

# Gravitational form factors on the lattice

JLab Theory Seminar  
Oct 30, 2023

**Dan Hackett (MIT → FNAL)**

Patrick Oare (MIT)

Dimitra Pefkou (MIT → Berkeley)

Phiala Shanahan (MIT)

# Outline

## Gravitational structure of hadrons

Gravitational form factors (GFFs)?

Why are GFFs interesting?

## GFFs on the lattice

Overview of calculation(s)

## Results

GFFs of proton, pion (w/ flavor decomp)

Experimental comparison

Densities, radii

[2307.11707](#)

### Gravitational form factors of the pion from lattice QCD

Daniel C. Hackett, Patrick R. Oare, Dimitra A. Pefkou, and Phiala E. Shanahan  
*Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, U.S.A.*

The two gravitational form factors of the pion,  $A^\pi(t)$  and  $D^\pi(t)$ , are computed as functions of the momentum transfer squared  $t$  in the kinematic region  $0 \leq -t < 2 \text{ GeV}^2$  on a lattice QCD ensemble with quark masses corresponding to a close-to-physical pion mass  $m_\pi \approx 170 \text{ MeV}$  and  $N_f = 2 + 1$  quark flavors. The flavor decomposition of these form factors into gluon, up/down light-quark, and strange quark contributions is presented in the  $\overline{\text{MS}}$  scheme at energy scale  $\mu = 2 \text{ GeV}$ , with renormalization factors computed non-perturbatively via the RI-MOM scheme. Using monopole and  $z$ -expansion fits to the gravitational form factors, we obtain estimates for the pion momentum fraction and  $D$ -term that are consistent with the momentum fraction sum rule and the next-to-leading order chiral perturbation theory prediction for  $D^\pi(0)$ .

[2310.08484](#)

### Gravitational form factors of the proton from lattice QCD

Daniel C. Hackett,<sup>1,2</sup> Dimitra A. Pefkou,<sup>3,2</sup> and Phiala E. Shanahan<sup>2</sup>

<sup>1</sup>*Fermi National Accelerator Laboratory, Batavia, IL 60510, U.S.A.*

<sup>2</sup>*Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, U.S.A.*

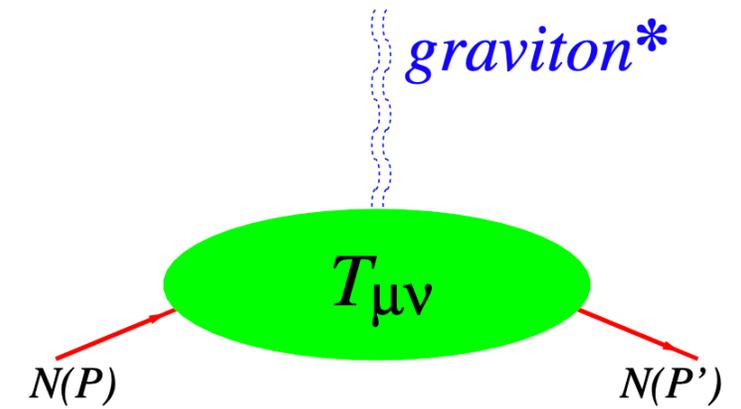
<sup>3</sup>*Department of Physics, University of California, Berkeley, CA 94720, U.S.A.*

The gravitational form factors (GFFs) of a hadron encode fundamental aspects of its structure, including its shape and size as defined from e.g., its energy density. This work presents a determination of the flavor decomposition of the GFFs of the proton from lattice QCD, in the kinematic region  $0 \leq -t \leq 2 \text{ GeV}^2$ . The decomposition into up-, down-, strange-quark, and gluon contributions provides first-principles constraints on the role of each constituent in generating key proton structure observables, such as its mechanical radius, mass radius, and  $D$ -term.

# Gravitational structure of hadrons

# Gravitational form factors (GFFs)

GFFs are EMT form factors



Schematically, for any hadron:

$$\langle \text{hadron}(p') | T(\Delta) | \text{hadron}(p) \rangle = \sum_i (\text{Lorentz structure})_i \text{GFF}_i(t = \Delta^2)$$

Graviton scattering  $\sim$  symmetric EMT

$$T^{\{\mu\nu\}} = \frac{2}{\sqrt{-g}} \frac{\delta S_{QCD}}{\delta g_{\mu\nu}} = 2 \text{Tr} \left[ -G^{\alpha\mu} G_{\alpha}^{\nu} + \frac{1}{4} g^{\mu\nu} G^{\alpha\beta} G_{\alpha\beta} \right] + \bar{q} \gamma^{\{\mu} i \overleftrightarrow{D}^{\nu\}} q$$
$$a^{\{\mu} b^{\nu\}} \equiv \frac{1}{2} (a^{\mu} b^{\nu} + a^{\nu} b^{\mu})$$

# Gravitational form factors

$$T^{\{\mu\nu\}} = 2 \text{Tr} \left[ -G^{\alpha\mu} G_{\alpha}^{\nu} + \frac{1}{4} g^{\mu\nu} G^{\alpha\beta} G_{\alpha\beta} \right] + \bar{q} \gamma^{\{\mu} i \overleftrightarrow{D}^{\nu\}} q$$

$$\begin{aligned} a^{\{\mu} b^{\nu\}} &\equiv \frac{1}{2} (a^{\mu} b^{\nu} + a^{\nu} b^{\mu}) \\ \overleftrightarrow{D} &= (\overrightarrow{D} - \overleftarrow{D})/2 \\ U, \bar{U} &= \text{Dirac spinors} \\ P &= (p' + p)/2 \\ \Delta &= p' - p \\ t &= \Delta^2 \end{aligned}$$

Nucleon:

$$\langle N(p') | T^{\{\mu\nu\}} | N(p) \rangle = \bar{U}(p') \left[ A(t) \frac{P^{\{\mu} P^{\nu\}}}{M} + J(t) \frac{i P^{\{\mu} \sigma^{\nu\} \rho} \Delta_{\rho}}{2M} + D(t) \frac{\Delta^{\{\mu} \Delta^{\nu\}} - g^{\mu\nu} \Delta^2}{4M} \right] U(p)$$

Pion:

$$\langle \pi(p') | T^{\{\mu\nu\}} | \pi(p) \rangle = A(t) 2P^{\mu} P^{\nu} + D(t) \frac{1}{2} (\Delta^{\mu} \Delta^{\nu} - g^{\mu\nu} \Delta^2)$$

Why are these interesting?

# Global properties

$$\langle N(p') | T^{\{\mu\nu\}} | N(p) \rangle = \bar{U}(p') \left[ A(t) \frac{P^{\{\mu} P^{\nu\}}}{M} + J(t) \frac{i P^{\{\mu} \sigma^{\nu\} \rho} \Delta_\rho}{2M} + D(t) \frac{\Delta^{\{\mu} \Delta^{\nu\}} - g^{\mu\nu} \Delta^2}{4M} \right] U(p)$$

$\partial_\mu T^{\mu\nu} = 0 \rightarrow$  GFFs are scale- and scheme-independent

Forward GFFs are fundamental, global properties:

$$A(0) = 1 \Leftrightarrow \langle p | T^{tt} | p \rangle = M$$

$$J(0) = \frac{1}{2} = \text{Total spin}$$

$$B(0) = 2J(0) - A(0) = 0 \quad \text{“vanishing of the anomalous gravitomagnetic moment”}$$

$$D(0) = ??? \quad (\text{internal forces})$$

Similar for pion, except no  $J$

# $D(0)$ : "the last global unknown"

[Polyakov Schweitzer 1805.06596]

<b>em:</b> $\partial_\mu J_{\text{em}}^\mu = 0$	$\langle N'   J_{\text{em}}^\mu   N \rangle$	$\longrightarrow$	$Q = 1.602176487(40) \times 10^{-19} \text{C}$ $\mu = 2.792847356(23) \mu_N$
<b>weak:</b> PCAC	$\langle N'   J_{\text{weak}}^\mu   N \rangle$	$\longrightarrow$	$g_A = 1.2694(28)$ $g_p = 8.06(55)$
<b>gravity:</b> $\partial_\mu T_{\text{grav}}^{\mu\nu} = 0$	$\langle N'   T_{\text{grav}}^{\mu\nu}   N \rangle$	$\longrightarrow$	$m = 938.272013(23) \text{ MeV}/c^2$ $J = \frac{1}{2}$ $D = ?$

Table I. The global properties of the proton defined in terms of matrix elements of the conserved currents associated with respectively electromagnetic, weak, and gravitational interaction. Notice the weak currents include the partially conserved axial current, and  $g_A$  or  $g_p$  are strictly speaking defined in terms of transition matrix elements in the neutron  $\beta$ -decay or muon-capture. The values of the properties are from the particle data book [107] and [108] (for  $g_p$ ) except for the unknown  $D$ -term.

# What is a hadron made of?

Decompose EMT into quark and glue:

$$T_g^{\{\mu\nu\}} = 2 \text{Tr} \left[ -G^{\alpha\mu} G_\alpha^\nu + \frac{1}{4} g^{\mu\nu} G^{\alpha\beta} G_{\alpha\beta} \right] \quad T_q^{\{\mu\nu\}} = \bar{q} \gamma^{\{\mu} i \overleftrightarrow{D}^{\nu\}} q$$

→ GFFs for glue and e/a quark:

$$\langle N(p') | T_{g,q}^{\{\mu\nu\}} | N(p) \rangle = \bar{U}(p') \left[ A_{g,q}(t) \frac{P^{\{\mu} p^{\nu\}}}{M} + J_{g,q}(t) \frac{i P^{\{\mu} \sigma^{\nu\}\rho} \Delta_\rho}{2M} \right. \\ \left. + D_{g,q}(t) \frac{\Delta^{\{\mu} \Delta^{\nu\}} - g^{\mu\nu} \Delta^2}{4M} + \bar{c}_{g,q}(t) M g^{\mu\nu} \right] U(p)$$

[Note: not individually conserved,  
decomposition is scale and scheme dependent]

“Extra” GFF ~ trace anomaly

$$\sum_q \bar{c}_q + \bar{c}_g = 0$$

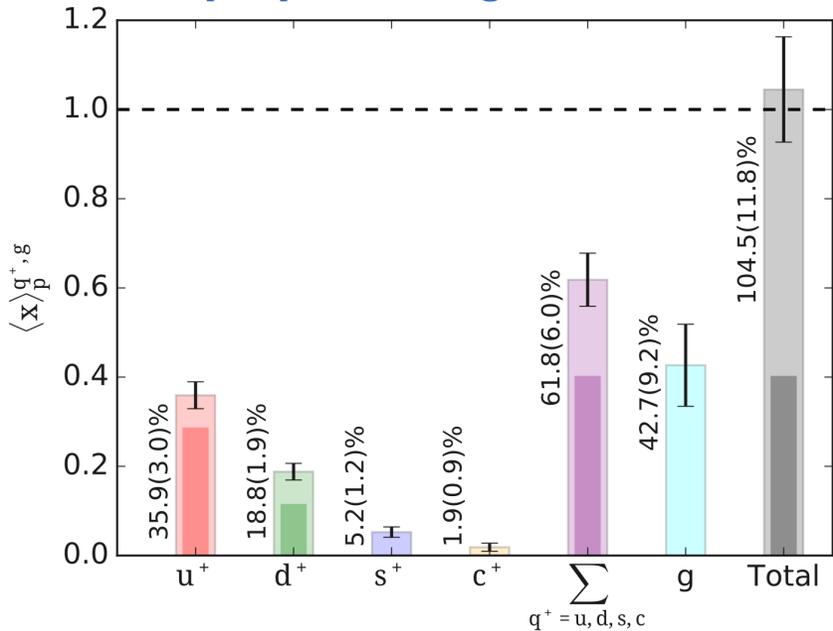
# Forward GFFs & decompositions

$$\langle N(p') | T_{g,q}^{\{\mu\nu\}} | N(p) \rangle = \bar{U}(p') \left[ A_{g,q}(t) \frac{P^{\{\mu\nu\}}}{M} + J_{g,q}(t) \frac{i P^{\{\mu\nu\}} \rho \Delta_\rho}{2M} + D_{g,q}(t) \frac{\Delta^{\{\mu\nu\}} - g^{\mu\nu} \Delta^2}{4M} + \bar{c}_{g,q}(t) M g^{\mu\nu} \right] U(p)$$

## Momentum fraction

$$A_{q,g}(0) = \langle x \rangle_{q,g}$$

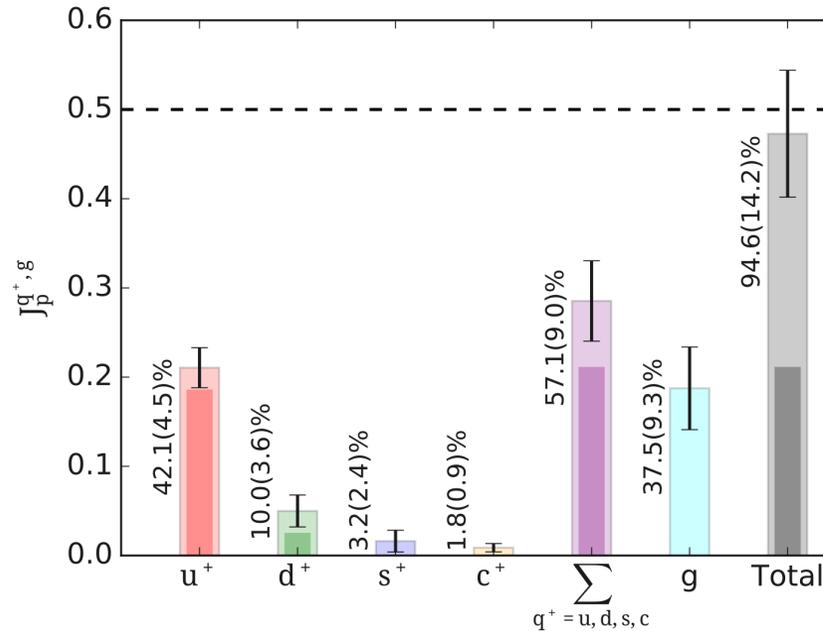
$$\sum_q A_q(0) + A_g(0) = 1$$



[ETMC 2003.08486]

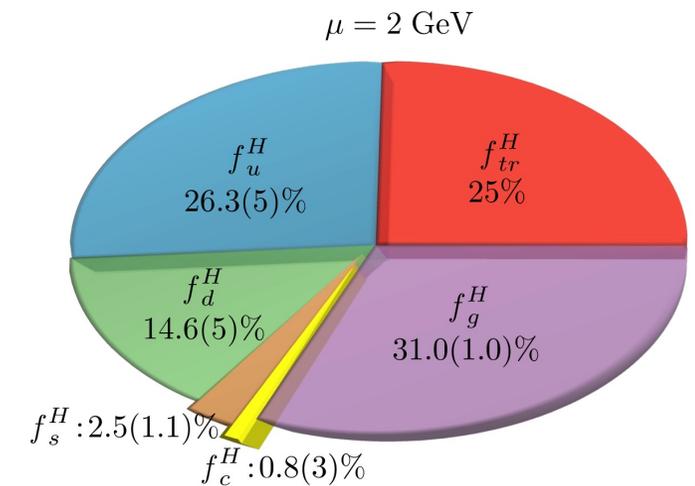
## Spin fraction

$$\sum_q J_q(0) + J_g(0) = 1/2$$



[ETMC 2003.08486]

...and involved in others, e.g. Ji's rest energy decomp



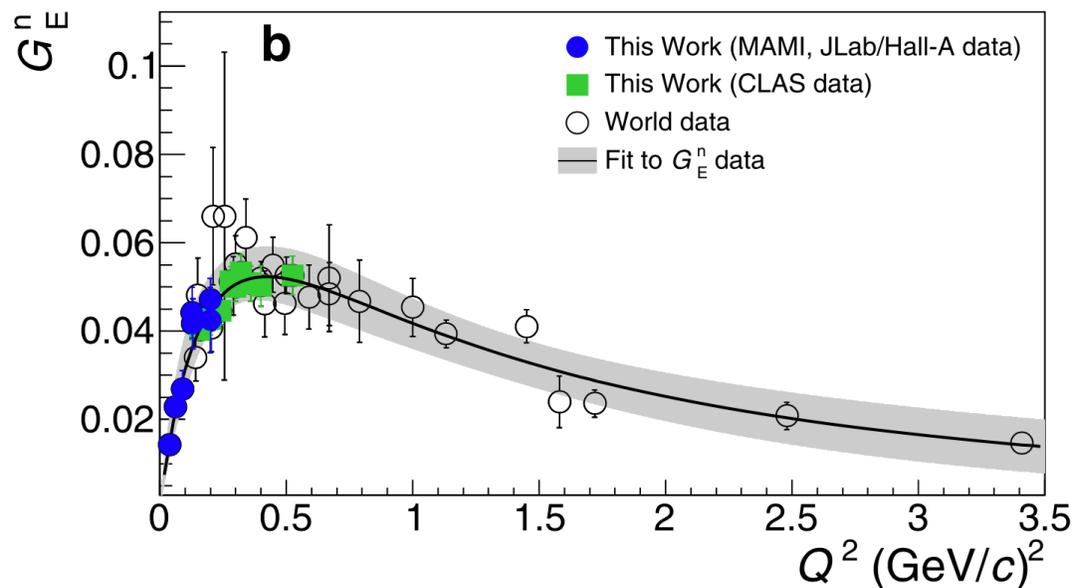
[Liu 2103.15768]

