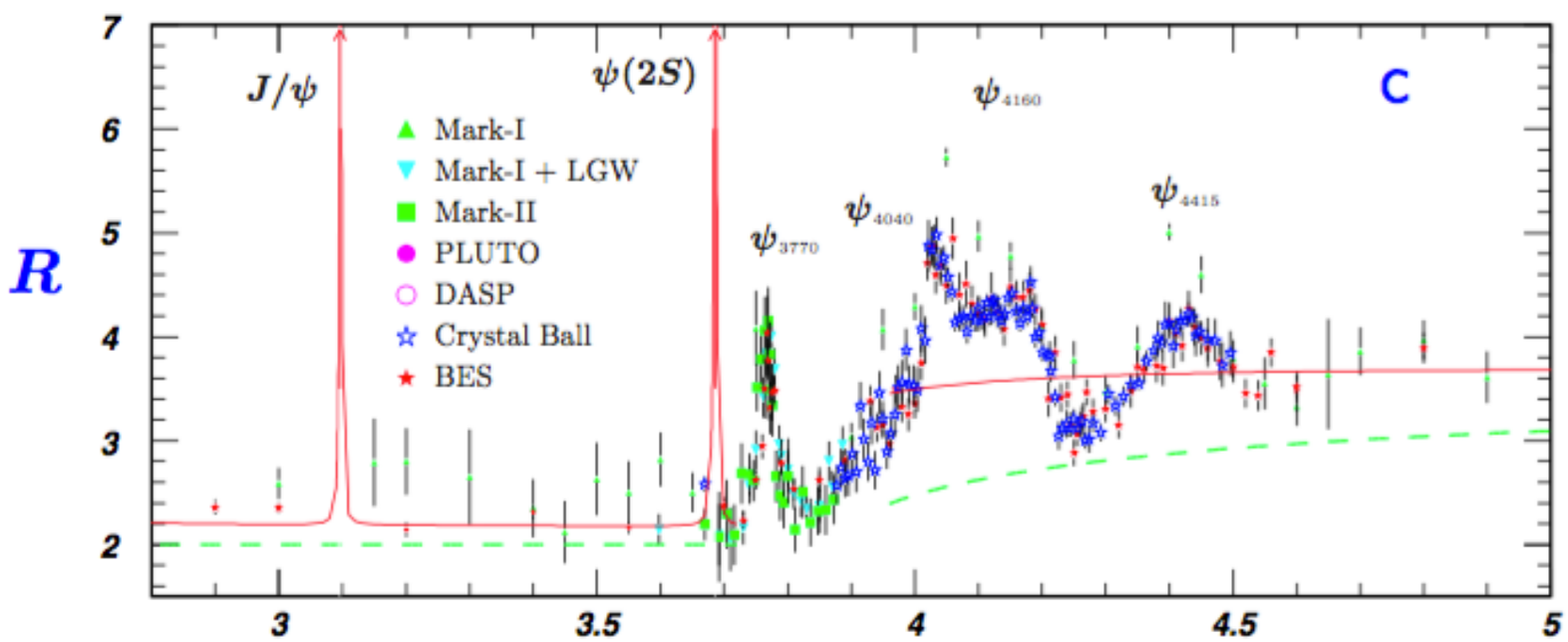

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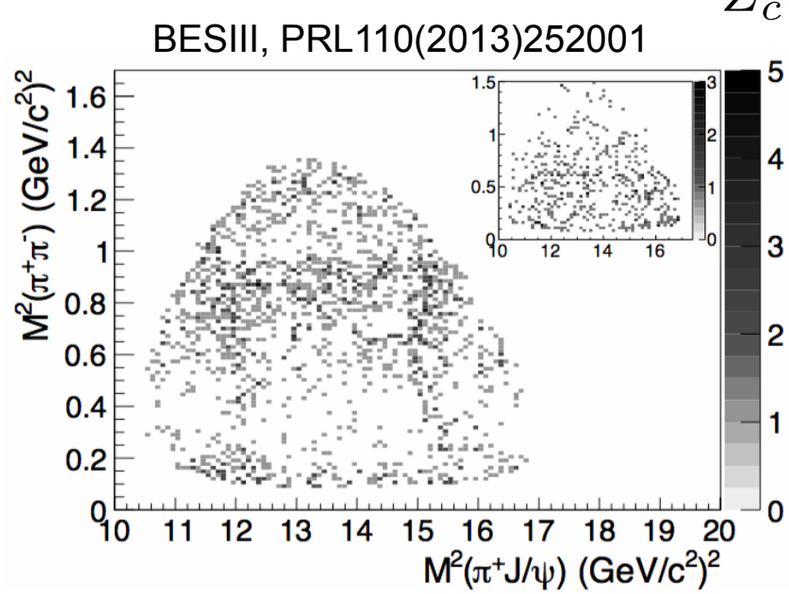

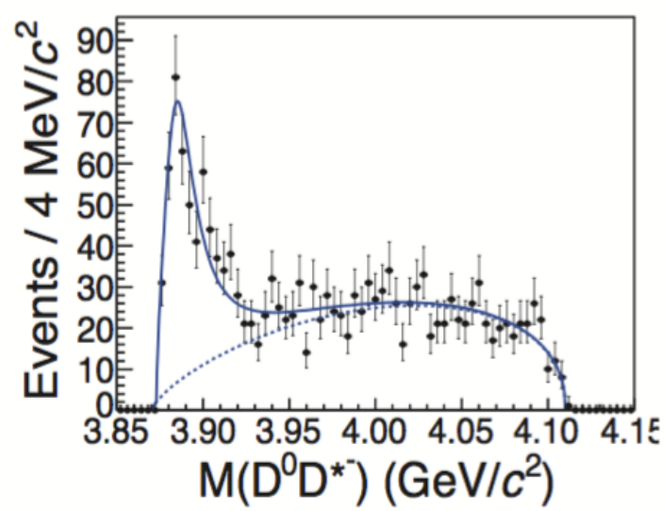

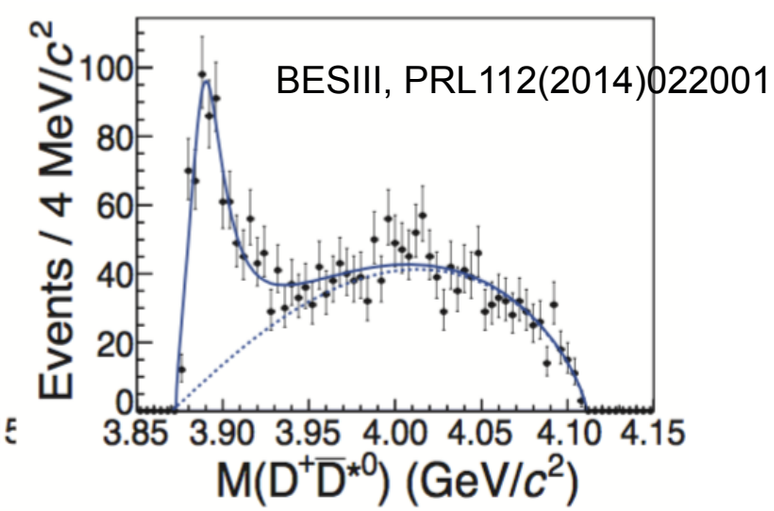

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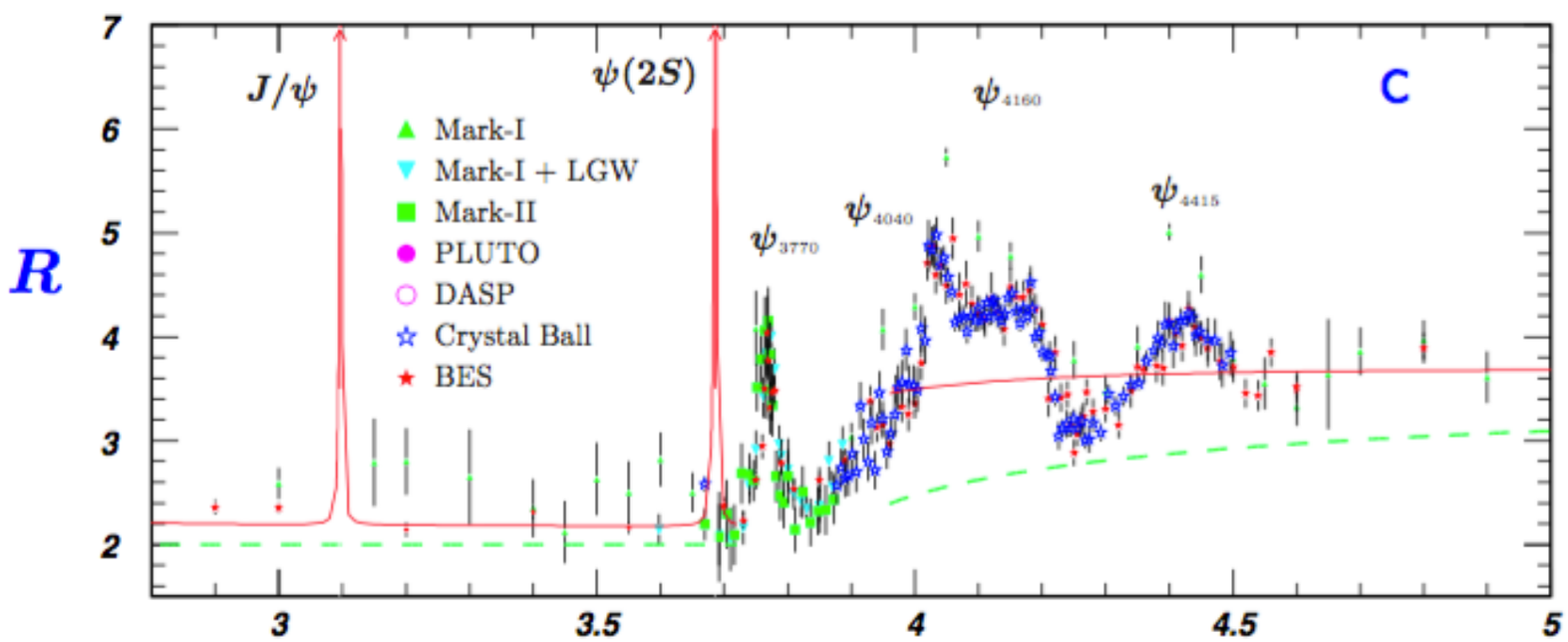
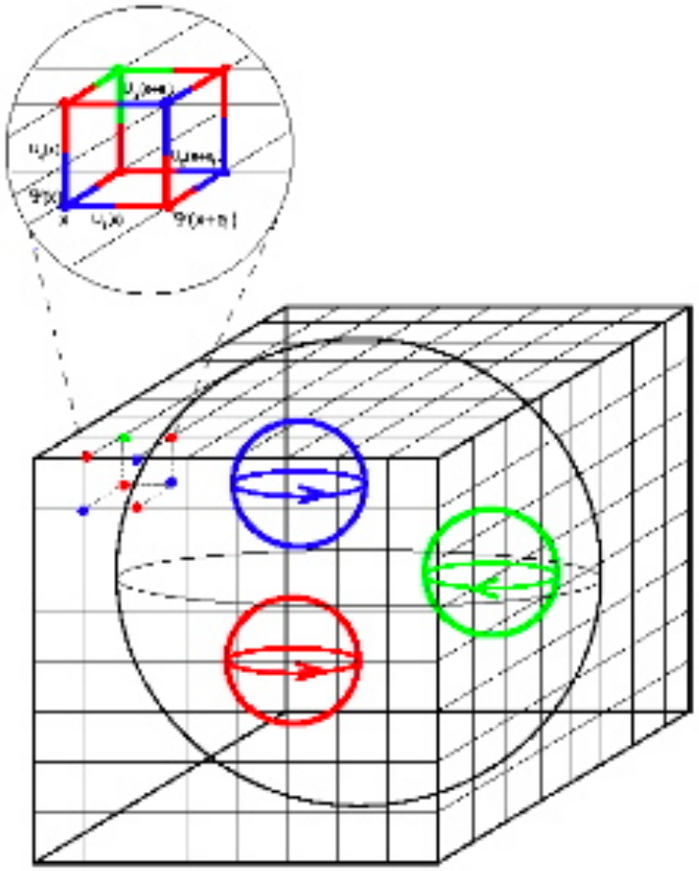


Mission

Few-body in Lattice QCD



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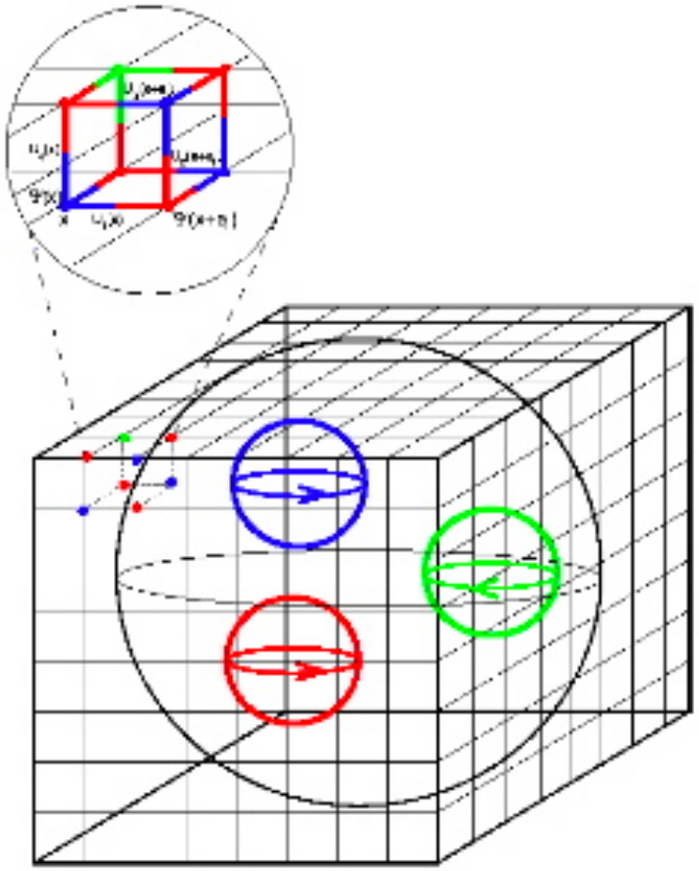

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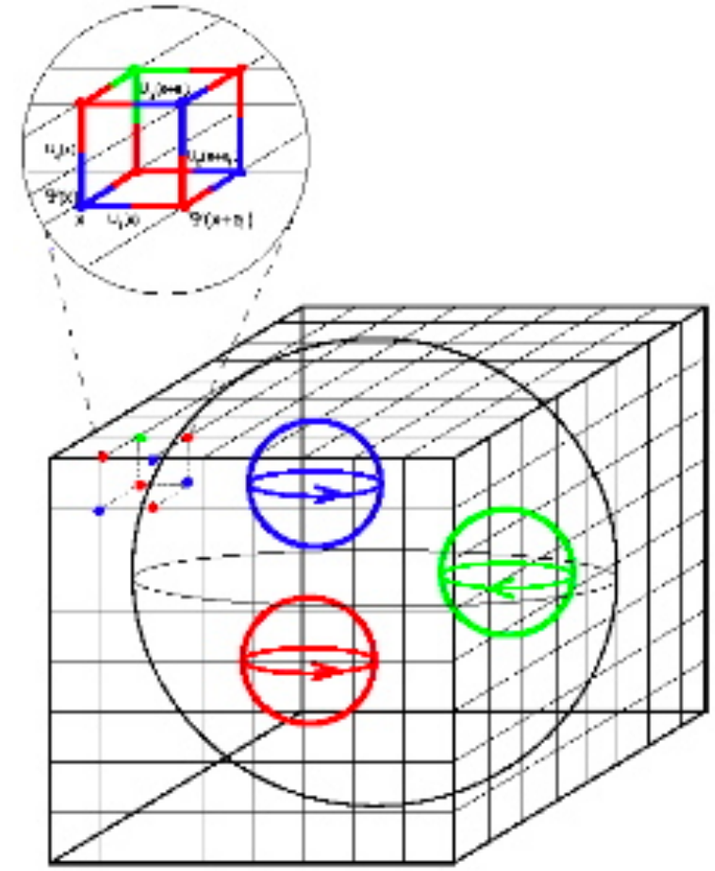