# What does a hadron look like?

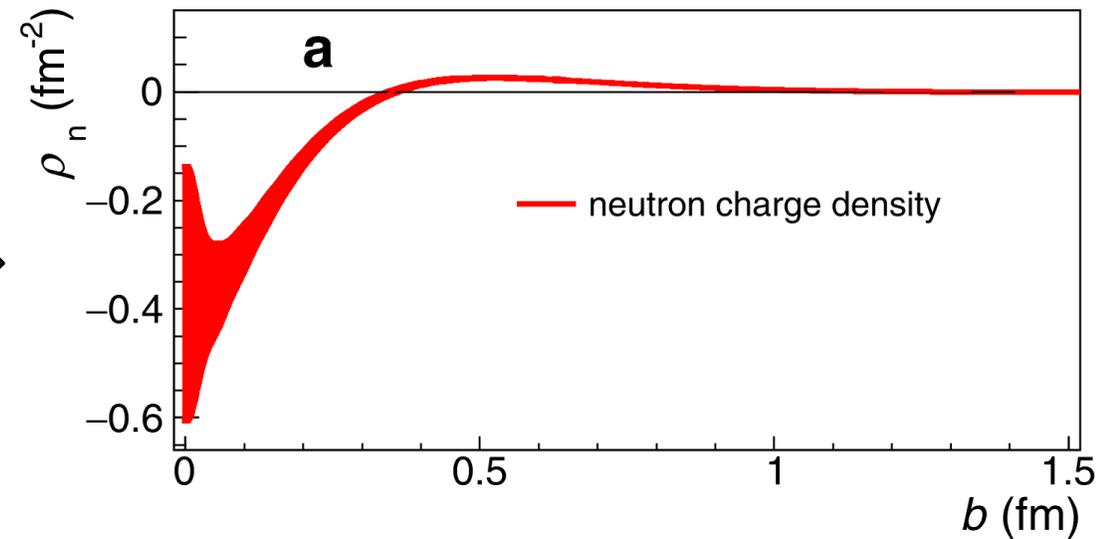
Fourier-transformed form factors provide information about spatial densities

**Example:** electric charge density in the neutron from  $G_E^n$

[[Atac, Constantinou, Meziani, Paolone, Sparveris 2103.10840](#)]



Fourier transform  
→



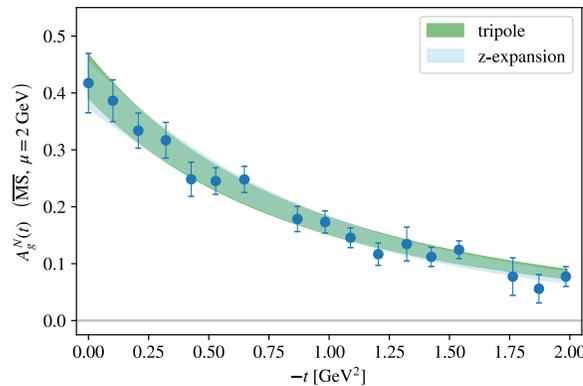
# Mass & mechanical densities

Applies also for GFFs: energy, pressure, shear forces

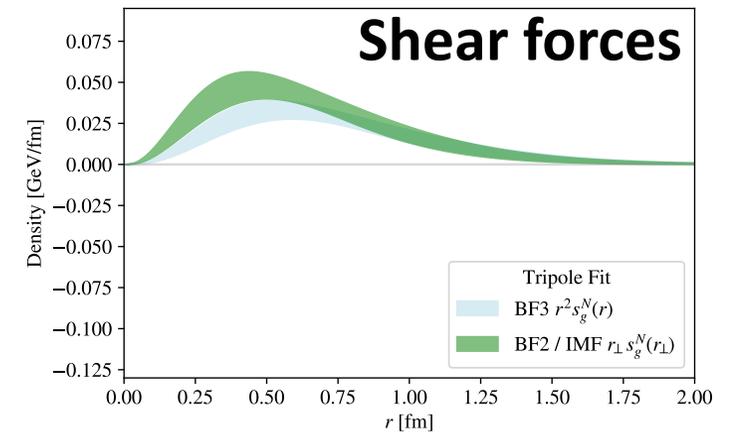
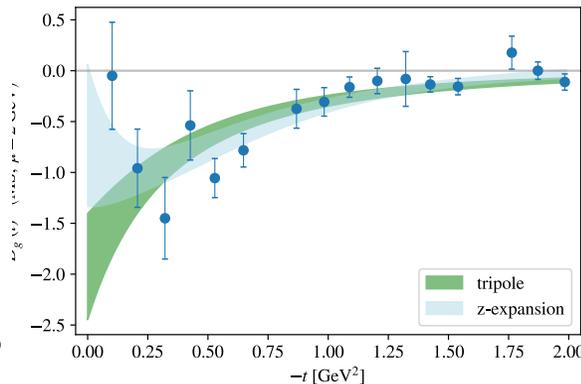
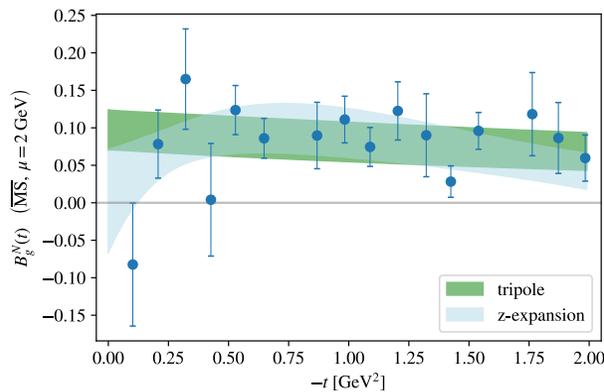
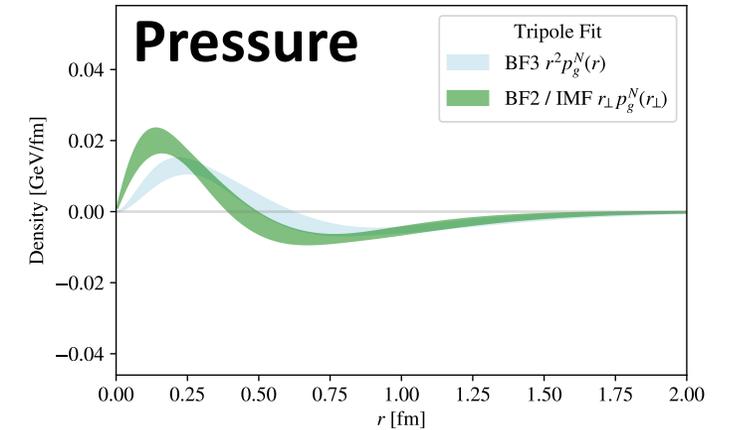
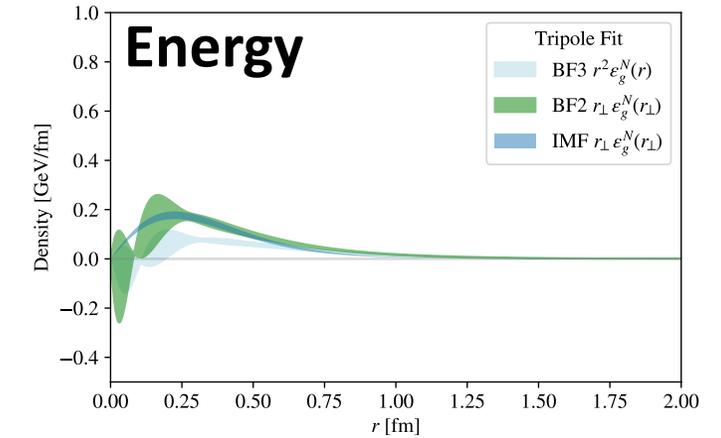
Example: pressure in the nucleon from  $D$

[\[Pefkou DH Shanahan 2107.10368\]](#)

(More on this later)



Fourier transform  
→



# Experimental accessibility?

GFFs related to Mellin moments of generalized parton distributions (GPDs)

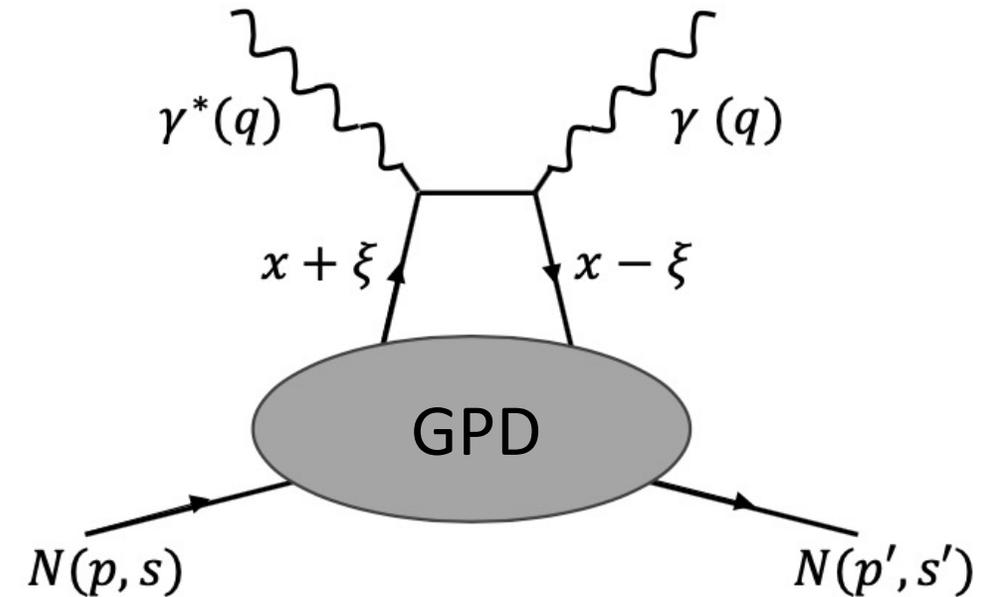
$$\int dx x^{n-1} \text{GPD}(x, \xi, t)$$

e.g. nucleon  $n = 2$

$$\int dx x H_{q,g}(x, \xi, t) = A_{q,g}(t) + \xi^2 D_{q,g}(t)$$

$$\int dx x E_{q,g}(x, \xi, t) = B_{q,g}(t) - \xi^2 D_{q,g}(t)$$

→ relate to experiment via factorization



# Experimental results

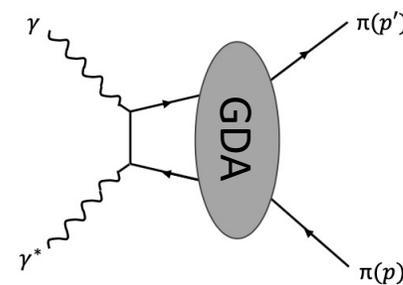
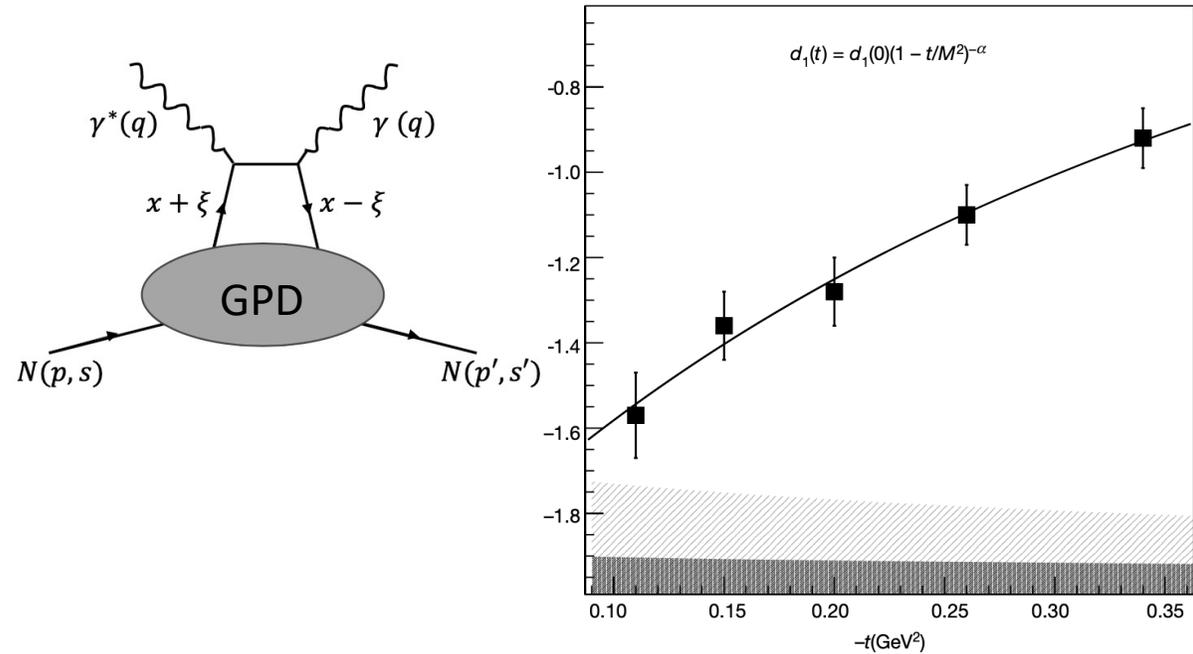
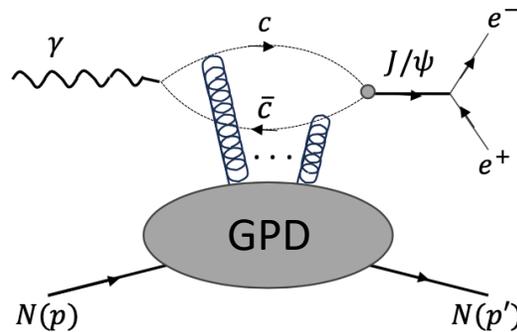
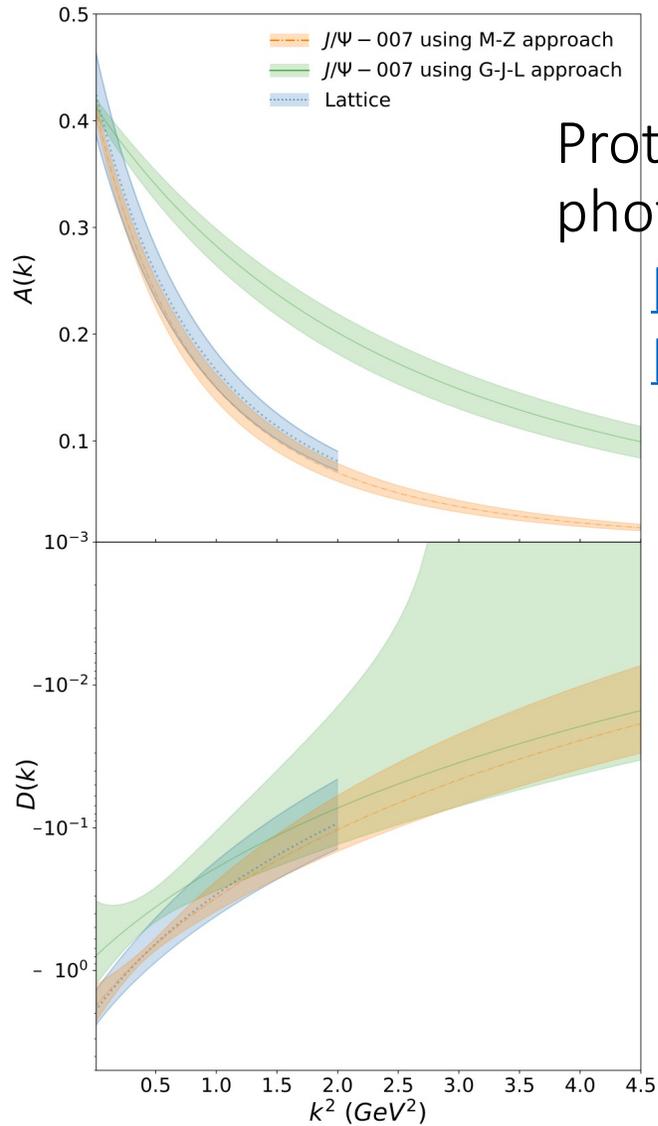
Proton: quark  $D$  from DVCS