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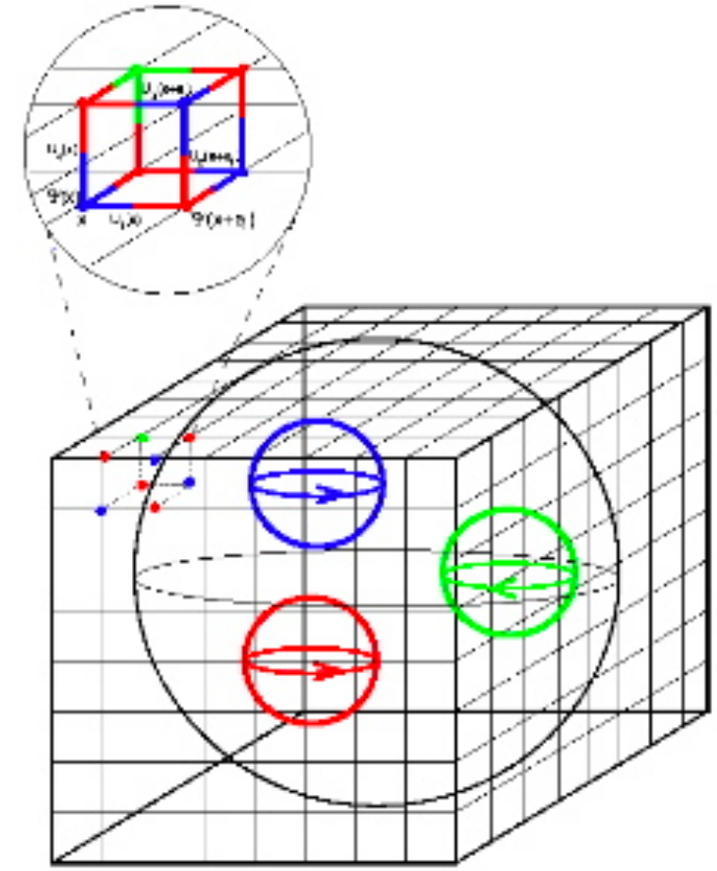

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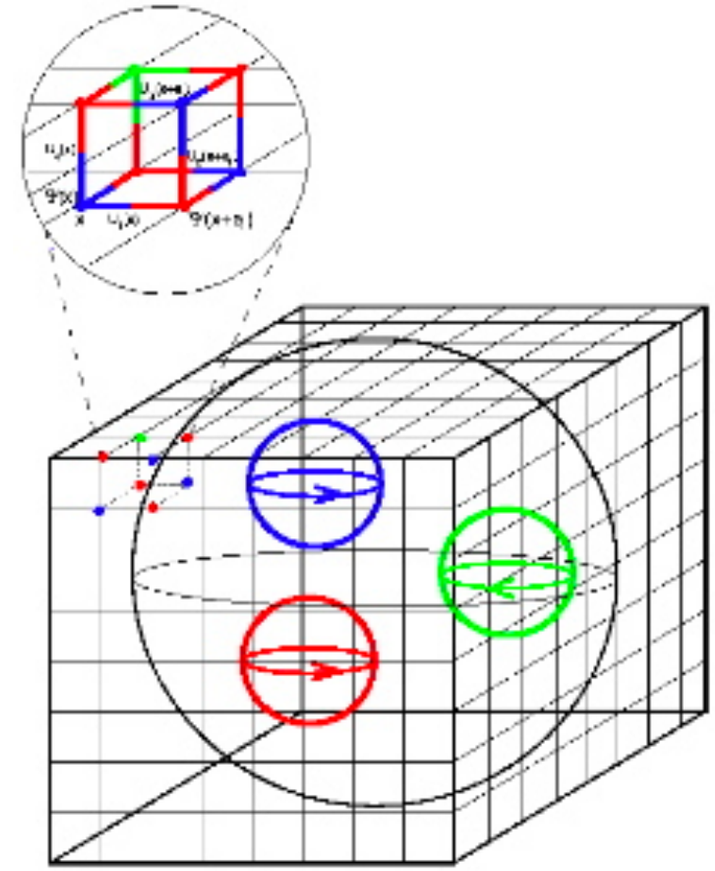

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A few wards about LQCD and Luescher formula

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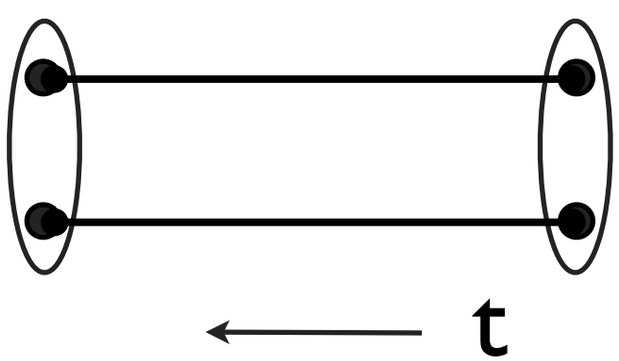

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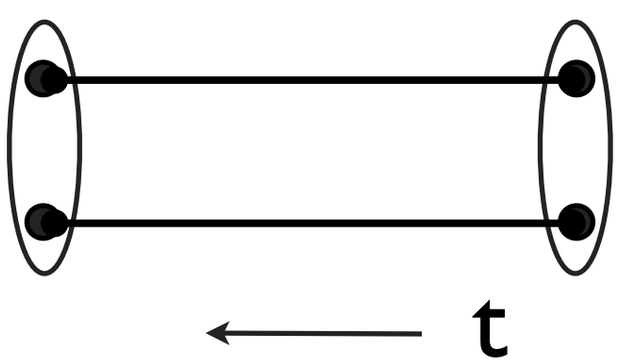

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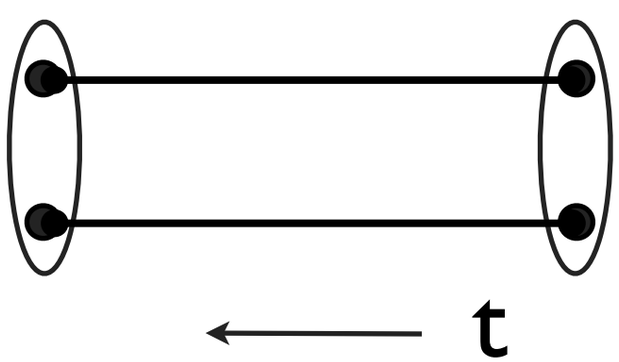

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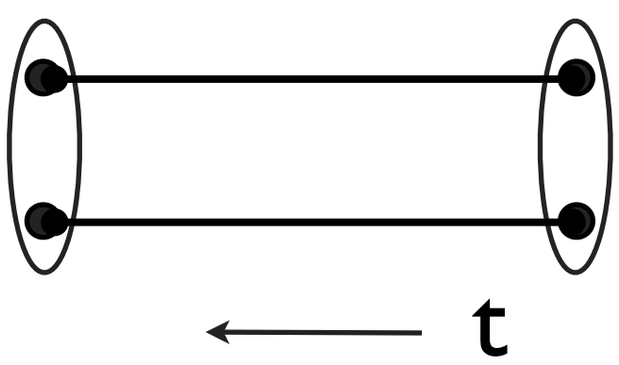
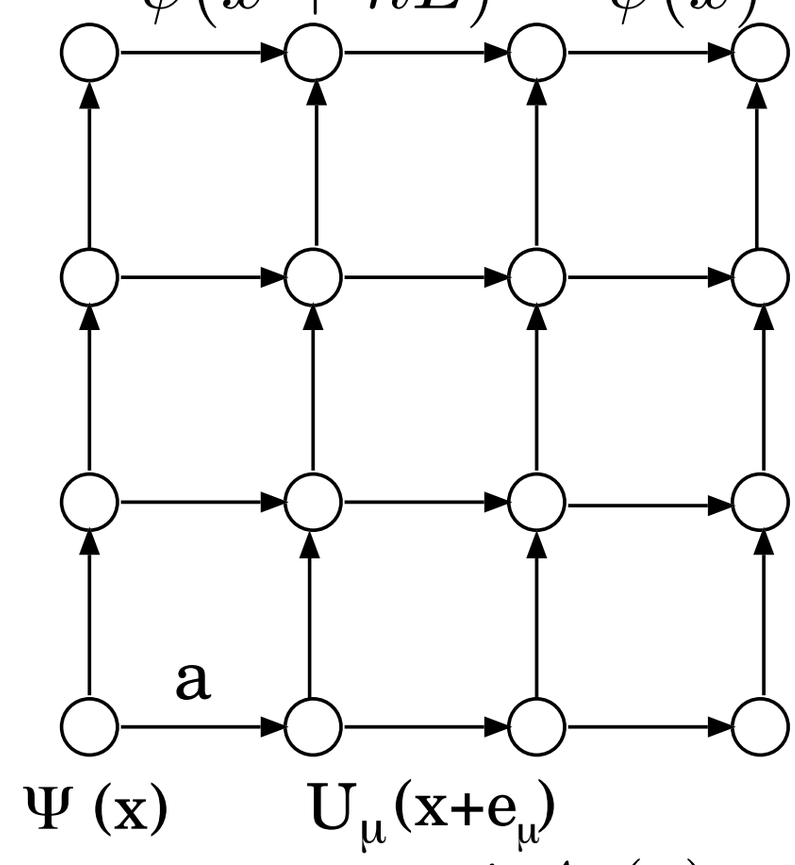


$$\psi(x+nL)=\psi(x)$$



$$U_\mu(n)=e^{iaA_\mu(n)}$$

Spectroscopy from Lattice QCD

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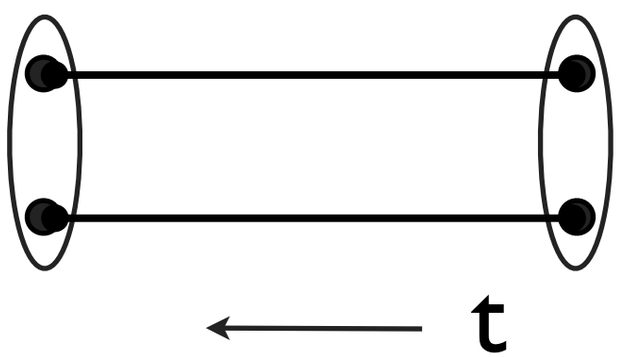

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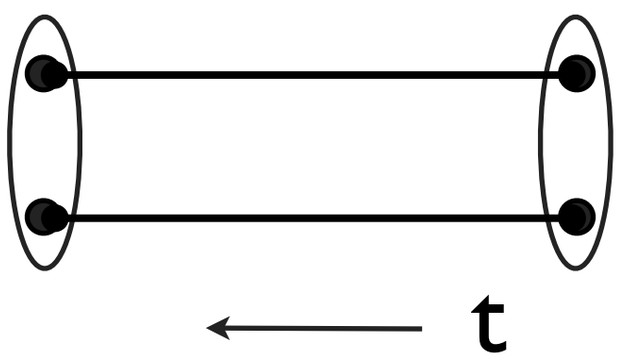

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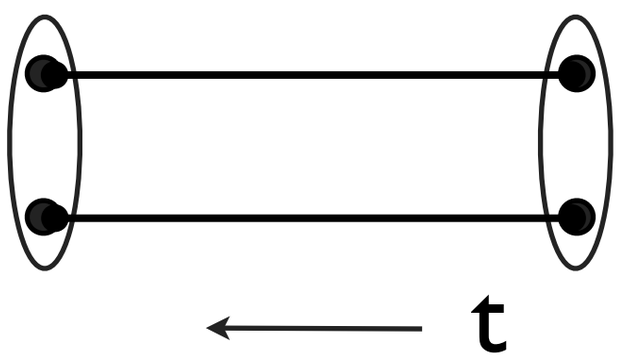

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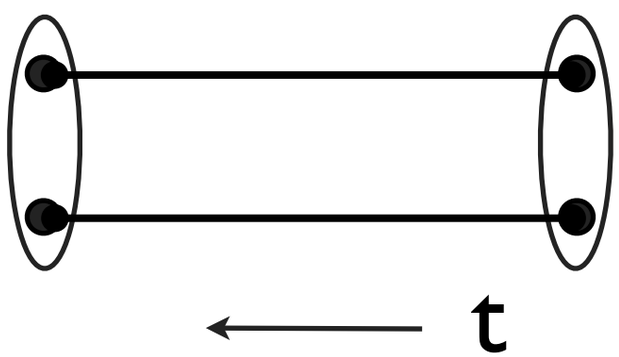

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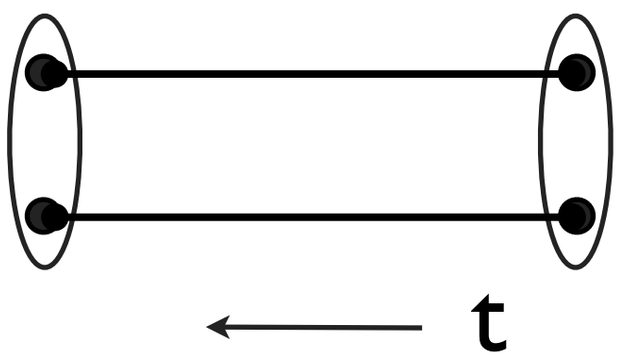

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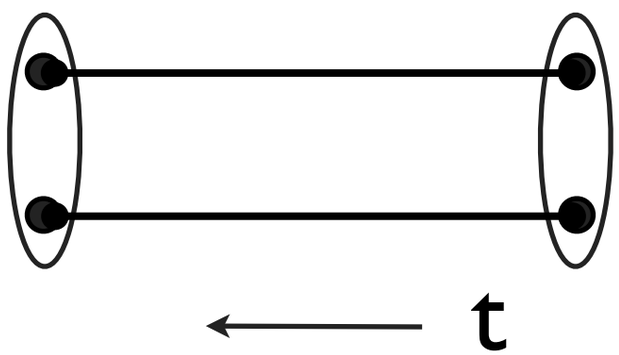

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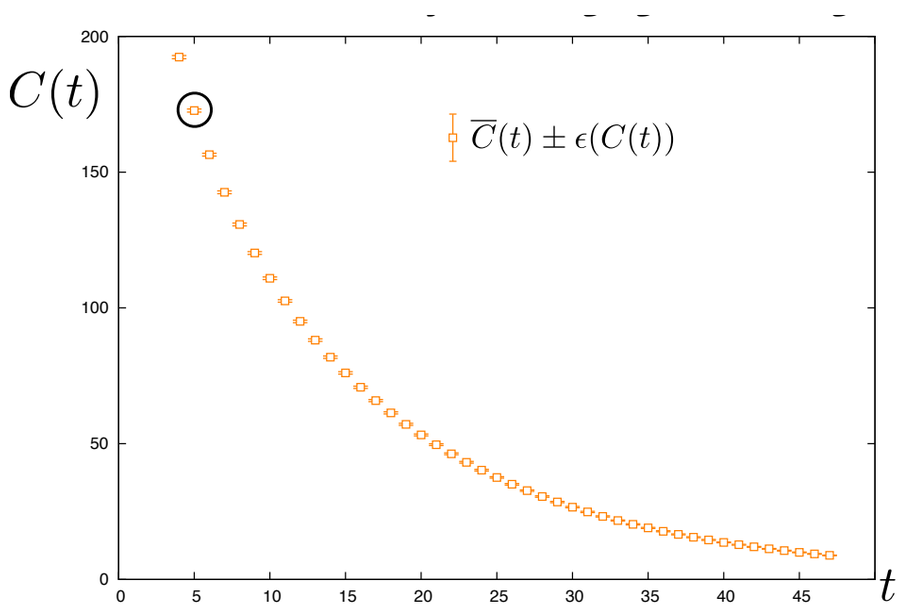