[\[Burkert Elouadrhiri Girod 2018\]](#)

Proton: glue from  $J/\Psi$  photoproduction

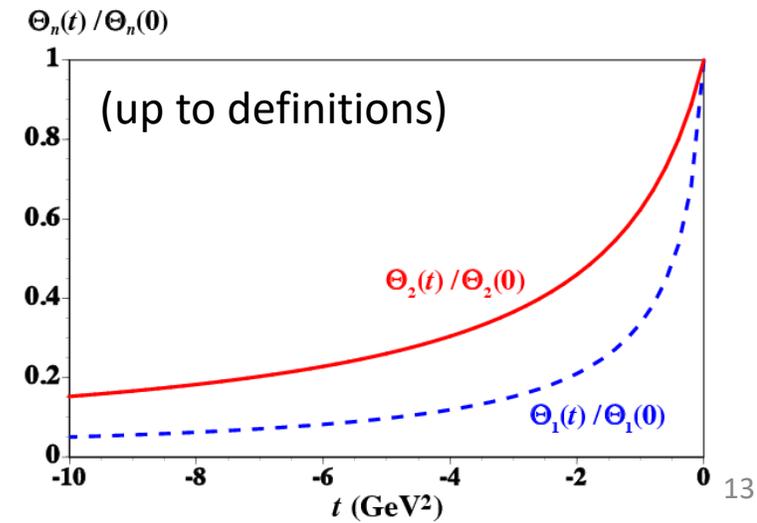
[\[Duran et al. 2207.05212\]](#)

[\[Guo et al. 2305.06992\]](#)



Pion: quark from  $\gamma\gamma \rightarrow \pi^0\pi^0$

[\[Kumano Song Teryaev 1711.08088\]](#)

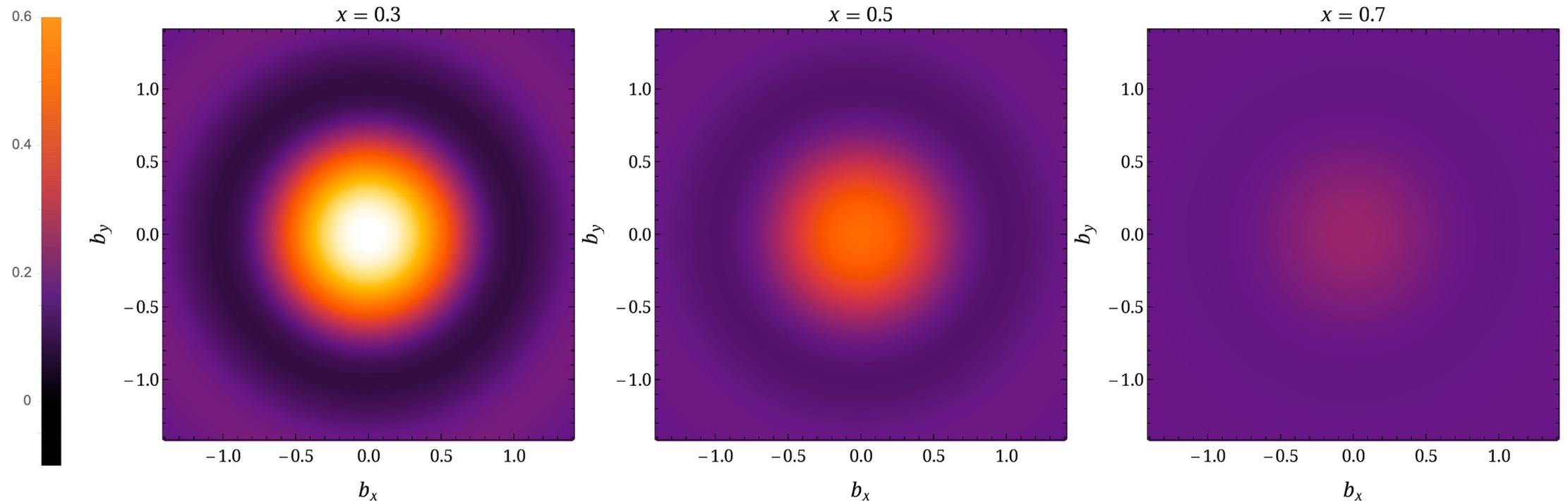


# Hadron tomography w/ GPDs

GPD  $\xrightarrow{FT}$  density of partons w/r/t impact parameter  $b$

$$q(x, b) = \int \frac{d\Delta}{(2\pi)^2} e^{iq \cdot b} H(x, \xi = 0, t = -\Delta^2)$$

Example: nucleon isovector density [[Lin 2008.12474](#)]



# GFFs on the lattice

# General idea: bare matrix elements

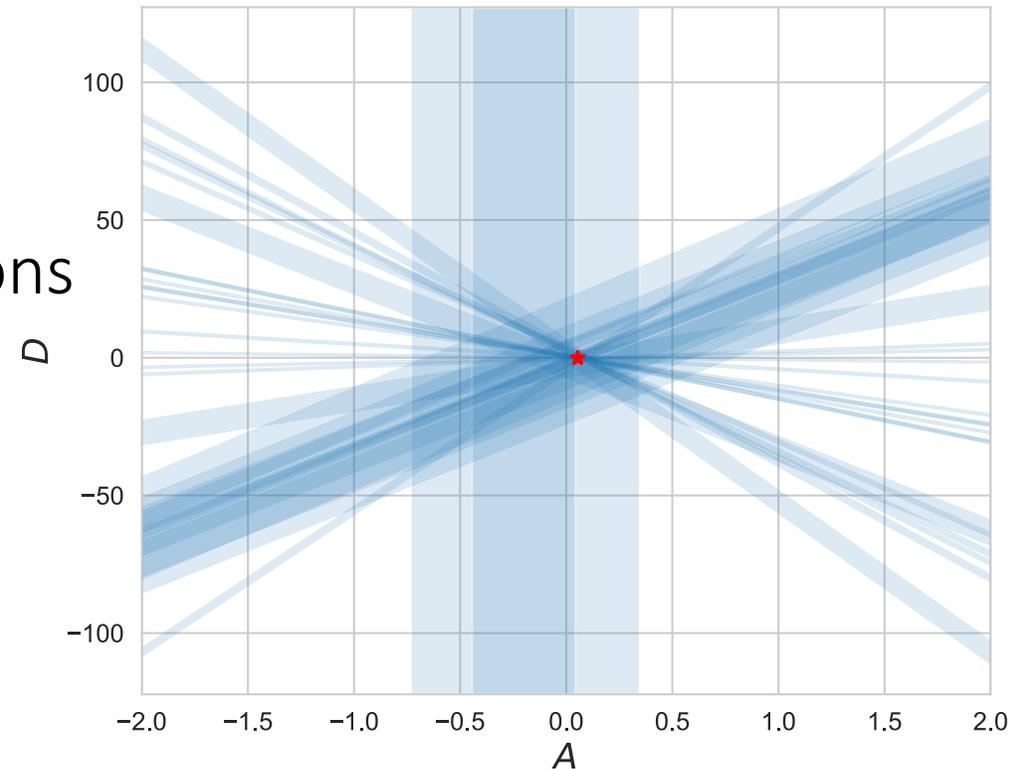
A three-point function for  $\Delta = p' - p$

$$\langle \chi(p', t_f) T^b \chi(p, 0) \rangle \sim Z_{p'} Z_p \langle p' | T^b | p \rangle e^{-E'(t_f - \tau) - E\tau} + (\text{excited states})$$

constrains the bare GFFs at  $t = \Delta^2$

$$\langle p' | T^b | p \rangle = c_A A^b(t) + c_J J^b(t) + c_D D^b(t)$$

⇒ measure and analyze many three-point functions



# General idea: renormalization

(Flavor singlet) EMTs mix & renormalize multiplicatively

$$\begin{bmatrix} T_q^{\overline{MS}} \\ T_g^{\overline{MS}} \end{bmatrix} = \begin{bmatrix} Z_{qq}^{\overline{MS}} & Z_{qg}^{\overline{MS}} \\ Z_{gq}^{\overline{MS}} & Z_{gg}^{\overline{MS}} \end{bmatrix} \begin{bmatrix} T_q^{\text{bare}} \\ T_g^{\text{bare}} \end{bmatrix}$$

Assert RI-MOM renormalization conditions at scale  $\mu^2 = p^2$

$$\langle q(p) T_i(0) \bar{q}(p) \rangle_{\text{lattice}} = Z_q R_{iq}^{\text{RI}} \langle q(p) T_{q,g}(0) \bar{q}(p) \rangle_{\text{tree}}$$

$$\langle A(p) T_i(0) A(p) \rangle_{\text{lattice}} = Z_g R_{ig}^{\text{RI}} \langle A(p) T_{q,g}(0) A(p) \rangle_{\text{tree}}$$

...then apply perturbative matching to  $\overline{MS}$  and run to  $\mu = 2 \text{ GeV}$

# Ensembles

Gauge action: tadpole-improved Luscher-Weisz

Fermion action: 2 + 1 flavors, stout-smearred clover

	$L/a$	$T/a$	$\beta$	$am_l$	$am_s$	$a$ [fm]	$m_\pi$ [MeV]
A	48	96	6.3	-0.2416	-0.2050	0.091(1)	169(1)
B	12	24	6.1	-0.2800	-0.2450	0.1167(16)	450(5)

## Bare matrix elements

Glue: 2511 configs  
Quarks: 1381 configs (subset)  
[“a091m170” (JLab/W&M/MIT/LANL)]

## Renormalization

Conn. quark: 240 configs  
Disco./glue: 20000 configs

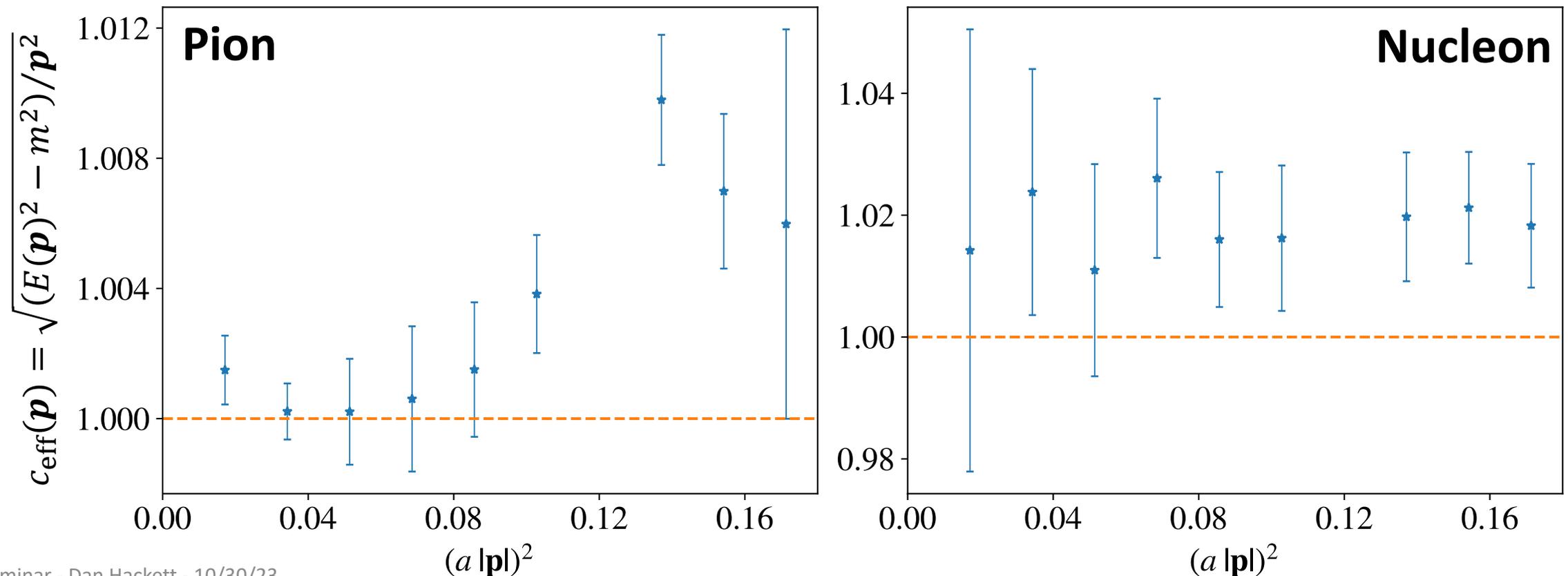
# Two-point functions

Compute on 2511 configs, 1024 srcs/cfg (2x offset  $4^3 \times 8$  grids)

Note: only one interpolating operator; both diagonal spin channels

Relativistic dispersion obeyed at  $\sim$  % level

→ Neglect errors in  $aM_\pi = 0.0779$  and  $aM_N = 0.4169$



# Lattice EMT operators

Quark:  $T_q^{\{\mu\nu\}} = \bar{q} \gamma^{\{\mu} i \overleftrightarrow{D}^{\nu\}} q$

Discretized covariant derivative

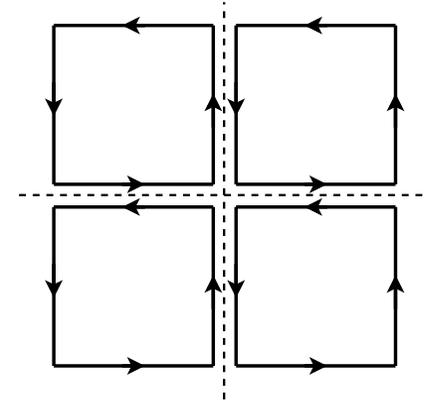
$$\overleftrightarrow{D} = (\overrightarrow{D} - \overleftarrow{D})/2$$

$$(\overrightarrow{D}_\mu \psi)(x) = \frac{1}{2} [U_\mu(x) \psi(x + \mu) - U_\mu^\dagger(x - \mu) \psi(x - \mu)]$$

$$(\overleftarrow{D}_\mu \bar{\psi})(x) = \frac{1}{2} [\bar{\psi}(x + \mu) U_\mu^\dagger(x) - \bar{\psi}(x - \mu) U_\mu(x - \mu)]$$

Glue:  $T_g^{\{\mu\nu\}} = \frac{2}{g^2} \text{Tr}[G^{\alpha\{\mu} G^{\nu\}\alpha}]$

$$G_{\mu\nu} \sim (Q_{\mu\nu} - Q_{\mu\nu}^\dagger)$$

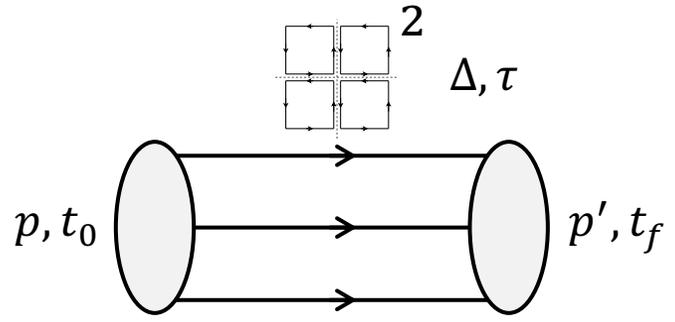
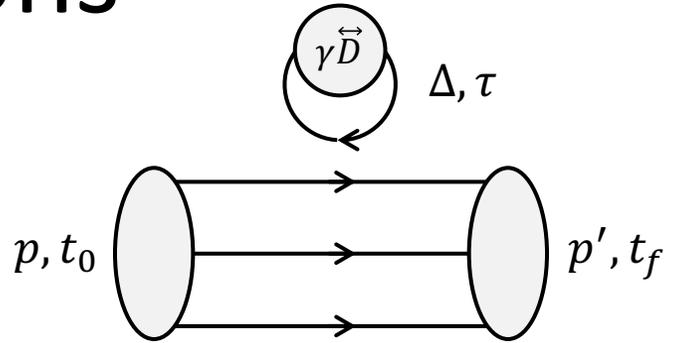
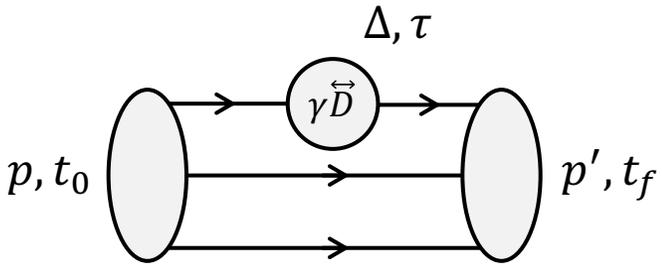