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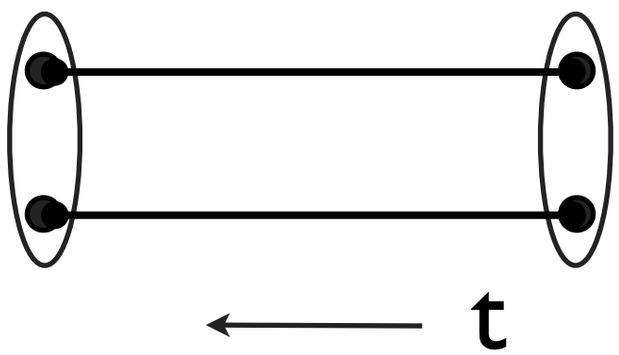

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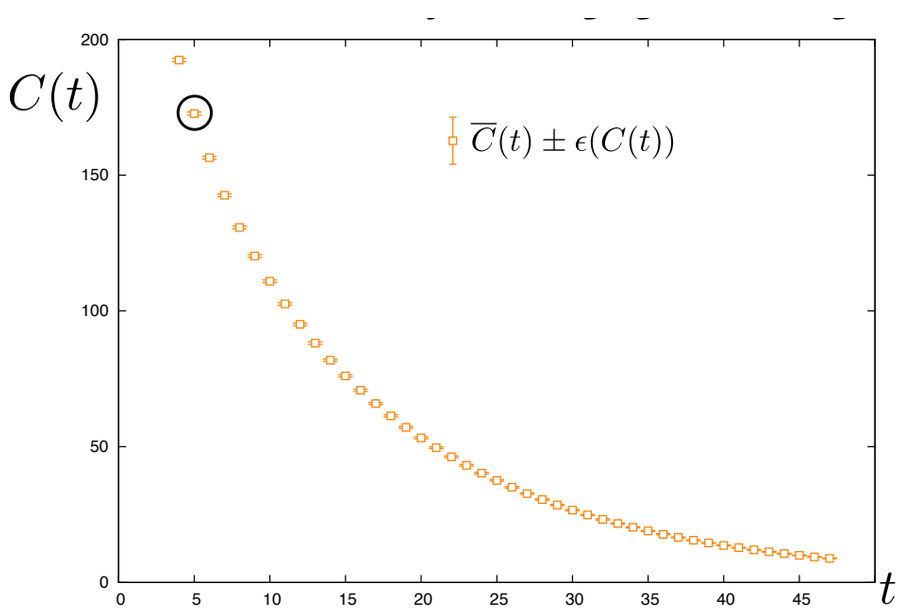
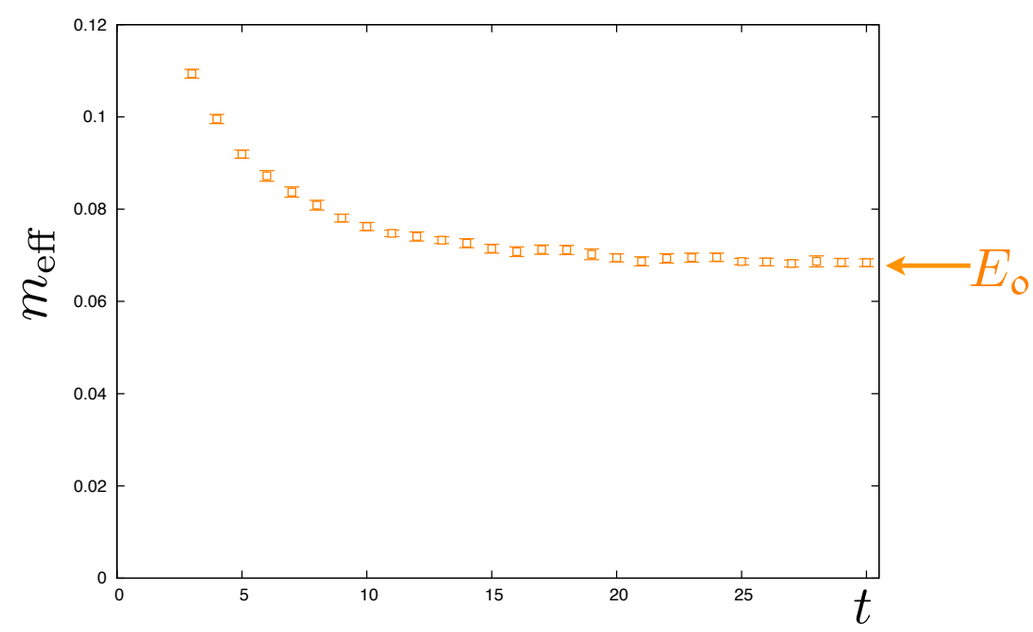
$$m_{\text{eff}}(t) = -\frac{1}{\delta t}\ln\frac{C(t+\delta t)}{C(t)}$$



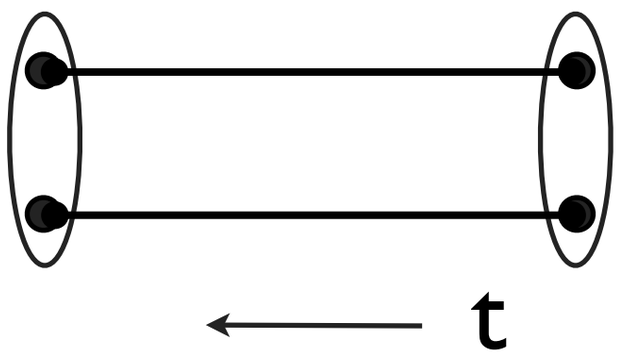


Spectroscopy from Lattice QCD

$$\mathcal{O}(t) = e^{t\hat{H}}\mathcal{O}(0)e^{-t\hat{H}}$$

$$\sum_n |n\rangle\langle n| = 1$$

$$\langle 0|\mathcal{O}(t)\mathcal{O}^\dagger(0)|0\rangle$$



$$\sim \sum_n z_n^* z_n e^{-E_n t}, \; z_n = \langle n|\mathcal{O}^\dagger(0)|0\rangle$$

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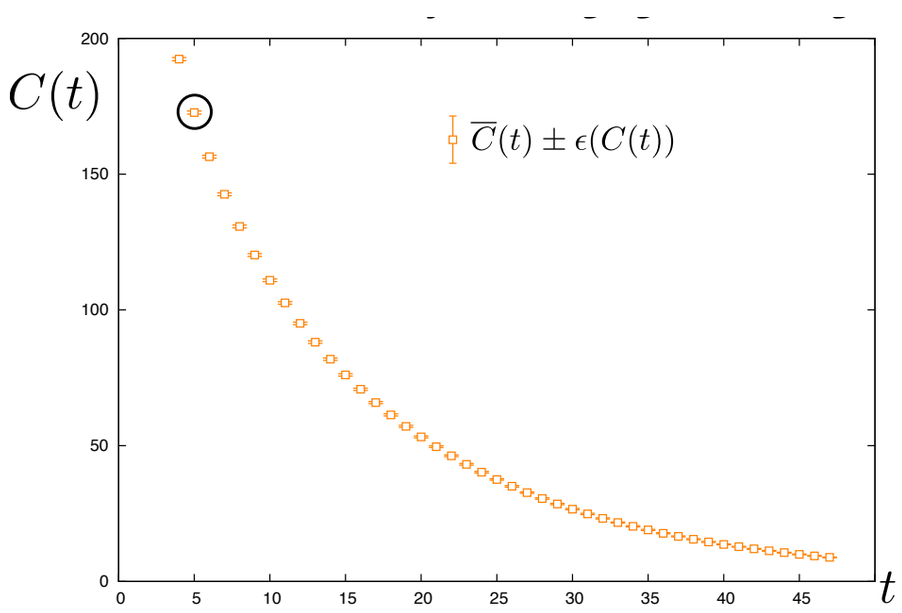

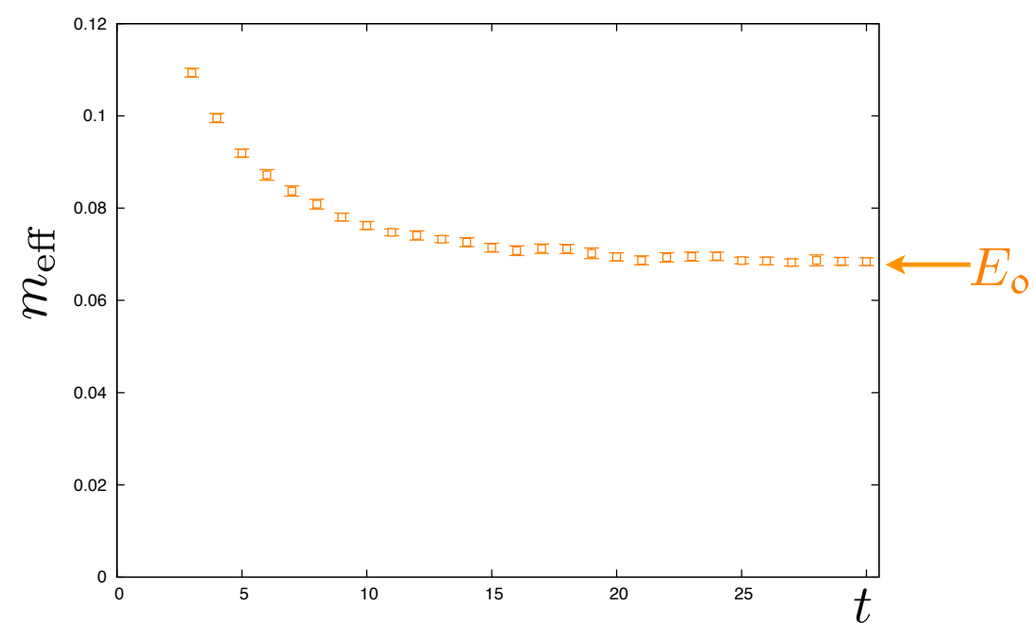

Short Summary of LQCD

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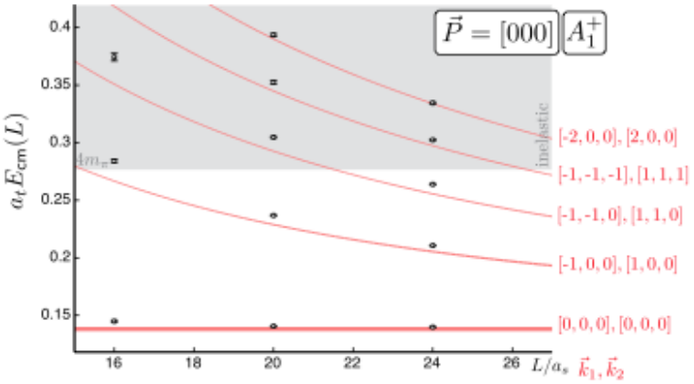

J. J. Dudek *et al.* (Hadron Spectrum Collaboration) Phys. Rev. D **86**, 034031 (2012).

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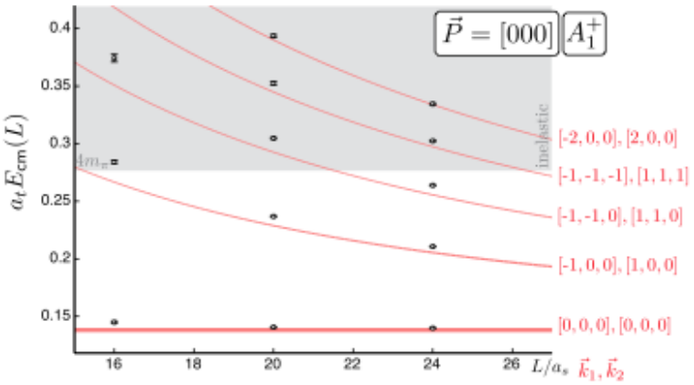

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- Periodic boundary condition: discrete energies

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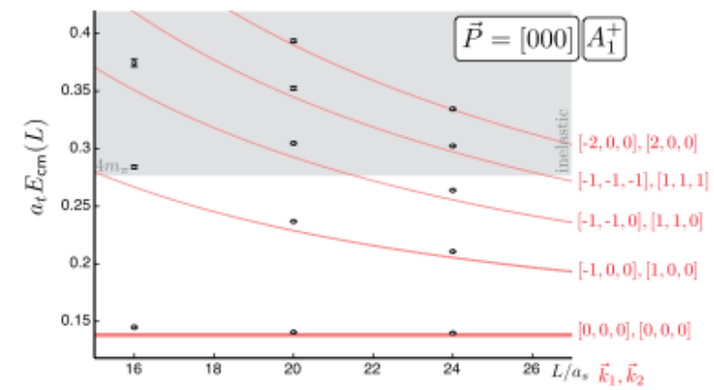

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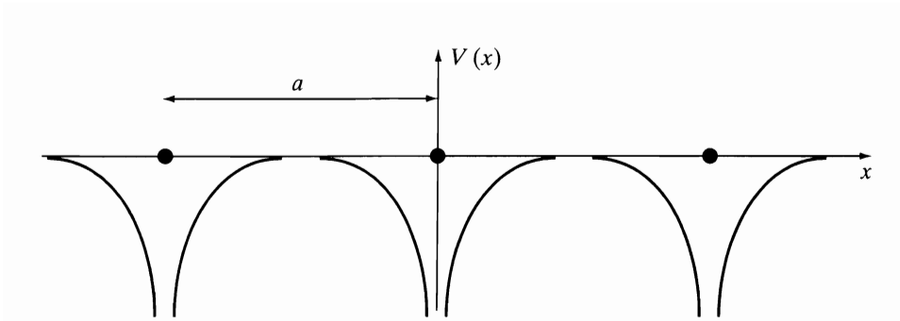
- Periodic boundary condition: discrete energies

- Euclidean space: need relation between spectrum and scattering amplitudes

- Taking advantage of periodic boundary condition

- Taking advantage of periodic boundary condition



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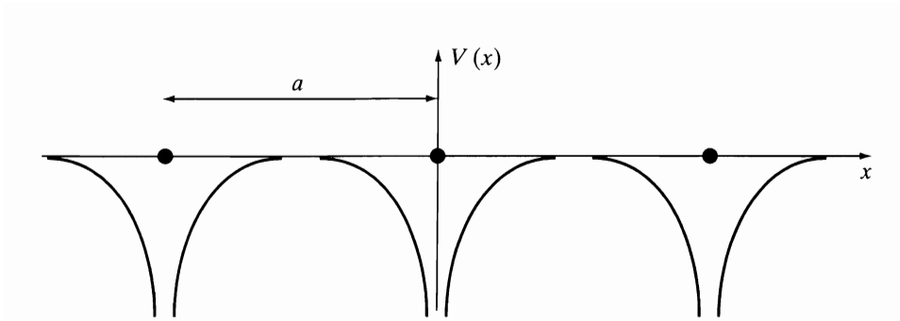

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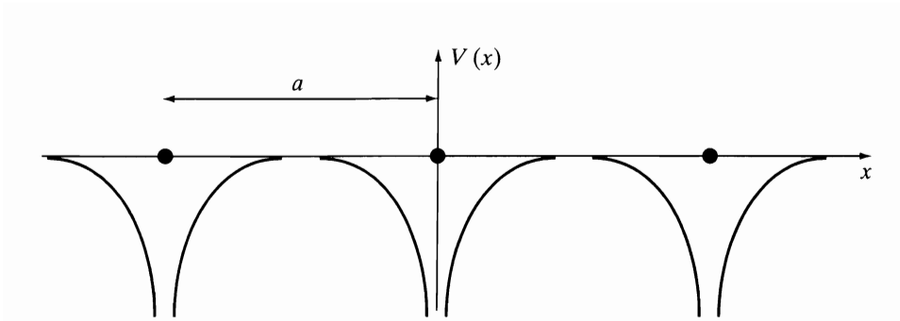
Luescher's formula:



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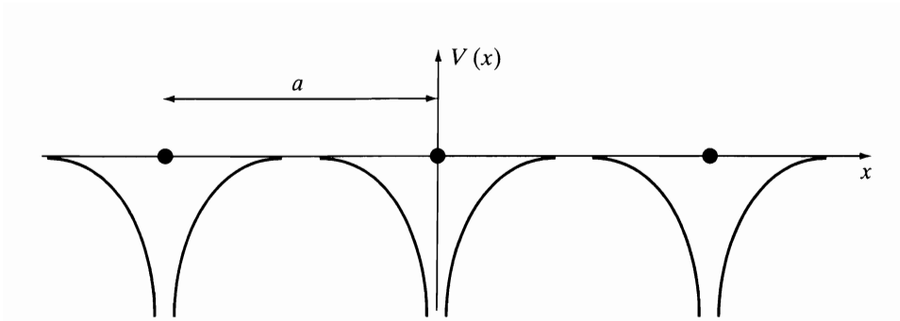

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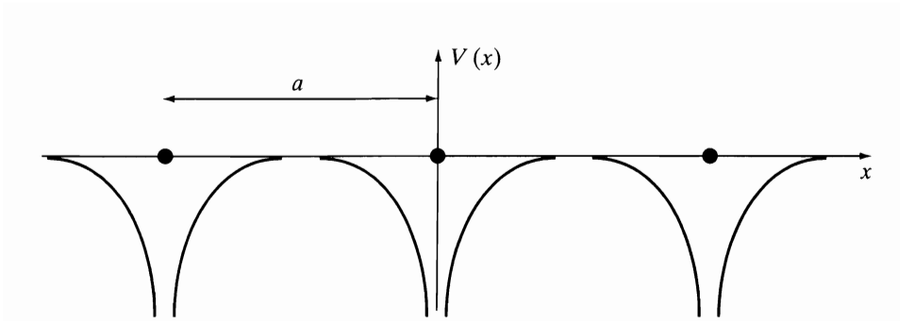

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Short-range Dynamics

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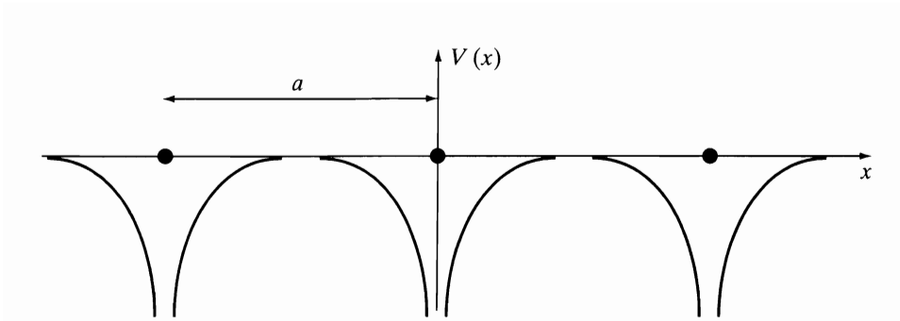

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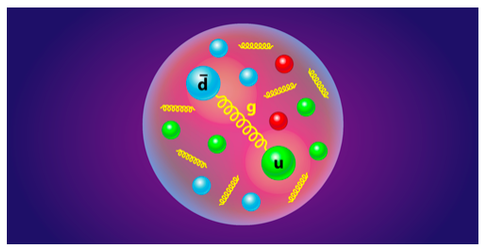
Short-range Dynamics

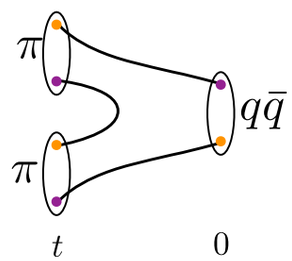
Long-range correlation in periodic lattice structure

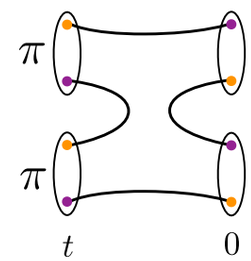
A few wards about LQCD and Luescher formula

- Access to two-body hadrons scattering in LQCD

Access to two-body hadrons scattering in LQCD







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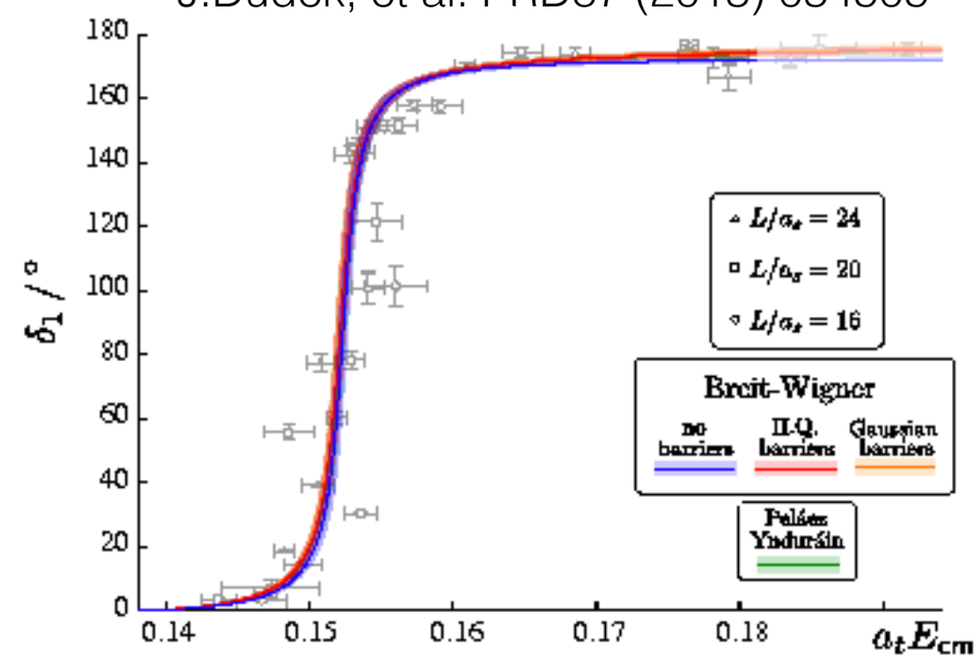

Access to two-body hadrons scattering in LQCD

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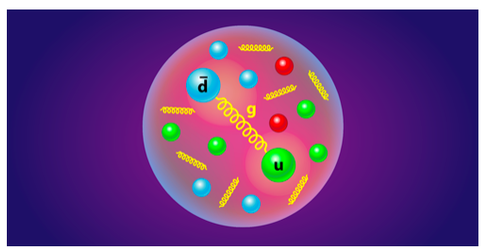
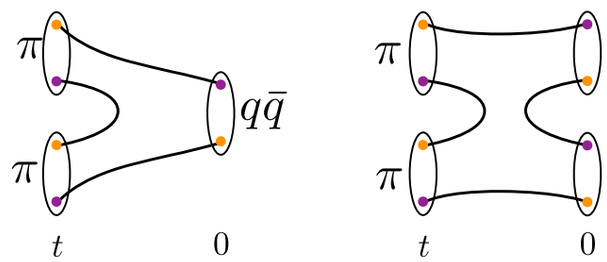
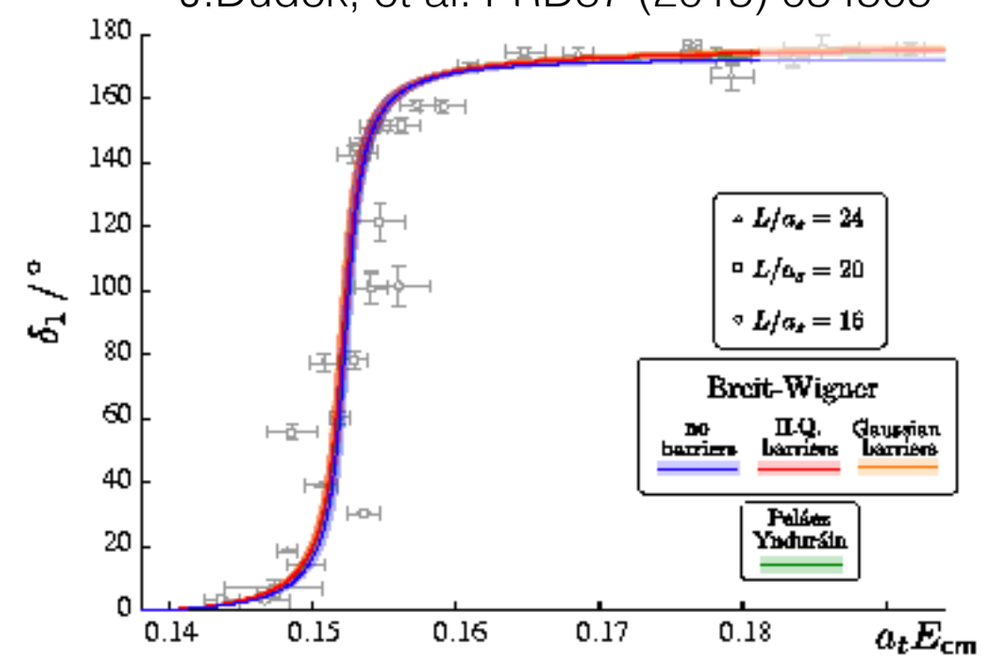
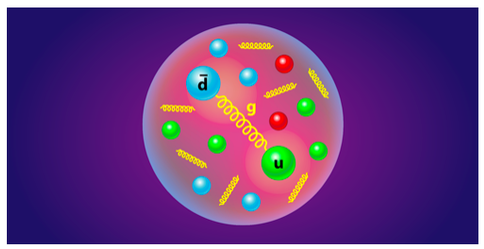


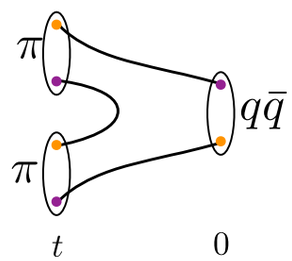


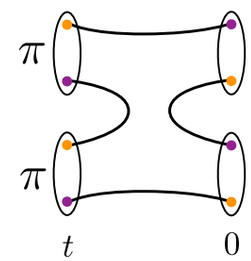


Luescher's formula is result of factorizing physics at two separate scales:

Access to two-body hadrons scattering in LQCD







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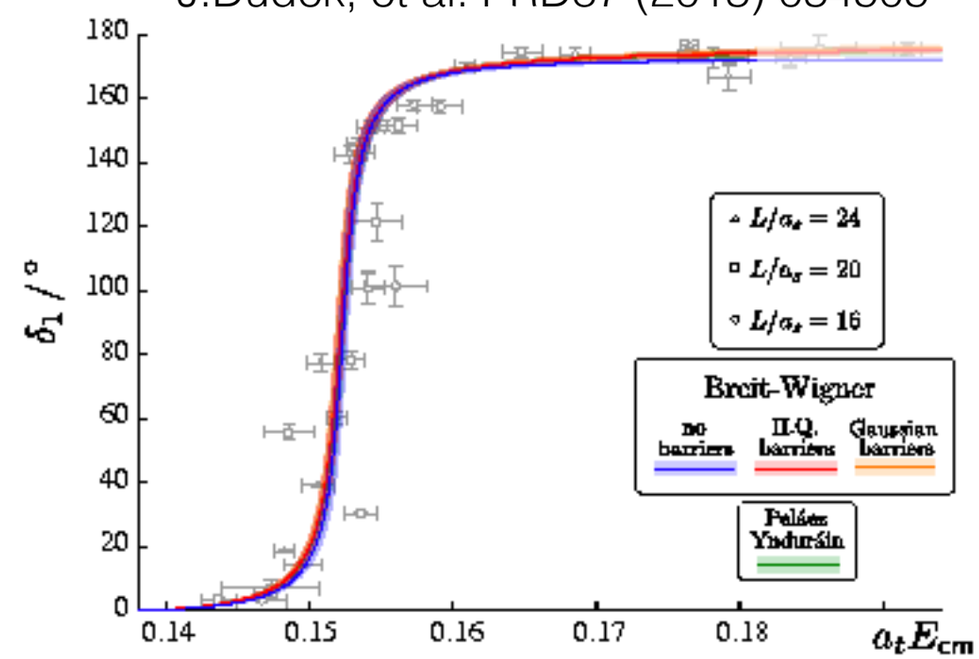

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- Short-range interaction within a box -> scattering phase shift

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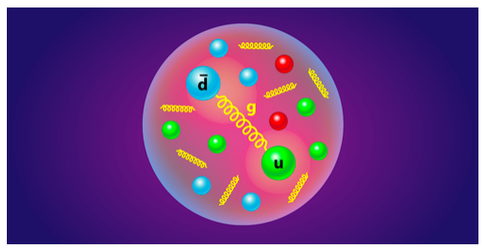

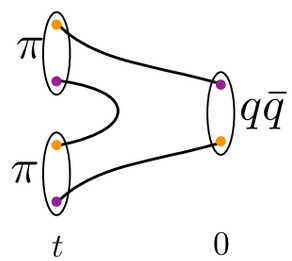

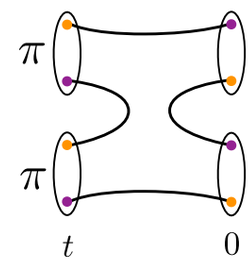

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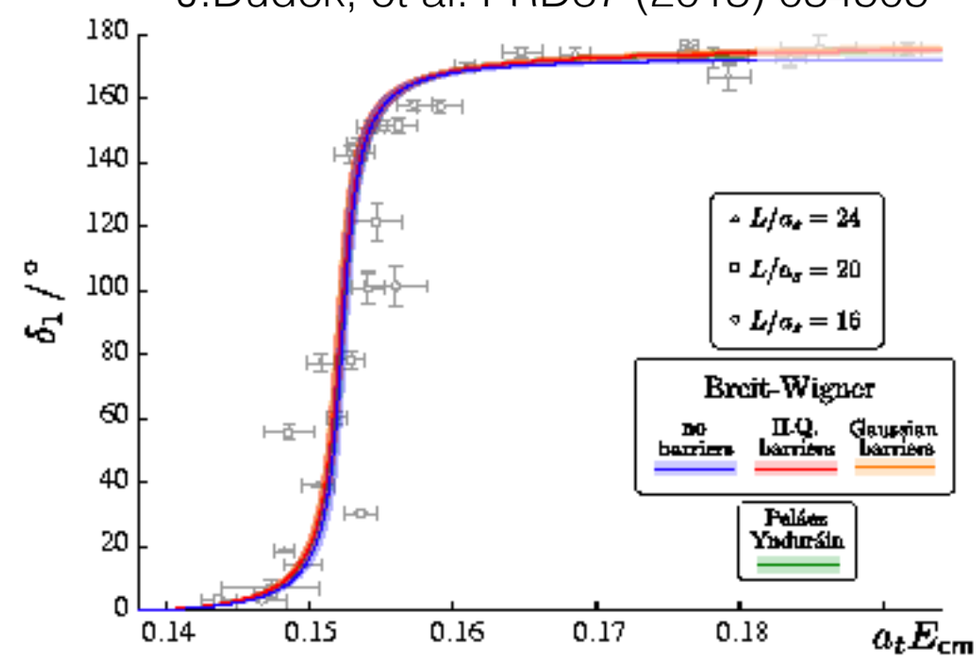

Luescher's formula is result of factorizing physics at two separate scales:

- Short-range interaction within a box -> scattering phase shift
- Long-range correlation between boxes -> zeta function

Mission

Few-body in Lattice QCD

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A ideal framework for Few-body in LQCD?

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Few-body in Lattice QCD

A ideal framework for Few-body in LQCD?