## Operator Bases

$$\tau_1^{(3)}: \quad \frac{1}{2} (T^{xx} + T^{yy} - T^{zz} + T^{tt}), \quad \frac{1}{\sqrt{2}} (T^{zz} + T^{tt}), \quad \frac{1}{\sqrt{2}} (T^{xx} - T^{yy})$$

$$\tau_3^{(6)}: \quad \left\{ \frac{i^{\delta_{\mu 0}}}{\sqrt{2}} (T^{\mu\nu} + T^{\nu\mu}), \quad 0 \leq \mu \leq \nu \leq 3 \right\}$$

# Three-point functions



## Connected Quark (*u, d*)

Sequential source (thru sink)

- 3 sink momenta
- Nucleon: 1 spin channel
- Sources / cfg varies w/  $t_f$

## Disconnected Quark (*u, d, s*)

- Hierarchical probing w/ 512 Hadamard vectors
- Pion: 1  $Z_4$  noise shot / cfg
- Nucleon: 2  $Z_4$  noise shots / cfg
- 1024 sources / cfg
- Nucleon: 4 spin channels

## Glue (disconnected)

Clover EMT

- Flowed to  $t/a^2 = 2$
- 1024 sources / cfg
- Nucleon: 4 spin channels

### Pion

$t_s$	6	8	10	12	14	16	18
$N_s$	6	16	16	16	32	32	32

### Nucleon

$t_s$	6	7	8	9	10	11	12	13	14	16	18
$N_s$	9	16	16	16	16	16	16	16	32	32	32

### Use all available data!

- all  $p^2 \leq 10 (2\pi/L)^2$
- all  $\Delta^2 \leq 25(2\pi/L)^2$
- all operators

# Extract bare matrix elements

1. Construct ratios

$$R(p, p'; \tau, t_f) = \frac{C^{3\text{pt}}(p, p'; t_f, \tau)}{C^{2\text{pt}}(p'; t_f)} \sqrt{\frac{C^{2\text{pt}}(p; t_f - \tau)}{C^{2\text{pt}}(p'; t_f - \tau)} \frac{C^{2\text{pt}}(p'; t_f)}{C^{2\text{pt}}(p; t_f)} \frac{C^{2\text{pt}}(p'; \tau)}{C^{2\text{pt}}(p; \tau)}}$$

$$= \# \langle p' | T | p \rangle + O\left(e^{-\Delta E \tau - \Delta E' (t_f - \tau)}\right)$$

2. Bin ratios together w/ same kinematic coeffs

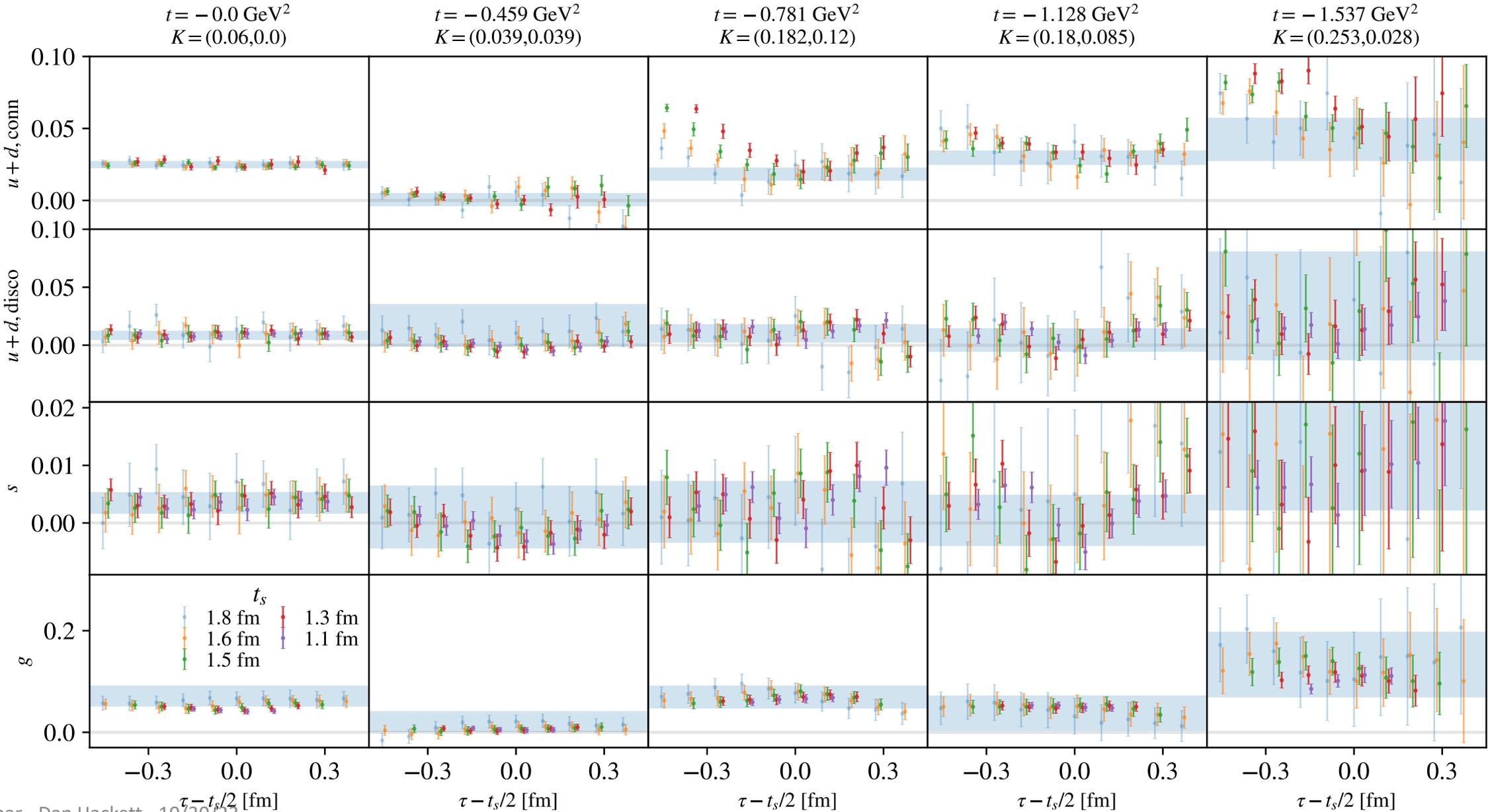
Nucleon	conn q:	6982	→	3081
	disc q/g:	1200296	→	11452
Pion	conn q:	3574	→	1379
	disc q/g:	157340	→	3364

3. Fit using summation method

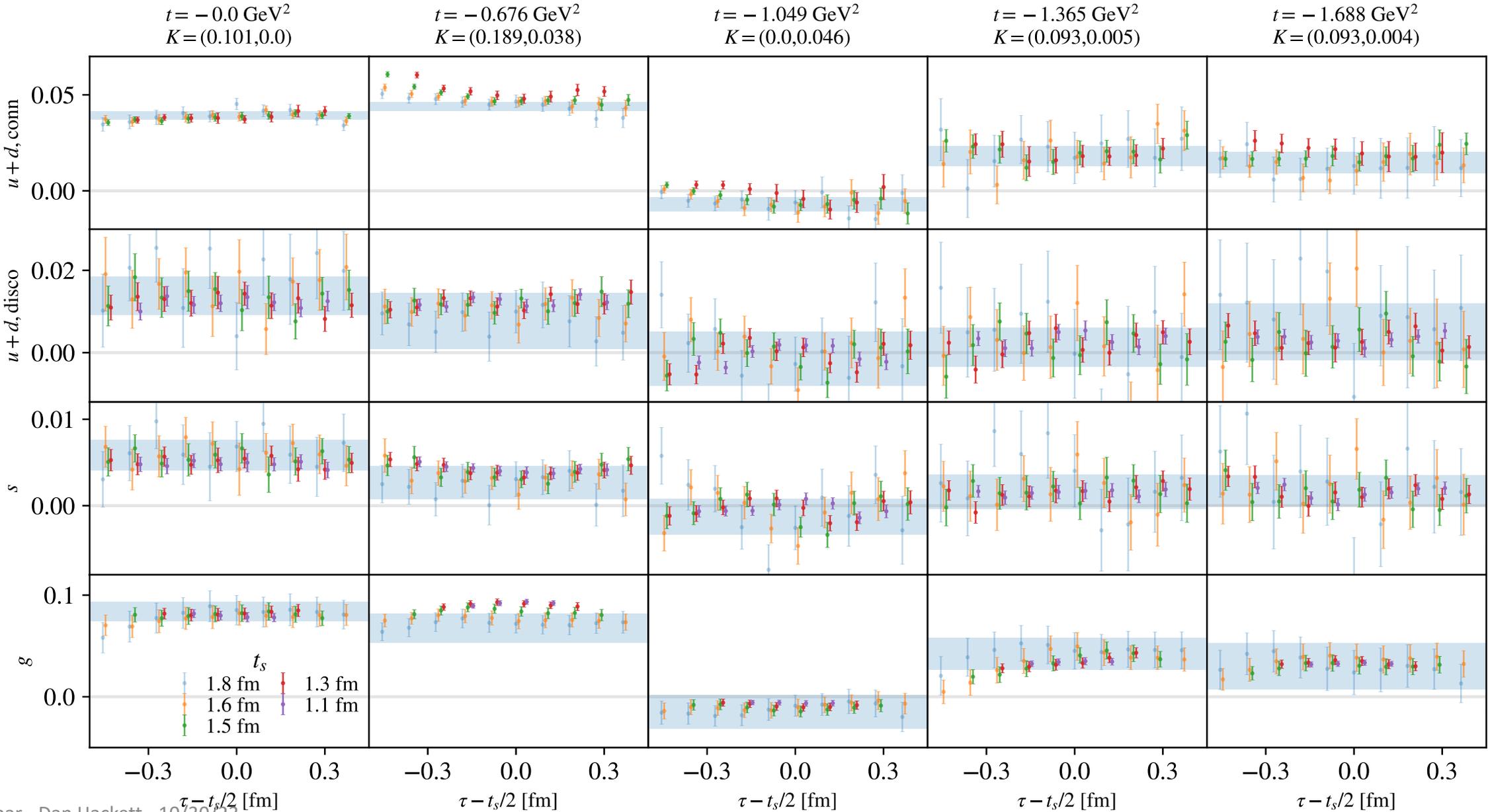
$$\Sigma(t_f) = \sum_{\tau=\tau_{\text{cut}}}^{t_f - \tau_{\text{cut}}} R(\tau, t_f) = (\text{const}) + \# \langle p' | T | p \rangle t_f + O(e^{-\delta E t_f})$$

... w/ Bayesian model averaging over fit ranges,  $\tau_{\text{cut}}$

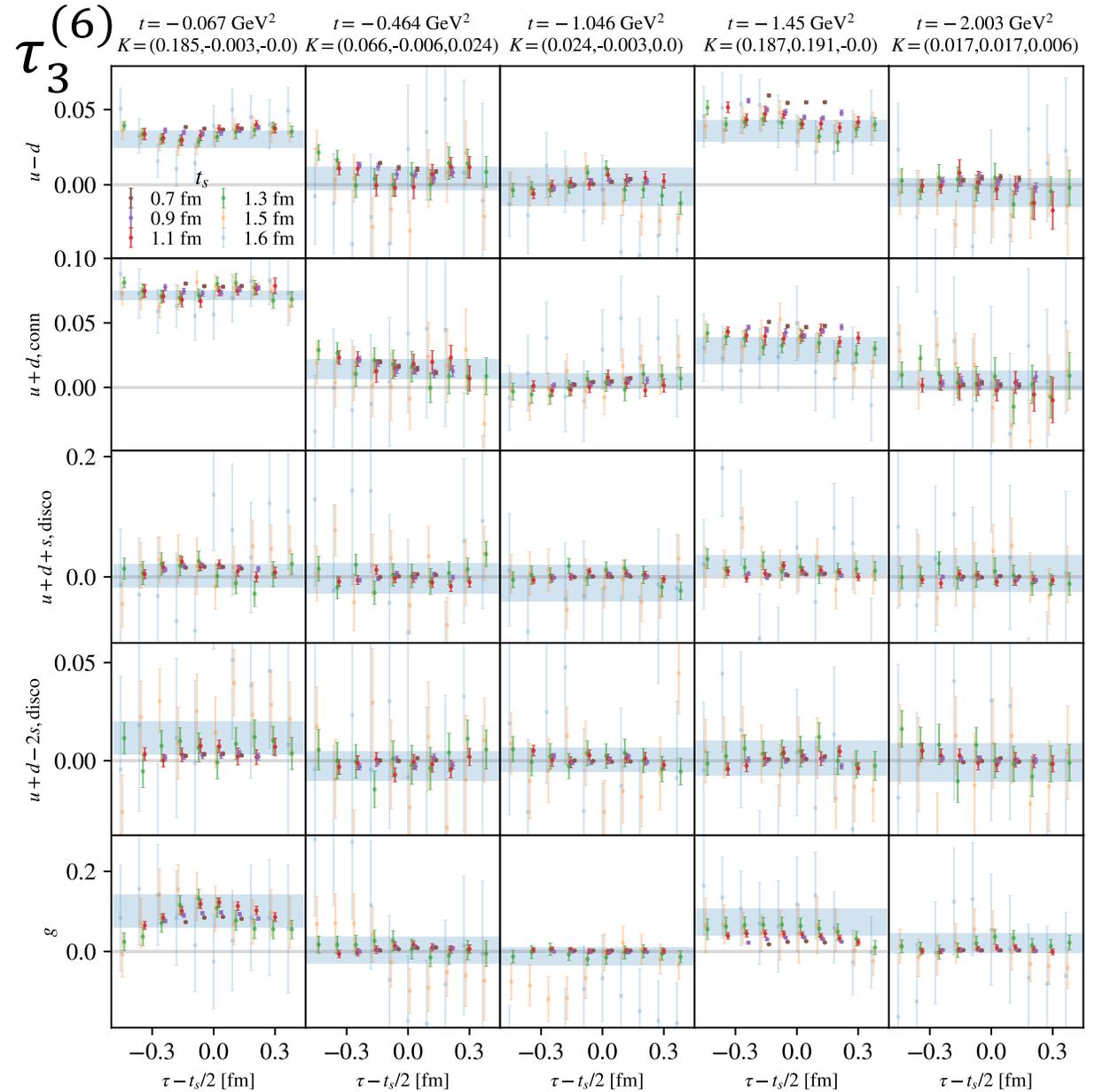
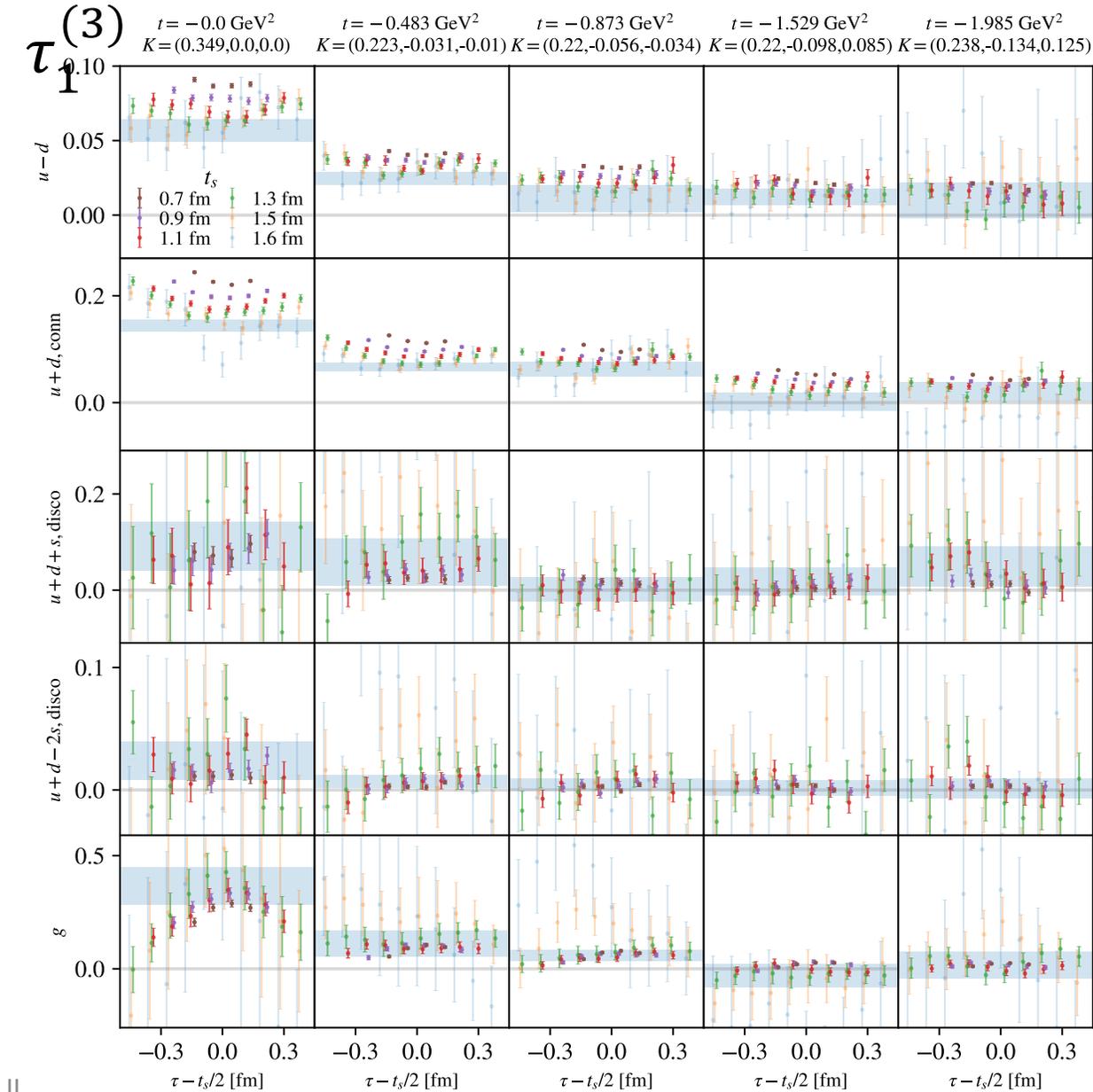
# Example pion ratios: $\tau_1^{(3)}$