- ☑ Describe relativistic strongly coupled few-body systems
- ☑ Satisfy periodic boundary condition -> discrete energy spectra
- ☑ Separate short-range dynamics within a box and long-range correlations between boxes
- ☑ Mathematically transparent -> user friendly

Variational approach to finite volume few-body problem: a not so perfect framework

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- ☑ **A non-relativistic framework**

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- ☑ Periodic boundary condition -> discrete energy spectra

Variational approach to finite volume few-body problem: a not so perfect framework

- A non-relativistic framework

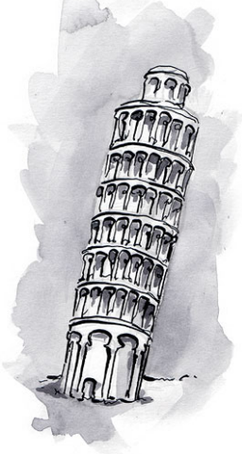
- Periodic boundary condition -> discrete energy spectra

- Clear demonstration of factorization of physics at two scales: short-range dynamics and long-range correlations

Variational approach to finite volume few-body problem: a not so perfect framework

$$[k^2 + \nabla^2 - V^{(L)}(\mathbf{r})]\Phi(\mathbf{r}) = 0$$

Variational approach to finite volume few-body problem: a not so perfect framework



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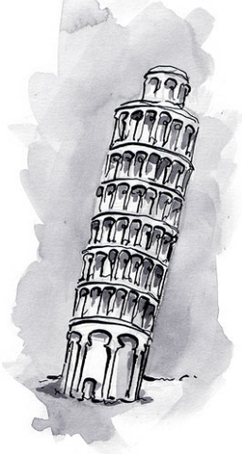

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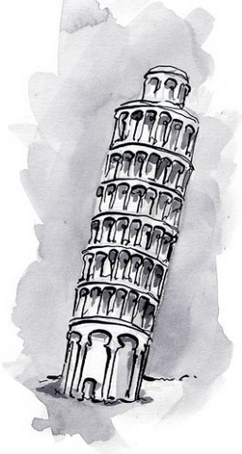

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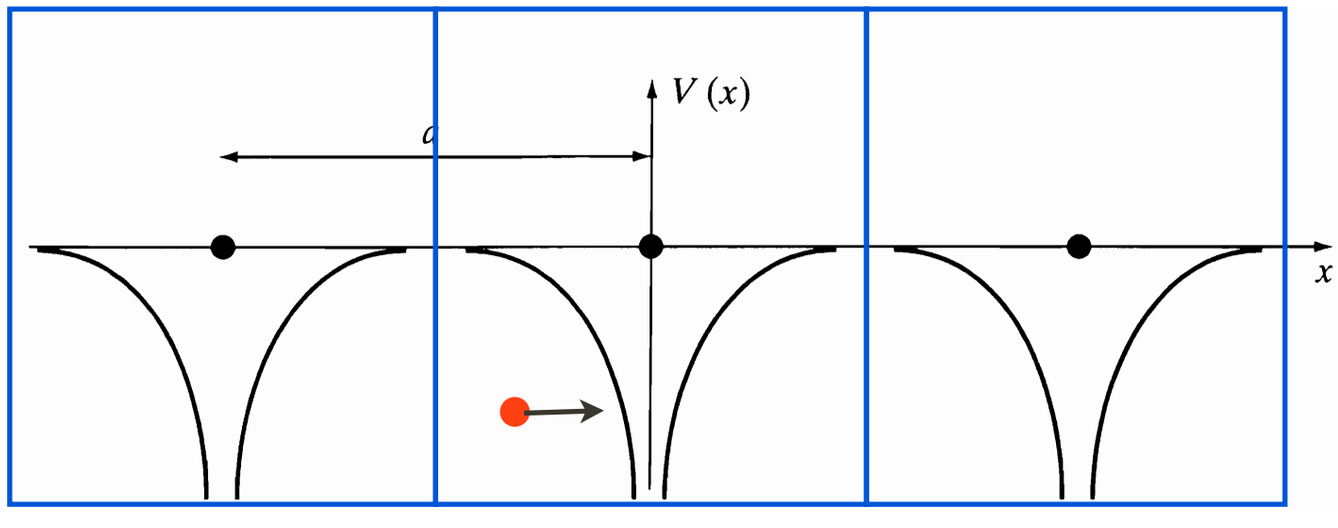

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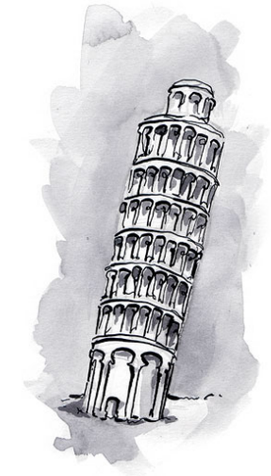

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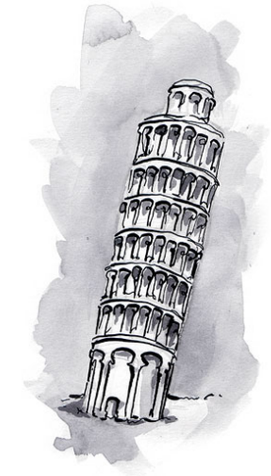

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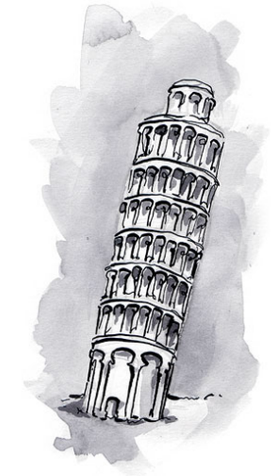

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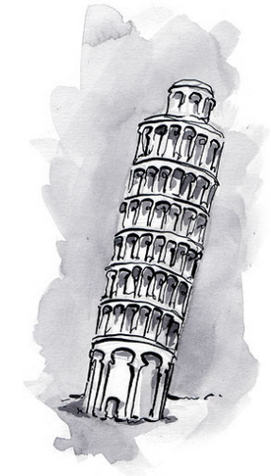

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- **Secular equations**

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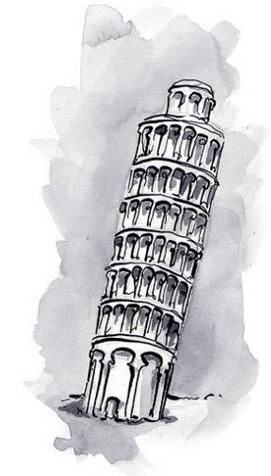

- **Periodic basis and variational principle**

$$\Phi(\mathbf{r}) = \sum_{[J]} c_{[J]} \Phi_{[J]}(\mathbf{r}) \qquad \Phi_{[J]}(\mathbf{r}) = \Phi_{[J]}(\mathbf{r} + \mathbf{n}L)$$

$$\longrightarrow \quad \Lambda = \sum_{[J'],[J]} c^*_{[J']} \langle \Phi_{[J']} | k^2 + \nabla^2 - V^{(L)} | \Phi_{[J]} \rangle c_{[J]}$$

$$\delta\Lambda = 0$$

- **Secular equations**

$$\sum_{[J]} \langle \Phi_{[J']} | k^2 + \nabla^2 - V^{(L)} | \Phi_{[J]} \rangle c_{[J]} = 0$$

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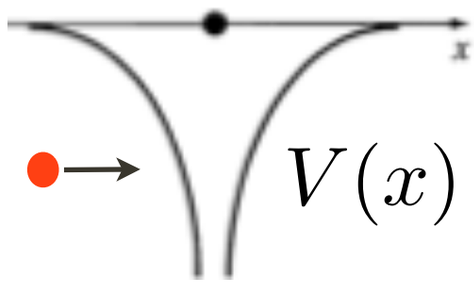

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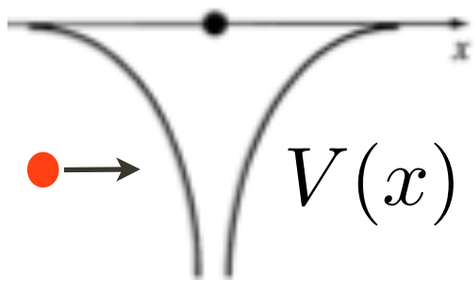

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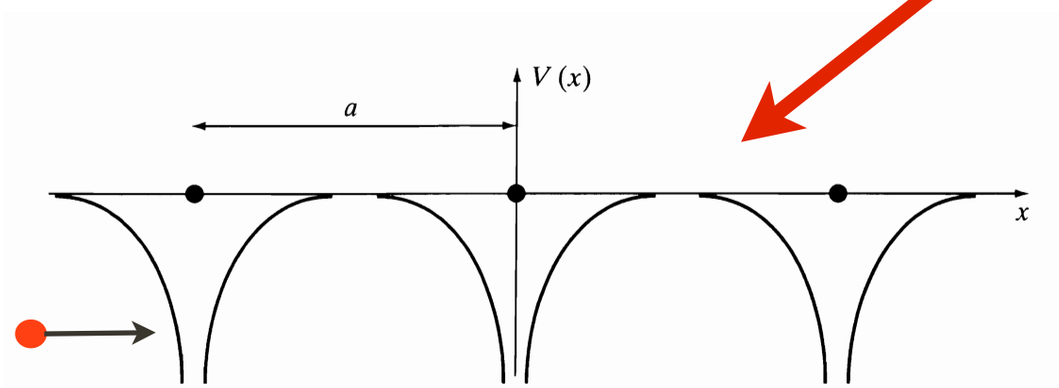

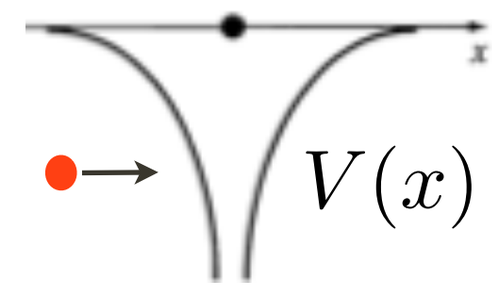

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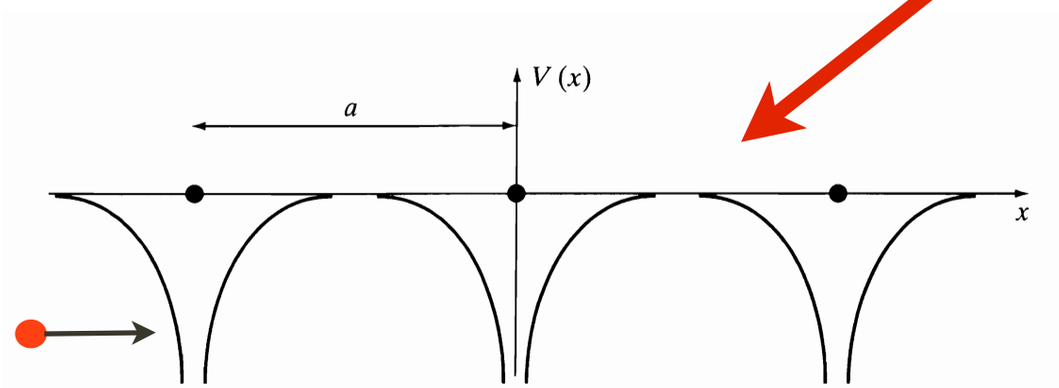

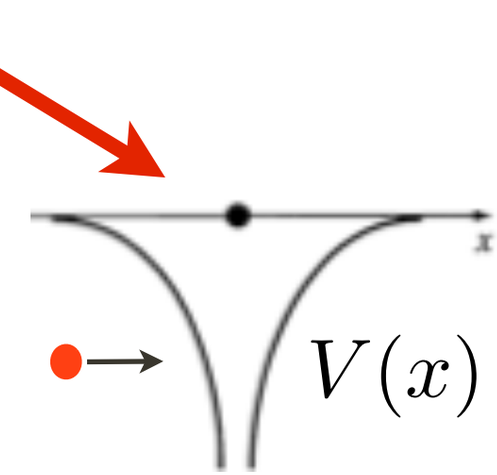

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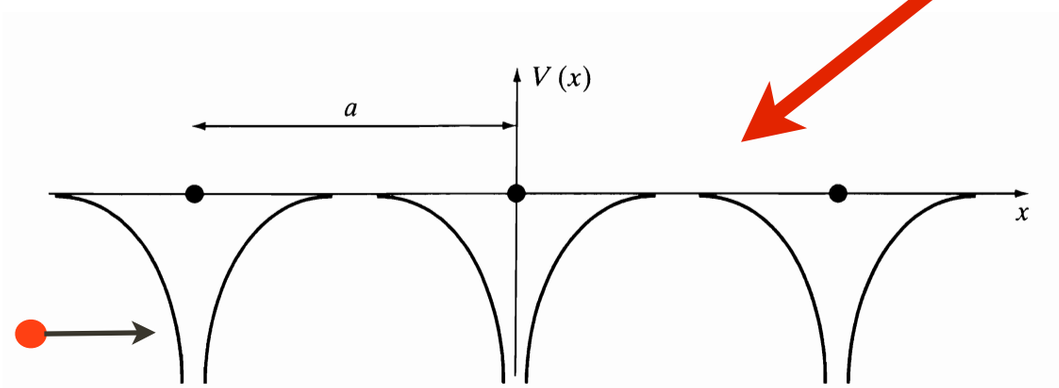

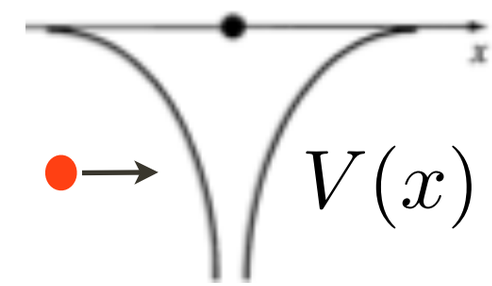

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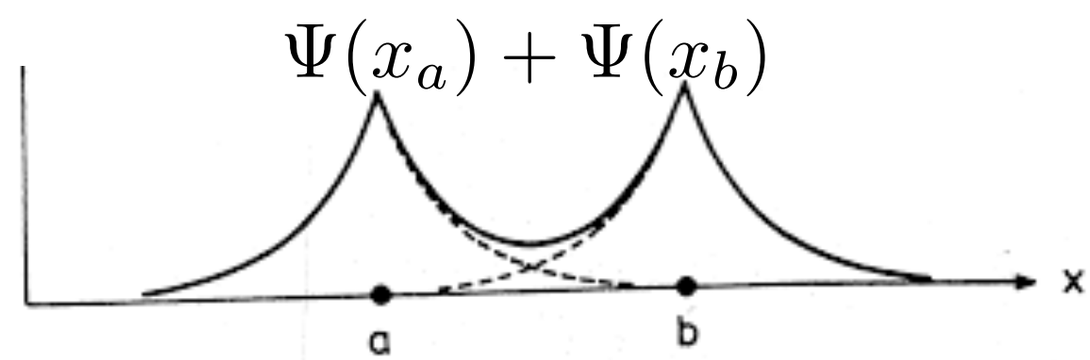

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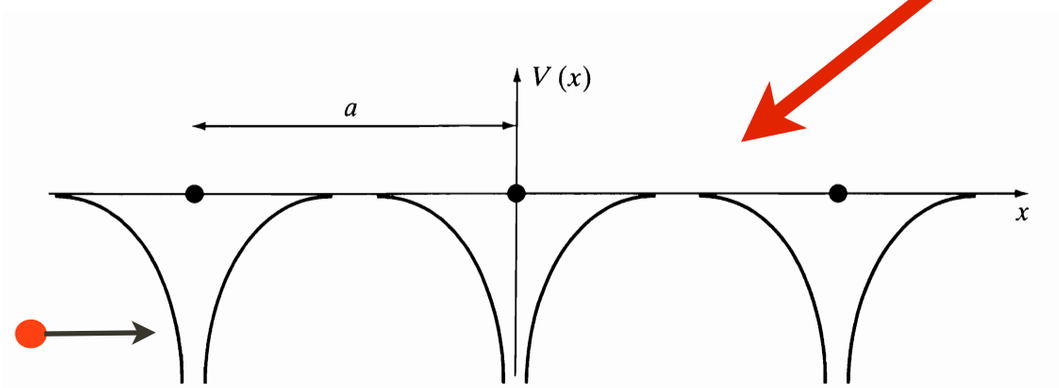

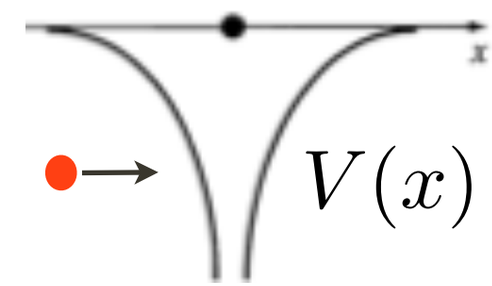

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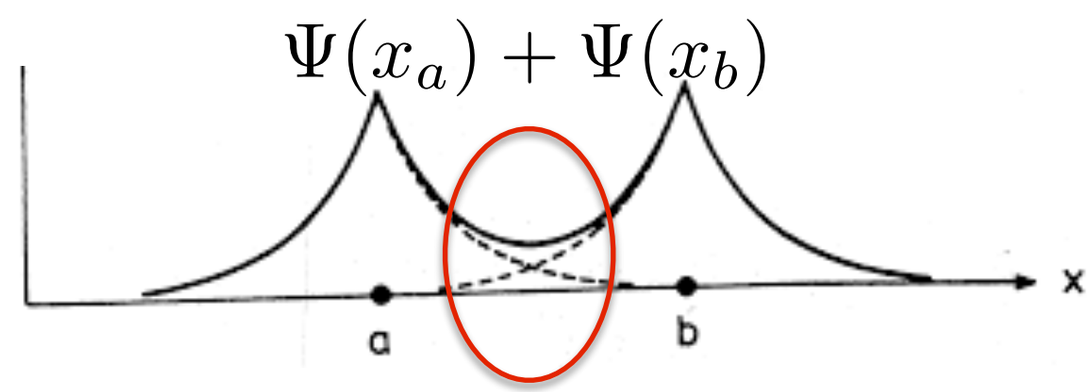

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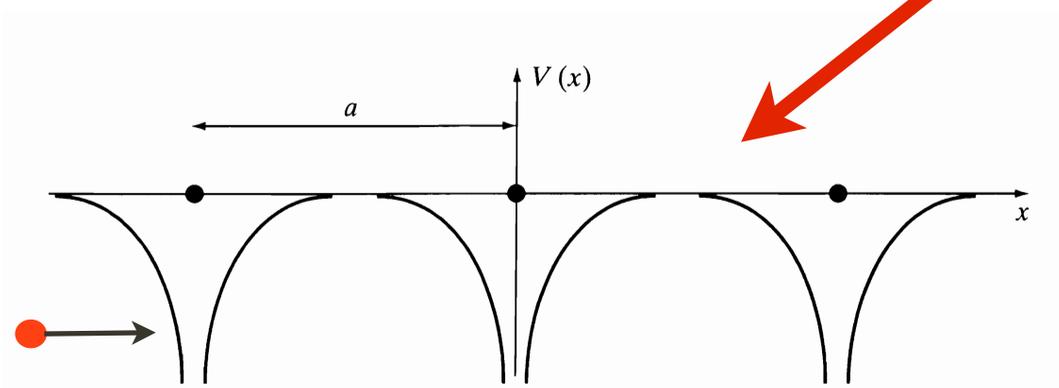

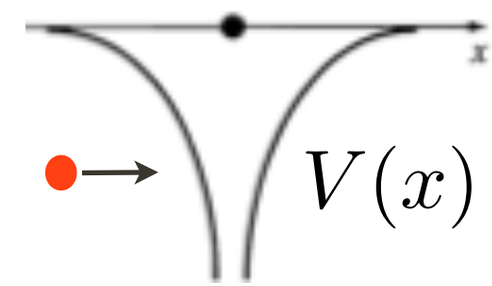

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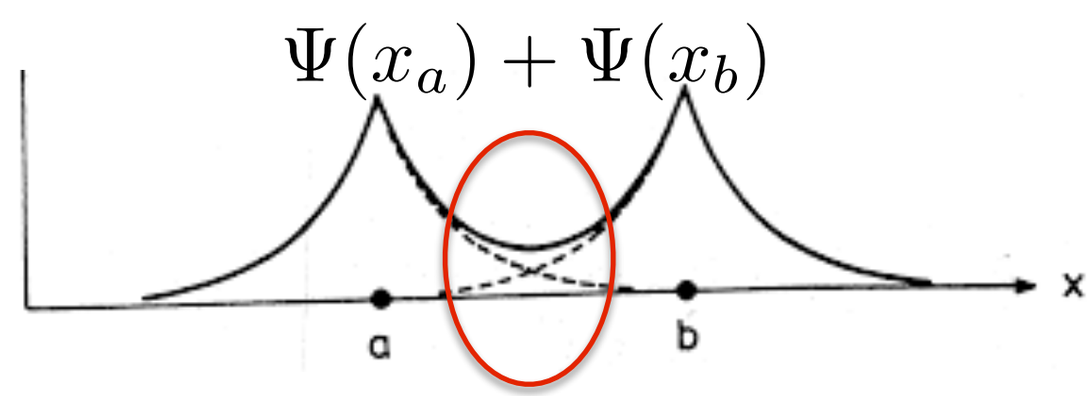

- Interference of local wave functions -> Long-range correlation

Variational approach to finite volume few-body problem: a not so perfect framework

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- ☑ Connection between short-range dynamics and long-range correlations is carried out by construction of basis functions

Variational approach to finite volume few-body problem: a not so perfect framework

- **Two-body Quantization conditions and Luescher formula:**

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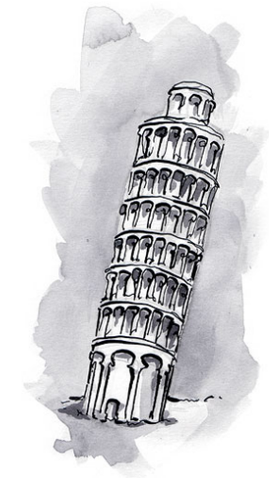

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Variational approach to finite volume few-body problem: a not so perfect framework

- **Extension to N-body problem with pair-wise interactions:**

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- **Require few-body solutions in infinite volume as input**

Sanity check in low dimension (1D)

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- ☑ A exactly solvable model in 1D

Sanity check in low dimension (1D)

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Sanity check in low dimension (1D)

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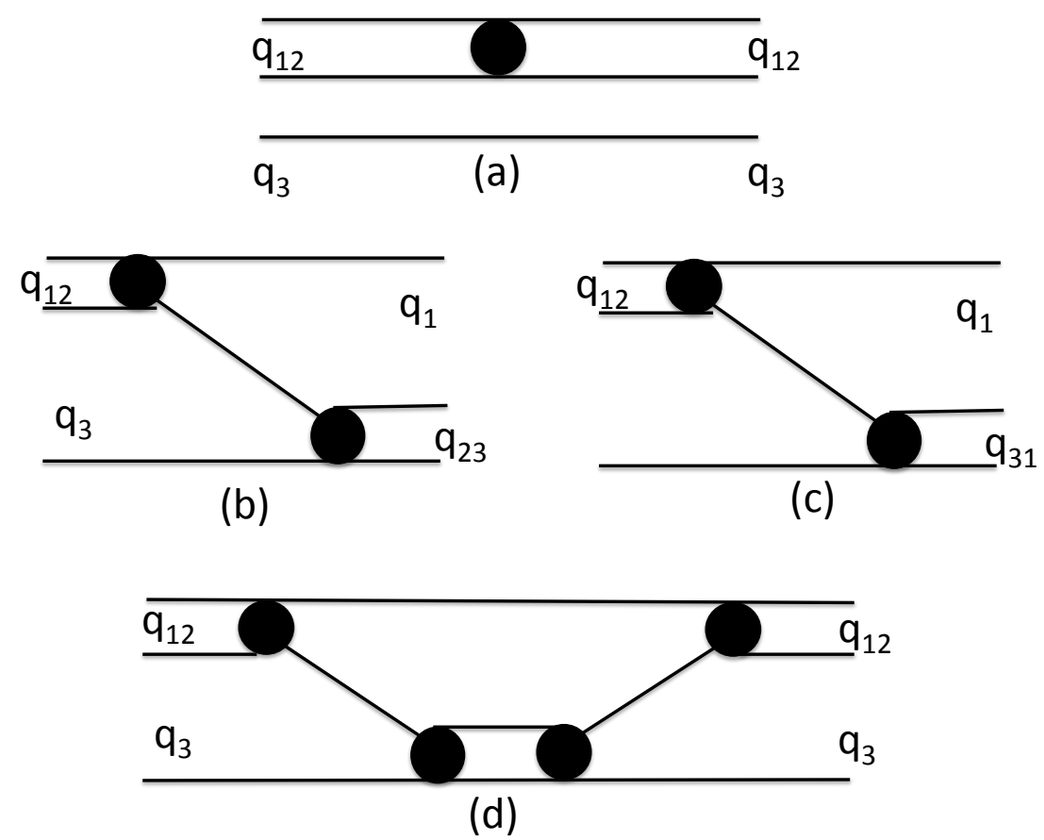

Sanity check in low dimension (1D)

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Sanity check in low dimension (1D)

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Sanity check in low dimension (1D)

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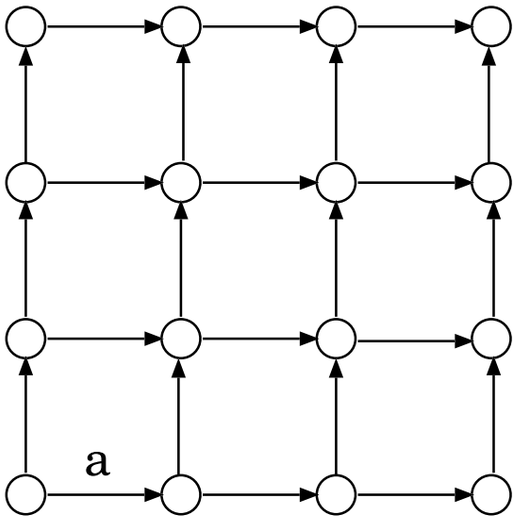

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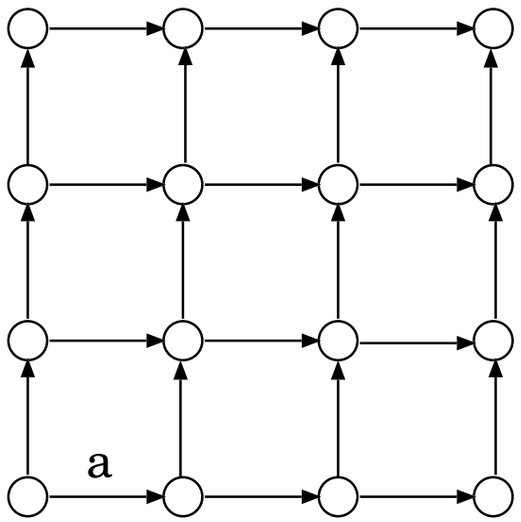

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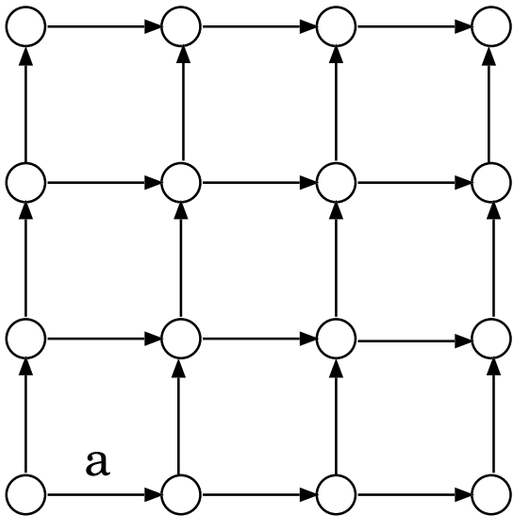

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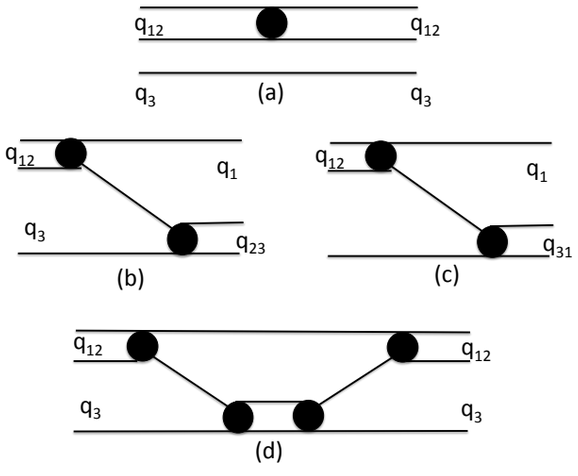

Sanity check in low dimension (1D)

☑ **One particle spectra -> mass**

$$C_{1b,n}(t) = \langle \widetilde{\phi}_n^*(t) \widetilde{\phi}_n(0) \rangle \propto e^{-E_{1b,n} t}$$

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$$\widetilde{\phi}_n(t) = \frac{1}{L}\sum_{x_1}\phi(x)e^{ix_1\frac{2\pi}{L}n}, \quad n \in \mathbb{Z}$$

Sanity check in low dimension (1D)

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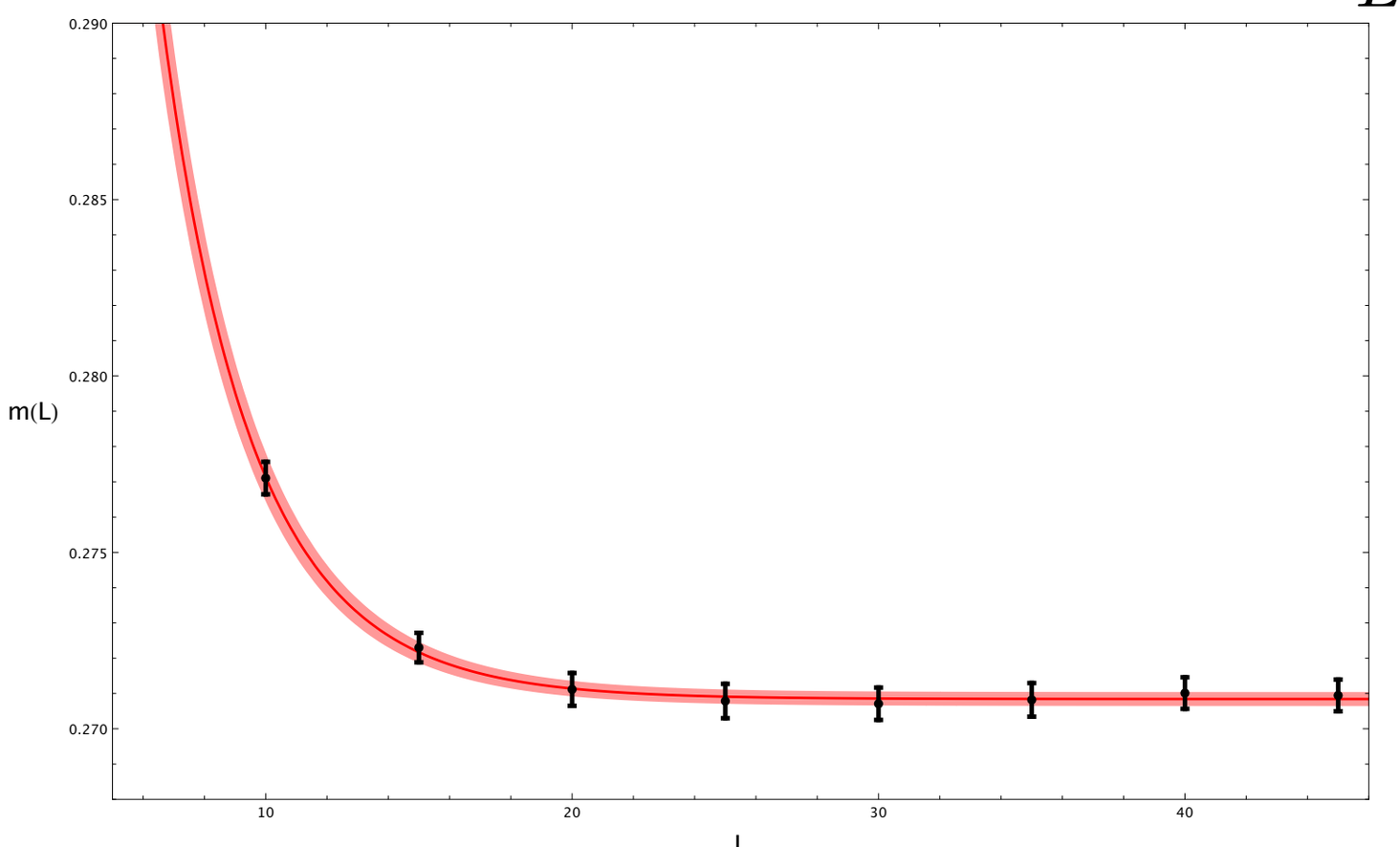