# Example pion ratios: $\tau_3^{(6)}$



# Example nucleon ratios



# Renormalization

Compute amputated 3pt functions, solve for:

Landau gauge

Flow to  $t/a^2 = 1.2$  to match physical scale

RI-MOM + matching to  $\overline{MS}$  gives:

$$R_{qq}^{\text{RI}}(\mu_R^2) = \frac{C_{q,\mu\nu}^{q,\text{amp}}}{Z_q \Lambda_{\mu\nu}^q} \Big|_{\tilde{p}^2 = \mu_R^2}, \quad \begin{aligned} q &= u + d + s \\ v &= u + d - 2s \end{aligned}$$

$$R_{gg}^{\text{RI}}(\mu_R^2) = \frac{C_{g,\mu\nu\alpha\beta}^{g,\text{amp}}}{Z_g \Lambda_{\mu\nu\alpha\beta}^g} \Big|_{\tilde{p}_\alpha=0, \tilde{p}^2 = \mu_R^2}, \quad \alpha=\beta, \alpha \neq \mu, \alpha \neq \nu$$

$$R_{qg}^{\text{RI}}(\mu_R^2) = \frac{C_{q,\mu\nu\alpha\beta}^{g,\text{amp}}}{Z_g \Lambda_{\mu\nu\alpha\beta}^g} \Big|_{\tilde{p}_\alpha=0, \tilde{p}^2 = \mu_R^2}, \quad \alpha=\beta, \alpha \neq \mu, \alpha \neq \nu$$

$$R_{gq}^{\text{RI}}(\mu_R^2) = \frac{C_{g,\mu\nu}^{q,\text{amp}}}{Z_q \Lambda_{\mu\nu}^q} \Big|_{\tilde{p}^2 = \mu_R^2}, \quad R_v^{\text{RI}}(\mu_R^2) = \frac{C_{v,\mu\nu}^{q,\text{amp}}}{Z_q \Lambda_{\mu\nu}^q} \Big|_{\tilde{p}^2 = \mu_R^2}$$

$$(Z_v^{\overline{MS}})^{-1}(\mu^2) = C_v^{\text{RI}/\overline{MS}}(\mu^2, \mu_R^2) R_v^{\text{RI}}(\mu_R^2)$$

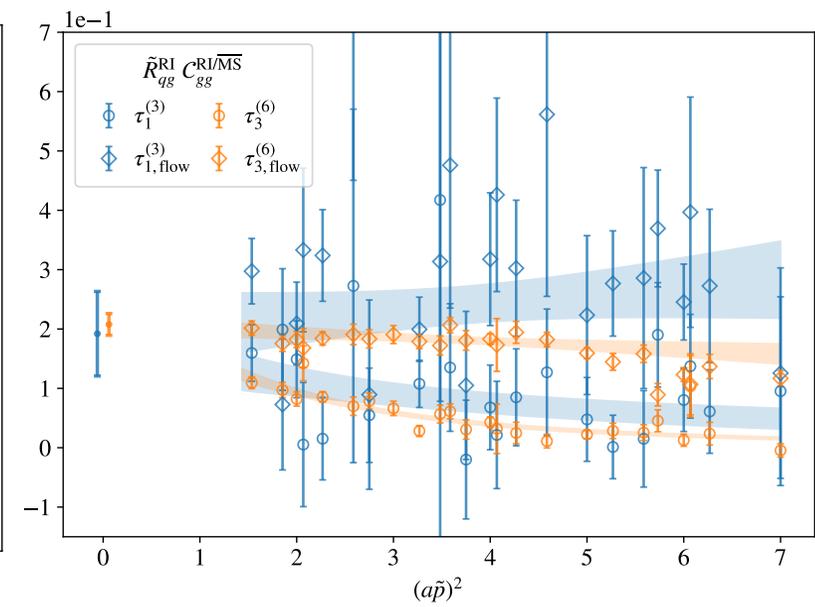
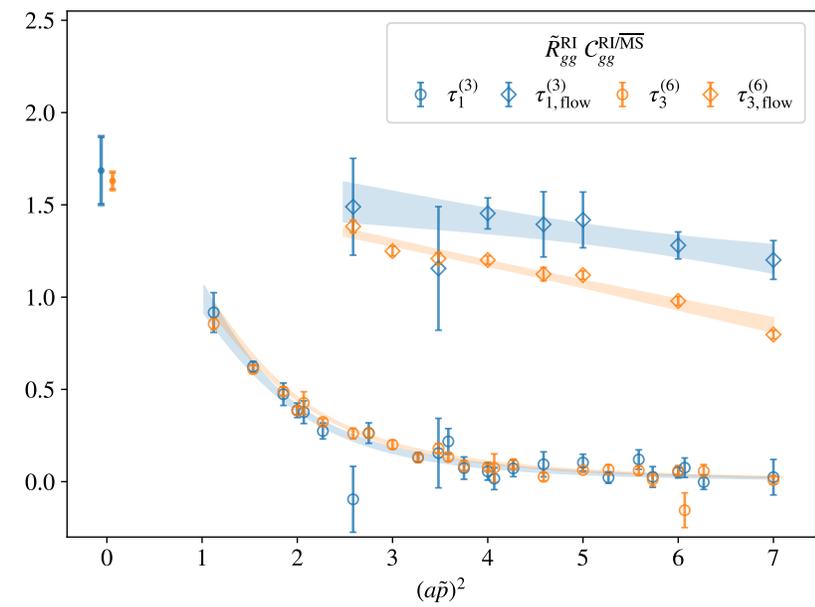
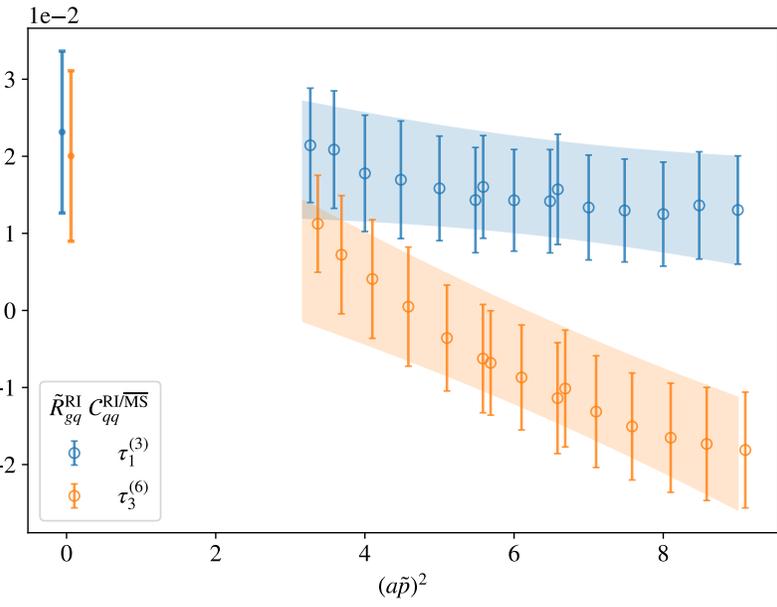
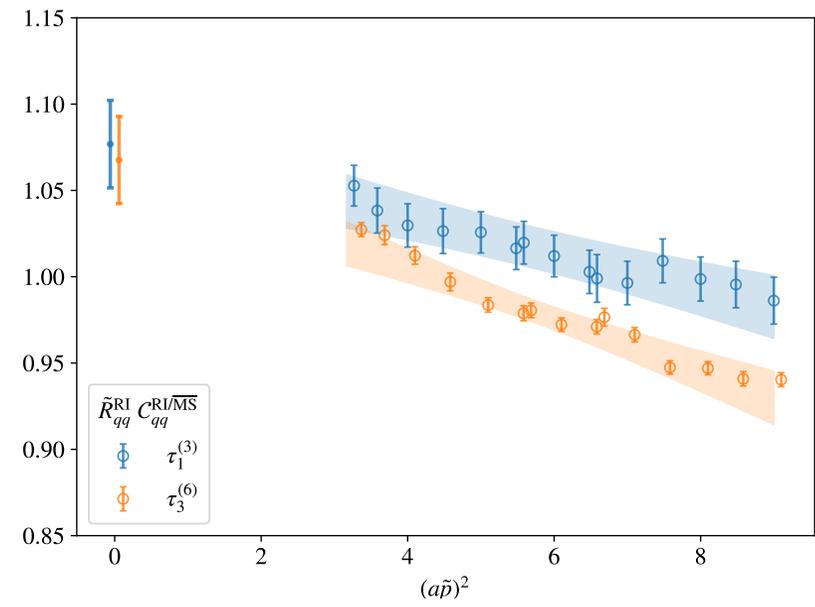
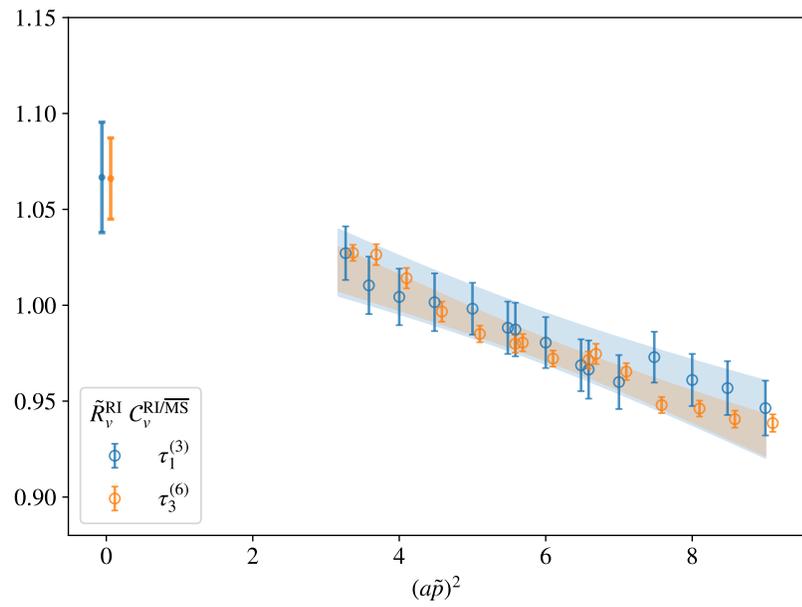
$$\begin{bmatrix} Z_{qq}^{\overline{MS}} & Z_{qg}^{\overline{MS}} \\ Z_{gq}^{\overline{MS}} & Z_{gg}^{\overline{MS}} \end{bmatrix}^{-1}(\mu^2) = \begin{bmatrix} R_{qq}^{\text{RI}} & R_{qg}^{\text{RI}} \\ R_{gq}^{\text{RI}} & R_{gg}^{\text{RI}} \end{bmatrix}(\mu_R^2) \begin{bmatrix} C_{qq}^{\text{RI}/\overline{MS}} & C_{qg}^{\text{RI}/\overline{MS}} \\ C_{gq}^{\text{RI}/\overline{MS}} & C_{gg}^{\text{RI}/\overline{MS}} \end{bmatrix}(\mu^2, \mu_R^2)$$

Model and fit residual  $(ap)^2$  dependence in each of product  $R^{\text{RI}} C^{\text{RI}/\overline{MS}}$

**Note:** repeat for each irrep!

# Fitting for renormalization coeffs

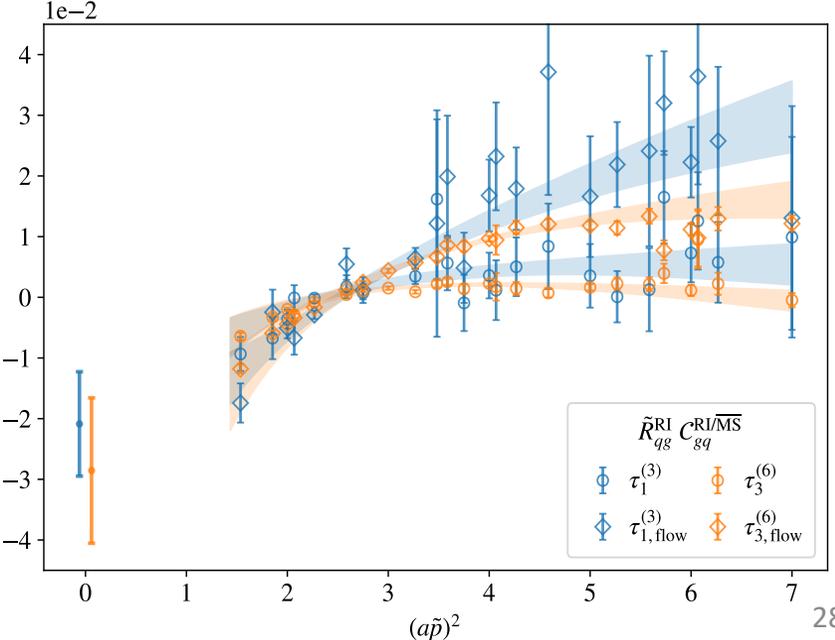
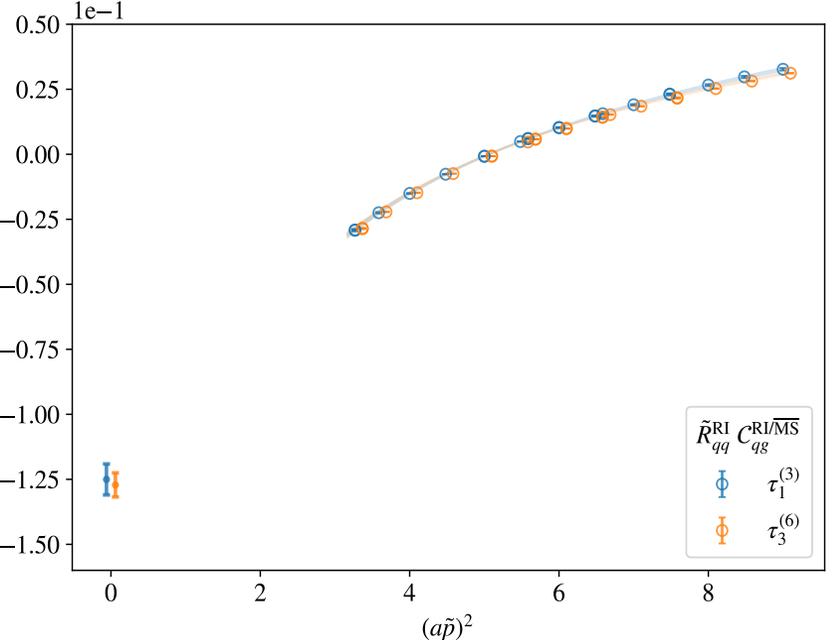
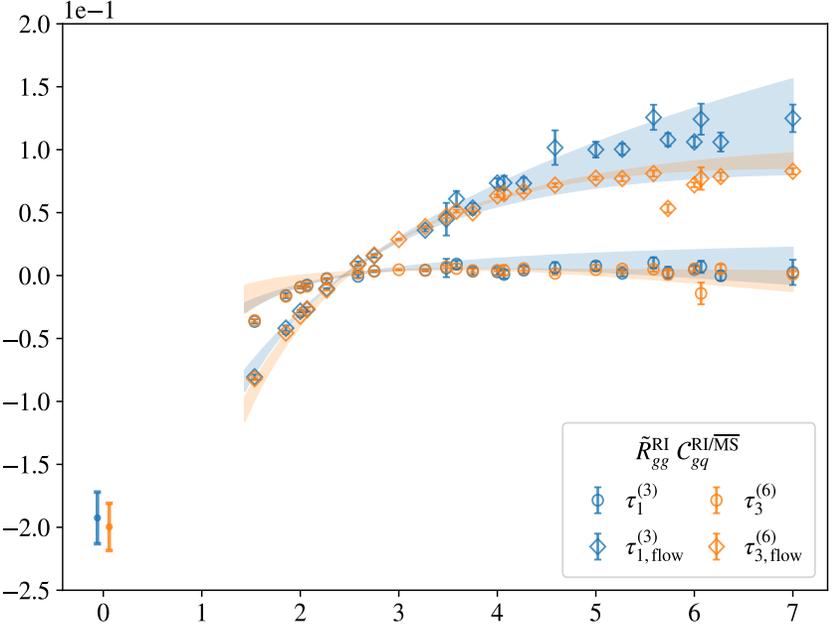
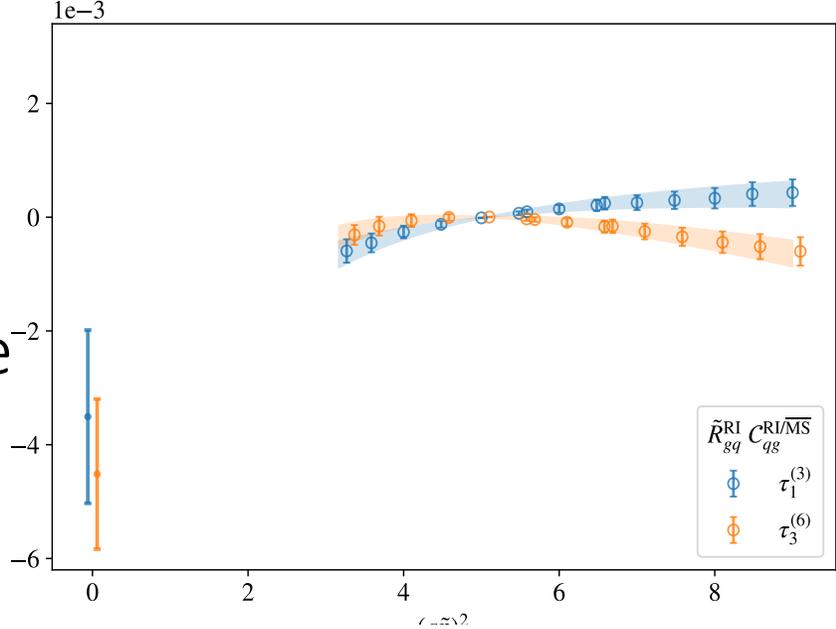
Model discretization artifacts as polynomials, inverse polynomials



# Fitting for renormalization coeffs

Model discretization artifacts as polynomials, inverse polynomials

+ logs for nonperturbative effects



# Computing the GFFs

Have, for each irrep:

- Bare matrix elements for glue & each quark flavor, binned by  $t$
- Mixing matrix + non-singlet  $Z$  factor

→ Renormalized set of linear constraints on GFFs in each  $t$ -bin

Fit to extract GFFs

Include data from both irreps

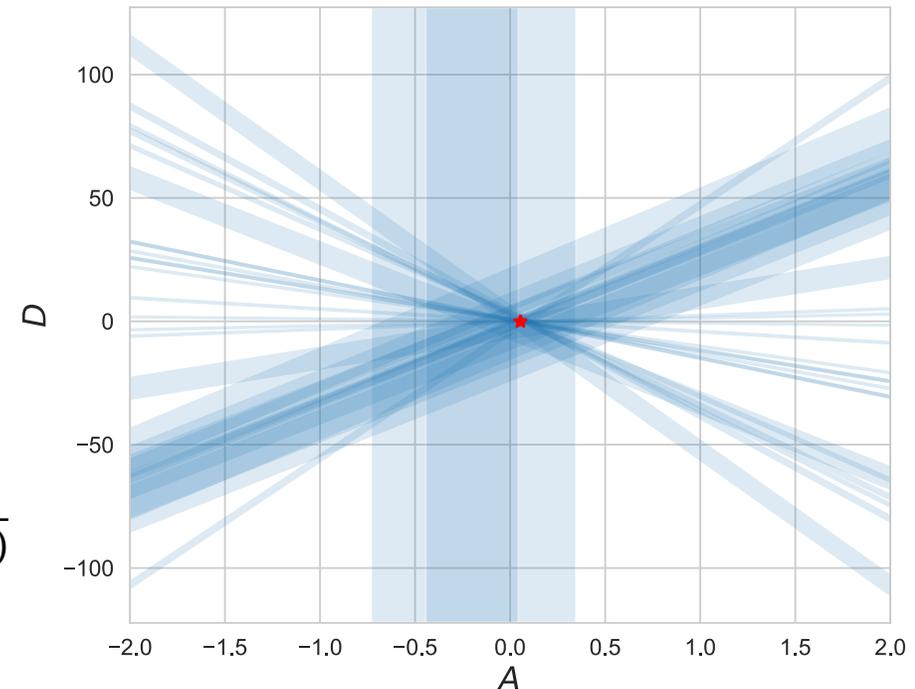
**Result:** GFFs for discrete values of  $t$

Fit GFFs to model functions

$$n\text{-pole } G(t) \sim \frac{\alpha}{(1-t/\Lambda^2)^3}$$

$$z\text{-expansion } G(t) \sim \sum_{k=0}^{k_{\max}=2} \alpha_k [z(t)]^k$$

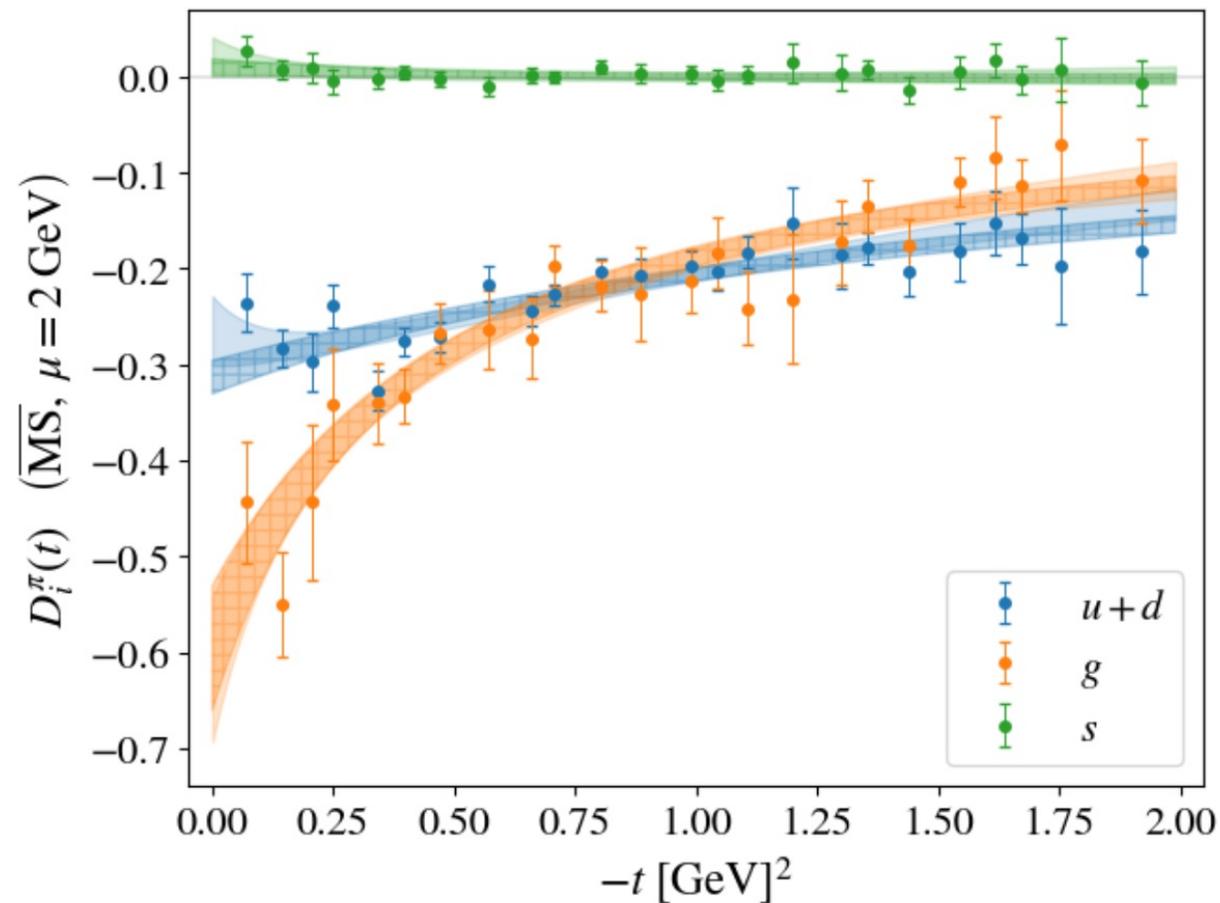
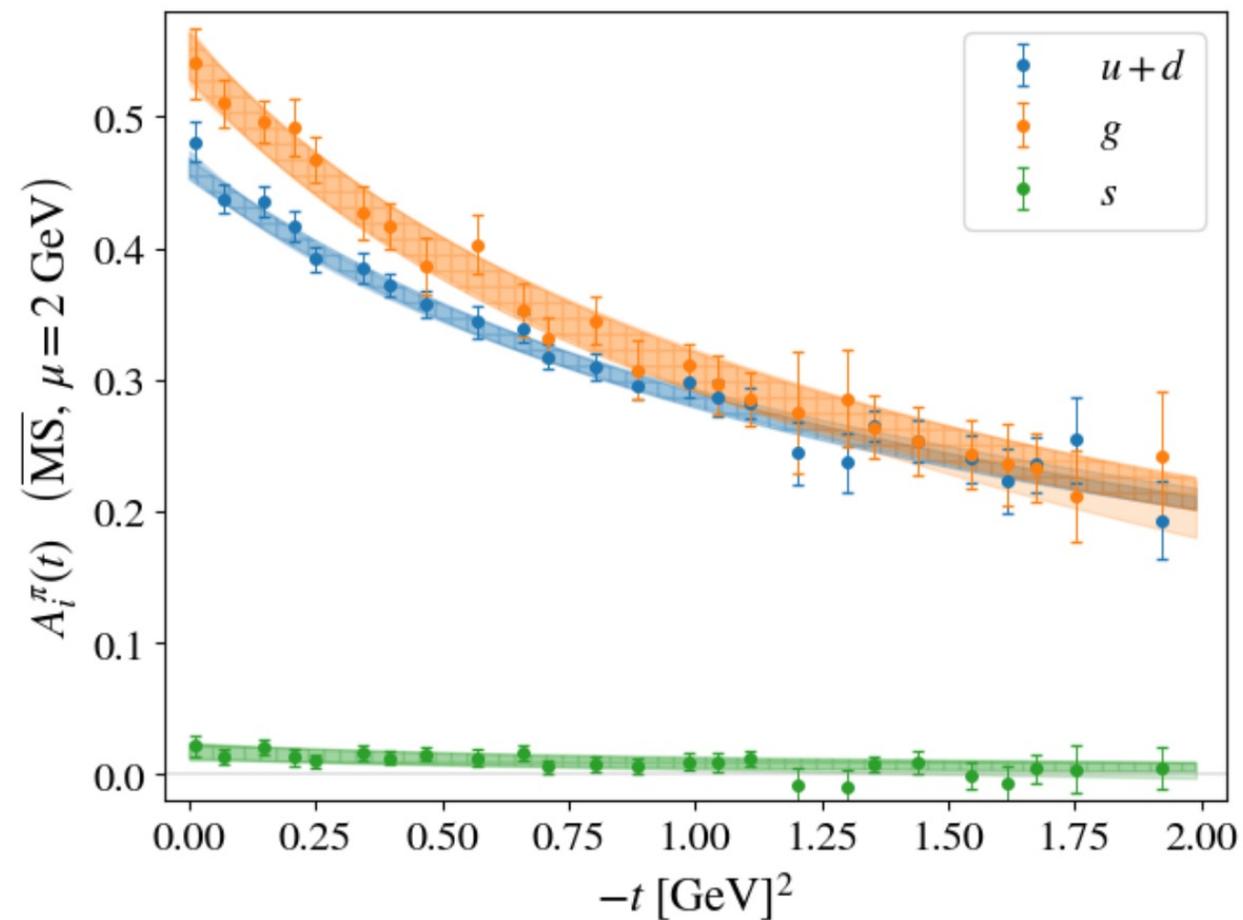
$$z(t) = \frac{\sqrt{t_{\text{cut}}-t} - \sqrt{t_{\text{cut}}-t_0}}{\sqrt{t_{\text{cut}}-t} + \sqrt{t_{\text{cut}}-t_0}} \quad t_{\text{cut}} = 4M_\pi^2 \quad t_0 = t_{\text{cut}}(1 - \sqrt{1 + (2 \text{ GeV}^2)})$$