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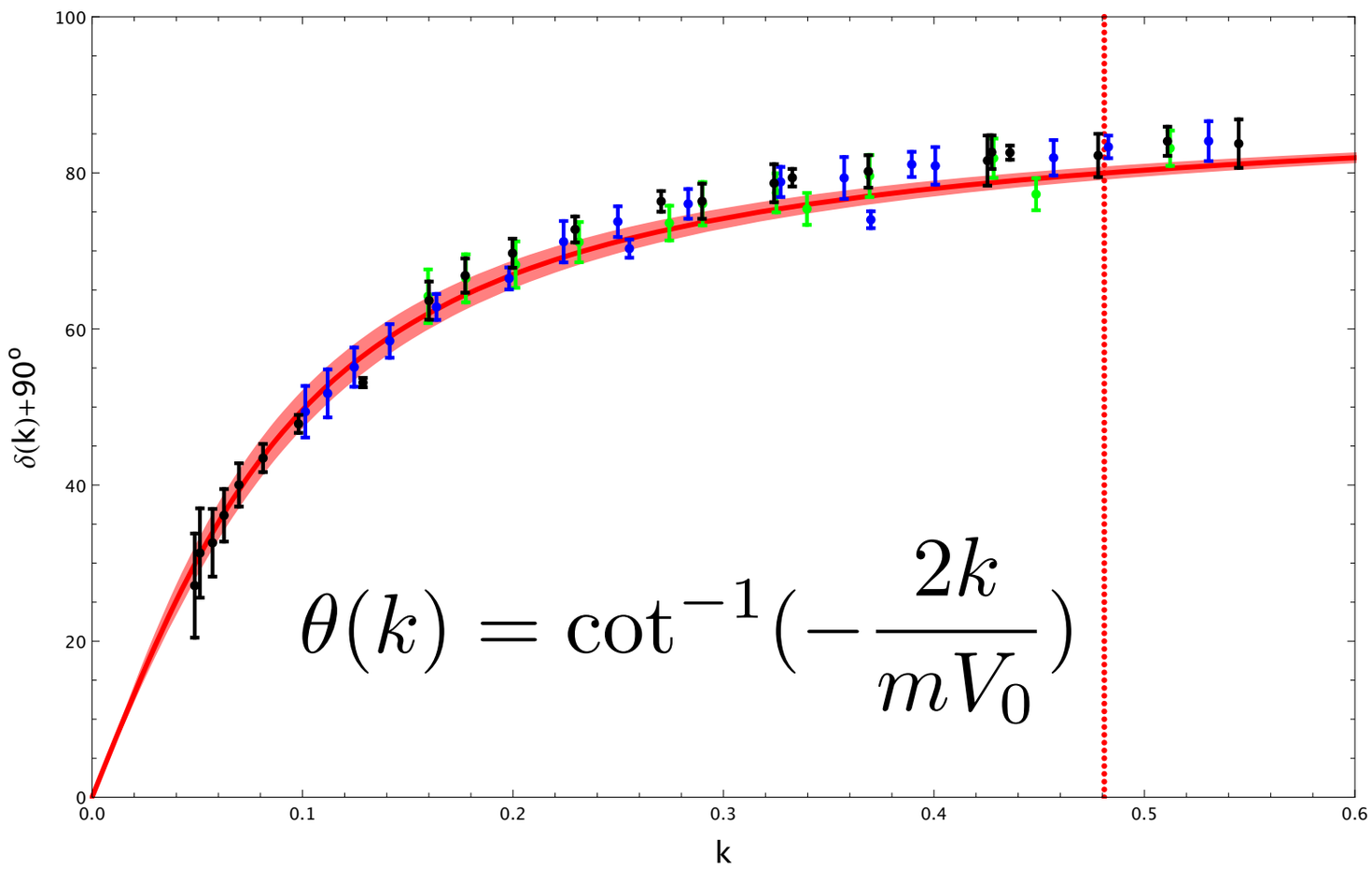

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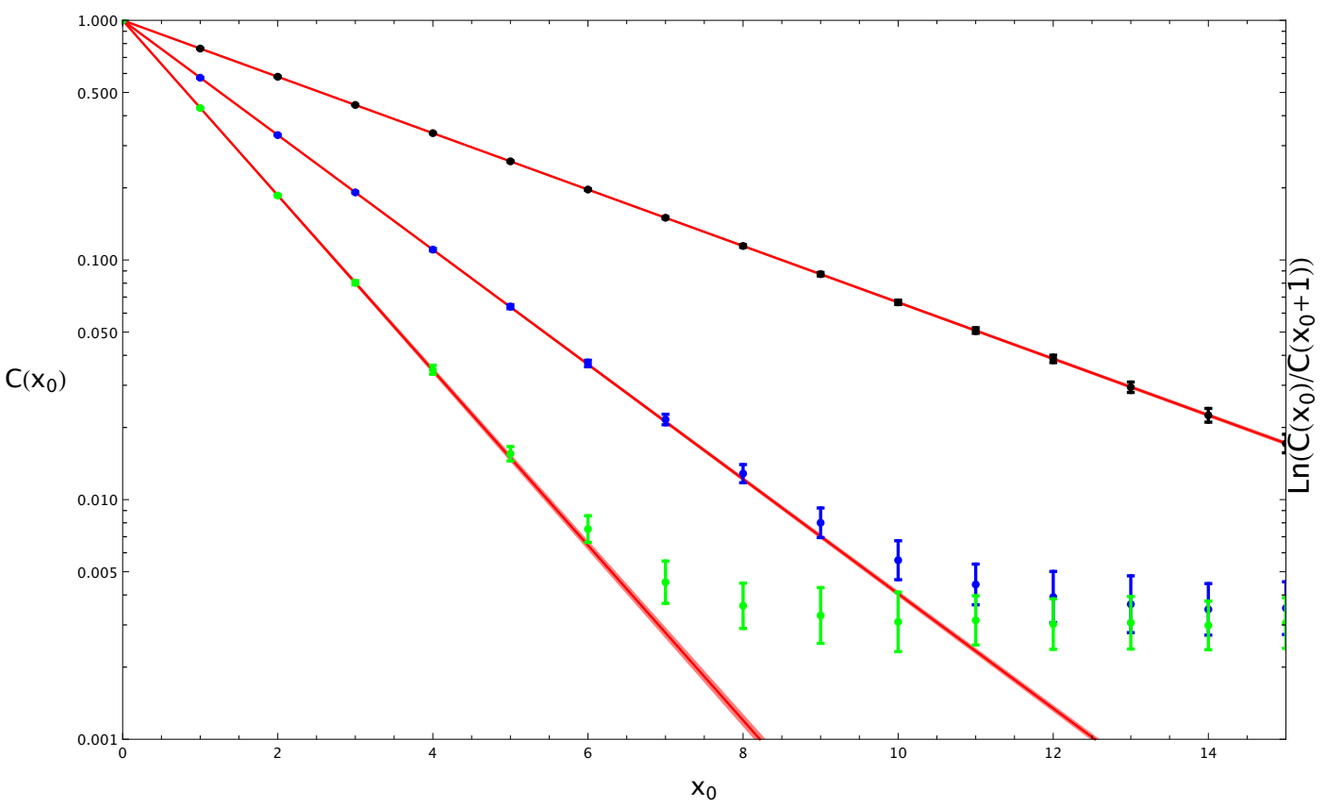

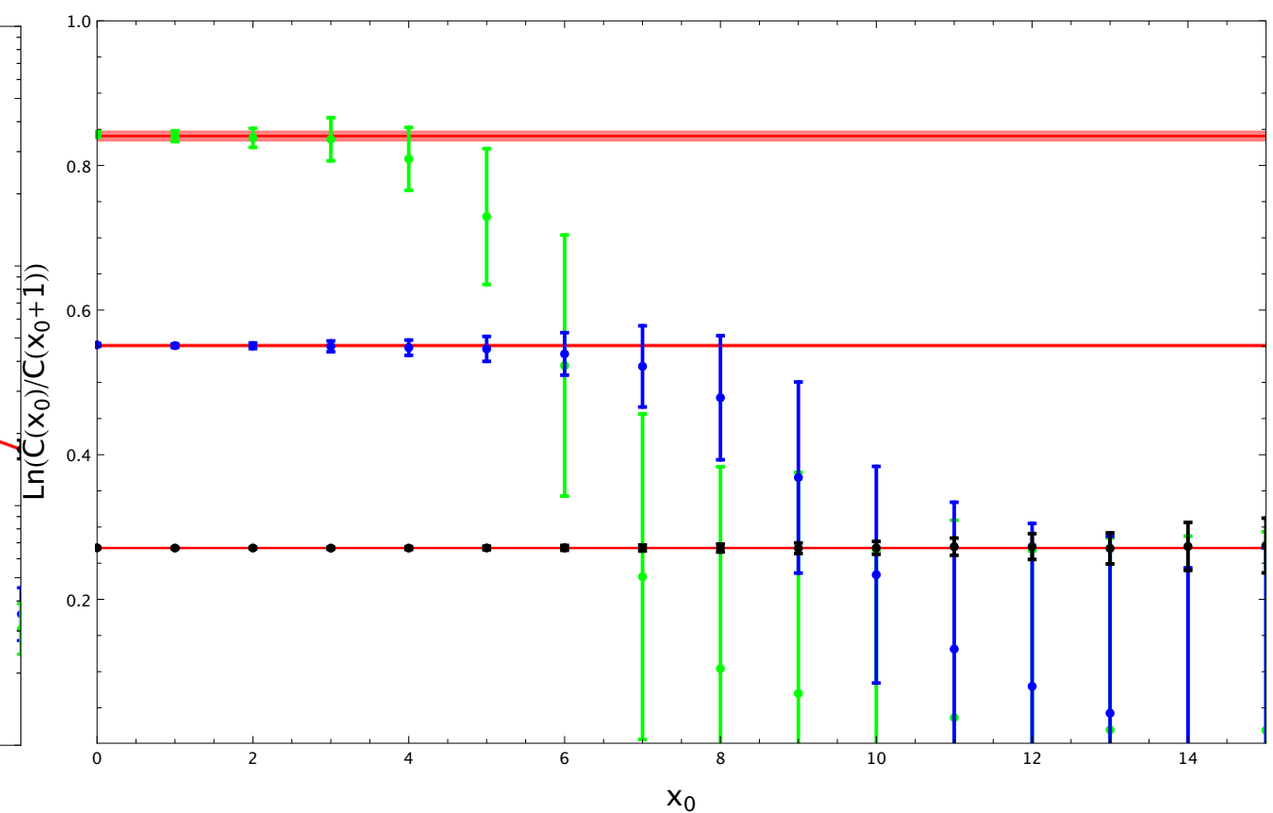

Sanity check in low dimension (1D)

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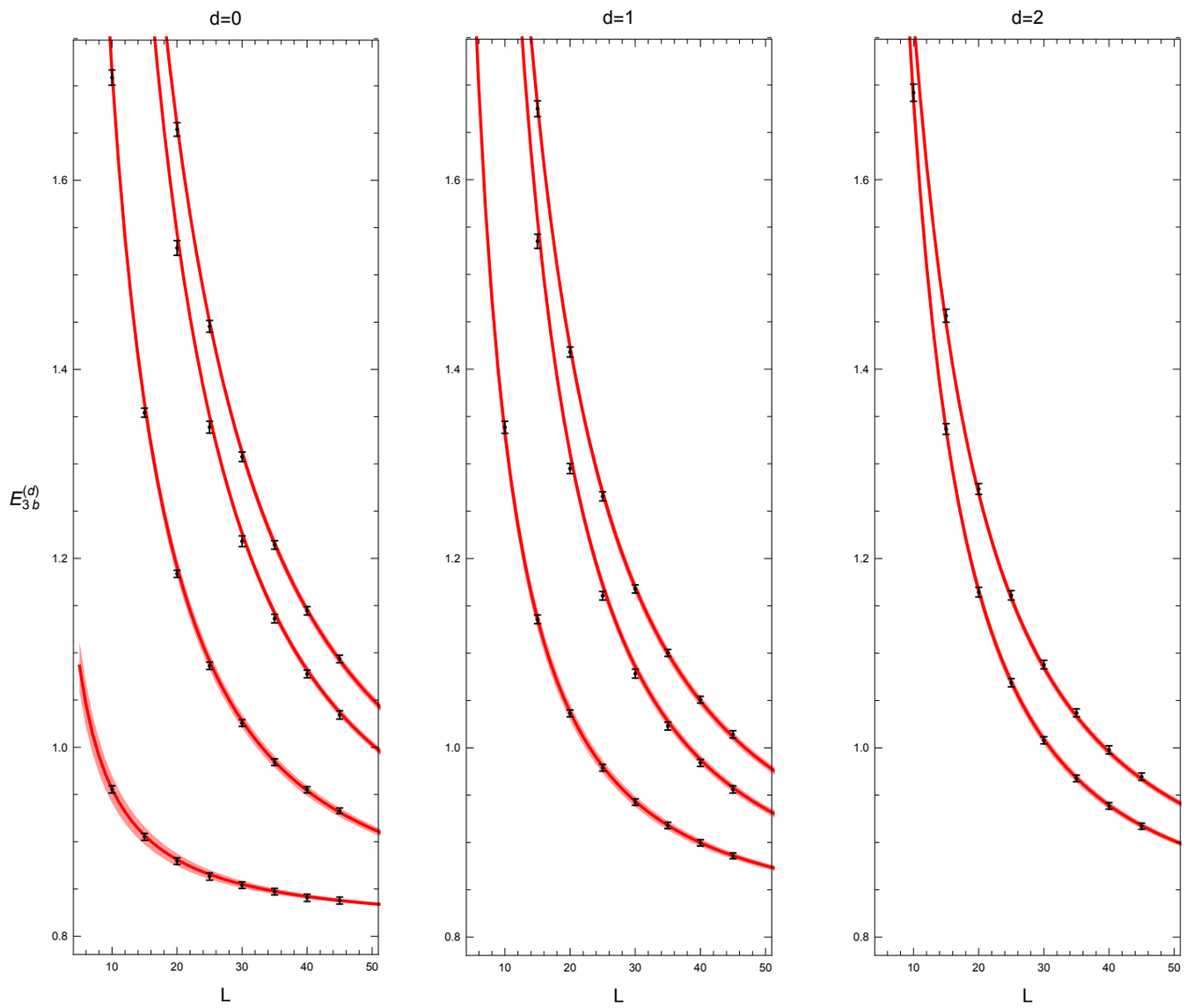