# Results

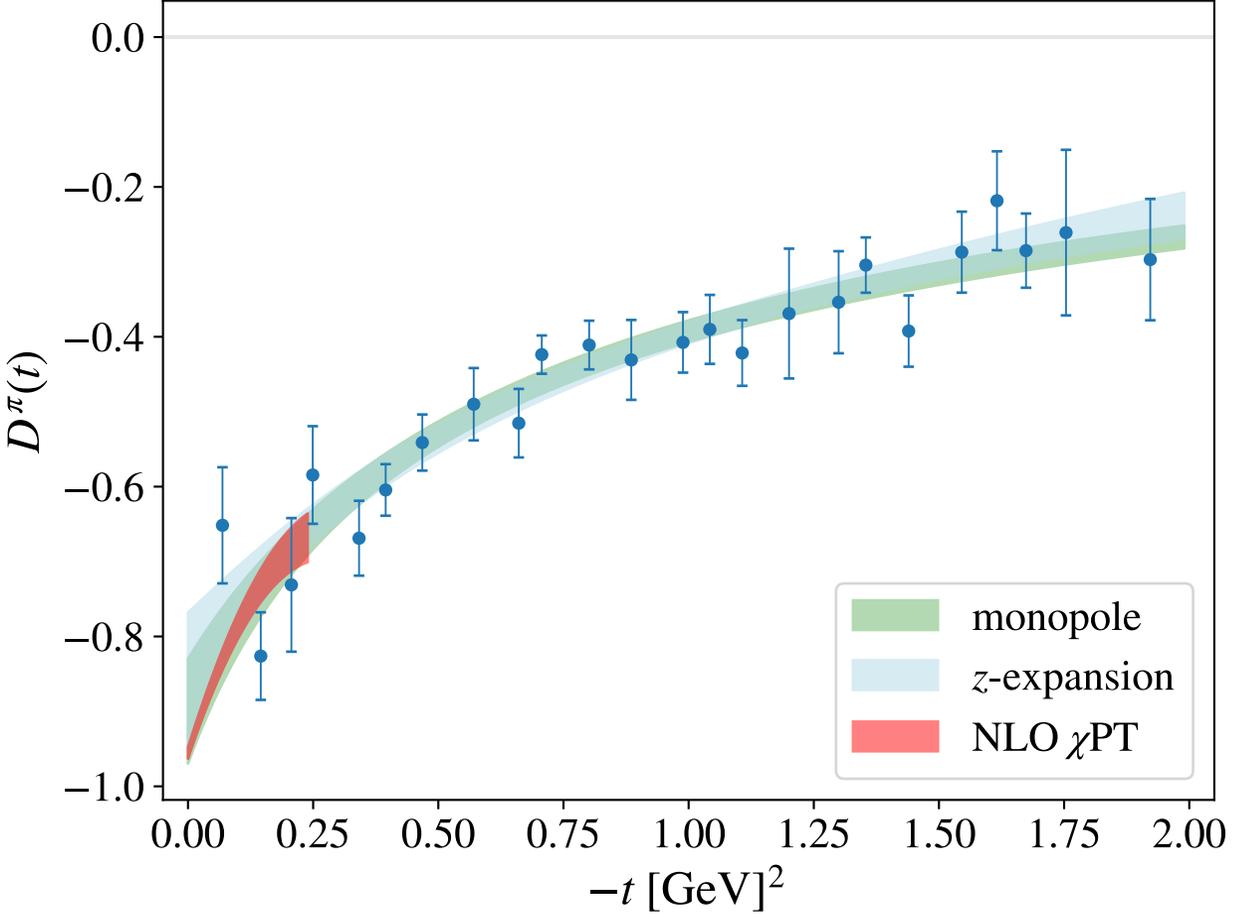
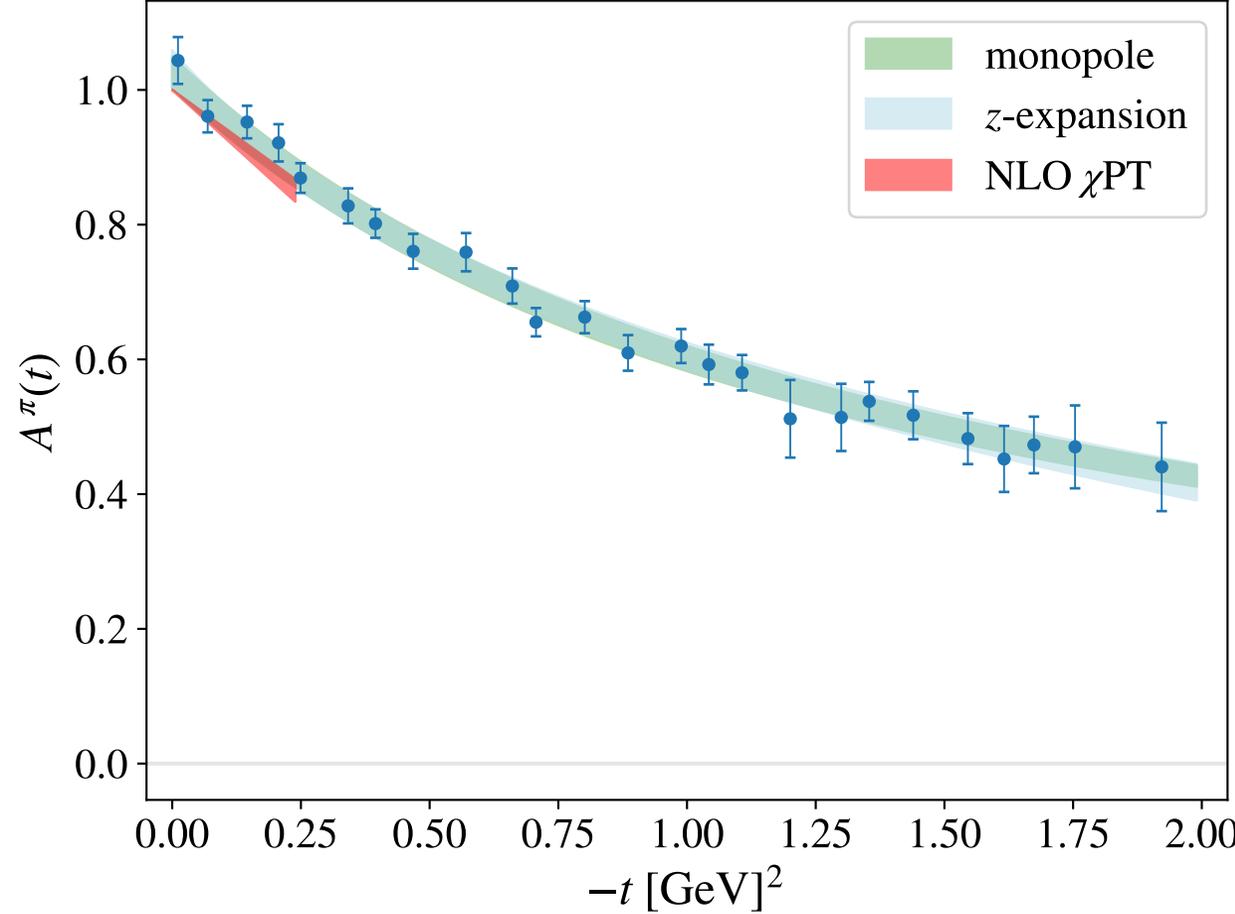
# Pion GFFs (flavor decomp)

Hatched bands: monopole      Solid bands: z-expansion



# Pion GFFs (total)

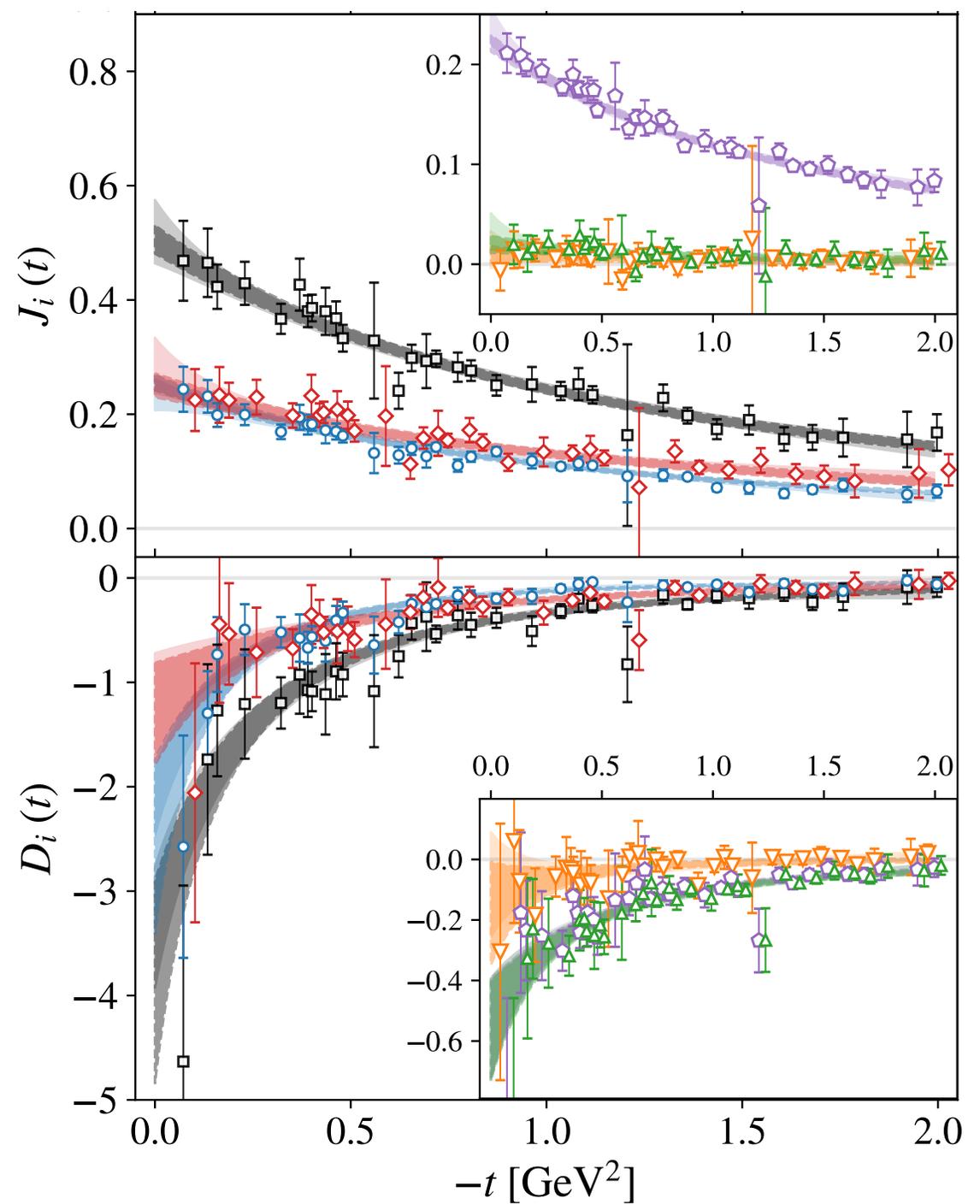
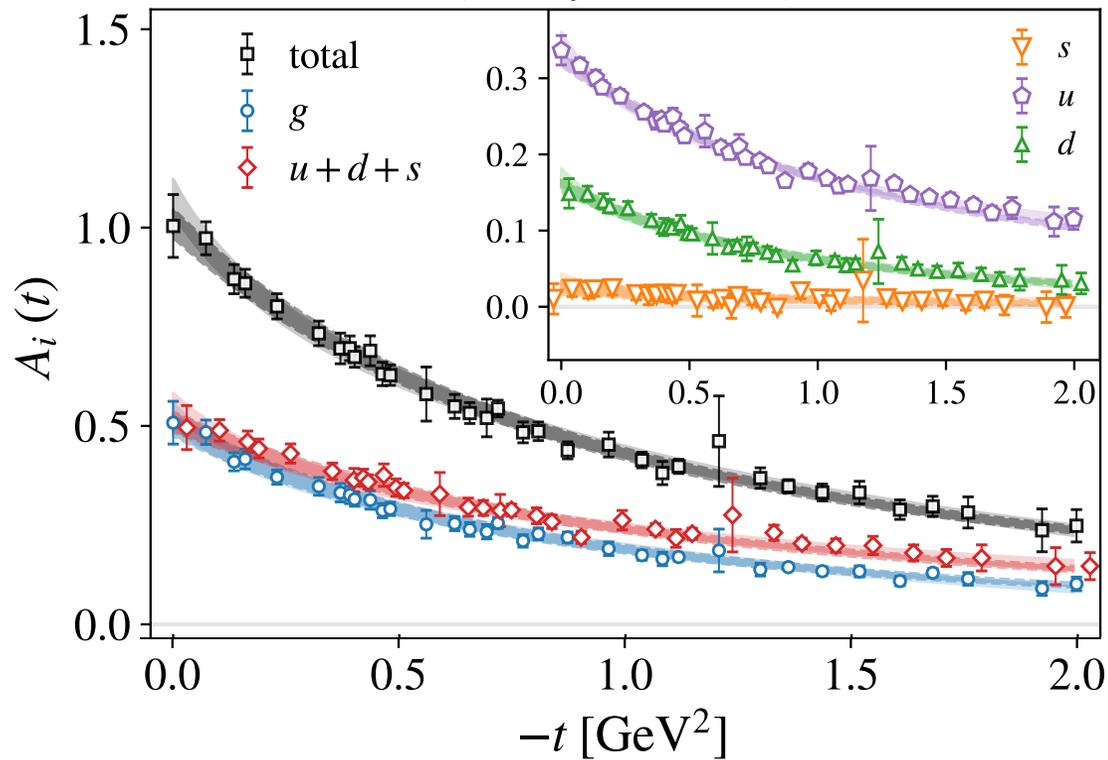
Error on  $\chi$ PT estimate due to different estimates for LECs [\[Donoghue Leutwyler 1991\]](#)



# Nucleon GFFs

Dark bands: dipole

Light bands: z-expansion



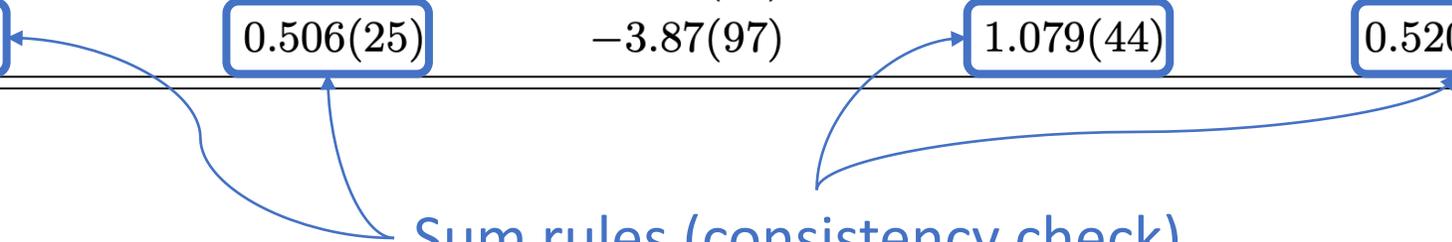
# Nucleon: forward limits

	Dipole			$z$ -expansion		
	$A_i$	$J_i$	$D_i$	$A_i$	$J_i$	$D_i$
$u$	0.3255(92)	0.2213(85)	-0.56(17)	0.349(11)	0.238(18)	-0.56(17)
$d$	0.1590(92)	0.0197(85)	-0.57(17)	0.171(11)	0.033(18)	-0.56(17)
$s$	0.0257(95)	0.0097(82)	-0.18(17)	0.032(12)	0.014(19)	-0.08(17)
$u + d + s$	0.510(25)	0.251(21)	-1.30(49)	0.552(31)	0.286(48)	-1.20(48)
$g$	0.501(27)	0.255(13)	-2.57(84)	0.526(31)	0.234(27)	-2.15(32)
Total	1.011(37)	0.506(25)	-3.87(97)	1.079(44)	0.520(55)	-3.35(58)

# Nucleon: forward limits

	Dipole			z-expansion		
	$A_i$	$J_i$	$D_i$	$A_i$	$J_i$	$D_i$
$u$	0.3255(92)	0.2213(85)	-0.56(17)	0.349(11)	0.238(18)	-0.56(17)
$d$	0.1590(92)	0.0197(85)	-0.57(17)	0.171(11)	0.033(18)	-0.56(17)
$s$	0.0257(95)	0.0097(82)	-0.18(17)	0.032(12)	0.014(19)	-0.08(17)
$u + d + s$	0.510(25)	0.251(21)	-1.30(49)	0.552(31)	0.286(48)	-1.20(48)
$g$	0.501(27)	0.255(13)	-2.57(84)	0.526(31)	0.234(27)	-2.15(32)
Total	1.011(37)	0.506(25)	-3.87(97)	1.079(44)	0.520(55)	-3.35(58)

Sum rules (consistency check)



# Nucleon: forward limits

	Dipole			z-expansion		
	$A_i$	$J_i$	$D_i$	$A_i$	$J_i$	$D_i$
$u$	0.3255(92)	0.2213(85)	-0.56(17)	0.349(11)	0.238(18)	-0.56(17)
$d$	0.1590(92)	0.0197(85)	-0.57(17)	0.171(11)	0.033(18)	-0.56(17)
$s$	0.0257(95)	0.0097(82)	-0.18(17)	0.032(12)	0.014(19)	-0.08(17)
$u + d + s$	0.510(25)	0.251(21)	-1.30(49)	0.552(31)	0.286(48)	-1.20(48)
$g$	0.501(27)	0.255(13)	-2.57(84)	0.526(31)	0.234(27)	-2.15(32)
Total	1.011(37)	0.506(25)	-3.87(97)	1.079(44)	0.520(55)	-3.35(58)

Sum rules (consistency check)

cf. global fit result

$$A_g(0) = 0.414(8)$$

[Hou et al. 1912.10053]