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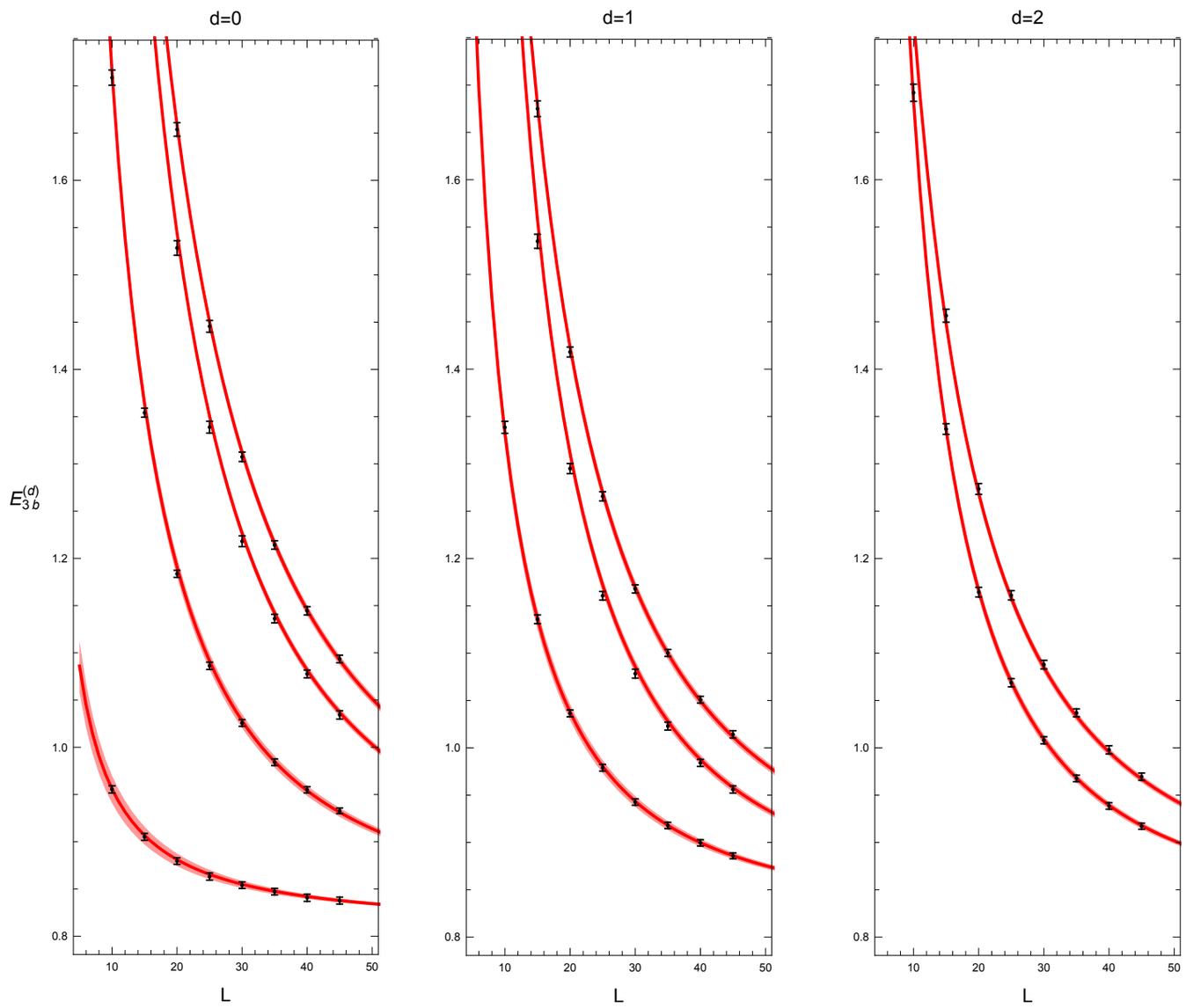

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Sanity check in low dimension (1D)

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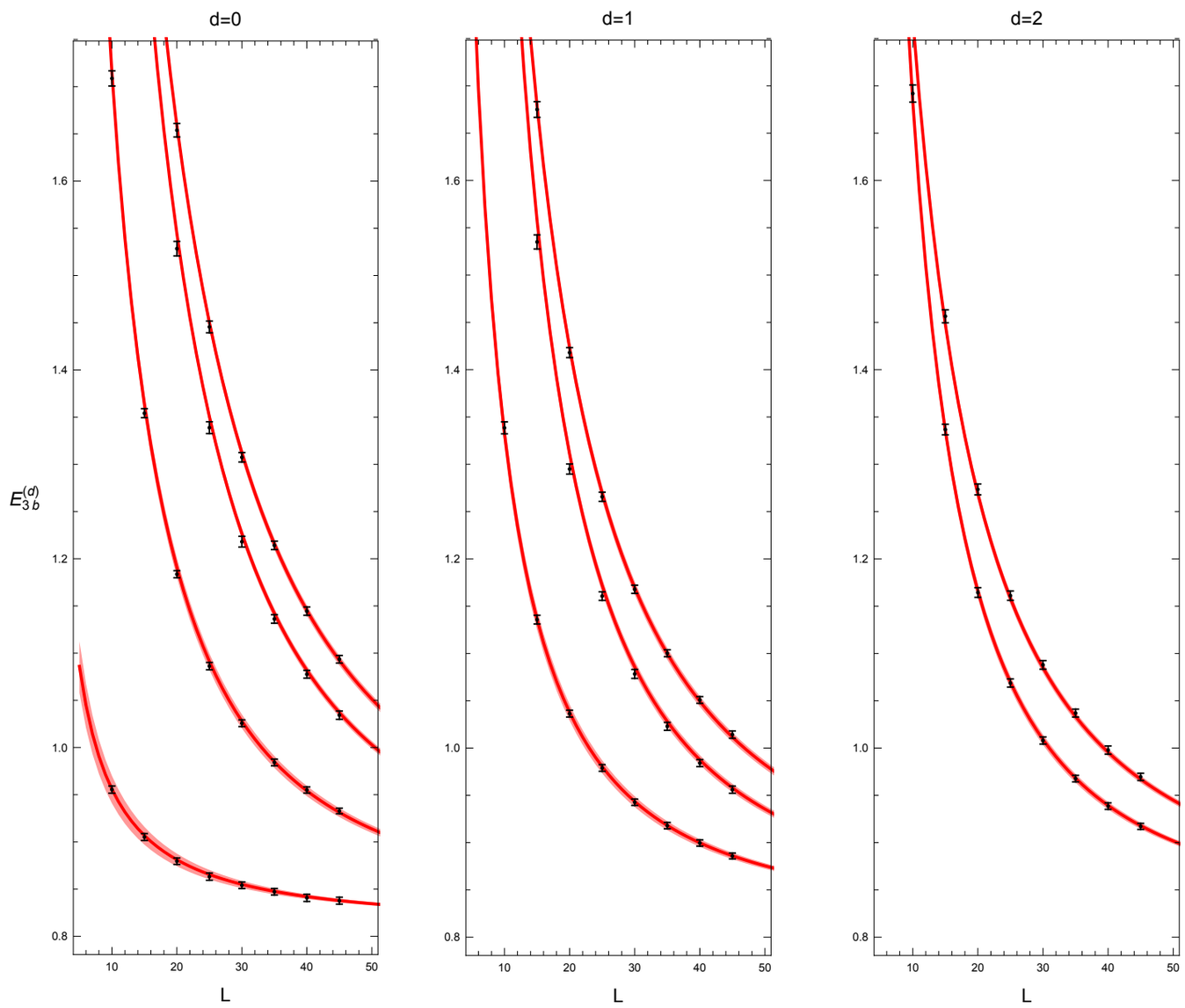

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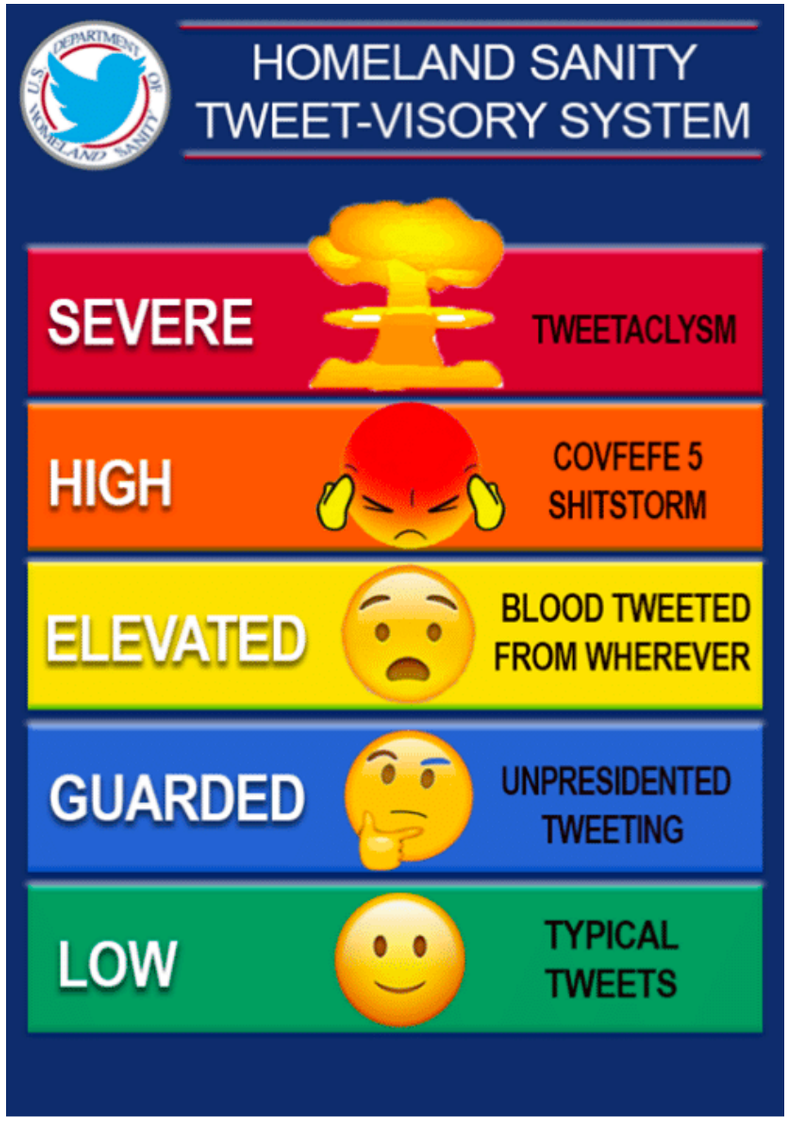

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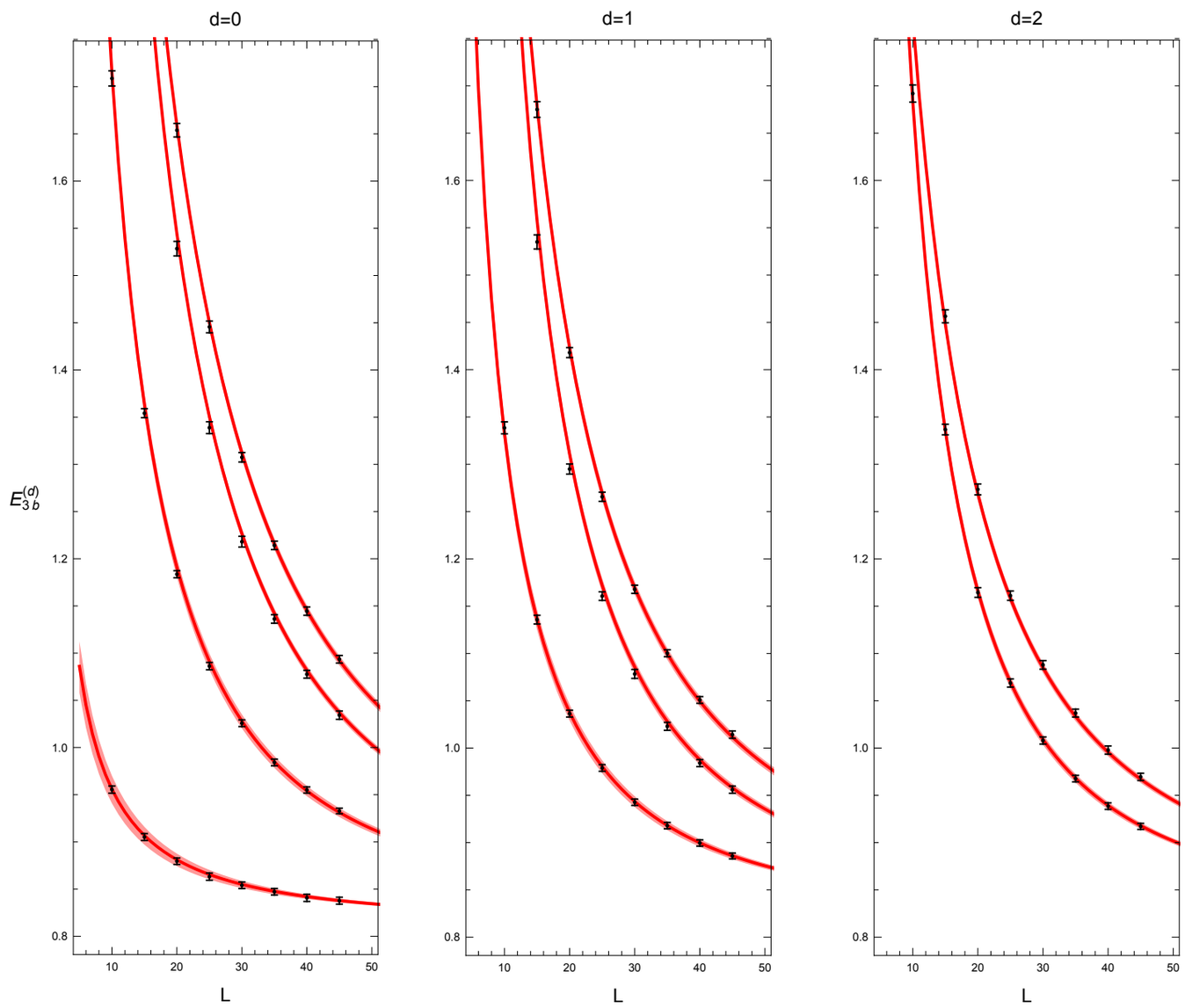

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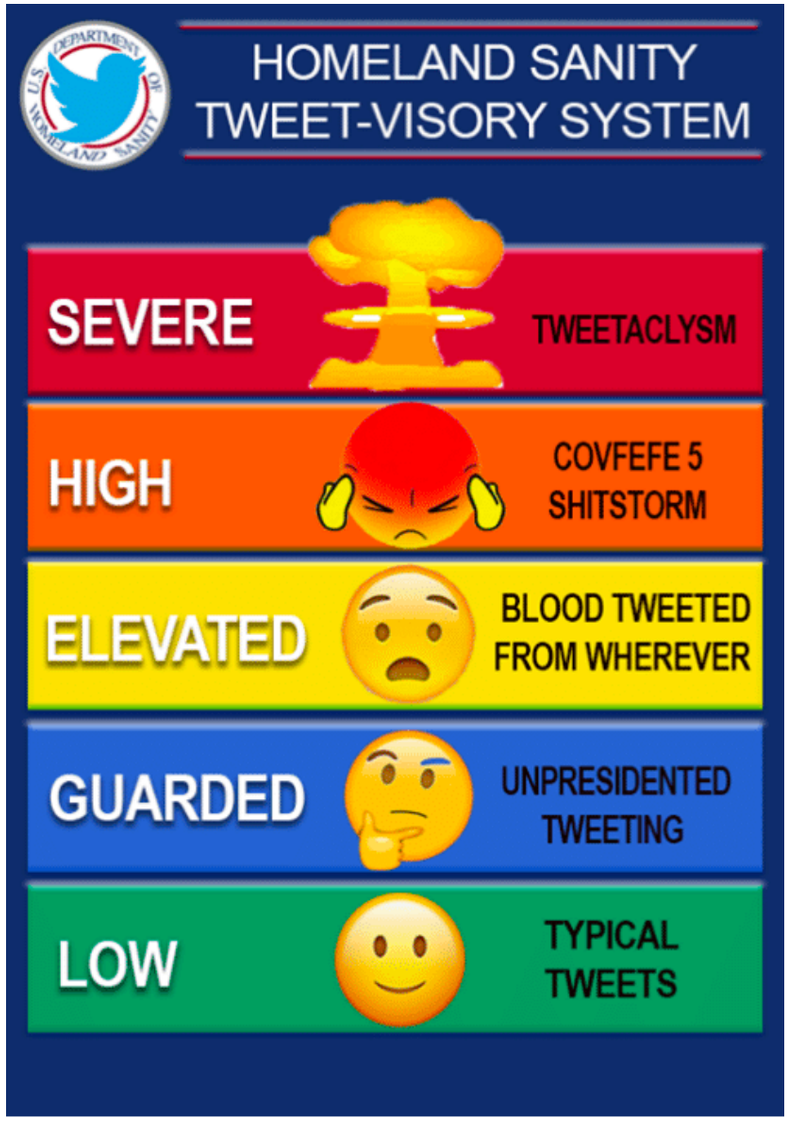

Summary and outlook

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