# Nucleon: forward limits

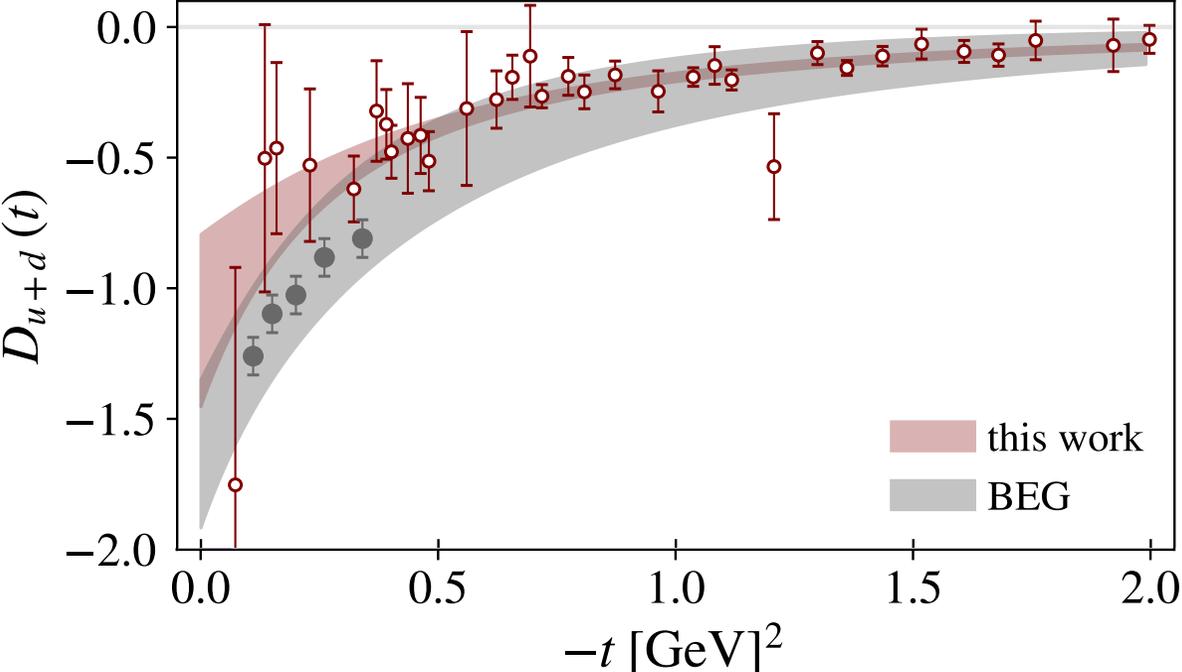
	Dipole			z-expansion		
	$A_i$	$J_i$	$D_i$	$A_i$	$J_i$	$D_i$
$u$	0.3255(92)	0.2213(85)	-0.56(17)	0.349(11)	0.238(18)	-0.56(17)
$d$	0.1590(92)	0.0197(85)	-0.57(17)	0.171(11)	0.033(18)	-0.56(17)
$s$	0.0257(95)	0.0097(82)	-0.18(17)	0.032(12)	0.014(19)	-0.08(17)
$u + d + s$	0.510(25)	0.251(21)	-1.30(49)	0.552(31)	0.286(48)	-1.20(48)
$g$	0.501(27)	0.255(13)	-2.57(84)	0.526(31)	0.234(27)	-2.15(32)
Total	1.011(37)	0.506(25)	-3.87(97)	1.079(44)	0.520(55)	-3.35(58)

Sum rules (consistency check)

cf. global fit result  
 $A_g(0) = 0.414(8)$   
 [Hou et al. 1912.10053]

First determination!  
 Satisfies  $\chi$ PT bound  
 $D(0)/M \leq -1.1(1) \text{ GeV}^{-1}$

# Nucleon vs. experiment



BEG = [\[Burkert Elouadrhiri Girod 2018\]](#) (DVCS)

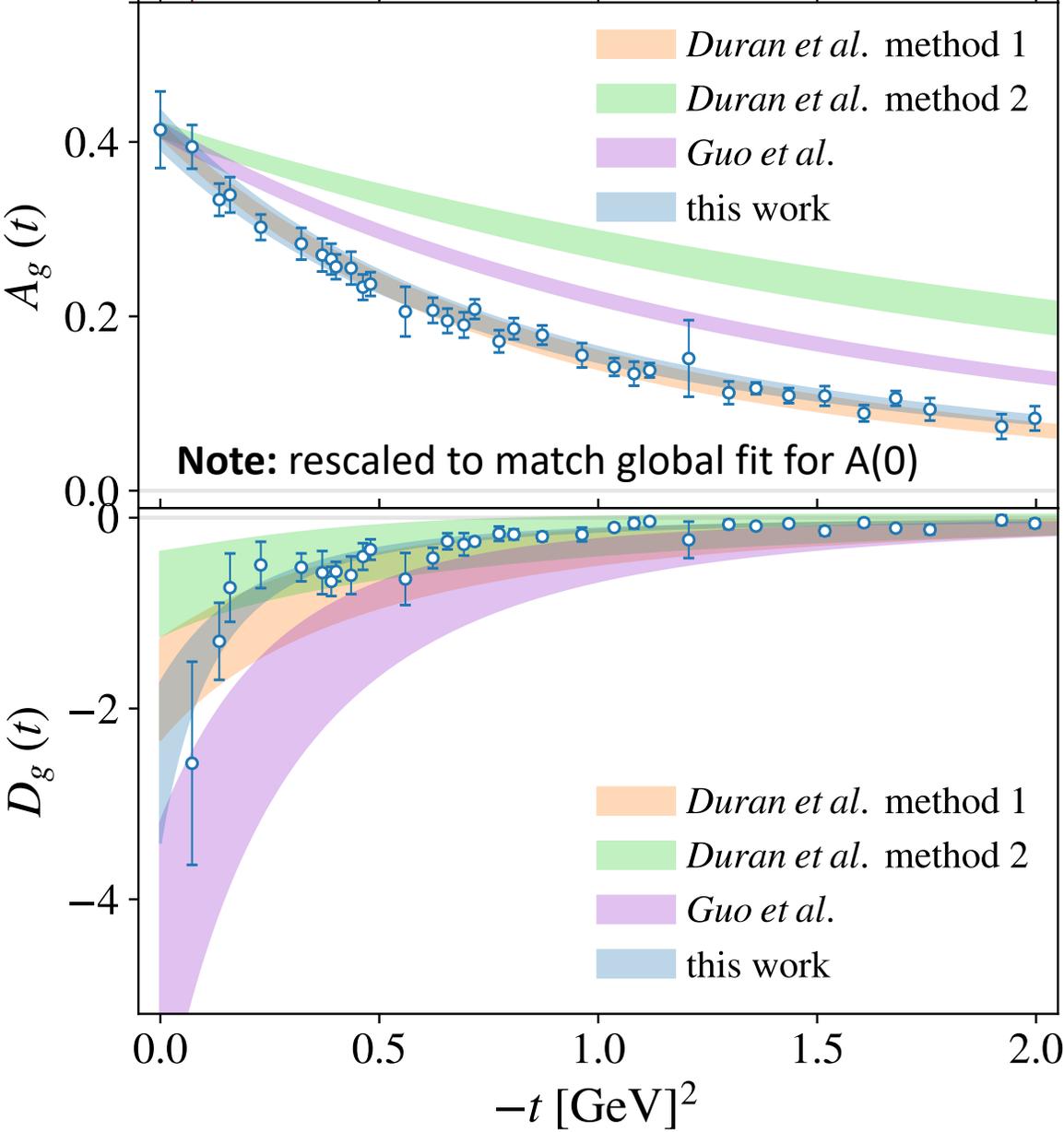
[\[Duran et al. 2207.05212\]](#) ( $J/\psi$ )

Method 1: holographic QCD (Mamo Jahed, PRD 21,22)

Method 2: GPD (Guo Ji Liu, PRD 2021)

[\[Guo et al. 2305.06992\]](#)

Updated GPD analysis + GlueX data



# Spatial densities

$$[f(t)]_{\text{FT}} = \int \frac{d^3 \Delta}{(2\pi)^3} e^{-i\Delta \cdot \mathbf{r}} f(t)$$

1. Parametrize  $T_{\mu\nu}(t)$  with GFFs
2. Fourier transform  $T_{\mu\nu}(t) \rightarrow T_{\mu\nu}(r)$
3. Identify

$$T_{\mu\nu}(r) = \begin{bmatrix} T_{tt}(r) & \\ & T_{ij}(r) \end{bmatrix} = \begin{bmatrix} \epsilon(r) & \\ & \left( \frac{r_i r_j}{r^2} - \frac{1}{d} \delta_{ij} \right) s(r) + \delta_{ij} p(r) \end{bmatrix}$$

→ Spatial densities

energy  $\epsilon(r) = M \left[ A(t) - \frac{t}{4M^2} (D(t) + A(t) - 2J(t)) \right]_{\text{FT}}$  shear forces  $s(r) = -\frac{1}{4M} r \frac{d}{dr} \frac{1}{r} \frac{d}{dr} [D(t)]_{\text{FT}}$

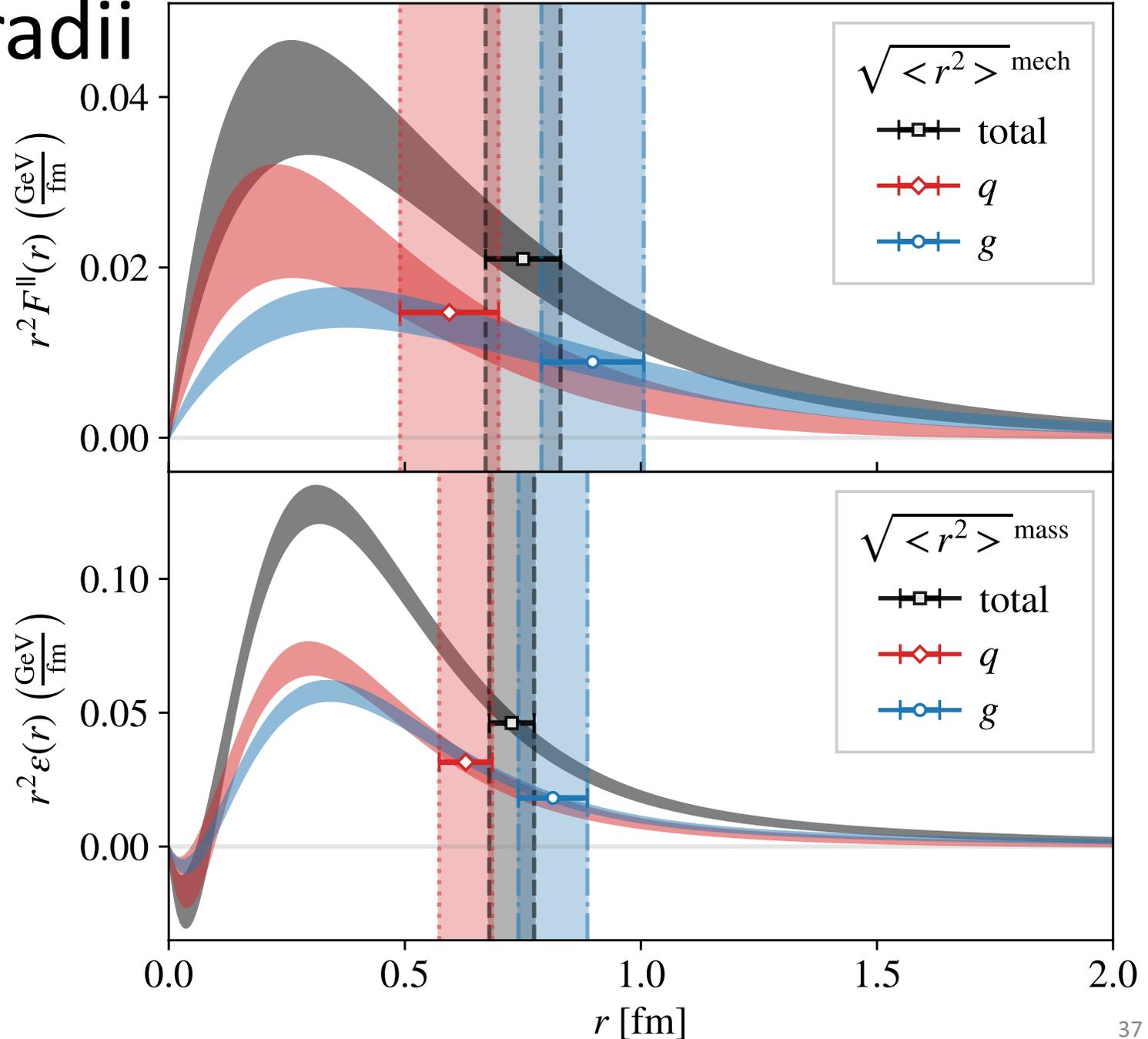
pressure  $p(r) = \frac{1}{6M} \frac{1}{r^2} \frac{d}{dr} r^2 \frac{d}{dr} [D(t)]_{\text{FT}}$  longitudinal force  $F^{\parallel}(r) = p(r) + 2s(r)/3$

**Caveat:** physical significance of these analogies is under debate

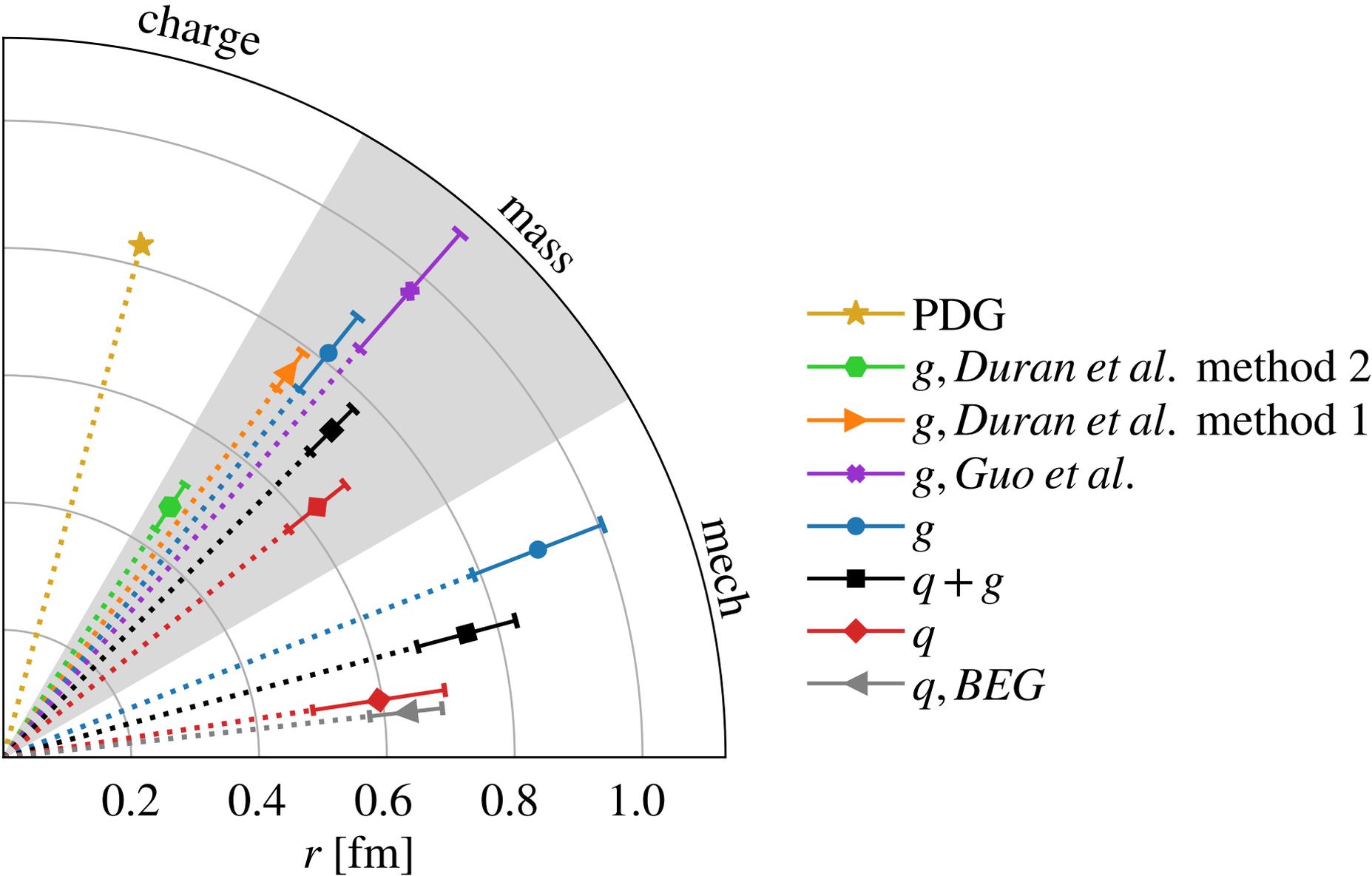
# Nucleon densities & radii

$$\langle r_i^2 \rangle^{\text{mass}} = \frac{\int d^3\mathbf{r} r^2 \epsilon_i(r)}{\int d^3\mathbf{r} \epsilon_i(r)}$$

$$\langle r_i^2 \rangle^{\text{mech}} = \frac{\int d^3\mathbf{r} r^2 F_i^{\parallel}(r)}{\int d^3\mathbf{r} F_i^{\parallel}(r)}$$



# How big is the nucleon?



# Conclusion

First complete flavor decomposition of the GFFs of the nucleon and pion

First determination of *total* GFFs

→ *physical* (i.e. RGI) densities, radii

New first-principles descriptions of size and shape of pion, proton

Nucleon results help discriminate between different experimental extractions

Towards a precision calculation of the GFFs, need:

Multiple ensembles to take continuum, physical-mass limits

Improved renormalization (GIRS, sum rules?)

Variational operator bases to fully control excited state effects

